RushHour Solver

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Algorithm(s): Dijkstra's Algorithm

Languages(s): Python, C++

Jam(s): http://www.mathsonline.org/game/jam.html

Results

Total solve time for all 40 jams from http://www.mathsonline.org/game/jam.html:

Language	Total Solve Time
C++	5.8 seconds
Python	10.45 seconds

Explanation

Dijkstra's algorithm

it is a breadth-first search algorithm on an unweighted graph using a priority queue.

Case #1: Simple Maze

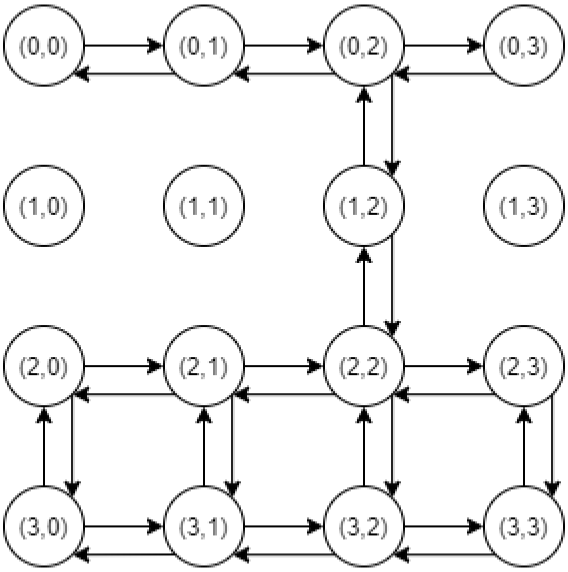
Suppose we have the following maze, where `W` represents a wall and `.` represents a tile you can walk on. We are trying to go from bottom left corner to top right corner.

```
. . . .
W W . W
. . . .
. . . .
```

One of the possible shortest paths is URRRURR:

```
. . _ _
W W | W
_ _ _ .
| . . .
```

Here is the graph for the maze:



Which can also be represented as:

```
graph = {
    (0, 0): {(0, 1), (1, 0)},
    (0, 1): {(0, 2), (0, 0), (1, 1)},
    (0, 2): {(1, 2), (0, 3), (0, 1)},
    (0, 3): {(1, 3), (0, 2)},
    (1, 0): set(),
    (1, 1): set(),
    (1, 2): {(1, 3), (1, 1), (0, 2), (2, 2)},
    (1, 3): set(),
    (2, 0): {(3, 0), (1, 0), (2, 1)},
    (2, 1): {(2, 0), (3, 1), (1, 1), (2, 2)},
    (2, 2): {(1, 2), (3, 2), (2, 3), (2, 1)},
    (2, 3): {(1, 3), (3, 3), (2, 2)},
    (3, 0): {(2, 0), (3, 1)},
    (3, 1): {(3, 0), (3, 2), (2, 1)},
    (3, 2): {(3, 1), (3, 3), (2, 2)},
    (3, 3): {(3, 2), (2, 3)}}
}
```

Which can be solved as follows:

```
# https://en.wikipedia.org/wiki/Dijkstra%27s_algorithm#Pseudocode
def shortest_path(prev, target):
    path = []
    u = target
    while u in prev:
        path.insert(0, u)
        u = prev[u]
    return path

def solve(graph, src, target):
    dist = dict((k, float('inf')) for k in graph.keys())
```

```
    prev = dict((k, None) for k in graph.keys())
    dist[src] = 0
    pq = [(0, src)]
    while pq:
        distu, u = heapq.heappop(pq)
        if u == target:
            return shortest_path(prev, u)
        for v in graph[u]:  # neighbors
            alt = distu + 1
            if alt < dist[v]:
                dist[v] = alt
                prev[v] = u
                heapq.heappush(pq, (alt, v))

# [(3, 0), (2, 0), (2, 1), (2, 2), (1, 2), (0, 2), (0, 3)]
print(solve(graph, ()))
```

Case #2: RushHour

Graph Representation

First let's discuss the graph representation for the RushHour game.

