LGCIV2041: Numerical Analysis of Civil Engineering Structures – Assignment

Handout: 19.02.2024 **Due:** Sunday 31.03.2024

Professors: João Pacheco de Almeida, Hadrien Rattez

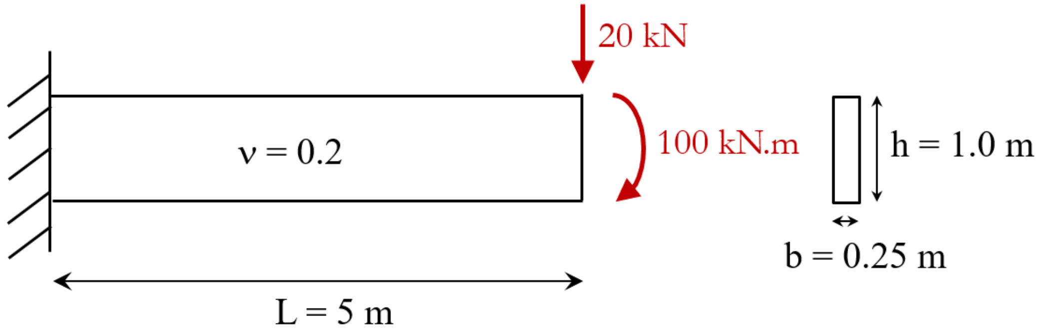
Teaching Assistant: Alexandre Sac-Morane

Instructions

- Assignment is graded to 100 points.
- The assignment is composed of 3 problems.
- The assignment should be solved in groups of 2 students.
- Please respect the due date indicated above, beyond which assignments will be accepted with penalty.
- Do not forget to download the supplementary files available in Moodle, "Python script to study frames.py" and "Solution_Methods_and_Nonlinearity_TO_COMPLETE.py".
- Hand in your solution together with the scripts and your input files.

Problem 1: Shear Locking, Reduced Integration, and Implementation of a Finite Element (30 points)

In the chapter of the lecture slides "A 2D Timoshenko Beam FE: Development, Characteristics, and Mesh Refinement", a Timoshenko beam theory was developed assuming a linear approximation both for the rotations $\theta(x)$ and the transverse displacements $u_{y0}(x)$. Similarly to the lecture, a cantilever is also herein investigated: 5 m long, rectangular cross section of width b = 0.25 m and height h = 1.0 m, shear coefficient k = 5/6, subjected to a tip downward force of 20 kN and a tip clockwise bending moment of 100 kN.m. Consider a Young's modulus E = 30 GPa and a Poisson's ratio ν = 0.2, which could represent concrete.



(a) Derive the exact solution for the transverse displacement field $u_{y0}(x)$ and for the rotational field $\theta(x)$. Compare them with the results obtained from the Python script distributed for the first exercise session ("Python script to study frames.py"), i.e. using Euler-Bernoulli beam theory, using a mesh of 1 and 4 elements. Comment.

(b) Implement the stiffness matrix corresponding to the Timoshenko beam in the Python script distributed for the first exercise session ("Python script to study frames.py"). Plot the transverse displacements and rotations for a mesh of 1, 4, 10, and 100 elements, together with the exact solution. Comment.

(10-point bonus: Show and comment on the evolution of the main static quantities, namely the bending moments and shear forces. Do they satisfy locally the equilibrium conditions? In particular: in the domain, between elements, and the natural boundary conditions.)

(c) Consider the following alternative cantilever lengths: L=1 m, 10 m, and 100 m. Plot the transverse displacement along the cantilever for a mesh of 100 elements, both for the Timoshenko and Euler-Bernoulli beam. Comment on the results, indicating which ones you would "trust" as an engineer without access to the exact solution.

(d) Redo the analyses of question (c) with Abaqus and compare the results.

(e) Compute and show the stiffness matrix considering selective reduced integration, as discussed in the lecture. Implement it in the Python script and plot again the transverse displacement for the same cases and mesh of question (c). Comment on the results obtained.

Problem 2: Large Displacements and Solution Methods for Nonlinear Problems (30 points)

Consider the column represented in Figure 1, which was modelled and analysed as a rigid bar during the exercise session.

