Make up quiz (4–6) Name: ____________________

Complete ONE of the following problems. Only one problem will be graded, and the score you receive will replace the score you received on the original quiz that covered the same material, unless your new score is lower.

4. Find the steady-state solution to the differential equation $y'' + 5y' + 2y = 9e^{-10t} + 1$

5. Find the inverse Laplace transform of

$$F(s) = \frac{2s^2 + 10}{s(s^2 + 4s + 10)}.$$

6. Find the Laplace transform of

$$g(t) = \begin{cases} 2 & \text{if } t < 1 \\ t^2 & \text{if } 1 < t < 3 \\ 0 & \text{if } 3 < t \end{cases}$$

Circle the problem you would like me to grade.

Solution:

4. The particular solution is of the form $Y = Ae^{-10t} + B$. Plugging this into the differential equation, we get $100Ae^{-10t} - 50Ae^{-10t} + 2Ae^{-10t} + 2B = 9e^{-10t} + 1$. So we have one equation of $100A - 50A + 2A = 9$, and another of $2B = 1$. Thus $A = 52/9$ and $B = 1/2$. The steady-state solution is therefore $y(t) = (52/9)e^{-10t} + 1/2$.

5. Using partial fractions, we get

$$F(s) = \frac{1}{s} + \frac{s+6}{s^2+4s+10} = \frac{1}{s} + \frac{s+2}{(s+2)^2+6} + \frac{4}{(s+2)^2+6}.$$

The inverse Laplace transform is

$$1 + e^{-2t}\cos(\sqrt{6}\,t) + \frac{4}{\sqrt{6}}e^{-2t}\sin(\sqrt{6}\,t).$$

6. In terms of Heaviside functions,

$$\begin{aligned} g(t) &= 2(1 - u_1(t)) + t^2(u_1(t) - u_3(t)) \\ &= 2 - 2u_1(t) + t^2u_1(t) - t^2u_3(t). \end{aligned}$$

The Laplace transform is

$$\begin{aligned} \mathcal{L}\{g(t)\} &= \mathcal{L}\{2\} - \mathcal{L}\{2u_1(t)\} + \mathcal{L}\{t^2u_1(t)\} - \mathcal{L}\{t^2u_3(t)\} \\ &= \frac{2}{s} - e^{-s}\mathcal{L}\{2\} + e^{-s}\mathcal{L}\{(t+1)^2\} - e^{-3s}\mathcal{L}\{(t+3)^2\} \\ &= \frac{2}{s} - e^{-s}\left(\frac{2}{s}\right) + e^{-s}\left(\frac{2}{s^3} + \frac{2}{s^2} + \frac{1}{s}\right) - e^{-3s}\left(\frac{2}{s^3} + \frac{6}{s^2} + \frac{9}{s}\right) \end{aligned}$$