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Executive Summary

This report contains a detailed analysis outlining our approach to the problem posed to the derivatives research group. Our goal was to develop a strategy to immunise a pension fund with $18 billion in AUM and back-test this strategy in MATLAB. Using a selection of 24 US bonds, we construct monthly portfolios that effectively mitigate exposure to interest rate risk or rho; the portfolio's performance was evaluated through simulation with hypothetical variations in the term structure over time. The immunisation is carried out with respect to the fund's two cash-flow streams, variable and fixed. We aimed to maintain a stable portfolio of securities while ensuring we had sufficient liquidity to pay the monthly liabilities.

We began our simulation on the 1st of December 2006 as we found this to be the date which yielded the most similar term structure to today's rate environment. As such, our back testing should characterise how this strategy would perform if executed in 2024. We focused mainly on selecting bonds with a high credit rating, high coupon rate and low maturity. We chose these criteria because it helped us minimise default risk and gave us more stable cash flows with shorter maturities that aligned better with our investment horizon. The immunisation process attempts to match the duration, convexity and the present value of the future liabilities of the fund. We have improved and added a number of processes to the original skeleton file in order to optimise the strategy's performance with respect to tracking errors. Operationally, our strategy also handles the payments of the fixed and floating liabilities each month, selling bonds if the value of the payment is not covered by the cash reserve; we also handle the maturity of bonds within the simulation by replacing them with cash.

Upon running the simulation and analysing the data, we found that the portfolio had a very high weighting in cash, with the remainder heavily invested into the shortest maturity bonds. This seemed to achieve great immunisation of our portfolio however, achieving a mean absolute tracking error of 4.7 basis points over the duration of the simulation.

Our report suggests that a successful immunisation strategy can be created using a relatively simple procedure consisting of only fixed-rate, investment-grade corporate and sovereign bonds. Therefore, this report would present a good starting point for any portfolio manager aiming to immunise and protect against interest rate risk.

In a real-world scenario, it is probable that the aim of the portfolio would extend beyond simply immunising the exposure to interest rate risk, and rather earning some return on the investment. Thus, it could be beneficial to utilise various other types of bonds such as callable or floating rate, as well as lower rating bonds which offer a superior yield.

Overall, the bond simulation's findings show how well a bond portfolio can be immunised by matching durations, convexity and the present value of the liabilities. This strategy lowers the risk brought on by changes in interest rates and may enhance portfolio performance over a longer period of time (Hayes, 2022).

Introduction and Background

The following report details the methodology used by the derivates research department to create a bond portfolio for a pension fund to reduce its rho exposure, i.e., to make the fund immune to interest rate risks. The performance of this portfolio is tested using hypothetical changes to the term structure of interest rates across time.

As both assets and liabilities of the fund are exposed to the uncertainty of interest rates, immunisation provides a strategy to mitigate this. It is achieved by firstly matching the duration of assets and liabilities in a portfolio. The aim is to ensure that interest rate fluctuations don't significantly impact the performance of the portfolio. Next, immunisation requires holding the present value of future liabilities in cash to ensure the fund can meet future obligations. Immunisation also factors in convexity, anticipating the magnitude and direction of potential changes in portfolio value in response to interest rate transformations. We note that this is an over-simplified setting for the pension fund industry. In reality, these funds balance exposure to a variety of risk sources while achieving returns consistent with their investors' expectations.

Our research targets a broad spectrum of funds looking for stability, consistency, and security in their portfolios with respect to interest rate risk. Our insights are mostly directed to institutional investors who manage substantial bond portfolios, such as pension fund managers, insurance firms, and asset management companies. These stakeholders have the responsibility of protecting and preserving capital that's entrusted to them, so they'll need to develop strategies to mitigate risk. Furthermore, our findings could also benefit individual investors and financial advisors seeking to consider interest rate risk in their investment strategies.

While immunisation strategies are useful tools for hedging rho, their effectiveness is dependent on a variety of economic factors such as interest rate environment and broader market conditions across asset classes. In this report we will document any limitations which we face in optimising this process.

We hope to replicate the methodology of portfolio managers in constructing and managing our immunised portfolio. This includes the initial allocations of our AUM, the start date of our simulation, bond selection, the simulation itself, the immunisation strategy, daily price and liability calculation and the tracking error of the portfolio.

Following this, our report analyses the results of our simulation and concludes with a summary of our findings and future recommendations.

Methodology

Throughout this section, we outline the challenges we faced in improving the skeleton file and the solutions and processes we implemented to overcome them.

3.1 Liability Calculation

Fixed and floating extractions from the pension fund occur over the course of the simulation. The proportion of assets under management (AUM) to hold in cash and proportion to invest in bonds is based on the liabilities that fall due over the 36-months. We aim to hold enough in cash to meet future short-term obligations, this is calculated by discounting fixed liabilities to find their present value on the initial day (t=1) of the simulation.

