Make up quiz (1–3) Name: ______________________________

Complete ONE of the following problems. Only one problem will be graded, and the score you receive will replace the score you received on the original quiz that covered the same material, unless your new score is lower.

1. Find all solutions to the differential equation $y' = 2(1+y)^2t^2$ AND find the specific solution when $y(1) = -1$. Write both answers in the form $y =$______.
2. Write an autonomous (depending on y only) differential equation that has a single, stable equilibrium at $y = b$. Solve it and show that all solutions approach the line $y = b$.
3. Solve the initial value problem $\frac{1}{2}y'' + y' + 100y = 0$, $y(0) = 0$, $y'(0) = 2$. Write your final answer in terms of real numbers only.

Circle the problem you would like me to grade.

Solution:

1. The differential equation is separable. After separating and integrating, we get $-\frac{1}{1+y} = \frac{2}{3}t^3 + c$. Solving for y, we get $y = -1/(\frac{2}{3}t^3 + c) - 1$. There is also a constant solution, $y = -1$. The solution to the initial value problem is also the constant solution $y = -1$.
2. One possibility is $y' = b - y$. The solution to this equation is $y = b - ce^{-t}$. This differential equation is valid for all t, so we can take the limit as $t \to \infty$, which gives $y = b$.
3. The characteristic equation has roots $-1 \pm \sqrt{199}\,i$. This means that the general solution is $y = e^{-t}\left(c_1\cos(\sqrt{199}\,t) + c_2\sin(\sqrt{199}\,t)\right)$. The first initial condition tells us $c_1 = 0$, and the second tells us $2 = \sqrt{199}\,c_2$, which means $c_2 = \sqrt{199}/2$.