(Problem 8 on page "11")

8. (a) $(-\sqrt{\frac{5}{2}}, 0)$ and $\sqrt{5}2, 0)$

(b) The slopes of the tangent lines at these point are both -4.

KEY : MATH 124 FINAL AUT 2009 (Problem 1 on page "2")

1. (16 points) Find the derivatives of the following functions. You do not have to simplify.

4 pts (a) $f(x) = \left(3\cos(x) + \frac{7}{x^2}\right)^4$

$$f'(x) = \underbrace{4\left(3\cos(x) + \frac{7}{x^2}\right)^3}_{2\text{ pts}} \left(\underbrace{-3\sin(x)}_{1\text{ pt}} \underbrace{- 2\cdot\frac{7}{x^3}}_{1\text{ pt}}\right)$$

4 pts (b) $g(x) = \frac{xe^x}{\arcsin x}$

$$g'(x) = \frac{(xe^x)'\arcsin x - xe^x(\arcsin x)'}{\arcsin^2 x}$$

2 pts: quotient rule (or product, if taken as $x\cdot\frac{e^x}{\arcsin x}$ or $\frac{x}{\arcsin x}\cdot e^x$),

$$= \boxed{\frac{(e^x + xe^x)\arcsin x - xe^x\frac{1}{\sqrt{1-x^2}}}{\arcsin^2 x}}$$

2 pts : 1 pt each correct derivative

continued on next page

1. continued.

5pts (c) $y = (\sqrt{x})^{\sqrt{x}}$ (Your final answer should be a function of x.)

+1pt $\{ \ln y = \sqrt{x} \ln \sqrt{x}$

+2pts $\{ \frac{1}{y}\frac{dy}{dx} = \frac{1}{2\sqrt{x}} \ln\sqrt{x} + \sqrt{x}\,\frac{1}{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}}$

+1pt $\{ \boxed{\frac{dy}{dx} = (\sqrt{x})^{\sqrt{x}}\,\frac{1}{2\sqrt{x}}\left(\ln\sqrt{x} + 1\right)}$

or equivalent expression

OR $y = x^{\frac{1}{2}\sqrt{x}}$

$\ln y = \frac{1}{2}\sqrt{x}\,\ln x$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\,\frac{1}{2\sqrt{x}}\ln x + \frac{1}{2}\sqrt{x}\cdot\frac{1}{x}$

$\boxed{\frac{dy}{dx} = x^{\frac{1}{2}\sqrt{x}}\,\frac{1}{2}\left[\frac{\ln x}{2\sqrt{x}} + \frac{\sqrt{x}}{x}\right]}$

or equivalent expression

4 pts (d) $h(t) = \tan^2(\ln t)$

$$h'(t) = \underbrace{2\tan(\ln t)}_{+2\text{pts}} \cdot \underbrace{\sec^2(\ln t)}_{+1\text{pt}} \cdot \underbrace{\frac{1}{t}}_{+1\text{pt}}$$

Problem 8 on page "10"

8. (14 points) Let $y = f(x) = e^x(x-1)^2$ on the domain of all real numbers.

(2pts) (a) Determine all x and y-intercepts for the curve.

x- intercept: set $y=0$, solve for x, $x=1$

$\boxed{(1,0)}$ 1pt

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y- intercept: set $x=0$, $y=1$,

$\boxed{(0,1)}$ 1pt

(2pts) (b) Determine any vertical asymptotes and horizontal asymptotes for the curve $y = f(x)$.

The function is differentiable every where.
so No vertical asymptote. 1pt

$\lim_{x\to+\infty} e^x(x-1)^2 = \infty$, $\lim_{x\to-\infty} e^x(x-1)^2 = 0$ 1pt

so $\boxed{y=0}$ is the only horizontal asymptote

(2pts) (c) Find all critical numbers for $f(x)$.

$$f'(x) = e^x(x-1)^2 + e^x(2(x-1))$$
$$= e^x(x^2-2x+1+2x-2)$$
$$= e^x(x^2-1)$$ 1pt

---

set $f'(x)=0$ solve for x.

we have $e^x(x^2-1)=0$ 1pt

$$x^2-1=0$$
$$x^2=1$$
$$x=\pm 1.$$

continued on next page

Critical numbers are $\boxed{x=1,}$ and $\boxed{x=-1}$

8. continued.

(3pts) (d) Find the intervals on which $f(x)$ is increasing, and the intervals on which $f(x)$ is decreasing. Determine x and y-coordinates of all local minimum(s) and local maximum(s).

Sign of $f'(x)$

x	$x<-1$	$x=-1$	$-1<x<1$	$x=1$	$x>1$
sign of $f'(x)$	+	0	−	0	+

(1pt)

$f(x)$ is increasing on $(-\infty, -1)$ and $(1, \infty)$ (1pt)

$f(x)$ is decreasing on $(-1, 1)$

local minimum pt is $\boxed{(1, 0)}$ (1pt)

local maximum pt. is $\boxed{(-1, 4e^{-1})}$ $4e^{-1} \approx 1.4715$

(3pts) (e) Find the intervals on which $f(x)$ is concave up and concave down. Find the x and y-coordinates of all of the inflection points.

$f''(x) = e^x(x^2-1) + e^x(2x) = e^x(x^2+2x-1)$ (1pt)

set $f''(x)=0$ solve for x, $x = -1-\sqrt{2}$, $x = -1+\sqrt{2}$

sign of $f''(x)$

x	$x<-1-\sqrt{2}$	$x=-1-\sqrt{2}$	$-1-\sqrt{2}<x<-1+\sqrt{2}$	$x=-1+\sqrt{2}$	$x>-1+\sqrt{2}$
sign of $f''(x)$	+	0	−	0	+

$f(x)$ is concave up on $(-\infty, -1-\sqrt{2})$ and $(-1+\sqrt{2}, +\infty)$

$f(x)$ is concave down on $(-1-\sqrt{2}, -1+\sqrt{2})$ (1pt)

$f(x)$ has two inflection pts.

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$\boxed{\left(-1-\sqrt{2}, \frac{e^{-1-\sqrt{2}}(2+\sqrt{2})^2}{5}\right)}$, $\boxed{\left(-1+\sqrt{2}, \frac{e^{-1+\sqrt{2}}(2-\sqrt{2})^2}{5}\right)}$ (1pt)

$(-2.414, 1.04256)$ $(0.414, 0.51924)$

8. continued.

2pts (f) Using (a)-(e), sketch the curve.