

1. (a) The augmented matrix $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 2 & 5 & -2 \\ 1 & 3 & 6 & -2 \end{array} \right]$ row reduces to $\left[\begin{array}{ccc|c} 1 & 0 & -3/2 & 1 \\ 0 & 1 & 5/2 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right]$. So the solution is

$$x_1 = \frac{3}{2}x_3 + 1, x_2 = -\frac{5}{2}x_3 - 1, x_3 = x_3.$$

- (b) No. Since the system of linear equations above has more than one solution, A must be singular, which means its columns are linearly dependent.

2. (a) The matrix $\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 3 & 4 & 0 & 1 & 0 \\ 4 & 5 & 8 & 0 & 0 & 1 \end{array} \right]$ row reduces to $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 & 2 & -1 \\ 0 & 0 & 1 & 1 & -3/2 & 1/2 \end{array} \right]$, so

$$A^{-1} = \begin{bmatrix} -2 & 1/2 & 1/2 \\ 0 & 2 & -1 \\ 1 & -3/2 & 1/2 \end{bmatrix}.$$

$$(b) (A^T)^{-1} = (A^{-1})^T = \begin{bmatrix} -2 & 0 & 1 \\ 1/2 & 2 & -3/2 \\ 1/2 & -1 & 1/2 \end{bmatrix}$$

3. (a) There are many ways to find the determinant of A . The easiest is to do cofactor expansion down the first column, where there is only one nonzero entry. Remember that it is the 1, 2 entry, so we need to put in a negative sign:

$$\det(A) = -1 \begin{vmatrix} 2 & 3 & 4 \\ 1 & 3 & 5 \\ 0 & 2 & 5 \end{vmatrix} = -1[(30 + 8) - (15 + 20)] = -3.$$

4. (a) True. If A is nonsingular, it has an inverse. Since $AA = A$, then $A^{-1}AA = A^{-1}A$. This simplifies to $A = I$.
- (b) True. If A and B are symmetric then $A^T = A$ and $B^T = B$, and then $(A + B)^T = A^T + B^T = A + B$, so $A + B$ is symmetric.
- (c) False. For example, if $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, then A and B are both nonsingular, but $A + B$ is zero, which is singular.
5. Since $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linearly dependent set, there are scalars a_1, a_2 , and a_3 which are not all zero, such that $a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + a_3\mathbf{v}_3 = \mathbf{0}$. Then there are scalars b_1, b_2, b_3, b_4 such that $b_1\mathbf{v}_1 + b_2\mathbf{v}_2 + b_3\mathbf{v}_3 + b_4\mathbf{v}_4 = \mathbf{0}$, namely $b_1 = a_1, b_2 = a_2, b_3 = a_3, b_4 = 0$. They are not all zero since one of a_1, a_2, a_3 is nonzero. So the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly dependent.

The answers to the alternate version of the midterm are not published. Please come talk to me if you would like to see them.