

1. Divide both sides by $2t$:

$$y' + \frac{1}{2t}y = 3t^{-1/2} + \frac{1}{2}t - \frac{1}{2}$$

Find the Integrating Factor:

$$\mu(t) = \exp \int \frac{1}{2t} dt = t^{1/2}$$

Multiply both sides by the integrating factor:

$$t^{1/2}y = 3 + \frac{1}{2}t^{3/2} - \frac{1}{2}t^{1/2}$$

Integrate both sides:

$$t^{1/2}y = \int 3 + \frac{1}{2}t^{3/2} - \frac{1}{2}t^{1/2} dt = 3t + \frac{1}{5}t^{5/2} - \frac{1}{3}t^{3/2} + c$$

Simplify to get the General Solution:

$$y = 3t^{1/2} + \frac{1}{5}t^2 - \frac{1}{3}t + ct^{-1/2}$$

The limit as $t \rightarrow \infty$ is ∞ no matter what the constant is, because t^2 grows faster than all the other terms, and it goes to ∞ .

2. Factor:

$$\frac{dy}{dx} = (2+x)y^2$$

Separate:

$$\frac{dy}{y^2} = (2+x)dx$$

Integrate:

$$-\frac{1}{y} = 2x + \frac{1}{2}x^2 + c$$

Solve for y :

$$y = -\frac{1}{2x + x^2/2 + c}$$

Use the initial condition:

$$1 = -\frac{1}{c}$$

So, $c = -1$, which gives our solution:

$$y = -\frac{1}{2x + x^2/2 - 1}$$

3. Let $Q(t)$ denote the quantity of dye in the tank. The volume of water in the tank increases at a rate of 1 L/min and starts at 150, so the volume of water in the tank is $150 + t$ for t from 0 to 250 (at which point the tank overflows).

Set up our differential equation

$$Q' = \text{rate in} - \text{rate out} = \left(\frac{3}{2}\right)(4) - \left(\frac{Q}{t+150}\right)(3) = 6 - \frac{3Q}{t+150}$$

The differential equation is linear but not separable, so we write it like this:

$$Q' - \frac{3}{t+150}Q = 6$$

The integrating factor is $(t+150)^3$, so we have

$$(t+150)^3 Q = \int 6(t+150)^3 dt = \frac{3}{2}(t+150)^4 + c.$$

Using our initial condition, we get

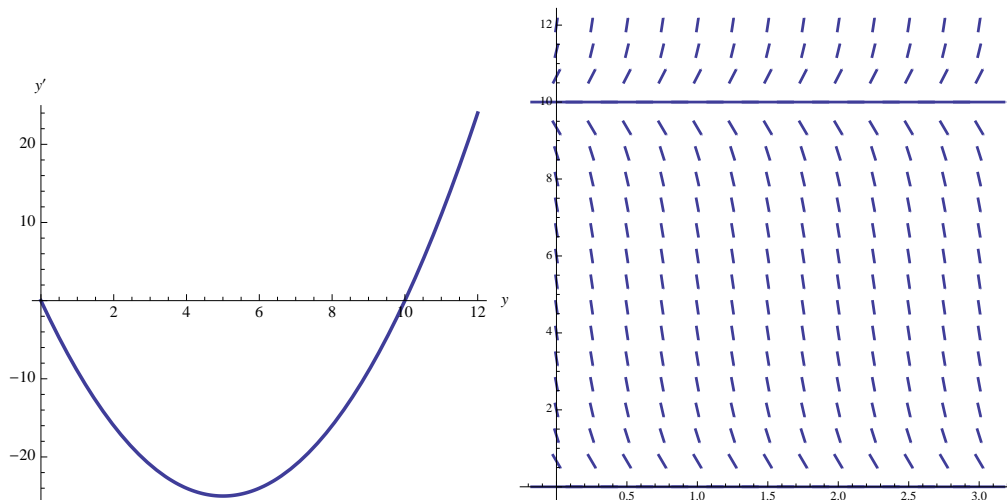
$$(150^3)(75) = \frac{3}{2}(150)^4 + c,$$

so $c = -506250000$. Now if we solve for Q , we get

$$Q(t) = \frac{3}{2}(t+150) - \frac{5.0625 \times 10^8}{(t+150)^3}$$

When it is about to overflow at $t = 250$, we have a quantity of $Q(250) = 592.09g$. But the question asked for concentration, which is quantity over volume, so we should divide by the current volume of the tank, which is 400 liters. So our answer is 1.48 g/L.

4.



As $t \rightarrow \infty$: $y \rightarrow \infty$ if $y_0 > 10$, $y \rightarrow 10$ if $y_0 = 10$, and $y \rightarrow 0$ if $0 \leq y < 10$.