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Background

Theorem 1. *Suppose we are given recurrence relation* $T(n) = a \times T(n/b) + f(n)$. *Let* $c = log_b(a)$. *Then*

1. *If* $f(n) = O(n^d)$ *for* $d < c$ *then* $T(n) = \Theta(n^c)$
2. *If* $f(n) = \Theta(n^c)$ *then* $T(n) = \Theta(n^c log(n))$
3. *If* $f(n) = \Omega(n^c)$ *then* $T(n) = \Theta(f)$

Note that Ω *is asymptotic lower bound,* O *is asymptotic upper bound and* Θ *is exact bound.*

Questions

1. Apply Master Theorem to find the running time of $T(n) = 2T(n/2) + O(1)$.

   Solution:

   Here $c = log_2(2) = 1$ and $d = 0$. Since $d < c$ then $T(n) = \Theta(n)$.

2. Apply Master Theorem to find the running time of $T(n) = 3T(n/2) + O(1)$.

   Solution:

   Here $c = log_2(3) = 1.58$ and $d = 0$. Since $d < c$ then $T(n) = \Theta(n^{1.58})$.

3. Apply Master Theorem to find the running time of $T(n) = 4T(n/2) + O(n)$.

   Solution:

   Here $c = log_2(4) = 2$ and $d = 1$. Since $d < c$ then $T(n) = \Theta(n^2)$.

4. Apply Master Theorem to find the running time of $T(n) = 3T(n/2) + O(n^2)$.

   Solution:

   Here $c = log_2(3) = 1.58$ and $d = 2$. We have case three, so $T(n) = O(n^2)$.

5. Apply Master Theorem to find the running time of $T(n) = 3T(n/3) + O(n)$.

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Solution:

Here $c = log_3(3) = 1$ and $d = 1$. We have case two, so $T(n) = O(nlog(n))$.

6. Apply Master Theorem to find the running time of $T(n) = 8T(n/2) + O(n^2)$.

Solution:

Here $c = log_8(2) = 3$ and $d = 2$. We have case one, so $T(n) = O(n^3)$.

7. Apply Master Theorem to find the running time of $T(n) = 8T(n/2) + O(n^3)$.

Solution:

Here $c = log_8(2) = 3$ and $d = 3$. We have case two, so $T(n) = O(n^3log(n))$.

8. Apply Master Theorem to find the running time of $T(n) = 8T(n/2) + O(n^7)$.

Solution:

Here $c = log_8(2) = 3$ and $d = 7$. We have case three, so $T(n) = O(n^7)$.

9. Apply Master Theorem to find the running time of $T(n) = 5T(n/3) + O(n^{1.5})$.

Solution:

Here $c = log_5(3) = 1.46$ and $d = 1.5$. We have case three, so $T(n) = O(n^{1.5})$.

10. Apply Master Theorem to find the running time of $T(n) = 5T(n/3) + O(n^{1.4})$.

Solution:

Here $c = log_5(3) = 1.46$ and $d = 1.4$. We have case one, so $T(n) = O(n^{1.46})$.

11. Apply Master Theorem to find the running time of $T(n) = 3T(n/3) + O(n)$.

Solution:

Here $c = log_3(3) = 1$ and $d = 1$. We have case two, so $T(n) = O(nlog(n))$.

12. Apply Master Theorem to find the running time of $T(n) = 9T(n/3) + O(n^2)$.

Solution:

Here $c = log_3(9) = 2$ and $d = 2$. We have case two, so $T(n) = O(n^2log(n))$.