Make up quiz (1–3) Name: ______________________

Complete ONE of the following problems. Only one problem will be graded, and the score you receive will replace the score you received on the original quiz that covered the same material, unless your new score is lower.

1. Solve the initial value problem $y' = te^{(t^2)} + 2ty, \quad y(2) = e^4$. Where is the solution valid?
2. Determine the critical (equilibrium) points of the following differential equation, and classify each as stable, semistable, or unstable.

$$y' = ay + by^2, \quad a > 0, \quad b > 0.$$

3. Solve the initial value problem $\frac{1}{2}y'' + y' + 50y = 0, \quad y(0) = 0, \quad y'(0) = 2$. Write your final answer in terms of real numbers only.

Circle the problem you would like me to grade.

Solution:

1. The differential equation is linear, with integrating factor $e^{(-t^2)}$. So $ye^{(-t^2)} = \int t = \frac{1}{2}t^2 + c$. Plugging in the initial condition, we get $c = -1$. So the final solution is $y = (\frac{1}{2}t^2 - 1)e^{t^2}$.
2. $y' = 0$ when $y = 0$ or $y = -a/b$. Since $ay + by^2$ is a concave up parabola, it is negative between these two solutions. Hence the smaller solution $y = -a/b$ is stable, and the larger $y = 0$ is unstable.
3. The characteristic equation has roots $-1 \pm \sqrt{99}\, i$. This means that the general solution is $y = e^{-t}\left(c_1 \cos(\sqrt{99}\, t) + c_2 \sin(\sqrt{99}\, t)\right)$. The first initial condition tells us $c_1 = 0$, and the second tells us $2 = \sqrt{99}\, c_2$, which means $c_2 = 2/\sqrt{99}$.