The fixed monthly extraction is $180m per month, this amount is extracted monthly regardless of interest rates and the value of the AUM across the period. The fixed extractions are treated as an annuity and their present values are found by discounting to the date of initiation using the RateSpecG2 rate in MATLAB. Given the certainty of this fixed monthly extraction, the G2++ rate is deemed a suitable discount rate.

The fixed monthly extract of $180m is discounted back to day 1 of the simulation across all 36 months.

Floating extractions are contingent on interest rate movements. Every monthly increase in interest by 10bps causes a -100k extraction from the bond portfolio and a +100k in cash allocation, a monthly decrease in interest rates by 10bps causes a reduction in cash and a +100k into the bond portfolio.

3.2 Initialisation

The first challenge we faced was improving the initial proportion of the AUM to hold in cash reserve. As mentioned previously, the goal of our research is to understand the challenges faced by pension fund, so we to simply hold enough cash to cover the present value of future liabilities .The present value of the liability is calculated as the sum of the PV of each monthly fixed extract. The proportion of the initial AUM (t=1) to allocate to cash is as follows:

$$Proportion\ to\ allocate\ to\ cash\!:\ 1-\left(\frac{PV\ of\ liability}{AUM(1)}\right)$$

The remainder was allocated to the bond portfolio:

$$Proportion\ to\ invest\ in\ bonds\!:\ \left(\frac{PV\ of\ liability}{AUM(1)}\right)$$

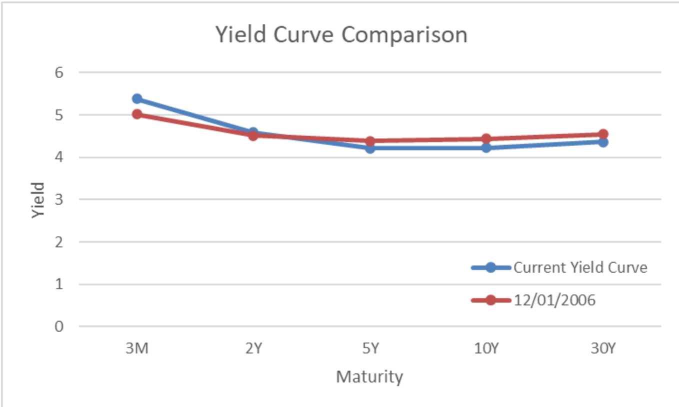
Actual cash held at any time 't' is the calculated as: $AUM(t) * Proportion\ to\ allocate\ to\ cash$

3.3 Selection of Start date

In picking our start date, we decided it best to choose a point at which the effective risk-free rates for various maturities (term structure) were the closest in proximity to those today. We had considered other methods such as simulating each possible 3 year period during the allocated timeframe to find the lowest cumulative difference in yields during the period, or where the shifts in the yield curve were closest to being parallel however ultimately decided against this. We resolved to employ the closest starting point, as we agreed this would be the best way to train our immunisation strategy without overfitting the code to the period selected, as the real simulation would be taking place from 2024 onwards, thus the interest rate trajectory during this time could look completely different to that on which we based our calculations during the period 2005-2010. We also thought that evaluating the bonds in a similar environment to their real trading timeframe would reduce any bias in bond characteristics. In practice, it is also quite unrealistic for the yield curve to shift in a parallel way in the market. This would require that all maturity yields rise or fall by the same amount, which is unrealistic given that they are affected by various factors.

We can see in the graph below that both yield curves are inverted, with short term yields higher than long term yields. This can likely be ascribed to the similarities in the interest rate environments between now and 2006. The Federal Reserve started raising interest rates in 2004, bringing them to a level of 5.25% by July 2006, or very close to the levels we see today. The rising cost of short-term debt pushes short term yields higher, causing an inversion of the yield curve

Our code simply loops through the daily yields of the 3m, 2y, 5y, 10y and 30y treasuries and finds the date on which the cumulative difference between these and the current yields on those bonds is the smallest. This happened to be the 1st of December, 2006 for the particular date on which we ran the code. However, this can be easily amended by changing the 'current' bond yields.



