

Mergers, Entry, and Consumer Welfare*

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Abstract

We analyze mergers and entry in a differentiated products oligopoly model of price competition. Any merger that does not yield efficiencies is unprofitable if it induces entry sufficient to preserve pre-merger consumer surplus. Thus, mergers occur in equilibrium only if barriers limit entry. Mergers that increase consumer surplus can occur in equilibrium for specific magnitudes of efficiencies and post-merger entry, and these combinations are identified from pre-merger market shares. The entry costs that would rationalize post-merger entry similarly can be bounded using pre-merger market shares. An application to the T-Mobile/Sprint merger illustrates the theoretical framework.

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1 Introduction

The antitrust review of mergers in the United States follows a set of evaluation criteria summarized in the 2010 Horizontal Merger Guidelines of the Department of Justice and Federal Trade Commission.¹ An important step examines whether entry is easy enough that the merging firms would find it unprofitable to raise prices or otherwise reduce competition. The Guidelines state that “[e]ntry is that easy if entry would be timely, likely, and sufficient...” and propose that entry barriers and the capabilities of prospective entrants be analyzed empirically.² However, little theoretical guidance is provided about how to interpret the results of the analysis, either on its own or in relation to other considerations such as market concentration, the degree of competition between the merging firms, and the magnitude of efficiencies.

In this paper, we provide a theoretical underpinning for entry analysis in merger review. We focus on the canonical model of Bertrand competition with logit demand.³ We start under the assumption that the merger does not generate efficiencies. Here we obtain a stark result: entry is never sufficient to eliminate the consumer surplus loss caused by profitable mergers. The result arises due to a selection effect, namely that entry sufficient to preserve consumer surplus renders mergers unprofitable. As we would expect firms to merge only if doing so is profitable, the presence of a merger suggests that barriers obstruct entry.⁴ The result extends to the nested logit demand system if consumer substitution between the products of the merging firms and those of the entrant is sufficiently strong. We interpret the analysis as calling into question entry as a stand-alone consideration in merger review, outside of exceptional cases.

Our revealed-preference argument can be overturned in the presence of an efficiency—which we model as higher qualities or lower marginal costs—because an efficiency increases the profitability of a merger. This motivates a joint analysis of entry and efficiencies. With both present, we prove that profitable mergers can increase consumer

¹Henceforth, the “Guidelines.”

²Guidelines, §9. See also Shapiro (2010) for a useful discussion. The standard of timely, likely, and sufficient also appears in the 2004 Merger Guidelines of the European Union.

³The Bertrand/logit model has long been a workhorse model for merger review in differentiated products settings (Werden and Froeb (1994, 2002)). It also provides the foundation for seminal academic research based on the more flexible random-coefficients logit model (e.g. Berry et al. (1995); Nevo (2001)). Miller and Sheu (2020) explain why the logit restrictions on consumer substitution often are reasonable in properly defined antitrust markets used for merger review.

⁴Farrell and Shapiro (1990) develop a similar revealed-preference argument in the context of merger efficiencies with Cournot competition: efficiencies may be possible to infer from the proposal of a merger that otherwise appears unprofitable. Werden (1991) discusses reasons that unprofitable mergers might sometimes be observed.

surplus, even if neither entry nor the efficiency eliminates consumer surplus loss on its own. Among other results, we show that any such merger must achieve at least a *minimum efficiency*, which is fully determined by pre-merger market shares. We also find that consumer surplus is not monotonically increasing in the size of the efficiency, as the efficiency can be large enough to deter post-merger entry, but too small to offset adverse competitive effects. Our integrated framework provides a theoretically-grounded way to interpret the empirical evidence gathered in the merger review process.

The paper proceeds as follows. We lay the foundation for our analysis in Section 2. We describe a three-stage oligopoly game of perfect information in which (1) two incumbents decide whether to merge, (2) a prospective entrant decides whether to incur a cost to enter, and (3) prices are determined by differentiated-products Bertrand competition with logit demand. The equilibrium concept is subgame perfection. Following standard practice in merger review, we focus especially on the case of *merger-induced entry*, in which entry is profitable if and only if a merger occurs.⁵

To make the analysis tractable, we write the third stage as an aggregative game, following Nocke and Schutz (2018). The contribution of each firm to equilibrium depends on its “type,” which summarizes the qualities and marginal costs of all its products. This allows us to characterize outcomes in terms of a simple firm-level primitive. Consumer surplus increases in the types of each firm in the market, and the profit of any firm increases in its type and decreases in the types of its competitors. A merger with neither entry nor efficiencies reduces consumer surplus and increases profit.

We provide our analysis of mergers and entry without efficiencies in Section 3. Our broad objective is to characterize the conditions under which profitable mergers generate merger-induced entry such that consumer surplus is at least weakly greater than in a scenario without merger and entry. We define the *compensating entrant* as the entrant that would equate pre-merger and post-merger consumer surplus, and show that its type can be calculated with pre-merger market shares. We also define the entrant that would make merger profit-neutral as the *best-case entrant*, as it provides the greatest possible consumer surplus in a subgame perfect equilibrium (SPE) featuring merger and merger-induced entry. We prove that the compensating entrant has a higher type than the best-case entrant, meaning that merger-induced entry is never sufficient to eliminate the adverse consumer surplus effects of profitable mergers. Restated, there is no SPE in which a merger occurs and consumer surplus does not fall. The result

⁵The Guidelines, §9, state: “[t]his section concerns entry or adjustments to pre-existing entry plans that are induced by the merger.”

extends to incumbent repositioning, entry by any number of firms, or any combination of repositioning and entry—none of these eliminate the adverse competitive effects of profitable mergers.

The integrated framework for mergers with entry and efficiencies is in Section 4. If entry costs are not too large, the SPE can feature merger with entry, merger without entry, or no merger (and no entry). The outcome depends on the type of the prospective entrant, T^F , and the magnitude of the efficiency, E . Three *neutrality curves* in (T^F, E) space characterize whether merger with entry would (i) increase consumer surplus, (ii) be profitable for the merging firms, and (iii) increase the profitability of entry. There exist combinations of (T^F, E) for which merger and entry occur in SPE and consumer surplus increases. The efficiency must be greater than an *efficiencies lower bound*, which depends on the type of the prospective entrant. The trough of this lower bound is the minimum efficiency that can be calculated from pre-merger market shares.

We also use the integrated framework to examine the profitability of entry. We show that an efficiency greater than an *efficiencies upper bound* is large enough to deter post-merger entry, but can be too small to offset adverse competitive effects. We also characterize numerically how mergers affect the profitability of entry, under the assumption of lower bound efficiencies—so that merger and entry are profitable and consumer surplus is preserved. If there are four symmetric incumbents, a merger increases the profitability of entry between 0 and 7.4 percent, depending on the entrant's type. These effects are larger in more concentrated markets, and smaller if efficiencies exceed the lower bound or with asymmetric merging firms. Merger-induced entry occurs in SPE if the profit gains are necessary and sufficient to recoup the entry cost.

The results described above are obtained under the assumption that demand is logit. In Section 5, we extend the analysis of mergers with entry to nested logit demand. We first consider the case in which the products of the merging firms and the prospective entrant are in the same nest, and thus are relatively close substitutes. In this case, our main result proves robust—there is no SPE in which a merger occurs and consumer surplus does not fall. If entry were to occur in a different nest, then our finding also extends, provided that the nesting parameter is not too large. We obtain this result by establishing the continuity of equilibrium with respect to the nesting parameter.

Section 6 contains two extensions. First, we provide some numerical evidence for the extent of possible outcomes that may result with post-merger entry. We use numerical experiments to evaluate how much best-case entry mitigates the adverse com-

petitive effects of mergers.⁶ Given our theoretical results, mitigation is guaranteed to be incomplete, but the magnitudes may be of practical interest to antitrust authorities nonetheless. We consider a range of hypothetical mergers and obtain the type of the best-case entrant for each. We find that best-case entry never counteracts more than 30 percent of the merging firms' price increase. If there are relatively few incumbents and the merging firms are small, then best-case entry can mitigate the bulk of consumer surplus loss, mainly by increasing product diversity. Otherwise, best case entry tends to mitigate less than half of consumer surplus loss.

Second, we provide an application to the T-Mobile/Sprint merger to demonstrate how the theoretical framework can be applied. In 2020, a Federal District Court ruled that the merger between two of the four largest mobile wireless operators in the United States could proceed, in part due to the expectation that DISH would successfully enter the market. We calibrate our model with publicly-available data on market shares, prices, and markups, and also a market elasticity of demand that appears in regulatory filings. Using a series of simulations, we show that there is no SPE featuring both merger and merger-induced entry by DISH. Merger-induced entry by DISH makes the merger unprofitable if efficiencies are small, but, with efficiencies large enough for merger profitability, the merger reduces the profitability of DISH entry.

We conclude in Section 7 with a short summary of the research project and a discussion about the role of entry in merger review.

1.1 Literature Review

Our research builds on a number of articles that consider the relationship between mergers and entry. The closest are Werden and Froeb (1998) and Spector (2003), which develop revealed preference arguments similar to ours. Werden and Froeb examine Bertrand competition with logit demand. For a large number of randomly-generated markets, they find that the vast majority of mergers are unprofitable if entry occurs, and further that mergers do not increase the entrant's profit by much. In explaining why they rely on simulation-based evidence, Werden and Froeb write:

Analytical methods are of little use with this model because products are differentiated and because predictions vary with demand parameters and

⁶We develop a partial calibration routine with which merger effects can be obtained from market shares. The routine itself provides a contribution to the antitrust literature by relaxing the data requirements for merger simulation. See also Nocke and Whinston (2020).

market shares.⁷

Our results are broadly consistent with those of Werden and Froeb, but they are sharper and (mostly) can be proven analytically. Further, we are able to incorporate merger efficiencies and provide an integrated framework for merger analysis. These innovations are possible due to the aggregative games framework of Nocke and Schutz (2018).

Spector (2003) examines a Cournot oligopoly model under the general assumptions employed in Farrell and Shapiro (1990). The main result is that—in the absence of marginal cost efficiencies—profitable mergers are incompatible with merger-induced entry that preserves consumer surplus. Spector concludes that “a merger unambiguously generating no synergies should be prevented, without delving into the question of entry.”⁸ We obtain the same theoretical result in the context of differentiated-products Bertrand competition. Further, our integrated framework allows us to characterize the minimum level of efficiencies necessary for pro-competitive profitable mergers.

Other articles explore mergers and entry under an assumption of free entry, under which fringe firms endogenously participate in the market both pre- and post-merger (e.g., Davidson and Mukherjee (2007); Anderson et al. (2020)). Among the main results are that (i) mergers do not affect consumers in long run equilibrium due to the fringe response, and (ii) mergers are unprofitable in the absence of efficiencies. Free entry is analogous to an assumption (in our setting) that compensating entry occurs post-merger. Whether such an assumption is appropriate depends on the situation. In our experience, the mergers that garner the most scrutiny are those where the prospective entrants are limited in number, have uncertain capability, and face entry costs. Our modeling framework is intended to inform merger review in such settings.⁹

To our knowledge, the integrated framework for mergers with entry and efficiencies that we provide is novel in the literature. Some of the concepts nonetheless are familiar. In the limit, as the type of the prospective entrant converges to zero, mergers reduce consumer surplus if and only if the merger efficiency is less than the *compensating efficiency*, which is derived elsewhere (e.g., Werden (1996); Nocke and Whinston (2020)). Indeed, the neutrality curve for consumer surplus can be interpreted as extending that concept to scenarios featuring entrants of varying types. Further, previous research points out that merger efficiencies reduce the scope for profitable entry

⁷Werden and Froeb (1998, p. 527).

⁸Spector (2003, p. 1597).

⁹See the Guidelines, §9, which state that “[w]here an identifiable set of firms appears to have necessary assets that others lack, or to have particularly strong incentives to enter, the Agencies focus their entry analysis on those firms.”

(Cabral (2003); Erkal and Piccinin (2010)).

Empirical research on mergers and entry is hampered by selection: observed mergers are (presumably) both profitable and competitively benign. Recent applications address the issue by estimating structural models of competition—including the distribution of entry costs and fixed costs—exploiting observed entry and exit in the data (Li et al. (2019); Ciliberto et al. (2020); Fan and Yang (2020)). Post-merger equilibrium then can be computed allowing for entry or incumbent repositioning. This empirical approach is complementary to our theoretical framework. First, these papers employ logit-based demand systems, such as the random coefficients logit model of Berry et al. (1995), so we suspect our results extend. Second, our theoretical approach informs the magnitude of entry costs (or fixed costs) that could generate merger-induced entry, whereas the empirical approach informs the realized magnitude of those costs.

Finally, our paper contributes to recent research that applies the aggregative games framework of Nocke and Schutz (2018) to antitrust. Nocke and Schutz (2019) provide conditions under which the change in the Herfindahl Index approximates the market power effects of a merger, and also examine merger efficiencies. Garrido (2019) explores endogenous product portfolios in a dynamic game. Nocke and Whinston (2020) derive the efficiencies necessary to counterbalance adverse merger effects. Alvarez et al. (2020) examine global beer mergers and the adequacy of divestitures.

2 Modeling Framework

In this section, we lay out our multi-stage model of merger and entry. We focus on the logit demand function as our baseline, and extend to the nested logit in Section 5.

2.1 Setup

We examine a three-stage game of perfect information. Let there be $f = 1, 2, \dots, F$ firms, with $F \geq 3$. Without loss of generality, the first $F - 1$ firms are incumbents, and firm F is a prospective entrant. Each firm is endowed with a type, $T^f \geq 0$, that summarizes the qualities and marginal costs of its products. The types remain fixed throughout the game unless specifically noted. The timing of the game is as follows:

1. Firms 1 and 2 decide whether to merge to form the combined firm, M . A merger commits these firms to maximize joint profits when setting prices in stage 3. The type of the merged firm is $T^M = T^1 + T^2 + E$ for efficiency $E \geq 0$.

2. Firm F observes whether merger occurs in stage 1 and decides whether to enter. If it enters, it incurs an entry cost, $\chi > 0$, the value of which is commonly known.
3. All firms observe whether merger and entry occur in stages 1 and 2. The incumbents and, if entry occurs, the entrant, form the set \mathcal{F} . The firms in \mathcal{F} choose prices simultaneously, consumers make purchasing decisions, and firms earn variable profit according to differentiated-products Bertrand equilibrium.

Our solution concept is subgame perfect equilibrium (SPE). In the next subsections, we examine the pricing subgame and then turn to the merger and entry decisions.

2.2 The Pricing Subgame

Our main analyses are conducted under the assumption that payoffs are determined by Bertrand competition among firms facing a logit demand system. Here we characterize equilibrium outcomes in the pricing subgame using a standard product-level formulation for multi-product firms, and then again using the aggregative game reformulation of Nocke and Schutz (2018). Although the reformulation requires some additional notation, it is essential for our subsequent results because it allows us to summarize all of a firm's product qualities and marginal costs in a single number. Finally, we establish some helpful equilibrium properties.

Product-level Representation

Let there be a finite and nonempty set of differentiated products \mathcal{J} available to consumers. Each consumer purchases a single product or forgoes a purchase by selecting the outside good. Let the indirect utility that consumer i receives from product $j \in \mathcal{J}$ be given by

$$u_{ij} = v_j - \alpha p_j + \epsilon_{ij},$$

where v_j and p_j are the quality and price of product j , α is a price coefficient, and ϵ_{ij} is a consumer-specific preference shock. The indirect utility provided by the outside good is $u_{i0} = v_0 + \epsilon_{i0}$, and we apply the standard normalization $v_0 = 0$.

We assume that the preference shocks are independently and identically distributed with a Type 1 extreme value distribution, and that consumers maximize utility. This

generates the logit market share:

$$s_j(p) = \frac{\exp(v_j - \alpha p_j)}{1 + \sum_{k \in \mathcal{J}} \exp(v_k - \alpha p_k)}, \quad (1)$$

for a vector of prices, p . The share of the outside good is $s_0(p) = 1/(1 + \sum_{k \in \mathcal{J}} \exp(v_k - \alpha p_k))$. Consumer surplus is given by

$$CS = \frac{1}{\alpha} \ln \left(1 + \sum_{j \in \mathcal{J}} \exp(v_j - \alpha p_j) \right). \quad (2)$$

On the supply-side of the model, let firms be indexed by f and the set of firms be denoted by \mathcal{F} . The products in \mathcal{J} are partitioned into a series of sets, where the set \mathcal{J}^f indicates the products sold by firm f . The profit of firm $f \in \mathcal{F}$ is

$$\pi^f(p) = \sum_{j \in \mathcal{J}^f} (p_j - c_j) s_j(p), \quad (3)$$

where c_j is the marginal cost of product j . We assume that each firm maximizes its profit conditional on the prices of other firms. The first order conditions for profit maximization take the form

$$\sum_{k \in \mathcal{J}^f} (p_k - c_k) \frac{\partial s_k(p)}{\partial p_j} + s_j(p) = 0. \quad (4)$$

A price vector that satisfies these first order conditions defines a Bertrand equilibrium.

Aggregative Game Reformulation

The primitives of the firm-level model are the vector of firm-specific types, $\{T^f\} \forall f \in \mathcal{F}$, and the price parameter, α . The type of firm f is defined as

$$T^f \equiv \sum_{j \in \mathcal{J}^f} \exp(v_j - \alpha c_j),$$

which represents the firm's contribution to consumer surplus if its prices equal its marginal costs. From these primitives, the Bertrand equilibrium can be characterized as a vector of " ι -markups," $\{\mu^f\} \forall f \in \mathcal{F}$, a vector of firm-level market shares, $\{s^f\} \forall f \in \mathcal{F}$, and a market aggregator, H . We define markups below, and let $s^f = \sum_{j \in \mathcal{J}^f} s_j$. The

aggregator is defined as $H \equiv 1 + \sum_{j \in \mathcal{J}} \exp(v_j - \alpha p_j)$, which is the denominator from the market share formula of the product-level model (see (1)).

We first derive a relationship between the ι -markups and firm-level market shares. The product-specific price derivatives for logit demand are

$$\frac{\partial s_j}{\partial p_k} = \begin{cases} -\alpha s_j(1 - s_j) & \text{if } k = j \\ \alpha s_j s_k & \text{if } k \neq j. \end{cases}$$

Substituting these demand derivatives into the first order conditions of (4) for some product j and rearranging gives

$$\alpha(p_j - c_j) = 1 + \alpha \sum_{k \in \mathcal{J}^f} (p_k - c_k) s_k. \quad (5)$$

The right-hand-side of this equation does not depend on the which product $j \in \mathcal{J}^f$ is referenced. This implies that the left-hand-side is equivalent for all products sold by firm f , meaning each firm imposes a common markup. Define the ι -markup of firm f as $\mu^f \equiv \alpha(p_j - c_j) \forall j \in \mathcal{J}^f$. Substituting back into equation (5) obtains

$$\mu^f = \frac{1}{1 - s^f} \quad (6)$$

Turning to the shares, we have $s^f = (1/H) \sum_{j \in \mathcal{J}^f} \exp(v_j - \alpha p_j)$ from (1), after substituting in for the definition of the aggregator, H . Adding and subtracting αc_j inside the exponential and applying the definitions of μ^f and T^f gives

$$s^f = \frac{T^f}{H} \exp(-\mu_f) \quad (7)$$

$$\iff \frac{T^f}{H} = s^f \exp\left(\frac{1}{1 - s^f}\right) \quad (8)$$

so that profit maximization implies that shares can be written $s^f = s(T^f/H)$. Plugging (7) into (6), we obtain that ι -markups satisfy

$$\mu_f \left(1 - \frac{T^f}{H} \exp(-\mu_f)\right) = 1. \quad (9)$$

Let the unique solution for μ_f from this expression be written as $m(T^f/H)$. The *markup fitting-in function*, $m(\cdot)$, has the properties that $m(0) = 1$ and $m'(\cdot) > 0$. To close the

system, the aggregator satisfies an adding-up constraint,

$$\frac{1}{H} + \sum_{f \in \mathcal{F}} s^f = 1, \quad (10)$$

which applies because market shares sum to one. A Bertrand equilibrium is defined by the shares, markups, and aggregator that satisfy (8)-(10).

