Answers to Math 308 Sample Midterm 1

1. (a) $x_1 = -72, x_2 = 23, x_3 = 9$. (b) Yes. One solution.

2.
$$A^{-1} = \begin{bmatrix} 0 & 0 & 1/4 \\ 3 & -2 & 0 \\ -1 & 1 & -1/4 \end{bmatrix}, \qquad (B^{-1}A)^{-1} = \begin{bmatrix} 0 & 0 & 1/4 \\ 2 & 3 & 0 \\ -1 & -1 & -1/4 \end{bmatrix}.$$

3. (a) -3, (b) $-1/3$.

4. (a) True, (b) False, (c) True, (d) False

5. If
$$\begin{aligned} a_1 &= x_1 \\ a_2 &= x_2 \\ a_3 &= x_3 \end{aligned}$$
is a solution to $a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + a_3\mathbf{v}_3 = \mathbf{0}$, then
$$\begin{aligned} a_1 &= x_1 \\ a_2 &= x_2 \\ a_3 &= x_3 \\ a_4 &= 0 \end{aligned}$$
is a solution to $a_1\mathbf{v}_1+a_2\mathbf{v}_2+a_3\mathbf{v}_3+a_4\mathbf{v}_4 = \mathbf{0}$. Since the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly independent, the only solution to $a_1\mathbf{v}_1+a_2\mathbf{v}_2+a_3\mathbf{v}_3+a_4\mathbf{v}_4 = \mathbf{0}$ is
$$\begin{aligned} a_1 &= 0 \\ a_2 &= 0 \\ a_3 &= 0 \\ a_4 &= 0 \end{aligned}.$$
So x_1, x_2, and x_3 must be 0, which means that
$$\begin{aligned} a_1 &= 0 \\ a_2 &= 0 \\ a_3 &= 0 \end{aligned}$$
is the only solution to $a_1\mathbf{v}_1 + a_2\mathbf{v}_2 + a_3\mathbf{v}_3 = \mathbf{0}$, which means that the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent.