RushHour Solver

Team Member(s): Nathan Esau

Algorithm(s): Dijkstra's Algorithm

Languages(s): C++, Python, Java

Jam(s): http://www.mathsonline.org/game/jam.html

Results

The C++ code is slower for project cases because it uses `map` instead of `unordered_map`. Using `unordered_map` and implementing custom hash functions would lead to better performance.

Language	40 Jam Cases	35 Project Cases
C++	5.8 seconds	21.8 seconds
Python	10.45 seconds	9.5 seconds
Java	11.73 seconds	12.3 seconds

Explanation

Dijkstra's algorithm

It is a breadth-first search algorithm on an unweighted graph using a priority queue.

Case #1: Simple Maze

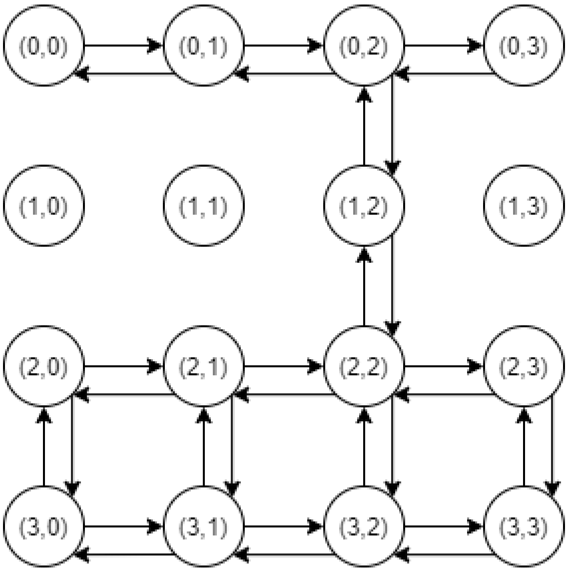
Suppose we have the following maze, where `W` represents a wall and `.` represents a tile you can walk on. We are trying to go from bottom left corner to top right corner.

```
. . . .
W W . W
. . . .
. . . .
```

One of the possible shortest paths is URRRURR:

```
. . _ _
W W | W
_ _ _ .
| . . .
```

Here is the graph for the maze:



Which can also be represented as:

```
graph = {
    (0, 0): {(0, 1), (1, 0)},
    (0, 1): {(0, 2), (0, 0), (1, 1)},
    (0, 2): {(1, 2), (0, 3), (0, 1)},
    (0, 3): {(1, 3), (0, 2)},
    (1, 0): set(),
    (1, 1): set(),
    (1, 2): {(1, 3), (1, 1), (0, 2), (2, 2)},
    (1, 3): set(),
    (2, 0): {(3, 0), (1, 0), (2, 1)},
    (2, 1): {(2, 0), (3, 1), (1, 1), (2, 2)},
    (2, 2): {(1, 2), (3, 2), (2, 3), (2, 1)},
    (2, 3): {(1, 3), (3, 3), (2, 2)},
    (3, 0): {(2, 0), (3, 1)},
    (3, 1): {(3, 0), (3, 2), (2, 1)},
    (3, 2): {(3, 1), (3, 3), (2, 2)},
    (3, 3): {(3, 2), (2, 3)}}
}
```

Which can be solved as follows:

```
# https://en.wikipedia.org/wiki/Dijkstra%27s_algorithm#Pseudocode
def shortest_path(prev, src, target):
    path = []
    u = target
    while u != src:
        path.insert(0, u)
        u = prev[u]
    path.insert(0, src)
    return path

def solve(graph, src, target):
```

```python
    dist = dict((k, float('inf')) for k in graph.keys())
    prev = dict((k, None) for k in graph.keys())
    dist[src] = 0
    pq = [(0, src)]
    while pq:
        distu, u = heapq.heappop(pq)
        if u == target:
            return shortest_path(prev, src, u)
        for v in graph[u]:  # neighbors
            alt = distu + 1
            if alt < dist[v]:
                dist[v] = alt
                prev[v] = u
                heapq.heappush(pq, (alt, v))

# [(3, 0), (2, 0), (2, 1), (2, 2), (1, 2), (0, 2), (0, 3)]
print(solve(graph, (3, 0), (0, 3)))
```

Case #2: RushHour

Graph Representation

First let's discuss the graph representation for the RushHour game.

