1. (a) Find the general solution to the following differential equation.

$$y'' + 10y' + y = 0$$

(b) Find the solution to the following initial value problem.

$$2y'' + 6y' + 17y = 0, \quad y(0) = 3, \quad y'(0) = -1$$

2. (a) Solve the following initial value problem.

$$y'' + y = t^3, \quad y(0) = 0, \quad y'(0) = 2$$

(b) Find the general solution to the following differential equation.

$$y'' + 2y' + y = 15e^t$$

3. Consider the following differential equation

$$t^2y'' + 2ty' - 2y = 0$$

The function $y_1 = t$ is a solution to this equation. Use reduction of order to find a solution that is not a constant multiple of y_1.

4. A spring of length 50 cm has a spring constant of 41 N/m. It has a damper on it that exerts a force proportional to and in the opposite direction of the velocity. When an object is moving 1 m/s, the damper exerts a force of 2 N. An object of mass 2 kg is attached to the unstretched spring and dropped. You are on the planet earth, so gravity induces an acceleration of 9.8 m/s^2.

   (a) Find a function that describes the position of the object at any given time.

   (b) Determine the amount of time it takes before the object's motion is confined to within 1 cm of the equilibrium.

   (c) Determine the amount of time it takes for the object to reach the equilibrium the first time.

   (d) Bonus[1]: Your physics teacher, who owns this spring, told you that it will be irreparably damaged if it is stretched to a total length of more than 135 cm. Did you break it?

[1]There will be no bonus questions on the actual exam

5. A ball weighing 16 lbs stretches a spring 16 ft. When the ball is moving 1 ft/s, the damping force is 1 lb. An external force of $2\cos t$ is applied to the ball.

   (a) Find a function describing the steady state of the system.

   (b) What is the amplitude of the steady state response?