Nathan Helms: CMSC208: HW1

1.
$$\sum_{i=1}^{n} i^3 = \frac{1}{4}n^2\left(n+1\right)^2$$

Base Case:
$$\sum_{i=1}^{1} i^3 = \frac{1}{4}1^2\left(1+1\right)^2 = \frac{1}{4} * 2^2 = \frac{4}{4} = 1$$

Assume:
$$\sum_{i=1}^{k} i^3 = \frac{1}{4}k^2\left(k+1\right)^2$$

What we Want:
$$\frac{1}{4}\left(k+1\right)^2\left(k+2\right)^2$$

Inductive Step:
$$\sum_{i=1}^{k+1} i^3 = \left(\sum_{i=1}^{k} i^3\right) + \left(k+1\right)^3$$

$$\sum_{i=1}^{k+1} i^3 = \frac{1}{4}k^2\left(k+1\right)^2 + \left(k+1\right)^3$$

$$\sum_{i=1}^{k+1} i^3 = \frac{4\left(k+1\right)^3}{4} + \frac{k^2\left(k+1\right)^2}{4}$$

$$\sum_{i=1}^{k+1} i^3 = \frac{\left(k+1\right)^2\left(4\left(k+1\right)+k^2\right)}{4}$$

$$\sum_{i=1}^{k+1} i^3 = \frac{\left(k+1\right)^2\left(4k+4+k^2\right)}{4}$$

$$\sum_{i=1}^{k+1} i^3 = \frac{\left(k+1\right)^2\left(k+2\right)^2}{4}$$

$$\sum_{i=1}^{k+1} i^3 = \frac{\left(k+1\right)^2\left(k+1+1\right)^2}{4}$$

Which is what we wanted to show

2. a) The sum of any three consecutive integers is even
Disproven by Counterexample:

$$4+5+6=15$$

which is an odd sum.
b) The product of any three consecutive integers is even
Proven by Construction:

$$4 * 5 * 6 = 120$$

$$1 * 2 * 3 = 6$$

$$2 * 3 * 4 = 24$$

which are all even products.

3. Applying the Pigeonhole principle to this problem leaves us with:

$$7/2 = 3.5$$

4. Compute the following

$$\sum_{i=1}^{10} (-1)^i \, i = 5$$

$$\sum_{i=1}^{1000} (-1)^i \, i = 500$$

5. Explain the symbolic expression:
For all positive integers, n, in set E, there exists two prime numbers, p1 and p2, of set P, that add together to equal n.

6. Proven by Construction:
if x = 0, and n = 1

$$(1+0)^1 \geq 1 + (1)(0) = 1 \geq 1 = TRUE$$

if x = 100, and n = 5

$$(1+100)^5 \geq 1 + (5)(100) = 10,510,100,501 \geq 500 = TRUE$$