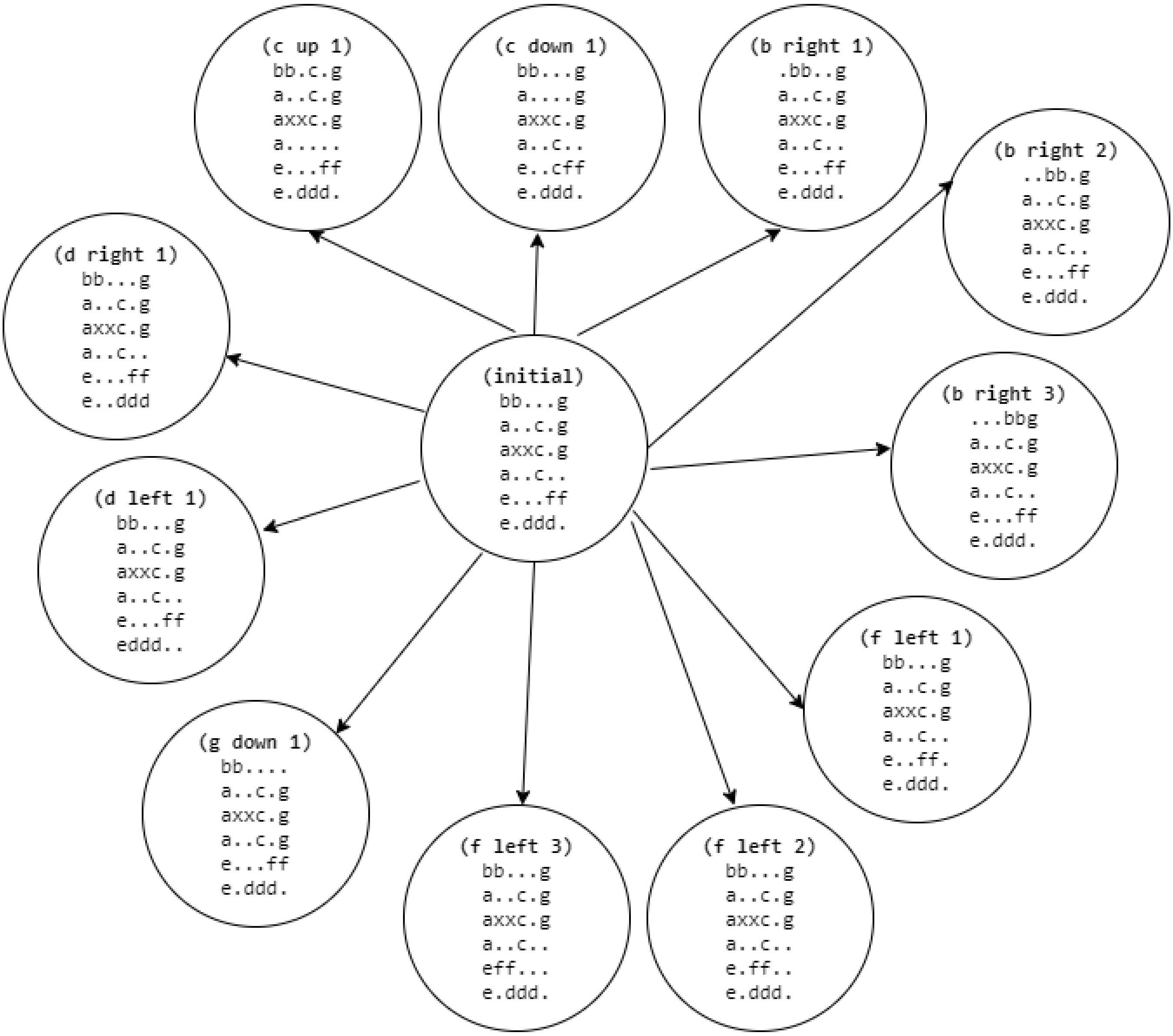
As an example, let's consider Jam 1.

```
bb...g
a..c.g
axxc.g
a..c..
e...ff
e.ddd.
```

