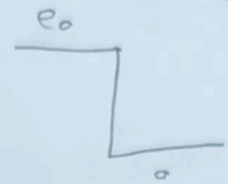
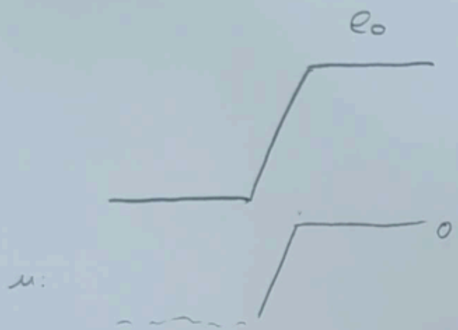
Idea: Look at simple wave evolution of

(1)

(2) and also of



(1): Will evolve as : So, if simple wave, λ^- is constant.

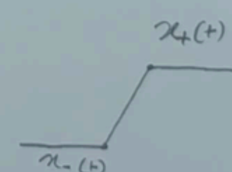


thus on the non constant parts, λ^+ verifies $(V_+ - \xi) = 0$, $\xi = \frac{x}{t}$

, also $\lambda^- = \lambda^-(0) = -c(e_0) \times \frac{2}{\gamma - 1}$

$$V_+ = ((1+\gamma)\lambda^+ + (3-\gamma)\lambda^-) \times \frac{1}{4} \Rightarrow \lambda^+ = \frac{1}{1+\gamma} \times \left[4\xi - \frac{2(3-\gamma)}{\gamma - 1} c_0\right]$$

And we can find the points where λ^+ is constant by solving $e = 0$ $(x^-(t))$ and $e = e_0$ $(x^+(t))$:



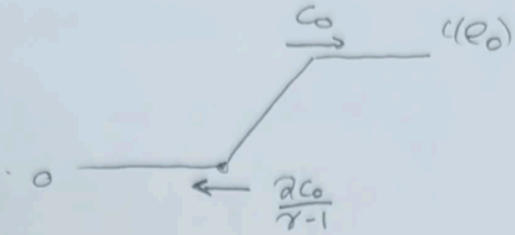
$e = 0 \iff \lambda^+ = \lambda^-$ thus

$$\frac{x_+(t)}{t} = \lambda^- \times [1+\gamma + 3-\gamma] \times \frac{1}{4} = \lambda^- = \frac{-2}{\gamma-1} c_0 t$$

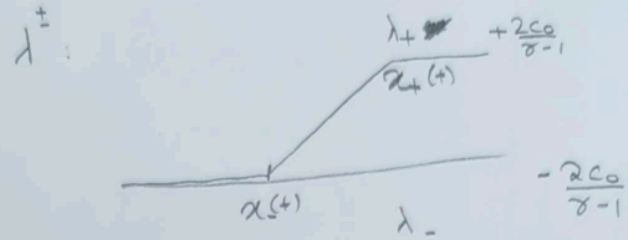
$e = e_0 \iff \lambda^+ = \frac{2}{\gamma - 1} c_0 = -\lambda^-$

thus $$\frac{x_-(t)}{t} = \lambda^- \left[3-\gamma - 1 - \gamma\right] \times \frac{1}{4} = \frac{-2}{\gamma-1} c_0 \times \frac{1}{4} \times 2 \times [1-\gamma] = c_0.$$

So:



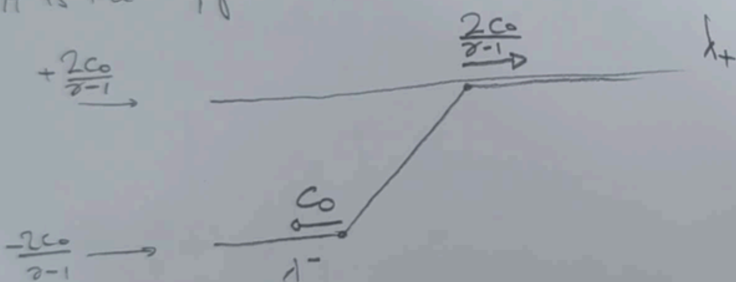
And for λ's:



For the case: $\rho(0)$:

$\delta u(0) = 0$!