Nocke and Schutz (2018) prove that a unique Bertrand equilibrium exists. The following equations characterize profit, consumer surplus, welfare, and dead-weight loss in equilibrium:

$$\pi^f \left(\frac{T^f}{H} \right) = \frac{1}{\alpha} (\mu^f - 1), \quad (11)$$

$$CS = \frac{1}{\alpha} \ln(H), \quad (12)$$

$$W = \frac{1}{\alpha} \left(\ln(H) + \sum_{f \in \mathcal{F}} (\mu^f - 1) \right), \quad (13)$$

$$DWL = \frac{1}{\alpha} \ln \left(1 + \sum_{f \in \mathcal{F}} T^f \right) - \frac{1}{\alpha} \left(\ln(H) + \sum_{f \in \mathcal{F}} (\mu^f - 1) \right). \quad (14)$$

Equation (11) is obtained by rearranging (5).¹⁰ That profit is a function of the ratio T^f/H follows from the observation that shares and markups are also functions of the same ratio. Equation (12) is obtained by substituting H into (2). Equation (13) is the sum of consumer surplus and profit. Finally, the first term on the right-hand-side of (14) is welfare under marginal cost pricing and the second term is realized welfare. It becomes relevant for our numerical analyses that the price coefficient, α , enters these objects as an inverse multiplicative constant. This allows for partial calibration and simulation, as percentage changes can be obtained without knowledge of α .

Equilibrium Properties

Before turning to the analysis of mergers and entry, we characterize some helpful properties of the ι -markup, profits, and prices. First, each firm's markup, profit, and share increase with its type and decrease with the types of competitors:

Proposition 1. *For every firm $f \in \mathcal{F}$, the ι -markup, μ^f ; profit, π^f ; and market share,*

¹⁰Notice that the left-hand side of (5) is μ^f and the right-hand side is $1 - \alpha\pi^f$.

$s_j \forall j \in \mathcal{J}^f$ are all increasing in the ratio T^f/H . Furthermore, these objects are also increasing in own-type, T^f , and decreasing in rivals' types, $T^g \forall g \neq f$.

Proof. See the Appendix, which follows Nocke and Schutz (2018a, Proposition 6).

Building on this result, consider a merger that enables a set of merging firms, $\mathcal{C} \subset \mathcal{F}$, to maximize joint profit. Further assume, for now, that the set of all firms is held fixed, meaning there is no entry. In the aggregative games formulation, the type of the merged firm is $T^M = \sum_{f \in \mathcal{C}} T^f$. The effect of such a merger is to increase the markup, profit, and prices of all firms:

Proposition 2. *Assume there are no efficiencies or entry. For every firm $f \in \mathcal{F}$, the ι -markup, μ^f , and profit, π^f , increase due to a merger among firms in $\mathcal{C} \subset \mathcal{F}$, but the market aggregator, H , and consumer surplus, CS , decrease.*

Proof. See the Appendix, which follows Anderson et al (2018, Section 4.3), or as a special case of Garrido (2020).

Finally, we can characterize the relationship between types and market shares. Given either shares or types, we can recover the other.

Proposition 3. *The unique Bertrand equilibrium is fully identified up to a multiplicative constant by either a vector of firm-level market shares, $\{s^f\} \forall f \in \mathcal{F}$, or a vector of firm types $\{T^f\} \forall f \in \mathcal{F}$. In particular, knowing either shares and types allows one to compute the other.*

Proof. See the Appendix. □

2.3 Subgame Perfect Equilibrium

We now turn to the entry and merger decisions. Firm F enters if it can earn positive profits in the Bertrand pricing stage, taking into account its entry costs and whether a merger has occurred. That is, entry occurs if the profit of firm F satisfies

$$\pi^F \left(\frac{T^F}{H_{*,e}} \right) - \chi \geq 0,$$

where we let $H_{*,e}$ be the market aggregator with entry, accounting for the observed action of firms 1 and 2 (denoted by $*$). Firms 1 and 2 merge if doing so increases their

combined profit in the pricing stage, taking into account the effect of the merger on the entry decisions. That is, a merger occurs if and only if it increases joint profits:

$$\pi^m \left(\frac{T^M}{H_{m,*}} \right) \geq \pi^1 \left(\frac{T^1}{H_{nm,*}} \right) + \pi^2 \left(\frac{T^2}{H_{nm,*}} \right), \quad (15)$$

where we let $H_{m,*}$ and $H_{nm,*}$ be the market aggregators with and without a merger, incorporating the best-response of the market outsider. A unique SPE of the three-stage game exists because the merger and entry decisions are made sequentially.

In much of our analysis, we focus on *merger-induced entry*, which arises if the equilibrium strategy of the prospective entrant is to enter if and only if merger occurs. Applying Propositions 1 and 2 verifies that a merger does indeed increase the profitability of entry and, as a corollary, for any merger there exists some entry cost such that merger-induced entry occurs. Thus, we state formally the following assumption and apply it in our subsequent analysis:

Assumption 1: *Entry is profitable if and only if a merger occurs in the first stage, meaning:*

$$\pi^F \left(\frac{T^F}{H_{nm,e}} \right) < \chi \leq \pi^F \left(\frac{T^F}{H_{m,e}} \right),$$

where $H_{nm,e}$ is the market aggregator with no merger, but with entry, and $H_{m,e}$ is the market aggregator with both merger and entry.

Assumption 1 allows us to leave aside cases that are less interesting for our analysis. In particular, we rule out instances in which entry costs are so low or so high that the entrant always enters or always refrains from entering, respectively. In those cases, merger has no impact on entry, which in turn means that firms 1 and 2 always elect to merge in order to reap the rewards of joint profit maximization (see Proposition 2).

We focus on how mergers affect equilibrium outcomes. To do so, we compare the equilibria in which a merger occurs in the first stage (i.e., the merger is profitable) to those that arise under a counterfactual without the merger. In all that follows, we will carry through the assumption that the merging entities both are active market participants, that is $T^1, T^2 > 0$. We refer to these scenarios as the *post-merger equilibria* and the *pre-merger equilibria*, respectively. We are now in position to state a useful intermediate result:

Proposition 4. *Let firm f be a non-merging incumbent with $T^f > 0$, and let $*$ denote either ‘merger, no entry’ or ‘merger, entry.’ If, for either case, any of the following conditions*

holds, then all of the following conditions hold for that case.

- (i) The merger does not affect the profitability of firm f .
- (ii) The merger does not affect consumer surplus.
- (iii) Market shares satisfy the following equality:

$$s_{nm,ne}^1 + s_{nm,ne}^2 = s_*^M + \sum_{\{f \in \mathcal{F}_* \setminus \mathcal{F}_{nm,ne} | f \neq m\}} s_*^f.$$

Proof. See the Appendix. □

As a corollary to the Proposition 4, a merger decreases consumer surplus if $s_{nm,ne}^1 + s_{nm,ne}^2 > s_{m,e}^M + s_{m,e}^F$, and increases consumer surplus if $s_{nm,ne}^1 + s_{nm,ne}^2 < s_{m,e}^M + s_{m,e}^F$. This suggests a simple rule-of-thumb that could be applied by antitrust authorities if post-merger market shares can be projected. The rule-of-thumb is applicable whether or not merger efficiencies exist. Also as a corollary, the interests of non-merging incumbents are not aligned with those of consumers: a merger is beneficial to consumers if and only if it is harmful to non-merging incumbents.

3 Merger Analysis with Entry

For the purposes of this section, we assume that the merger does not affect the qualities or marginal costs of the merging firms' products. Absent such efficiencies, any merger is profitable and reduces consumer surplus if entry does not occur (Prop 2). Thus, we focus on how the type of the prospective entrant, T^F , affects equilibrium outcomes. With the aggregative games formulation, our formal assumption about efficiencies is:

Assumption 2: Merger does not generate efficiencies, meaning $T^M = T^1 + T^2$.

Our results center on two objects of interest: the type of entrant that would exactly compensate consumers for the welfare losses of a merger (the *compensating entrant*) and the type of entrant that would exactly make a merger profit-neutral for the merging firms (the *best-case entrant*). A merger followed by merger-induced entry benefits consumers if and only if the type of the prospective entrant exceeds that of the compensating entrant. Similarly, a merger is profitable if and only if the type of the prospective entrant is less than that of the best-case entrant.

We focus first on the compensating entrant:

Proposition 5. *Under Assumptions 1 and 2, if consumer surplus is unaffected by merger then the type of the entrant is given by $T^F = \tilde{T}^F$, where \tilde{T}^F satisfies:*

$$\frac{\tilde{T}^F}{T^1 + T^2} = \frac{(s_{nm,ne}^1 + s_{nm,ne}^2 - s_{m,e}^M) \exp\left(\frac{1}{1-s_{nm,ne}^1 - s_{nm,ne}^2 + s_{m,e}^M}\right)}{s_{nm,ne}^1 \exp\left(\frac{1}{1-s_{nm,ne}^1}\right) + s_{nm,ne}^2 \exp\left(\frac{1}{1-s_{nm,ne}^2}\right)}, \quad (16)$$

and

$$s_{m,e}^M \exp\left(\frac{1}{1-s_{m,e}^M}\right) = s_{nm,ne}^1 \exp\left(\frac{1}{1-s_{nm,ne}^1}\right) + s_{nm,ne}^2 \exp\left(\frac{1}{1-s_{nm,ne}^2}\right). \quad (17)$$

Furthermore, $T^F < \frac{1}{2}(T^1 + T^2)$ and $s_{m,e}^F < \frac{1}{2}(s_{nm,ne}^1 + s_{nm,ne}^2)$.

Proof. See the Appendix. □

We refer to \tilde{T}^F as the type of the compensating entrant. Together, the above and Proposition 3 imply that the type of the compensating entrant is determined solely by pre-merger market shares.¹¹ Thus, it can be obtained from data typically available to antitrust authorities. The second claim of the proposition establishes that the compensating entrant's type is less than the average of the merging firms' types, and its market share in the event of merger and entry is less than the average of the merging firms' pre-merger market shares.

We now turn to the best-case entrant—an entrant that leaves the merger profit-neutral and thus maximizes consumer surplus conditional on a merger in equilibrium. As entry affects the profit of incumbents through the aggregator, under Assumption 1 there exists a cut-off rule such that a merger occurs only if the entrant has a sufficiently small type. The cutoff type, \bar{T}^F , characterizes the best-case entrant. Formally,

Proposition 6. *Under Assumption 1, there is a cutoff level, \bar{T}^F , such that the merger occurs in the first stage if and only if $T^F \leq \bar{T}^F$.*

Proof. See the Appendix. □

A key question is whether the best-case entrant type exceeds the compensating entrant type. This leads to our main analytical result, which addresses whether a merger

¹¹Proposition 5 expresses \tilde{T}^F in terms of the merging firms pre-merger shares and types. However, Proposition 3 applies, so the types can be recovered from pre-merger shares.

can simultaneously be profitable for the merging firms and preserve the aggregator, such that consumers are unharmed:

Theorem 1. *Under Assumptions 1 and 2, no SPE exists in which a merger occurs and consumer surplus does not decrease.*

Proof. See the Appendix. □

An implication is that $\bar{T}^F < \tilde{T}^F$, i.e., the best-case entrant has a smaller type than the compensating entrant. Thus, no two incumbents would merge in the first stage if the merger would induce entry that is sufficient to preserve consumer surplus at the pre-merger level, because doing so would reduce the joint profit of the incumbents.

Additionally, a profitable merger lowers the market aggregator, so Proposition 1 implies that it also increases incumbents' markups, even with entry. As marginal costs are fixed, incumbents' prices must also increase. We state these implications formally:

Corollary 1. *Under Assumptions 1 and 2, all incumbent markups and prices increase in any SPE with a merger.*

Finally, the three-stage game we have considered can be augmented such that multiple prospective entrants and an option for *incumbent repositioning* are incorporated into the second stage. By incumbent repositioning, we mean costly investments by one or more non-merging firms that improve product qualities or reduce marginal costs, and thus increase firm type. Let Assumption 1 include repositioning, i.e., repositioning is profitable only if a merger occurs. Theorem 1 extends to this richer environment:

Corollary 2. *In an augmented game with an arbitrary number of prospective entrants and incumbent repositioning, under Assumptions 1 and 2, no SPE exists in which a merger occurs and consumer surplus does not decrease.*

This follows as if a merger is to leave consumer surplus unchanged, then it must be that the market aggregator is unchanged. However, the proof for Theorem 1 shows that if the aggregator remains the same due to an induced action of other firms, then the merger is unprofitable. That proof does not rely on a specific form of entry or repositioning, only that the induced reaction restores the aggregator.

4 Merger Analysis with Entry and Efficiencies

We now let the merger improve the qualities or reduce the marginal costs of the merging firms' products. With such efficiencies, Theorem 1 does not apply, and profitable mergers can either increase or decrease consumer surplus.

4.1 Analysis of SPE

We provide an integrated framework that characterizes how outcomes depend on the type of the prospective entrant, T^F , and the level of efficiencies, E . To do, we adjust our assumption on merger-induced entry so as to accommodate that (i) pre-merger entry is not profitable, (ii) mergers can increase or decrease entrant profitability, and (iii) any increase would make entry profitable. Formally, we state the following two assumptions:

Assumption 1b: *Entry is just unprofitable if a merger does not occur in the first stage, meaning:*

$$\pi^F \left(\frac{T^F}{H_{nm,e}} \right) - \chi = \epsilon,$$

where $H_{nm,e}$ is the aggregator with no merger, but with entry, and ϵ is an arbitrarily small, positive number.

Assumption 2b: *Merger can have an efficiency, meaning $T^M = T^1 + T^2 + E$, for $E \geq 0$.*

Figure 1 introduces the integrated framework graphically, using numerical results that we generate for a market with four incumbents and an outside good, each with a market share of 0.20.¹² The vertical axis represents the efficiency of the merger, E , and the horizontal axis represents the type of the prospective entrant, T^F . On these axes, the best-case entrant (\bar{T}^F) and the compensating entrant (\tilde{T}^F) are as defined in our previous analysis of mergers without an efficiency. The *compensating efficiency* (\bar{E}) is the efficiency that makes merger without entry neutral for consumer surplus (Werden (1996), Nocke and Whinston (2020)). We define \hat{T}^F and \underline{E} later.

Three *neutrality curves* are plotted in the (E, T^F) space:

¹²The figure is meant to be illustrative about the analytical results that we derive; Appendix Figures C.1, C.2, and C.3 provide analogous numerical results under alternative market structures. We describe the numerical methodology used to create these figures in Appendix B.

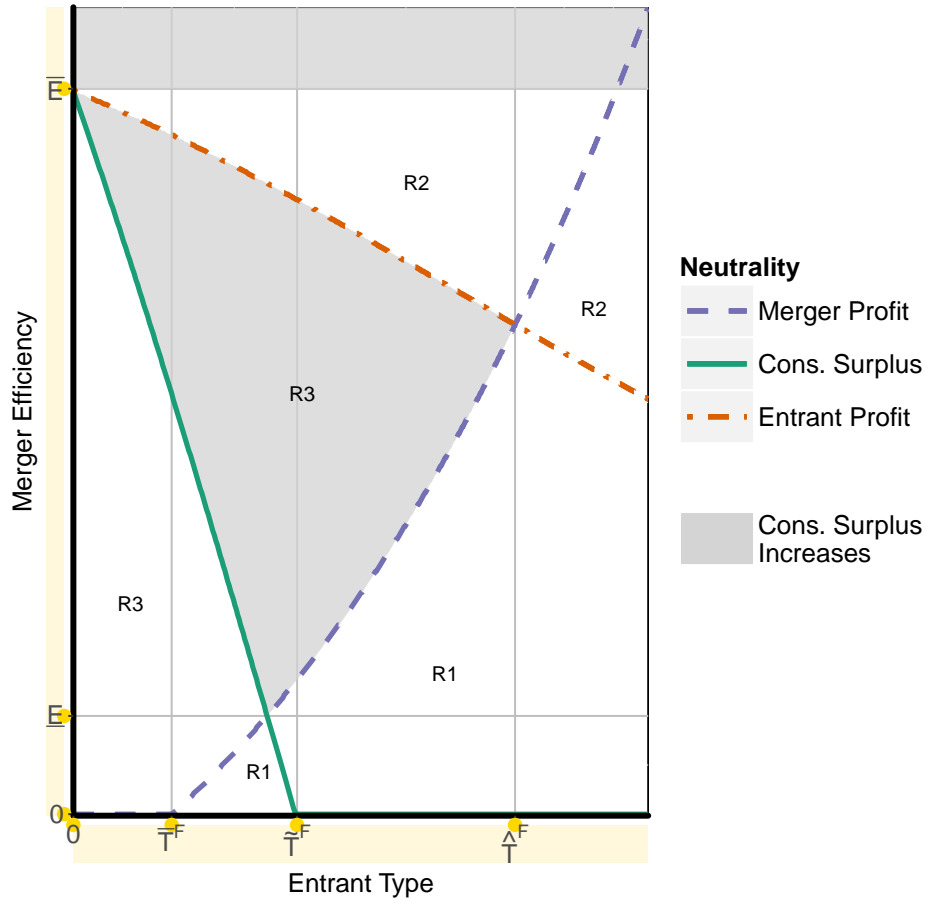


Figure 1: Merger Analysis with Entry and Efficiency

Notes: The figure illustrates the integrated framework for merger analysis with entry and efficiencies. The results are generated numerically given pre-merger market shares of 0.20 for each of four incumbents and an outside good.

1. Consumer surplus neutrality is plotted as the solid green curve. Consumer surplus increases with merger and entry if (E, T^F) fall above the curve, and decreases otherwise. The comparison is between the pre-merger equilibrium and a Bertrand equilibrium with merger and entry, which may or may not arise in SPE.
2. Neutrality for merger profitability is the dashed purple line. Merger with entry is profitable for the merging firms if (E, T^F) fall above the curve, and unprofitable otherwise. Again, the comparison is between the pre-merger equilibrium and a Bertrand equilibrium with merger and entry, which may or may not arise in SPE.
3. Neutrality for entrant profitability is the dot-dash orange line. Under Assumption 1b, merger-induced entry is profitable for the prospective entrant if (E, T^F) fall

below this curve, and unprofitable otherwise. The comparison is between an equilibrium without merger but with entry, and an equilibrium with both merger and entry. The former does not arise in SPE, and the latter may or may not.

Under Assumption 1b, the SPE of the three-stage game can feature no merger, merger without entry, or merger with entry. The neutrality curves show how different (E, T^F) combinations correspond to the different outcomes. Regions that yield no merger are marked with ‘R1.’ Regions that yield merger without entry are marked with ‘R2.’ Here merger with entry would increase consumer surplus, but entry does not occur in SPE. Thus, consumer surplus increases in R2 if $E > \bar{E}$ and decreases if $E < \bar{E}$. Regions that yield merger with entry are marked with ‘R3.’ The gray shading shows the combinations of (E, T^F) for which merger increases consumer surplus in SPE.

