# 1 Analytic Functions

We look at different properties of analytic functions.

1. We show that if $f(z)$ is analytic then it "only" depends on $z$. For this question, we'll assume the opposite and write $f(z, \overline{z})$.

   (a) Using the fact that $z = x + iy$ and $z = x - iy$, as well as the chain rule, show that

   $$\frac{\partial f}{\partial z} = \frac{1}{2}\left(\frac{\partial f}{\partial x} - i\frac{\partial f}{\partial y}\right) \qquad \frac{\partial f}{\partial \overline{z}} = \frac{1}{2}\left(\frac{\partial f}{\partial x} - i\frac{\partial f}{\partial y}\right)$$

   (b) Compute the difference quotient with $\Delta y = 0$ to show that $f'(z) = \frac{\partial f}{\partial x}$.

   (c) Compute the difference quotient with $\Delta x = 0$ to show that $f'(z) = -i\frac{\partial f}{\partial y}$.

   (d) Use the above three facts to conclude $\frac{\partial f}{\partial \overline{z}} = 0$.

   *Note:* If you want to see more of this, it's found in Complex Analysis by Lars V. Ahlfors, who also happens to be the first winner of the Fields medal!

# 2 Finding Solutions to Polynomials

1. Find all solutions to $p(z) = (z^3 + 1)(z^2 - 2) = z^5 - 2z^3 + z^2 - 2$.
2. Find all solutions to $p(z) = (z^2 - 3i)(z^2 - 3) = z^4 + -3z(1 + i) + 9i$

# 3 Multiplication by a Complex Number

This is not quite relevant to the class but I think it's an interesting concept.

1. Argue that, if $S$ is a circle and $z_0$ is a complex number, then $z_0 S$ is also a circle.

   *Hint:* Use the fact that multiplication by a complex number $z_0 = \alpha + i\beta$ looks like a linear map $\begin{pmatrix} \alpha & -\beta \\ \alpha & \beta \end{pmatrix}$. That is, $z_0 z = \begin{pmatrix} \alpha & -\beta \\ \beta & \alpha \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$