Chapter 1: Introduction

Math 307 Spring 2012

Slides available on the class website

What is a differential equation?

$$(1-x^2)y'' - xy' + 3y = 0$$

$$\frac{dy}{dx} = \frac{3x^2+4+2}{2(y-1)}$$

An equation that involves a derivative

Mathematical Modeling

or: finally, we can factor in air resistance (kind of)

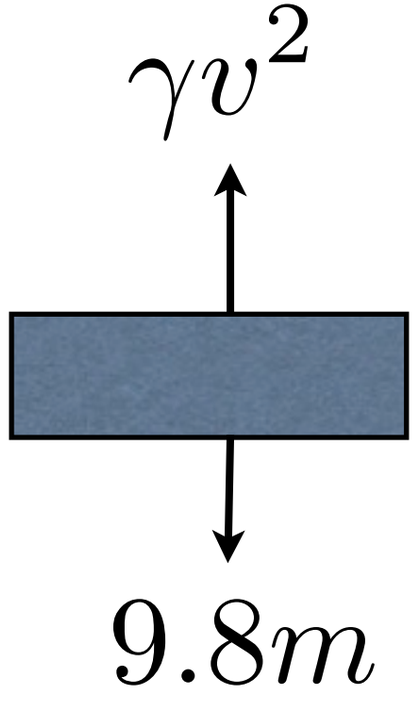
Determine the how the velocity of a falling object changes over time.

Ignoring air resistance

- Newton's first law: $F = ma$
- In diff eq form: $v' = F/m$
- Force from gravity: $F = 9.8m$
- Final equation: $v' = 9.8$
- Solution: $v(t) = 9.8t + c$

Air resistance

- Assume the drag force is proportional to the velocity squared.



