Mergers, Entry, and Consumer Welfare[*]

Peter Caradonna[†]

California Institute of Technology

Nathan H. Miller[‡]

Georgetown University

Gloria Sheu[§]

Federal Reserve Board

June 27, 2022

Abstract

We analyze mergers and entry in oligopoly models of differentiated-products price competition. Under logit or constant elasticity of substitution demands, entry that restores pre-merger consumer surplus renders merger unprofitable. Thus, by revealed preference, it can be appropriate to infer entry barriers in merger review. The result extends to nested and random coefficients demand systems unless the entrant is a distant competitor of the merging firms. We develop modeling frameworks to guide empirical analysis in settings where theory is not dispositive. Applying these to the T-Mobile/Sprint merger, we find the Court may have erred in treating DISH as a merger-induced entrant.

[†]California Institute of Technology, Division of the Humanities and Social Sciences, 1200 East California Boulevard, Pasadena, CA 91125. Email: ppc@caltech.edu.

[‡]Georgetown University, McDonough School of Business, 37th and O Streets NW, Washington DC 20057. Email: nathan.miller@georgetown.edu.

[§]Board of Governors of the Federal Reserve System, 20th Street and Constitution Avenue NW, Washington DC 20551. Email: gloria.sheu@frb.gov.

1 Introduction

The antitrust review of mergers in the European Union and the United States follows formal merger guidelines promulgated by their respective antitrust authorities. One step examines post-merger entry by competitors. The EU and US merger guidelines propose that entry that is "timely, likely, and sufficient" might deter or counteract any anticompetitive actions of the merging firms.[1] In practice, the associated analyses tends to be empirical: the antitrust authorities collect testimony and documents from industry participants and prospective entrants about their past and future plans and their assessments of any regulatory, investment, or other entry barriers. Another indicator can be whether entry has been observed in the recent past.[2]

In this paper, we examine mergers and entry with a more theoretical lens. We prove that, under certain conditions, entry sufficient to restore pre-merger consumer surplus renders the merger unprofitable. By revealed preference, one can infer in merger review that barriers preclude entry at a scale that would protect consumers, and the empirical analysis of entry proposed in the guidelines is redundant and unnecessary. For settings in which theory is not dispositive, we derive a set of results to help interpret the empirical evidence described in the guidelines. These results demonstrate how entry analysis can incorporate relevant considerations about the market and the merger, such as the number of competitors, their market shares, or whether the merging firms would benefit from more efficient production.

Our focus is on differentiated-products Bertrand competition. In this context, prices after merger and entry may be higher than in the pre-merger equilibrium, hence, making entry profitable, yet consumer surplus may also be higher due to the additional product variety supplied by the entrant. We derive analytical results on mergers and entry by reformulating the model as an aggregative game, following Nocke and Schutz (2018). The model of Bertrand competition is often used in merger review (Miller and Sheu (2021)).[3]

---

[1] For the EU, see Section 6 of the Guidelines on the Assessment of Horizontal Mergers under the Council Regulation on the Control of Concentrations between Undertakings. For the US, see §9 of the 2010 Horizontal Merger Guidelines of the Department of Justice (DOJ) and Federal Trade Commission (FTC). The standard of timely, likely, and sufficient is also used by other antitrust authorities, including the Australian Competition and Consumer Commission and the Canadian Competition Bureau.

[2] Shapiro and Shelanski (2021) determine that this empirical approach has "made it harder for merging firms to mount an entry defense in the absence of actual, recent, and successful instances of entry," based on a review of all merger cases decided in the federal courts of the US since 2000. In the 1980s and 1990s, federal courts ruled against the government in a series of merger trials on the basis that post-merger entry would counteract any anticompetitive actions (Werden and Froeb (1998)).

[3] Other often-used models include Cournot (e.g., Farrell and Shapiro (1990)) and efficient second-score auctions (e.g., Miller (2014)). Entry analysis under Cournot is a simpler case because consumers do not value product variety, and Spector (2003) provides an analogous revealed preference argument. With efficient second-

The first part of the paper (Section 2) starts under the assumption of multinomial logit (MNL) or constant elasticity of substitution (CES) demand. These demand systems provide an appropriate baseline for the analysis because they exhibit the Independence of Irrelevant Alternatives (IIA) property.[4] With IIA, entry does not affect the relative market shares of incumbents, so the entrant and merging firms are neither particularly close nor particularly distant competitors. In this context, we prove that entry sufficient to restore consumer surplus renders the merger unprofitable, and mergers do not occur unless entry barriers exist that prevent entry or limit its scope.

We then explore the robustness of the result using the nested MNL, nested CES, and random coefficients logit (RCL) demand models. In the nested models, we prove merger profitability is incompatible with entry that restores consumer surplus unless (i) the entrant and the merged firm are in different nests, and (ii) the nests are sufficiently differentiated. In that case, the products of the entrant and merged firm are distant enough substitutes that merger is profitable despite entry. Although analytical results are unavailable for RCL demand, a numerical exercise suggests that this intuition extends. Finally, we examine the Miller/Coors merger in the beer industry, using the RCL demand model of Miller and Weinberg (2017). For a range of entrant marginal costs and qualities that we interpret as spanning what can be accomplished with the existing production technology, no entrant restores consumer surplus without making the merger unprofitable.

The second part of the paper (Section 3) provides results to help guide analysis when theory alone is not dispositive. This might be the case if the merging firms do not engage in profit maximization (e.g., if managers engage in "empire-building") or if the profitability of the merger is not due solely to the reduction in competition.

First, we define the *compensating entrant* as a hypothetical entrant with the marginal costs and product qualities such that it exactly restores pre-merger consumer surplus. With MNL and CES demand, the compensating entrant can be characterized using commonly-available data on the pre-merger equilibrium. Entry then eliminates the adverse competitive effects of a merger if and only if the entrant's capabilities exceed those of the compensating entrant. Compensating entry does not eliminate the merging firms' price increases, as consumers benefit from the additional variety it provides. If all adverse price effects are to be eliminated, entry at a much large scale is necessary. With symmetric merging firms, such an entrant

score auctions, mergers do not increase the profitability of entry.

[4]Models of this type are common in merger review, in part because they connect market shares and the Herfindahl-Hirschman index (HHI) with demand substitution. The discussion in Shapiro (2010), for example, explains how the change in HHI relates to diversion between the merging firms.

obtains a larger market share than the merged firm, and the merged firm obtains a share that is half of the combined pre-merger shares of the merging firms.

Second, we consider the case in which the merger generates efficiencies in the form of marginal cost reductions or quality improvements for the merging firms. Efficiencies increase the profitability of merger, raise consumer surplus, and reduce the profitability of entry.[5] With MNL and CES demand, we prove that profitable mergers can increase consumer surplus, even if neither entry nor the efficiency eliminates consumer surplus loss on its own. Among other results, we show that any such merger must achieve at least a *minimum efficiency*, the magnitude of which can be calculated with data on the pre-merger equilibrium. We also find that consumer surplus may not be monotonically increasing in the size of the efficiency, as the efficiency can be large enough to deter post-merger entry, but too small to offset adverse competitive effects.

Third, we examine in greater depth whether mergers are likely to induce entry. Given the demand models we consider, any merger that leads the merging firms to restrict output also increases the profitability of entry. Whether this actually induces entry depends on the magnitude of entry and fixed costs that the entrant must incur. Specifically, entry occurs if the (discounted) entry and fixed costs falls between the (discounted) profits that the entrant could earn, without and with the merger. This bounds approach can inform the empirical assessment of entry barriers.

We use these results in an application to the T-Mobile/Sprint merger (Section 4). A Federal District Court ruled that the merger between the mobile wireless operators could proceed, in part due to the expectation that DISH would successfully enter the market. We calibrate our model with publicly-available data on market shares, prices, and markups, and also a market elasticity of demand that appears in regulatory filings. Using a series of simulations, some of which account for the divestiture of the Boost brand, we show that there is no equilibrium with both the merger and merger-induced entry. That is, the merger is unprofitable if it causes DISH to enter, unless merger efficiencies are large, in which case the merger does not cause DISH to enter. We interpret the fact that DISH has not entered the market, now more than two years after the T-Mobile/Sprint merger closed, as consistent with our analysis.[6]

---

[5]That efficiencies reduce the scope for profitable entry has been explored previously in different modeling contexts (e.g, Cabral (2003); Erkal and Piccinin (2010)).

[6]To the extent that evidence pointed toward DISH entry, our analysis suggests that it would have been more appropriate to treat DISH as an existing market participant, rather than as a merger-induced entrant. The US Horizontal Merger Guidelines, §9, state that "Firms that have, prior to the merger, committed to entering the market also will normally be treated as market participants."

We conclude with a short discussion of policy implications (Section 5). Motivating the discussion is that the antitrust authorities in the EU and US are evaluating the efficacy of merger review as currently practiced: the European Commission is conducting a study about the accuracy of its entry assessments in recent merger decisions[7] and, in the US, the DOJ and FTC have launched a review of their merger guidelines.

1.1 Literature Review

Our research builds on a number of articles that consider the relationship between mergers and entry. The closest is Werden and Froeb (1998), which examines the cases of Bertrand competition with logit and nested logit demand. For a large number of randomly-generated markets, they find that most mergers are unprofitable if entry occurs, and further that mergers do not increase the entrant's profit by much. In explaining why they rely on Monte Carlo simulations, Werden and Froeb write:

> Analytical methods are of little use with this model because products are differentiated and because predictions vary with demand parameters and market shares.[8]

Our results are broadly consistent with those of Werden and Froeb, but they are sharper and (mostly) can be proven analytically using the aggregative games framework of Nocke and Schutz (2018). Further, we provide a more complete analysis of nested logit demand,[9] extend the results to CES, nested CES, and random coefficients logit demand, and also provide a set of results to inform empirical analysis.

Also along these lines, Spector (2003) examines mergers and entry in a Cournot model with the general assumptions of Farrell and Shapiro (1990), and proves that mergers are unprofitable if merger-induced entry restores consumer surplus.

Other articles explore mergers under the free entry assumption that fringe firms endogenously participate in the market both pre- and post-merger (e.g., Davidson and Mukherjee (2007); Anderson et al. (2020)). Among the main results are that (i) mergers do not affect consumers in long run equilibrium due to the fringe response, and (ii) mergers are unprofitable in the absence of efficiencies. Free entry is analogous to an assumption (in our setting)

---

[7]See https://ec.europa.eu/commission/presscorner/detail/en/mex_21_3182, last accessed June 6, 2022.

[8]Werden and Froeb (1998, p. 527).

[9]Werden and Froeb focus on a single value of the nesting parameter and consider only entry into the nest of the merging firms.

that compensating entry occurs post-merger. Whether such an assumption is appropriate depends on the empirical setting. In our experience, the mergers that garner the most scrutiny are those where the prospective entrants are limited in number, have uncertain capability, and face entry costs. The results that we develop are intended to inform merger review in such settings.

Empirical research on mergers and entry is hampered by selection: observed mergers are (presumably) both profitable and competitively benign. Recent applications address the issue by estimating structural models of competition—including the distribution of entry costs and fixed costs—exploiting observed entry and exit in the data (Li et al. (2022); Ciliberto et al. (2021); Fan and Yang (2021)). Post-merger equilibrium then can be computed allowing for entry or incumbent repositioning. This empirical approach is complementary to our theoretical framework. First, these papers employ logit-based demand systems, such as the random coefficients logit model of Berry et al. (1995), so we suspect our results extend. Second, our theoretical approach informs the magnitude of entry costs (or fixed costs) that could generate merger-induced entry, whereas the empirical approach informs the realized magnitude of those costs.

Finally, our paper contributes to recent research that applies the aggregative games framework of Nocke and Schutz (2018) to antitrust. Nocke and Schutz (2019) provide conditions under which the change in the HHI approximates the market power effects of a merger, and also examine merger efficiencies. Garrido (2019) explores endogenous product portfolios in a dynamic game. Nocke and Whinston (2022) derive the efficiencies necessary to counterbalance adverse merger effects. Alviarez et al. (2020) examine global beer mergers and the adequacy of divestitures.

2   A Model of Mergers and Entry

2.1   Setup

We examine a three-stage game of perfect information. There are $f = 1, 2, \ldots, F$ firms, with $F \geq 3$. Without loss of generality, the first $F - 1$ firms are incumbents, and firm F is a prospective entrant. The timing of the game is as follows:

1. Firms 1 and 2 decide whether to merge to form the combined firm, M. A merger commits these firms to maximize joint profits when setting prices in stage 3.

2. Firm F observes whether merger occurs in stage 1 and decides whether to enter. If it enters, it incurs an entry cost, $\chi > 0$, the value of which is commonly known.

3. All firms observe whether merger and entry occur in stages 1 and 2. The incumbents and, if entry occurs, the entrant, form the set $\mathcal{F}$. The firms in $\mathcal{F}$ choose prices simultaneously, consumers make purchasing decisions, and firms earn variable profit according to differentiated-products Bertrand equilibrium.

In adopting this three-stage structure, we follow the theoretical literature on mergers and entry (e.g., Werden and Froeb, 1998; Spector, 2003). In Appendix B, we consider an alternative structure with delayed or probabilistic entry and obtain numerical results that are similar to the analytical results presented in this section.

Our solution concept is subgame perfect equilibrium (SPE). Whether merger and entry occur in equilibrium is determined by the payoffs available to firms in the pricing stage of the game. The interesting case for antitrust enforcement is that of *merger-induced entry*, which we define as entry that occurs if and only if merger occurs. This requires $\Pi_{nm}^F < \chi \leq \Pi_{m}^F$, where Π^F is the profit of the entrant and the subscripts nm and m refer to "no merger" and "merger," respectively. Indeed, if the entry decision is unaffected by the merger decision, then the enforcement decision is straight-forward as the merger is profitable and reduces consumer surplus relative to a counterfactual in which the merger is prohibited.

To make progress, we focus on assumptions that are commonly maintained in the industrial organization literature and employed in antitrust practice. On the supply-side, we assume that profit functions take the form

$$\Pi^f(p) = \sum_{j \in \mathcal{J}^f} (p_j - c_j) q_j(p), \tag{1}$$

where $\mathcal{J}^f$ is the set of products sold by firm f, p_j and c_j are the price and marginal cost of product j, and $q_j(p)$ represents the quantity demanded of product j as a function of all prices. The first order condition for profit maximization for any $j \in \mathcal{J}^f$ is

$$q_j(p) + \sum_{k \in \mathcal{J}^f} (p_k - c_k) \frac{\partial q_k(p)}{\partial p_j} = 0. \tag{2}$$

Prices that satisfy this equation for all products constitute a Bertrand equilibrium of the pricing subgame.

We consider MNL, CES, nested logit (NMNL), nested CES (NCES), and random coefficients logit (RCL) demand. Among these, MNL and CES demand provide a useful baseline because they exhibit the IIA property. Thus, entry does not affect the relative market shares of incumbents, with market shares being in terms of unit sales for MNL and in terms of revenues for CES. Relatedly, the diversion ratio[10] from any product k to any other product j is proportional to product j's market share:

$$\frac{\frac{\partial s_j}{\partial p_k}}{\frac{\partial s_k}{\partial p_k}} \equiv DIV_{k \to j} = \frac{s_j}{1 - s_k}.$$

The NMNL, NCES, and RCL demand systems allow for more flexible substitution patterns. We will refer to two products as "close substitutes" if $DIV_{k \to j} > \frac{s_j}{1 - s_k}$ and "distant substitutes" if $DIV_{k \to j} < \frac{s_j}{1 - s_k}$. With this terminology in hand, the entrant and the merging firms are neither close nor distant competitors with MNL and CES.

2.2 MNL and CES Demand

2.2.1 Bertrand equilibrium in the pricing subgame

With MNL demand, each consumer purchases a single product j from the set $\mathcal{J}$ or forgoes a purchase by selecting the outside good ($j = 0$). The indirect utility that consumer i receives from product $j \in \mathcal{J}$ is

$$u_{ij} = v_j - \alpha p_j + \epsilon_{ij} \tag{3}$$

where v_j and p_j are the quality and price of product j, α is a price coefficient, and ϵ_{ij} is a consumer-specific preference shock. The indirect utility provided by the outside good is $u_{i0} = \epsilon_{i0}$, where we apply the standard normalization $v_0 = p_0 = 0$. The preference shocks are iid with a Type 1 extreme value distribution, which yields a closed-form solution for market shares (in terms of unit sales):

$$s_j(p) = \frac{\exp(v_j - \alpha p_j)}{1 + \sum_{k \in \mathcal{J}} \exp(v_k - \alpha p_k)}. \tag{4}$$

Mapping back to the profit function of equation (1), we have $q_j(p) = s_j(p)M$, where M is the density of consumers. We normalize M to one for simplicity.

---

[10]See Miller and Sheu (2021) for a discussion of how diversion ratios are employed in merger review and Conlon and Mortimer (2021) for a useful theoretical analysis.

With CES demand, consumer utility takes the form

$$u_i = \left(\sum_{j \in \mathcal{J}, j=0} v_j^{\frac{1}{\sigma}} q_j^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

where q_j is the quantity consumed of product j, v_j is that product's quality, and $\sigma > 1$ is the elasticity of substitution between products in the utility function. Consumers choose quantities to maximize utility subject to a budget constraint of $\sum_j p_j q_j = Y$, where Y is total income. We apply a standard set of normalizations: $v_0 = p_0 = 1, Y = 1$. This obtains a closed form solution for market shares (in terms of revenue):

$$s_j(p) = \frac{v_j p_j^{1-\sigma}}{1 + \sum_{k \in \mathcal{J}} v_k p_k^{1-\sigma}}. \tag{5}$$

Mapping back to the profit function, we have $q_j(p) = s_j(p)/p_j$.

We represent Bertrand equilibrium using the *type aggregation* property of these demand systems (Nocke and Schutz (2018)). In particular, equilibrium outcomes depend on a firm-level primitive—the firm type—that summarizes the qualities and marginal costs of each firm's products. The types take the form:

$$T^f \equiv \begin{cases} \sum_{j \in \mathcal{J}^f} \exp(v_j - \alpha c_j) & \text{(MNL)} \\ \sum_{j \in \mathcal{J}^f} v_j c_j^{1-\sigma} & \text{(CES)}. \end{cases} \tag{6}$$

Further, with these demand systems, each firm finds it optimal to apply the same markup to all of its products. It is convenient to define firm-specific "ι-markups":

$$\mu^f \equiv \begin{cases} \alpha(p_j - c_j) & \forall j \in \mathcal{J}^f \quad \text{(MNL)} \\ \sigma \frac{p_j - c_j}{p_j} & \forall j \in \mathcal{J}^f \quad \text{(CES)}. \end{cases} \tag{7}$$

By inspection, the ι-markups are proportional to the actual markups.