Figure 1 is generated numerically but its main properties are general. In the following three propositions we formally establish the existence of the neutrality curves and enumerate some of their properties. We use the subscripts ‘nm’, ‘nme’, and ‘me’ to help distinguish between Bertrand outcomes with no merger, no merger but with entry, and merger with entry, respectively.

Proposition 7. *For any $T^F \in [0, \tilde{T}^F]$ there exists a unique E such that the merger with entry is neutral for consumer surplus (i.e., $CS_{nm,ne} = CS_{m,e}$) These combinations define a neutrality curve with the following properties:*

- (i) *The curve is downward-sloping in (T^F, E) space.*
- (ii) *If $T^F = 0$ then $E = \bar{E}$, and if $T^F = \tilde{T}^F$ then $E = 0$.*
- (iii) *For any (T^F, E) on the curve, market shares satisfy $s_{nm,ne}^1 + s_{nm,ne}^2 = s_{m,e}^F + s_{m,e}^M$, i.e., the combined pre-merger market shares of the merging firms equal the combined post-merger market shares of the entrant and the merged firm.*

Proof. See the Appendix. □

Proposition 8. *For any $T^F \geq \bar{T}^F$, there exists a unique E such that the merger is profit neutral (i.e., $\pi_{nm,ne}^1 + \pi_{nm,ne}^2 = \pi_{m,e}^M$). These combinations define a neutrality curve with the following properties:*

- (i) *The curve is upward-sloping in (T^F, E) space.*
- (ii) *If $T^F = \bar{T}^F$ then $E = 0$, and there exists some T^F such that $E = \bar{E}$.*

(iii) For any (T^F, E) on the curve, market shares satisfy

$$s_{m,e}^M = 1 - \frac{(1 - s_{nm,ne}^1)(1 - s_{nm,ne}^2)}{1 - s_{nm,ne}^1 s_{nm,ne}^2}. \quad (18)$$

where $s_{nm,ne}^1$ and $s_{nm,ne}^2$ are the pre-merger shares, and $s_{m,e}^M$ is the post-merger share.

Proof. See the Appendix. □

Proposition 9. For any $T^F > 0$ there exists a unique E such that the merger is neutral for the entrant's profit (i.e., $\pi_{nm,e}^F = \pi_{m,e}^F$). These combinations define a neutrality curve with the following properties:

- (i) The curve is downward-sloping in (T^F, E) space.
- (ii) As $T^F \rightarrow^+ 0$, $E \rightarrow \bar{E}$. Further, E is strictly positive for all $T^F > 0$.
- (iii) For any (T^F, E) on the curve, with $T^F > 0$, merger is consumer surplus neutral:

$$CS_{nm,e} = CS_{m,e}.$$

Moreover, $CS_{nm,ne} < CS_{nm,e}$ and thus this curve lies above the CS-neutrality line for nm, ne versus m, e .

Proof. See the Appendix. □

4.2 Implications

These propositions have a number of immediate implications. Each of the regions in Figure 1 exists generally. The shaded triangular R3 region, in particular, exists and is bounded by the three neutrality curves. Thus, there exist combinations of (T^F, E) for which merger and entry occur in SPE and consumer surplus increases. The upper envelope of the neutrality curves for consumer surplus and merger profitability form an *efficiencies lower bound* for the shaded R3 region. For merger and entry to occur in SPE and increase consumer surplus, the efficiency must be larger than this bound. The minimum of this bound—which we refer to as the *minimum efficiency* and denote \underline{E} —occurs at the crossing of the neutrality functions. Any profitable merger that increases consumer surplus must have $E \geq \underline{E}$. We characterize the minimum efficiency shortly.

We define the *maximum entrant* (\hat{T}^F) as the largest entrant type that falls in the shaded R3 region. Merger with entry does not occur in SPE if the prospective entrant has $T^F > \hat{T}^F$ because the merger would be unprofitable (low E), merger-induced entry would be unprofitable (high E), or both would be unprofitable (moderate E).

The regions R2 and R3 are divided by the neutrality curve for entrant profitability. Thus, this neutrality curve provides an *efficiencies upper bound* for the shaded R3 region. For any (T^F, E) combination in the shaded R3 region with $T^F > 0$, there exists some (T^F, E^*) with $E^* > E$ that is in R2. As consumer surplus increases in the shaded R3 region but decreases in R2 below \bar{E} , it follows that consumer surplus is not monotonically increasing in the efficiency. For any (T^F, E) combination in R2 where $E < \bar{E}$, the efficiency is large enough to deter post-merger entry, but too small to eliminate consumer surplus loss absent entry.

We now characterize the compensating efficiency and minimum efficiency formally. The compensating efficiency also is derived in Nocke and Whinston (2020), and is included here for completeness.

Proposition 10. *Under Assumptions 1b and 2b, the compensating efficiency, \bar{E} , that must be attained in order for a merger without entry to preserve consumer surplus satisfies:*

$$\frac{T^1 + T^2 + \bar{E}}{T_1 + T_2} = \frac{(s_{nm,ne}^1 + s_{nm,ne}^2) \exp\left(\frac{1}{1-(s_{nm,ne}^1 + s_{nm,ne}^2)}\right)}{s_{nm,ne}^1 \exp\left(\frac{1}{1-s_{nm,ne}^1}\right) + s_{nm,ne}^2 \exp\left(\frac{1}{1-s_{nm,ne}^2}\right)}. \quad (19)$$

Furthermore, the minimum efficiency that must be attained in order for a profitable merger with entry to preserve consumer surplus is given by:

$$\underline{E} = H \sum_{i \in \{1,2\}} s_{nm,ne}^i \exp\left(\frac{1}{1-s_{nm,ne}^i}\right) + s_{m,e}^M \exp\left(\frac{1}{1-s_{m,e}^M}\right), \quad (20)$$

where

$$s_{m,e}^M = 1 - \frac{(1-s_{nm,ne}^1)(1-s_{nm,ne}^2)}{1-s_{nm,ne}^1 s_{nm,ne}^2}. \quad (21)$$

Proof. See the Appendix. □

Any merger consistent with SPE reduces consumer surplus if it generates an efficiency $E < \underline{E}$. With an efficiency satisfying $\underline{E} < E < \bar{E}$, a merger might increase

consumer surplus, depending on the type of the prospective entrant and the magnitude of entry costs. With $E > \bar{E}$, a merger increases consumer surplus. The thresholds \underline{E} and \bar{E} are determined by the pre-merger shares and thus can be obtained from data typically available to antitrust authorities.¹³

4.3 The Likelihood of Merger-Induced Entry

Our analysis in this section, thus far, has been premised on the idea that if the merger increases the profitability of entry, then entry occurs. This is implemented with an assumption that entry costs just exceed the profit that the entrant could obtain without the merger (Assumption 1b). In real-world settings, entry costs may be larger, and potentially much larger. Thus, merger review often involves some empirical assessment of entry costs. The Guidelines state that:

Entry is likely if it would be profitable, accounting for the assets, capabilities, and capital needed and the risks involved, including the need for the entrant to incur costs that would not be recovered if the entrant later exits.¹⁴

Our purely theoretical approach does not inform the entry costs that arise in real-world markets. However, it does allow us to quantify how a merger affects the entrant's profit opportunity, and thus it can be helpful in informing priors about the likelihood of merger-induced entry. To that end, we generate numerical results under different assumptions on market structure. We focus initially on the case of four symmetric incumbents and an outside good that receives a market share of 0.20 in the initial equilibrium—the same assumptions used to generate Figure 1.¹⁵

Figure 2 summarizes the results. The left panel plots the profit that the entrant would receive both with and without the merger, assuming no merger efficiencies, as a function of the entrant's type. Both curves are upward-sloping, as higher-type entrants obtain higher profit, from Proposition 1. For any given entrant type, profit is higher with the merger, which is guaranteed from Proposition 2. A necessary condition for

¹³Proposition 10 expresses \bar{E} in terms of pre-merger shares and types, and \underline{E} in terms of pre-merger shares and the aggregator. However, the types and the aggregator each can be recovered from pre-merger shares, by Proposition 3 and (10), respectively.

¹⁴Guidelines, §9.2

¹⁵We apply the partial calibration and simulation techniques explained in Appendix B. Thus, market shares are sufficient to generate the results.

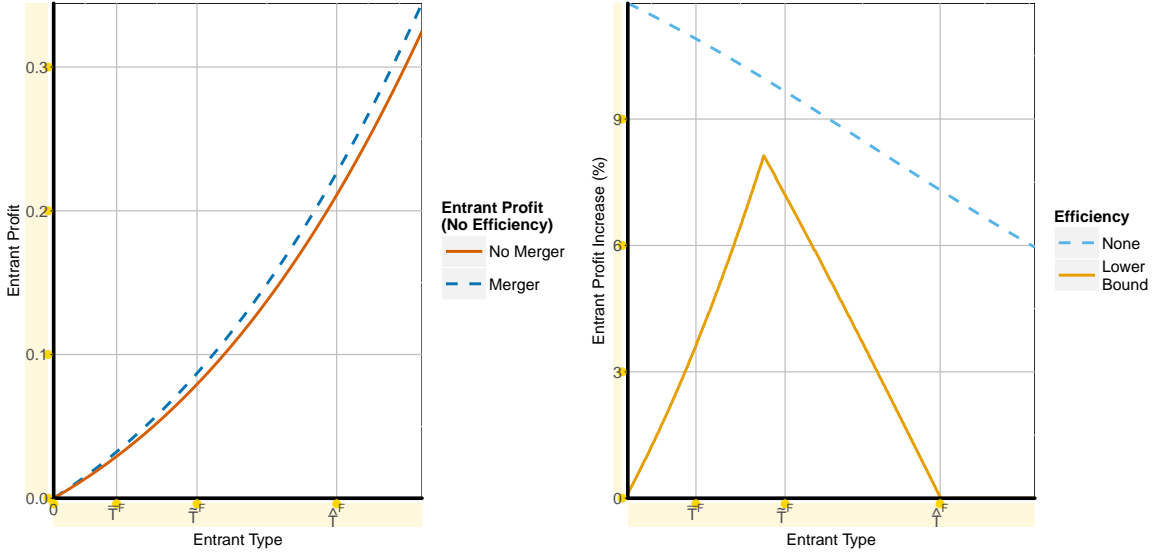


Figure 2: The Entrant's Profit Opportunity

Notes: The left panel plots the profit that the prospective entrant would receive both without and with the merger, assuming no merger efficiencies. The right panel plots the percentage change in entrant profit due to a merger without efficiencies, along with the analogous percentage change due to a merger with lower bound efficiencies. The figures are generated numerically for a market with four incumbents and an initial outside good share of 0.20.

merger-induced entry to occur is for the entry cost to fall between the two lines, i.e.,

$$\pi^F \left(\frac{T^F}{H_{nm,e}} \right) < \chi \leq \pi^F \left(\frac{T^F}{H_{m,e}} \right),$$

as stated in Assumption 1. This places bounds on the entry costs necessary to generate merger-induced entry that are perhaps surprisingly tight. For example, merger takes the profit of the compensating entrant ($T^F = \tilde{T}^F$) from 0.079 to 0.087, a 10 percent increase. Thus, a necessary condition for compensating entry is $\chi \in [0.079, 0.087]$.¹⁶

Analytical results already obtained indicate that at least lower-bound efficiencies are necessary if the merger-induced entry in question is to be (i) sufficient to preserve consumer surplus and (ii) consistent with profitable merger. The presence of such effi-

¹⁶In practical settings, we would conceptualize χ as incorporating upfront entry costs (EC) and the present value of fixed costs (FC). With that decomposition, the necessary condition for merger-induced entry is

$$\pi^F \left(\frac{T^F}{H_{nm,e}} \right) < EC + (1 - \delta)FC \leq \pi^F \left(\frac{T^F}{H_{m,e}} \right),$$

where δ is the discount rate.

ciencies reduces the entrant's profit opportunity, by Proposition 1. A tighter necessary condition can be obtained by incorporating lower bound efficiencies.

The right panel of Figure 2 plots the percentage change in entrant profit due to a merger, both without efficiencies and with lower bound efficiencies. The first line slopes down—higher type entrants benefit less from merger without efficiencies. The second line, which incorporates lower bound efficiencies, takes a \wedge shape. This reflects the influence of neutrality curves for consumer surplus and merger profitability (Figure 1). On the left, lower bound efficiencies are decreasing in the entrant's type, as less is required to preserve consumer surplus. Thus, the entrant's profit opportunity slopes upward initially. On the right, lower bound efficiencies are increasing in the entrant's type, as more is required to preserve merger profitability, and this causes the entrant's profit opportunity to slope downward. The peak occurs at the crossing of the neutrality curves. Thus, mergers appear to have small effects on the profits of low-type and high-type entrants, and somewhat greater effects for entrants with more moderate types. In our numerical example, the effect at its peak is a 7.4 percent increase in profit.

Although these results are specific to one selected market structure, analogous results can be generated for any arbitrary set of pre-merger market shares. Appendix Figures C.4-C.6 reproduce the right panel for a variety of alternative market structures. The main qualitative features are similar to those of Figure 2. Mergers appear to have a larger impact on the entrant's profit opportunity in more concentrated markets and with greater symmetry among the merging firms.

5 Nested Logit Demand

We have established that, under Bertrand/logit competition, profitable mergers do not induce entry sufficient to preserve consumer surplus, absent an efficiency. We now re-examine that result using a nested logit demand system, which allows for more flexible consumer substitution patterns. We first represent the model as an aggregative game and examine the case in which the merging firms and the prospective entrant have products in the same nest. We then provide a result about the continuity of equilibrium with respect to the nesting parameter, which allows us to extend our analysis to other nests if this parameter is not too large.

5.1 Notation and Compensating Entry

Let each inside product belong to a particular nest, $g \in \mathcal{G}$. There is an additional nest labeled 0 that contains only the outside good. The indirect utility that consumer i receives from the inside product j in group $g(j)$ is

$$u_{ij} = v_j - \alpha p_j + \zeta_{ig(j)} + (1 - \sigma)\epsilon_{ij}$$

where ϵ_{ij} is iid extreme value and $\zeta_{ig(j)}$ has the unique distribution such that $\zeta_{ig(j)} + (1 - \sigma)\epsilon_{ij}$ is also iid extreme value (Berry (1994); Cardell (1997)). The nesting parameter, $\sigma \in [0, 1)$, characterizes the correlation in preferences for products of the same nest. With $\sigma = 0$ the model collapses to the flat logit model.

Following Nocke and Schutz (2018), we assume that each firm sells products located in a single nest, which allows for the Bertrand pricing game to be reformulated as an aggregative game. The primitives are the price coefficient and nesting parameter, (α, σ) , and a vector of firm-specific types, $\{T^f\} \forall f \in \mathcal{F}$, where

$$T^f \equiv \sum_{j \in \mathcal{J}^f} \exp\left(\frac{v_j - \alpha c_j}{1 - \sigma}\right).$$

For the sake of brevity, we provide the relationships that arise in Bertrand equilibrium and defer the derivations to the appendix:

Proposition 11. *With nested logit demand, the following equations hold in Bertrand*

equilibrium:

$$\mu^f = \frac{1}{1 - \sigma s^{f|g} - (1 - \sigma)s^f} \quad (22)$$

$$1 = \mu^f \left(1 - \sigma \frac{T^f}{H_g} \exp(-\mu^f) - (1 - \sigma) \frac{T^f}{H_g} \frac{H_g^{1-\sigma}}{H} \exp(-\mu^f) \right) \quad (23)$$

$$\frac{T^f}{H_g} = s^{f|g} \exp \left(\frac{1}{1 - \sigma s^{f|g} - (1 - \sigma)s^f} \right) \quad (24)$$

$$s^f = s^{f|g} \bar{s}_g \quad (25)$$

$$\bar{s}_g = \frac{H_g^{1-\sigma}}{H} \quad (26)$$

$$1 = \sum_{f \in \mathcal{F}_g} s^{f|g} \quad (27)$$

$$H = 1 - \sum_{f \in \mathcal{F}} s^f \quad (28)$$

$$\pi^f = \left(\frac{1 - \sigma}{\alpha} \right) \mu^f s^f \quad (29)$$

$$CS = \frac{1}{\alpha} \ln(H) \quad (30)$$

where T^f is the type of the firm, s^f is the share of the firm, $s^{f|g}$ is the share of the firm within its nest, \bar{s}_g is the share of the group, μ^f is an ι -markup, H_g is a group aggregator, H is the market aggregator, π^f is the profit of the firm, and CS is consumer surplus.

Proof. See the Appendix.

We now proceed with an analysis of mergers and entry. We maintain Assumptions 1 and 2, which restrict attention to merger-induced entry and zero efficiencies, respectively. We also assume that the products of the merging firms are in the same nest. A closed-form expression for the type of the compensating entrant is available if entry occurs in the nest of the merging firms:

Proposition 12. *Under Assumptions 1 and 2 and with nested logit demand, if consumer surplus is unaffected by merger when post-merger entry occurs in the nest of the merging firms, then the type of the entrant is given by $T^F = \tilde{T}^F$, where \tilde{T}^F satisfies:*

$$\frac{\tilde{T}^F}{T^1 + T^2} = \frac{(s_{nm,ne}^{1|g} + s_{nm,ne}^{2|g} - s_{m,e}^{M|g}) \exp \left(\frac{1}{1 - (s_{nm,ne}^{1|g} + s_{nm,ne}^{2|g} - s_{m,e}^{M|g})(\sigma + (1 - \sigma)\bar{s}_g)} \right)}{s_{nm,ne}^1 \exp \left(\frac{1}{1 - s_{nm,ne}^{1|g}(\sigma + (1 - \sigma)\bar{s}_g)} \right) + s_{nm,ne}^2 \exp \left(\frac{1}{1 - s_{nm,ne}^{2|g}(\sigma + (1 - \sigma)\bar{s}_g)} \right)}, \quad (31)$$

and

$$s_{m,e}^{M|g} \exp \left(\frac{1}{1 - s_{m,e}^{M|g}(\sigma + (1 - \sigma)\bar{s}_g)} \right) = s_{nm,ne}^{1|g} \exp \left(\frac{1}{1 - s_{nm,ne}^{1|g}(\sigma + (1 - \sigma)\bar{s}_g)} \right) + s_{nm,ne}^{2|g} \exp \left(\frac{1}{1 - s_{nm,ne}^{2|g}(\sigma + (1 - \sigma)\bar{s}_g)} \right). \quad (32)$$

Further, the share of the group, \bar{s}_g , does not change due to the merger.

Proof. See the Appendix.

Together, Proposition 12 and Proposition 11 imply that the type of the compensating entrant is determined by σ and pre-merger market shares. In the proof of the proposition, we establish that consumer surplus neutrality implies that

$$s_{nm,ne}^1 + s_{nm,ne}^2 = s_{m,e}^F + s_{m,e}^M,$$

so the rule-of-thumb proposed in Section 2.3 extends. A merger benefits consumers if the combined shares of the merging firms and the entrant increase, and harms consumers if the combined shares decrease. Finally, we note that if $\sigma = 0$ then the equations in the proposition collapse to those of Proposition 5 for logit demand.

We now formally extend the main result:

Theorem 2. *Under assumptions 1 and 2, and with nested logit demand, no SPE exists in which a merger induces entry into the nest of the merging firms and consumer surplus does not decrease.*

Proof. See the Appendix.

Thus, if compensating entry does occur then the merger is unprofitable and does not arise in SPE. As with the logit case, it is easily verified that any merger that occurs in SPE also increases price and markups, and also that the theorem extends to the multiple entrants and incumbent repositioning.