3.4 Bond selection

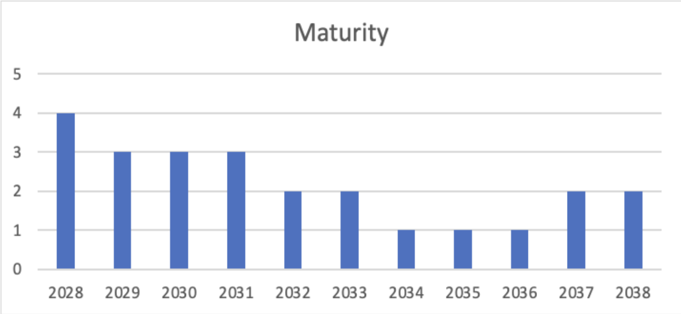
We used UCD's Bloomberg suite to find our selection of suitable bonds to use in our simulation. We handpicked our bonds using a common criterion that we came up with based on our project goals and outside research. To start, all of our bonds are fixed-rate instruments issued in the United States, and they are all investment-grade bonds with a Moody's rating of Aa3 or better. All the bonds we selected, identified below, are also coupon-bearing bonds with semi-annual frequency. Here we display the Issuer, Price, Moody's Credit Rating, Coupon Rate, and Maturity for each bond.

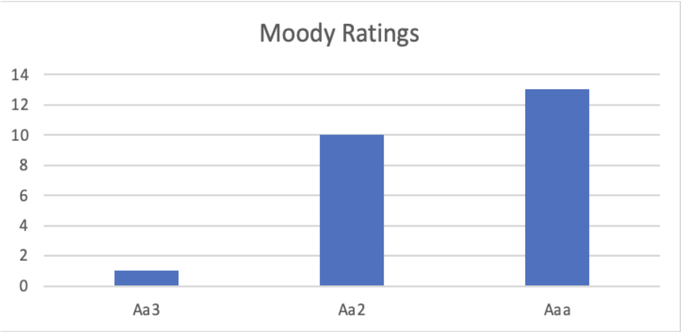
Bond Selection Table (source: Bloomberg)

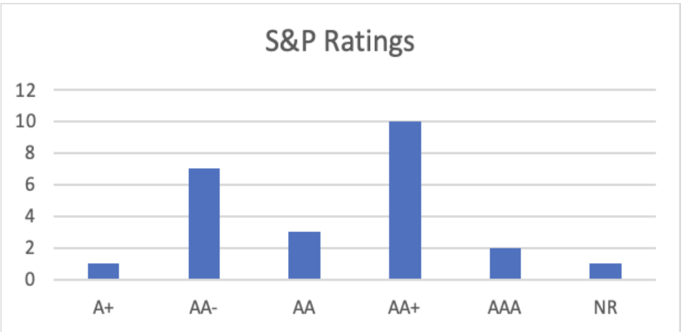
Ticker	Coupon	Maturity	Ratings	Price (USD)
FHLB	0.0613	1/12/28	Aaa	105.77
FNMA	0.06	1/21/28	Aaa	104.80
FNMA	0.0615	1/28/28	Aaa	104.65
CL	0.0645	6/16/28	Aa3	105.19
T	0.0525	2/15/29	Aaa	107.26
FNMA	0.0625	5/15/29	Aaa	112.66
JNJ	0.0695	9/1/29	Aaa	108.52
FHLB	0.07125	2/15/30	Aaa	111.97
FFCB	0.0735	5/8/30	Aaa	115.17
FNMA	0.0725	5/15/30	Aaa	115.85
FHLMC	0.0675	3/15/31	Aaa	111.85
AARP	0.075	5/1/31	Aa2	113.92
CVX	0.08625	11/15/31	Aa2	126.84
FHLB	0.0651	1/2/32	Aaa	108.63
CVX	0.08625	4/1/32	Aa2	126.84
CVX	0.0784	2/15/33	Aa2	121.21
FFCB	0.0548	4/12/33	Aaa	102.00
BRK	0.065	10/1/34	Aa2	115.69
NEE	0.0565	2/1/35	Aa2	112.45
XOM	0.061	4/1/36	Aa2	103.10
XOM	0.0675	8/1/37	Aa2	111.85
JNJ	0.0595	8/15/37	Aaa	110.02
WMT	0.062	4/15/38	Aa2	113.02
XOM	0.06375	6/15/38	Aa2	113.06

To select bonds for our simulation, we researched which bond characteristics were the most and least important to consider in a problem like this. As evident in our final list of bonds, we selected bonds with a very **high credit rating**. Since our region is the United States, it was relatively easy to find such bonds. This is important because selecting bonds with high credit ratings minimises default risk, ensuring more stable cash flows. Our second priority was a **high coupon rate**, which gives us stable and regular cash flows. Bonds with high coupon rates also tend to have shorter durations, which helps reduce risk and allows us to more easily reach our goal. We also looked for **shorter maturities** in order to better align the duration of our bonds to our investment horizon. These types of bonds provide more flexibility and are generally less sensitive to changes in interest rates (Fidelity, no date). Our fourth and fifth priorities, **low duration and convexity** go hand in hand with our other considerations. On top of this, we also aimed to **diversify** our portfolio in the best way possible, avoiding multiple bonds of the same company and picking a wide variety of different maturities, however we did select a variety of bonds from the same issuers which provided good yields whilst being of high credit ratings - particularly the MBS products which we utilised a number of (Dbouk and Kryzanowski, 2009). These investments are comprised of a collection of mortgages bought from the banks that initially issued them. The asset-backed securities are typically very safe investments since they have fixed-rates and often come with prepayment penalties, ensuring the collection of interest. Moreover, they help with portfolio diversification from corporate and government investment options (Johnson, 2024).