The Bertrand equilibrium can be characterized as a vector of ι-markups, $\{\mu^f\} \; \forall f \in \mathcal{F}$, a vector of firm-level market shares, $\{s^f\} \; \forall f \in \mathcal{F}$, and a market aggregator, H. A firm's market share is the combined market share of its products, $s^f = \sum_{j \in \mathcal{J}^f} s_j$, and the aggregator is the denominator from equations (4) and (5),

$$H \equiv \begin{cases} 1 + \sum_{j \in \mathcal{J}} \exp(v_j - \alpha p_j) & \text{(MNL)} \\ 1 + \sum_{j \in \mathcal{J}} v_j p_j^{1-\sigma} & \text{(CES)}. \end{cases} \tag{8}$$

In equilibrium, the ι-markups satisfy

$$1 = \begin{cases} \mu_f \left(1 - \frac{T^f}{H} \exp(-\mu_f)\right) & \text{(MNL)} \\ \mu^f \left(1 - \frac{\sigma-1}{\sigma} \frac{T^f}{H} \left(1 - \frac{\mu^f}{\sigma}\right)^{\sigma-1}\right) & \text{(CES)}. \end{cases} \tag{9}$$

Let the unique solution for μ_f from this expression be written as $m(T^f/H)$, where $m(\cdot)$ is the *markup fitting-in function*. Equilibrium market shares satisfy

$$s^f = S\left(\frac{T^f}{H}\right) \equiv \begin{cases} \frac{T^f}{H} \exp\left(-m\left(\frac{T^f}{H}\right)\right) & \text{(MNL)} \\ \frac{T^f}{H} \left(1 - \frac{1}{\sigma} m\left(\frac{T^f}{H}\right)\right)^{\sigma-1} & \text{(CES)}. \end{cases} \tag{10}$$

The system is closed with the constraint that market shares must sum to one:

$$\frac{1}{H} + \sum_{f \in \mathcal{F}} s^f = 1. \tag{11}$$

A unique solution to this system of equations is guaranteed to exist. Finally, firm-level profit (in equilibrium) and consumer surplus can be expressed:

$$\Pi^f = \pi\left(\frac{T^f}{H}\right) \equiv \begin{cases} \frac{1}{\alpha}\left(m\left(\frac{T^f}{H}\right) - 1\right) & \text{(MNL)} \\ \frac{1}{\sigma-1}\left(m\left(\frac{T^f}{H}\right) - 1\right) & \text{(CES)} \end{cases} \tag{12}$$

and

$$CS(H) \equiv \begin{cases} \frac{1}{\alpha} \log(H) & \text{(MNL)} \\ H^{\frac{1}{\sigma-1}} & \text{(CES)}. \end{cases} \tag{13}$$

Nocke and Schutz (2018, Proposition 6) establish that the markups, market shares, and profit of any firm f increase in T^f and the ratio T^f/H, but decrease in T^g for any $g \neq f$. Thus, firms that produce at lower cost, have more valuable products, and maintain larger product portfolios (higher T^f) or that face less competition (lower H or T^g) fare better in equilibrium.

2.2.2 Mergers and entry in SPE

We now turn to the entry and merger decisions. In the second stage of the game, firm F enters if it can earn positive profits in the Bertrand pricing stage, taking into account its type, T^F, its entry costs, χ, and whether a merger has occured in the first stage of the game. That

is, entry occurs if the profit of firm F satisfies

$$\pi \left(\frac{T^F}{H_{*,e}} \right) - \chi \geq 0,$$

where we let $H_{*,e}$ be the market aggregator with entry, accounting for the observed merger decision of firms 1 and 2 (denoted by $*$).

In the first stage of the game, Firms 1 and 2 merge if doing so increases their combined profit in the pricing stage, taking into account the effect of the merger on the entry decisions. That is, a merger occurs if and only if it increases joint profits:

$$\pi \left(\frac{T^M}{H_{m,*}} \right) \geq \pi \left(\frac{T^1}{H_{nm,*}} \right) + \pi \left(\frac{T^2}{H_{nm,*}} \right), \tag{14}$$

where $T^M = T^1 + T^2$ is the type of the merged firm and $H_{m,*}$ and $H_{nm,*}$ are the aggregator with and without a merger, incorporating the best-response of the prospective entrant. A unique SPE exists because the merger and entry decisions are made sequentially and a unique equilibrium exists in the pricing subgame.

We now are in a position to state the first result:

Proposition 1. *With MNL or CES demand, any profitable merger lowers the aggregator relative to a counterfactual in which merger is prohibited. Thus, no SPE exists in which a merger occurs and consumer surplus does not decrease as a result.*

Proof. See the Appendix. □

In this model, mergers increase markups and profit and decrease consumer surplus, all else equal. The softer competitive environment makes entry more profitable and, should entry occur, it benefits consumers and reduces the profit of incumbents. The proposition establishes that if the effect of a merger would be to induce entry sufficient to preserve consumer surplus, then the merger is unprofitable and does not occur in equilibrium. Thus, if merger does occur then at least one of the following three must be true: (i) the marginal costs, quality, and product portfolio of the entrant are insufficient to preserve consumer surplus, (ii) the entry cost is large enough to deter post-merger entry, or (iii) the entry cost is small enough that entry is not merger-induced, and instead would occur with or without the merger.

Because a profitable merger lowers the market aggregator, it also increases incumbents' markups. Thus, under our assumption that the merger does not affect marginal costs, prices also increase. Formally,

Corollary 1. *With MNL or CES demand, any profitable merger increases the markups and prices of incumbents, relative to a counterfactual in which merger is prohibited.*

We note that these results are robust to an extension featuring multiple prospective entrants and an option for *incumbent repositioning*. By incumbent repositioning, we mean costly investments by one or more non-merging incumbents that improve product quality, reduce marginal costs, or expand product portfolios. The proof of Proposition 1 shows that if consumer surplus is unaffected by the merger due to induced actions by other firms, then the merger is unprofitable. Thus, the proof is fairly general and does not rely on a specific form of entry or repositioning.

2.3 Nested Logit and Nested CES

We now examine the NMNL and NCES models. Products are grouped into exhaustive and mutually exclusive sets, or "nests," with products in the same nest being close substitutes and products in different nests being distant substitutes. Let each product $j \in \mathcal{J}$ belong to a nest, $g(j) \in \mathcal{G}$, and let the set of products in nest g be $\mathcal{J}_g$. We assume that there is an additional nest ($g = 0$), that contains only the outside good ($j = 0$).

With NMNL demand, each consumer purchases a single product $j \in \mathcal{J}$ or forgoes a purchase by selecting the outside good ($j = 0$). The indirect utility that consumer i receives from product $j \in \mathcal{J}$ in nest $g(j)$ is

$$u_{ij} = v_j - \alpha p_j + \zeta_{ig(j)} + (1 - \rho)\epsilon_{ij}$$

where ϵ_{ij} is iid Type I extreme value and $\zeta_{ig(j)}$ has the unique distribution such that $\zeta_{ig(j)} + (1 - \rho)\epsilon_{ij}$ is also iid Type I extreme value (Berry (1994); Cardell (1997)). The nesting parameter, $\rho \in [0, 1)$, characterizes the correlation in preferences for products of the same nest; larger values correspond to more substitution within nests, and less substitution between nests. With $\rho = 0$ the model collapses to MNL demand. Market shares are given by:

$$s_j(p) = \frac{\exp(v_j - \alpha p_j)}{\sum_{k \in g(j)} \exp(v_k - \alpha p_k)} \frac{\left(\sum_{k \in g(j)} \exp(v_k - \alpha p_k)\right)^{1-\rho}}{1 + \sum_{g \in \mathcal{G}} \left(\sum_{k \in g} \exp(v_k - \alpha p_k)\right)^{1-\rho}} \tag{15}$$

where the first ratio is the share of product j within its nest and the second ratio is the combined share all products $k \in g(j)$.

11

With NCES demand, consumer utility takes the form

$$u_i = \left(\sum_{g \in \mathcal{G}} Q_g^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}$$

where

$$Q_g = \left(\sum_{j \in \mathcal{J}_g} v_j^{\frac{1}{\sigma}} q_j^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

and $\sigma \geq \gamma > 1$ are the elasticities of substitution between products in each of the nests. If $\sigma = \gamma$, this collapses to the CES model. Market shares are given by

$$s_j = \frac{v_j p_j^{1-\sigma}}{\left(\sum_{k \in \mathcal{J}_{g(j)}} v_k p_k^{1-\sigma} \right)^{\frac{\sigma-\gamma}{\sigma-1}} \left(1 + \sum_{g \in \mathcal{G}} \left(\sum_{k \in \mathcal{J}_g} v_k p_k^{1-\sigma} \right)^{\frac{\gamma-1}{\sigma-1}} \right)}. \tag{16}$$

In Bertrand equilibrium, both NMNL and NCES demand exhibit the type aggregation and common markup properties if all products of any given firm are located in a single nest (Nocke and Schutz (2019)). We restrict attention to that special case because it allows us to make analytical progress. The firm types take the form

$$T^f \equiv \begin{cases} \sum_{j \in \mathcal{J}^f} \exp\left(\frac{v_j - \alpha c_j}{1-\rho} \right) & \text{(NMNL)} \\ \sum_{j \in \mathcal{J}^f} v_j c_j^{1-\sigma} & \text{(NCES)} \end{cases} \tag{17}$$

and thus firms with lower costs, more valuable products, and broader product portfolios have larger types. The ι-markups are defined as

$$T^f \equiv \begin{cases} \sum_{j \in \mathcal{J}^f} \exp\left(\frac{v_j - \alpha c_j}{1-\rho} \right) & \text{(NMNL)} \\ \sum_{j \in \mathcal{J}^f} v_j c_j^{1-\sigma} & \text{(NCES).} \end{cases} \tag{18}$$

The Bertrand equilibrium can be characterized as a vector of ι-markups, $\{\mu^f\} \; \forall f \in \mathcal{F}$, a vector of firm-level market shares, $\{s^f\} \; \forall f \in \mathcal{F}$, a vector of nest-level aggregators $\{H_g\} \; \forall g \in \mathcal{G}$, and a market aggregator, H. As substantial notation is required, we defer these characterizations to Appendix A.

Our results for entry extend straightforwardly to the case in which the merging firms and the entrant have products in the same nest.

Proposition 2. *Suppose the products of the merging firms and the entrant are in the same*

12

nest. Under NMNL or NCES demand, if the merger is profitable then it lowers the aggregator relative to a counterfactual in which merger is prohibited. Thus, no SPE exists in which a merger occurs and consumer surplus does not decrease as a result.

Proof. See the Appendix. □

Thus, profitable mergers are incompatible with merger-induced entry sufficient to preserve consumer surplus if the products of the merging firms and entrant are in the same nest, i.e., their products are close substitutes.

We now analyze the case in which the entrants' products are distant substitutes for those of the merging firms. One technical advantage of the revealed preference approach underpinning our Propositions 1 and 2 is that it is robust to mild perturbations of the underlying demand framework. We leverage this to obtain an analogous result in the case of NMNL or NCES demands with entry into an arbitrary nest, provided the appropriate nesting parameter (ρ in the case of NMNL, σ for NCES) is not too large. Formally, we establish the continuity of the unique Bertrand equilibrium as a function of the relevant nesting parameter in a neighborhood of the value for which the demand system collapses to its non-nested counterpart ($\rho = 0$ for NMNL, $\sigma = \gamma$ for NCES).

We first state the continuity result formally:

Lemma 1. *For any fixed vector of model primitives, the mapping taking ρ (resp. σ) to the unique Bertrand equilibrium of the pricing game with NMNL (resp. NCES) demand is continuous on a neighborhood of 0 (resp. γ).*

Proof. See the Appendix. □

By appeal to this result, we obtain a "robust" analogue of Proposition 2 that does not depend on whether the merging firms' and the prospective entrant's product lines belong to the same nest.

Proposition 3. *Under NMNL (resp. NCES) demand, there exists $\bar{\rho} > 0$ (resp. $\bar{\sigma} > \gamma$) such that, for any $\rho \leq \bar{\rho}$ (resp. $\sigma \leq \bar{\sigma}$), if the merger is profitable then it lowers the aggregator relative to a counterfactual in which merger is prohibited. Thus, no SPE exists in which a merger occurs and consumer surplus does not decrease as a result.*
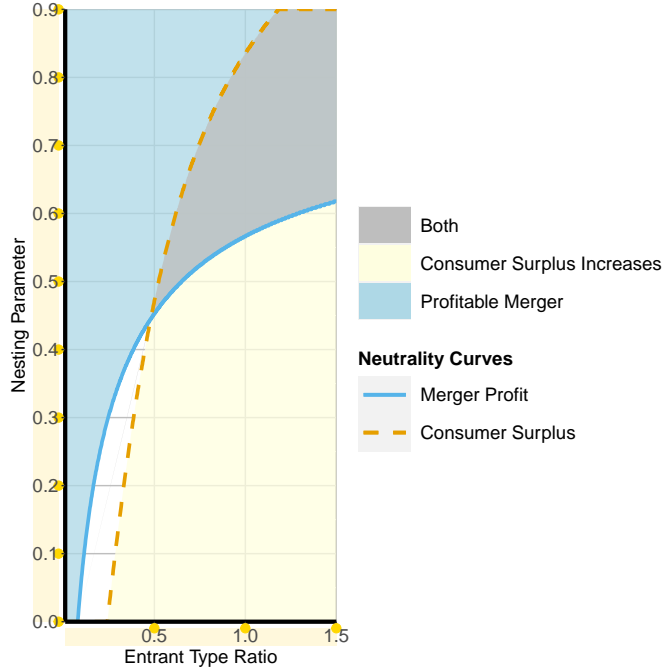
Proof. See the Appendix. □

Figure 1: Numerical Analysis of Mergers and Entry with NMNL Demand

Notes: The plot shows the nesting parameters (ρ) and entrant type ratios (T^F/T^1) for which merger with entry increases consumer surplus (shaded yellow), increases the merging firms' profit (shaded blue), or both (shaded gray). The corresponding neutrality curves for merger profitability and consumer surplus are plotted as solid blue and dashed orange lines, respectively.

We use numerical simulations to examine how large the nesting parameter must be in order to generate the possibility of merger-induced entry that is sufficient to preserve consumer surplus. We focus on NMNL demand and consider settings with four, six, and eight incumbents of equal market share, evenly split between two nests, and with an outside good that has a 20 percent market share. We calibrate incumbent types using the market shares, for $\rho \in [0, 0.9]$. We then simulate a merger between two incumbents given entrant types $T^F \in [0, 1.50 \times T^1]$, under the assumption that the merging firms' products are in one nest and the entrants' products are in the other nest. Finally, we identify the (ρ, T^F) combinations for which merger that induces entry is profitable and increases consumer surplus.

Figure 1 plots the results for the case of six incumbents. The vertical axis is ρ (the "nesting parameter") and the horizontal axis is T^F/T^1 (the "entrant type ratio"). A merger that induces entry is profitable for combinations of the nesting parameter and the entrant type ratio that fall above the solid blue neutrality curve, and increases consumer surplus for combinations that fall below the dashed orange neutrality curve. Thus, the shaded gray area

between these curves provides the region for which a merger could increase consumer surplus in SPE. As shown, the minimum ρ under which consumer surplus increases is around 0.45. Similar results obtain for the cases of four and eight incumbents. We interpret the results as suggesting that the entrant must be substantially differentiated from the merging firms in order for merger-induced entry to eliminate the consumer surplus loss of a merger in SPE.[11]

2.4 Random Coefficients Logit

The RCL model does not exhibit the type aggregation or common markup properties. Thus, our analytical results do not extend, and we proceed with a numerical analysis and an empirical application. The numerical analysis is intended to explore whether merger-induced entry sufficient to restore the consumer surplus loss caused by a profitable merger requires an entrant with products that are distant substitutes to those of the merging firms. The empirical application, which uses the demand estimates of Miller and Weinberg (2017) on the US beer industry, demonstrates a way to evaluate merger-induced entry that might be useful in other settings.

In the numerical analysis, we use the following specification for the indirect utility that consumer i receives from product j:

$$u_{ij} = (1 + \beta_i)v_j - \alpha p_j + \epsilon_{ij}, \tag{19}$$

where ϵ_{ij} is iid Type I extreme value and $\beta_i \sim N(0,1)$ is a consumer-specific valuation for quality. There are two single-product incumbents, each with $v_j = 4$ and $mc_j = 2$. We consider four values of the price parameter, $\alpha = (1, 2, 3, 4)$, with the larger values implying more elastic demand. With $\alpha = 4$, the pre-merger equilibrium features prices of 2.36, incumbent market shares of 6.4%, and a diversion ratio between incumbents of 45%. With $\alpha = 1$, these statistics are 3.72, 30%, and 72%, respectively.

We consider entrants with marginal costs and qualities that range between -2 and 8. With a step size of 0.05, this yields 40,401 entrants. We simulate a merger between the incumbents under the assumption that the merger induces entry by one of the entrants. Iterating through the entrants, we determine whether consumer surplus and the merging firms' profit increase relative to the pre-merger baseline.

Figure 2 summarizes the results. In each panel, the shaded gray region provides the entrant qualities and marginal costs for which the merger is profitable and increases consumer

---

[11]We address the likelihood that entry into another nest would occur in Section 3.3.

surplus. In the top left panel ($\alpha = 4$), this region features entrant marginal costs that are close to zero or negative and entrant quality that is substantially less than that of the merging firms.[12] Comparing across panels, as demand becomes less elastic and incumbent market powers grows, the gray region requires even lower entrant marginal costs and qualities. In the bottom right panel ($\alpha = 1$), the region does not exist within the considered marginal cost and quality ranges.

We interpret the numerical results as suggesting that the intuition behind our results for the NMNL and NCES models extends to the RCL model. That is, merger-induced entry sufficient to preserve consumer surplus can be compatible with a profitable merger if the entrant is different enough from the merging firms. An interesting observation is that the model informs the entrant characteristics under which merger-induced entry that restores consumer surplus can arise in SPE. In empirical settings, then, knowledge of the feasible production technologies can be paired with the model to determine whether merger-induced entry is plausible. For example, it might be possible to rule out merger-induced entry if the model indicates that the entrants marginal costs would have to be negative, or if its quality would have to be implausibly high.

To illustrate, we use the demand estimates of Miller and Weinberg (2017) for the US beer industry. The data include monthly and quarterly observations over 2005-2011 on the prices and quantities of 39 products sold by the major brewers (13 flagship brands $\times$ 3 package-size categories) in each of 39 distinct geographic regions.[13] In order to reduce computational time, we restrict attention to Boston, the first region alphabetically. We also restrict attention to the second quarter of 2008, which is the last full quarter before the consummation of the Miller/Coors merger. We use the RCNL-2 specification of the demand model, in which consumer income affects preferences for price, package size, and calories. With the notation of Miller and Weinberg, the quality of product j and period t can be defined as

$$v_{ij} \equiv \sigma_j^D + \tau_t^D + \xi_{jt}$$

where the terms on the right-hand-side are a product fixed effect, a period fixed effect, and a structural error term. We recover product qualities using the demand estimates, and marginal costs from the first order conditions implied by Bertrand competition.

We simulate the Miller/Coors merger with a series of entrants that differ in their marginal

---

[12]We suspect that a similar region exists for entrant costs and quality that are both much higher than the merging firms, but computing equilibrium in that parameter range is difficult for numerical reasons.

[13]These data are constructed from the IRI Marketing Data Set (Bronnenberg et al. (2008)).

cost and quality.[14] We assume that all entrants have the same product portfolio as Coors (e.g., as Coors sells a "Light 12 pack," so too does the entrant). However, we allow the entrant's marginal costs to be up to 50% lower than those of Coors, or up to 100% higher. We also allow for the entrant's qualities to be substantially higher than the highest-quality product in the market, or substantially lower than the lowest-quality product. Specifically, letting the maximum and minimum observed product quality be $\overline{v}$ and $\underline{v}$, we consider entrant qualities between $\underline{v} - (\overline{v} - \underline{v})$ and $\overline{v} + (\overline{v} - \underline{v})$. We assume that marginal costs and qualities outside these ranges are infeasible given the existing production technology.

Figure 3 shows the results. The vertical axis provides the entrant's quality and the horizontal axis provides its marginal cost. As shown, there is no overlap between the region in which consumer welfare increases due to the merger (shaded yellow) and the region in which the merger increases the profit of the merging firms (shaded blue). Therefore, in this case, merger-induced entry sufficient to preserve consumer surplus is incompatible with merger profitability. Under the maintained assumptions, it is possible to infer by revealed preference that barriers prevent entry by firms that are sufficiently capable to eliminate consumer surplus loss that otherwise would occur.

3   Frameworks for Empirical Analysis

Our analysis in the preceding section indicates that merger-induced entry sufficient to preserve pre-merger consumer surplus can be inconsistent with SPE in a set of fairly standard models of oligopoly price competition. Empirical analysis may nonetheless be necessary if the merger generates efficiencies, if it conveys benefits that accrue outside the market under examination, or if firms fail to maximize profit due to a managerial principle-agent problem or other considerations. In this section, we provide a number of results that are intended to help guide empirical analysis in such settings.

3.1   The Compensating Entrant

The existing literature defines the compensating efficiency of a merger as the magnitude of efficiencies, such as marginal cost reductions or quality improvements, such that the merger does not affect prices and consumers surplus (Werden (1996); Nocke and Whinston (2022)).