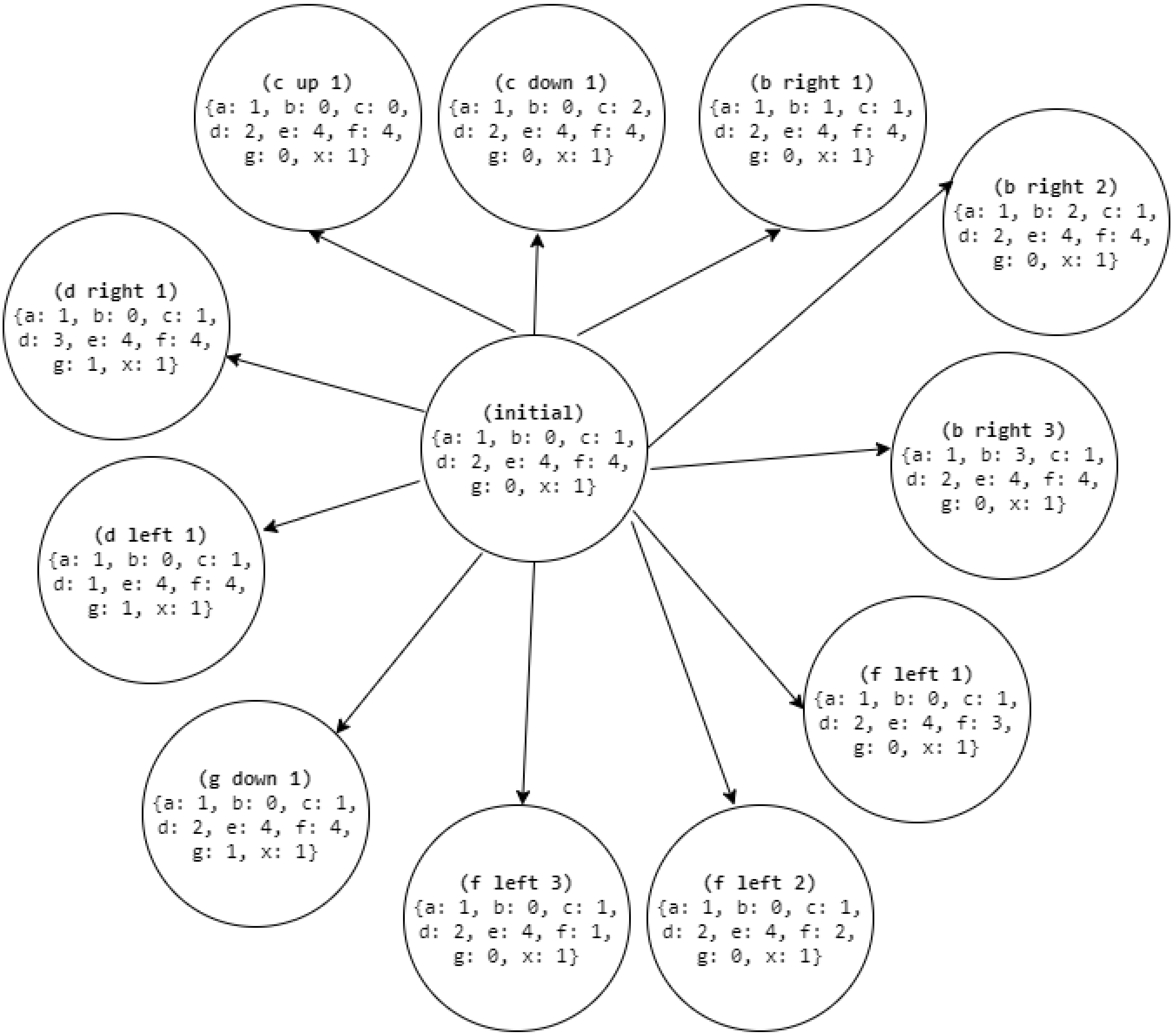
Here we have:

car	orientation	size	fixed position	variable position
a	vertical	3	column 0	row 1
b	horizontal	2	row 0	column 0
c	vertical	3	column 3	row 1
d	horizontal	3	row 5	column 2
e	vertical	2	column 0	row 4
f	horizontal	2	row 4	column 4
g	vertical	3	column 5	row 0
x	horizontal	2	row 2	column 1

The initial graph is shown below. The vertex is the initial board and the edges are the boards 1 move away from the initial board.



We can represent this more succinctly using the variable position like this:



Algorithm

Let's make a few modifications to the maze solver from above. Here is a full solution in 100 lines of Python code.

```
class Metadata:
    def __init__(self, grid):
        cl = set([t for r in grid for t in r if t != '.'])
        tl = dict((c, [(i, j) for i in range(6) for j in range(6)
                       if grid[i][j] == c]) for c in cl)
        v = dict((car, tl[car][0][0] != tl[car][1][0]) for car in cl)
        self.grid = grid
        self.cars = cl
        self.orientation = v
        self.size = dict((car, len(tl[car])) for car in cl)
        self.fixed_position = dict((car, tl[car][0][1]
                                    if v[car] else tl[car][0][0]) for car in cl)
        self.node_count = 0

def convert_to_grid(u, metadata):  # convert variable position to grid
```

```
    grid = [['.' for _ in range(6)] for _ in range(6)]
    for car in metadata.cars:
        orientation = metadata.orientation[car]
        size = metadata.size[car]
        fp = metadata.fixed_position[car]
        vp = u[car]
        if car == 'x' and vp + size > 6:
            size -= 1
        for d in range(size):
            grid[vp+d if orientation else fp][fp if orientation else vp+d] = car
    return grid

def get_neighbors(u, metadata):
    neighbors = []
    grid = convert_to_grid(u, metadata)
    for car in metadata.cars:
        orientation = metadata.orientation[car]
        size = metadata.size[car]
        fp = metadata.fixed_position[car]
        vp = u[car]
        for np in range(vp-1, -1, -1):
            if orientation and grid[np][fp] != '.' or \
                    not orientation and grid[fp][np] != '.':
                break
            nb = dict((k, v) if k != car else (k, np) for k, v in u.items())
            neighbors.append(nb)
        for np in range(vp+size, 7):
            if np < 6 and orientation and grid[np][fp] != '.' or \
                    np < 6 and not orientation and grid[fp][np] != '.' or \
                    np == 6 and car != 'x':
                break
            nb = dict((k, v) if k != car else (k, np-size+1)
                      for k, v in u.items())
            neighbors.append(nb)
    metadata.node_count += len(neighbors)
    return neighbors

def shortest_path(prev, src, target):
    path = []
    u = target
    uhash = hash(frozenset(u.items()))
    while uhash in prev:
        path.insert(0, u)
        u = prev[uhash]
        uhash = hash(frozenset(u.items()))
    path.insert(0, src)
    return path

def solve(src, metadata):  # no graph or target parameters
    dist = {}
    prev = {}
    dist[hash(frozenset(src.items()))] = 0
    q = [(0, src)]
    while q:
```

```
        q = sorted(q, key=lambda it: it[0])
        distu, u = q.pop(0)
        if u['x'] == 5:
            return shortest_path(prev, src, u)
        for v in get_neighbors(u, metadata):  # neighbors
            vhash = hash(frozenset(v.items()))
            alt = distu + 1
            if vhash not in dist or alt < dist[vhash]:
                dist[vhash] = alt
                prev[vhash] = u
                q.append((alt, v))

def print_solution(path, metadata, microseconds):
    print(f"\n{'='*10}\nsolved jam in {microseconds} microseconds\n{'='*10}")
    for i, u in enumerate(path):
        grid = convert_to_grid(u, metadata)
        print(f"\ni = {i}\n" + '\n'.join([''.join(row) for row in grid]))

from datetime import datetime
jam = 'bb...g\na..c.g\naxxc.g\na..c..\ne...ff\ne.ddd.'
grid = [list(line) for line in jam.splitlines()]
cl = set([t for r in grid for t in r if t != '.'])
tl = dict((c, [(i, j) for i in range(6) for j in range(6)
               if grid[i][j] == c]) for c in cl)
v = dict((car, tl[car][0][0] != tl[car][1][0]) for car in cl)
src = dict((car, tl[car][0][0] if v[car] else tl[car][0][1]) for car in cl)
metadata = Metadata(grid)
start = datetime.now()
path = solve(src, metadata)
end = datetime.now()
print_solution(path, metadata, (end - start).microseconds)
```

Let's compare the `solve` function for rush hour to the `solve` function for the maze:

```
def maze_solve(graph, src, target):
    dist = dict((k, float('inf')) for k in graph.keys())
    prev = dict((k, None) for k in graph.keys())
    dist[src] = 0
    pq = [(0, src)]
    while pq:
        distu, u = heapq.heappop(pq)
        if u == target:
            return shortest_path(prev, src, u)
        for v in graph[u]:  # neighbors
            alt = distu + 1
            if alt < dist[v]:
                dist[v] = alt
                prev[v] = u
                heapq.heappush(pq, (alt, v))

def rushhour_solve(src, metadata):
    dist = {}
```

```
    prev = {}
    dist[hash(frozenset(src.items()))] = 0
    q = [(0, src)]
    while q:
        q = sorted(q, key=lambda it: it[0])
        distu, u = q.pop(0)
        if u['x'] == 5:
            return shortest_path(prev, u)
        for v in get_neighbors(u, metadata):  # neighbors
            vhash = hash(frozenset(v.items()))
            alt = distu + 1
            if vhash not in dist or alt < dist[vhash]:
                dist[vhash] = alt
                prev[vhash] = u
                q.append((alt, v))
```

As you can see, the main solve logic (i.e. BFS with priority queue) is still in fact. A few comments:

- `target`: for rush hour the 'x' car should be on the right side of the board.
- `vhash`: we need to define a custom hash function for the variable position.
- `get_neighbors`: we need to determine all the possible boards one move away. that requires some logic.
- `metadata`: we cache information about the size, orientation and fixed position of cars as these don't change.

Solution

For Jam 1, the final node count is 11586 using Dijkstra's Algorithm.

