1. Let

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 1 & 0 & 3 \\ 1 & 1 & 3 \end{bmatrix}$$

.

(a) Find $\det(A)$.

(b) Compute A^{-1}.

(c) Find a matrix B and a vector C such that

$$BA = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}, AC = \begin{bmatrix} 4 \\ 3 \\ 3 \end{bmatrix}$$

.

(d) Let F be any matrix row-equivalent to A. Find the nullspace and range of F.

(e) Let

$$V_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, V_2 = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}, V_3 = \begin{bmatrix} 4 \\ 3 \\ 3 \end{bmatrix}$$

.

G is a 3×3 matrix with $GV_1 = 2V_1, GV_2 = 3V_2, GV_3 = 7V_3$. Find G.

2. Let

$$A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 5 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

.

Find the eigenvalues of A, and for each eigenvalue find a basis of the corresponding eigenspace. Is A diagonalisable? If so, find a diagonal matrix D similar to A. (That is, find a diagonal D such that, for some invertible matrix P, $AP = PD$ or $A = PDP^{-1}$.

3. (a) Let

$$B = \begin{bmatrix} 2 \\ 5 \\ 7 \\ 9 \end{bmatrix}$$

. Suppose A is a 4×3 matrix. Can the matrix equation $AX = B$, where X is a vector in $\mathbb{R}^3$, have zero, one, or infinitely many solutions? For each possible number of solutions, find a 4×3 matrix A such that $AX = B$ has precisely that many solutions.

(b) Let

$$B = \begin{bmatrix} 7 \\ 9 \\ 6 \end{bmatrix}$$

. Suppose A is a 3×4 matrix. Can the matrix equation $AX = B$, where X is a vector in $\mathbb{R}^4$, have zero, one, or infinitely many solutions? For each possible number of solutions, find a 3×4 matrix A such that $AX = B$ has precisely that many solutions.

4. (a) Find a spanning set for $\mathbb{R}^3$ which is not linearly independent.

   (b) TRUE OR FALSE: Suppose A is a singular 4×4 matrix, and B is row-equivalent to A. Then $\det(A)=\det(B)$.

   (c) Suppose C is a diagonalisable 6×6 matrix with eigenvalue 7 with algebraic multiplicity 2. Find $\text{rank}(C - 7I)$.

   (d) Suppose A is a singular 2×2 matrix which is not diagonalisable. Find the eigenvalues of A, and the characteristic polynomial of A. Give an example of such a matrix.