---

[14]Our analysis focuses on how mergers and entry can be evaluated with RCL demand, so we do not consider the efficiencies and coordinated effects of the merger. Nocke and Whinston (2022) take a similar approach in using the data to explore the connection between HHI and merger price effects.

Compensating efficiencies have been used in merger review and relied upon by courts to determine whether efficiencies are sufficient to offset a loss of competition (Miller and Sheu (2021)). We develop similar concepts for entry analysis, focusing first on consumer surplus and then on the merging firms' prices.

We refer to the entrant that leaves consumer surplus unchanged due a merger without efficiencies as the *compensating entrant.* If such a merger induces entry by a firm with a type lower than that of the compensating entrant then consumer surplus decreases, and the opposite result obtains if the entrant's type is higher than the compensating entrant. With MNL, the type of the compensating entrant is a function of pre-merger market shares only. With CES, knowledge of the parameter σ also is required, so additional data on the pre-merger equilibrium is necessary.[15] Entry by such a firm would not occur in equilibrium in our model of the previous section. However, such entry could occur in an alternative situation if the merging firms earned, for example, profits elsewhere in a separate market.

In our formalization, we use s^1 and s^2 to refer to the merging firms' market shares in an equilibrium without merger (or entry), and use s^M and s^F to refer to the market shares of the merged firm and the entrant in an equilibrium with merger and entry.

Proposition 4. *With MNL demand, the type of the compensating entrant, $\widetilde{T}^F$, satisfies*

$$\frac{\widetilde{T}^F}{T^1 + T^2} = \frac{(s^1 + s^2 - s^M) \exp\left(\frac{1}{1 - s^1 - s^2 + s^M}\right)}{s^1 \exp\left(\frac{1}{1 - s^1}\right) + s^2 \exp\left(\frac{1}{1 - s^2}\right)} \tag{20}$$

where s^M is the unique solution to

$$s^M \exp\left(\frac{1}{1 - s^M}\right) = s^1 \exp\left(\frac{1}{1 - s^1}\right) + s^2 \exp\left(\frac{1}{1 - s^2}\right). \tag{21}$$

With CES demand, the type of the compensating entrant satisfies

$$\frac{\widetilde{T}^F}{T^1 + T^2} = \frac{(s^1 + s^2 - s^M)\left(\sigma + \frac{s^1 + s^2 - s^M}{1 - (s^1 + s^2 - s^M)}\right)^{\sigma - 1}}{s^1 \left(\sigma + \frac{s^1}{1 - s^1}\right)^{\sigma - 1} + s^2 \left(\sigma + \frac{s^2}{1 - s^2}\right)^{\sigma - 1}} \tag{22}$$

---

[15]Expressions for the compensating entrant are possible to derive for NMNL and NCES demand, although they are more complicated than the MNL and CES expressions provided.

where s^M is the unique solution to

$$s^M \left(\sigma + \frac{s^M}{1 - s^M} \right)^{\sigma - 1} = s^1 \left(\sigma + \frac{s^1}{1 - s^1} \right)^{\sigma - 1} + s^2 \left(\sigma + \frac{s^2}{1 - s^2} \right)^{\sigma - 1}. \tag{23}$$

Furthermore, with MNL and CES demand, $\widetilde{T}^F < \frac{1}{2} \left(T^1 + T^2 \right)$ and $s^F < \frac{1}{2} \left(s^1 + s^2 \right)$.

Proof. See the Appendix. □

We view the compensating entrant as a way to gauge the capabilities of a prospective entrant. A prospective entrant would restore pre-merger consumer surplus only if its marginal costs, qualities, and product portfolio correspond to a type greater than that of the prospective entrant. Further, the final statement of the proposition provides a simple rule-of-thumb for merger review: if the prospective entrant would capture a market that exceeds the average pre-merger market share of the merging firms then consumer surplus is higher with merger and entry than without merger (or entry).

Although compensating entry restores pre-merger consumer surplus, it does not eliminate the price increases of the merging firms. Thus, some of the benefits of entry are due to the greater product diversity that it introduces. To see why this is the case, note that if the merger does not affect consumer surplus then the aggregator, H, also remains unchanged (equation (13)). Then, because equilibrium markups are a function of the ratio T^f / H and we have $T^M = T^1 + T^2$, the equilibrium markups and prices of the merging firms increase. (It also is the case that the markups and prices of non-merging incumbents are constant.) We state this result formally:

Corollary 2. *With MNL or CES demand, a merger that induces compensating entry increases the markups and prices of the merging firms, relative to a counterfactual in which merger is prohibited. It does not affect the markups or prices of non-merging incumbents.*

Indeed, the type of the entrant that eliminates the price increases of the merging firms is considerably larger than that of the compensating entrant. The reason is that with MNL or CES demand the entrant steals share from all incumbents, in proportion to their share, rather than competing predominantly with the merging firms. With symmetric merging firms ($T^1 = T^2$), we obtain a clean analytical result: if entry eliminates all price increases in the post-merger equilibrium then the merging firms' profit in post-merger equilibrium is half of the merging firms' combined profit in the pre-merger equilibrium. Further, the entrant's market share is strictly larger than that of the merging firms.

19

Proposition 5. *With MNL or CES demand and symmetric merging firms ($T^1 = T^2$), if the markups and prices of the merging firms after merger and entry are unchanged relative to a scenario with neither merger nor entry, then $s^M = s^1 = s^2$ and $\Pi^M = \Pi^1 = \Pi^2$. Furthermore, we have $s^F > s^M$.*

Proof. See the Appendix. □

The last statement of the proposition is conservative, as an entrant that eliminates all price increases obtains a post-merger share approaching half of the market, possibly larger than the market share of all incumbents combined. In the proof, we observe that the aggregator, H, must be twice as large in the post-merger equilibrium than in the pre-merger equilibrium. Thus, the entrant steals half of the pre-merger market share of the merging firms and half of the pre-merger outside good market share (by equation (11)). It also steals a substantial portion of the non-merging incumbents' market share, though this is somewhat less than half due to these firms' price responses. Merger-induced entry by such a large firm is probably unlikely in most practical settings.

3.2 Merger Efficiencies

Merger efficiencies, such as marginal cost reductions or quality improvements, increase the profitability of merger, raise consumer surplus, and reduce the profitability of entry. With MNL and CES demand, we show that a profitable merger can increase consumer surplus if it generates efficiencies *and* merger-induced entry in the same market, even if neither the efficiencies nor the entry are sufficient in isolation. However, this is not guaranteed, and many combinations of efficiency and (prospective) entrants fail to restore pre-merger consumer surplus, do not produce entry in SPE, or deter merger in the first stage of the game. This suggests that it may be appropriate to analyze efficiencies and entry jointly, and we provide an integrated framework to help guide empirical inquiry.

To start, we reexamine the SPE of the three-stage game in which the type of the merged firm becomes:

$$T^M = T^1 + T^2 + E$$

for some efficiency $E \geq 0$ that represents lower marginal cost, higher quality, a broader product portfolio, or some combination of these three factors. For simplicity, we assume that entry occurs if and only if the merger increases the profitability of entry, even if only by an

infinitesimal amount. We revisit the likelihood with which merger-induced entry occurs in Section 3.3.

Figure 4 provides a graphical representation of the integrated framework.[16] Our formal analysis, which proves that the main qualitative features of the graph are general, appears in Appendix C. The vertical axis represents the efficiency of the merger, and the horizontal axis represents the type of the prospective entrant.

Three *neutrality curves* are plotted in the (E, T^F) space:

1. Consumer surplus neutrality is plotted as the solid green curve. Consumer surplus increases with merger and entry if (E, T^F) fall above the curve, and decreases otherwise. The comparison is between the pre-merger equilibrium and a Bertrand equilibrium with merger and entry, which may or may not arise in SPE.

2. Neutrality for merger profitability is the dashed purple line. Merger with entry is profitable for the merging firms if (E, T^F) fall above the curve, and unprofitable otherwise. Again, the comparison is between the pre-merger equilibrium and a Bertrand equilibrium with merger and entry, which may or may not arise in SPE.

3. Neutrality for entrant profitability is the dot-dash orange line. Merger-induced entry is profitable for the prospective entrant if (E, T^F) fall below this curve, and unprofitable otherwise. The comparison is between an equilibrium without merger but with entry, and an equilibrium with both merger and entry.

The SPE of the three-stage game can feature no merger, merger without entry, or merger with entry. The neutrality curves show how different (E, T^F) combinations correspond to the different outcomes. Regions that yield no merger are marked with 'R1.' Regions that yield merger without entry are marked with 'R2.' In R2, merger with entry would increase consumer surplus, but entry does not occur in SPE. Thus, consumer surplus increases in R2 if $E > \bar{E}$ and decreases if $E < \bar{E}$. Regions that yield merger with entry are marked with 'R3.' The gray shading shows the combinations of (E, T^F) for which merger increases consumer surplus in SPE. This last region exists for any parameterization of MNL or CES demand. Formally,

Proposition 6. *With MNL or CES demand, there exist combinations of efficiencies and entrant types, (E, T^F), for which merger-induced entry can occur in SPE, such that consumer surplus is not lower than in a counterfactual with neither merger nor entry.*

---

[16]We use an MNL model with four incumbents and an outside good, each with a market share of 0.20. Similar results obtain for alternative market shares. Appendix E describes our numerical methods.

Proof. The proposition follows from the three propositions in Appendix C. □

The upper envelope of the neutrality curves for consumer surplus and merger profitability bound the efficiencies needed for merger-induced entry to preserve consumer surplus in SPE. Larger efficiencies are required for relatively low-type entrants, in order to preserve consumer surplus, but also for relatively high-type entrants, in order to maintain merger profitability. The minimum of this bound—which we refer to as the *minimum efficiency* and denote $\underline{E}$—occurs at the crossing of the neutrality functions. Any profitable merger that increases consumer surplus must have $E \geq \underline{E}$.

As in the previous subsection, we use s^1 and s^2 to refer to the merging firms' market shares in an equilibrium without merger (or entry), and use s^M to refer to the market share of the merged firm in an equilibrium with merger and entry.

Proposition 7. *With MNL demand, the minimum efficiency that must be attained in order for a profitable merger with entry to preserve consumer surplus is given by:*

$$\underline{E} = H\left(s^M \exp\left(\frac{1}{1-s^M}\right) - \sum_{i\in\{1,2\}} s^i \exp\left(\frac{1}{1-s^i}\right)\right) \tag{24}$$

where

$$s^M = 1 - \frac{(1-s^1)(1-s^2)}{1-s^1 s^2}. \tag{25}$$

With CES demand, the minimum efficiency is given by:

$$\underline{E} = \frac{H}{(\sigma-1)^{\sigma-1}}\left(s^M\left(\sigma + \frac{s^M}{1-s^M}\right)^{\sigma-1} - s^1\left(\sigma + \frac{s^1}{1-s^1}\right)^{\sigma-1} - s^2\left(\sigma + \frac{s^2}{1-s^2}\right)^{\sigma-1}\right) \tag{26}$$

where

$$s^M = \frac{\sigma}{\sigma-1}\left(1 - \frac{\left(1 - \frac{\sigma-1}{\sigma}s^1\right)\left(1 - \frac{\sigma-1}{\sigma}s^2\right)}{1 - \frac{\sigma-1}{\sigma}s^1 s^2}\right). \tag{27}$$

Proof. See the Appendix. □

Figure 4 also shows regions in which merger-induced entry does not occur. One interesting case is that of an the entrant with $T^F > \widehat{T}^F$. Large efficiencies are necessary to maintain merger profitability given such an entrant. However, if efficiencies are that large then the merger actually reduces the profitability of entry. Thus, if merger occurs, merger-induced entry does not. Another interesting case is that of efficiencies that fall between the compensating efficiency and the neutrality curve for entrant profitability. Such efficiencies are too small to preserve consumer surplus, but sufficiently large to make merger-induced entry unprofitable. Thus, consumers are better off with larger efficiencies or with smaller efficiencies.

22

3.3 Likelihood of Entry

Merger review often involves some empirical assessment of entry costs that is intended to inform the likelihood of merger-induced entry. We develop how modeling can place bounds on the entry costs under which merger-induced entry can occur in SPE. This approach can be applied with a number of different demand systems and parameterizations. To provide an illustration, we focus on MNL demand with four symmetric incumbents and an outside good that receive market shares of 0.20 each, applying the calibration and simulation techniques described in Appendix E. Similar results obtain for different market structures.

Figure 5 summarizes our results. The left panel plots the profit that the entrant would receive both with and without the merger, assuming no merger efficiencies, as a function of the entrant's type. Both curves are upward-sloping, as higher-type entrants obtain higher profit, and profit is higher with the merger. A necessary condition for merger-induced entry to occur is that the entry cost falls between the two lines, i.e.,

$$\pi \left(\frac{T^F}{H_{nm,e}} \right) < \chi \leq \pi \left(\frac{T^F}{H_{m,e}} \right).$$

Note that in an empirical application, it might be appropriate to decompose χ into upfront entry costs (EC) and the present value of fixed costs (FC), such that $\chi \equiv EC + (1 - \delta)FC$, where δ is the applicable discount rate.

Our first observation is that the numerical analysis places bounds on the entry costs consistent with merger-induced entry that are quite tight. For instance, a merger increases the profit of the compensating entrant ($T^F = \tilde{T}^F$) by about 10 percent, from 0.079 to 0.087. Thus, empirical analysis that places $\chi \in [0.079, 0.087]$ would support the possibility of merger-induced entry sufficient to preserve consumer surplus. In contrast, entry costs that fall above or below this range to do not generate merger-induced entry, because entry is unprofitable in the former case and profitable regardless of the merger decision in the latter case. (Whether entry costs can be placed within such a tight window is unclear to us, and probably depends on the empirical setting.)

Still, our prior analyses suggest that the bounds on entry costs might be even tighter than described above. The reason is that, for many mergers, if merger-induced entry is to occur in SPE then either (1) the entrant must be a distant competitor to the merging firms, or (2) the merger must create efficiencies. In the first case, diversion from the merging firms to the entrant is smaller than with MNL demand, and the merger has a smaller effect

on the profitability of entry.[17] In the second case, at least lower-bound merger efficiencies are necessary to preserve consumer surplus and maintain merger profitability, and merger efficiencies reduce the profitability of entry.

We explore the effect of efficiencies in greater detail. The right panel of Figure 5 plots the percentage change in entrant profit due to a merger, both without efficiencies and with lower bound efficiencies. The first line slopes down—higher type entrants benefit less from merger without efficiencies. The second line, which incorporates lower bound efficiencies, takes a $\wedge$ shape. This reflects the influence of neutrality curves for consumer surplus and merger profitability (Figure 4). On the left, lower bound efficiencies are decreasing in the entrant's type, as less is required to preserve consumer surplus. Thus, the entrant's profit opportunity slopes upward initially. On the right, lower bound efficiencies are increasing in the entrant's type, as more is required to preserve merger profitability, and this causes the entrant's profit opportunity to slope downward. The peak occurs at the crossing of the neutrality curves. Thus, mergers appear to have small effects on the profits of low-type and high-type entrants, and somewhat greater effects for entrants with more moderate types. In our numerical example, the effect at its peak is a 7.4 percent increase in profit.

If one takes a stance on the empirical distributions of the demand parameters and the entry costs, then it is possible to use the bounds analysis that we propose above to inform the likelihood with which entry occurs. That is the approach of Werden and Froeb (1998), which uses a Monte Carlo experiment. Summarizing results for the case of MNL demand and mergers without efficiencies, the authors write that "the entry opportunity is probably insufficient for the vast majority of the mergers." We view the Monte Carlo experiment as suggesting that the bounds on entry costs tend to be tight generally, not just in the numerical examples we provide.[18] Whether this is true in any particular setting, however, can be determined given knowledge of demand and supply conditions. Thus, we view the bounds approach as an appropriate guide to empirical inquiry about the likelihood of merger-induced entry, in settings for which merger-induced entry cannot be ruled out on theoretical grounds.

---

[17]The effect of a merger on the entrant's profit is roughly proportional to the diversion from the merging firms to the entrant. To see this, consider a market with single-product firms. Then, differentiating the profit of the entrant with respect to a price of the merging firms obtains, after a few algebraic steps,

$$\frac{\partial \pi^F}{\partial p^1} = s^1 \frac{\mu^F}{\mu^1} DIV_{1 \to F}$$

so that the effect depends on (i) the size of the merging firm, (ii) relative markups, and (iii) diversion.

[18]As the Werden and Froeb simulations do not allow for efficiencies, our theoretical analysis applies, and merger-induced entry sufficient to restore pre-merger consumer surplus does not occur in SPE.

4 Application to T-Mobile/Sprint

We apply the empirical frameworks to the T-Mobile/Sprint merger, which combined two of the four national providers of mobile wireless telecommunications service. The Department of Justice and the Federal Communications Commission (FCC) approved the merger conditional on certain behavioral remedies and the divestiture of Boost—a Sprint prepaid brand—to DISH, a prospective entrant. The merger then was challenged unsuccessfully in Federal District Court by several states. The Court's decision addressed whether adverse competitive effects from the loss of competition would be offset by efficiencies related to network capacity and by DISH's entry.

We analyze the merger using a Bertrand/logit model of competition among the four national providers: Sprint, T-Mobile, Verizon, and AT&T. We calibrate the model using publicly-available data on market shares, prices, and markups. We also use a market elasticity of demand that appears in regulatory filings.[19] Details on the data and calibration process are provided in Appendix E.

Figure 6 provides graphs that we have developed earlier in this paper. Starting with the left panel, we obtain values of $\underline{E}$ and $\bar{E}$ that are equivalent to marginal cost reductions of 1.6 and 4.0 percent, respectively, if one holds quality constant. These bound the efficiencies necessary to generate a pro-competitive merger with induced entry. That is, if the efficiency is less than 1.6 percent then the merger harms consumers, and if the efficiency is greater than 4.0 percent then merger-induced entry never occurs (though consumers benefit). These bounds are substantially tighter for most prospective entrant types, by inspection of the panel. The right panel shows that the merger increases the profitability of entry by at most 5.1 percent, assuming lower bound efficiencies.

Entry requires a specific set of assets, including spectrum, that is necessary for the transmission of a wireless signal. Thus, the litigation focused specifically on one prospective entrant, DISH, that had acquired a substantial portfolio of spectrum licenses in prior FCC auctions. The Court's decision states that:

> DISH is well positioned to become a fourth [mobile network operator] in the
> market, and its extensive preparations and regulatory remedies indicate that it

---

[19]The market elasticity of demand is the percentage change in the (combined) share of the inside products due to a one percent change in the weighted-average price. Letting ϵ be the market elasticity of demand, with logit demand we have $\epsilon = -\alpha s_0 \bar{p}$, where $\bar{p}$ is the weighted-average price. In calibration, the market elasticity determines the the outside good's share, which is unobservable from the data.

can sufficiently replace Sprint's competitive impact....[20]

We interpret this language as indicating a belief that DISH could offer service at a quality and cost that is similar to Sprint. As our calibration obtains a Sprint type of $T^1 = 5.00$, we assume a DISH type of $T^F = 5.00$. Applying Proposition 4, this exceeds the type of the compensating entrant because the T-Mobile type is $T^2 = 6.14$.

We first simulate the T-Mobile/Sprint merger under the assumption of merger-induced entry by DISH. The results indicate that the prices of T-Mobile and Sprint increase by 3.1 percent and 4.4 percent, respectively. Consumer surplus nonetheless increases by 5.0 percent due to the diversity that DISH introduces.

Whether merger-induced entry would occur is another matter. Indeed, we find no SPE featuring merger-induced entry because the DISH type is too high, as we have $T^F = 5.00 > \hat{T}^F = 4.63$. The logic that rules out DISH as a merger-induced entrant depends on the level of the merger efficiency. With a small efficiency, say a 2% reduction in marginal costs, the merger would induce entry by DISH given small enough entry barriers, but DISH's entry would make the merger unprofitable. With a large efficiency, say a 3.8% reduction in marginal costs, the merger would be profitable even with merger-induced entry by DISH, but merger reduces the profitability of DISH entry. In neither scenario does both merger and merger-induced entry occur.