Jam 1: Initial

```
bb...g
a..c.g
axxc.g
a..c..
e...ff
e.ddd.
```

Jam 1: Move #1: `ff` goes left two squares.

```
bb...g
a..c.g
axxc.g
a..c..
eff...
e.ddd.
```

Jam 1: Move #2: `ggg` goes down two squares.

```
bb....
a..c..
axxc..
a..c.g
eff..g
e.dddg
```

Jam 1: Move #3: `bb` goes right 1 square.

```
.bb...
a..c..
axxc..
a..c.g
eff..g
e.dddg
```

Jam 1: Move #4: `aaa` goes up 1 square.

```
abb...
a..c..
axxc..
...c.g
eff..g
e.dddg
```

Jam 1: Move #5: `ee` goes up 1 square.

```
abb...
a..c..
axxc..
e..c.g
eff..g
..dddg
```

Jam 1: Move #6: `dd` goes left two squares.

```
abb...
a..c..
axxc..
e..c.g
eff..g
ddd..g
```

```
```

Jam 1: Move #7: `ccc` goes down two squares.

```
abb...
a.....
axx...
e..c.g
effc.g
dddc.g
```

Jam 1: Move #8: `x` goes right three squares.

```
abb...
a.....
a....x
e..c.g
effc.g
dddc.g
```

Appendix

Java code examples

Maze solver:

```
// https://en.wikipedia.org/wiki/Dijkstra%27s_algorithm#Pseudocode
public class Solver {

    static List<Integer[]> shortest_path(Map<Integer[], Integer[]> prev,
                Integer[] src, Integer[] target) {
        List<Integer[]> path = new ArrayList<>();
        Integer[] u = target;
        while (!u.equals(src)) {
            path.add(0, u);
            u = prev.get(u);
        }
        path.add(0, src);
        return path;
    }

    static List<Integer[]> solve(Map<Integer[], Set<Integer[]>> graph,
                Integer[] src, Integer[] target) {

        Map<Integer[], Integer> dist = new HashMap<Integer[], Integer>();
        Map<Integer[], Integer[]> prev = new HashMap<Integer[], Integer[]>();

        for (Entry<Integer[], Set<Integer[]>> entry : graph.entrySet()) {
```

```
            dist.put(entry.getKey(), Integer.MAX_VALUE);
            prev.put(entry.getKey(), null);
        }

        List<Map.Entry<Integer, Integer[]>> q = new ArrayList<>();
        q.add(new AbstractMap.SimpleEntry<>(0, src));

        while (!q.isEmpty()) {
            Collections.sort(q, new Comparator<Map.Entry<Integer, Integer[]>>() {
                @Override
                public int compare(Map.Entry<Integer, Integer[]> a1,
                        Map.Entry<Integer, Integer[]> a2) {
                    return a1.getKey().compareTo(a2.getKey());
                }
            });

            Entry<Integer, Integer[]> entry = q.remove(0);
            Integer distu = entry.getKey();
            Integer[] u = entry.getValue();

            if (u.equals(target)) {
                return shortest_path(prev, src, u);
            }

            for (Integer[] v : graph.get(u)) {
                Integer alt = distu + 1;
                if (alt < dist.get(v)) {
                    dist.put(v, alt);
                    prev.put(v, u);
                    q.add(new AbstractMap.SimpleEntry<>(alt, v));
                }
            }
        }

        return Arrays.asList();
    }

    public static void main(String[] args) {
        Integer[] e00 = new Integer[]{0, 0};
        Integer[] e01 = new Integer[]{0, 1};
        Integer[] e02 = new Integer[]{0, 2};
        Integer[] e03 = new Integer[]{0, 3};
        Integer[] e10 = new Integer[]{1, 0};
        Integer[] e11 = new Integer[]{1, 1};
        Integer[] e12 = new Integer[]{1, 2};
        Integer[] e13 = new Integer[]{1, 3};
        Integer[] e20 = new Integer[]{2, 0};
        Integer[] e21 = new Integer[]{2, 1};
        Integer[] e22 = new Integer[]{2, 2};
        Integer[] e23 = new Integer[]{2, 3};
        Integer[] e30 = new Integer[]{3, 0};
        Integer[] e31 = new Integer[]{3, 1};
        Integer[] e32 = new Integer[]{3, 2};
        Integer[] e33 = new Integer[]{3, 3};
```

```
        Map<Integer[], Set<Integer[]>> graph = Map.ofEntries(
            Map.entry(e00, new HashSet<>(Arrays.asList(e01, e10))),
            Map.entry(e01, new HashSet<>(Arrays.asList(e02, e00, e11))),
            Map.entry(e02, new HashSet<>(Arrays.asList(e03, e13, e02))),
            Map.entry(e03, new HashSet<>(Arrays.asList(e13, e02))),
            Map.entry(e10, new HashSet<>()),
            Map.entry(e11, new HashSet<>()),
            Map.entry(e12, new HashSet<>(Arrays.asList(e13, e11, e02, e22))),
            Map.entry(e13, new HashSet<>()),
            Map.entry(e20, new HashSet<>(Arrays.asList(e30, e10, e21))),
            Map.entry(e21, new HashSet<>(Arrays.asList(e20, e31, e11, e22))),
            Map.entry(e22, new HashSet<>(Arrays.asList(e12, e32, e23, e21))),
            Map.entry(e23, new HashSet<>(Arrays.asList(e13, e33, e22))),
            Map.entry(e30, new HashSet<>(Arrays.asList(e20, e31))),
            Map.entry(e31, new HashSet<>(Arrays.asList(e30, e32, e21))),
            Map.entry(e32, new HashSet<>(Arrays.asList(e31, e33, e22))),
            Map.entry(e33, new HashSet<>(Arrays.asList(e32, e23)))
        );

        // [(3, 0), (3, 1), (2, 1), (2, 2), (1, 2), (0, 2), (0, 3)]
        for (Integer[] entry : solve(graph, e30, e03)) {
            System.out.println(Arrays.toString(entry));
        }
    }
```

