Quiz 4 Name: ________________________________

Find the general solution to the following differential equation:

$$y'' + 2y' + 6y = 6t^2 + 4t$$

Solution: The general solution to the homogeneous version of the equation is

$$c_1 e^{-t} \cos(\sqrt{5}t) + c_2 e^{-t} \sin(\sqrt{5}t).$$

The particular solution will have the form

$$Y = At^2 + Bt + C.$$

After computing Y' and Y'' and substituting into the differential equation, we get

$$2A + 4At + 2B + 6At^2 + 6Bt + 6C = 6t^2 + 4t.$$

Grouping the t^2, t, and constant terms, we get a system of three equations:

$$\begin{aligned} 6A \qquad\qquad &= 6 \\ 4A + 6B \qquad &= 4 \\ 2A + 2B + 6C &= 0 \end{aligned}$$

So, $A = 1$, $B = 0$, and $C = -1/3$. The final general solution is

$$c_1 e^{-t} \cos(\sqrt{5}t) + c_2 e^{-t} \sin(\sqrt{5}t) + t^2 - 1/3.$$