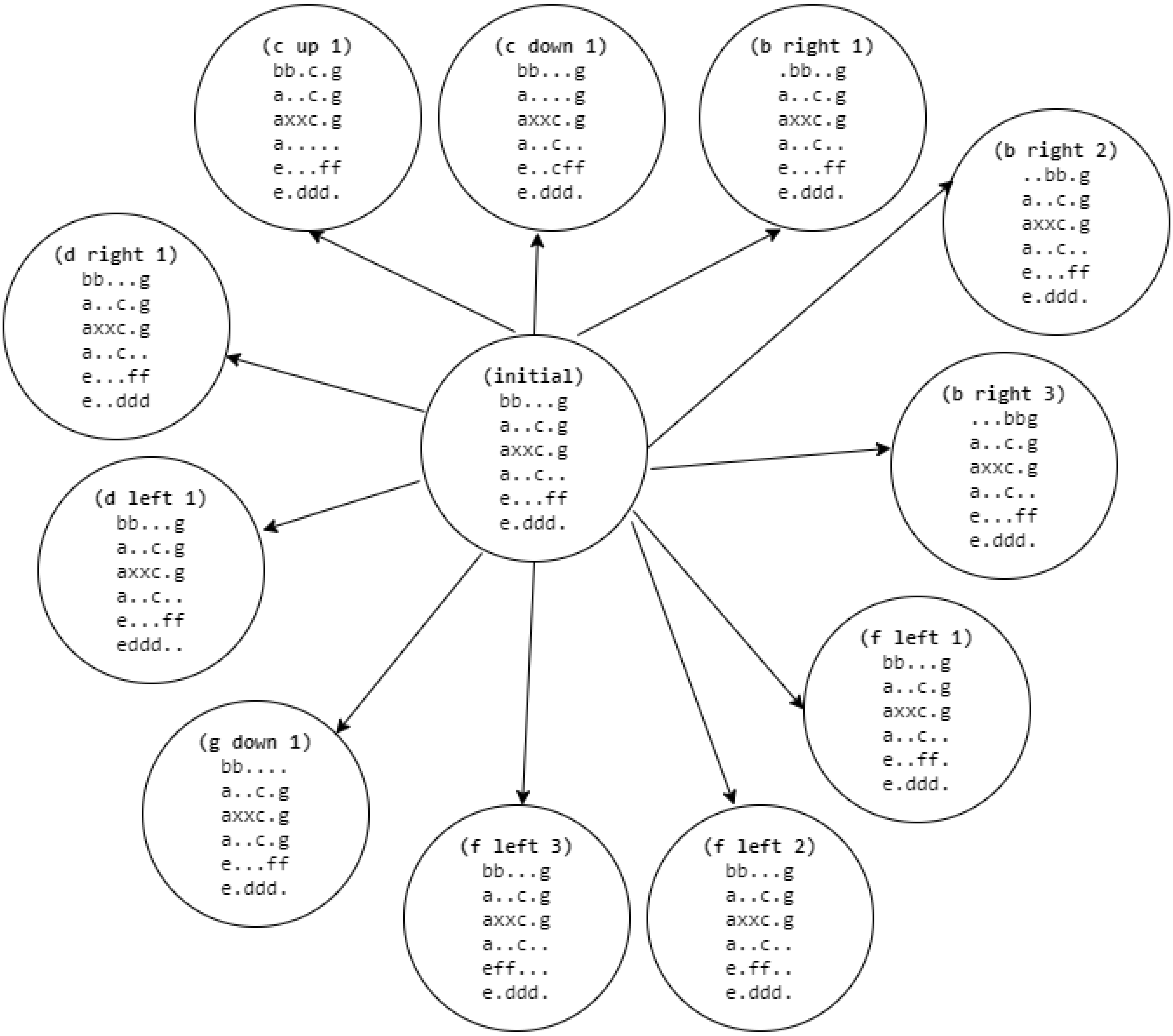
As an example, let's consider Jam 1.

```
bb...g
a..c.g
axxc.g
a..c..
e...ff
e.ddd.
```