On top of all of this, we avoided selecting callable bonds for several reasons. First of all, callable bonds can be called by the issuer before the maturity date, making them riskier than non-callable bonds. This adds extra uncertainty, which is not ideal for our risk mitigation strategies as it could disrupt the crucial matching of asset and liability durations. While callable bonds often generally more substantial rates due to the added risk, our goal with this project isn't to make a lot of money, but rather minimise risk. Creating this criteria helped us keep on track and select a good combination of bonds that is as immune to interest rate movements as possible. We included some graphs below to tell a more visual story of our criteria. As you can see, we chose bonds with unique coupon rates, maturities, and credit ratings. We thought it would be productive to include both S&P and Moody's ratings – while it seems as though our bond credit ratings are higher on Moody's ratings, the S&P ratings have smaller and more specific categories, showing our commitment to diversification in the portfolio. Note: The initial weighting of the bond portfolio was selected according to our immunisation strategy, which will be discussed below in section 3.7.







3.5 Simulation

3.5.1 Simulation Introduction

A key aspect of the simulation is the dynamic management of the bond portfolio to minimise tracking errors against liabilities. The script employs immunisation strategies to select an optimal combination of bonds that effectively hedges against interest rate risk. This involves determining portfolio weights based on bond characteristics such as duration and convexity. By continuously adjusting the portfolio composition, the script aims to achieve the maximise interest rate risk reduction possible.
See the end of this section for a flow chart describing the simulation.

3.5.2 Daily Changes

The code operates on a daily (business days) time step, iterating through each period to simulate the evolution of the bond portfolio. During each iteration, it recalculates bond prices based on prevailing interest rates, updates the present value of liabilities to reflect changes in market conditions, and computes tracking error against the new portfolio value against the new discounted liabilities.
Note: We adjusted how the current_days_ir_simulation variable increases after each iteration to ensure rates were adjusted for the use of only business days.

3.5.3 Monthly Changes

On a monthly basis, the simulation re-immunises, which we will discuss in a later section. However, it also carries out a number of operational activities. It manages cash flows, including both fixed and floating cash inflows/outflows, ensuring that the portfolio remains sufficiently liquid to meet its obligations. Bonds may be bought or sold as needed to maintain liquidity and alignment with investment objectives.

3.5.4 Yield Calculation

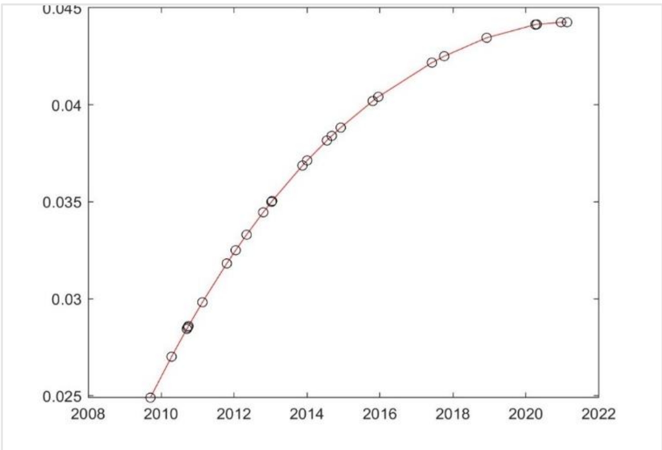
Interest rate movements play a central role in the simulation, influencing bond prices, yields, and credit spreads. The script incorporates the Nelson-Siegel model (seen below) to simulate interest rate dynamics.

$$R(m) = \beta_0 + \left(\beta_1 + \frac{\beta_2}{k}\right)\frac{1-\exp(-mk)}{mk} - \frac{\beta_2}{k}\exp(-mk)$$

capturing both level, slope, and curvature factors that drive yield curve changes. By modelling these factors, the script can generate realistic scenarios for interest rate movements, enabling users to assess the impact on bond prices and portfolio performance. This dynamic approach allows for the adaptation of investment strategies in response to changing market conditions. Below, we have provided an example of a yield curve generated during our simulation, it exhibits the typical concave shape of term structures.