5.2 Continuity of Equilibrium and Implications

One technical advantage of the revealed preference approach underpinning our Theorems 1 and 2 is that it is robust to mild perturbations of the underlying demand framework. In this section, we consider the potential for merger-induced entry into a nest

different than that of the merging parties. We find that even still, an analogue of Theorem 2 obtains, so long as the demand system's nesting parameter σ is not 'too large.' This is established by first proving the continuity of the equilibrium variables in the underlying nesting parameter about a neighborhood of zero. Armed with this result, the strict inequalities characterizing the impossibility underlying Theorem 1 corresponding to the case of $\sigma = 0$ obtain for a nested framework, with entry into an arbitrary nest.

Theorem 3. *Fix any vector of model primitives, and consider the mapping taking $\sigma \in [0, 1)$ to the unique differentiated Bertrand equilibrium of the pricing game for the nested logit demand system with nesting parameter σ . This mapping is continuous on a neighborhood of 0.*

Proof. See the Appendix.

By appeal to this result, we obtain the following 'robust' analogue of Theorem 2.

Theorem 4. *There exists $\varepsilon > 0$ such that, for nested logit demand with any $\sigma \in [0, \varepsilon)$, Assumptions 1 and 2 imply that there does not exist any SPE in which a merger induces entry and consumer surplus does not decrease.*

Proof. See the Appendix.

Thus, in our context, the model does not rule out that consumers benefit from a profitable merger (without efficiencies) that induces entry by a firm with products that are relatively weak substitutes for those of the merging firms.¹⁷ However, intuition suggests that the effect of the merger on the profit of such an entrant is modest, so we conjecture that merged-induced entry that occurs outside the merging firm's nest is unlikely in most settings, especially with higher levels of σ .

6 Extensions

We provide two extensions. In the first, we use numerical techniques to explore the extent to which best-case entry mitigates the consumer surplus loss and price increases that arise due to a merger without efficiencies, in the baseline logit model. Given our

¹⁷In the legal context, the products in the nest of the merging firms might constitute a relevant market, in the sense that a hypothetical monopolist of products in the nest would impose a small but significant and non-transitory increase in price. This is especially the case with higher levels of σ . Thus, our modeling result suggests that merger-induced entry that eliminates the consumer surplus loss of a profitable merger may necessarily occur outside the relevant market.

theoretical results, mitigation is guaranteed to be incomplete, but the magnitudes may be of practical interest to antitrust authorities nonetheless. Second, we provide an application to the T-Mobile/Sprint merger to demonstrate how the theoretical framework can be applied.

6.1 Numerical Examination of Best-Case Entry for Logit

Implementation uses a partial calibration of the model in which firm types are recovered from data on market shares, $\{s^f\} \forall f \in \mathcal{F}$. In the data generating process, we assume that the merging firms have the same pre-merger market share; we obtain results with shares of 0.01, 0.02, ..., 0.40. We assume the outside good has a share of 0.20, and assign the remaining share evenly to non-merging incumbents.¹⁸ We consider separately cases with three, five, seven, and nine incumbents. The calibration and simulation methodologies are provided in Appendix B.

Figure 3 characterizes the proportion of consumer surplus loss that is mitigated by best case entry, as a function of the initial market shares of the merging firms. Each panel considers a different number of incumbents. Within each panel, mitigation takes a U-shape, such that best-case entry mitigates the least amount of consumer surplus loss for moderately-sized mergers. Comparing across panels, mitigation is greater if there are fewer incumbents, and this is most pronounced for small mergers. Finally, the mitigation of consumer surplus loss approaches 50 percent from below in all panels, as the initial shares of the merging firms reach their maximum.

Figure 4 plots the proportion of the merging firms' price increases that are mitigated by the best-case entry. The panels again consider different numbers of incumbents, and the effects are similar in each. Entry does little to offset the price effects of smaller mergers, and becomes somewhat more important for larger mergers. Mitigation approaches 30 percent as the initial shares of the merging firms reach their maximum.¹⁹ Taken together, these results indicate that best case entry mitigates only a fraction of the adverse competitive effects caused by mergers without efficiencies.

¹⁸For example, if we allow for five incumbents, one pre-merger equilibrium we examine features shares of 0.10 for each merging firm and shares of 0.20 for non-merging firms.

¹⁹Entry improves consumer surplus through product diversity and lower prices. For mergers between small incumbents and markets with few incumbents, the diversity effect appears to be more important.

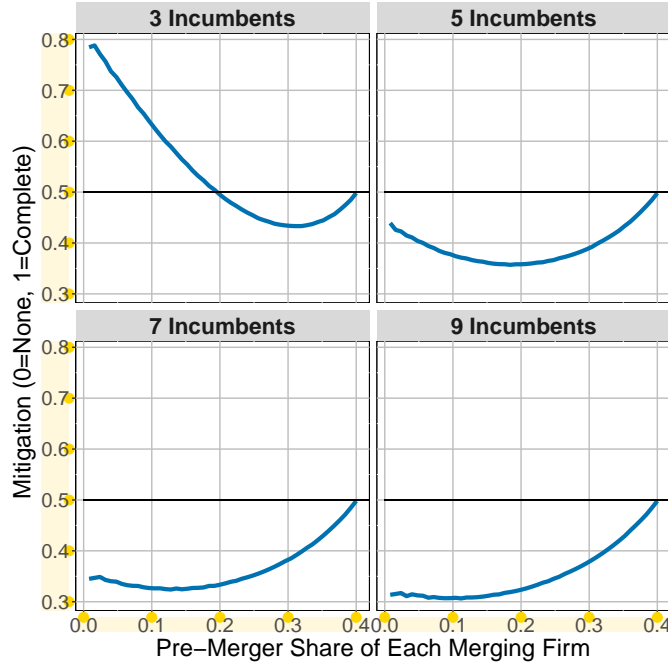


Figure 3: Best-Case Mitigation of Consumer Surplus Loss

6.2 Application to T-Mobile/Sprint

We apply the integrated framework to the T-Mobile/Sprint merger, which combined two of the four national providers of mobile wireless telecommunications service.²⁰ The Department of Justice and the Federal Communications Commission (FCC) approved the merger conditional on certain behavioral remedies and the divestiture of Boost—a Sprint prepaid brand—to DISH, a prospective entrant. The merger then was challenged unsuccessfully in Federal District Court by several states. The Court’s decision addressed whether adverse competitive effects from the loss of competition would be offset by efficiencies related to network capacity and by DISH’s entry.

We analyze the merger using a Bertrand/logit model of competition among the four national providers: Sprint, T-Mobile, Verizon, and AT&T. We calibrate the model using publicly-available data on market shares, prices, and markups. We also use a market elasticity of demand that appears in regulatory filings.²¹ Details on the data and calibration process are provided in Appendix B.

²⁰The merger was announced in April 2018 and closed in April 2020.

²¹The market elasticity of demand is the percentage change in the (combined) share of the inside products due to a one percent change in the weighted-average price. Letting ϵ be the market elasticity of demand, with logit demand we have $\epsilon = -\alpha s_0 \bar{p}$, where \bar{p} is the weighted-average price. In calibration, the market elasticity determines the the outside good’s share, which is unobservable from the data.

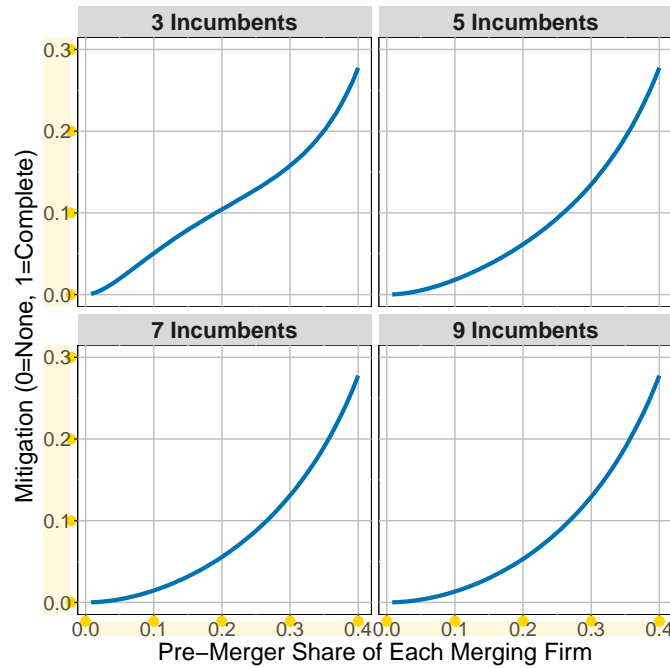


Figure 4: Best-Case Mitigation of Price Increases

Figure 5 provides graphs from the integrated framework.²² Starting with the left panel, we obtain values of \underline{E} and \bar{E} that are equivalent to marginal cost reductions of 1.6 and 4.0 percent, respectively, if one holds quality constant. These bound the efficiencies necessary to generate a pro-competitive merger with induced entry. That is, if the efficiency is less than 1.6 percent then the merger harms consumers, and if the efficiency is greater than 4.0 percent then merger-induced entry never occurs (though consumers benefit). These bounds are substantially tighter for most prospective entrant types, by inspection of the panel. The right panel shows that the merger increases the profitability of entry by at most 5.1 percent, assuming lower bound efficiencies.

Entry requires a specific set of assets, including spectrum, which is necessary for the transmission of a wireless signal. Thus, the litigation focused specifically on one prospective entrant, DISH, that had acquired a substantial portfolio of spectrum licenses in prior FCC auctions. The Court's decision states that:

DISH is well positioned to become a fourth [mobile network operator] in the market, and its extensive preparations and regulatory remedies indicate that it can sufficiently replace Sprint's competitive impact....²³

²²See also Appendix Figure C.8.

²³See p. 117 of the opinion.

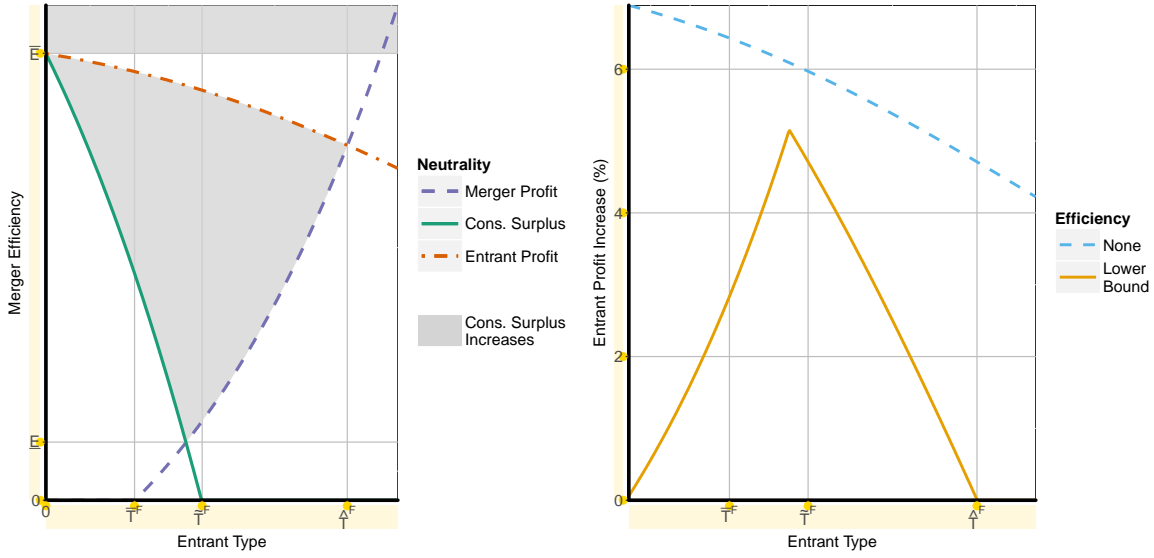


Figure 5: Application to T-Mobile/Sprint Merger

Notes: The left panel plots the neutrality curves in the integrated framework and the right panel plots the percentage change in entrant profit due to a merger without efficiencies, along with the analogous percentage change due to a merger with lower bound efficiencies. The figure is generated numerically given market shares for the mobile wireless telecommunications industry.

We interpret this language as indicating a belief that DISH could offer service at a quality and cost that is similar to Sprint. As our calibration obtains a Sprint type of $T^1 = 5.00$, we assume a DISH type of $T^F = 5.00$.

Under this assumption, there is no SPE in which DISH enters in response to the merger. The reason is that the DISH type is too large to be consistent with merger-induced entry—specifically, we have $T^F = 5.00 > \hat{T}^F = 4.63$. The chain of logic that rules out DISH as a merger-induced entrant depends on the level of efficiencies that the merger generates. With a small efficiency, say a 2% reduction in marginal costs, the merger would induce entry by DISH given small enough entry barriers, but DISH's entry would make the merger unprofitable. With a larger efficiency, say a 3.8% reduction in marginal costs, the merger would be profitable even with merger-induced entry by DISH, but merger reduces the profitability of DISH entry. Thus, in neither scenario does SPE feature both merger and merger-induced entry.

Our main goal in providing this empirical application is to highlight how the modeling framework can help assess entry in a real-world setting. There are two main caveats to the conclusions we reach: First, our Bertrand/logit model does not incorporate all of the market realities, and indeed is somewhat less sophisticated than the nested logit

models that appear in regulatory filings.²⁴ Second, we have not attempted to incorporate the divestiture of the Boost brand to DISH. Both caveats could be addressed with access to more detailed data.

7 Conclusion

We have sought to characterize the extent to which post-merger entry can mitigate the adverse effects of otherwise anticompetitive mergers. With the canonical model of Bertrand competition among firms facing logit demand, we establish that entry alone is never sufficient to eliminate the consumer surplus losses or price effects of profitable mergers. Together, entry and efficiencies can combine to eliminate consumer surplus losses, even if neither is sufficient in isolation. The presences of efficiencies, however, also reduces the profitability of entry. Whether a post-merger equilibrium that features entry, efficiencies, and consumer gains is *likely* to arise is ultimately an empirical question that depends on the magnitude of entry costs among other factors; we provide an integrated framework to help guide empirical analysis.

At a high level, we interpret our results as indicating that economic models can inform the efficacy of entry in mitigating the adverse competitive effect of mergers. In the specific context of Bertrand competition with logit demand, establishing conditions under which entry yields a pro-competitive merger requires a joint analysis of market shares, merger efficiencies, the capabilities of prospective entrants, and entry costs. This suggests a role for entry in merger analysis that is more nuanced than is discussed in the 2010 Horizontal Merger Guidelines.

The generality of our framework has some limitations. The revealed preference arguments we develop may not apply to mergers that reduce fixed costs, or to mergers that affect competition in one market but yield efficiencies in multiple markets. We also do not incorporate that entry costs could be endogenous due to strategic actions by incumbents to deter or punish entry. On other dimensions, however, similar logic appears to extend. We cover the nested logit case, for example, and previous research provides results for Cournot competition (Spector (2003)). With procurement auctions, mergers do not increase the profitability of entry if the auction is efficient (e.g., as in Miller (2014)), so merger-induced entry does not occur. A full analysis of merger-induced entry for the alternative case in which buyers set reserve prices to discipline post-merger

²⁴In order to implement such richer models, we would require access to the confidential data used to generate those filings.

market power (e.g., Waehrer and Perry (2003); Loertscher and Marx (2019)) has not been conducted, and would represent an useful contribution to the literature.

References

- Aliprantis, C. D. and K. C. Border (2006). *Infinite Dimensional Analysis: a Hitchhiker's Guide*. Springer.
- Alviarez, V., K. Head, and T. Mayer (2020). Global giants and local stars: How changes in brand ownership affect competition. Working Paper.
- Anderson, S. P., N. Erkal, and D. Piccinin (2020). Aggregative games and oligopoly theory: Short-run and long-run analysis. *RAND Journal of Economics* 51(2), 470–495. Working Paper.
- Berry, S., J. Levinsohn, and A. Pakes (1995). Automobile prices in market equilibrium. *Econometrica* 63(4), 847–890.
- Berry, S. T. (1994). Estimating Discrete-Choice Models of Product Differentiation. *RAND Journal of Economics* 25(2), 242–262.
- Cabral, L. (2003). Horizontal mergers with free-entry: Why cost efficiencies may be a weak defense and asset sales a poor remedy. *International Journal of Industrial Organization* 21, 607–623.
- Cardell, S. N. (1997). Variance Components Structures for the Extreme-Value and Logistic Distributions with Applications to Models of Heterogeneity. *Econometric Theory* 13, 185–213.
- Ciliberto, F., C. Murry, and E. Tamer (2020). Market structure and competition in airline markets.
- Davidson, C. and A. Mukherjee (2007). Horizontal mergers with free entry. *International Journal of Industrial Organization* 25, 157–172.
- Erkal, N. and D. Piccinin (2010). Welfare-reducing mergers in differentiated oligopolies with free entry. *Economic Record* 86(273), 178–184.
- Fan, Y. and C. Yang (2020). Merger, product variety and firm entry: The retail craft beer market in California.
- Farrell, J. and C. Shapiro (1990). Horizontal mergers: An equilibrium analysis. *American Economic Review* 80(1), 107–126.

- Garrido, F. (2019). Mergers between multi-product firms with endogenous variety: Theory and an application to the RTE-cereal industry. Georgetown University dissertation.
- Garrido, F. (2020). An aggregative approach to price equilibrium among multi-product firms with nested demand. *Available at SSRN 3647311*.
- Hirsch, M. W. (2012). Differential topology. Springer Science & Business Media.
- Li, S., J. Mazur, J. Roberts, Y. Park, A. Sweeting, and J. Zhang (2019). Repositioning and market power after airline mergers.
- Loertscher, S. and L. M. Marx (2019). Merger review for markets with buyer power. *Journal of Political Economy* 127(6), 2967–3017.
- Mas-Colell, A. (1974). A note on a theorem of f. browder. *Mathematical Programming* 6(1), 229–233.
- Miller, N. H. (2014). Modeling the effects of mergers in procurement. *International Journal of Industrial Organization* 37, 201–208.
- Miller, N. H. and G. Sheu (2020). Quantitative methods for evaluating the unilateral effects of mergers. Working paper.
- Nevo, A. (2001). Measuring market power in the ready-to-eat cereal industry. *Econometrica* 69(2), pp. 307–342.
- Nocke, V. and N. Schutz (2018). Multiproduct-firm oligopoly: An aggregative games approach. *Econometrica* 86(2), 523–557.
- Nocke, V. and N. Schutz (2019). An aggregative games approach to merger analysis in multiproduct-firm oligopoly. Working Paper.
- Nocke, V. and M. Whinston (2020). Concentration screens for horizontal mergers. Working paper.
- Shapiro, C. (2010). The 2010 Horizontal Merger Guidelines: From hedgehog to fox in forty years. *Antitrust Law Journal* 77, 49–107.
- Spector, D. (2003). Horizontal mergers, entry, and efficiency defences. *International Journal of Industrial Organization* 21, 1591–1600.

- Waehrer, K. and M. K. Perry (2003). The effects of mergers in open-auction markets. *RAND Journal of Economics* 34(2), 287–304.
- Werden, G. J. (1991). Horizontal mergers: Comment. *American Economic Review* 81(4), 1002–1006.
- Werden, G. J. (1996). A robust test for consumer welfare enhancing mergers among sellers of differentiated products. *The Journal of Industrial Economics* 44(4), pp. 409–413.
- Werden, G. J. and L. M. Froeb (1994). The effects of mergers in differentiated products industries: Logit demand and merger policy. *Journal of Law, Economics, and Organization* 10(407), 407–426.
- Werden, G. J. and L. M. Froeb (1998). The entry-inducing effects of horizontal mergers: An exploratory analysis. *The Journal of Industrial Economics* 46(4), pp. 525–543.
- Werden, G. J. and L. M. Froeb (2002). The antitrust logit model for predicting unilateral competitive effects. *Antitrust Law Journal* 70(1), 257–260.