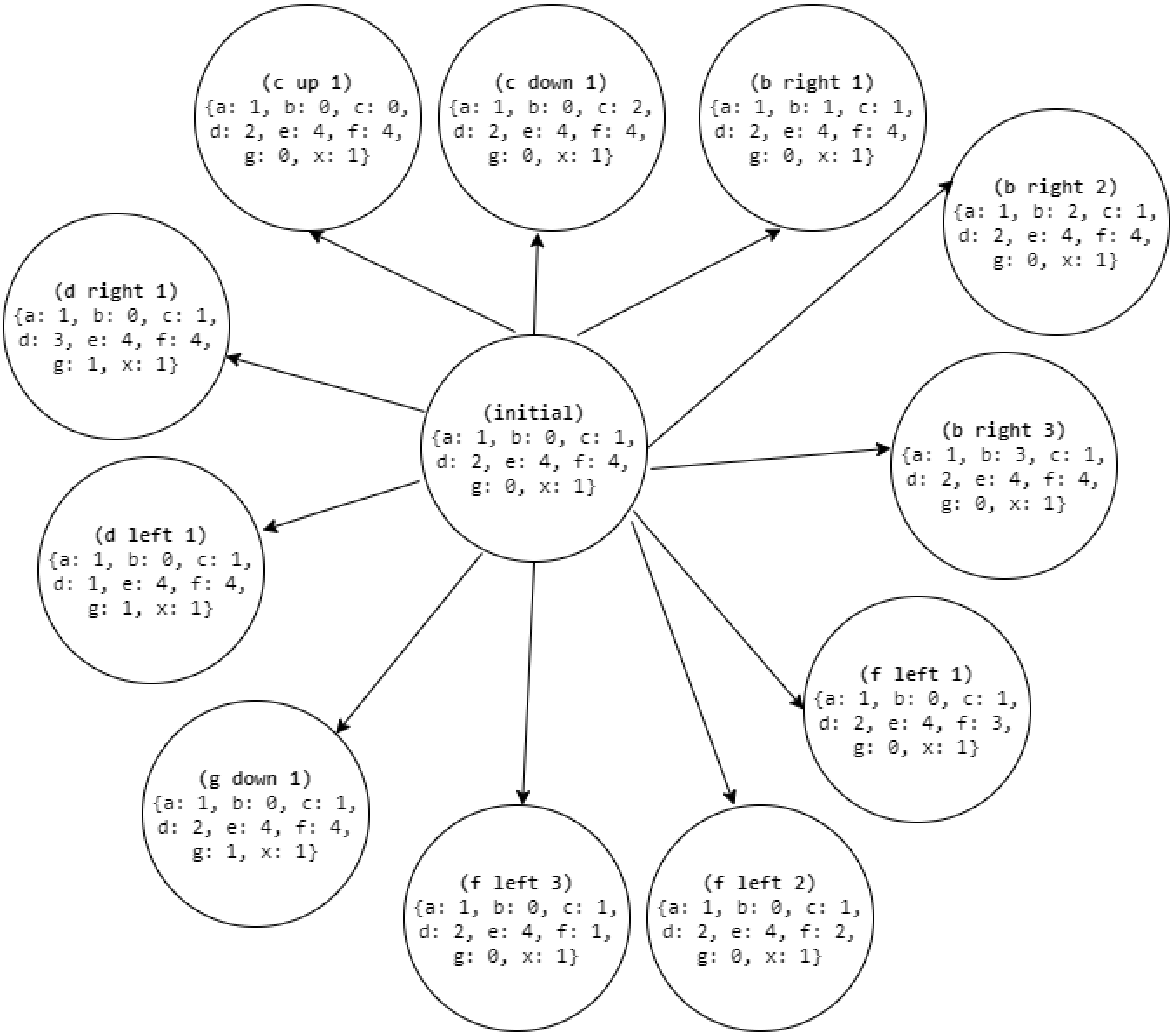
Here we have:

