

ANSWERS TO DIFFERENCE EQUATION HOMEWORK

MATH 308

Suppose that $p(k)$ corresponds to balls in the clubhouse and $q(k)$ is balls on the grass. Then

$$p(k+1) = .5p(k) + .9q(k)$$

$$q(k+1) = .5p(k) + .1q(k)$$

$$\mathbf{x}_0 = \begin{bmatrix} 700 \\ 0 \end{bmatrix}$$

So $A = \begin{bmatrix} .5 & .9 \\ .5 & .1 \end{bmatrix}$. Also, $\mathbf{u}_1 = \begin{bmatrix} 1.8 \\ 1 \end{bmatrix}$ is an eigenvector of A corresponding to eigenvalue 1, and $\mathbf{u}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is an eigenvector corresponding to eigenvalue -0.4 . We can write \mathbf{x}_0 in terms of these eigenvectors as follows:

$$\mathbf{x}_0 = 250\mathbf{u}_1 + 250\mathbf{u}_2.$$

Therefore the equation for \mathbf{x}_k is

$$\begin{aligned} \mathbf{x}_k &= A^k \mathbf{x}_0 \\ &= A^k(250\mathbf{u}_1) + A^k(250\mathbf{u}_2) \\ &= 250\mathbf{u}_1 + 250(-0.4)^k \mathbf{u}_2. \end{aligned}$$

The limit as $t \rightarrow \infty$ is $250\mathbf{u}_1 = \boxed{\begin{bmatrix} 450 \\ 250 \end{bmatrix}}$