A Appendix for Online Publication

A.1 Proof of Proposition 1

Proof. Let $x^f = T^f/H$. After applying the implicit function theorem to (9), we find that

$$m'(x^f) = \frac{m(x^f)^2 \exp(-m(x^f))}{1 + m(x^f)^2 x^f \exp(-m(x^f))}.$$

By inspection, this expression is positive. Furthermore, the derivative of π^f with respect to x^f is equal to $(1/\alpha)$ times the derivative of μ^f , meaning it is also positive if $\alpha > 0$. That is, it is positive so long as consumers earn negative utility from price.

By combining (6) and (8), solving for s^f , and taking the derivative with respect to x^f , we find that

$$\frac{ds^f}{dx^f} = \frac{m'(x^f)}{m(x^f)^2},$$

which is positive because $m'(x^f)$ is positive. Additionally, by applying the implicit function theorem to the adding-up constraint (10), we find that

$$\frac{dH}{dT^f} = \frac{\frac{ds^f}{dx^f}}{\frac{1}{H} + \sum_{g \in \mathcal{F}} x^g \frac{ds^g}{dx^g}},$$

which is also positive given that $ds^f/dx^f \forall f \in \mathcal{F}$ is positive. Computations for types T^f and T^g give

$$\frac{dx^f}{dT^f} = \frac{1}{H} \left(1 - \frac{T^f}{H} \frac{dH}{dT^f} \right),$$

which can be confirmed as positive after substituting in for dH/dT^f , and

$$\frac{dx^f}{dT^g} = -\frac{T^f}{H^2} \frac{dH}{dT^g},$$

which is negative because dH/dT^g is positive. Applying the chain rule gives the desired results. □

A.2 Proof of Proposition 2

Proof. By Proposition 1, the markups for the merging firms increase as $T^m > \max_{f \in \mathcal{C}} \{T^f\}$. Given marginal costs remain fixed, the corresponding equilibrium prices increase. However, as the aggregator, H , is decreasing in prices, Proposition 1 also implies that the markups (and thus prices) of non-merging incumbents rise as well, further decreasing the aggregator. Thus, the value T^f/H is higher for all firms in the post-merger equilibrium, so Proposition 1 implies that profits are higher as well. Finally, given (12),

consumer surplus is lower, post merger, because the aggregator is lower. \square

A.3 Proof of Proposition 3

Proof. Given market shares, the aggregator can be recovered from (10). The firm types then satisfy (8). There is a unique positive solution because the expression $x \exp(1/(1-x))$ is increasing if $x \in [0, 1)$. Conversely, given a vector of firm types, the markups μ^f are uniquely determined by (9) up to the value of H . This value is then uniquely determined by (10), and shares may then be obtained from (8). \square

A.4 Proof of Proposition 4

Proof. (i) \implies (ii): Suppose (i) holds, that is:

$$\pi_{nm,ne}^f = \pi_*^f$$

By (11), $\mu_{nm,ne}^f = \mu_*^f$, and by (6), $s_{nm,ne}^f = s_*^f$. Because $T^f = T_{nm,ne}^f = T_*^f$ by hypothesis, (8) implies:

$$\frac{T^f}{H_{nm,ne}} = s_{nm,ne}^f \exp\left(\frac{1}{1-s_{nm,ne}^f}\right) = s_*^f \exp\left(\frac{1}{1-s_*^f}\right) = \frac{T^f}{H_*}$$

and thus $H_{nm} = H_m$, which implies (ii).

(ii) \implies (i): Suppose now that $H_{nm,ne} = H_* = H$. By (8), we obtain $s_{nm,ne}^f = s_*^f$ for every $f \in \mathcal{F}_{nm,ne}$ immediately, and (i) follows by chain of substitutions identical to the above.

(ii) \implies (iii): Suppose now that $H_{nm,ne} = H_* = H$. From (10):

$$\frac{1}{H} + \sum_{f \in \mathcal{F}_{nm,ne}} s_{nm,ne}^f = \frac{1}{H} + \sum_{f \in \mathcal{F}_*} s_*^f \iff \sum_{f \in \mathcal{F}_{nm,ne}} s_{nm,ne}^f = \sum_{f \in \mathcal{F}_*} s_*^f,$$

which implies (iii) immediately upon cancelling terms (via appeal to (ii) implying (i) and hence to the shares also coinciding across scenarios).

(iii) \implies (ii): We proceed by contraposition. Thus suppose that the merger affects consumer surplus: $H_{nm,ne} \neq H_*$. Let f belong to both $\mathcal{F}_{nm,ne}$ and \mathcal{F}_* , i.e. let f denote any firm other than 1, 2, M or potentially F . By (8), we have:

$$\frac{T^f}{H_{nm,ne}} = s_{nm,ne}^f \exp\left(\frac{1}{1-s_{nm,ne}^f}\right)$$

and

$$\frac{T^f}{H_*} = s_*^f \exp\left(\frac{1}{1-s_*^f}\right).$$

For both equations, the right-hand side is strictly increasing in the relevant share, and thus for all such f :

$$\frac{1}{H_{nm,ne}} > \frac{1}{H_*} \iff s_{nm,ne}^f > s_*^f$$

Thus:

$$\frac{1}{H_{nm,ne}} + \sum_{f \in \mathcal{F}_{nm,ne} \cap \mathcal{F}_*} s_{nm,ne}^f \neq \frac{1}{H_*} + \sum_{f \in \mathcal{F}_{nm} \cap \mathcal{F}_*} s_*^f,$$

and it follows by (10) that (iii) cannot hold. \square

A.5 Proof of Proposition 5

Proof. Evaluating (8) for firm F after the merger and firms 1 and 2 before the merger, and dividing the result for firm F by the sum of the results for firms 1 and 2 gives (16), after substituting in for $s_{m,e}^F = s_{nm,ne}^1 + s_{nm,ne}^2 - s_{m,e}^M$ which obtains from Proposition 4. Under Assumption 2, $T^M = T^1 + T^2$. Substituting into this sum for types using $T^f = s^f H \exp(1/(1 - s^f))$ gives (17), which has a unique positive solution because the expression $x \exp(1/(1 - x))$ is increasing if $x \in [0, 1)$.

To establish the final claim, let s^a be the average of s^1 and s^2 , calculated as $(s^1 + s^2)/2$. Let T^a be the type that generates a share of s^a given aggregator H , which can be found by solving the equation $T^a/H = s^a \exp(1/(1 - s^a))$ holding H fixed. (Note that if $s^1 \geq s^2$, then $s^1 \geq s^a \geq s^2$ and $T^1 \geq T^a \geq T^2$, the latter due to the monotonicity of shares.) We wish to characterize the relationship between the entrant's type T^F and this "average" type T^a . In order for consumers to be unharmed, H must be unchanged due to the merger. Therefore, since $T^m > T^1$ and $T^m > T^2$, $T^m/H > T^1/H$ and $T^m/H > T^2/H$. In turn, this means that $s^m > s^1$ and $s^m > s^2$, since shares are increasing in T^f/H . Adding these inequalities gives $2s^m > s^1 + s^2$, and then dividing by two gives $s^m > s^a$. As discussed in the previous section, if H remains the same, then $s^F + s^m = s^1 + s^2$, which also means that $s^F + s^m = 2s^a$. In order for this equality to hold when we also know that $s^m > s^a$, it must be that $s^F < s^a$. By the monotonicity of shares, this means that $T^F < T^a$. \square

A.6 Proof of Proposition 6

Proof. If Assumption 1 holds, the right hand side of expression (15) does not depend on T^F . Suppose, for given type T^F , F enters the market conditional upon the merger. Then $H_{m,*} = H_{m,e}$. We first verify there exists some \bar{T}^F that makes the merger profit-neutral:

$$\pi^M \left(\frac{T^M}{H_{m,e}} \right) = \pi^1 \left(\frac{T^1}{H_{nm,ne}} \right) + \pi^2 \left(\frac{T^2}{H_{nm,ne}} \right),$$

Clearly, if $T^F = 0$ then $\pi^m(T^M/H_{m,e}) = \pi^m(T^M/H_{m,ne})$ and thus the merger is profitable by hypothesis. Conversely, consider the limit as $T^F \rightarrow \infty$.²⁵ First, suppose that $\lim_{T^F \rightarrow \infty} T^F/H_{m,e} < \infty$. In this case, since T^F is going to infinity, it must be the case that $H_{m,e}$ is too. Thus for firm M , $T^M/H_{m,e} \rightarrow 0$; by (8) $s^M \rightarrow 0$ and hence so too does π^M . On the other hand, suppose that $\lim_{T^F \rightarrow \infty} T^F/H_{m,e} = \infty$. Then by (8), $s^F \rightarrow 1$, and hence by (10), $H \rightarrow \infty$. An analogous argument then establishes that $\pi^M \rightarrow 0$. Thus we conclude that as $T^F \rightarrow \infty$, the left-hand side of the merger profitability inequality monotonically decreases from above the (constant in T^F) right-hand side, to 0. Hence at least one such \bar{T}^F exists. However, by Proposition 1, π^M is globally decreasing in T^F , all else equal, and thus \bar{T}^F is unique. \square

A.7 Proof of Theorem 1

Proof. We maintain Assumption 1. Suppose, for purposes of contradiction, there exists a SPE in which firms 1 and 2 merge, and consumers surplus does not decrease. Thus the merger must increase joint profits:

$$\pi^m \left(\frac{T^1 + T^2}{H_{m,e}} \right) \geq \pi^1 \left(\frac{T^1}{H_{nm,ne}} \right) + \pi^2 \left(\frac{T^2}{H_{nm,ne}} \right),$$

where $H_{nm,ne}$ denotes the aggregator with no merger and no entry. By hypothesis, consumers surplus does not fall, hence by (12), we have $H_{nm,e} \leq H_{m,e}$. Furthermore, by Proposition 1, π^M is decreasing in H , meaning that

$$\pi^M \left(\frac{T^1 + T^2}{H_{nm,ne}} \right) \geq \pi^1 \left(\frac{T^1}{H_{nm,ne}} \right) + \pi^2 \left(\frac{T^2}{H_{nm,ne}} \right). \quad (\text{A.1})$$

Define $x^i = T^i/H_{nm,ne}$ for $i \in \{1, 2\}$, and let $\phi(\mu) = \mu e^{-\mu}$. Then (A.1) can be expressed as:

$$(x^1 + x^2)\phi(m(x^1 + x^2)) \geq x^1\phi(m(x^1)) + x^2\phi(m(x^2)),$$

where $m(\cdot)$ denotes the markup fitting-in function. This expression is equivalent to:

$$\sum_{i \in \{1, 2\}} x^i [\phi(m(x^i)) - \phi(m(x^1 + x^2))] \leq 0,$$

which is an impossibility. The function $\phi(\cdot)$ is decreasing on $[1, \infty)$, and for all i , $m(t^1 + t^2) > m(t^i) > 1$, forcing the sum to be component-wise strictly positive. \square

²⁵It is straightforward to show that, for any f , T^f/H is monotonically increasing in T^f (e.g. Nocke and Schutz (2018) Online Appendix p. 110). Thus, in particular, the limits we consider exist, allowing for the possibility of infinite limits.

A.8 Proof of Proposition 7

Proof. We first wish to show that, for all choices of T^F , there is a unique efficiency E that makes the merger CS-neutral. Fix T^F and suppose that the merger is CS-neutral, i.e. $H_{nm,ne} = H_{m,e} = H$. Since types are unchanged across market structures, the adding up constraint implies that $s_{nm,ne}^1 + s_{nm,ne}^2 = s_{m,e}^F + s_{m,e}^M$. Moreover, $s_{m,e}^F$ depends only on T^F and H , not E . In the m, e equilibrium:

$$\frac{T^1 + T^2 + E}{H} = s_{m,e}^M \exp\left(\frac{1}{1 - s_{m,e}^M}\right),$$

the left-hand side of which is strictly increasing in E . However, the right-hand is fixed by the nm, ne equilibrium and the constant-in- E , for fixed H $s_{m,e}^F$ term, and hence is constant in E , under the hypothesis of CS-neutrality. Thus there can be only one E satisfying the above.

We note that (iii) follows directly from Proposition 4. We now prove (i) then (ii).

(i) We claim that the CS-neutrality line in the box is downward-sloping. Firstly, if consumer surplus is unchanged across equilibria, then $H_{nm,ne} = H_{m,e} = H$. Note that since no types change across equilibria, by (iii) we have $s_{nm,ne}^1 + s_{nm,ne}^2 = s_{m,e}^F + s_{m,e}^M$. In the m, e equilibrium:

$$\frac{T^1 + T^2 + E}{H} = s_{m,e}^m \exp\left(\frac{1}{1 - s_{m,e}^m}\right)$$

and

$$\frac{T^F}{H} = s_{m,e}^F \exp\left(\frac{1}{1 - s_{m,e}^F}\right)$$

Suppose T^F is increased. We have shown this does not change H as H is pinned down by its value in the nm, ne equilibrium under the assumption of CS neutrality. Thus an increase in T^F leads to a higher equilibrium value of $s_{m,e}^F$. But we have already shown an increase in s^F leads to an equivalent decrease in $s_{m,e}^M$ as the sum is fixed by $s_{nm,ne}^1$ and $s_{nm,ne}^2$ which are constant because we are only changing values affecting the m, e equilibrium. Thus the value of E that ensures CS-neutrality decreases.

(ii) Follows immediately from the definitions of these objects. □

A.9 Proof of Proposition 8

Proof. We first verify (iii). Suppose, then that the merger is profit-neutral, that is:

$$\pi_{nm,ne}^1 + \pi_{nm,ne}^2 = \pi_{m,e}^M.$$

By (11),

$$\mu_{m,e}^M - 1 = (\mu_{nm,ne}^1 - 1) + (\mu_{nm,ne}^2 - 1)$$

and hence:

$$\mu_{m,e}^M + 1 = \mu_{nm,ne}^1 + \mu_{nm,ne}^2.$$

By substituting using (6) and solving for $s_{m,e}^M$ in terms of $s_{nm,ne}^1$ and $s_{nm,ne}^2$, we obtain:

$$s_{m,e}^M = 1 - \frac{(1 - s_{nm,ne}^1)(1 - s_{nm,ne}^2)}{1 - s_{nm,ne}^1 s_{nm,ne}^2},$$

which establishes (iii).

We now pause to show that, for all choices of T^F , there is a unique efficiency E that makes the merger profit-neutral. Under the hypothesis the merger is profit-neutral, we have via (iii) that the merged party's market share is fully determined by the equilibrium shares of the merging parties in the nm, ne equilibrium. For fixed T^F , let E solve:

$$\frac{T^1 + T^2 + E}{H_{m,e}} = s_{m,e}^M \exp\left(\frac{1}{1 - s_{m,e}^M}\right).$$

Suppose there exists a second solution to this, E' ; without loss of generality suppose $E' > E$. Then $T^{M'} > T^M$, hence by Proposition 1, $s_{m,e}^{M'} > s_{m,e}^M$, which poses a contradiction because s^M is pinned down by the pre-merger values via (iii).

(i) We claim the merger profit-neutrality curve is upward sloping. By (iii):

$$s_{m,e}^M = \frac{(1 - s_{nm,ne}^1)(1 - s_{nm,ne}^2)}{1 - s_{nm,ne}^1 s_{nm,ne}^2},$$

and, in particular, on the profit-neutrality curve $s_{m,e}^M$ is a fixed function of the ex-ante quantities $(s_{nm,ne}^1, s_{nm,ne}^2)$. Thus, suppose T^F increases. This leads to a decrease in $s_{m,e}^f$ for all $f \neq F$; in particular, absent the constraint of profit-neutrality, $s_{m,e}^M$ would decrease. However, by (iii) it cannot since it must be a fixed function of nm, ne variables. Thus the effect of an increase in T^F must be precisely offset by must be precisely offset by an increase in E , as:

$$\frac{T^1 + T^2 + E}{H_{m,e}} = s_{m,e}^m \exp\left(\frac{1}{1 - s_{m,e}^m}\right).$$

(ii) The first claim follows immediately from the definitions of \bar{T}^F . For the latter claim, suppose that $E = \bar{E}$, and observe that if $T^F = 0$, then the merger is profitable. Conversely, suppose $T^F \rightarrow \infty$, and first suppose further that $\lim_{T^F \rightarrow \infty} T^F / H_{m,e} < \infty$. In this case, since T^F is going to infinity, it must be the case that $H_{m,e}$ is too. Thus for firm M , $T^M / H_{m,e} \rightarrow 0$; by (8) $s^M \rightarrow 0$ and hence so too does π^M . On the other hand, suppose that $\lim_{T^F \rightarrow \infty} T^F / H_{m,e} = \infty$. Then by (8), $s^F \rightarrow 1$, and hence by (10),

$H \rightarrow \infty$. An analogous argument then establishes that $\pi^M \rightarrow 0$. Thus we conclude that as $T^F \rightarrow \infty$, the merged entrant's profits monotonically decreases to 0.²⁶ Since pre-merger, the entrant is not in the market, the pre-merger profits of the merging entities are unaffected by T^F , there exists some T^F for which (T^F, \bar{E}) makes the merger profit-neutral; by Proposition 1, π^M is globally decreasing in T^F , and hence this \bar{T}^F is unique. \square

A.10 Proof of Proposition 9

Proof. We first wish to show that, for all choices of $T^F > 0$, there is a unique efficiency E that makes the merger cause the entrant to be profit-neutral. We consider two equilibria, nm, e and m, e , where the entrant has type $T^F > 0$. If the entrant's profits are unchanged, then by Proposition 4, $H_{nm, e} = H_{m, e} = H$, and:

$$s_{nm, e}^1 + s_{nm, e}^2 = s_{m, e}^M$$

In the m, e equilibrium:

$$\frac{T^1 + T^2 + E}{H} = s_{m, e}^M \exp\left(\frac{1}{1 - s_{m, e}^M}\right),$$

the left-hand side of which is strictly increasing in E . However, the right-hand side is an injective function of $s_{m, e}^M$ and $s_{m, e}^M$ is fixed by the nm, e equilibrium and hence is constant in E , under the hypothesis of entrant profit-neutrality. Thus there can be only one E satisfying the above.

(i) We now show the entrant profit neutrality curve is downward sloping. By Proposition 4, we know that $H_{nm, e} = H_{m, e} = H$ and $s_{nm, e}^1 + s_{nm, e}^2 = s_{m, e}^m$. In equilibrium:

$$\frac{T^1 + T^2 + E}{H} = s_{m, e}^m \exp\left(\frac{1}{1 - s_{m, e}^m}\right).$$

By Proposition 1, increase in T^F for fixed E leads to a decrease in $s_{nm, e}^1$ and $s_{nm, e}^2$. But this implies a decrease in $s_{m, e}^m$ as it is simply the sum of these terms, and hence by the above equation, there must be a commensurate decrease in E .

(ii) We now momentarily consider three market structures: $\mathcal{F}_{nm, ne}$, $\mathcal{F}_{nm, e}$, and $\mathcal{F}_{m, e}$. The entry neutrality line is determined by profit-neutrality across $\mathcal{F}_{nm, e}$ and $\mathcal{F}_{m, e}$; the CS neutrality line is determined by surplus remaining constant across $\mathcal{F}_{nm, ne}$ and $\mathcal{F}_{m, e}$. We claim that if the two curves intersect for some (T^F, E) then $T^F = 0$. By Proposition 4, $CS_{nm, e} = CS_{m, e}$. By hypothesis, $CS_{m, e} = CS_{nm, ne}$. Hence in particular, $H_{nm, ne} =$

²⁶It is straightforward to show that, for any f , T^f/H is monotonically increasing in T^f (e.g. Nocke and Schutz (2018) Online Appendix p. 110). Thus, in particular, the limits we consider exist and our cases are exhaustive.