Our analysis thus far has not incorporated the divestiture of the Boost brand to DISH, which was intended to help create the conditions under which merger-induced entry would occur. Our publicly-available data are insufficiently rich to calibrate the "type" associated with Boost. Thus, we explore a range of possibilities in which we transfer some of the merged firm's type to DISH (as would happen with divestiture); we look at transfers that range between 0% and 50% of the Sprint type. For each transfer, we also consider efficiencies between 0% and 100% of the compensating efficiency. Across these combinations of divestitures and efficiencies, we find no SPE featuring merger-induced entry in which pre-merger consumer surplus is preserved. Thus, incorporating the divestiture does not appear change our results.[21]

To the extent that evidence pointed toward DISH entry, our analysis suggests that it may have been more appropriate to obtain effects of the T-Mobile/Sprint merger by comparing an

---

[20]See p. 117 of the opinion.

[21]The settlement also provided DISH roaming access to the 3G network of T-Mobile, our modeling still does not account for all of the nuances of setting. We do note, however, that T-Mobile reneged on this commitment within a year after the merger was approved, and DISH still not has entered the market, aside from the Boost brand.

equilibrium with DISH entry and without the merger to an equilibrium with DISH entry and the merger. Thus, we proceed by treating DISH as an incumbent.[22] Our simulation results indicate that, absent efficiencies, the merger increases the prices of T-Mobile and Sprint by 4.0 percent and 5.2 percent, respectively, and decreases consumer surplus by 1.7 percent.

Our main goal in providing this empirical application is to highlight how the modeling can help assess entry in real-world settings. Our analysis is admittedly somewhat more stylized than the nested logit models that appear in regulatory filings (those model could be brought to bear on the question with richer data). Subject to that caveat, the results we obtain suggest that the Court may have erred in treating DISH as a merger-induced entrant. We interpret the fact that DISH has not entered the market, now more than two years after the merger closed, as consistent with our analysis.

5 Conclusion

We conclude by describing what we view as the policy ramifications of our research. A coherent pattern emerges from both our work and the Monte Carlo findings of Werden and Froeb (1998), the theoretical results of Spector (2003) for Cournot competition, and from models of efficient second score auctions (e.g., Miller (2014)), where mergers do not increase entrant profits. Across the wide range of modeling frameworks considered in these articles, profitable mergers are incompatible with merger-induced entry sufficient to restore consumer surplus unless merger efficiencies exist or the entrant is substantially differentiated from the merging firms. As it seems unlikely that a merger would be the deciding factor for entry by a highly differentiated competitor, it appears reasonable to infer that entry barriers typically would prevent post-merger entry or limit its scope if merger profitability derives mainly from a reduction in competition.

This leads us to conclude that, as a matter of economic theory, there may be little reason, outside of exceptional cases, to consider merger-induced entry as a standalone justification for an otherwise anticompetitive merger. For such mergers, the empirical analysis of the sort proposed in the merger guidelines may be redundant and unnecessary. We are not the first to reach this conclusion, though we have shown the generality of the result in several differentiated-products Bertrand settings.[23]

---

[22]The US Horizontal Merger Guidelines, §9, is explicit on this point, stating that "Firms that have, prior to the merger, committed to entering the market also will normally be treated as market participants."

[23]Werden and Froeb (1998, p. 541) state that "the best way for courts to treat entry in many merger cases may be not to consider it at all." Spector (2003, p. 1597) states that "a merger unambiguously generating no

A more nuanced role for entry in merger review may nonetheless be appropriate for cases in which merger profitability derives in part from efficiencies, such as fixed cost savings, marginal cost savings, or quality improvements. In such instances, our model suggests that entry and efficiencies should be actively incorporated into the analysis of merger harm, rather than being tackled as separate questions. The frameworks we introduce are intended to guide these efforts, as they can be taken directly to data on objects like market shares. Given our findings based on firms' revealed preferences, additional work on subjects such as (i) why a merger remains profitable in the face of entry, and (ii) why barriers to entry are likely to be overcome after a merger but not before the merger would be helpful. In our view, modeling can help provide structure in examining these issues and in identifying which types of data are necessary for testing and quantification.

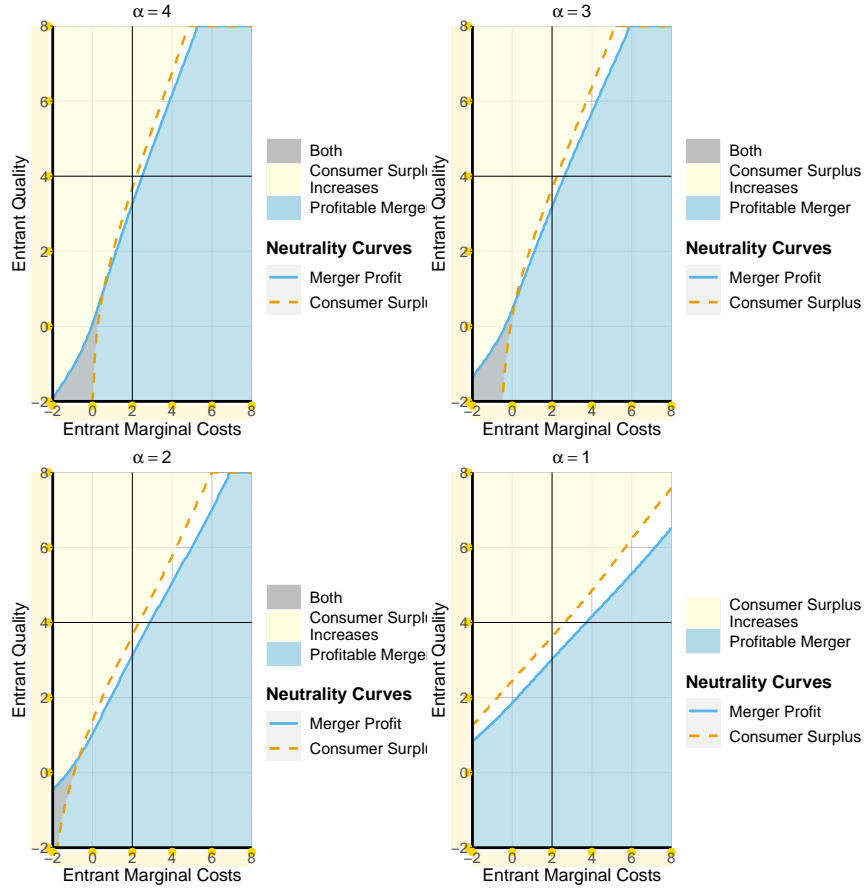synergies should be prevented, without delving into the question of entry."

Figure 2: Numerical Results for RCL Demand with $\alpha = (4, 3, 2, 1)$

Notes: The panels shows the combinations of entrant quality and marginal cost for which merger with entry increases consumer surplus (shaded yellow), increases the merging firms' profit (shaded blue), or both (shaded gray). The corresponding neutrality curves for merger profitability and consumer surplus are plotted as solid blue and dashed orange lines, respectively. The marginal cost and quality of the merging firms are plotted with the black vertical and horizontal lines.
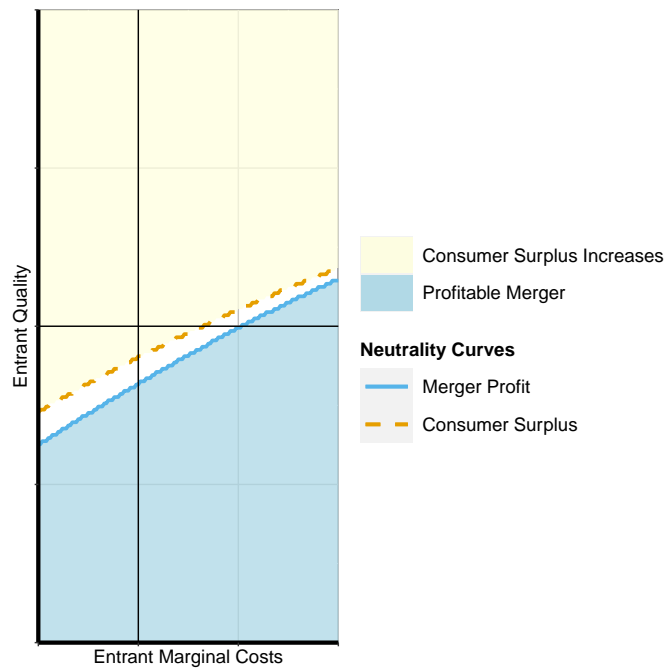
Figure 3: Entry Analysis for the Miller/Coors Merger in the Beer Industry

Notes: The plot shows the combinations of entrant quality and marginal cost for which merger with entry increases consumer surplus (shaded yellow) and increases the merging firms' profit (shaded blue). The corresponding neutrality curves for merger profitability and consumer surplus are plotted as solid blue and dashed orange lines, respectively. The marginal cost and quality of Coors, the smaller of the two merging firms, are plotted with the black vertical and horizontal lines.
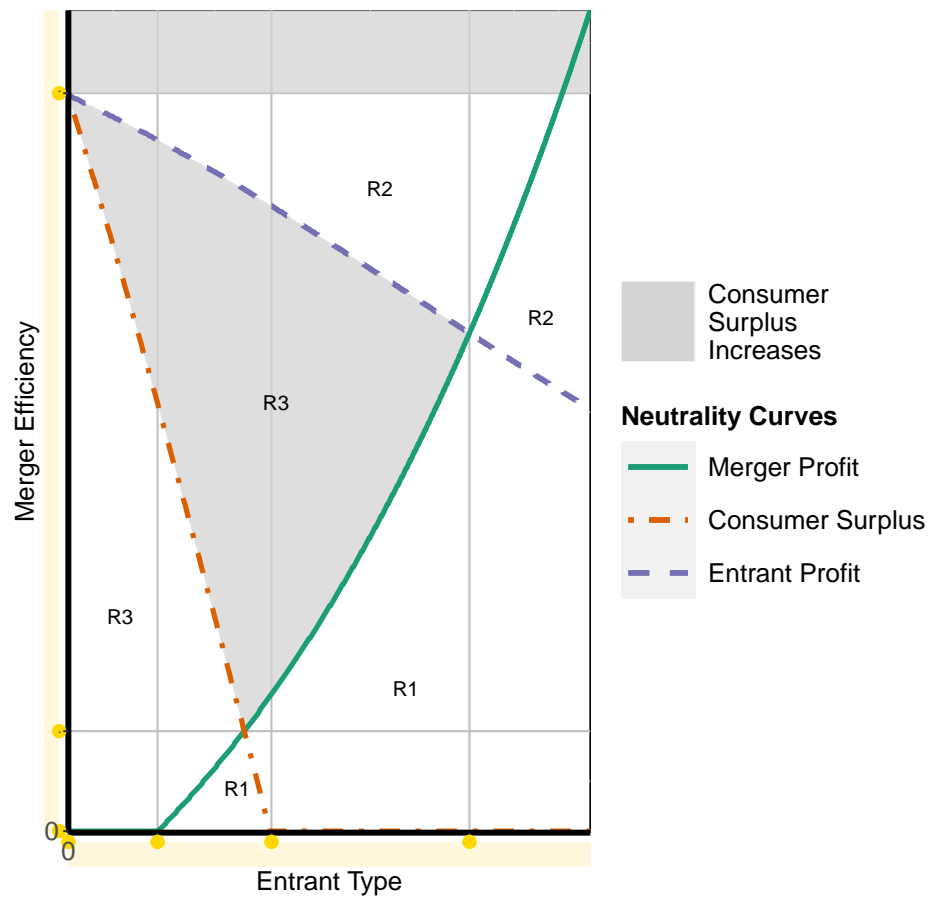
Figure 4: Merger Analysis with Entry and Efficiency

Notes: The figure illustrates the integrated framework for merger analysis with entry and efficiencies. The results are generated numerically given pre-merger market shares of 0.20 for each of four incumbents and an outside good.
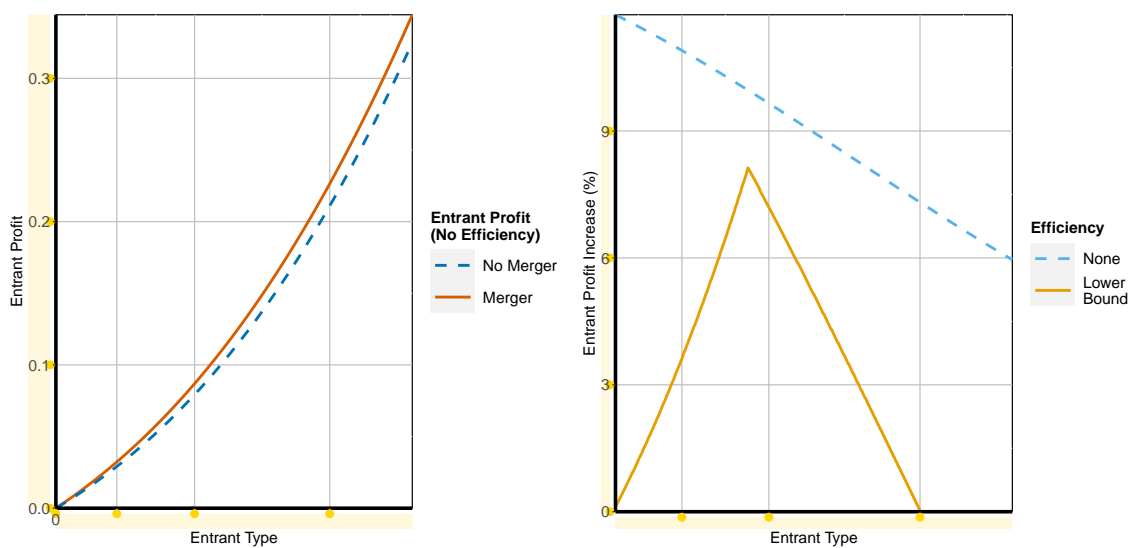
Figure 5: The Entrant's Profit Opportunity

Notes: The left panel plots the profit that the prospective entrant would receive both without and with the merger, assuming no merger efficiencies. The right panel plots the percentage change in entrant profit due to a merger without efficiencies, along with the analogous percentage change due to a merger with lower bound efficiencies. The figures are generated numerically for a market with four incumbents and an initial outside good share of 0.20.

Figure 6: Application to T-Mobile/Sprint Merger
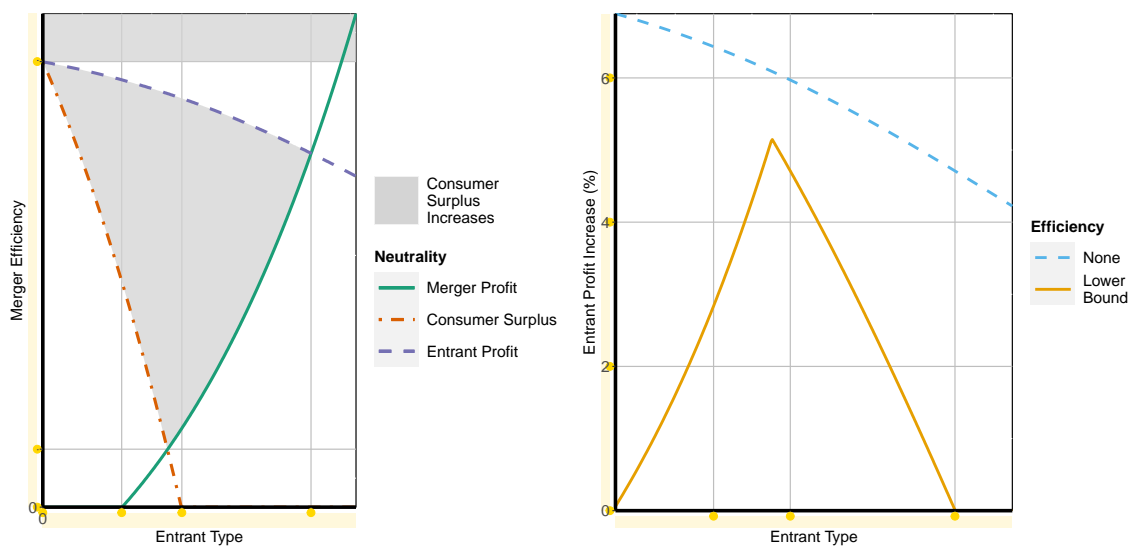
Notes: The left panel plots the neutrality curves in the integrated framework and the right panel plots the percentage change in entrant profit due to a merger without efficiencies, along with the analogous percentage change due to a merger with lower bound efficiencies. The figure is generated numerically given market shares for the mobile wireless telecommunications industry.

References

Aliprantis, C. D. and K. C. Border (2006). *Infinite Dimensional Analysis: A Hitchhiker's Guide*. Springer Science & Business Media.

Alviarez, V., K. Head, and T. Mayer (2020). Global giants and local stars: How changes in brand ownership affect competition. Working Paper.

Anderson, S. P., N. Erkal, and D. Piccinin (2020). Aggregative games and oligopoly theory: Short-run and long-run analysis. *RAND Journal of Economics 51*(2), 470–495.

Berry, S., J. Levinsohn, and A. Pakes (1995). Automobile prices in market equilibrium. *Econometrica 63*(4), 847–890.

Berry, S. T. (1994). Estimating Discrete-Choice Models of Product Differentiation. *RAND Journal of Economics 25*(2), 242–262.

Bronnenberg, B. J., M. W. Kruger, and C. F. Mela (2008). The IRI marketing data set. *Marketing Science 27*(4), 745–748.

Cabral, L. (2003). Horizontal mergers with free-entry: Why cost efficiencies may be a weak defense and asset sales a poor remedy. *International Journal of Industrial Organization 21*, 607–623.

Cardell, S. N. (1997). Variance Components Structures for the Extreme-Value and Logistic Distributions with Applications to Models of Heterogeneity. *Econometric Theory 13*, 185–213.

Ciliberto, F., C. Murry, and E. Tamer (2021). Market structure and competition in airline markets. *Journal of Political Economy 129*(11), 2995–3038.

Conlon, C. T. and J. H. Mortimer (2021). Empirical properties of diversion ratios. *RAND Journal of Economics 52*(4), 693–726.

Davidson, C. and A. Mukherjee (2007). Horizontal mergers with free entry. *International Journal of Industrial Organization 25*, 157–172.

Erkal, N. and D. Piccinin (2010). Welfare-reducing mergers in differentiated oligopolies with free entry. *Economic Record 86*(273), 178–184.

Fan, Y. and C. Yang (2021). Estimating discrete games with many firms and many decisions: An application to merger and product variety. Working Paper.

Farrell, J. and C. Shapiro (1990). Horizontal mergers: An equilibrium analysis. *American Economic Review 80*(1), 107–126.

Garrido, F. (2019). Mergers between multi-product firms with endogenous variety: Theory and an application to the RTE-cereal industry. Georgetown University dissertation.

Hirsch, M. W. (2012). *Differential topology*, Volume 33. Springer Science & Business Media.

Li, S., J. Mazur, J. Roberts, Y. Park, A. Sweeting, and J. Zhang (2022). Repositioning and market power after airline mergers. *RAND Journal of Economics 53*(1), 166–199.

Mas-Colell, A. (1974). A note on a theorem of F. Browder. *Mathematical Programming 6*(1), 229–233.

Miller, N. H. (2014). Modeling the effects of mergers in procurement. *International Journal of Industrial Organization 37*, 201–208.

Miller, N. H. and G. Sheu (2021). Quantitative methods for evaluating the unilateral effects of mergers. *Review of Industrial Organization 58*(1), 143–177.

Miller, N. H. and M. C. Weinberg (2017). Understanding the price effects of the MillerCoors joint venture. *Econometrica 85*(6), 1763–1791.

Nocke, V. and N. Schutz (2018). Multiproduct-firm oligopoly: An aggregative games approach. *Econometrica 86*(2), 523–557.

Nocke, V. and N. Schutz (2019). An aggregative games approach to merger analysis in multiproduct-firm oligopoly. Working Paper.

Nocke, V. and M. Whinston (2022). Concentration screens for horizontal mergers. *American Economic Review 112*(6), 1915–1948.