RushHour solver:

```
public class Solver {

    static Character[][] convertToGrid(Map<Character, Integer> u,
            Metadata metadata) {

        Character[][] grid = new Character[6][6];

        for (int i = 0; i < 6; i++) {
            for (int j = 0; j < 6; j++) {
                grid[i][j] = '.';
            }
        }

        for (Character car : metadata.cars) {
            Boolean orientation = metadata.orientation.get(car);
            Integer size = metadata.size.get(car);
            Integer fp = metadata.fixedPosition.get(car);
            Integer vp = u.get(car);

            if (car.equals('x') && vp + size > 6) {
                size -= 1;
            }

            for (int d = 0; d < size; d++) {
```

```
                if (orientation) {
                    grid[vp + d][fp] = car;
                } else {
                    grid[fp][vp + d] = car;
                }
            }
        }

        return grid;
    }

    static List<Map<Character, Integer>> getNeighbors(Map<Character, Integer> u,
            Metadata metadata) {

        var neighbors = new ArrayList<Map<Character, Integer>>();
        Character[][] grid = convertToGrid(u, metadata);

        for (Character car : metadata.cars) {
            Boolean orientation = metadata.orientation.get(car);
            Integer size = metadata.size.get(car);
            Integer fp = metadata.fixedPosition.get(car);
            Integer vp = u.get(car);

            for (int np = vp - 1; np >= 0; np -= 1) {
                if ((orientation && grid[np][fp] != '.')
                        || (!orientation && grid[fp][np] != '.')) {
                    break;
                }

                var neighbor = new HashMap<Character, Integer>();
                for (Entry<Character, Integer> entry : u.entrySet()) {
                    if (entry.getKey().equals(car)) {
                        neighbor.put(entry.getKey(), np);
                    } else {
                        neighbor.put(entry.getKey(), entry.getValue());
                    }
                }

                neighbors.add(neighbor);
            }

            for (int np = vp + size; np <= 6; np++) {
                if ((np < 6 && orientation && grid[np][fp] != '.')
                        || (np < 6 && !orientation && grid[fp][np] != '.')
                        || (np == 6 && !car.equals('x'))) {
                    break;
                }

                var neighbor = new HashMap<Character, Integer>();
                for (Entry<Character, Integer> entry : u.entrySet()) {
                    if (entry.getKey().equals(car)) {
                        neighbor.put(entry.getKey(), np - size + 1);
                    } else {
                        neighbor.put(entry.getKey(), entry.getValue());
```

```
                    }
                }

                neighbors.add(neighbor);
            }
        }

        metadata.node_count += neighbors.size();
        return neighbors;
    }

    static List<Map<Character, Integer>> shortest_path(
            Map<Map<Character, Integer>,
            Map<Character, Integer>> prev,
            Map<Character, Integer> src,
            Map<Character, Integer> target) {

        var path = new ArrayList<Map<Character, Integer>>();
        Map<Character, Integer> u = target;
        while (prev.containsKey(u)) {
            path.add(0, u);
            u = prev.get(u);
        }

        path.add(0, src);
        return path;
    }

    static List<Map<Character, Integer>> solve(Map<Character, Integer> src,
            Metadata metadata) {

        var dist = new HashMap<Map<Character, Integer>, Integer>();
        var prv = new HashMap<Map<Character, Integer>, Map<Character, Integer>>();
        dist.put(src, 0);

        List<Map.Entry<Integer, Map<Character, Integer>>> q = new ArrayList<>();
        q.add(new AbstractMap.SimpleEntry<>(0, src));

        while (!q.isEmpty()) {
            Collections.sort(q, new Comparator<
                    Map.Entry<Integer, Map<Character, Integer>>>() {
                @Override
                public int compare(Map.Entry<Integer, Map<Character, Integer>> a1,
                        Map.Entry<Integer, Map<Character, Integer>> a2) {
                    return a1.getKey().compareTo(a1.getKey());
                }
            });

            Map.Entry<Integer, Map<Character, Integer>> item = q.remove(0);

            Integer distu = item.getKey();
            Map<Character, Integer> u = item.getValue();

            if (u.get('x') == 5) {
```

```
                return shortest_path(prv, src, u);
            }

            for (var v : getNeighbors(u, metadata)) {
                Integer alt = distu + 1;
                if (!dist.containsKey(v) || alt < dist.get(v)) {
                    dist.put(v, alt);
                    prv.put(v, u);
                    q.add(new AbstractMap.SimpleEntry<>(alt, v));
                }
            }
        }

        return Arrays.asList();
    }
}
```

