# SOLUTION TO 1.5.67

NATHAN GRIGG

*Proof.* Let $A$ and $B$ be upper triangular $n \times n$ matrices. Then the $(i, j)$th entry in $AB$ is

$$(AB)_{ij} = \sum_{k=1}^{n} A_{ik} B_{kj}.$$

I need to prove that if $i > j$ then this sum is 0. If $k < i$ then since $A$ is upper triangular, $A_{ik} = 0$. If $k \geq i$ then because $i > j$, $k > j$. So since $B$ is upper triangular, $B_{kj} = 0$. So no matter what $k$ is, $A_{ik} B_{kj} = 0$, which means that

$$\sum_{k=1}^{n} A_{ik} B_{kj} = 0 + 0 + \cdots + 0 = 0.$$

So $AB$ is also upper triangular. □