$$i\hbar\partial_t\psi = \left[\frac{-\hbar^2}{2m}\nabla^2 + V + g|\psi|^2\right]\psi$$

NLSE to PDE

1

$$\psi = Ae^{iS} \longrightarrow \vec{\nabla}\psi = \left[\vec{\nabla}A + iA\vec{\nabla}S\right]e^{iS}$$

$$\nabla^2\psi = \left[\nabla^2 A + i\vec{\nabla}A.\vec{\nabla}S + iA\nabla^2 S + i\vec{\nabla}S.\vec{\nabla}A - A|\vec{\nabla}S|^2\right]e^{iS}$$

$$g|\psi|^2\psi = gA^3e^{iS}$$

$$i\hbar\partial_t\psi = \left[i\hbar\partial_t A - \hbar A\partial_t S\right]e^{iS}$$

Real Part:

$$-\hbar A\partial_t S = \frac{-\hbar^2}{2m}\nabla^2 A + \frac{\hbar^2}{2m}A|\vec{\nabla}S|^2 + gA^3 \,(1) + VA$$

Imaginary Part:

$$i\hbar\partial_t A = \frac{-\hbar^2}{2m}\times 2\,\vec{\nabla}A\,\vec{\nabla}S - \frac{\hbar^2}{2m}A\nabla^2 S \quad (2)$$

(2): $\rho = A^2$, $\vec{u} = \frac{\hbar}{m}\vec{\nabla} S$

$$\partial_t A^2 = -\frac{\cancel{\hbar}}{\cancel{m}} \vec{\nabla}\rho \cdot \vec{u} - \frac{\cancel{\hbar}}{\cancel{m}} \rho \nabla \cdot \vec{u}$$

$$\partial_t A^2 = -\vec{\nabla}\rho \cdot \vec{u} - \rho \nabla \cdot \vec{u}$$

$$\partial_t \rho + \nabla \cdot (\rho \vec{u}) = 0$$

(1) Neglect term in $\nabla^2 A$.

$$-\hbar \partial_t S = \frac{\hbar^2}{2m} |\vec{\nabla} S|^2 + g A^2 + V$$

Take ~~divergence~~ grad:

$$-\hbar \partial_t \vec{\nabla} S = \frac{\hbar^2}{2m} \vec{\nabla}(|\vec{\nabla} S|^2) + g \vec{\nabla}\rho + \vec{\nabla} V$$

$$-\partial_t \vec{u} = \frac{1}{2}\vec{\nabla}(\vec{u}^2) + \frac{g}{m} \vec{\nabla}\rho + \frac{\vec{\nabla} V}{m}$$

Because $\vec{\nabla} \wedge \vec{u} = 0$, $\frac{1}{2}\vec{\nabla} \vec{u}^2 = (\vec{u} \cdot \vec{\nabla})\vec{u}$

$$\begin{cases} \partial_t u + (\vec{u} \cdot \vec{\nabla})\vec{u} + \frac{g}{m} \vec{\nabla}\rho = -\frac{\vec{\nabla} V}{m} \\ \partial_t \rho + \vec{\nabla} \cdot (\rho \vec{u}) = 0 \end{cases}$$