Previous weeks: http://individual.utoronto.ca/nathangs/teaching/tutoring.html

1 Functions and Their Inverses

Recall the definition of a function:

Definition 1. Given two sets A and B a function f from A to B is denoted $f : A \to B, a \in A \mapsto f(a) \in B$. It's defining property is that every input has a unique output. That is, if $a \in A$ and $f(a) = b_1 \in B$ and $f(a) = b_2 \in B$, then $b_1 = b_2$. A is called the domain and B is called the codomain.

Remark. The textbook uses the term "independent" variables for elements in the domain of f and "dependent" variables for elements in the range of f. **Don't** confuse this with the physical meaning of "independent" and "dependent"! For example, you could walk in a straight line for 30 seconds and write the distance you've travelled as a function of time. This aligns with our physical understanding of the terms independent and dependent. However, you could just as easily write time as a function of the distance you've travelled, and so in this case you'd call "distance" the independent variable and "time" the dependent variable. However, clearly the passage of time does not depend on you walking around! Quite an unfortunate usage of English in math.

1. Which of the following equations can be written as a function?
   (a) $4x + 2y = 6$
   (b) $y - 3x^2 + x = 8$
   (c) $x^2 + y^2 = 1$
   (d) $x^2 + x - 2 + y^3 = 0$
2. State the domain and range for the following functions
   (a) $f(x) = x + 2$
   (b) $f(x) = |x| - 2$
   (c) $f(x) = \sqrt{x - 1}$
   (d) $f(x) = \sqrt{1 - x^2}$
   (e) $f(x) = \frac{1}{x}$
   (f) $f(x) = x^{1/3}$
3. The **inverse** of a function $f : A \to B$ is a function $g : B \to A$ such that $g(f(a)) = a$ for all $a \in A$. Find the inverse function for each of the following functions:
   (a) $f(x) = \sqrt{x}$
   (b) $g(x) = x + 1$
   (c) $g(x) = \frac{1}{x}$

2 Logic

Mathematics, at its core, is based on logic. The general idea of mathematics is that we start with statements we assume to be true, **axioms**, and see how many additional facts we can derive from those axioms. We look at some basic logic now.

A logical argument is the combination of "premises", statements we assume to be true, and a "conclusion", a statement we are trying to say is true, given the premises.

Example 1. It will either rain or snow tomorrow. It's too warm for snow. Therefore, it will rain.

Here, we have two premises

1. It will either rain or snow tomorrow
2. It's too warm for snow

and one conclusion: It will rain.

A "valid" logical argument is one in which, if all of the "premises" are true, the conclusion **must** be true. In every other situation we have an invalid logical argument.

1. In each of the following logical arguments: (i) identify the premises and conclusion and (ii) decide if the argument is valid or not, and justify why.
   (a) If today is Sunday, then I don't have to go to work today. Today is Sunday. Therefore, I don't have to go to work today.
   (b) Either the butler is guilty or the maid is guilty. Either the maid is guilty or the cook is guilty. Therefore, either the butler is guilty or the cook is guilty.
   (c) A number n is either positive, negative, or zero. n is negative. Therefore, n is positive.
   (d) Every root of a quadratic function $f(x) = ax^2+bx+c, a \neq 0$ must be of the form $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$. r satisfies $f(r) = 0$. Thereofre, $r = \frac{-b+\sqrt{b^2-4ac}}{2a}$ or $r = \frac{-b-\sqrt{b^2-4ac}}{2a}$