SOLUTION TO 3.2 18-19

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18. Let $\mathbf{a}$ be a fixed vector in $\mathbb{R}^3$. Show that $W = \{\mathbf{x} : \mathbf{a}^{\mathrm{T}}\mathbf{x} = 0\}$ is a subspace of $\mathbb{R}^3$.

Proof. We need to show that W satisfies the three properties of Theorem 2.

(1) The vector $\mathbf{0}$ is in W because $\mathbf{a}^{\mathrm{T}}\mathbf{0} = 0$.
(2) If $\mathbf{x}$ and $\mathbf{y}$ are two vectors in W then $\mathbf{a}^{\mathrm{T}}\mathbf{x} = 0$ and $\mathbf{a}^{\mathrm{T}}\mathbf{y} = 0$, so $\mathbf{a}^{\mathrm{T}}(\mathbf{x}+\mathbf{y}) = \mathbf{a}^{\mathrm{T}}\mathbf{x} + \mathbf{a}^{\mathrm{T}}\mathbf{y} = 0 + 0 = 0$.
(3) If c is any scalar and $\mathbf{x}$ is in W, then again $\mathbf{a}^{\mathrm{T}}\mathbf{x} = 0$, so $\mathbf{a}^{\mathrm{T}}(c\mathbf{x}) = c(\mathbf{a}^{\mathrm{T}}\mathbf{x}) = c(0) = 0$.

Since W satisfies all three properties of Theorem 2, W is a subspace of $\mathbb{R}^3$. □

19. Describe W if $\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

Solution. In this case, W is all solutions to the equation $x + 2y + 3z = 0$. W is a plane through the origin with normal vector $\mathbf{a}$. □