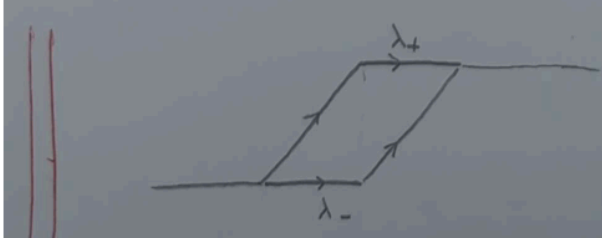
It is the opposite, λ^+ constant and λ^- varying:



So, for an initial profile:

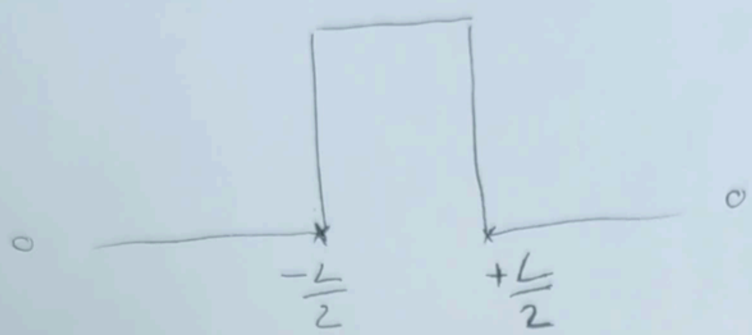
We evolve through simple waves till:

At this moment, the λ^s are like:



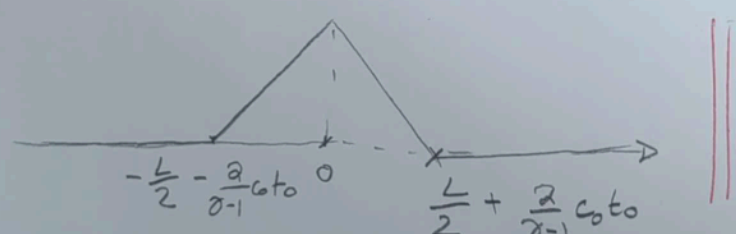
And we'll need to do a full treatment

Now we imagine that the density is:



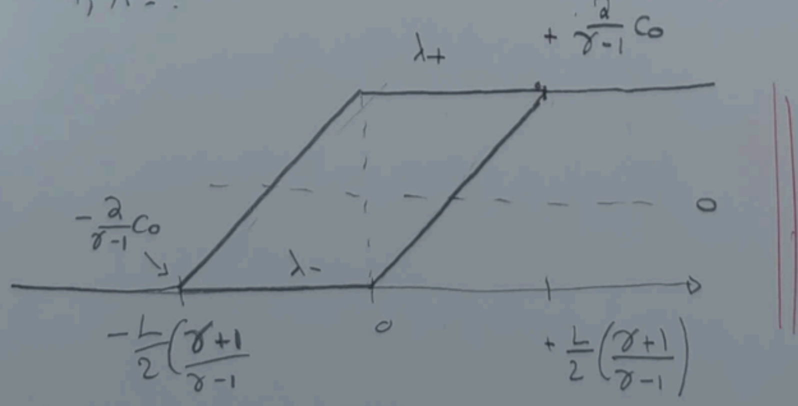
So the time t_0 where simple wave treatment stops working is when $c_0 \times t_0 = \frac{L}{2}$, $t_0 = \frac{L}{2c_0}$

At this moment, we have c distributed like:



and $\frac{L}{2} + \frac{2}{\gamma-1}c_0t_0 = \frac{L}{2}\left[1 + \frac{2}{\gamma-1}\right] = \frac{L}{2}\left(\frac{\gamma+1}{\gamma-1}\right)$

And λ_+, λ_-:



So we can determine λ_+, λ_- as a function of x in this initial condition : Calling $L_0 = L\left(\frac{\gamma+1}{\gamma-1}\right)$

for $-\frac{L_0}{2} < x < 0$: $\begin{cases} \lambda_- = -\dfrac{2c_0}{\gamma-1} \\ \lambda_+ = \dfrac{8c_0}{L_0(\gamma-1)}x + \dfrac{2c_0}{\gamma-1} \end{cases}$

for $0 < x < \frac{L_0}{2}$ $\begin{cases} \lambda_- = +\dfrac{8c_0}{L_0(\gamma-1)}x - \dfrac{2c_0}{\gamma-1} \\ \lambda_+ = \dfrac{2c_0}{\gamma-1} \end{cases}$

In $\lambda_\pm$ space these are initial conditions :