Make up quiz (4–6)

4. Determine the particular solution to the following differential equation:

$$y'' + 3y' + 2y = 2te^{4t}$$

Solution: Using the method of undetermined coefficients, we know that the particular solution should be of the form $Y = Ate^{4t} + Be^{4t}$.

Plugging this into the equation, we get

$$18Ate^{4t} + (11A + 22B)e^{4t} = 2te^{4t}.$$

This gives us two equations: $18A = 2$ and $11A + 22B = 0$. So $A = 1/9$ and $B = -1/18$. Thus the steady state solution is $\frac{1}{9}te^{4t} - \frac{1}{18}e^{4t}$.

5. Solve the following differential equation using Laplace transforms:

$$y'' - 3y' + 2y = 8e^{6t}, \qquad y(0) = 0, \qquad y'(0) = 7$$

Solution: Taking Laplace transforms, we get $s^2Y - 3sY + 2Y - 7 = \dfrac{8}{s-6}$. We solve for Y and get $Y = \dfrac{7s - 34}{(s-6)(s-1)(s-2)} = \dfrac{2/5}{s-6} - \dfrac{27/5}{s-1} + \dfrac{5}{s-2}$.

When we take the inverse Laplace transform, we get

$$y(t) = \frac{2}{5}e^{6t} - \frac{27}{5}e^{t} + 5e^{2t}.$$

6. Find the Laplace transform of $g(t)$ and the inverse Laplace transform of $h(s)$:

$$g(t) = \begin{cases} -t^2 & \text{if } t \leq 6 \\ e^t & \text{if } t > 6 \end{cases} \qquad\qquad h(s) = \frac{e^{-7s}}{s+2}$$

Solution: Write $g(t) = -t^2 + t^2u_6(t) + e^tu_6(t)$. Then

$$\begin{aligned} \mathcal{L}\{g(t)\} &= -\frac{2}{s^3} + e^{-6s}\mathcal{L}\left\{(t+6)^2\right\} + e^{-6s}\mathcal{L}\left\{e^{t+6}\right\} \\ &= -\frac{2}{s^3} + e^{-6s}\mathcal{L}\left\{t^2 + 12t + 36\right\} + e^{-6s}\mathcal{L}\left\{e^6e^t\right\} \\ &= -\frac{2}{s^3} + e^{-6s}\left(\frac{2}{s^3} + \frac{12}{s^2} + \frac{36}{s}\right) + e^{-6s}\frac{e^6}{s-1} \end{aligned}$$

Next, $\mathcal{L}^{-1}\left\{\dfrac{e^{-7s}}{s+2}\right\} = e^{-2(t-7)}u_7(t)$