Shapiro, C. (2010). The 2010 Horizontal Merger Guidelines: From hedgehog to fox in forty years. *Antitrust Law Journal 77*, 49–107.

Shapiro, C. and H. Shelanski (2021). Judicial response to the 2010 Horizontal Merger Guidelines. *Review of Industrial Organization 58*, 51–79.

Spector, D. (2003). Horizontal mergers, entry, and efficiency defences. *International Journal of Industrial Organization 21*, 1591–1600.

Werden, G. J. (1996). A robust test for consumer welfare enhancing mergers among sellers of differentiated products. *The Journal of Industrial Economics 44*(4), pp. 409–413.

Werden, G. J. and L. M. Froeb (1998). The entry-inducing effects of horizontal mergers: An exploratory analysis. *The Journal of Industrial Economics 46*(4), pp. 525–543.

Appendix

For Online Publication

A Notes on Aggregative Games

In this appendix, we derive the aggregative games formulation of the Bertrand model with MNL, CES, NMNL, and NCES demand. We focus especially on the MNL and CES models in order to provide something of a "practitioner's guide" for those who previously have not studied aggregative games.

A.1 MNL Demand

We take as given the profit function and first order conditions of (1) and (2), the indirect utility of (3), and the market shares of (4). In this framework, it is well known that consumer surplus is given by

$$CS = \frac{1}{\alpha} \ln \left(1 + \sum_{j \in \mathcal{J}} \exp(v_j - \alpha p_j) \right). \tag{A.1}$$

The primitives of the aggregative game reformulation are the vector of firm-specific types, $\{T^f\} \ \forall f \in \mathcal{F}$, and the price parameter, α. Equation (6) defines the type of each firm f as

$$T^f \equiv \sum_{j \in \mathcal{J}^f} \exp(v_j - \alpha c_j),$$

which represents the firm's contribution to consumer surplus if its prices equal its marginal costs. From these primitives, the Bertrand equilibrium can be characterized as a vector of "ι-markups," $\{\mu^f\} \ \forall f \in \mathcal{F}$, a vector of firm-level market shares, $\{s^f\} \ \forall f \in \mathcal{F}$, and a market aggregator, H. We define markups below, and let $s^f = \sum_{j \in \mathcal{J}^f} s_j$. The aggregator is defined as $H \equiv 1 + \sum_{j \in \mathcal{J}} \exp(v_j - \alpha p_j)$, which is the denominator from the market share formula of the product-level model (see (4)).

We first derive a relationship between the ι-markups and firm-level market shares. The product-specific price derivatives for logit demand are

$$\frac{\partial s_j}{\partial p_k} = \begin{cases} -\alpha s_j (1 - s_j) & \text{if } k = j \\ \alpha s_j s_k & \text{if } k \neq j. \end{cases}$$

Substituting these demand derivatives into the first order conditions of (2) for some product

j and rearranging gives

$$\alpha(p_j - c_j) = 1 + \alpha \sum_{k \in \mathcal{J}^f} (p_k - c_k)s_k. \tag{A.2}$$

The right-hand side of this equation does not depend on the which product $j \in \mathcal{J}^f$ is referenced. This implies that the left-hand side is equivalent for all products sold by firm f, meaning each firm imposes a common markup (in levels) across all of its products. Following (7), define the ι-markup of firm f as $\mu^f \equiv \alpha(p_j - c_j) \; \forall j \in \mathcal{J}^f$. Substituting back into (A.2) obtains an equilibrium relationship between markups and shares:

$$\mu^f = \frac{1}{1 - s^f}. \tag{A.3}$$

We also have $s^f = (1/H) \sum_{j \in \mathcal{J}^f} \exp(v_j - \alpha p_j)$ from (4), after substituting in for the definition of the aggregator, H. Adding and subtracting αc_j inside the exponential and applying the definitions of μ^f and T^f gives

$$s^f = \frac{T^f}{H} \exp(-\mu_f) \tag{A.4}$$

$$\Longleftrightarrow \quad \frac{T^f}{H} = s^f \exp\left(\frac{1}{1 - s^f}\right). \tag{A.5}$$

Plugging (A.4) into (A.3), we obtain that equilibrium ι-markups satisfy (9):

$$\mu_f \left(1 - \frac{T^f}{H} \exp(-\mu_f)\right) = 1.$$

Let the unique solution for μ_f from this expression be written as $m(T^f/H)$. This *markup fitting-in function*, $m(\cdot)$, has the properties that $m(0) = 1$ and $m'(\cdot) > 0$. Plugging $\mu_f = m(T^f/H)$ into (A.4) yields the expression for equilibrium market shares provided in (10). Equilibrium market shares can be written $s^f = S(T^f/H)$, and thus equilibrium profit can be written $\Pi^f = \pi(T^f/H)$. To close the system, the aggregator satisfies an adding-up constraint of (11). The expressions for equilibrium profit and consumer surplus provided in (12) obtain immediately.

A.2 CES

Derivation of the CES aggregative game mirrors that of MNL case, except the CES demand derivatives and formula for shares must be used instead. With CES, the pricing first order condition for product j becomes

$$\sigma \frac{p_j - c_j}{p_j} = 1 + (\sigma - 1) \sum_{k \in \mathcal{J}^f} s_k \frac{p_k - c_k}{p_k}, \tag{A.6}$$

which is the counterpart to the MNL equation (A.2). We again see that the right-hand side of this equation does not depend on the identity of $j \in \mathcal{J}^f$, which in turn implies that each firm charges a constant percentage markup across all of its products.

Once we define the ι-markup as $\mu^f = \sigma(p_j - c_j)/p_j$ following (7), we obtain

$$\mu^f = \frac{1}{1 - \frac{\sigma-1}{\sigma} s^f} \tag{A.7}$$

after substituting into the pricing first order condition. Take the share equation (5) and multiply and divide it by $c_j^{1-\sigma}$. We can then substitute in the definitions of the aggregator H, μ^f, and the type T^f. Summing across the shares for the products sold by firm f gives

$$s^f = \frac{T^f}{H} \left(1 - \frac{\mu^f}{\sigma} \right)^{\sigma-1} \tag{A.8}$$

for firm-level revenue shares. Substituting this share into the markup expression in (A.7) gives the markup fitting-in function,

$$1 = \mu^f \left(1 - \frac{\sigma-1}{\sigma} \frac{T^f}{H} \left(1 - \frac{\mu^f}{\sigma} \right)^{\sigma-1} \right) \tag{A.9}$$

which appears in (9). The model is closed with the adding-up constraint given by (11).

A.3   NMNL

With NMNL demand, the following equations hold in Bertrand equilibrium:

$$\mu^f = \frac{1}{1 - \rho s^{f|g} - (1 - \rho)s^f} \tag{A.10}$$

$$1 = \mu^f \left(1 - \rho \frac{T^f}{H_g} \exp(-\mu^f) - (1 - \rho)\frac{T^f}{H_g}\frac{H_g^{1-\rho}}{H} \exp(-\mu^f) \right) \tag{A.11}$$

$$\frac{T^f}{H_g} = s^{f|g} \exp\left(\frac{1}{1 - \rho s^{f|g} - (1 - \rho)s^f} \right) \tag{A.12}$$

$$\bar{s}_g = \frac{H_g^{1-\rho}}{H} \tag{A.13}$$

$$s^f = s^{f|g}\bar{s}_g \tag{A.14}$$

$$1 = \sum_{f \in \mathcal{F}_g} s^{f|g} \tag{A.15}$$

$$\frac{1}{H} = 1 - \sum_{f \in \mathcal{F}} s^f \tag{A.16}$$

$$\pi^f = \frac{1 - \rho}{\alpha}\mu^f s^f \tag{A.17}$$

$$CS = \frac{1}{\alpha} \ln(H) \tag{A.18}$$

where T^f is the type of the firm, s^f is the share of the firm, $s^{f|g}$ is the share of the firm within its nest, $\bar{s}_g$ is the share of the nest, μ^f is the ι-markup of the firm, H_g is a nest aggregator, H is the market aggregator, π^f is the profit of the firm, and CS is consumer surplus.

Firm types are defined as in (18). Firm share is given by $s^f = \sum_{j \in \mathcal{J}^f} s_j$, as in the MNL and CES models. Firm share within its nest is given by $s^{f|g} = \sum_{j \in \mathcal{J}^f} s_{j|g}$, where the share of a product within a nest is

$$s_{j|g} = \frac{\exp\left(\frac{v_j - \alpha p_j}{1 - \rho} \right)}{H_g}. \tag{A.19}$$

The aggregators are defined as $H_g \equiv \sum_{j \in \mathcal{J}_g} \exp((v_j - \alpha p_j)/(1 - \rho))$ and $H \equiv 1 + \sum_{g \in \mathcal{G}} H_g^{1-\rho}$. The markup is defined as $\mu^f \equiv (\alpha/(1 - \rho))(p_j - c_j)$ for all $j \in \mathcal{J}^f$.

The pricing first order condition for good j can be written as

$$\frac{\alpha}{1 - \rho}(p_j - c_j) = 1 + \frac{\alpha\rho}{1 - \rho} \sum_{k \in \mathcal{J}^f} (p_k - c_k)s_{k|g} + \alpha \sum_{k \in \mathcal{J}^f} (p_k - c_k)s_k, \tag{A.20}$$

under the assumption that firm f owns products only in nest g. We again see that the right-hand side of this condition does not depend on the identity of $j \in \mathcal{J}^f$. Substituting in for the definition of μ^f gives (A.10).

Next, adding and subtracting αc_j inside the exponential on the right-hand side of (A.19) and applying the definitions of μ^f, T^F, and H_g obtains

$$s^{f|g} = \frac{T^f}{H_g} \exp(-\mu^f), \tag{A.21}$$

which rearranges to (A.12). Then (A.11) can be obtained by plugging (A.12) and (A.13) back into (A.10). Next, (A.15) and (A.16) are adding-up constraints that close the model, (A.17) is obtained by plugging μ^f into the profit function, and (A.13), (A.14), and (A.18) follow directly from the NMNL functional form.

A.4 NCES

With NCES, the following equations hold in Bertrand equilibrium:

$$\mu^f = \frac{1}{1 - \frac{\gamma-1}{\sigma}s^f - \frac{\sigma-\gamma}{\sigma}s^{f|g}} \tag{A.22}$$

$$1 = \mu^f \left(1 - \frac{\gamma-1}{\sigma} \frac{T^f}{H_g^{\frac{\sigma-\gamma}{\sigma-1}} H} \left(1 - \frac{\mu^f}{\sigma}\right)^{\sigma-1} - \frac{\sigma-\gamma}{\sigma} \frac{T^f}{H_g} \left(1 - \frac{\mu^f}{\sigma}\right)^{\sigma-1} \right) \tag{A.23}$$

$$\frac{T^f}{H_g} = s^{f|g} \left(1 - \frac{\mu^f}{\sigma}\right)^{1-\sigma} \tag{A.24}$$

$$s^f = \frac{T^f}{H_g^{\frac{\sigma-\gamma}{\sigma-1}} H} \left(1 - \frac{\mu^f}{\sigma}\right)^{\sigma-1} \tag{A.25}$$

$$1 = \sum_{f \in \mathcal{F}_g} s^{f|g} \tag{A.26}$$

$$\frac{1}{H} = 1 - \sum_{f \in \mathcal{F}} s^f \tag{A.27}$$

$$\pi^f = \frac{1}{\sigma} \mu^f s^f \tag{A.28}$$

$$CS = H^{1/(\gamma-1)} \tag{A.29}$$

where T^f is the type of the firm, s^f is the share of the firm, $s^{f|g}$ is the share of the firm within its nest, μ^f is the ι-markup of the firm, H_g is a nest aggregator, H is the market aggregator, π^f is the profit of the firm, and CS is consumer surplus.

Firm types are defined as in (18). Firm share is given by $s^f = \sum_{j \in \mathcal{J}^f} s_j$, as in the MNL and CES models. Firm share within its nest is given by $s^{f|g} = \sum_{j \in \mathcal{J}^f} s_{j|g}$, where the share of a product within a nest is

$$s_{j|g} = \frac{v_j p_j^{1-\sigma}}{\sum_{k \in \mathcal{J}_g} v_k p_k^{1-\sigma}} \tag{A.30}$$

The aggregators are defined as $H_g \equiv \sum_{j \in \mathcal{J}_g} v_j p_j^{1-\sigma}$ and $H \equiv 1 + \sum_{g \in \mathcal{G}} H_g^{\gamma-1/\sigma-1}$. The markup is defined as $\mu^f \equiv \sigma(p_j - c_j)/p_j$ for all $j \in \mathcal{J}^f$, same as with CES demand.

The pricing first order condition for good j can be written as

$$\sigma \frac{p_j - c_j}{p_j} = 1 + \sum_{k \in \mathcal{J}^f} \frac{p_k - c_k}{p_k}[(\gamma - 1)s_k + (\sigma - \gamma)s_{k|g}] \tag{A.31}$$

under the assumption that firm f owns products only in nest g. We again see that the right-hand side of this condition does not depend on the identity of $j \in \mathcal{J}^f$. Substituting in for the definition of μ^f gives (A.22).

Next, multiplying and dividing by $c_j^{1-\sigma}$ on the right-hand side of (A.30) and applying the definitions of μ^f, T^F, and H_g obtains

$$s^{f|g} = \frac{T^f}{H_g}\left(1 - \frac{\mu^f}{\sigma}\right)^{\sigma-1}. \tag{A.32}$$

which rearranges to (A.24). Applying the same computation to (16) gives (A.25). Then (A.23) can be obtained by plugging (A.24) and (A.25) back into (A.22). Next, (A.26) and (A.27) are adding-up constraints that close the model, (A.28) is obtained by plugging μ^f into the profit function, and (A.29) follows directly from the NCES functional form.

B Delayed and Probabilistic Entry

Our baseline model considers a three-stage game in which (1) firms decide to merge, (2) an outside decides to enter, and (3) payoffs are realized according to a differentiated pricing game. In this appendix, we consider two variants. The first is a model of *delayed entry* in which incumbents obtain payoffs for N periods before entry occurs (if it does occur). The second is a model of *probabilistic entry* in which entry occurs in the second stage with some fixed probability p if it is profitable, and with probability zero otherwise.

With delayed and probabilistic entry, a merger that induces entry increases the net present value of the merging firms if and only if

$$\frac{1-\theta}{1-\delta}\pi_{m,ne}^M + \frac{\theta}{1-\delta}\pi_{m,e}^M \geq \sum_{i=1,2} \frac{1}{1-\delta}\pi_{nm,ne}^i \tag{B.1}$$

where δ is a discount factor, $\theta = \delta^N$ with delayed entry, and $\theta = p$ with probabilistic entry. Similarly, a merger that induces entry increases the net present value of consumer surplus if and only if

$$\frac{1-\theta}{1-\delta}CS^{m,ne} + \frac{\theta}{1-\delta}CS^{m,e} \geq \frac{1}{1-\delta}CS^{nm,ne} \tag{B.2}$$

As these equations nest both delayed and probabilistic entry, we proceed by analyzing mergers and entry in the two models jointly.

With $\theta = 1$, the analytical results from in the main body of the paper obtain, and with MNL or CES demands merger-induced entry sufficient to preserve consumer surplus renders merger unprofitable. At the other end, entry is irrelevant with $\theta = 0$.

With $\theta \in (0, 1)$, our intuition is that Proposition 1 extends for most reasonable parameterizations. The reason is that as θ decreases from one, the profitability of the merger increases but so does the consumer surplus loss. Given the strict inequalities we obtain, the first of these effects would have to be considerably stronger than the second to generate a profitable, pro-competitive merger. Our examination of the implied relationships indicates this is unlikely to be the case.

In support of this conjecture, we conduct numerical simulations using a model with two incumbents and MNL demand. We consider market shares for the incumbents that range from 0.01 to 0.80. After calibrating incumbent types, we examine entrants with types that range between that of the compensating entrant (Proposition 4) and ten times that of the merged firm. Finally, for each of these, we scale θ between zero and one in increments of 0.01. We find no cases in which a profitable merger increases consumer surplus.

This is not to claim that profitable, pro-competitive mergers cannot be found with *unreasonable* parameterizations. Indeed, for any initial set of incumbent types and MNL or CES demands, we can prove that there exists some θ and entrant type T^F for which a profitable merger improves consumer surplus. As one example, suppose that two incumbents each have of a market share of 0.40 initially. The implied types are $T^1 = T^2 = 10.59$. Further let $\theta = 0.099$, which obtains with 21.96 years of delay (given $\delta = 0.90$) or with a probability of post-merger entry just less than 10 percent. If, in addition, the entrant's type exceeds 3.59×10^{102}, then a profitable, pro-competitive merger obtains.[24] This entrant captures a market share of 0.996; the incumbents' combined market share decreases to 0.004 and the share of the outside good is approximately zero.

We now formally state that with delayed and probabilistic entry, the model can generate profitable, pro-competitive mergers.

Proposition B.1. *Fix an initial market structure comprising $f = 1, \ldots, F - 1$ incumbents and their types, and consider a merger of firms 1 and 2. With MNL and CES demands, there exists a θ and entrant type T^F such that merger with induced entry increases the present value of consumer surplus and the merging firms' profit.*

Proof. See Appendix D. □

For intuition, if θ is small enough—i.e., entry is sufficiently delayed or unlikely—then a merger increases the present value of the merging firms' profit, even if this profit is approximately zero in every period after entry occurs. Thus for any baseline calibration, by choosing a small enough θ, the profit and surplus inequalities 'decouple,' in the sense that the profit inequality holds for *any* value of entrant type. However, as consumer surplus increases to infinity with the type of the entrant, one can then always find some sufficiently capable entrant such that the present value of consumer surplus increases. The numerical results we describe above suggest that this theoretical possibility is not practically relevant for merger review.

---

[24]For comparison, there are approximately 2.40×10^{67} atoms in the Milky Way galaxy.

C   Neutrality Curves

In this appendix, we characterize the neutrality curves introduced in Section 3.2. The results are presented using MNL demand in the interests of brevity. The proofs appear in Appendix D. Analogous results also hold, *mutatis mutandis*, for CES demands. Statements and proofs of the CES results are available upon request from the authors.

Lemma C.1. *Let firm f be a non-merging incumbent with $T^f > 0$, and let $*$ denote either 'merger, no entry' or 'merger, entry.' Then if any of the following conditions holds, all of the following conditions hold for that case.*

 (i) *The merger does not affect the profitability of firm f.*

 (ii) *The merger does not affect consumer surplus.*

 (iii) *Market shares satisfy the following equality:*

$$s^1_{nm,ne} + s^2_{nm,ne} = s^M_* + \sum_{\{f \in \mathcal{F}_* \backslash \mathcal{F}_{nm,ne} | f \neq m\}} s^f_*.$$

Proposition C.1. *For any $T^F \in [0, \tilde{T}^F]$ there exists a unique E such that the merger with entry is neutral for consumer surplus (i.e., $CS_{nm,ne} = CS_{m,e}$). These combinations define a neutrality curve with the following properties:*

 (i) *The curve is downward-sloping in (T^F, E) space.*

 (ii) *If $T^F = 0$ then $E = \bar{E}$, and if $T^F = \tilde{T}^F$ then $E = 0$.*

 (iii) *For any (T^F, E) on the curve, market shares satisfy $s^1_{nm,ne} + s^2_{nm,ne} = s^F_{m,e} + s^M_{m,e}$, i.e., the combined pre-merger market shares of the merging firms equal the combined post-merger market shares of the entrant and the merged firm.*

Proposition C.2. *For any $T^F \geq \bar{T}^F$, there exists a unique E such that the merger is profit neutral (i.e., $\pi^1_{nm,ne} + \pi^2_{nm,ne} = \pi^M_{m,e}$). These combinations define a neutrality curve with the following properties:*

 (i) *The curve is upward-sloping in (T^F, E) space.*

 (ii) *If $T^F = \bar{T}^F$ then $E = 0$, and there exists some T^F such that $E = \bar{E}$.*

 (iii) *For any (T^F, E) on the curve, market shares satisfy*

$$s^M_{m,e} = 1 - \frac{(1 - s^1_{nm,ne})(1 - s^2_{nm,ne})}{1 - s^1_{nm,ne} s^2_{nm,ne}}. \tag{C.1}$$

where $s^1_{nm,ne}$ and $s^2_{nm,ne}$ are the pre-merger shares, and $s^M_{m,e}$ is the post-merger share.

Proposition C.3. *For any $T^F > 0$ there exists a unique E such that the merger is neutral for the entrant's profit (i.e., $\pi_{nm,e}^F = \pi_{m,e}^F$). These combinations define a neutrality curve with the following properties:*

(i) *The curve is downward-sloping in (T^F, E) space.*

(ii) *As $T^F \to^+ 0$, $E \to \bar{E}$. Further, E is strictly positive for all $T^F > 0$.*

(iii) *For any (T^F, E) on the curve, with $T^F > 0$, merger is consumer surplus neutral:*

$$CS_{nm,e} = CS_{m,e}.$$

Moreover, $CS_{nm,ne} < CS_{nm,e}$ and thus this curve lies above the CS-neutrality line for nm, ne versus m, e.

D Proofs

D.1 Proof of Proposition 1

D.1.1 Technical Preliminaries

Lemma D.1. *In Bertrand equilibrium with MNL demand, all firms with positive share have markups such that $\mu^f \in (1, \infty)$. If we instead have CES demand, all firms with positive share have markups such that $\mu^f \in (1, \sigma)$.*

Proof. In equilibrium in the MNL case we have that

$$\mu^f = \frac{1}{1 - s^f}$$

from (A.3). There is an outside good with positive share, so $s^f < 1$ for all active firms. Thus we have that $\mu^f > 1$, since the denominator in the expression above, $1 - s^f$, is less than one for all positive values of s^f. We also have that μ^f approaches infinity as s^f approaches 1.

In equilibrium in the CES case we have that

$$\mu^f = \frac{1}{1 - \frac{\sigma-1}{\sigma} s^f} = \frac{\sigma}{\sigma - s^f(\sigma - 1)}$$

from (A.7). Given that there is an outside good with positive share, $s^f < 1$ for all active firms. Thus, the first equality implies that $\mu^f > 1$, since the denominator $1 - ((\sigma - 1)/\sigma)s^f$ is less than one for all positive values of s^f. The second equality implies that μ^f is bounded above by σ as s^f approaches 1. $\square$

Note that by the same logic, the markups for the NMNL model are such that $\mu^f \in (1, \infty)$ and for the NCES model are such that $\mu^f \in (1, \sigma)$.

Lemma D.2. *Define the function*

$$\phi(x) \equiv \begin{cases} xe^{-x} & \textit{(MNL or NMNL)} \\ x\left(1 - \frac{x}{\sigma}\right)^{\sigma-1} & \textit{(CES or NCES)} \end{cases} \tag{D.1}$$

where the first specification applies to the MNL and NMNL models, and the second applies to the CES and NCES models. This function $\phi(\cdot)$ is decreasing on $(1, \infty)$ for the MNL/NMNL specification and decreasing on $(1, \sigma)$ for the CES/NCES specification.

Proof. The derivative for the MNL/NMNL specification is

$$\frac{d}{dx}\phi(x) = (1 - x)\exp(-x).$$

This derivative is negative if and only if $1 - x$ is negative. This in turn is true if $x > 1$.

For the CES/NCES specification, we employ a change of variables by defining $\tilde{x} = x/\sigma$. The derivative of the redefined function has the same sign as the original, since σ is positive. We have that $\phi(\tilde{x}) = \sigma\tilde{x}(1 - \tilde{x})^{\sigma-1}$. Then the derivative is

$$\frac{d}{d\tilde{x}}\phi(\tilde{x}) = \sigma(1 - \tilde{x})^{\sigma-1}\left[1 - \frac{\tilde{x}(\sigma - 1)}{1 - \tilde{x}}\right].$$

This derivative is negative in the relevant range if and only if the term in brackets is negative, because $(1 - \tilde{x})$ is positive for all $x \in (1, \sigma)$. The term in brackets is negative if and only if $\tilde{x} > 1/\sigma$. We know that $x > 1$, so this condition is met. $\square$

D.1.2 Proof of Proposition 1

Proof. Suppose, for purposes of contradiction, there exists a SPE in which firms 1 and 2 merge, and consumers surplus does not decrease. Thus the merger must increase joint profits:

$$\pi^M\left(\frac{T^1 + T^2}{H_{m,e}}\right) \geq \pi^1\left(\frac{T^1}{H_{nm,ne}}\right) + \pi^2\left(\frac{T^2}{H_{nm,ne}}\right),$$

where $H_{nm,ne}$ denotes the aggregator with no merger and no entry. By hypothesis, consumers surplus does not fall, hence we have $H_{nm,e} \leq H_{m,e}$. Furthermore, by Nocke and Schutz (2018, Proposition 6), π^M is decreasing in H all else equal, meaning that

$$\pi^M\left(\frac{T^1 + T2}{H_{nm,ne}}\right) \geq \pi^1\left(\frac{T^1}{H_{nm,ne}}\right) + \pi^2\left(\frac{T2}{H_{nm,ne}}\right). \tag{D.2}$$

Multiplying the markup and firm share shows that firm profit is given by

$$\pi^f = \begin{cases} \frac{1}{\alpha}\mu^f\frac{T^f}{H}\exp(-\mu^f) & \text{(MNL)} \\ \frac{1}{\sigma}\mu^f\frac{T^f}{H}\left(1 - \frac{\mu^f}{\sigma}\right)^{\sigma-1} & \text{(CES)}. \end{cases}$$

Then (D.2) is satisfied, after canceling certain constants, if and only if

$$(T^1 + T^2)\phi\big(m(T^1 + T^2, H^{nm,ne})\big) \geq T^1\phi\big(m(T^1, H^{nm,ne})\big) + T^2\phi\big(m(T^2, H^{nm,ne})\big),$$

where $\phi(\cdot)$ is defined as in D.1, and $m(\cdot)$ denotes the markup fitting-in function for the MNL or CES, as appropriate. This expression is equivalent to:

$$\sum_{i \in \{1,2\}} T^i\big[\phi\big(m(T^i, H^{nm,ne})\big) - \phi\big(m(T^1 + T^2, H^{nm,ne})\big)\big] \leq 0,$$

which is an impossibility. The function $\phi(\cdot)$ is decreasing for all possible markup values for both the MNL and CES cases according to Lemma D.2. Furthermore, for all i, $m(T^1 + T^2) > m(T^i)$, since Nocke and Schutz (2018, Proposition 6) implies that markups are increasing in type for fixed H. Therefore, the sum above is component-wise strictly positive, which is a contradiction. $\square$

D.2 Proof of Proposition 2

Proof. The proof mirrors that for Proposition 1, but within a nest. Suppose, for purposes of contradiction, there exists an SPE in which firms 1 and 2 merge, and consumers are un-harmed. Thus the merger must increase joint profits:

$$\pi^M(T^1 + T^2, H_g^{m,e}) \geq \pi^1(T^1, H_g^{nm,ne}) + \pi^2(T^2, H_g^{nm,ne}),$$

where $H_g^{nm,ne}$ denotes the nest-level aggregator with no merger and no entry, while $H_g^{m,e}$ is the same object but for a merger with entry. The products in all other nests remain the same, meaning that the resulting overall aggregator is a function of activity from nest g, so we have dropped H in order to save on notation.

By hypothesis, consumers are unharmed, hence we have $H_g^{m,e} \geq H_g^{nm,ne}$. Furthermore, profits are decreasing in H_g according to Nocke and Schutz (2018, Proposition 6), extended to NMNL and NCES in their Appendix (pp. 104-106). Therefore, we have

$$\pi^M(T^1 + T^2, H_g^{nm,ne}) \geq \pi^1(T^1, H_g^{nm,ne}) + \pi^2(T^2, H_g^{nm,ne}). \tag{D.3}$$

Multiplying the markup and firm share shows that firm profit is given by

$$\pi^f \equiv \begin{cases} \frac{1-\rho}{\alpha}\mu^f \frac{T^f}{H_g} \exp(-\mu^f)\bar{s}_g & \text{(NMNL)} \\ \frac{1}{\sigma}\mu^f \frac{T^f}{H_g^{\frac{\gamma-\sigma}{1-\sigma}} H} \left(1 - \frac{\mu^f}{\sigma}\right)^{\sigma-1} & \text{(NCES)}. \end{cases}$$

Substituting for profit in the inequality expression D.3 with $\phi(\cdot)$ from (D.1) and canceling gives the condition

$$(T^1 + T^2)\phi\big(m(T^1 + T^2, H_g^{nm,ne})\big) \geq T^1\phi\big(m(T^1, H_g^{nm,ne})\big) + T^2\phi\big(m(T^2, H_g^{nm,ne})\big),$$

where $m(\cdot)$ denotes the markup fitting-in function for the NMNL or NCES, as appropriate. The profit inequality in (D.3) is satisfied if and only if this condition holds. Note that this condition is analogous to that in the non-nested proof for Proposition 1. Markups are also increasing in type, all else equal (again referencing Nocke and Schutz (2018, Proposition 6)). Thus, we also arrive at a contradiction in the nested case as well. □

D.3 Proof of Lemma 1

Traditionally, the continuity of a fixed point as a function of some set of parameters is established via an appeal to an appropriate form of the implicit function theorem. However, this requires one to consider parameters on the interior of their domain, whereas here we wish to establish continuity precisely on the boundary. Thus we instead employ an approach dating back to Mas-Colell (1974) utilizing a generalization of the implicit function theorem known as the regular value theorem (see Hirsch (2012), Theorem 1.4.1) which remains valid for problems on the boundary.

D.3.1 NMNL Preliminaries

We will prove the Lemma 1 by first establishing two technical lemmas. Define:[25]

$$\Omega_g(H, H_g; \rho) = \frac{1}{H_g} \sum_{f \in \mathcal{F}_g} \sum_{j \in \mathcal{J}^f} \exp\left[\frac{\delta_j - \alpha c_j}{1 - \rho} - \tilde{m}^f\left(\frac{\rho}{H_g} + (1 - \rho)\frac{1}{H_g^\rho H} ; \rho \right) \right].$$

where the the function $\tilde{m}^f(X; \rho)$ defined as the solution in μ^f, for fixed ρ to:

$$\frac{\mu^f - 1}{\mu^f} \frac{1}{T^f \exp\left(-\mu^f\right)} = X. \tag{D.4}$$

Let: $\Omega : \mathbb{R}_{++}^{G+1} \times [0, 1) \to \mathbb{R}^{G+1}$ via:

$$\Omega\big((H_g)_{g \in \mathcal{G}}, H; \rho\big) = \begin{bmatrix} \Omega^1(H_1, H; \rho) - 1 \\ \vdots \\ \Omega^G(H_G, H; \rho) - 1 \\ 1 + \sum_{g \in \mathcal{G}} H_g^{1-\rho} - H \end{bmatrix}.$$

The set of equilibria, treating ρ as a free parameter, are precisely the solutions to:

$$\Omega\big((H_g)_{g \in \mathcal{G}}, H; \rho\big) = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{D.5}$$

---

[25] See equation (xxxi) in Nocke and Schutz (2018) Appendix (p. 70) for reference.

The differential of Ω, evaluated at a solution to (D.5), is of the form:

$$D\Omega((H_g)_{g\in\mathcal{G}}, H; \rho) = \left(\begin{array}{ccc|c|c} & \Lambda & & \Theta & * \\ \hline (1-\rho)H_1^{-\rho} & \cdots & (1-\rho)H_G^{-\rho} & -1 & -\sum_{g\in\mathcal{G}} H_g^{1-\rho}\ln H_g \end{array}\right) \quad \text{(D.6)}$$

where Λ is a $G \times G$ diagonal matrix with:

$$\Lambda_{gg} = \frac{1}{H_g}\left(\frac{\rho}{H_g} + \frac{\rho(1-\rho)}{H_g^\rho H}\right)B_g - \frac{1}{H_g},$$

and Θ is the $G \times 1$ matrix with:

$$\Theta_g = \frac{\partial\Omega_g}{\partial H} = \frac{(1-\rho)}{H_g^\rho H^2}B_g,$$

where the expression B_g is given by:

$$B_g = \frac{1}{H_g}\sum_{f\in\mathcal{F}_g}\sum_{j\in\mathcal{J}^f}\exp\left[\frac{\delta_j - \alpha c_j}{1-\rho} - \tilde{m}^f\left(\frac{\rho}{H_g} + (1-\rho)\frac{1}{H_g^\rho H}; \rho\right)\right]\tilde{m}^{f\prime}\left(\frac{\rho}{H_g} + \frac{(1-\rho)}{H_g^\rho H}\right).$$

We now turn to our first technical lemma.

Lemma D.3. *For some $\varepsilon > 0$, the differential $D\Omega$, evaluated at any solution to (D.5) with $\rho \in [0, \varepsilon)$, is of rank $G + 1$.*

Proof. We break down the proof into steps.

1. **Rank at least G**: Firstly, by direct observation, the upper-left $G \times G$ block Λ is diagonal. Moreover, each diagonal element is strictly negative (see Nocke and Schutz (2018) Online Appendix, Lemma XXIII proof). Hence the first G columns of $D\Omega$ are linearly independent, evaluated at any solution to (D.5).

2. **Removal of Nuisance Terms**: Suppose we evaluate $D\Omega$ at the unique solution to (D.5) with $\rho = 0$. Then, in particular:

$$\Lambda_{gg}\big|_{\rho=0} = -\frac{1}{H_g},$$

   and

$$\Theta_g\big|_{\rho=0} = \frac{1}{H^2}B_g\big|_{\rho=0}.$$

3. **Contradiction Hypothesis**: Suppose, for sake of contradiction, that the $G+1$st column of $D\Omega$ evaluated at the unique solution to (D.5) where $\rho = 0$ is a linear combination of

49

the first G columns. Then there exist $(a_g)_{g=1}^{G}$ such that:

$$(\forall g) \quad \Lambda_{gg}|_{\rho=0} a_g = \Theta_g|_{\rho=0},$$

and which satisfy:

$$\sum_{g=1}^{G} a_g = -1. \tag{$*$}$$

Using the results of the preceding step, we can back out these weights:

$$(\forall g) \quad a_g = -\frac{H_g}{H^2} B_g|_{\rho=0}.$$

4. **Algebra**: Then, plugging in to ($*$), we obtain:

$$\sum_{g \in \mathcal{G}} \sum_{f \in \mathcal{F}_g} \sum_{j \in \mathcal{J}^f} \exp\left[\delta_j - \alpha c_j - \tilde{m}^f(1/H)\right] \tilde{m}^{f'}(1/H) = H^2.$$

Since we're at an equilibrium (i.e. a solution to (D.5)) we can simplify this using the usual system of equations that hold in an equilibrium. In particular:

$$\sum_{g \in \mathcal{G}} \sum_{f \in \mathcal{F}_g} T^f \exp\left(-\mu^f\right) \tilde{m}^{f'}(1/H) = H^2.$$

5. **Dealing with $\tilde{m}^{f'}$**: Recall $\tilde{m}^f$ is the implicit solution to (D.4). In particular,

$$\frac{d\tilde{m}^f}{dX} = \frac{T^f \tilde{m}^f \exp\left(-\tilde{m}^f\right)}{1 - XT^f\left[\exp\left(-\tilde{m}^f\right) - \tilde{m}^f \exp\left(-\tilde{m}^f\right)\right]}.$$

For the H under consideration, let us define $\mu^f = \tilde{m}^f(1/H)$. Then this derivative, evaluated at $X = 1/H$, is:

$$\frac{T^f \mu^f \exp\left(-\mu^f\right)}{1 - \frac{1}{H}T^f\left[\exp\left(-\mu^f\right) - \mu^f \exp\left(-\mu^f\right)\right]}.$$

Now, as we are working at an equilibrium, it must be the case that $T^f \mu^f \exp\left(-\mu^f\right) = H(\mu^f - 1)$, hence our expression for the derivative at $1/H$ may be simplified to:

$$\frac{H\mu^f(\mu^f - 1)}{1 + \mu^f(\mu^f - 1)}.$$

6. **Simplifying** Plugging in the result of Step 5 into that of Step 4 and dividing both sides

by H yields:

$$\sum_{g \in \mathcal{G}} \sum_{f \in \mathcal{F}_g} T^f \exp\left(-\mu^f\right) \left[\frac{\mu^f(\mu^f - 1)}{1 + H\mu^f(\mu^f - 1)}\right] = H.$$

Note that the square bracketed term lies strictly within $[0, 1)$ for all $\mu > 1$. Thus:

$$\sum_{g \in \mathcal{G}} \sum_{f \in \mathcal{F}_g} T^f \exp\left(-\mu^f\right) \left[\frac{\mu^f(\mu^f - 1)}{1 + \mu^f(\mu^f - 1)}\right] < \sum_{g \in \mathcal{G}} \sum_{f \in \mathcal{F}_g} T^f \exp\left(-\mu^f\right)$$

$$= \sum_{g \in \mathcal{G}} \sum_{f \in \mathcal{F}_g} H_g s^{f|g}$$

$$= \sum_{g \in \mathcal{G}} H_g \sum_{f \in \mathcal{F}_g} s^{f|g}$$

$$= \sum_{g \in \mathcal{G}} H_g$$

$$< 1 + \sum_{g \in \mathcal{G}} H_g$$

$$= H.$$

Thus $(*)$ can never hold for any (a_g), and the first $G + 1$ columns of $D\Omega$, at the solution to (21) where $\rho = 0$, are linearly independent. By continuity of these terms in ρ, the same must be true for some small enough open set of ρ's containing 0, and the result follows.

