On problems involving bases, there will be more than one correct response.

1. (a) and (b) are the same subspace. Two possible bases are

$$\left\{\begin{bmatrix}1\\-2\\-3\\0\end{bmatrix},\begin{bmatrix}2\\-4\\6\\1\end{bmatrix},\begin{bmatrix}0\\0\\9\\1\end{bmatrix}\right\}\text{ and }\left\{\begin{bmatrix}1\\-2\\0\\0\end{bmatrix},\begin{bmatrix}0\\0\\1\\0\end{bmatrix},\begin{bmatrix}0\\0\\0\\1\end{bmatrix}\right\}$$

(c) $\left\{\begin{bmatrix}4\\1\\0\\0\\0\end{bmatrix},\begin{bmatrix}-3\\0\\2\\-3\\1\end{bmatrix}\right\}$. (d) A has rank 3 and nullity 2. B has rank 3 and nullity 1.

2. (a) $\left\{\begin{bmatrix}1\\1\\1\\0\end{bmatrix},\begin{bmatrix}-1\\0\\1\\0\end{bmatrix}\right\}$ (b) $\left\{\begin{bmatrix}1/\sqrt{3}\\1/\sqrt{3}\\1/\sqrt{3}\\0\end{bmatrix},\begin{bmatrix}-1\sqrt{2}\\0\\1/\sqrt{2}\\0\end{bmatrix}\right\}$

3. (a) $\begin{bmatrix}1\\5\end{bmatrix}$ (b) 3,4,5 or 6 (c) 4×5

   (d) $\vec{\mathbf{v}}_4$ together with any one of the other three.

4. (a) Suppose that $\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}$ and $\begin{bmatrix}y_1\\y_2\\y_3\\y_4\end{bmatrix}$ are in W. Then the sum is $\begin{bmatrix}x_1+y_1\\x_2+y_2\\x_3+y_3\\x_4+y_4\end{bmatrix}$. Since $x_2+x_3+x_4=0$ and $y_2+y_3+y_4=0$, then $(x_2+y_2)+(x_3+y_3)+(x_4+y_4)=0$. So the sum is in W, which means that W is closed under addition.

   (b) Since W is defined by a homogeneous system of equations, it is the null space of $\begin{bmatrix}0 & 1 & 1 & 2\end{bmatrix}$. A basis is $\{[1\ 0\ 0\ 0]^T,[0\ -1\ 1\ 0]^T,[0\ -1\ 0\ 1]^T\}$.

5. $A=\begin{bmatrix}1&-1\\1&1\\1&2\\1&3\end{bmatrix}$ and $\vec{\mathbf{b}}=\begin{bmatrix}0\\3\\6\\10\end{bmatrix}$. $A^TA=\begin{bmatrix}4&5\\5&15\end{bmatrix}$ and $A^T\vec{\mathbf{b}}=\begin{bmatrix}19\\45\end{bmatrix}$. The solution to the system of equations is $a=12/7$ and $b=17/7$, so the equation of the line is $y=\frac{12}{7}+\frac{17}{7}y$.

6. (a) $A\vec{\mathbf{v}}_1=\vec{\mathbf{0}}$ and $A\vec{\mathbf{v}}_2=\vec{\mathbf{0}}$.

   (b) By row reducing A, we see that the dimension of $\mathcal{N}(A)$ is 2. Since $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$ are are linearly independent, and any two linearly independent vectors in a 2-dimensional space form a basis, this set is a basis for $\mathcal{N}(A)$.