$H_{nm,e} = H$. Then for each $f \in \mathcal{F}_{nm,e} \setminus \{F\}$, we have:

$$s_{nm,e}^f \exp\left(\frac{1}{1 - s_{nm,e}^f}\right) = \frac{T^f}{H} = s_{nm,ne}^f \exp\left(\frac{1}{1 - s_{nm,ne}^f}\right)$$

and hence $s_{nm,e}^f = s_{nm,ne}^f$. By the adding up constraint:

$$\sum_{f \in \mathcal{F}_{nm,ne}} s^f = \sum_{f \in \mathcal{F}_{nm,e}} s^f$$

and thus $s^F = 0$ and hence so too is T^F . Of course, if $T^F = 0$, then entry accomplishes nothing and for all active firms nm, e and nm, ne equilibria are functionally equivalent. Thus as $T^F \rightarrow^+ 0$, the associated efficiency tends to \bar{E} by definition.

To establish that for all $T^F > 0$, E is strictly positive, observe that as F is in the market both before and after the merger, if there were no efficiencies, then Proposition 2 implies that $\pi_{m,e}^F > \pi_{nm,e}^F$. Since $\pi_{m,e}^F$ is decreasing in $T^M = T^1 + T^2 + E$, and $\pi_{nm,ne}^F$ is constant in E , if there is any E that makes F profit-neutral, it must be unique. However, such an E exists by an argument analogous to that appearing in the proof of Proposition 6 or Proposition 8.(ii).

(iii) Follows from Proposition 4, and the immediate observation that entry increases consumer surplus. \square

A.11 Proof of Proposition 10

Proof. We start with result for the compensating efficiency. From (12), we know that consumer surplus is increasing in the aggregator H . Thus, for consumer surplus to be unaffected by a merger, H must remain constant. By (8), a constant H implies that the shares of non-merging firms are unaffected by the merger, and then for market shares to sum to one, we have $s_{m,e}^M = s_{nm,ne}^1 + s_{nm,ne}^2$. Evaluating (8) for the merged firm (and substituting in for $T^M = T^1 + T^2 + \bar{E}$ and $s_{m,e}^M = s_{nm,ne}^1 + s_{nm,ne}^2$) and then dividing by the sum of the two pre-merger analogues for firms 1 and 2 gives the desired result. See also the proof of Proposition 2 in Nocke and Whinston (2020).

Turning to the minimum efficiency, let (\underline{E}, T^F) be such that (i) the merger is profit-neutral and (ii) consumer surplus is unchanged due to the merger. From (ii), we have $H_{nm,ne} = H_{m,e} = H$, by inspection of (12). From (8), we also have

$$T^M = T^1 + T^2 + \underline{E} = H s_{m,e}^M \exp\left(\frac{1}{1 - s_{m,e}^M}\right)$$

Plugging in for T^1 and T^2 using (8) and solving for \underline{E} yields (20). From (i), we obtain (21), as Proposition 8 applies. \square

A.12 Proof of Proposition 11

Proof. Define two types of aggregators, one for each group,

$$H_g = \sum_{j \in \mathcal{J}_g} \exp \left(\frac{v_j - \alpha p_j}{1 - \sigma} \right), \quad (\text{A.2})$$

and one for the market,

$$H = 1 + \sum_{g \in \mathcal{G}} H_g^{1-\sigma}. \quad (\text{A.3})$$

With the nested logit assumptions, the share of a product within its nest is

$$s_{j|g}(p) = \frac{\exp \left(\frac{v_j - \alpha p_j}{1 - \sigma} \right)}{H_g}, \quad (\text{A.4})$$

and the share of nest g is

$$\bar{s}_g(p) = \frac{H_g^{1-\sigma}}{H},$$

which implies an unconditional share of product j of

$$s_j(p) = s_{j|g}(p) \bar{s}_g(p).$$

We have $s^{f|g} = \sum_{j \in \mathcal{F}_g} s_{j|g}(p)$ and $s^f = \sum_{f \in \mathcal{F}_g} s_j$.

The profit function is given by (3). Substituting the nested logit demand derivatives into the first order conditions for profit maximization (see (4)) and rearranging obtains an ι -markup that is common to all products owned by the same firm:

$$\mu^f \equiv \left(\frac{\alpha}{1 - \sigma} \right) (p_j - c_j) = \frac{1}{1 - \sigma s^{f|g} - (1 - \sigma) s^f},$$

which is (22). Next, adding and subtracting αc_j inside the exponential on the RHS of (A.4) and applying the definitions of μ^f and T^f obtains

$$\begin{aligned} s^{f|g} &= \frac{T^f}{H_g} \exp(-\mu^f), \\ \iff \frac{T^f}{H_g} &= s^{f|g} \exp \left(\frac{1}{1 - s^{f|g}} \right) \end{aligned} \quad (\text{A.5})$$

We have derived (24)-(26), and (23) can be obtained by plugging (24) back into (22). Next, (27) and (28) are adding-up constraints that close the model, (29) is obtained by plugging μ^f into the profit function, and (30) is a characteristic of nested logit demand. \square

A.13 Proof of Proposition 12

Proof. Let the nest of the merging firms be g . From 30, if a merger is neutral for consumer surplus then it does not affect H . Merger also does not affect any group aggregator $H_{g'}$, for $g' \neq g$, by inspection of (A.2). Thus, (A.3) implies that H_g is unaffected. Furthermore, (24)-(26), together with constant H_g and H , imply that the shares for all non-merging incumbents within group g remain the same before and after the merger. Therefore, in order for market shares to add up to one, it must be that the combined post-merger conditional share of the merged firm and the entrant equals the sum of their pre-merger conditional shares:

$$s^{F|g} + s^{M|g} = s^{1|g} + s^{2|g}.$$

Given that H and all H_g are unaffected by the merger, this also implies that

$$s^F + s^M = s^1 + s^2.$$

Next, evaluating (24) for firm F after the merger and firms 1 and 2 before the merger, and dividing the result for firm F by the sum of the results for firms 1 and 2 gives (31), after substituting in for $s^{F|g} = s^{1|g} + s^{2|g} - s^{M|g}$. As there are no merger efficiencies, $T^M = T^1 + T^2$. Substituting into this sum for types using (24) gives (32). \square

A.14 Proof of Theorem 2

Proof. Suppose, for purposes of contradiction, there exists an SPE in which firms 1 and 2 merge, and consumers are unharmed. Thus the merger must increase joint profits:

$$\pi^m(T^1 + T^2, H_g^{m,e}) \geq \pi^1(T^1, H_g^{nm,ne}) + \pi^2(T^2, H_g^{nm,ne}),$$

where $H_g^{nm,ne}$ denotes the group aggregator with no merger and no entry, while $H_g^{m,e}$ is the same object but for a merger with entry. The products in all other nests remain the same, meaning that the resulting overall aggregator is a function of activity from group g , so we have dropped H in order to save on notation.

By hypothesis, consumers are unharmed, hence we have $H_g^{m,e} \geq H_g^{nm,ne}$. Furthermore, profits are decreasing in H_g , meaning that

$$\pi^m(T^1 + T^2, H_g^{nm,ne}) \geq \pi^1(T^1, H_g^{nm,ne}) + \pi^2(T^2, H_g^{nm,ne}).$$

Equations (29) and (A.5) imply that profit is given by

$$\pi^f = ((1 - \sigma)/\alpha)\mu^f(T^f/H_g)\exp(-\mu^f)\bar{s}_g.$$

Substituting for profit in the preceding inequality, the $((1 - \sigma)/\alpha)$, H_g , and \bar{s}_g cancel. The expression becomes

$$(T^1 + T^2)\mu^m \exp(-\mu^m) \geq (T^1)\mu^1 \exp(-\mu^1) + (T^2)\mu^2 \exp(-\mu^2),$$

or, rearranging,

$$\sum_{i=1,2} T^i(\mu^m \exp(-\mu^m) - \mu^i \exp(-\mu^i)) \geq 0.$$

However, we know from Equation (22) that the markup is at least 1, which means that the function $\mu^f \exp(-\mu^f)$ is decreasing in the markup. Furthermore, the markup is increasing in own-firm type. This means that the left hand side of the above expression is negative, which generates a contradiction. \square

A.15 Proof of Theorem 3

We will prove the theorem by first establishing two technical lemmas. Define:²⁷

$$\Omega_g(H, H_g; \sigma) = \frac{1}{H_g} \sum_{f \in \mathcal{F}_g} \sum_{j \in \mathcal{J}^f} \exp \left[\frac{\delta_j - \alpha c_j}{1 - \sigma} - \tilde{m}^f \left(\frac{\sigma}{H_g} + (1 - \sigma) \frac{1}{H_g^\sigma H} ; \sigma \right) \right].$$

where the the function $\tilde{m}^f(X; \sigma)$ defined as the solution in μ^f , for fixed σ to:

$$\frac{\mu^f - 1}{\mu^f} \frac{1}{T^f \exp(-\mu^f)} = X. \quad (\text{A.6})$$

Let: $\Omega : \mathbb{R}_{++}^{G+1} \times [0, 1) \rightarrow \mathbb{R}^{G+1}$ via:

$$\Omega((H_g)_{g \in \mathcal{G}}, H; \sigma) = \begin{bmatrix} \Omega^1(H_1, H; \sigma) - 1 \\ \vdots \\ \Omega^G(H_G, H; \sigma) - 1 \\ 1 + \sum_{g \in \mathcal{G}} H_g^{1-\sigma} - H \end{bmatrix}.$$

The set of equilibria, treating σ as a free parameter, are precisely the solutions to:

$$\Omega((H_g)_{g \in \mathcal{G}}, H; \sigma) = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}. \quad (\text{A.7})$$

The differential of Ω , evaluated at a solution to (A.7), is of the form:

$$D\Omega((H_g)_{g \in \mathcal{G}}, H; \sigma) = \left(\begin{array}{ccc|c|c} \Lambda & & & \Theta & * \\ \hline (1-\sigma)H_1^{-\sigma} & \cdots & (1-\sigma)H_G^{-\sigma} & -1 & -\sum_{g \in \mathcal{G}} H_g^{1-\sigma} \ln H_g \end{array} \right) \quad (\text{A.8})$$

²⁷See equation (xxxi) in Nocke and Schutz (2018) Appendix (p. 70) for reference.

where Λ is a $G \times G$ diagonal matrix with:

$$\Lambda_{gg} = \frac{1}{H_g} \left(\frac{\sigma}{H_g} + \frac{\sigma(1-\sigma)}{H_g^\sigma H} \right) B_g - \frac{1}{H_g},$$

and Θ is the $G \times 1$ matrix with:

$$\Theta_g = \frac{\partial \Omega_g}{\partial H} = \frac{(1-\sigma)}{H_g^\sigma H^2} B_g,$$

where the expression B_g is given by:

$$B_g = \frac{1}{H_g} \sum_{f \in \mathcal{F}_g} \sum_{j \in \mathcal{J}^f} \exp \left[\frac{\delta_j - \alpha c_j}{1-\sigma} - \tilde{m}^f \left(\frac{\sigma}{H_g} + (1-\sigma) \frac{1}{H_g^\sigma H} ; \sigma \right) \right] \tilde{m}^{f'} \left(\frac{\sigma}{H_g} + \frac{(1-\sigma)}{H_g^\sigma H} \right)$$

We now turn to our first technical lemma.

Lemma A.1. *For some $\varepsilon > 0$, the differential $D\Omega$, evaluated at any solution to (A.7) with $\sigma \in [0, \varepsilon)$, is of rank $G + 1$.*

Proof. We break down the proof into steps.

1. **Rank at least G :** Firstly, by direct observation, the upper-left $G \times G$ block Λ is diagonal. Moreover, each diagonal element is strictly negative (see Nocke and Schutz (2018) Online Appendix, Lemma XXIII). Hence the first G columns of $D\Omega$ are linearly independent, evaluated at any solution to (A.7).
2. **Removal of Nuisance Terms:** Suppose we evaluate $D\Omega$ at the unique solution to (A.7) with $\sigma = 0$. Then, in particular:

$$\Lambda_{gg}|_{\sigma=0} = -\frac{1}{H_g},$$

and

$$\Theta_g|_{\sigma=0} = \frac{1}{H^2} B_g|_{\sigma=0}.$$

3. **Contradiction Hypothesis:** Suppose, for sake of contradiction, that the $G + 1$ st column of $D\Omega$ evaluated at the unique solution to (A.7) where $\sigma = 0$ is a linear combination of the first G columns. Then there exist $(a_g)_{g=1}^G$ such that:

$$(\forall g) \quad \Lambda_{gg}|_{\sigma=0} a_g = \Theta_g|_{\sigma=0},$$

and which satisfy:

$$\sum_{g=1}^G a_g = -1. \tag{*}$$

Using the results of the preceding step, we can back out these weights:

$$(\forall g) \quad a_g = -\frac{H_g}{H^2} B_g|_{\sigma=0}.$$

4. **Algebra:** Then, plugging in to (*), we obtain:

$$\sum_{g \in \mathcal{G}} \sum_{f \in \mathcal{F}_g} \sum_{j \in \mathcal{J}^f} \exp \left[\delta_j - \alpha c_j - \tilde{m}^f(1/H) \right] \tilde{m}^{f'}(1/H) = H^2$$

Since we're at an equilibrium (i.e. a solution to (A.7)) we can simplify this using the usual system of equations that hold in an equilibrium. In particular:

$$\sum_{g \in \mathcal{G}} \sum_{f \in \mathcal{F}_g} T^f \exp(-\mu^f) \tilde{m}^{f'}(1/H) = H^2.$$

5. **Dealing with $\tilde{m}^{f'}$:** Recall \tilde{m}^f is the implicit solution to (A.6). In particular,

$$\frac{d\tilde{m}^f}{dX} = \frac{T^f \tilde{m}^f \exp(-\tilde{m}^f)}{1 - XT^f [\exp(-\tilde{m}^f) - \tilde{m}^f \exp(-\tilde{m}^f)]}.$$

For the H under consideration, let us define $\mu^f = \tilde{m}^f(1/H)$. Then this derivative, evaluated at $X = \frac{1}{H}$, is:

$$\frac{T^f \mu^f \exp(-\mu^f)}{1 - \frac{1}{H} T^f [\exp(-\mu^f) - \mu^f \exp(-\mu^f)]}.$$

Now, as we are working at an equilibrium, it must be the case that $T^f \mu^f \exp(-\mu^f) = H(\mu^f - 1)$, hence our expression for the derivative at $\frac{1}{H}$ may be simplified to:

$$\frac{H \mu^f (\mu^f - 1)}{1 + \mu^f (\mu^f - 1)}.$$

6. **Simplifying** Plugging in the result of Step 5 into that of Step 4 and dividing both sides by H yields:

$$\sum_{g \in \mathcal{G}} \sum_{f \in \mathcal{F}_g} T^f \exp(-\mu^f) \left[\frac{\mu^f (\mu^f - 1)}{1 + H \mu^f (\mu^f - 1)} \right] = H.$$

Note that the square bracketed term lies strictly within $[0, 1)$ for all $\mu > 1$. Thus:

$$\begin{aligned}
\sum_{g \in \mathcal{G}} \sum_{f \in \mathcal{F}_g} T^f \exp(-\mu^f) \left[\frac{\mu^f(\mu^f - 1)}{1 + \mu^f(\mu^f - 1)} \right] &< \sum_{g \in \mathcal{G}} \sum_{f \in \mathcal{F}_g} T^f \exp(-\mu^f) \\
&= \sum_{g \in \mathcal{G}} \sum_{f \in \mathcal{F}_g} H_g s^{f|g} \\
&= \sum_{g \in \mathcal{G}} H_g \sum_{f \in \mathcal{F}_g} s^{f|g} \\
&= \sum_{g \in \mathcal{G}} H_g \\
&< 1 + \sum_{g \in \mathcal{G}} H_g \\
&= H.
\end{aligned}$$

Thus $(*)$ can never hold for any (a_g) , and thus the first $G + 1$ columns of $D\Omega$, at the solution to (21) where $\sigma = 0$, are linearly independent. By continuity of these terms in σ , the same must be true for some small enough open set of σ 's containing 0, and the result follows. \square

We now establish the following immediate corollary:

Lemma A.2. *Let $\hat{\Omega} : \mathbb{R}_{++}^{G+1} \rightarrow \mathbb{R}^{G+1}$ denote the restriction of Ω to the open set $\mathbb{R}_{++}^{G+1} \times \{0\}$. Then $D\hat{\Omega}$ is of full rank at the unique solution to (A.7) in this domain.*

Proof. By direct calculation:

$$D\hat{\Omega} = \left(\begin{array}{c|c} \hat{\Lambda} & \hat{\Theta} \\ \hline 1 & \dots & 1 & -1 \end{array} \right)$$

where $\hat{\Lambda}_{gg} = -\frac{1}{H_g}$ and $\hat{\Theta}_g = \frac{1}{H^2} B_g|_{\sigma=0}$, hence an identical argument to the prior lemma yields the result. \square

We now prove Theorem 3.

Proof. Let $\varepsilon > 0$ be any such value such that the conclusions of Lemma A.1 hold, and by abuse of notation, denote the restriction of Ω to $\mathbb{R}_{++}^{G+1} \times [0, \varepsilon')$ for any $0 < \varepsilon' < \varepsilon$ simply by Ω . By Lemma A.1, 0 is a regular value of Ω on this domain, and by Lemma A.2, 0 is also a regular value of Ω restricted to the boundary of the domain. Thus by the Regular Value Theorem (see Hirsch (2012) Theorem 1.4.1, see also Mas-Colell (1974) Theorem 2), $\Omega^{-1}(0)$ is a C^1 submanifold of $\mathbb{R}_{++}^{G+1} \times [0, \varepsilon')$, with boundary precisely equal to the

unique equilibrium at $\sigma = 0$. Consider the (necessarily unique) connected component of $\Omega^{-1}(0)$ that intersects $\mathbb{R}_{++}^{G+1} \times \{0\}$. Since this component is a connected C^1 manifold with boundary, it is C^1 -diffeomorphic to $[0, 1]$ (Hirsch (2012) Exercise 1.5.9).²⁸ Since the Regular Value Theorem guarantees its intersection with the slice $\mathbb{R}_{++}^{G+1} \times \{0\}$ is transverse, the restriction of this component to $\mathbb{R}_{++}^{G+1} \times [0, \varepsilon'']$ for some $0 < \varepsilon'' < \varepsilon'$ is diffeomorphic to $[0, 1]$, and hence is compact.