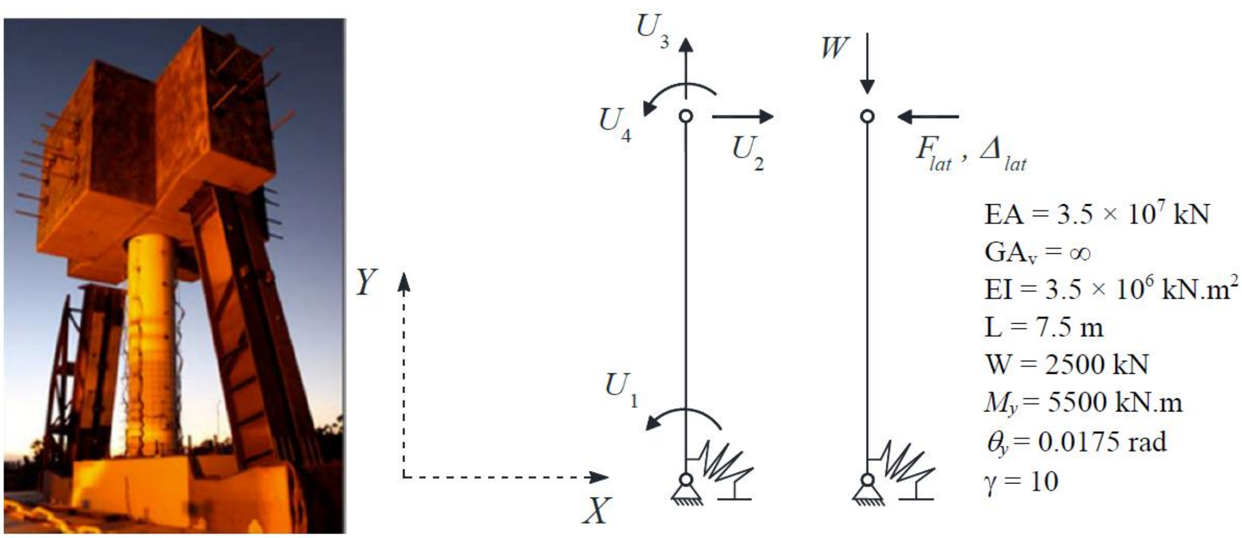


Figure 1. Bridge pier and structural model with global degrees of freedom, loading and properties.

This problem is a continuation of exercises session and studies the same structure. However, instead of a rigid bar, an elastic Euler-Bernoulli beam is now used to model the behaviour of the column. Further, it is assumed that a nonlinear plastic hinge develops at its base, which follows the moment-rotation response given by the Ramberg-Osgood law (similar to the exercise session). The curve for a specific set of parameters My, θ_y, and γ is illustrated on the right-hand side of Figure 2.

$$\frac{\theta}{\theta_y} = \frac{M}{M_y}\left[1+\left(\frac{M}{M_y}\right)^{\gamma-1}\right]$$

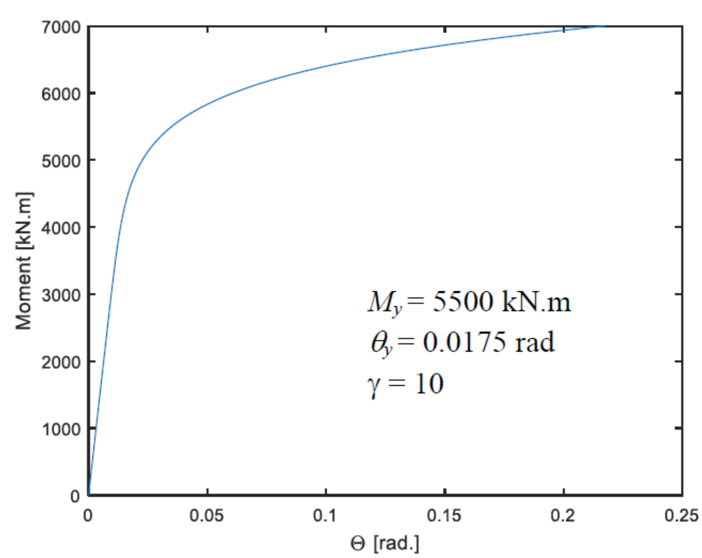


Figure 2. Ramberg-Osgood law used for the moment-rotation relation of the plastic hinge.

The Python script provided for the exercise sessions (‘Solution_Methods_and_Nonlinearity_TO_COMPLETE.py’) allows solving the structural problem when linear geometry is considered. Both the classical Newton-Raphson and a displacement-control method were implemented, which can be selected in the part of the script after the comment ‘INPUT PARAMETERS: START’.

(a) Compute the force-displacement response using the Python script and compare it, in the same graph, with the response obtained: (i) during the exercise session, with the rigid bar assuming geometric linearity; (ii) with the distributed Abaqus input file. Comment.

(b) You are now asked to extend the provided script to account for large displacements according to the corotational formulation, as addressed in the lectures. In particular: complete the two parts of the code corresponding to the computation of the element resisting forces in the global reference system **p** and the element stiffness matrix in the global reference system **k**, for large displacements. Using displacement-control, push until a lateral displacement $\Delta lat = (-)1.2$ m. Provide a physical explanation for the difference in the results with respect to linear geometry. Hand in the completed Python script.

(c) Compare the force-displacement curve with large displacements obtained in (b) with the response obtained: (i) during the exercise session, with the rigid bar considering nonlinear geometric effects; (ii) with the distributed Abaqus input file, but now also considering nonlinear geometric effects. Comment.

(d) Identify and comment on the limitations of using the classical Newton-Raphson method in capturing the response, both with and without geometric nonlinear effects, with respect to that of the displacement-control method. Illustrate your response with the required plots.

Problem 3: Material Nonlinearity (40 points)

Coming soon…