3.5.5 Credit Spreads

In calculating our credit spreads, we decided against using negative spreads on days on which the corporate yield on our bonds was lower than that of Treasuries, or our 'risk-free rate'. Instead, we capped the spread to be 0% on these days. According to 'Negative Credit Spreads: Liquidity and Limits to Arbitrage' (Bhanot & Guo, 2011), the negative credit spreads on certain corporate bonds can be ascribed to asset-specific liquidity, or lack thereof. It was found that once the bid-ask spread was accounted for, the observed 'negative' spread was in fact positive in most cases investigated. Furthermore, trading volume on those days was also found to be smaller, suggesting it was smaller lots that determined the price. This is corroborated by the fact that Credit Default Swap prices for the corporate bonds were higher than those of sovereign debt in most cases (7 of 10), indicating that the negative spread was an asset-specific liquidity issue rather than one of default risk.



3.5.6 Duration Calculation

We calculated Duration by finding the present value-weighted average time of the bond's cash flows. For each cash flow, the present value is multiplied by the time until receipt, and these values are then summed and divided by the bond's price. We also calculated duration using MATLAB's bnddury function. We opted to use the latter as it provided a more flexible process for use across our bonds. For example, its ability to handle calculation for bonds with different basis'.

$$D = \frac{\sum_{t=1}^{n} \frac{CF_t}{(1+y)^t} \times t}{P}$$

3.5.7 Convexity Calculation

We calculated convexity by finding the present value-weighted average time squared of the bond's cash flows. Similar to Duration, for each cash flow, the present value is multiplied by the squared time until receipt, and these values are then summed and divided by the bond's price (seen below). In the script, for our convexity calculation, we tried both a manual calculation and by using the MATLAB bndconvy function. We opted to use the latter as it provided a more flexible process for use across our bonds. For example, its ability to handle calculation for bonds with different basis'.

$$Convexity = \frac{1}{P \times (1+y)^2} \sum_{t=1}^{T} \frac{CF_t}{(1+y)^t} . (t^2 + t)$$

3.6 PV of Liability

Liabilities are calculated by considering fixed cash flows over the remaining months of the simulation. To calculate the present value of our liabilities, we used the corresponding discount rates for each cash flow and calculated the sum of each discounted future payment.

3.7 Payment of Liabilities

3.7.1 Fixed Liabilities

If there is sufficient cash available in our reserve, fixed liabilities are settled directly. However, if cash is insufficient, we handle this scenario by selling bonds to raise the necessary funds.

3.7.2 Floating Liabilities

The floating liability is interest rate dependent. A cash inflow or outflow is received or paid based on a 10 bp increase or decrease in our weighted average portfolio yield. If a cash outflow could not be covered by the cash reserve, bonds are be sold to meet the outflow demand.

3.8 Monthly Re-Immunisation Strategy

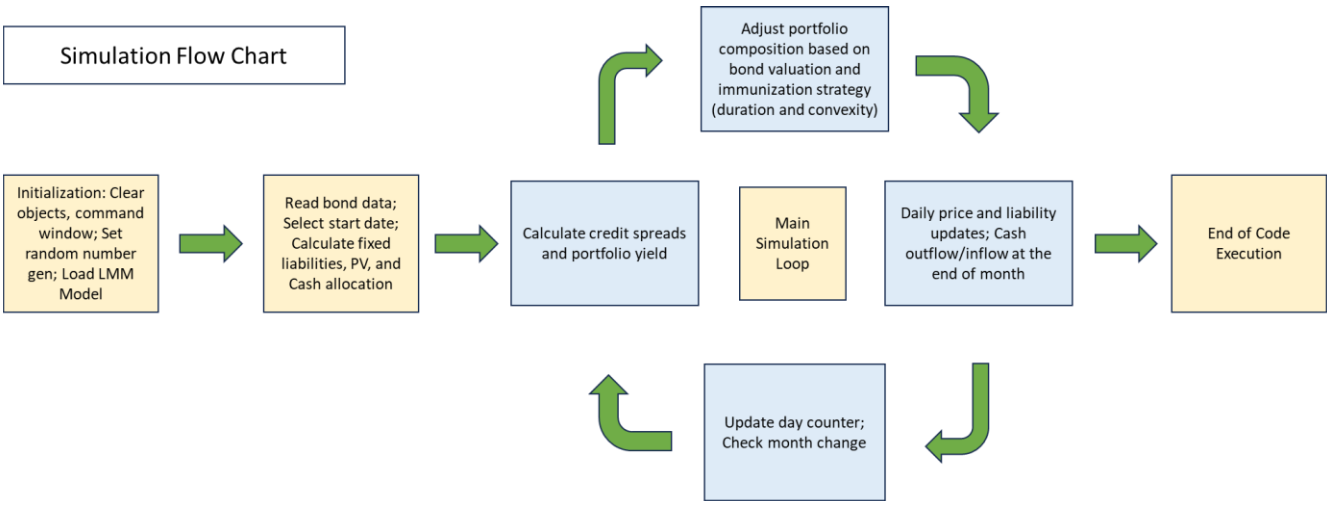
We adjust the portfolio composition by calculating the current portfolio weighted average yield, incorporating credit spreads. We determine the number of units of each bond to purchase and adjust cash and invested values accordingly. At the end of each month, the portfolio undergoes re-immunisation. This process involves discounting the cash flows of each bond using its corresponding yield curve, calculating duration and convexity, and selecting an optimal combination of bonds to align with the immunisation strategy. The chosen bonds are used to adjust the portfolio composition, ensuring alignment with cash inflows and outflows. Finally, we evaluate different combinations of bonds to find the most suitable immunisation strategy; the script calculates the portfolio's total exposure relative to present value liabilities and implements the chosen strategy by adjusting bond units in the portfolio. The price, duration, and convexity of bonds were also considered. For each bond, we calculated its price by discounting the bond's cash flows using its corresponding yield curve. It discounts both coupon payments and the bond's face value at maturity. The present value of each cash flow is then summed to obtain the bond's price.