However, $\Omega^{-1}(0)|_{\mathbb{R}_{++}^{G+1} \times [0, \varepsilon'']}$ is also the graph of the function $e : [0, \varepsilon''] \rightarrow \mathbb{R}_{++}^{G+1}$ that takes a nesting parameter value and maps it to the unique differentiated Bertrand-Nash pricing game of the model. Thus we may view e as taking $[0, \varepsilon'']$ into some compact $K \subseteq \mathbb{R}_{++}^{G+1}$ such that (i) $K \times [0, \varepsilon'']$ contains $\Omega^{-1}(0)|_{\mathbb{R}_{++}^{G+1} \times [0, \varepsilon'']}$, and (ii) the graph of e is a closed subset of $K \times [0, \varepsilon'']$.²⁹ But then by the Closed Graph Theorem (Aliprantis and Border (2006) Theorem 2.58), this map is continuous on $[0, \varepsilon'']$. \square

A.16 Proof of Theorem 4

Proof. Suppose, for purposes of contradiction, there exists an SPE in which firms 1 and 2 merge, and consumers are unharmed, where the nesting parameter $\sigma = 0$. Thus the merger must increase joint profits, and, moreover, as $\sigma = 0$, profits depend only on the aggregator H :

$$\pi^m(T^1 + T^2, H_{m,e}) \geq \pi^1(T^1, H_{nm,ne}) + \pi^2(T^2, H_{nm,ne}),$$

where $H_{nm,ne}$ denotes the aggregator with no merger and no entry, while $H_{m,e}$ is the same object but for a merger with entry, where the entry is in to an unspecified nest (which of course does not matter for the value of H here).

By hypothesis, consumers are unharmed, hence we have $H_{m,e} \geq H_{nm,ne}$. Furthermore, profits are decreasing in H , meaning that

$$\pi^m(T^1 + T^2, H_{nm,ne}) \geq \pi^1(T^1, H_{nm,ne}) + \pi^2(T^2, H_{nm,ne}).$$

Equations (29) and (A.5) imply that profit is given by

$$\pi^f = \frac{1}{\alpha} \frac{T^f}{H} \mu^f \exp(-\mu^f).$$

Substituting for profit in the preceding inequality, we obtain:

$$(T^1 + T^2) \mu^m \exp(-\mu^m) \geq (T^1) \mu^1 \exp(-\mu^1) + (T^2) \mu^2 \exp(-\mu^2),$$

²⁸It cannot be diffeomorphic to $[0, 1]$ as from the Regular Value theorem, its boundary is given precisely by its intersection with the boundary of the domain, and at $\sigma = 0$ the equilibrium is unique.

²⁹It suffices to let K be the projection of $\Omega^{-1}(0)|_{\mathbb{R}_{++}^{G+1} \times [0, \varepsilon'']}$ onto \mathbb{R}_{++}^{G+1} to satisfy both these properties. In particular, this set is compact by continuity of the projection, and the graph of e is closed in $\mathbb{R}_{++}^{G+1} \times [0, \varepsilon'']$ hence it is closed in the subspace topology on $K \times [0, \varepsilon'']$.

or, rearranging,

$$\sum_{i=1,2} T^i (\mu^m \exp(-\mu^m) - \mu^i \exp(-\mu^i)) \geq 0.$$

However, we know from Equation (22) that the markup is at least 1, which means that the function $\mu^f \exp(-\mu^f)$ is decreasing in the markup. By assumption that at least one of T^1 or T^2 are strictly greater than zero (i.e. are market participants), and since the markup is increasing in own-firm type we see the left hand side of the above expression is strictly negative, which generates a contradiction. Thus, for $\sigma = 0$, if consumer surplus is unhurt then:

$$\pi^m(T^1 + T^2, H_g^{m,e}, H) < \pi^1(T^1, H_g^{nm,ne}, H) + \pi^2(T^2, H_g^{nm,ne}, H)$$

Suppose then we consider a sequence of nesting parameter values $(\sigma_n)_{n \in \mathbb{N}}$ such that $\sigma_n \rightarrow 0$. By Theorem 3, and the continuous dependence of profits and markups on the underlying equilibrium variables $(H_g)_{g \in \mathcal{G}}$ and H , we obtain a sequence of profits for the individual merging parties and the merged entity which converge to their $\sigma = 0$ values as $\sigma_n \rightarrow 0$. In particular, for n large enough it then must be the case that:

$$\pi^m(T^1 + T^2, H_g^{m,e}(\sigma_n), H(\sigma_n)) < \pi^1(T^1, H_g^{nm,ne}(\sigma_n), H(\sigma_n)) + \pi^2(T^2, H_g^{nm,ne}(\sigma_n), H(\sigma_n)),$$

establishing the result. \square

B Numerical Methods

B.1 Basics of Calibration and Simulation

The numerical results presented in this paper can be generated with data on pre-merger market shares, with the exception of the T-Mobile/Sprint application. Proposition 3 indicates that it is possible to recover types from market shares, and vice-versa. To implement the former—a calibration step—first obtain the market aggregator from (10), and the ι -markups from (6). We recover firm types from a rearranged (8):

$$T^f = \frac{s^f H}{\exp(-\mu^f)}.$$

To implement the latter—a simulation step—use a nonlinear equation solver to recover the shares and the market aggregator, given these types. There are $F + 1$ nonlinear equations that must be solved simultaneously. One of these is the adding-up constraint of (10), and the others are obtained by plugging (6) into (8), which yields

$$s^f = \frac{T^f}{H} \exp\left(-\frac{1}{1 - s^f}\right).$$

If one knows the types, and thus also the aggregator, then markups, profit, con-

sumer surplus, and welfare are identified up to a multiplicative constant (see (11)-(14)). Thus, the outcomes that arise with different firm types can be meaningfully compared. This allows for merger simulation with or without entry and efficiencies. The ratio of outcomes is identified, even with the partial calibration approach, because the multiplicative constant cancels.

A full calibration also recovers the multiplicative constant—the price parameter, α . This can be accomplished with data on one margin, for example. See also the Nocke and Schutz (2018) appendix. Then markups, profit, consumer surplus, and welfare also are obtained (not just up to a multiplicative constant). However, these objects are not necessary for our purposes, so we use partial calibration.

B.2 The Integrated Framework

We discuss how to construct the figures shown in Section 4. We start with knowledge of pre-merger market shares. Our baseline is four incumbents and an outside good, each with a market share of 0.20. We calibrate the types and obtain pre-merger profit and consumer surplus.³⁰ We consider a large number of entrants, defined by their types, T_i^F , such that entrant $i = 0$ has $T^F = \epsilon$ for some small ϵ and each other entrant $i > 0$ has a type $T_i^F = T_{i-i}^F + \epsilon$. We simulate a ‘no merger, entry’ equilibrium for each of these entrants and save the profit of the entrant. Then we simulate a series of ‘merger, entry’ equilibrium for each these entrants. The merger simulations involve replacing the types of the merging firms (T^1, T^2) with the type of the merged firm ($T^M = T^1 + T^2 + E$) and adding the type of the entrant (T^F). In the series of merger simulation for a given entrant type, we start E at $E = \bar{E}$ and then gradually decrease E so as to identify the levels of efficiency that (i) entrant profit is the same as in the ‘no merger, entry’ scenario, (ii) consumer surplus is the same as in the pre-merger scenario, and (iii) the merged firm has the same profit as the merging firms (together) in the pre-merger scenario. This allows us to trace out the neutrality curves of Figure 1. Then, Figure 2 can be constructed by comparing the entrants’ profit in the ‘no merger, entry’ scenario to profit in the ‘merger, entry’ scenario, with and without lower bound efficiencies.

B.3 Best-Case Entry

We discuss how to construct the figures shown in Section 6.1. We start again with knowledge of pre-merger market shares. We calibrate the types and obtain pre-merger profit, markups and consumer surplus. We also simulate a merger without entry by replacing the types of the merging firms (T^1, T^2) with the type of the merged firm ($T^M = T^1 + T^2 + E$) and again obtain profit, markups, and consumer surplus. We then simulate a series of merger with entry, using a gradually increasing T^F , until the merged firm has the same profit as the merging firms (together) in the pre-merger scenario. This identifies the best-case entrant. We store the markups and consumer

³⁰Again, the welfare statistics are identified up to a multiplicative constant.

surplus of merger with best-case entry. Repeating over the many sets of pre-merger market shares discussed in Section 6.1 generates the data used in Figures 3 and 4.

We calculate the percentage of consumer surplus loss that is mitigated by best-case entry according to:

$$1 - \frac{\Delta CS^{m,e}}{\Delta CS^{m,ne}} = 1 - \frac{\alpha \Delta CS^{m,e}}{\alpha \Delta CS^{m,ne}}, \quad (\text{B.1})$$

where ΔCS refers the change in consumer surplus due to the merger. The left-hand-side of the equation is the object of interest, and it can be recovered because both the numerator and denominator on the right-hand-side can be recovered without knowledge of α . (The α term cancels with the $(1/\alpha)$ term in that appears in the consumer surplus expression.) A similar approach can be taken for other welfare statistics.

We also calculate the percentage of the merging firms' price increase that is mitigated by entry:

$$1 - \frac{\Delta p^{m,e}}{\Delta p^{m,ne}} = 1 - \frac{\Delta \mu^{m,e}}{\Delta \mu^{m,ne}}, \quad (\text{B.2})$$

where again the left-hand-side is the object of interest, and both the numerator and denominator on the right-hand-side can be recovered with simulation. Interestingly, we can characterize the *mitigation* of price increases using this method, even though the price increases are not identified.

Properties of the Best-Case Entrant

Figure C.7 provides information on the best-case entrant. The dashed line plots the ratio of the entrant type to the type of a merging firm. The entrant type is less than 20 percent of the merging firm type for all mergers considered. The solid line plots the entrant share in the post-merger equilibrium, which is substantially less than half the pre-merger share of a merging firm for all the mergers considered. Thus, modest levels of entry can eliminate the profitability of mergers. However, such entry does little to offset price increases, and may leave consumer surplus losses mostly unmitigated.

B.4 Application to T-Mobile/Sprint

Our primary source of data is the 2017 Annual Report of the Federal Communications Commission (FCC) on competition in the mobile wireless sector.³¹ We obtain the following information:

- Among national providers, Verizon, AT&T, T-Mobile, and Sprint account for 35.0, 32.4, 17.1, and 14.3 percent of total connections at end-of-year 2016, respectively. See Figure II.B.1 on page 15.

³¹Annual Report and Analysis of Competitive Market Conditions With Respect to Mobile Wireless, Including Commercial Mobile Services, FCC-17-126.

- The average revenue per user (ARPU) in 2016:Q4 for Verizon, AT&T, T-Mobile, and Sprint is 37.52, 36.58, 33.80, 32.03, respectively. See Figure III.A.1 on page 42. Following common practice, we use the ARPU as a measure of price.
- The EBITDA per subscriber in 2016 for Verizon, AT&T, T-Mobile, and Sprint is 22.71, 18.30, 11.80, 13.00, respectively. See Figure II.D.1 on page 24. We interpret the EBITDA as providing the markup.

Finally, we obtain a market elasticity of -0.3 from regulatory filings. The market elasticity is defined theoretically as $\epsilon = -\alpha s_0 \bar{p}$, where \bar{p} is the weighted-average price.

The main distinction between the T-Mobile/Sprint application and our other numerical results is that we do not observe pre-merger market shares. The reason is that the FCC data on total connections does not incorporate the consumer option to purchase the outside good. Thus, we use a full calibration approach in which we use the market elasticity and a markup (we use the T-Mobile ARPU) to recover the outside good share and the price coefficient. We obtain an outside good share of 0.084. With this in hand, the pre-merger market for T-Mobile, for example, is $17.1/(1 - 0.084)$. With the pre-merger market shares in hand, Figures 5 and C.8 can be created using the methods described above.

C Additional Figures

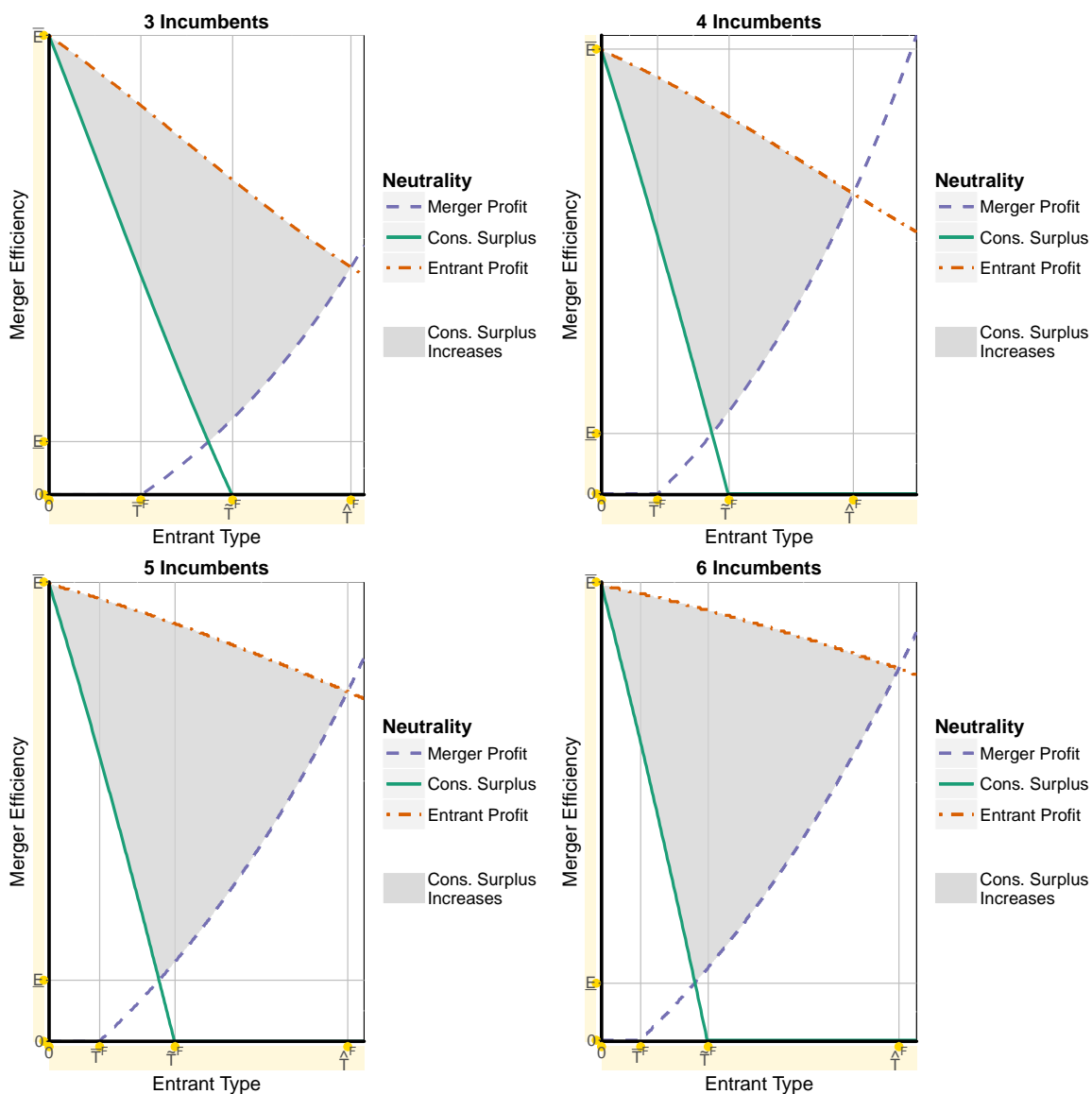


Figure C.1: Integrated Framework Varying the Number of Incumbents

Notes: The figure illustrates the integrated framework for merger analysis with entry and efficiencies for markets with three, four, five, and six incumbents. The results are generated numerically given pre-merger market shares. In each case, the outside good is assigned a market share of 0.20 and the remaining 0.80 share is split evenly among the incumbents.

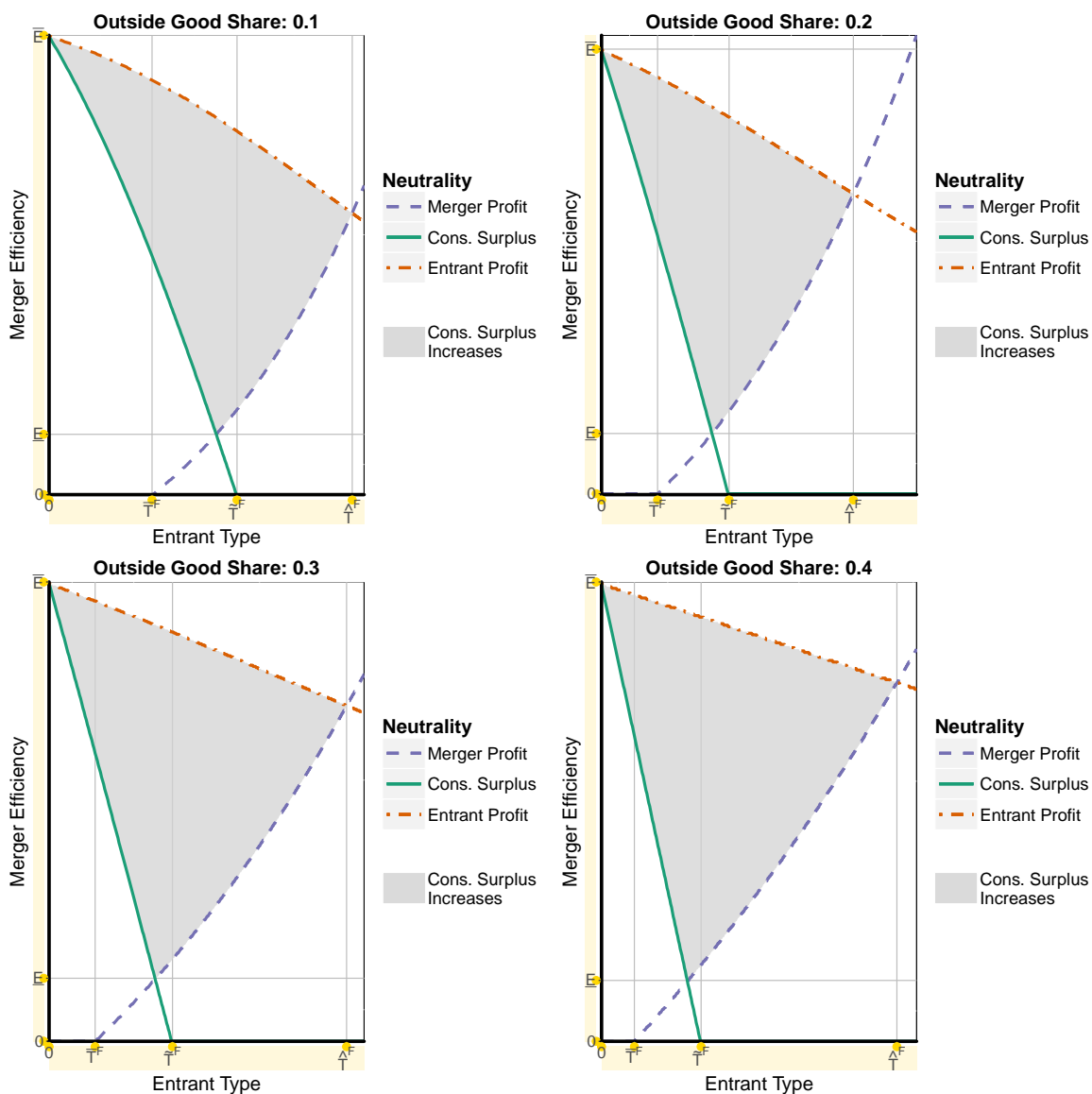


Figure C.2: Integrated Framework Varying the Outside Good Share

Notes: The figure illustrates the integrated framework for merger analysis with entry and efficiencies for markets in which the outside good receives a pre-merger market share of 0.10, 0.20, 0.30, and 0.40. The results are generated numerically given pre-merger market shares. In each case, the remaining share (0.90, 0.80, 0.70, and 0.60) is split evenly among four incumbents.