Converting path to solution format:

```
static String convertPathToInstruction(List<Map<Character, Integer>> path,
        Metadata metadata) {

        String instruction = "";

        for (int i = 1; i < path.size(); i++) {
            var currEntry = path.get(i);
            var prevEntry = path.get(i-1);
            for (Character car : currEntry.keySet()) {
                if (!currEntry.get(car).equals(prevEntry.get(car))) {
                    Integer d = currEntry.get(car) - prevEntry.get(car);
                    Boolean v = metadata.orientation.get(car);
                    String direction =
                        (v && d > 0) ? "D" :
                        (v && d < 0) ? "U" :
                        (d > 0) ? "R" :
                        "L";
                    instruction += String.format("%s%s%d\n", car, direction,
                        (car.equals(metadata.x)) ? Math.abs(d) - 1 : Math.abs(d)
                    );
                }
            }
        }

        return instruction;
    }
```

Solutions

35 Project Cases

- A01: [AR1, PU1, BU1, RL2, CL3, QD2, OD3, XR3]
- A02: [EL1, PD2, XR0, AD1, OL3, BU1, CU2, XR3]
- A03: [PU3, CR2, OU2, AR3, OD3, XR0, BU4, XL0]
- A04: [OD3, XL0, AU3, XR0, OU3, QL3, RL2, PD3, XR3]
- A05: [AR1, PU1, RL1, EL3, FL3, QD2, GD1, OD3, XR3]
- A06: [XL0, EU3, DR1, FU1, RL3, PD2, QD1, OD2, XR4]
- A07: [CD3, FD1, DD1, BR2, EU1, XR0, AD3, XL0, ED1, BL3, DU1, EU1, XR3]
- A08: [AL3, BL2, CU1, DU2, EU2, GL1, XR2, IU2, PL1, QL1, OD3, XR1]
- A09: [XR0, PU3, QL1, DD2, EL1, XR0, AD1, BL1, CL1, FU2, OD1, XR2]
- A10: [CL1, XR1, OU1, QR2, BD4, AR1, DR1, PU2, QL3, XL1, EU2, FL1, HL1, OD3, CR1, EU2, XR3]
- B11: [OD3, XL0, BU2, QL1, EU3, QR1, RR1, BD3, XR0, OU3, QL3, PD1, AR3, PU1, QR3, OD3, XL0, BU4, XR0, OU3, QL3, RL3, PD3, ED2, XR3]
- B12: [BR2, PU1, QL2, CU3, QR2, RR3, PD3, BL1, CU1, XR2, AD1, BL2, PU3, QL1, RL1, OD3, XR1]
- B13: [DD2, ED1, OU1, FR1, XL2, DU2, GU3, FL2, HL2, CD3, KL1, OD3, BR2, DU1, GU1, XR4]
- B14: [DU1, EU1, GU2, HL2, IU1, KR3, ID1, HR2, DD3, ED3, XL1, HL2, BD2, CL3, BU2, FU2, XR4]
- B15: [AL1, BR1, IR1, HR1, QD1, RD1, XL1, EU1, FU1, GL2, OD1, PD1, DR2, EU2, FU2, XR1, QU1, RU1, HL2, IL2, OD1, PD1, XR2]
- B16: [PD1, DD2, EL2, FD1, XL2, CD1, BR1, PU3, GR2, FD1, QL2, CD3, QR2, FU1, GL2, PD3, XR2, PU3, QL1, OD3, XR1]
- B17: [BL1, PU2, FU2, GU2, QR3, RR3, DD2, ER1, XR0, AD4, BL1, EL1, XL0, OR2, DU4, ER1, XR0, AU2, QL3, RL3, PD2, FD1, GD1, XR3]
- B18: [QR2, RR3, PD1, DR3, XL0, BD4, AR1, CR1, XR0, PU3, QL3, OD2, AR3, CR3, OU2, QR3, PD3, XL0, BU4, XR0, PU3, QL3, RL3, OD3, XR3]
- B19: [DU2, EL2, XL1, AD1, BL1, JU1, FU1, OR2, AD3, ER2, XR1, DD4, BL2, EL2, XL1, AU4, ER2, XR1, DU2, OL2, FD1, XR2]
- B20: [CD1, BR2, DU2, EU2, QL1, PD1, FL3, CD1, ED1, XR4]
- C21: [PD2, QR2, XL0, BD4, AR1, XR0, PU3, QL3, OD1, AR3, OU1, QR3, PD3, XL0, BU4, XR0, PU3, QL3, RL3, OD3, XR3]
- C22: [BU1, FU1, GR1, XL0, AD3, XR0, BD1, OL3, PU1, EL1, HU2, QR2, AD1, DD1, EL2, GR1, PD2, CL3, PU2, HU2, ER3, AU1, DU1, QL3, PD3, XR3]
- C23: [QL2, DD1, EL1, PD1, OR1, AU1, XL2, AD1, OL1, PU1, ER1, DU1, QR3, CD1, AD1, BL3, AU1, DU2, EL4, PD1, OR1, AU1, CU2, QL1, FL4, PD2, CD1, DD2, XR4]
- C24: No solution
- C25: [CL1, EU1, OU1, PD1, QR2, XL0, BD4, AR1, DR1, XR0, PU3, QL3, GU3, IL1, QR3, PD3, AL1, DL1, XL0, BU4, HL2, QL2, ED2, OD3, CR1, GU1, XR4]
- C26: [DU1, ED1, RL1, GU1, HR1, CD3, XR0, AD1, OL3, PU1, RR2, AD3, RL2, DU1, XL0, CU4, XR0, HL1, GD1, RR3, AU3, EU1, FU1, HL3, FD1, RL2, PD3, XR2]
- C27: [PU2, ER1, OD2, BR2, CR2, DU2, XR0, AD4, XL0, DD2, BL3, CL3, DU1, OU2, EL4, DD1, OD2, CR3, DU2, ER1, XR0, AU3, EL1, FU1, RL3, PD3, OD1, XR3]
- C28: [BU1, ER1, GU3, EL1, FR2, HU1, RR3, CD1, DL1, AD3, DR1, XR0, OR1, CU4, DL1, XL0, AU3, RL3, FL2, HD1, ER1, GD3, EL2, PD3, OR2, AU1, XR2, AD1, OL1, BU1, XR1]
- C29: [QU1, DR1, CR1, FR2, ER1, OD3, XL0, AD1, PL1, QU2, DR1, BD3, DL1, QD1, PR1, AU1, XR2, OU3, CL1, EL1, AD4, CR1, XL1, OD1, PL3, QU1, DR1, BU4, DL1, FL1, QD3, XR3
- D30: [AR1, DU2, XL0, QU2, FR3, QD2, XR0, DD4, AL1, XL0, QU3, EL3, QD3, BD1, CL1, PU1, FR1, BD1, XR2, QU3, ER1, DU3, EL1, QD3, AR1, DU1, XL2, QU2, BU1, FL4, QD2, BD1, XR2, QU1, RL1, PD2, XR1]

