Worksheet 1 — Math 126 — Summer 2010

This worksheet should help with your geometric visualization and understanding, which will help you with other problems in this chapter. Also, there may be quiz or test problems which are similar some of these questions.

1. Suppose that $\mathbf{a}$ and $\mathbf{b}$ are nonzero vectors.
   (a) Show by examples that $\text{comp}_{\mathbf{a}}\mathbf{b}$ and $\text{comp}_{\mathbf{b}}\mathbf{a}$ can be the same and can be different. What conditions on $\mathbf{a}$ and $\mathbf{b}$ will guarantee they are the same?
   (b) Your friend who skips class frequently says, "I'm confused. Isn't $\mathbf{a}\cdot\mathbf{b} = \mathbf{b}\cdot\mathbf{a}$? If that is true, how can $\text{comp}_{\mathbf{a}}\mathbf{b}$ and $\text{comp}_{\mathbf{b}}\mathbf{a}$ be different?" What is your answer?
   (c) Show by examples that $\text{proj}_{\mathbf{a}}\mathbf{b}$ and $\text{proj}_{\mathbf{b}}\mathbf{a}$ can be the same and can be different. What conditions on $\mathbf{a}$ and $\mathbf{b}$ will guarantee they are the same?

2. Decide for each expression below whether it is a vector (**V**), a scalar (**S**), or nonsense (**N**). Note that $\mathbf{a}, \mathbf{b}, \mathbf{u}$, and $\mathbf{v}$ are vectors, while c and d are scalars.

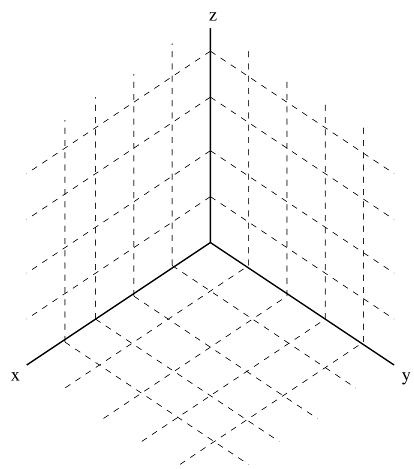
	Circle one:		
(a) $\mathbf{a}\cdot(\mathbf{u} - c\mathbf{v})$	**V**	**S**	**N**
(b) $\mathbf{a}\cdot(\mathbf{b} + c)$	**V**	**S**	**N**
(c) $(c + d)\cdot\mathbf{a}$	**V**	**S**	**N**
(d) $\mathbf{uv}$	**V**	**S**	**N**
(e) $\dfrac{\mathbf{a}}{c}$	**V**	**S**	**N**
(f) $\dfrac{c}{\mathbf{a}}$	**V**	**S**	**N**

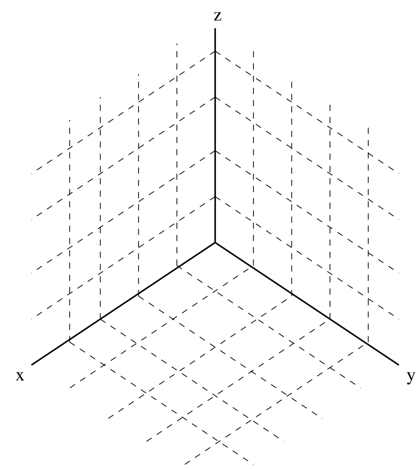
3. Determine whether each of the following statements is true or false. If it is true, prove it. If it is false (that is, doesn't hold for every case), give a counterexample. Note that $\mathbf{a}$ and $\mathbf{b}$ are vectors and c is a scalar.

   (a) If $\mathbf{a} \cdot \mathbf{b} = \mathbf{0}$, then at least one of $\mathbf{a}$ or $\mathbf{b}$ must be the zero vector.

   (b) If $c\mathbf{a} = \mathbf{0}$, then either $c = 0$ or $\mathbf{a} = \mathbf{0}$.

Here are two three-dimensional coordinate grids for drawing the graphs in problems 4 and 5.





4. Find the equation and sketch the graph of a plane that is parallel to the yz-coordinate plane and contains the point $(2, 1, 3)$. How is this plane related to the other two coordinate planes, the xy-coordiante plane and the xz-coordinate plane?

5. Graph the plane P given by the equation $x + z = 2$.
   Is P parallel to any of the coordinate planes?
   Is P perpendicular to any of the coordinate planes?
   Is P parallel to any of the coordinate axes?
   Is P perpendicular to any of the coordinate axes?
   What fact about the equation for P immediately gives you the anwer to all of these questions?

6. Decide by yourself whether each of the following is true or false. Compare answers with one or two neighbors, then confirm your answers by using pieces of paper and/or a desktop as models for planes, and pens and/or pencils as models for lines.

   (a) Two lines perpendicular to the same plane are parallel.

   (b) Two lines parallel to the same plane are parallel.

   (c) Two planes perpendicular to the same (third) plane are parallel.

   (d) Two planes parallel to the same (third) plane are parallel.

   (e) Two lines perpendicular to the same (third) line are parallel

   (f) Two lines parallel to the same (third) line are parallel.

   (g) Two planes either intersect or are parallel

   (h) Two planes perpendicular to the same line are parallel.

   (i) Two planes parallel to the same line are parallel.