car	orientation	size	fixed position	variable position
a	vertical	3	column 0	row 1
b	horizontal	2	row 0	column 0
c	vertical	3	column 3	row 1
d	horizontal	3	row 5	column 2
e	vertical	2	column 0	row 4
f	horizontal	2	row 4	column 4
g	vertical	3	column 5	row 0
x	horizontal	2	row 2	column 1

The initial graph is shown below. The vertex is the initial board and the edges are the boards 1 move away from the initial board.



We can represent this more succinctly using the variable position like this:



Algorithm

Let's make a few modifications to the maze solver from above. Here is a full solution in 100 lines of Python code.

```
class Metadata:
    def __init__(self, grid):
        cl = set([t for r in grid for t in r if t != '.'])
        tl = dict((c, [(i, j) for i in range(6) for j in range(6)
                       if grid[i][j] == c]) for c in cl)
        v = dict((car, tl[car][0][0] != tl[car][1][0]) for car in cl)
        self.grid = grid
        self.cars = cl
        self.orientation = v
        self.size = dict((car, len(tl[car])) for car in cl)
        self.fixed_position = dict((car, tl[car][0][1]
                                    if v[car] else tl[car][0][0]) for car in cl)
        self.node_count = 0

def convert_to_grid(u, metadata):  # convert variable position to grid
```

```
    grid = [['.' for _ in range(6)] for _ in range(6)]
    for car in metadata.cars:
        orientation = metadata.orientation[car]
        size = metadata.size[car]
        fp = metadata.fixed_position[car]
        vp = u[car]
        if car == 'x' and vp + size > 6:
            size -= 1
        for d in range(size):
            grid[vp+d if orientation else fp][fp if orientation else vp+d] = car
    return grid

def get_neighbors(u, metadata):
    neighbors = []
    grid = convert_to_grid(u, metadata)
    for car in metadata.cars:
        orientation = metadata.orientation[car]
        size = metadata.size[car]
        fp = metadata.fixed_position[car]
        vp = u[car]
        for np in range(vp-1, -1, -1):
            if orientation and grid[np][fp] != '.' or \
                    not orientation and grid[fp][np] != '.':
                break
            nb = dict((k, v) if k != car else (k, np) for k, v in u.items())
            neighbors.append(nb)
        for np in range(vp+size, 7):
            if np < 6 and orientation and grid[np][fp] != '.' or \
                    np < 6 and not orientation and grid[fp][np] != '.' or \
                    np == 6 and car != 'x':
                break
            nb = dict((k, v) if k != car else (k, np-size+1)
                      for k, v in u.items())
            neighbors.append(nb)
    metadata.node_count += len(neighbors)
    return neighbors

def shortest_path(prev, src, target):
    path = []
    u = target
    uhash = hash(frozenset(u.items()))
    while uhash in prev:
        path.insert(0, u)
        u = prev[uhash]
        uhash = hash(frozenset(u.items()))
    path.insert(0, src)
    return path

def solve(src, metadata):  # no graph or target parameters
    dist = {}
    prev = {}
    dist[hash(frozenset(src.items()))] = 0
    q = [(0, src)]
    while q:
```

```
        q = sorted(q, key=lambda it: it[0])
        distu, u = q.pop(0)
        if u['x'] == 5:
            return shortest_path(prev, src, u)
        for v in get_neighbors(u, metadata):  # neighbors
            vhash = hash(frozenset(v.items()))
            alt = distu + 1
            if vhash not in dist or alt < dist[vhash]:
                dist[vhash] = alt
                prev[vhash] = u
                q.append((alt, v))

def print_solution(path, metadata, microseconds):
    print(f"\n{'='*10}\nsolved jam in {microseconds} microseconds\n{'='*10}")
    for i, u in enumerate(path):
        grid = convert_to_grid(u, metadata)
        print(f"\ni = {i}\n" + '\n'.join([''.join(row) for row in grid]))

from datetime import datetime
jam = 'bb...g\na..c.g\naxxc.g\na..c..\ne...ff\ne.ddd.'
grid = [list(line) for line in jam.splitlines()]
cl = set([t for r in grid for t in r if t != '.'])
tl = dict((c, [(i, j) for i in range(6) for j in range(6)
               if grid[i][j] == c]) for c in cl)
v = dict((car, tl[car][0][0] != tl[car][1][0]) for car in cl)
src = dict((car, tl[car][0][0] if v[car] else tl[car][0][1]) for car in cl)
metadata = Metadata(grid)
start = datetime.now()
path = solve(src, metadata)
end = datetime.now()
print_solution(path, metadata, (end - start).microseconds)
```