$\square$

We now establish the following immediate corollary:

Lemma D.4. *Let $\hat{\Omega} : \mathbb{R}_{++}^{G+1} \to \mathbb{R}^{G+1}$ denote the restriction of Ω to the (relatively) open set $\mathbb{R}_{++}^{G+1} \times \{0\}$. Then $D\hat{\Omega}$ is of full rank at the unique solution to (D.5) in this domain.*

Proof. By direct calculation:

$$D\hat{\Omega} = \left(\begin{array}{ccc|c} & \hat{\Lambda} & & \hat{\Theta} \\ \hline 1 & \cdots & 1 & -1 \end{array}\right)$$

where $\hat{\Lambda}_{gg} = -1/H_g$ and $\hat{\Theta}_g = (1/H^2)B_g|_{\rho=0}$, hence an identical argument to the prior lemma yields the result. $\square$

D.3.2 NCES Preliminaries

For NCES we define the function $\tilde{m}^f(X; \sigma)$ as the solution in μ^f, for fixed σ to:

$$\frac{\mu^f - 1}{\mu^f} \frac{1}{T^f\left(1 - \frac{\mu^f}{\sigma}\right)^{\sigma-1}} = X. \tag{D.7}$$

Define:

$$\Omega_g(H, H_g; \sigma) = \frac{1}{H_g} \sum_{f \in \mathbb{F}_g} \sum_{j \in \mathbb{J}^f} \delta_j c_j^{1-\sigma} \left[1 - \frac{1}{\sigma} \tilde{m}^f \left(\frac{\gamma - \sigma}{\sigma} \frac{1}{H_g} + \frac{\gamma - 1}{\sigma} \frac{1}{H_g^{\frac{\sigma-\gamma}{\sigma-1}} H}\right)\right]^{\sigma-1}. \tag{D.8}$$

The solutions to (D.8) are equivalent to solving $\Omega_g(H_g, H; \sigma) = 1$. Let: $\Omega : \mathbb{R}_{++}^{G+1} \times [\gamma, \infty) \to \mathbb{R}^{G+1}$ via:

$$\Omega\big((H_g)_{g \in \mathbb{G}}, H; \sigma\big) = \begin{bmatrix} \Omega^1(H_1, H; \sigma) - 1 \\ \vdots \\ \Omega^G(H_G, H; \sigma) - 1 \\ \sum_{g \in \mathbb{G}} H_g^{\frac{\gamma-1}{\sigma-1}} - H \end{bmatrix}.$$

The set of equilibria, treating σ as a free parameter, are precisely the solutions to:

$$\Omega\big((H_g)_{g \in \mathbb{G}}, H; \sigma\big) = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{D.9}$$

The differential of Ω is of the form:

$$D\Omega\big((H_g)_{g \in \mathbb{G}}, H; \sigma\big) = \left(\begin{array}{ccc|c|c} & & & \Theta & * \\ & \Lambda & & & \\ & & & & \\ \hline \frac{\gamma-1}{\sigma-1} H_1^{\frac{\gamma-\sigma}{\sigma-1}} & \cdots & \frac{\gamma-1}{\sigma-1} H_G^{\frac{\gamma-\sigma}{\sigma-1}} & -1 & * \end{array} \right) \tag{D.10}$$

where Λ is a $G \times G$ diagonal matrix with:

$$\Lambda_{gg} = \frac{-1}{H_g} + \frac{1-\sigma}{\sigma} \left[-\frac{\gamma-\sigma}{\sigma} \frac{1}{H_g^2} + \frac{\gamma-1}{\sigma} \frac{\gamma-\sigma}{\sigma-1} \frac{1}{H H_g^{\frac{1-\gamma+2\sigma}{\sigma-1}}} \right] B_g \tag{D.11}$$

at any solution to (D.9), where:

$$B_g = \frac{1}{H_g} \sum_{f \in \mathbb{F}_g} \sum_{j \in \mathbb{J}^f} \delta_j c_j^{1-\sigma} \left[1 - \frac{1}{\sigma} \tilde{m}^f \left(\frac{\gamma-\sigma}{\sigma} \frac{1}{H_g} + \frac{\gamma-1}{\sigma} \frac{1}{H_g^{\frac{\sigma-\gamma}{\sigma-1}} H}\right)\right]^{\sigma-2} \tilde{m}^{f\prime} \left(\frac{\gamma-\sigma}{\sigma} \frac{1}{H_g} + \frac{\gamma-1}{\sigma} \frac{1}{H_g^{\frac{\sigma-\gamma}{\sigma-1}} H}\right) \tag{D.12}$$

and Θ is a $G \times 1$ matrix with:

$$\Theta_g = \frac{\partial \Omega_g}{\partial H} = -\frac{1-\sigma}{\sigma} \frac{\gamma-1}{\sigma} \frac{1}{H_g^{\frac{\sigma-\gamma}{\sigma-1}} H^2} B_g. \tag{D.13}$$

Lemma D.5. *For some $\varepsilon > 0$, the differential $D\Omega$, evaluated at any solution to* (D.9) *with $\sigma \in [\gamma, \gamma + \varepsilon)$ is of rank $G + 1$.*

Proof. We again break down the proof into steps.

1. **Rank at least G**: Firstly, by direct observation, the upper-left $G \times G$ block Λ is diagonal. Moreover, each diagonal element is strictly negative (see Nocke and Schutz (2018) Online Appendix, Lemma XXIII proof). Hence the first G columns of $D\Omega$ are linearly independent, evaluated at a solution to (D.9).

2. **Removal of Nuisance Terms**: Suppose we evaluate $D\Omega$ at the unique solution to (D.9) with $\sigma = \gamma$. Then (D.11) becomes:

$$\Lambda_{gg}\big|_{\sigma=\gamma} = -\frac{1}{H_g}$$

and (D.13),

$$\Theta_g\big|_{\sigma=\gamma} = \frac{(\gamma-1)^2}{\gamma^2} \frac{1}{H^2} B_g\big|_{\sigma=\gamma}.$$

3. **Contradiction Hypothesis**: Suppose, for sake of contradiction, that the $G+1$st column of $D\Omega$ is a linear combination of the first G columns when evaluated at the unique solution with $\sigma = \gamma$. Since Λ is diagonal, this means that there exist real numbers $\{a_g\}_{g \in \mathbb{G}}$ such that $a_g \Lambda_{gg}\big|_{\sigma=\gamma} = \Theta_g\big|_{\sigma=\gamma}$ (from the first G rows), and $\sum_g a_g = -1$ (the $G+1$st row). From these equations we can solve for a_g:

$$\begin{aligned} a_g &= \frac{\Theta_g\big|_{\sigma=\gamma}}{\Lambda_{gg}\big|_{\sigma=\gamma}} \\ &= -\frac{(\gamma-1)^2}{\gamma^2} \frac{H_g}{H^2} B_g\big|_{\sigma=\gamma}. \end{aligned} \tag{D.14}$$

4. **Algebra**: Plugging in (D.14) for the contradiction hypothesis that $\sum_g a_g = -1$, we obtain:

$$\frac{(\gamma-1)^2}{\gamma^2} \sum_{g \in \mathbb{G}} \sum_{f \in \mathbb{F}_g} \sum_{j \in \mathbb{J}^f} \delta_j c_j^{1-\sigma} \left[1 - \frac{1}{\gamma} \tilde{m}^f \left(\frac{\gamma-1}{\gamma} \frac{1}{H} \right) \right]^{\sigma-2} \tilde{m}^{f'} \left(\frac{\gamma-1}{\gamma} \frac{1}{H} \right) = H^2. \tag{$*$}$$

5. **Dealing with $\tilde{m}^{f'}$**: Consider the $\tilde{m}^{f'}$ term now. We know $\tilde{m}^f(X)$ is the solution (in

μ^f) to (D.7). Thus, by direct computation:

$$\frac{d\tilde{m}^f}{dX} = \frac{\tilde{m}^f T^f \left(1 - \frac{\tilde{m}^f}{\gamma}\right)^{\gamma-1}}{1 - XT^f\left[\left(1 - \frac{\tilde{m}^f}{\gamma}\right)^{\gamma-1} - \tilde{m}^f\frac{\gamma-1}{\gamma}\left(1 - \frac{\tilde{m}^f}{\gamma}\right)^{\gamma-2}\right]}. \tag{D.15}$$

Considering some fixed solution to (D.9) at $\sigma = \gamma$, define $\mu^f = \tilde{m}^f\left(((\gamma-1)/\gamma)(1/H)\right)$, and let $X = ((\gamma-1)/\gamma)(1/H)$. Then (D.15) becomes:

$$\frac{\mu^f T^f \left(1 - \frac{\mu^f}{\gamma}\right)^{\gamma-1}}{1 - \frac{\gamma-1}{\gamma}\frac{1}{H}T^f\left[\left(1 - \frac{\mu^f}{\gamma}\right)^{\gamma-1} - \mu^f\frac{\gamma-1}{\gamma}\left(1 - \frac{\mu^f}{\gamma}\right)^{\gamma-2}\right]}$$

which, given we are at a solution to (D.9), simplifies to:

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{H(\mu^f - 1)}{1 - (\mu^f - 1)\left[\frac{1}{\mu^f} - \frac{\gamma-1}{\gamma}\frac{1}{1-\mu^f/\gamma}\right]}.$$

6. Simplifying: Plugging in to $(*)$ we obtain:

$$\sum_{g\in\mathbb{G}}\sum_{f\in\mathbb{F}_g}\sum_{j\in\mathbb{J}^f}\delta_j c_j^{1-\sigma}\left[1 - \frac{\mu^f}{\gamma}\right]^{\sigma-2}\frac{(\gamma-1)}{\gamma}\frac{(\mu^f - 1)}{1 - (\mu^f - 1)\left[\frac{1}{\mu^f} - \frac{\gamma-1}{\gamma}\frac{1}{1-\mu^f/\gamma}\right]} = H. \tag{D.16}$$

Simplifying yields:

$$\sum_{g\in\mathbb{G}}\sum_{f\in\mathbb{F}_g}T^f\left[1 - \frac{\mu^f}{\gamma}\right]^{\sigma-1}\underbrace{\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{1}{1-\frac{\mu^f}{\gamma}}\right)\frac{(\mu^f - 1)}{1 - (\mu^f - 1)\left[\frac{1}{\mu^f} - \frac{\gamma-1}{\gamma}\frac{1}{1-\mu^f/\gamma}\right]}}_{\equiv \chi^f} = H. \tag{D.17}$$

By Lemma D.1, in any solution $\mu^f \in [1, \gamma)$ and hence for all g and all $f \in \mathcal{F}_g$, χ^f lives within $[0, 1)$. Thus, considering the left-hand side of (D.17):

$$\sum_{g\in\mathbb{G}}\sum_{f\in\mathbb{F}_g}T^f\left[1 - \frac{\mu^f}{\gamma}\right]^{\sigma-1}\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{1}{1-\frac{\mu^f}{\gamma}}\right)\frac{(\mu^f - 1)}{1 - (\mu^f - 1)\left[\frac{1}{\mu^f} - \frac{\gamma-1}{\gamma}\frac{1}{1-\mu^f/\gamma}\right]}$$

$$< \sum_{g\in\mathbb{G}}\sum_{f\in\mathbb{F}_g}T^f\left[1 - \frac{\mu^f}{\gamma}\right]^{\sigma-1}$$

$$= \sum_{g\in\mathbb{G}}\sum_{f\in\mathbb{F}_g}H_g s^{f|g}$$

$$= \sum_{g\in\mathbb{G}}H_g$$

$$= H,$$

a contradiction of (D.17). Thus the Jacobian $D\Omega$, evaluated at any solution to (D.9) with $\sigma = \gamma$, is of full rank.

$\square$

Lemma D.6. *Let $\hat{\Omega} : \mathbb{R}_{++}^{G+1} \to \mathbb{R}^{G+1}$ denote the restriction of Ω to the (relatively) open set $\mathbb{R}_{++}^{G+1} \times \{\gamma\}$. Then $D\hat{\Omega}$ is of full rank at the unique solution to (D.9) in this domain.*

Proof. By direct calculation:

$$
D\hat{\Omega} = \left(\begin{array}{ccc|c}
 & \hat{\Lambda} & & \hat{\Theta} \\
\hline
1 & \cdots & 1 & -1
\end{array} \right)
$$

where $\hat{\Lambda}_{gg} = -1/H_g$ and $\hat{\Theta}_g = ((\gamma-1)^2/\gamma^2)(1/H^2)B_g\big|_{\sigma=\gamma}$, hence an identical argument to the prior lemma yields the result. $\square$

D.3.3 Proof of Lemma 1

Proof. We state the proof for the NMNL case; the NCES case follows, *mutatis mutandis.* Let $\varepsilon > 0$ be any such value such that the conclusions of Lemmas D.3 and D.4 hold, and by abuse of notation, denote the restriction of Ω to $\mathbb{R}_{++}^{G+1} \times [0, \varepsilon')$ for any $0 < \varepsilon' < \varepsilon$ simply by Ω. By Lemma D.3, 0 is a regular value of Ω on this domain, and by Lemma D.4, 0 is also a regular value of Ω restricted to the boundary of this domain. Thus by the Regular Value Theorem (see Hirsch (2012) Theorem 1.4.1, see also Mas-Colell (1974) Theorem 2), $\Omega^{-1}(0)$ is a C^1 submanifold of $\mathbb{R}_{++}^{G+1} \times [0, \varepsilon')$, with boundary precisely equal to the unique equilibrium at $\rho = 0$. Consider the (necessarily unique) connected component of $\Omega^{-1}(0)$ that intersects $\mathbb{R}_{++}^{G+1} \times \{0\}$. Since this component is a connected C^1 manifold with boundary, it is C^1-diffeomorphic to $[0,1)$ (Hirsch (2012) Exercise 1.5.9).[26] Since the Regular Value Theorem guarantees its intersection with the slice $\mathbb{R}_{++}^{G+1} \times \{0\}$ is transverse, the restriction of this component to $\mathbb{R}_{++}^{G+1} \times [0, \varepsilon'']$ for some $0 < \varepsilon'' < \varepsilon'$ is diffeomorphic to $[0,1]$, and hence is compact.

However, $\Omega^{-1}(0)|_{\mathbb{R}_{++}^{G+1} \times [0, \varepsilon'']}$ is also the graph of the function $e : [0, \varepsilon''] \to \mathbb{R}_{++}^{G+1}$ that takes a nesting parameter value and maps it to the unique equilibrium of the associated differentiated Bertrand-Nash pricing game. By the preceding argument, we may without loss restrict the codomain of e to be some compacta $K \subseteq \mathbb{R}_{++}^{G+1}$ such that (i) $K \times [0, \varepsilon'']$ contains $\Omega^{-1}(0)|_{\mathbb{R}_{++}^{G+1} \times [0, \varepsilon'']}$, and (ii) the graph of e is a closed subset of $K \times [0, \varepsilon'']$.[27] But then by the

---

[26]It cannot be diffeomorphic to $[0,1]$ as from the Regular Value theorem, its boundary is given precisely by its intersection with the boundary of the domain, and at $\rho = 0$ the equilibrium is unique.

[27]It suffices to let K be the projection of $\Omega^{-1}(0)|_{\mathbb{R}_{++}^{G+1} \times [0, \varepsilon'']}$ onto $\mathbb{R}_{++}^{G+1}$ to satisfy both these properties. In particular, this set is compact by continuity of the projection, and the graph of e is closed in $\mathbb{R}_{++}^{G+1} \times [0, \varepsilon'']$ hence it is closed in the subspace topology on $K \times [0, \varepsilon"]$.

Closed Graph Theorem (Aliprantis and Border (2006) Theorem 2.58), this map is continuous on $[0, \varepsilon'']$. $\qquad \square$

D.4 Proof of Proposition 3

Proof. We know from Proposition 1 that in the MNL and CES cases, if consumer welfare remains unchanged after a merger, then the profits of the merging firms must fall. Thus, in the NMNL (resp. NCES) model, for $\rho = 0$ (resp. $\sigma = \gamma$), if consumer surplus is unhurt then:

$$\pi^M(T^1 + T^2, H_g^{m,e}, H) < \pi^1(T^1, H_g^{nm,ne}, H) + \pi^2(T^2, H_g^{nm,ne}, H)$$

Suppose then we consider a sequence of nesting parameter values $(\rho_n)_{n \in \mathbb{N}}$ such that $\rho_n \to 0$ (resp. $(\sigma_n)_{n \in \mathbb{N}}$ such that $\sigma_n \to \gamma$). By Lemma 1, and the continuous dependence of profits and markups on the underlying equilibrium variables $(H_g)_{g \in \mathcal{G}}$ and H, we obtain a sequence of profits for the individual merging parties and the merged entity which converge to their $\rho = 0$ values as $\rho_n \to 0$ (resp. $\sigma = \gamma$ values as $\sigma_n \to \gamma$). In the NMNL model, for n large enough it then must be the case that:

$$\pi^M(T^1 + T^2, H_g^{m,e}(\rho_n), H(\rho_n)) < \pi^1(T^1, H_g^{nm,ne}(\rho_n), H(\rho_n)) + \pi^2(T^2, H_g^{nm,ne}(\rho_n), H(\rho_n)),$$

establishing the result. The sequence of profits would generate an analogous inequality in the NCES model. $\qquad \square$

D.5 Proof of Proposition 4

Proof. Begin by rearranging (10) to solve for firm type, giving

$$T^f = \begin{cases} H s^f \exp\left(\frac{1}{1-s^f}\right) & \text{(MNL)} \\ H s^f (\sigma - 1)^{1-\sigma} \left(\sigma + \frac{s^f}{1-s^f}\right)^{\sigma-1} & \text{(CES)} \end{cases} \tag{D.18}$$

after substituting in for markups. Then evaluate this type equation for firm F after the merger and firms 1 and 2 before the merger, substituting in for the entrant share using $s^F = s^1 + s^2 - s^M$, which obtains from Lemma C.1. Dividing the result for firm F by the sum of the results for firms 1 and 2 gives (20) and (22) for MNL and CES, respectively.

Without efficiencies, $T^M = T^1 + T^2$. Substituting into this sum for types using (D.18) gives (21) and (23). These two expressions have unique positive solutions because the expressions $x \exp(1/(1-x))$ and $x(\sigma + x/(1-x))^{\sigma-1}$ are increasing if $x \in [0, 1)$.

To establish the final claim, let s^a be the average of s^1 and s^2, calculated as $(s^1 + s^2)/2$. Let T^a be the type that generates a share of s^a given aggregator H, which can be found by solving (10) holding H fixed. (Note that if $s^1 \geq s^2$, then $s^1 \geq s^a \geq s^2$ and $T^1 \geq T^a \geq T^2$, the latter due to the monotonicity of shares in terms of T^f/H.) We wish to characterize the relationship between the entrant's type T^F and this "average" type T^a. In order for consumers

to be unharmed, H must be unchanged due to the merger. Therefore, since $T^M > T^1$ and $T^M > T^2$, $T^M/H > T^1/H$ and $T^M/H > T^2/H$. In turn, this means that $s^M > s^1$ and $s^M > s^2$, since shares are increasing in T^f/H. Adding these inequalities gives $2s^M > s^1 + s^2$, and then dividing by two gives $s^M > s^a$. As shown by Lemma C.1, if H remains the same, then $s^F + s^M = s^1 + s^2$, which also means that $s^F + s^M = 2s^a$. In order for this equality to hold when we also know that $s^M > s^a$, it must be that $s^F < s^a$. By the monotonicity of shares, this means that $T^F < T^a$. $\qquad\square$

D.6 Proof of Proposition 5

Proof. Let the identical pre-merger types of firm 1 and firm 2 be denoted by T, meaning that $T^M = 2T$. If markups for the merging firms are constant before and after the merger, then the monotonicity of markups in T^f/H (Nocke and Schutz (2018, Proposition 6)), means that the post-merger aggregator must be twice the pre-merger aggregator (the latter denoted by H), in order for the merging firm's ratio of type to aggregator to remain constant at T/H. By (10) and (12), shares and profits are only functions of T^f/H, markups, and fixed parameters, which establishes that $s^M = s^1 = s^2$ and $\pi^M = \pi^1 = \pi^2$ when the merging firms do not change their markups post-merger.

In turn, the post-merger share of the outside good drops from $1/H$ to $1/2H$, and the shares of all non-merging incumbents fall because their ratio of type to aggregator decreases from T^f/H to $T^f/2H$. We also know that the merger has caused two firms, 1 and 2, to be replaced by one firm, M, that is only half their combined size in terms of share. Therefore, by the adding up constraint in (11), we know that the share of the entrant must be large enough to compensate for the loss of one firm of size s^M and for the shrinking of all the other incumbent firms. Thus, $s^F > s^M$. $\qquad\square$

D.7 Proof of Proposition 7

Proof. Let $(\underline{E}, T^F)$ be such that (i) the merger is profit-neutral and (ii) consumer surplus is unchanged due to the merger. From (ii), we know that the aggregator is constant at some level H. From (D.18), we also have

$$T^M = T^1 + T^2 + \underline{E} = \begin{cases} Hs^M \exp\left(\frac{1}{1-s^M}\right) & \text{(MNL)} \\ Hs^M(\sigma-1)^{1-\sigma}\left(\sigma + \frac{s^M}{1-s^M}\right)^{\sigma-1} & \text{(CES).} \end{cases}$$

Plugging in for T^1 and T^2 again using (D.18) and solving for $\underline{E}$ yields (24) and (26). From (i), we obtain (25) and (27). We derive these expressions by evaluating the profit functions in (12) for the merged firms before and after the merger, plugging into $\pi^M = \pi^1 + \pi^2$, and substituting in for markups using (A.3) and (A.7), for MNL and CES, respectively (see Proposition C.2, which works out the MNL case in more detail). $\qquad\square$

D.8 Proof of Lemma C.1

Proof. **(i)** $\implies$ **(ii)**: Suppose (i) holds, that is:

$$\pi_{nm,ne}^f = \pi_*^f$$

By (12), $\mu_{nm,ne}^f = \mu_*^f$, and by (A.3), $s_{nm,ne}^f = s_*^f$. Because $T^f = T_{nm,ne}^f = T_*^f$ by hypothesis, (10) implies:

$$\frac{T^f}{H_{nm,ne}} = s_{nm,ne}^f \exp\left(\frac{1}{1 - s_{nm,ne}^f}\right) = s_*^f \exp\left(\frac{1}{1 - s_*^f}\right) = \frac{T^f}{H_*}$$

and thus $H_{nm} = H_m$, which implies (ii).