$$v' = \frac{9.8m - \gamma v^2}{m}$$

$$= 9.8 - \frac{\gamma v^2}{m}$$

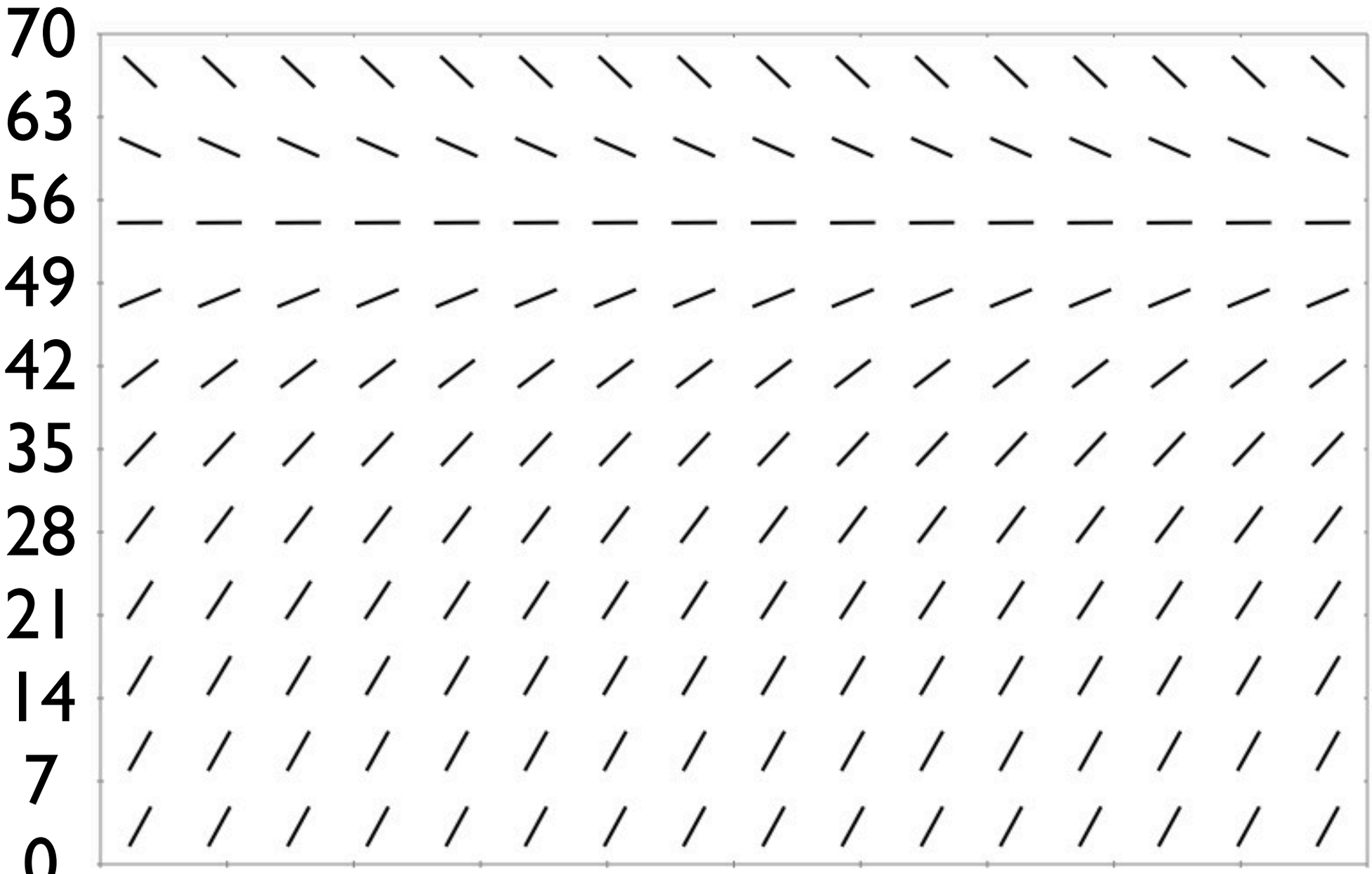
Analyze it

- Suppose an object has mass 60 kg and γ is 0.2 kg/m:

$$v' = 9.8 - v^2/300$$

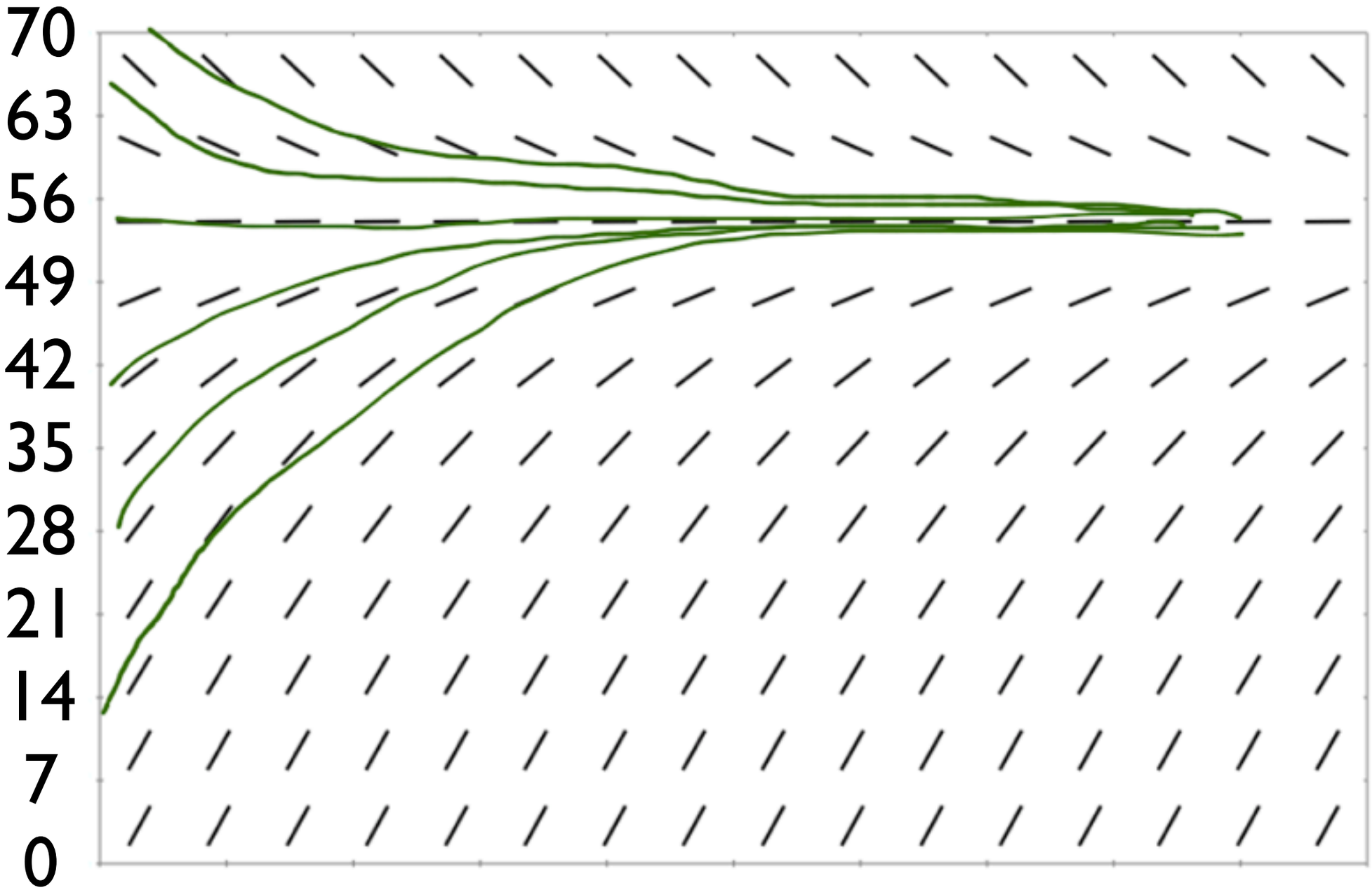
- What is happening?
- How can you tell if the speed is increasing or decreasing?
- What is terminal velocity?

