1. (a) Two of the many possibilities: $\left\{\begin{bmatrix}1\\3\\1\end{bmatrix}, \begin{bmatrix}1\\3\\2\end{bmatrix}\right\}$; $\left\{\begin{bmatrix}1\\3\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$.

   (b) One possibility: $\left\{\begin{bmatrix}-2\\0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\-2\\0\\1\end{bmatrix}\right\}$.

   (c) See above

   (d) 2

2. (a) $\left\{\begin{bmatrix}1\\1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\1\\1\\-3\end{bmatrix}, \begin{bmatrix}1\\2\\3\\4\end{bmatrix}\right\}$

   (b) $\frac{1}{2}\begin{bmatrix}1\\1\\1\\1\end{bmatrix} + \frac{1}{2}\begin{bmatrix}1\\1\\1\\-3\end{bmatrix}$.

   (c) 3

3. (a) $T\left(\begin{array}{c}a\\b\end{array}\right) = \begin{bmatrix}0\\\frac{1}{2}a+\frac{1}{3}b\\a+\frac{2}{3}b\end{bmatrix}$.

   (b) $A = \begin{bmatrix}0 & 0\\1/2 & 1/3\\1 & 2/3\end{bmatrix}$

4. $A = \begin{bmatrix}1 & 0\\1 & 1\\1 & 2\end{bmatrix}$, $\vec{\mathbf{b}} = \begin{bmatrix}1\\3\\9\end{bmatrix}$. $A^{\mathrm{T}}A = \begin{bmatrix}3 & 3\\3 & 5\end{bmatrix}$, $A^{\mathrm{T}}\vec{\mathbf{b}} = \begin{bmatrix}13\\21\end{bmatrix}$. $\vec{\mathbf{x}} = \begin{bmatrix}1/3\\4\end{bmatrix}$.

   So the linear fit equation is $y = 1/3 + 4x$.

5. (a) For example, $\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}2\\0\end{bmatrix}$.

   (b) 2

   (c) No because it does not span R^3.

6. Let $\vec{\mathbf{x}}$ and $\vec{\mathbf{y}}$ be in R^2 and let a be a scalar. Then $T(\vec{\mathbf{x}})+T(\vec{\mathbf{x}}) = \vec{\mathbf{0}}+\vec{\mathbf{0}} = \vec{\mathbf{0}}$, and $T(\vec{\mathbf{x}}+\vec{\mathbf{y}}) = \vec{\mathbf{0}}$, so they are the same. Also, $T(a\vec{\mathbf{x}}) = \vec{\mathbf{0}}$ and $aT(\vec{\mathbf{x}}) = a\vec{\mathbf{0}} = \vec{\mathbf{0}}$, so they are the same. Therefore T is a linear transformation.