Supplemental Exercises for Matrix Algebra

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A. Write the matrix and vector equations that are equivalent to the system of equations in Problem 4 of section 1.3.

B. Solve two systems of equations at once! Consider the systems of equations

$$\begin{aligned} 2x + 3y + z &= 1 \\ x + y - z &= -1 \end{aligned} \qquad \text{and} \qquad \begin{aligned} 2x + 3y + z &= 2 \\ x + y - z &= 0 \end{aligned}$$

Row reduce the double-augmented matrix

$$\begin{bmatrix} 2 & 3 & 1 & 1 & 2 \\ 1 & 1 & -1 & -1 & 0 \end{bmatrix},$$

and check that the fourth column tells you the solutions to the first system of equations and the fifth column tells you the solutions to the second system. Also, write down what similarities you notice about the two different sets of solutions.

C. Let

$$D = \begin{bmatrix} -1/2 & 3/2 \\ 1/2 & -1/2 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}, \text{ and } \mathbf{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

Show that DA is the same as the reduced row echelon form of A. Solve the equation $A\mathbf{x} = \mathbf{b}$ by instead solving $DA\mathbf{x} = D\mathbf{b}$. (Optional: Show that if you row reduce $\begin{bmatrix} A \mid I_2 \end{bmatrix}$ you get $\begin{bmatrix} A' \mid D \end{bmatrix}$, where A' is the the reduced row echelon form of A.)

D. The matrix

$$S = \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix}$$

is a reflection matrix. That is, when a vector in $\mathbf{R}^3$ is multiplied by S, the vector is reflected across a certain plane. Find the equation for that plane.

E. Suppose that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linearly dependent set. Prove that no matter what $\mathbf{v}_4$ is, the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly dependent.

F. The *trace* of a matrix is the sum of the entries along the main diagonal. For example, the trace of $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $a + d$. Prove that if A is a 2×2 matrix with $\det(A) = 1$ and trace$(A) > 2$, then A has two distinct real eigenvalues.