Let's compare the `solve` function for rush hour to the `solve` function for the maze:

```
def maze_solve(graph, src, target):
    dist = dict((k, float('inf')) for k in graph.keys())
    prev = dict((k, None) for k in graph.keys())
    dist[src] = 0
    pq = [(0, src)]
    while pq:
        distu, u = heapq.heappop(pq)
        if u == target:
            return shortest_path(prev, u)
        for v in graph[u]:  # neighbors
            alt = distu + 1
            if alt < dist[v]:
                dist[v] = alt
                prev[v] = u
                heapq.heappush(pq, (alt, v))

def rushhour_solve(src, metadata):
    dist = {}
```

```
    prev = {}
    dist[hash(frozenset(src.items()))] = 0
    q = [(0, src)]
    while q:
        q = sorted(q, key=lambda it: it[0])
        distu, u = q.pop(0)
        if u['x'] == 5:
            return shortest_path(prev, u)
        for v in get_neighbors(u, metadata):  # neighbors
            vhash = hash(frozenset(v.items()))
            alt = distu + 1
            if vhash not in dist or alt < dist[vhash]:
                dist[vhash] = alt
                prev[vhash] = u
                q.append((alt, v))
```

As you can see, the main solve logic (i.e. BFS with priority queue) is still in fact. A few comments:

- `target`: for rush hour the 'x' car should be on the right side of the board.
- `vhash`: we need to define a custom hash function for the variable position.
- `get_neighbors`: we need to determine all the possible boards one move away. that requires some logic.
- `metadata`: we cache information about the size, orientation and fixed position of cars as these don't change.

Solution

Jam 1: Initial

```
bb...g
a..c.g
axxc.g
a..c..
e...ff
e.ddd.
```

Jam 1: Move #1: `ff` goes left two squares.

```
bb...g
a..c.g
axxc.g
a..c..
eff...
e.ddd.
```

Jam 1: Move #2: `ggg` goes down two squares.

```
bb....
a..c..
axxc..
a..c.g
eff..g
e.dddg
```

Jam 1: Move #3: `bb` goes right 1 square.

```
.bb...
a..c..
axxc..
a..c.g
eff..g
e.dddg
```

Jam 1: Move #4: `aaa` goes up 1 square.

```
abb...
a..c..
axxc..
...c.g
eff..g
e.dddg
```

Jam 1: Move #5: `ee` goes up 1 square.

```
abb...
a..c..
axxc..
e..c.g
eff..g
..dddg
```

Jam 1: Move #6: `dd` goes left two squares.

```
abb...
a..c..
axxc..
e..c.g
eff..g
ddd..g
```

Jam 1: Move #7: `ccc` goes down two squares.

```
abb...
a.....
axx...
e..c.g
effc.g
dddc.g
```

Jam 1: Move #8: `x` goes right three squares.

```
abb...
a.....
a....x
e..c.g
effc.g
dddc.g
```

Appendix

Detailed Runtime Results

todo

Solutions

todo

Java code examples

todo