Direction Fields



$$v' = 9.8 - v^2/300$$

Direction Fields



$$v' = 9.8 - v^2/300$$

Direction Field Plotter

http://staff.washington.edu/grigg/slopefield

Is this a good model?

- Does it agree with experimental results?
- What assumptions does our model make?
- Where does the model break down?

Classifying Differential Equations

- If it looks like [----] then do [----].
- Learn to recognize patterns.
- Call the equations by their names.

Ordinary or Partial?

Harder

- Ordinary differential equation (ODE): involves a function of one variable.
- Partial differential equation (PDE): involves a function of 2+ variables (and partial derivatives).
- In this class, we will study ODEs only.

Single equation or system of equations?

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- We will study a single function at a time.
- You might have several functions that depend on one another.

First order, second order, or higher?

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- Order refers to the highest derivative that appears in the equation.

$$(1 - x^2)y'' - xy' + 3y = 0$$ Second order

$$\frac{dy}{dx} = \frac{3x^2 + 4 + 2}{2(y - 1)}$$ First Order

Linear or nonlinear?

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- Linear:

$$t^2y'' + y' + (\sin t)y = \sqrt{t}$$

- Nonlinear:

$$y' = \sin y \qquad (y')^2 = t$$

$$\sqrt{y'} + y = 0 \qquad yy' = t^2$$

Red marks what makes each equation nonlinear

Homogeneous or non-homogeneous

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- This is a way to classify linear equations
- Homogeneous

$$ty'' + (\sin t)y' + y = 0$$ Zero

- Non-homogeneous

$$ty'' + (\sin t)y' + y = t$$ Nonzero

Classify it!

$ty' + y = 1$

Ordinary, linear, first order, non-homogeneous, single equation

$yy'' = 0$

Ordinary, nonlinear, 2nd order, single equation

$$\begin{cases} y_1' + y_2' = \cos t \\ y_1 + y_2 = \sin t \end{cases}$$

Ordinary, first order, linear, non-homogeneous, system of equations