

**Instructions****You should:**

- write complete solutions or you may not receive credit.
- box your final answer.
- check that your exam contains a total of 1 pages.

**You may:**

- use ten sheets of notes and a calculator.
- write on the backs of the pages if you need more room.
- raise your hand if you have a question.

**Please do not:**

- come to the front of the room before you finish the exam.
  - share notes or calculators.
  - use any electronic device other than a calculator.
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1. Consider the differential equation  $2ty' + y = 6t^{1/2} + t^2 - t$ .
  - (a) Find the general solution to the differential equation.
  - (b) What is  $\lim_{t \rightarrow \infty} y(t)$ ? Your answer may depend on the constant of integration.
2. Solve the initial value problem

$$y' = 2y^2 + xy^2, \quad y(0) = 1.$$

3. A tank with a capacity of 400 liters is initially filled with 150 liters of water that contains 75 grams of dye. Water with a dye concentration of  $3/2$  g/L pours into the tank at a rate of 4 liters per minute, and the mixed solution leaves at a rate of 3 liters per minute. Create an equation that models the amount of dye in the water before the tank overflows. Then find the concentration of dye in the tank right before it overflows.
4. Consider the autonomous differential equation  $y' = y^2 - 10y$ .
  - (a) Sketch a graph of  $y$  vs  $y'$  for positive values of  $y$ . Label key points.
  - (b) Sketch a slope field for positive values of  $t$  and  $y$  and label equilibrium points as stable, unstable, or neither.
  - (c) Suppose  $y(0) = y_0$ . Describe the behavior of  $y(t)$  as  $t \rightarrow \infty$  and how it depends on  $y_0$ .