Math 307 G - Spring 2010
Final Exam
June 7, 2010

Name: _____ Student number: _____

1	10	
2	10	
3	10	
4	12	
5	14	
6	14	
Total	70	

- Complete all questions.
- You may use a calculator during this examination. Other electronic devices are not allowed, and should be turned off for the duration of the exam.
- If you use a trial-and-error or guess-and-check method, or read a numerical solution from a graph on your calculator, when an algebraic method is available, you will not receive credit.
- You may use one hand-written 8.5 by 11 inch page of notes.
- Show all work for full credit.
- You have 110 minutes to complete the exam.

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1. (a) Find the general solution of

$$t^2 y' = 1 - 2ty, \qquad t > 0.$$

(5 points)

(b) Solve the initial value problem

$$\frac{dy}{dx} = e^{2y-3x}, \qquad y(0) = 0.$$

(5 points)

2. Suppose we have a tank containing $Q(t)$ lbs of salt dissolved in 3 gallons of water after t minutes. Water containing a constant concentration of γ lbs/gal of salt enters the tank at a rate of 2 gal/min, and the well-stirred mixture leaves the tank at the same rate. If there is 1 lb of salt in the tank initially, and 13/16 lbs of salt in the tank after $\ln(8)$ min, what is γ?
(10 points)

3. (a) Find the general solution of

$$y'' - 10y' + 25y = e^{5t}.$$

(5 points)

(b) For the following equation, y_1 is a solution. Use the method of reduction of order to find a second solution.

$$t^2y'' - 7ty' + 15y = 0, \quad t > 0, \qquad y_1 = t^3.$$

(5 points)

4. An object weighing 8 lbs hangs from a spring with spring constant k lb/ft. It is pulled down 1 ft and set into motion with an initial downward velocity of 1 ft/sec. If the object's motion has amplitude $5/4$ ft, what is k? Use $g = 32$ ft/sec^2.

   (12 points)

5. An object weighing 4 lb stretches a spring 4/5 ft in a medium which exerts a damping force of 2 lb when the speed of the object is 4 ft/sec. The object is pushed up 6 in and given an initial upward velocity of 2 ft/sec.

   (a) Determine the position of the object at any given time.
   (6 points)

   (b) When does the object first return to equilibrium position?
   (8 points)

6. Solve the initial value problem

$$y'' + 2y' + 2 = g(t), \qquad y(0) = 0, \quad y'(0) = 0,$$

where

$$g(t) = \begin{cases} -1 & \text{if } 0 \le t < 1 \\ t - 2 & \text{if } 1 \le t < 3 \\ -\frac{1}{3}t + 2 & \text{if } 3 \le t < 6 \\ 0 & \text{if } 6 \le t. \end{cases}$$

(14 points)

Laplace transforms:

$$\begin{aligned}
\mathcal{L}\{t^n\} &= \frac{n!}{s^{n+1}} \\
\mathcal{L}\{e^{at}\sin(bt)\} &= \frac{b}{(s-a)^2+b^2} \\
\mathcal{L}\{e^{at}\cos(bt)\} &= \frac{s-a}{(s-a)^2+b^2} \\
\mathcal{L}\{u_c(t)\} &= \frac{e^{-cs}}{s}
\end{aligned}$$

If $\mathcal{L}\{f(t)\} = F(s)$, then

$$\begin{aligned}
\mathcal{L}\{u_c(t)f(t-c)\} &= e^{-cs}F(s) \\
\mathcal{L}\{e^{ct}f(t)\} &= F(s-c)
\end{aligned}$$