Math 308 Sample Midterm 1

Print Your Name

Student ID #

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

Directions

- Please check that your exam contains a total of 6 pages.
- Write complete solutions or you may not receive credit.
- This exam is closed book. You may use one 8.5×11 sheet of notes and a calculator.
- You may not share notes or calculators. You may not use a graphing calculator or any electronic device other than a calculator.
- If you need more room, use the backs of the pages and indicate to the reader that you have done so.
- Raise your hand if you have a question.

Signature. Please sign below to indicate that you have not and will not give or receive any unauthorized assistance on this exam.

Signature: ______________________

1. (10 points) Consider the following system of linear equations.

$$\left\{\begin{array}{rcrcrcr} x_1 & + & 2x_2 & + & 3x_3 & = & 1 \\ 2x_1 & + & 5x_2 & + & 3x_3 & = & -2 \\ x_1 & & & + & 8x_3 & = & 0 \end{array}\right.$$

   (a) Solve this system of linear equations.

   (b) Let A be the coefficient matrix corresponding to the system of equations. Does A have an inverse? How many solutions are there to the equation $A\mathbf{x} = \mathbf{0}$, where $\mathbf{x}$ is a vector in $\mathbf{R}^3$.

2. (10 points) Let

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 3 & 1 & 3 \\ 4 & 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

(a) Find the inverse of A or state that A is not invertible.

(b) Use your answer above to calculate $(B^{-1}A)^{-1}$.

3. (10 points) Let

$$A = \begin{bmatrix} 1 & 2 & 3 & 3 \\ 2 & 1 & 4 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 3 \end{bmatrix}.$$

(a) Find the determinant of A.

(b) What is the determinant of A^{-1}?

4. (10 points) For each statement, tell whether it is true or false and explain why. You do not need to prove anything, but your explanation should be clear.

   (a) Let A be an $n \times n$ matrix. True or false: If $A^2 = I_n$ then $A^8 = I_n$

   (b) True or false: A consistent 4×2 linear system of equations can never have a unique solution.

   (c) An $n \times n$ matrix A is called diagonal if $A_{ij} = 0$ whenever $i \neq j$. True or false: if A and B are $n \times n$ diagonal matrices, then $AB = BA$.

   (d) True or false: A 2×4 linear system of equations always has infinitely many solutions.

5. (10 points) Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, and $\mathbf{v}_4$ be vectors in $\mathbf{R}^m$. Prove that if the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly independent, then the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is also linearly independent.