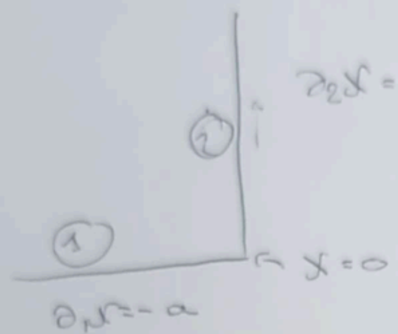
Boundaries through Euler Poisson Darboux ($\gamma = \frac{5}{3}$)



$\leftarrow \; x = a(\lambda_- + \lambda_0)$

$\uparrow$

$x = -a(\lambda_+ - \lambda_0)$

$$x = \frac{f(\lambda_+) + g(\lambda_-)}{\lambda_+ - \lambda_-} \qquad (f(\lambda_0) = 0 \;,\; g(-\lambda_0) = 0)$$

$$f(\lambda_+) = a(\lambda_0^2 - \lambda_+^2) \quad , \; g(\lambda_-) = a(\lambda_0^2 - \lambda_-^2)$$

$$\rightarrow \; x = \frac{a}{\lambda_+ - \lambda_-}\left(2\lambda_0^2 - \lambda_+^2 - \lambda_-^2\right)$$

Now, $x_+(t)$ is on the boundary when λ_0 varies so (8)

We have $x_+ - V_1 t = \partial_1 x$

$x_+ - V_2 t = \partial_2 x$

$$\partial_2 X = a$$

$$\partial_1 X = \left(\frac{-2a\lambda_0}{\lambda_0 - \lambda_\bullet} - \frac{a(\lambda_\bullet + \lambda_0)}{\lambda_0 - \lambda_\bullet} \right)$$

$$= \frac{-a}{\lambda_0 - \lambda_\bullet} \left[2\lambda_0 + \cancel{a}(\lambda_\bullet + \lambda_0) \right]$$

$$= \frac{-a}{\lambda_0 - \lambda_\bullet} \times \left[3\lambda_0 + \lambda_\bullet \right]$$

We call $\lambda' = \lambda_0 - \lambda_\bullet$

$$\left[\begin{array}{l} \partial_1 X = \dfrac{-a}{\lambda'} \left[4\lambda_0 - \lambda' \right] \\ \partial_2 X = a \end{array} \right.$$

And $V_1 = \frac{2}{3}\lambda_0 + \frac{1}{3}\lambda_\bullet = \lambda_0 - \frac{1}{3}\lambda'$

$$V_2 = \frac{2}{3}\lambda_\bullet + \frac{1}{3}\lambda_0 = \lambda_0 - \frac{2}{3}\lambda'$$

$$V_1 - V_2 = \frac{1}{3}\lambda'$$

$$x - V_1 t = \partial_1 x$$

$$x - V_2 t = \partial_2 x.$$

We invert: $\begin{pmatrix} 1 & -V_1 \\ 1 & -V_2 \end{pmatrix}\begin{pmatrix} x \\ t \end{pmatrix} = \begin{pmatrix} \partial_1 x \\ \partial_2 x \end{pmatrix}$, $\begin{vmatrix} 1 & -V_1 \\ 1 & -V_2 \end{vmatrix} = (V_1 - V_2)$

$$x = \frac{\partial_2 x\, V_1 - \partial_1 x\, V_2}{V_1 - V_2} \qquad t = \frac{\partial_2 x - \partial_1 x}{V_1 - V_2}.$$

$$\text{So, } t = \frac{a + \frac{a}{\lambda'}\left[4\lambda_0 - \lambda'\right]}{\frac{1}{3}\lambda'} = \frac{12 a \lambda_0}{\lambda'^2}$$

$$\text{So } \boxed{\lambda'^2 = \frac{12 a \lambda_0}{t}}$$ (Logique : $t \to \infty$, $\lambda_\cdot \to \lambda_0$)

$$x = \frac{a\left(\lambda_0 - \frac{1}{3}\lambda'\right) + \frac{a}{\lambda'}\left[4\lambda_0 - \lambda'\right]\overset{\lambda_0 - \frac{2}{3}\lambda'}{\left[\cancel{\text{[illegible]}}\right]}}{\frac{1}{3}\lambda'}$$

4

$$x = 3a\frac{\lambda_0}{\lambda'} - a + \frac{3a}{\lambda'^2}\left[4\lambda_0 - \lambda'\right]\left[\lambda_0 - \frac{2}{3}\lambda'\right]$$

$$= \frac{3a\lambda_0}{\lambda'} - a + \frac{12a\lambda_0^2}{\lambda'^2} - \frac{3a}{\lambda'}\left[\lambda_0 + \frac{8}{3}\lambda_0\right] + 2a$$

$$= \frac{12a\lambda_0^2}{\lambda'^2} + a - \frac{8a\lambda_0}{\lambda'}$$

Replacing it with t:

$$x(t) = \lambda_0 t + a - \frac{8a\lambda_0}{\sqrt{12a\lambda_0}} \times \sqrt{t}$$

$$\lambda_0 = 3c_0$$

$$x(t) = 3c_0 t + a - 4\sqrt{a\,c_0 t}$$

$$3c_0 t + a - 4\sqrt{ac_0 t}$$

Matches perfectly with result derived through ODE on x_t !

changing times to $t' = c_0 t + \frac{a}{2}$