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1. Fit an AR(1) model to the data by implementing the least square method presented on page 10 of the lecture notes.

Solution:

The time series data is plotted in Figure 1.

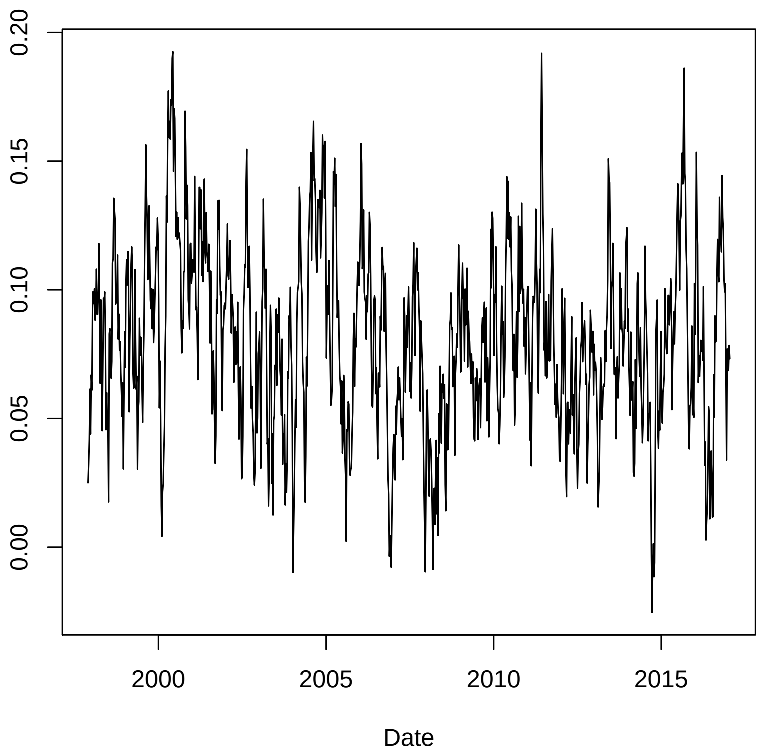

Figure 1: Data for Homework 1

The least squares method is implemented below in `R` and the parameter estimates are calculated for the provided time series.

```
> fit.ar1 <- function(x) {
+
+   N <- length(x)
+
+   # based on (3.17) and (3.18)
+   phi1 = sum(x[2:N] * x[1:(N-1)]) / sum(x[1:(N-1)]^2)
+   sigma2 = sum((x[2:N] - phi1 * x[1:(N-1)])^2) / (N - 1)
+
+   names(phi1) <- "ar1"
+
```

```
+   list(coef = phi1, sigma2 = sigma2)
+ }
> y <- tsdata$Series
> x <- tsdata$Series - mean(y)
> model1 <- fit.ar1(x)
```

This gives $\hat{\phi}_1 = 0.83453413$ and $\hat{\sigma}_a^2 = 0.00037948$.

2. Fit an AR(1) model to the data using a package.

```
> model2 <- arima(x, c(1,0,0), include.mean = FALSE, method = 'CSS')
> model3 <- arima(x, c(1,0,0), include.mean = FALSE, method = 'CSS-ML')
```

(i) If method `CSS` is used, we get $\hat{\phi}_1 = 0.83453412$ and $\hat{\sigma}_a^2 = 0.00037948$.

(ii) If method `CSS-ML` is used (i.e. the default method), we get $\hat{\phi}_1 = 0.83576532$ and $\hat{\sigma}_a^2 = 0.00038004$.

3. Compare the fitted models.

Solution:

The fitted models are compared in Table 1.

Model	$\hat{\phi}_1$	$\hat{\sigma}_a^2$	Call
`model1`	0.83453413	0.00037948	`fit.ar1`
`model2`	0.83453412	0.00037948	`arima(method = 'CSS')`
`model3`	0.83576532	0.00038004	`arima(method = 'CSS-ML')`

Table 1: Comparison of fitted models