Quiz 6 Name: ____________________

1. Find the Laplace transform of

$$g(t) = \begin{cases} e^{2t} & \text{if } t < 3 \\ t & \text{if } t > 3 \end{cases}$$

Solution: First write $g(t)$ in terms of Heaviside functions:

$$g(t) = [1 - u_3(t)](e^{2t}) + u_3(t)(t) = e^{2t} + u_3(t)(t - e^{2t}).$$

The Laplace transform of e^{2t} is $\dfrac{1}{s-2}$. For the other piece,

$$\begin{aligned} \mathcal{L}\{u_3(t)(t - e^{2t})\} &= e^{-3s}\mathcal{L}\{(t+3) - e^{2(t+3)}\} \\ &= e^{-3s}\left(\frac{1}{s^2} + \frac{3}{s} - \mathcal{L}\{e^{2t}e^6\}\right) \\ &= e^{-3s}\left(\frac{1}{s^2} + \frac{3}{s} - \frac{e^6}{s-2}\right). \end{aligned}$$

So the final answer is

$$\frac{1}{s-2} + e^{-3s}\left(\frac{1}{s^2} + \frac{3}{s} - \frac{e^6}{s-2}\right).$$

2. Find the inverse Laplace transform of

$$F(s) = \frac{2e^{-4s}}{s+4}$$

Solution: The inverse Laplace transform of $\dfrac{2}{s+4}$ is $2e^{-4t}$. Because of the e^{-4s}, we change t to $t-4$ and multiply by $u_4(t)$:

$$f(t) = u_4(t)\left(2e^{-4(t-4)}\right).$$