- D31: [BD1, CL1, PU3, FR1, KU1, HR4, DD1, EL1, KD1, FL1, PD1, CR1, BU1, OD3, XR2, OU3, ER1, DU3, EL1, OD3, AR1, DU1, XL2, BD1, CL1, PU1, FR1, KU1, OU1, HL4, KD1, FL1, PD3, CR1, BU1, OD1, XR4]
- D32: [BL1, PU3, ER1, GU3, QR1, ID1, DL1, EL2, HU3, QR2, ER2, FR3, RD3, BL1, GD2, HU1, XR2, AD1, BL2, RU3, DR1, IU3, DL1, RD3, BR2, AU1, IU1, XL2, RU2, GU1, FL4, RD2, GD1, XR2, RU1, QL3, GD1, EL1, PD3, XR1]
- D33: [RL1, PU1, CU1, XR0, HR1, DD1, QR3, AD3, RL2, BU1, XL0, EU3, IR1, AD1, QL3, PD1, DU2, FL3, DD2, QR1, BD1, RR3, EU1, XR0, AU4, FL1, XL0, ED1, RL1, PU1, QR2, IL1, ED3, QL3, CD2, DU1, XR0, AD1, RL2, BU1, HL1, PD3, XR3]
- D34: [PD1, AR1, BU1, FU3, GR1, KD1, QL1, EU2, DR3, ED2, BD1, AL1, PU1, QR3, KU1, GL1, OD3, XR0, KU3, XL0, OU3, QL3, PD1, AR1, BU1, EU2, DL4, ED2, BD1, AL1, PU1, QR3, OD3, AL2, BU1, FU1, XR2, KD1, AL1, OU3, QL1, PD3, XR1]
- D35: [CR1, DD1, RR1, XR0, OD1, PL1, AL1, QU1, RR2, EU3, RL1, QD1, AR1, PR1, OU1, RL2, DU1, GR4, DD1, RR1, OD1, PL1, AL1, QU1, RR2, BD3, ED3, RL1, QD1, AR1, PR1, CL2, XL1, OU1, RL2, DU3, RR1, FL1, GL1, QD2, OD1, PL1, DU1, XR3]