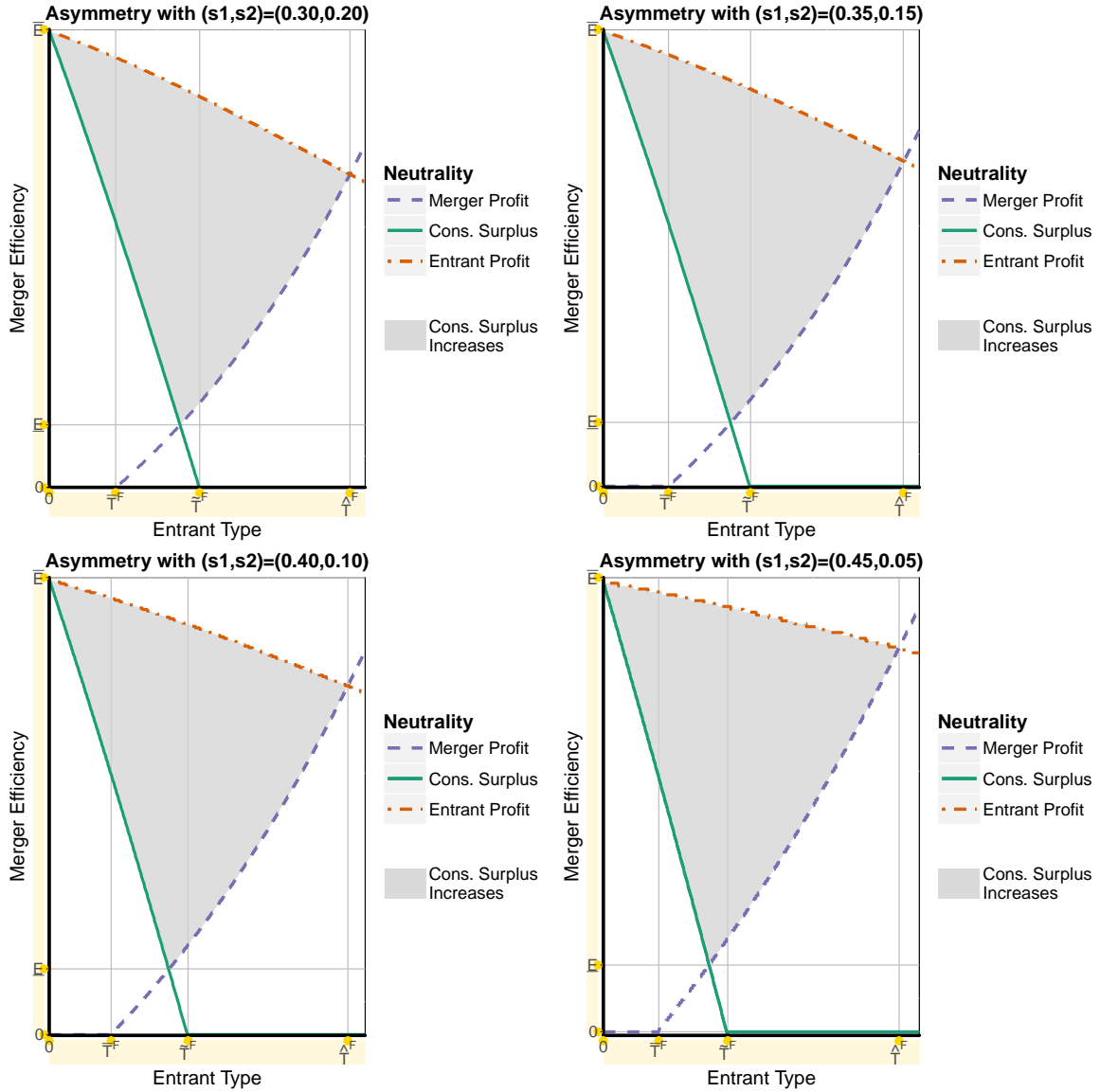


Figure C.3: Integrated Framework with Asymmetric Oligopoly

Notes: The figure illustrates the integrated framework for merger analysis with entry and efficiencies for markets with asymmetric oligopoly. The results are generated numerically. The pre-merger market shares are 0.15 for each of two non-merging incumbents, 0.20 for the outside good, and according to the panel titles for the merging firms.

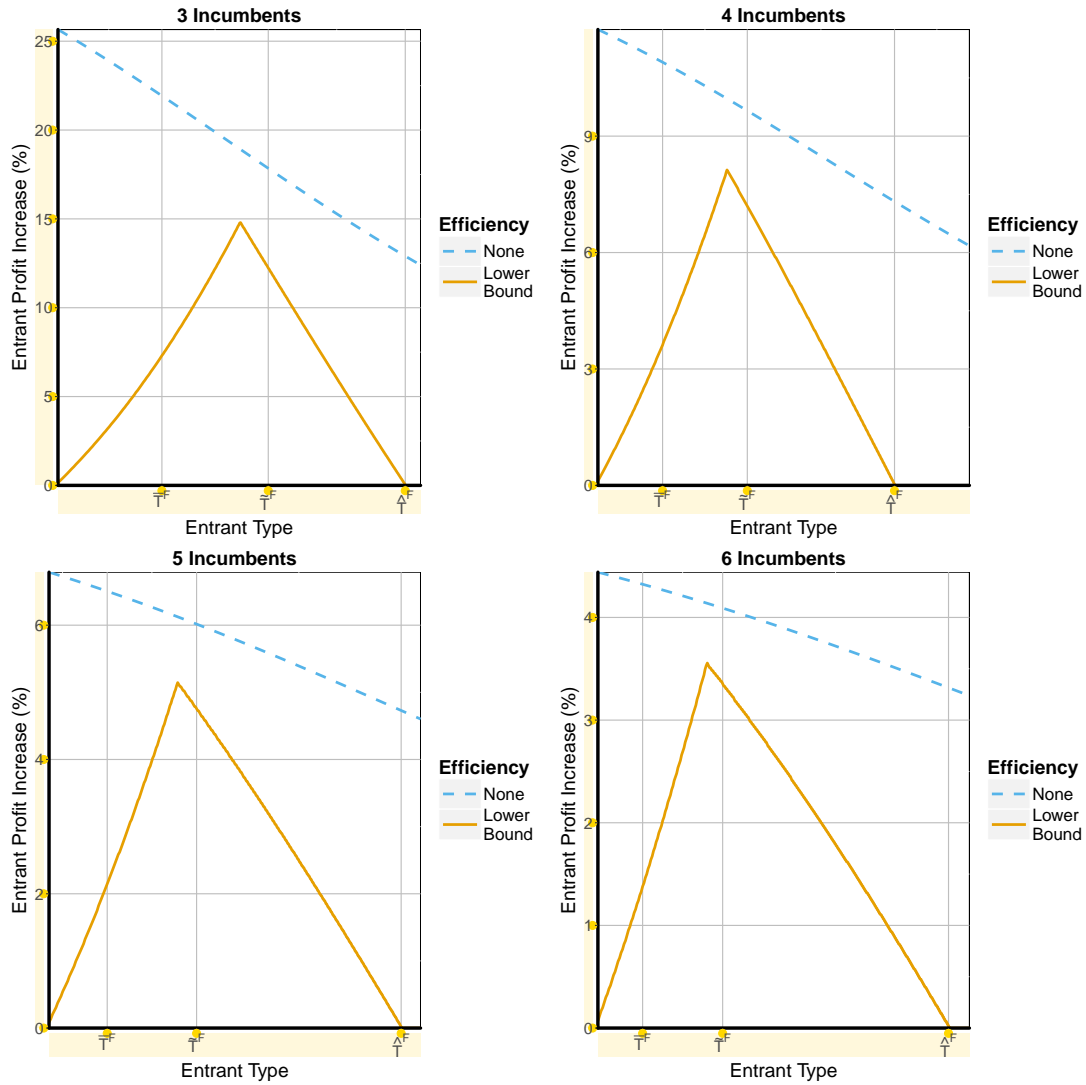


Figure C.4: The Entrant's Profit Opportunity by the Number of Incumbents

Notes: The panels plot the percentage change in entrant profit due to a merger without efficiencies, along with the analogous percentage change due to a merger with lower bound efficiencies. The results are generated numerically for markets with three, four, five, and six incumbents, and an initial outside good share of 0.20.

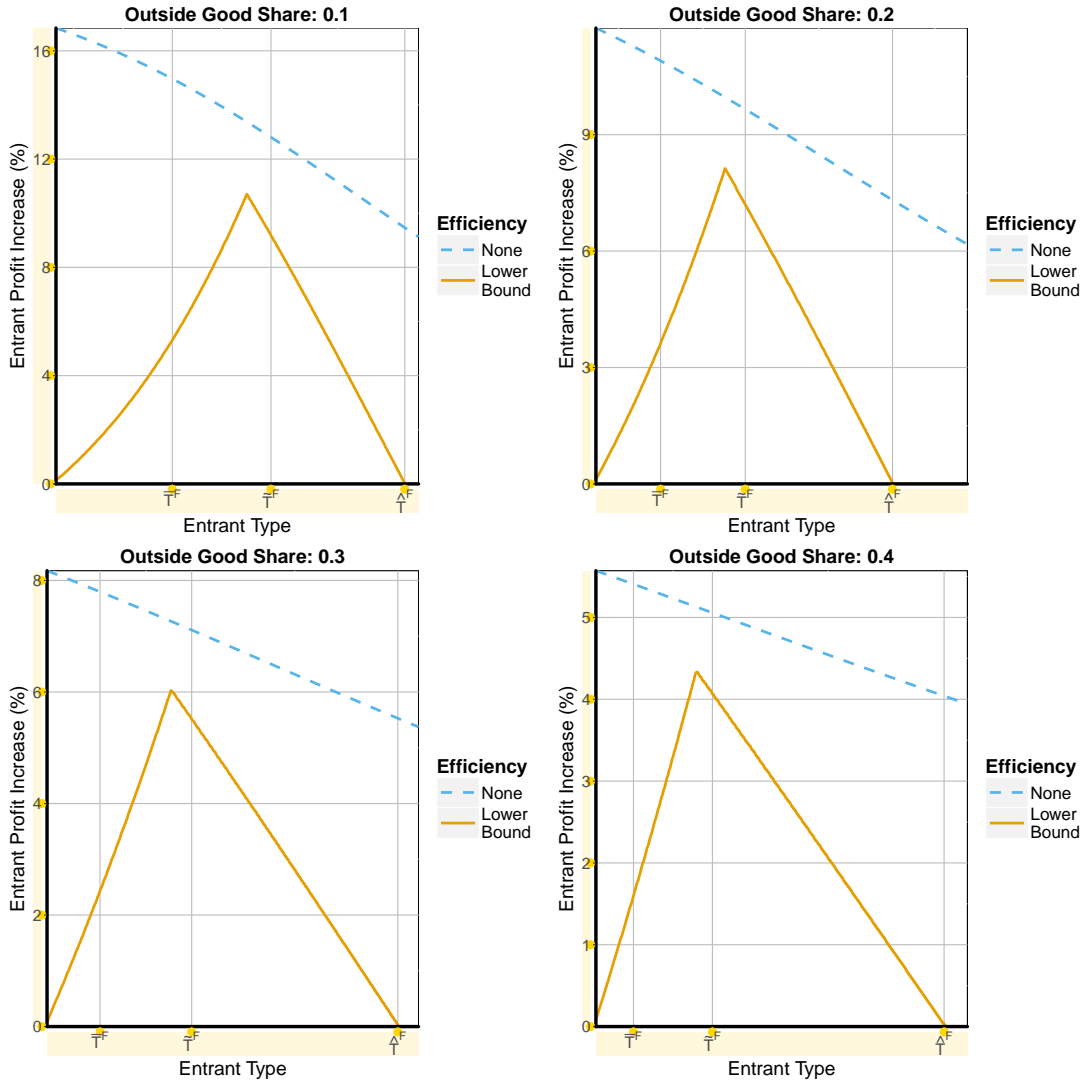


Figure C.5: The Entrant's Profit Opportunity Varying the Outside Good Share

Notes: The panels plot the percentage change in entrant profit due to a merger without efficiencies, along with the analogous percentage change due to a merger with lower bound efficiencies. The results are generated numerically for markets with four incumbents and an initial outside good share of 0.10, 0.20, 0.30, and 0.40.

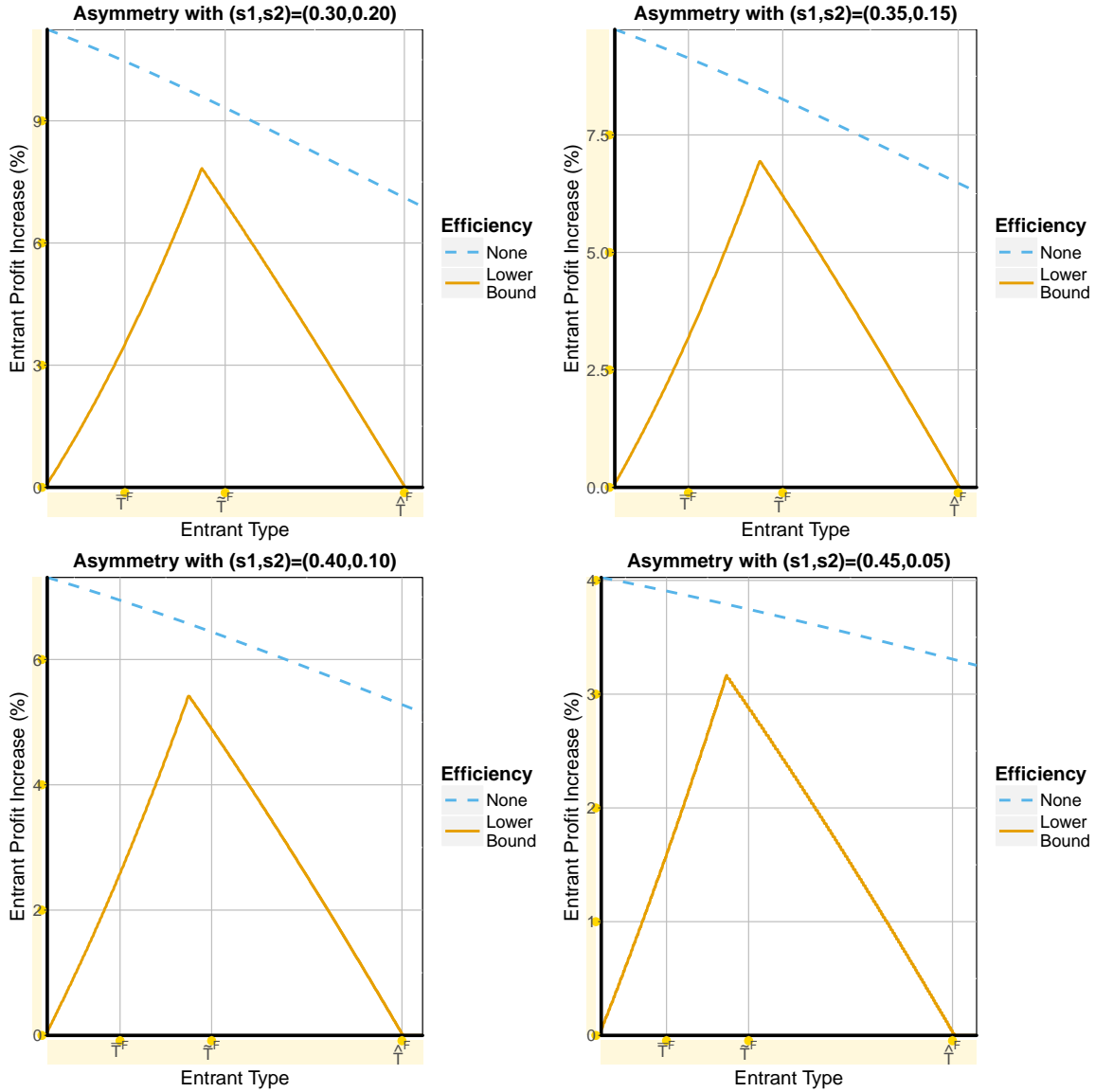


Figure C.6: The Entrant's Profit Opportunity with Asymmetric Oligopoly

Notes: The panels plot the percentage change in entrant profit due to a merger without efficiencies, along with the analogous percentage change due to a merger with lower bound efficiencies. The results are generated numerically. The pre-merger market shares are 0.15 for each of two non-merging incumbents, 0.20 for the outside good, and according to the panel titles for the merging firms.

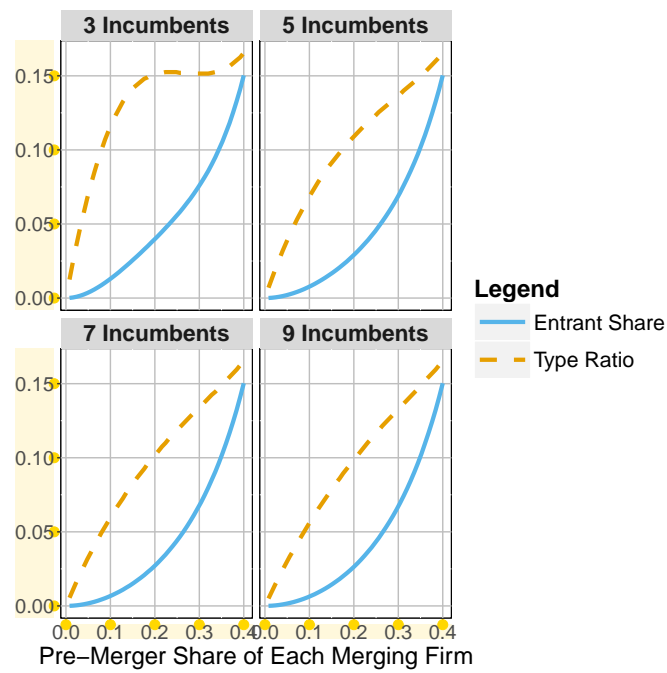


Figure C.7: Properties of the Best-Case Entrant

Notes: The figure examines two properties of the entrant that makes a merger revenue-neutral: the entrant share in the post-merger equilibrium and the ratio of the entrant's type to that of a merging firm.

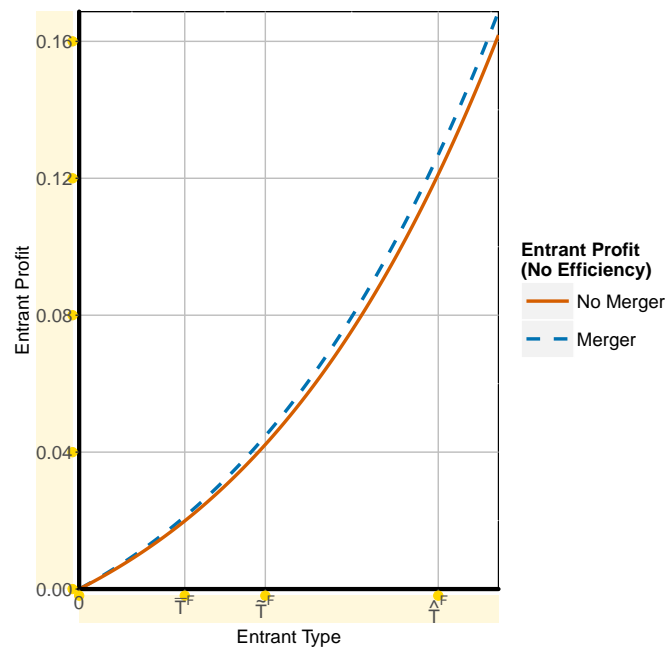


Figure C.8: Entry Profit with Sprint/T-Mobile

Notes: The figure plots entrant profit with and without the Sprint/T-Mobile merger, under the assumption that the merger does not generate an efficiency. Merger-induced entry occurs if entry costs fall between the two lines. The figure is generated numerically given market shares for the wireless telecommunications industry.