Makeham's User Guide

This guide provides important details regarding the implementation of Makeham's Law in R.

Select Period

An important implementation detail is regarding the select period. In all cases, a select period is assumed by default when using an actuarial function. This means that

- ${}_tp_{[x]}$ is implemented such that using a non-zero s arugment results in ${}_tp_{[x]+s}$
- $A_{[x]}$ is implemented such that using a non-zero s arugment results in $A_{[x]+s}$
- $\mu_{[x]}$ is implemented such that using a non-zero s argument results in $\mu_{[x]+s}$

  $\vdots$

For instance, lets say that the select period is 2 and the value of A_{20} is wanted. Calling `Ax(20,s=2)` actually gives the value $A_{[20]+2}$ and not A_{20}. Therefore, this value would have to be calculated as $A_{[18]+2}$ which is `Ax(18,s=2)`

To generalize the model to any select period, numerical integration was used in the implementation of several functions. For instance, the functions implementing ${}_tp_x$ and A_x use numerical integration. Although this provides for flexibility in changing the model parameters, the disadvantages of such an approach are

- Running code such as building life tables takes noticeably longer when a large select period is used, such as $d = 10$
- In addition to a function using numerical integration potentially being slow, it is also less accurate than solving an integral before programming the function

Optional arguments

Typically, as is the case with the $A_{[x]}$ function, rather than implementing new functions such as $\bar{A}_{[x]}$, these are optional parameters to the existing function. For instance, $\bar{A}_{[x]}$ can be calculate as `Ax(x,c=1)` where c is an optional parameter indicating that a continuous expected present value should be calculated.

```
> library(makehams)
> head(createLifeTable(x=20))
```

```
   x   l[x]+0   l[x]+1      lx+2 x+2
1 NA       NA       NA 100000.00  20
2 NA       NA       NA  99975.04  21
3 20 99995.08 99973.75  99949.71  22
4 21 99970.04 99948.40  99923.98  23
5 22 99944.63 99922.65  99897.79  24
6 23 99918.81 99896.43  99871.08  25
```

Another table that can be readily accessed is the insurance table.[1]

```
> head(createInsuranceTable(x=20))
```

```
   x       A[x]     A[x]+1       Ax+2     5E[x]   5E[x]+1      5Ex+2 x+2
1 20 0.04917546 0.05143193 0.05377599 0.7825547 0.7825077 0.7824769  22
2 21 0.05139908 0.05376425 0.05622182 0.7825368 0.7824872 0.7824536  23
3 22 0.05373095 0.05620990 0.05878622 0.7825168 0.7824641 0.7824275  24
4 23 0.05617607 0.05877410 0.06147464 0.7824942 0.7824381 0.7823980  25
5 24 0.05873967 0.06146230 0.06429274 0.7824688 0.7824089 0.7823650  26
6 25 0.06142720 0.06428015 0.06724641 0.7824403 0.7823761 0.7823278  27
```

Recursions

The following recursion relationships hold

$$\begin{aligned}
A_{[x]+d} &= A_{x+d} \\
A_{[x]+d-1} &= q_{[x]+d-1}v + p_{[x]+d-1}v(A_{x+d}) \\
A_{[x]+d-2} &= q_{[x]+d-2}v + p_{[x]+d-2}v(A_{[x]+d-1}) \\
&\vdots \\
A_{[x]} &= q_{[x]}v + p_{[x]}v(A_{[x]+1})
\end{aligned}$$

where A_{x+d} can be calculated recursively using

$$A_x = vq_x + vp_xA_{x+1} \tag{1}$$

Therefore the approach is to

- Calculate A_{x+d} for $x = \omega - d - 1$ to $x - d$
- Calculate $A_{[x]+d-t}$ for $t = 1$ to d using $x = \omega - d - 1$ to $x - d$

[1]note that the optional argument s is the select already used. For $A[x]$ this is 0 and for $A[x] + d$ this is d