3.9 Tracking Error

Tracking error was calculated using the formula:

TE_i = Bond Portfolio Value$_i$ – PV of Liability$_i$

In the results section, we will discuss the different measures of tracking error we employed to evaluate the performance of our strategy.



Results

In this section, we will discuss the results of our simulation with respect to the key measure of immunisation performance, which is tracking error.

We will explore the average values of tracking error relative to other strategy values and also examine the changes in tracking error over time. Following this, we examine the impact of introducing bonds which mature in the simulation, and finally, we discuss the overall distribution of the tracking error generated by our immunisation. As our start date was selected based on the similarity of its rates environment to today, these results should be highly generalizable to deployment of our strategy in 2024.

Tracking Error Statistics

To present the notional tracking error calculated on each day of the iteration, we have used several relative measures.

1. Average absolute tracking error relative to bond portfolio value.

$$TE^{bond} = \frac{1}{T}\sum_{t=1}^{T}\frac{|TE_t|}{Bond\ Port\ Value_t}$$

2. Average absolute tracking error relative to AUM.

$$TE^{AUM} = \frac{1}{T}\sum_{t=1}^{T}\frac{|TE_t|}{AUM_t}$$

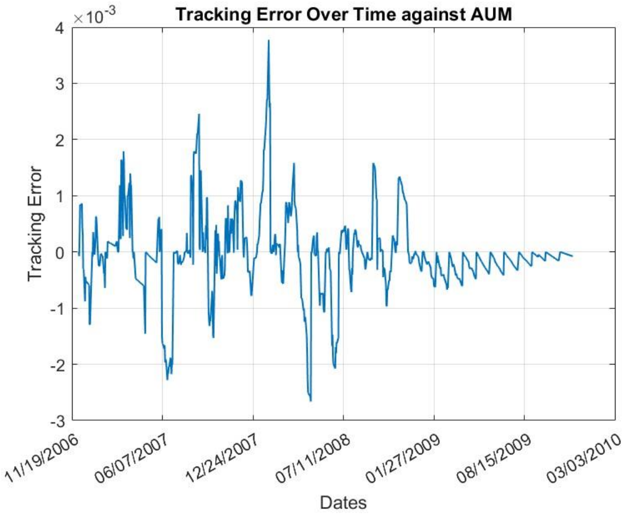
3. Average absolute tracking error relative to the present value of liabilities

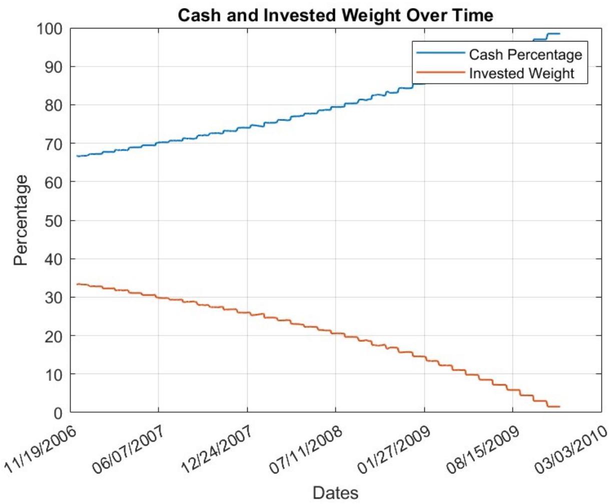
$$TE^{PVL} = \frac{1}{T}\sum_{t=1}^{T}\frac{|TE_t|}{PV\ Liability_t}$$

Below we have summarised the values of each of these statistics.

