THIS SAMPLE TEST covered only Chapter 15. To conserve paper (if you print this out), I have removed the space provided for working the problems which would be on an actual test.

1. *(11 points total)* Suppose $\iint_D f(x,y)\,dA = \int_0^4 \int_{\sqrt{y}}^2 f(x,y)\,dx\,dy$.

   (a) *(4 points)* Reverse the order of integration in the double integral.

   (b) *(7 points)* Suppose a thin plate with mass density $\rho = ke^y$ occupies the region D, and has total mass m. Find the x-coordinate of the center of mass of the plate. (Your answer will be in terms of k and m.)

2. *(10 points)* A thin plate occupies the region D that lies above the line $y = x$ and inside the circle of radius 2 centered at (2,0). The mass density of the plate is inversely proportional to distance from the origin (that is, the density $= k/r$). Find the total mass of the plate. (Your answer will be in terms of k.)

3. *(11 points total)* Let E be the region in the first octant (that is, $x, y, z \geq 0$) that is inside the sphere $x^2 + y^2 + z^2 = 1$ and below the plane $z = y$.

   (a) *(8 points)* Write $\iiint_E f(x,y,z)\,dV$ as an iterated integral with respect to $dx\,dy\,dz$ (in that order).

   (b) *(3 points)* Explain why it probably would not be a good idea to integrate with respect to z first in the integral in part (a). Make specific use of one or more of the bounding surfaces of E in your explanation.

4. *(10 points)* Set up an integral in spherical coordinates for the volume of the region that is outside the sphere $x^2 + y^2 + z^2 = 1$ and inside the sphere $x^2 + y^2 + z^2 = 2z$. *Note that you are not asked to evaluate the integral.*

5. *(8 points)* Let R be the region in the xy-plane that is bounded by the ellipse

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1.$$

   Use the change of coordinates $2u = x$ and $3v = y$ to evaluate the following integral.

$$\iint_R \left(\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 \right) dA$$