Resonance and beats

Suppose you have an object of mass 1 kg hanging from a spring with damping coefficient 1/10 N/(m/s) and spring coefficient 1 N/m. If the mass is oscillating, what is the pseudo frequency (in radians per second)?

Answer $\sqrt{3.99}/2 \approx 0.999$

Now suppose there is a forcing function $2\sin(t)$. Solve the differential equation and determine the amplitude of the steady state response. How does this amplitude compare to the amplitude of the forcing function?

Answer The steady state solution is $-20\cos(t)$, which has amplitutde 20.

What if you change the forcing function to $2\sin(2t)$. Now what is the amplitude of the steady state response?

Answer The steady state solution is $\frac{5}{113}(\cos t + 25\sin t)$, which has amplitude $\frac{5\sqrt{226}}{113} \approx 0.66$.

Suppose you have an object of mass 1 kg hanging from a spring with spring coefficient 1 N/m, without any damping or forcing function. What is the frequency of oscillation (in radians per second)?

Answer 1

Suppose there is a forcing function of $\cos(11t/10)$, and at time $t = 0$, the mass is at $u = 0$ with $u' = 0$. Determine the position of the mass at any time.

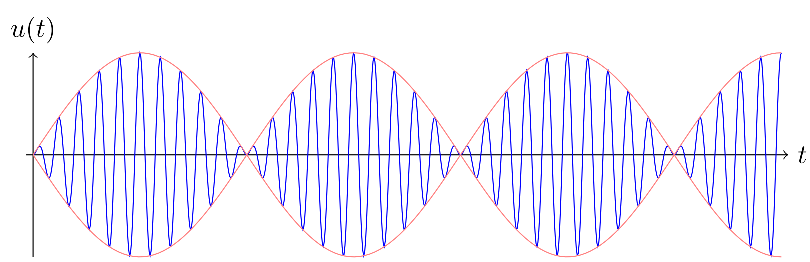
Answer $y = -\frac{100}{21}(\cos(\frac{11}{10}t) - \cos t)$.

Use the trig identity

$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

to rewrite your answer as a product of two sin functions. Sketch a graph. This is an example of a *beat*. A similar thing happens when you play two tones at very close frequencies; you can then hear the amplitude of the sound increase and decrease in a periodic way.

Answer $u(t) = \frac{200}{21}\sin(\frac{21}{20}t)\sin(\frac{1}{20}t)$



The red line is the graph of $\pm\frac{200}{21}\sin(\frac{1}{20}t)$. The graph of u oscillates between these functions.