Tracking Error Statistics	
TE^{bond}	23.963 bps
TE^{AUM}	4.762 bps
TE^{PVL}	23.945 bps

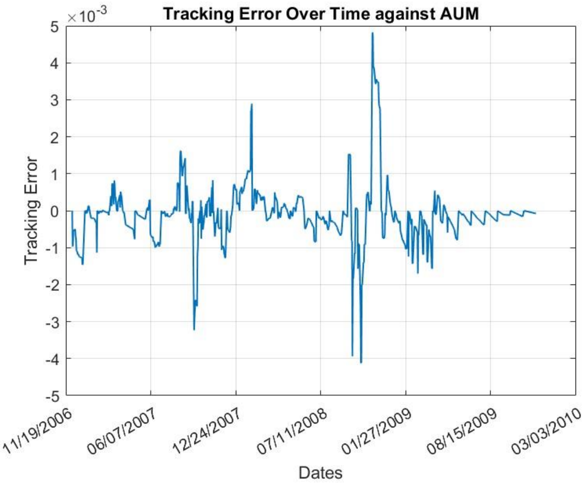
The average absolute monthly tracking error relative to bond portfolio value, AUM, and PV Liability is very small. To examine the immunisation's performance over time, we have produced the following performance plots.





As you can see, the tracking error against AUM has experienced high levels of variance in the earlier years of the simulation. However, this has stabilised dramatically in 2009. Our hypothesis is that as our first bond matures just after the simulation (in early 2010), in the final year of the simulation, the immunised portfolio places a large weight in this near-maturity bond as it is exposed to minimal interest rate risk. Another reason for the reduced tracking error towards the end of the simulation is evident from the plot of cash vs invested percentages. As the portfolio nears the end of the simulation, over 90% of the portfolio weight is invested in cash, which leads to reduced variance in tracking error.

To examine the effect of near-maturity bonds on the variance of the tracking error, we modified the code to handle bonds which mature during the simulation. We added an FNMA 7.125, which had simulation maturity on 15/07/2007; we also added a CVX 8, which would have simulation maturity at the end of 2008. These bonds get replaced by cash on their maturity date. After adding two new bonds that matured within the simulation, we achieved the following results:



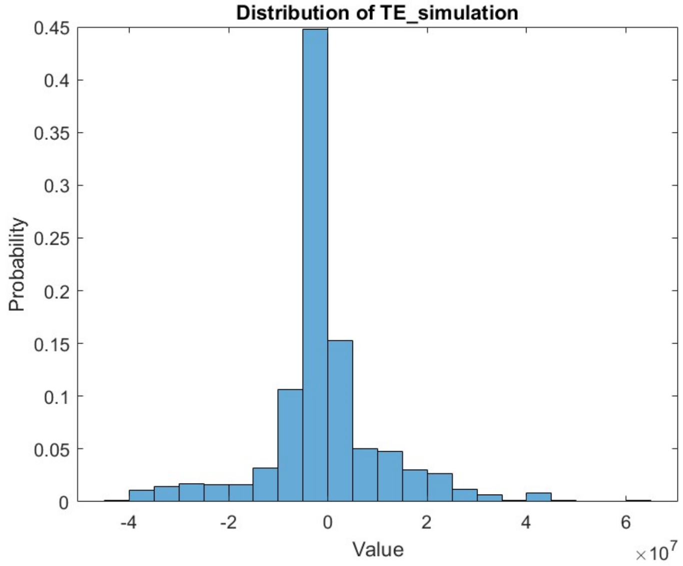
As you can see, there has been a dramatic reduction in the variance of tracking errors in the periods leading up to the maturity of the new bonds. However, we note the sharp increases in tracking error after the maturity of the new bonds. In fact, relative to the 4.762 bps tracking error without the new bonds, this new strategy only reduced the tracking error to 4.32 bps. It is also worth noting that by introducing these new bonds, operational activities for the pension fund will be added as they will have to settle the matured bonds and receive large repayments of the face value. This will also lead to a reduction in the number of bonds the portfolio is holding, so the fund may wish to purchase new, longer-maturity bonds. So overall, we would suggest that the pension fund understand the trade-off in increased operational activities and the small reduction in tracking error.

Distribution of Tracking Error

Below we have presented a table of summary statistics and a distribution plot of the tracking error so the pension fund can better quantify their expectations of tracking error.