(ii) $\implies$ **(i)**: Suppose now that $H_{nm,ne} = H_* = H$. By (10), we obtain $s_{nm,ne}^f = s_*^f$ for every $f \in \mathcal{F}_{nm,ne}$ immediately, and (i) follows by a chain of substitutions identical to the above.

(ii) $\implies$ **(iii)**: Suppose now that $H_{nm,ne} = H_* = H$. From (11):

$$\frac{1}{H} + \sum_{f \in \mathcal{F}_{nm,ne}} s_{nm,ne}^f = \frac{1}{H} + \sum_{f \in \mathcal{F}_*} s_*^f \iff \sum_{f \in \mathcal{F}_{nm,ne}} s_{nm,ne}^f = \sum_{f \in \mathcal{F}_*} s_m^*,$$

which implies (iii) immediately upon cancelling terms (via appeal to (ii) implying (i) and hence to the shares also coinciding across scenarios).

(iii) $\implies$ **(ii)**: We proceed by contraposition. Thus suppose that the merger affects consumer surplus: $H_{nm,ne} \neq H_*$. Let f belong to both $\mathcal{F}_{nm,ne}$ and $\mathcal{F}_*$, i.e. let f denote any firm other than $1, 2, M$ or potentially F. By (10), we have:

$$\frac{T^f}{H_{nm,ne}} = s_{nm,ne}^f \exp\left(\frac{1}{1 - s_{nm,ne}^f}\right)$$

and

$$\frac{T^f}{H_*} = s_*^f \exp\left(\frac{1}{1 - s_*^f}\right).$$

For both equations, the right-hand side is strictly increasing in the relevant share, and thus for all such f:

$$\frac{1}{H_{nm,ne}} > \frac{1}{H_*} \iff s_{nm,ne}^f > s_*^f.$$

Thus:

$$\frac{1}{H_{nm,ne}} + \sum_{f \in \mathcal{F}_{nm,ne} \cap \mathcal{F}_*} s_{nm,ne}^f \neq \frac{1}{H_*} + \sum_{f \in \mathcal{F}_{nm} \cap \mathcal{F}_*} s_*^f,$$

and it follows by (11) that (iii) cannot hold. $\square$

D.9 Proof of Proposition B.1

For brevity we focus on MNL demand. An analogous proof for CES demand can be provided upon request by the authors. We first show that as the type of any one firm goes to infinity, so too does the market aggregator.

Lemma D.7. *Fix any market structure $\mathcal{F}_*$ and vector of model primitives. For any $f \in \mathcal{F}_*$,*

$$\lim_{T^f \to \infty} H_* = \infty$$

Proof. First note that $\lim_{T^f \to \infty} T^f / H_* = \infty$, as established in the proof of Proposition C.2. Thus, as

$$s^f = \frac{T^f}{H_*} \exp\left[\frac{-1}{1 - s^f}\right],$$

as T^f goes to infinity, s^f goes to one. But as:

$$\frac{1}{H_*} + \sum_{f \in \mathcal{F}_*} s^f = 1,$$

it follows that $H_* \to \infty$. $\qquad\square$

We now prove the proposition.

Proof. Fix an arbitrary market structure $\mathcal{F}_{nm,ne}$ and associated $\mathcal{F}_{m,ne}$. The merger is profitable with delayed or probabilistic entry if and only if:

$$(1 - \theta)\left[\frac{1}{1 - s_{m,ne}^M}\right] + \theta\left[\frac{1}{1 - s_{m,e}^M}\right] \geq \frac{1}{1 - \underline{s}_{m,e}^M} \qquad (D.19)$$

where

$$\underline{s}_{m,e}^M = 1 - \frac{(1 - s^1)(1 - s^2)}{1 - s^1 s^2}.$$

is the market share of the merged firm in a counterfactual with entry that makes the merger exactly neutral for stage-game profit.[28] We obtain (D.19) by substituting in for profit using (12) and (A.3). Note that $f(x) = \frac{1}{1-x}$ is increasing, and as $s_{m,ne}^M > \underline{s}_{m,e}^M$,

$$\frac{1}{1 - s_{m,ne}^M} > \frac{1}{1 - \underline{s}_{m,e}^M}.$$

Define:

$$(1 - \theta^*) \equiv \frac{\frac{1}{1 - \underline{s}_{m,e}^M}}{\frac{1}{1 - s_{m,ne}^M}}.$$

---

[28]We derive the expression for $\underline{s}_{m,e}^M$ in Proposition C.2.

Thus for the choice $\theta = \theta^*$, the profit inequality reduces to:

$$\theta^* \left[\frac{1}{1 - s_{m,e}^M} \right] \geq 0,$$

which always holds strictly. The definition of θ^* does not depend on T^F. Thus, for $\theta = \theta^*$, the type of the entrant does not affect whether the merger is profitable. Assuming that $\theta = \theta^*$, we turn to consumer surplus, which weakly increases if and only if:

$$(1 - \theta^*) \ln H_{m,ne} + \theta^* \ln H_{m,e} \geq \ln H_{nm,ne}$$

The only term in this inequality that depends on T^F is $H_{m,e}$. Furthermore, if we send T^F to infinity, then $H_{m,e}$ also goes to infinity, by Lemma D.7. Therefore, for some large enough T^F, and for $\theta = \theta^*$, the merger is profitable and consumer surplus strictly increases. $\qquad\square$

D.10 Proof of Proposition C.1

Proof. We first show that, for all choices of T^F, there is a unique efficiency E that makes the merger CS-neutral. Fix T^F and suppose that the merger is CS-neutral. Then $H_{nm,ne} = H_{m,e} = H$. Since types are unchanged across market structures, by (10) and (11) it follows that:

$$s_{nm,ne}^1 + s_{nm,ne}^2 = s_{m,e}^F + s_{m,e}^M. \tag{D.20}$$

This establishes claim (iii). Clearly $s_{nm,ne}^1$ and $s_{nm,ne}^2$ do not depend upon E. Moreover, by (9) and (10), $s_{m,e}^F$ depends only on T^F and H, not E. Then, by appeal to (10) and (A.3), the only term in (D.20) that depends on E is pinned down by:

$$\frac{T^1 + T^2 + E}{H} = s_{m,e}^M \exp\left(\frac{1}{1 - s_{m,e}^M} \right), \tag{D.21}$$

the left-hand side of which is strictly increasing in E. However, by (D.20), the right-hand side does not depend on E and hence there can be only one such value for E.

We now establish that the CS-neutrality curve is downward-sloping. To this end, suppose consumer surplus is unchanged across the nm, ne and m, e equilibria, and hence that (D.21) obtains. By an identical argument, for the entrant F:

$$\frac{T^F}{H} = s_{m,e}^F \exp\left(\frac{1}{1 - s_{m,e}^F} \right). \tag{D.22}$$

Suppose T^F is increased. This does not change H, as it is pinned down by its value in the nm, ne equilibrium (which does not depend upon T^F) and our hypothesis of CS neutrality. Then by (D.22), an increase in T^F leads to a higher equilibrium share $s_{m,e}^F$. But by (D.20) this implies a corresponding, equivalent decrease in $s_{m,e}^M$ as $s_{nm,ne}^1$ and $s_{nm,ne}^2$ do not depend upon T^F or E. By (D.21), we then conclude the CS neutrality curve is downward sloping.

Finally, claim (ii) follows immediately from the above, and the definitions of these objects. $\qquad\square$

D.11 Proof of Proposition C.2

Proof. We first establish claim (iii). Suppose that the merger is profit-neutral:

$$\pi^1_{nm,ne} + \pi^2_{nm,ne} = \pi^M_{m,e}.$$

By (12), it follows that:

$$\mu^M_{m,e} + 1 = \mu^1_{nm,ne} + \mu^2_{nm,ne}.$$

By substituting using (A.3) and solving for $s^M_{m,e}$ in terms of $s^1_{nm,ne}$ and $s^2_{nm,ne}$, we obtain:

$$s^M_{m,e} = 1 - \frac{(1 - s^1_{nm,ne})(1 - s^1_{nm,ne})}{1 - s^1_{nm,ne}s^2_{nm,ne}}$$

as desired.

We now show that for all values of T^F, there is a unique efficiency E that makes the merger profit-neutral. Suppose then for some T^F, that there exists some efficiency E is such that the merger is profit-neutral. Then by (10) and (A.3), E satisfies:

$$\frac{T^1 + T^2 + E}{H_{m,e}} = s^M_{m,e} \exp\left(\frac{1}{1 - s^M_{m,e}}\right).$$

However, (iii) implies the right-hand side is constant in E, as it is a function solely of the pre-merger equilibrium quantities $s^1_{nm,ne}$ and $s^2_{nm,ne}$, which do not depend on E. In the Online Appendix (p.110) of Nocke and Schutz (2018), it is shown that for any firm f, T^f/H is increasing in T^f. This implies that if there exists any such E, then it is necessarily unique. To show such an E exists, it suffices to show that the left-hand side (i.e. $T^M/H_{m,e}$) is unbounded above in T^M. Suppose, for sake of contradiction, this is not the case. Then as $T^M/H_{m,e}$ is increasing and bounded above, it converges to some limit $K < \infty$. Since $T^M \to \infty$, this implies $\lim_{T^M \to \infty} H_{m,e} = \infty$ as well. Thus for any $g \in \mathcal{F}_{m,e}$, $g \neq M$, (10) and (A.3) imply that $s^g_{m,e} \to 0$. As g was arbitrary, by (11), $s^M_{m,e} \to 1$ and hence by (10) and (A.3) $T^M/H_{m,e} \to \infty$, a contradiction. Thus $\lim_{T^M \to \infty} T^M/H_{m,e} = \infty$, and in particular, for any such T^F, there exists an E such that the merger is profit-neutral.

We now establish claim (i), that the merger profit-neutrality curve is upward sloping. Suppose that, for T^F, E is such that the merger is profit-neutral. Then, as noted prior, E must satisfy:

$$\frac{T^1 + T^2 + E}{H_{m,e}} = s^M_{m,e} \exp\left(\frac{1}{1 - s^M_{m,e}}\right),$$

where, by (iii), the right-hand side is a constant function in E. Suppose T^F increases. This increases $H_{m,e}$. Since the right-hand side of the above is constant in T^F and $H_{m,e}$, for the

equality to hold, the unique solution in E must increase (given the left-hand side is increasing and unbounded in E).

For (ii), the first claim follows immediately from the definitions of $\bar{T}^F$. For the latter claim, suppose that $E = \bar{E}$, and observe that if $T^F = 0$, then the merger is profitable. Conversely, suppose $T^F \to \infty$. Then, as shown above, $T^F/H_{m,e} \to \infty$ as well. By (10), $s^F_{m,e} \to 1$, and hence $s^M_{m,e}$ and $\pi^M_{m,e} \to 0$. Thus we conclude that as $T^F \to \infty$, the merged entrant's profits monotonically decreases to 0. Since pre-merger, the entrant is not in the market, the pre-merger profits of the merging entities are unaffected by T^F, there exists some T^F for which $(T^F, \bar{E})$ makes the merger profit-neutral; as $\pi^M_{m,e}$ is globally decreasing in T^F, this $\bar{T}^F$ is unique. $\qquad\square$

D.12 Proof of Proposition C.3

Proof. We first establish that, for all choices of $T^F > 0$, there is a unique efficiency E that makes the merger cause the entrant to be profit-neutral. Fix T^F and consider the associated nm, e and m, e equilibria. If the entrant's profits are equal across both equilibria, then by Lemma C.1, $H_{nm,e} = H_{m,e} = H$, and:

$$s^1_{nm,e} + s^2_{nm,e} = s^M_{m,e}.$$

In the m, e equilibrium:

$$\frac{T^1 + T^2 + E}{H} = s^M_{m,e} \exp\left(\frac{1}{1 - s^M_{m,e}}\right),$$

the left-hand side of which is strictly increasing in E. However, the right hand side is injective in $s^M_{m,e}$, and $s^M_{m,e}$ is fixed by the nm, e equilibrium and hence its equilibrium is fixed under the hypothesis of entrant profit-neutrality. Thus there can be only one E satisfying the above.[29]

We consider now claim (i), that the entrant profit neutrality curve is downward sloping. By Lemma C.1, we know that $H_{nm,e} = H_{m,e} = H$ and $s^1_{nm,e} + s^2_{nm,e} = s^M_{m,e}$. In equilibrium:

$$\frac{T^1 + T^2 + E}{H} = s^M_{m,e} \exp\left(\frac{1}{1 - s^M_{m,e}}\right).$$

By Proposition 6 of Nocke and Schutz (2018), an increase in T^F for fixed E leads to a decrease in $s^1_{nm,e}$ and $s^2_{nm,e}$. But this implies a decrease in $s^M_{m,e}$ as it is the sum of these terms. Thus there must be a commensurate decrease in E.

We now establish claim (ii). Consider the following three market structures: $\mathcal{F}_{nm,ne}$, $\mathcal{F}_{nm,e}$, and $\mathcal{F}_{m,e}$. The entry neutrality line is determined by profit-neutrality across $\mathcal{F}_{nm,e}$ and $\mathcal{F}_{m,e}$; the CS neutrality line is determined by surplus remaining constant across $\mathcal{F}_{nm,ne}$ and $\mathcal{F}_{m,e}$. We first claim that if the two curves intersect for some (T^F, E) then $T^F = 0$.

---

[29]Here, as H is fixed by the nm, e equilibrium value, the left hand side is unbounded in E.

By Lemma C.1, $CS_{nm,e} = CS_{m,e}$; by hypothesis, $CS_{m,e} = CS_{nm,ne}$. Hence in particular, $H_{nm,ne} = H_{nm,e} = H$. Then for each $f \in \mathcal{F}_{nm,e} \setminus \{F\}$, we have:

$$s_{nm,e}^f \exp\left(\frac{1}{1 - s_{nm,e}^f}\right) = \frac{T^f}{H} = s_{nm,ne}^f \exp\left(\frac{1}{1 - s_{nm,ne}^f}\right)$$

and hence $s_{nm,e}^f = s_{nm,ne}^f$. By the adding up constraint:

$$\sum_{f \in \mathcal{F}_{nm,ne}} s^f = \sum_{f \in \mathcal{F}_{nm,e}} s^f$$

and thus $s^F = 0$ and hence so too is T^F. Thus consider $T^F \to^+ 0$. If $T^F > 0$, then $CS_{nm,e} > CS_{nm,ne}$, however, $\lim_{T^F \to^+ 0} CS_{nm,e} = CS_{nm,ne}$. Thus as $T^F \to^+ 0$, the associated efficiency tends to $\bar{E}$ by definition.

Suppose now that $T^F > 0$. We will establish that the unique E such that (T^F, E) is entrant profit-neutral must be strictly positive. Suppose, for sake of contradiction, that $E = 0$. Since $T^M > \max\{T^1, T^2\}$, following the merger the markups for the merging firms increase. Given marginal costs remain fixed, the corresponding equilibrium prices increase and hence the effect of the merger on H is an unambiguous decrease. But this implies then $\pi_{m,e}^F > \pi_{nm,e}^F > 0$, a contradiction. By an argument analogous to that appearing in the proof of Proposition C.2, an E such that the merger is profit-neutral for F must exist, thus we conclude $E > 0$.

Finally, claim (iii) follows from Proposition C.1, and the immediate observation that, ceteris paribus, entry increases consumer surplus. $\square$

E   Numerical Methods

In this appendix, we describe how a model of Bertrand competition with MNL demand can be calibrated based on data on market shares, and then simulated to obtain the percentage changes in markups, profit, and consumer surplus due to a merger. The NMNL is analogous if one has knowledge of the nesting parameter. We then detail the data sources and methods that are used in the application to the T-Mobile/Sprint merger that is presented in Section 4.

E.1   Calibration and Simulation

With MNL demand, it is possible to recover types from market shares, and vice-versa. To implement the former—a calibration step—first obtain the market aggregator from (11), and the ι-markups from (A.3). Firm types then are given by a rearranged (10):

$$T^f = \frac{s^f H}{\exp\left(-\mu^f\right)}.$$

To implement the latter—a simulation step—use a nonlinear equation solver to recover the shares and the market aggregator, given a set of types. There are $F + 1$ nonlinear equations that must be solved simultaneously. One of these is the adding-up constraint of (11), and the others are obtained by plugging (A.3) into (10), which yields

$$s^f = \frac{T^f}{H} \exp\left(-\frac{1}{1 - s^f}\right).$$

If one knows the types, and thus also the aggregator, then markups, profit, and consumer surplus are identified up to a multiplicative constant (see (9), (12), and (13)). An implication is that the outcomes that arise with different firm types can be meaningfully compared—the ratio of outcomes is identified because the multiplicative constant cancels.

A full calibration also recovers the multiplicative constant—the price parameter, α. This can be accomplished with data on one margin, for example. See also the Nocke and Schutz (2018) Online Appendix. Then markups, profit, and consumer surplus also are obtained (not just up to a multiplicative constant). However, these objects are not necessary for our purposes, so we use partial calibration.

An observation is that our market shares, $\{s^f\} \ \forall f \in \mathcal{F}$, assign a positive share to the outside good. Thus, they differ from the antitrust market shares described in the US Horizontal Merger Guidelines, which assign zero weight to products that are outside the relevant market.[30] Nonetheless, it is possible to convert antitrust market shares into our market shares using information that often is available during merger review. For example, suppose one has information on the diversion ratio that characterizes substitution from firm k to firm j. Then, in the context of MNL (and CES) we have

$$\frac{\frac{\partial s^j}{\partial p^k}}{\frac{\partial s^k}{\partial p^k}} \equiv DIV_{k \to j} = \frac{s_j}{1 - s_k}. \tag{E.1}$$

Letting the relevant antitrust market comprise the products of firms $f \in \mathcal{F}$, we have

$$\hat{s}^f = \frac{s^f}{1 - s^0} \tag{E.2}$$

where $\hat{s}^f$ is the antitrust market share and s^0 is the outside good share in our context. The system of equations in (E.1) and (E.2) identifies s^0 and $\{s^f\} \ \forall f \in \mathcal{F}$ from data on diversion, $DIV_{k \to j}$, for some $j \neq k$, and the antitrust market shares, $\{\hat{s}^f\} \ \forall f \in \mathcal{F}$.

---

[30]See the US Horizontal Merger Guidelines §5 for a discussion of market shares.

E.2 Application to T-Mobile/Sprint

Our primary source of data is the 2017 Annual Report of the FCC on competition in the mobile wireless sector.[31] We obtain the following information:

- Among national providers, Verizon, AT&T, T-Mobile, and Sprint account for 35.0, 32.4, 17.1, and 14.3 percent of total connections at end-of-year 2016, respectively. See Figure II.B.1 on page 15.

- The average revenue per user (ARPU) in 2016:Q4 for Verizon, AT&T, T-Mobile, and Sprint is 37.52, 36.58, 33.80, 32.03, respectively. See Figure III.A.1 on page 42. Following common practice, we use the ARPU as a measure of price.

- The EBITDA per subscriber in 2016 for Verizon, AT&T, T-Mobile, and Sprint is 22.71, 18.30, 11.80, 13.00, respectively. See Figure II.D.1 on page 24. We interpret the EBITDA as providing the markup.

Finally, we obtain a market elasticity of -0.3 from regulatory filings. The market elasticity is defined theoretically as $\epsilon = -\alpha s_0 \bar{p}$, where $\bar{p}$ is the weighted-average price.

The main distinction between the T-Mobile/Sprint application and our other numerical results is that we do not observe pre-merger market shares. The reason is that the FCC data on total connections does not incorporate the consumer option to purchase the outside good. Thus, we use a full calibration approach in which we use the market elasticity and a markup (we use the T-Mobile ARPU) to recover the outside good share and the price coefficient. We obtain an outside good share of 0.084. With this in hand, the pre-merger market for T-Mobile, for example, is $17.1/(1 - 0.084)$. With the pre-merger market shares, Figure 6 can be created using the methods described above.

---

[31]*Annual Report and Analysis of Competitive Market Conditions With Respect to Mobile Wireless, Including Commercial Mobile Services*, FCC-17-126.