Estimating a Linear Stochastic Discount Factor

MATH 364 Final Project

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I. Introduction

Investment pricing must account for both the risk associated with returns and the opportunity cost of the capital needed to fund the investment. Discounting investment payoffs using the time value of money captures the latter. The stochastic discount factor (SDF) improves on this approach, by defining the discount factor as a random variable that can capture the conditional relationship between the investment return and the utility of that return. This SDF is not known, however; it must be estimated using predictors of investment risk. We estimate the SDF as a linear function of company fundamentals and macroeconomic variables. We, then, leverage properties of the factor to extract an implicit estimation of the tangent portfolio. We test an investment strategy using this portfolio on a universe of 65 real estate companies between 2015 and 2023. We find that this portfolio outperforms the universe index.

II. Existing Research

Research has previously explored several methods of leveraging the SDF to create an investment strategy. Taking a statistical approach, [2] creates a modified version of principal component analysis that takes advantage of SDF theory. This procedure allows PCA factor loadings to vary, conditional on asset characteristics and macroeconomic factors. From these loadings, one can extract a portfolio that maximizes returns conditional on these factors.

Taking a computationally different but theoretically similar approach, [1] uses machine learning to estimate the SDF as a function of several asset-specific and macroeconomic variables. It does so by minimizing the mispricing error of the most mispriced assets. From this estimation, it extracts the implicit weights of the tangent portfolio then shows the superior performance of this portfolio compared to several benchmarks.

III. Model Formulation

Taking a similar approach to [1], we estimate the SDF as a function of several company-specific and macroeconomic variables. We do so by taking advantage of the relationship between the SDF and minimizing the sum of squared mispricing errors. From this factor, we extract the implied tangent portfolio weights and evaluate that portfolio's performance.

A. The SDF

The SDF is an extension of the time value of money discount rate. The discount rate,

$$d = \frac{1}{1 + r_f},$$

where r_f is the risk-free rate of return over a given period, captures the effects of the time value of money on an asset's price. Using this factor, an asset with deterministic payoffs $c_i,\ 1 \leq i \leq n$ at the end of each period i should have price

$$p = \sum_{i=1}^{n} c_i d^i.$$

This factor captures the risk-free payoff the investor forgoes by buying the asset.

This factor can be understood more generally to capture difference in the utility of wealth between two states of the world: the present and one step in the future. Because money can grow in time, money today is worth more than money tomorrow, so we get the common inequality $d < 1$.

This factor, however, assumes that the payoff and the utility of wealth are independent (or, even more simplistically, deterministic). The expression above can more generally be expressed as

$$p = \sum_{i=1}^{n} E[c(\omega_i) d(\omega_i)],$$

where the values c and d have become random variables dependent on the state of the world at time i, ω_i. This expectation only expands to the deterministic sum above if the payoff and the discount factor are independent.

The payoff and the discount factor are rarely independent, especially in the context of public equities. Consider a catastrophic event in the market: the marginal utility of wealth in this scenario would increase, as it will take less money to buy the same securities; the payoff on a given stock will decrease with the market. Now consider the opposite event: the marginal utility of wealth in this scenario would decrease relatively as more money is now required to buy the same securities, and the payoff of that same stock would increase with the market. Clearly the utility of wealth and the payoff are not independent in these scenarios.

The deterministic equation overlooks significant effects of this dependence. The SDF addresses these shortcomings of a deterministic discount factor. The SDF at time t is the value $M_{t+1}(\omega)$ such that

$$E[M_{t+1} r^e_{i,t+1}] = 0, \tag{1}$$

where $r_{t,i}(\omega)$ is the random variable representing the excess return on an asset i at time t. That is, there is no investment with non-zero expected return; all assets are priced correctly.