Tracking Error Summary Statistics	
Mean	-520413
Min	- 40470424.9
Max	60209648
Volatility	12020681
Skewness	0.27
Kurtosis	6.6022

Given that we selected our start date, our simulation is run in a similar interest rate environment to today. The fund should expect tracking error statistics like those above using our immunisation process. On average, tracking error has been negative with a mean of -520413, which is very small; in fact, it represents only 0.29 bps of the initial AUM. The min and max values of this distribution are consistent with the large peaks and troughs observed in the previous section, and the volatility represents the standard deviation of the distribution. Interestingly, the distribution has a skewness less than ± 0.5, indicating that it is approximately symmetric around its mean. On the other hand, the high level of kurtosis indicates a Leptokurtic distribution with thin tails and concentrated around its mean. Below, you can see a plot of the approximate distribution to expect if our immunisation was deployed in today's rates environment.



So overall the tracking error we generated from our immunisation is very small relative to AUM, bond portfolio value and PV Liability. While the tracking error can be reduced marginally by adding bonds which mature during the simulation, it may not be worthwhile for the fund as the reduction is so small. The distribution of our tracking error is highly leptokurtic, which indicates that it is tightly centred around its mean. Therefore, when the immunisation is performed in today's rates environment, we expect the overall tracking error to be very low. We feel that all these points are positive indicators that we have achieved our objective of immunising the pension fund against interest rate risks.

Limitations and Recommendations

We initially ran our simulation using only bonds which expired following the simulation period. This simplified the code and process of picking bonds and exchanging them for cash. However, we then trialled using bonds which expire within the simulation time frame. Although this provided a slight reduction in tracking error, it also introduces some potential operational difficulties in cashing the bonds during the period of the simulation. Also, using this approach we found that a large portion of our capital was invested only into these short duration bonds. Although this provides the best results in terms of tracking error minimisation, we should note than in real life applications we may want to diversify our holdings somewhat to minimise the risk of any drastic changes in price or default. Holding a large portion of our portfolio in few bonds could also pose some issues regarding liquidity if we want to exit these positions. However, considering the issuers of the bonds we have used, this shouldn't pose too much of an issue in real life applications.
Some further limitations of our study are that we employed a fixed investment strategy based on predefined rules and assumptions, lacking dynamic adjustments or optimisation techniques that real portfolio managers might use in response to evolving market conditions. Additionally, the simulation operates on static input data and lacks real-time integration, hindering its ability to reflect current market conditions accurately. We also have assumed efficient and rational market behaviour that may not always hold true in practice, as real-world markets can exhibit irrational behaviour, volatility, and unexpected events that the simulation does not account for.

We also note that the purpose of this portfolio optimisation was to minimise tracking error. This goal meant that our approach was significantly different than if we were aiming to maximise returns simultaneously for example. For the purpose of our simulation, we only used standard corporate bonds and treasuries, all rated

Aa3 (Moody's) or better. However, incorporating various other instruments such as the callable, floating rate or convertible bonds and extending the scope of our selection to lower-rated bonds could provide superior returns for a relatively small amount of default risk. Furthermore, this default risk could be hedged against using Credit Default Swaps. As our bonds and liabilities were both USD-denominated, hedging FX risk was not a concern. We also deemed the pricing of the various other types of bonds as beyond the scope of this assignment, however important to mention their use in real life applications.

We must also mention the various other immunisation methods which given more time, could be useful to implement in this assignment. Using simple methods such as barbells or bullets can be useful as a bond investment strategy (McFarlane, 2022), however in the interest of minimising tracking error, strategies where one invests in only very short- and long-dated bonds (barbell), and ones where all bonds mature at the same time (bullet) didn't seem completely suitable to our needs. Barbell structures are suitable for more volatile interest rate environments; however, considering the nature of this assignment and having to extract and pay out a fixed amount on a regular basis, the reinvestment risk present with investing in very long-duration bonds seemed unsuitable.

Principal Component Analysis is a concept often used in portfolio optimisation (Barber, 1996). It is a dimensionality reduction method which simplifies the factors that affect the interest rate term structure and thus simplify the immunisation problem. This approach could be useful in the simulation ran in this assignment as it can also be used to find factors driving changes in bond prices and yields and its functionality can be extended to modelling yield curves as an alternative to the Nelson Siegel model. Finding bonds which are least sensitive to these changes can reduce the volatility of a bond portfolio.

Conclusion

To conclude, whilst we have mentioned numerous improvements which could be made to the simulation and code, our tracking error of 4.7 basis points suggests successful immunisation of the portfolio over the period tested. Our selection of start date, as well as our choice of investment grade, high-yield bonds, resulted in a suitable combination to allow for the effective completion of the task. We note that basing our investments in the US is likely beneficial, as the universe of high rated, high yield bonds is much larger than we are likely to find in other regions. Considering the performance of our strategy, it would be a suitable starting point for a pension fund or other asset manager aiming to immunise their portfolio against interest rate fluctuations.