Mergers, Entry, and Consumer Welfare[*]

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April 28, 2020

First Draft: February 7, 2020

Abstract

We analyze mergers and entry in a differentiated products oligopoly model of price competition. We prove that any merger among incumbents is unprofitable if it spurs entry sufficient in magnitude to preserve consumer surplus. Thus, mergers occur in equilibrium only if barriers limit entry. Numerical simulations indicate that with profit-neutral mergers—the best-case for consumers—entry mitigates under 30 percent of the adverse price effects and, in most cases, under 50 percent of the consumer surplus loss. The results suggest a limited and conditional role for entry analysis in merger review.

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1 Introduction

The antitrust review of mergers in the Unites States follows a set of evaluation criteria summarized in the 2010 Horizontal Merger Guidelines of the Department of Justice and Federal Trade Commission. An important step examines entry—specifically, the question of whether entry is easy enough that the merging firms would find it unprofitable to raise prices or otherwise reduce competition. In this framework, the existence of entry that meets the standard of being timely, likely, and sufficient in its magnitude to alleviate consumer harm is grounds for approving a merger.[1]

In this paper, we reexamine mergers and entry in a differentiated products oligopoly model of price competition. We obtain a simple result: post-merger entry is never sufficient to fully mitigate the price increases and consumer surplus loss caused by profitable mergers. The result arises due to a selection effect, namely that entry sufficient to offset adverse competitive effects also renders mergers unprofitable. As firms would typically merge only if it is profitable, the existence of a merger proposal suggests a perception among the merging firms that barriers obstruct entry.[2]

Our analysis is based on a three-stage game of oligopoly in which (1) two market incumbents decide whether to merge, (2) a market outsider decides whether to incur a cost to enter the market, and (3) prices are set according to differentiated products Bertrand competition with logit demand.[3] If the merger decision does not affect the entry decision—which occurs if entry costs are low enough or high enough—then merger is both profitable and harmful for consumer welfare. More interesting is the case of *merger-induced entry*, in which entry is profitable if and only if merger occurs. Thus, our focus is on whether mergers can coexist with merger-induced entry that leaves consumers unharmed relative to a scenario without merger or entry.[4]

To make the analysis tractable, we write the model as an aggregative game, follow-

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[1]See Shapiro (2010) for a useful discussion of the 2010 Horizontal Merger Guidelines. The standard of timely, likely, and sufficient also appears in the 2004 Merger Guidelines of the European Union.

[2]Farrell and Shapiro (1990) develop a similar revealed-preference argument in the context of merger efficiencies with Cournot competition: efficiencies may be possible to infer from the proposal of a merger that otherwise appears unprofitable. Werden (1991) discusses reasons unprofitable mergers might sometimes be observed.

[3]The Bertrand logit has long been a workhorse model for merger review in differentiated products settings (Werden and Froeb (1994, 2002)). It also provides the foundation for seminal academic research based on the more flexible random-coefficients logit model (e.g. Berry et al. (1995); Nevo (2001)). Miller and Sheu (2020) explain why the logit restrictions on consumer substitution often are reasonable in properly defined antitrust markets used for merger review.

[4]The 2010 Horizontal Merger Guidelines, §9, states: "[t]his section concerns entry or adjustments to pre-existing entry plans that are induced by the merger."

ing Nocke and Schutz (2018a). The contribution of each firm to equilibrium is fully determined by its "type," a scalar that captures the quality and marginal cost of all its products. This allows us to characterize outcomes in terms of a simple firm-level primitive, rather than as depending on the marginal costs and qualities of every individual product. The profits of any firm decrease in the types of its competitors. Thus, the profitability of merger depends on the type of the entrant (if any). Further, there exists some critical entrant type that makes the merger profit-neutral.

We refer to the critical entrant type as characterizing *best-case entry* because it provides the greatest possible consumer surplus in a subgame perfect equilibrium (SPE) featuring merger and merger-induced entry. This raises a natural question: can merger followed by best-case entry preserve (or improve upon) pre-merger levels of consumer surplus? Analyzing the structure of the model provides an answer in the negative: merger-induced entry is never sufficient to eliminate the adverse consumer surplus effects of profitable mergers. Restated, there is no SPE in which a merger occurs and consumer surplus does not fall as a result. The result extends to incumbent repositioning, entry by any number of outsiders, or any combination of repositioning and entry—none of these eliminate the adverse effects of profitable mergers.

We use numerical analyses to inform the extent to which best-case entry mitigates the consumer harm caused by mergers. We develop a partial calibration routine with which merger effects can be obtained from market shares.[5] We then consider a range of hypothetical mergers, varying the size of the merging firms, and obtain the type of the best-case entrant in each case. The results suggest that best-case entry never counteracts more than 30 percent of the merging firms' price increase. If there are few incumbents and the merging firms are small, then best-case entry can mitigate nearly 80 percent of consumer surplus loss, mainly by increasing product diversity. Otherwise, the mitigation of consumer surplus loss tends to be well less than 50 percent. Mergers with best-case entry can increase total welfare relative to the pre-merger equilibrium, but appear to do so only if the merging firms are sufficiently small.

Our results suggest a more nuanced and conditional role for entry analysis than is discussed in the 2010 Horizontal Merger Guidelines. In particular, we verify numerically that a *combination* of merger efficiencies and merger-induced entry can be compatible with profitable mergers that increase consumer surplus.[6] An implication

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[5]The partial calibration routine itself provides a contribution to the antitrust literature by relaxing the data requirements for merger simulation. See also Nocke and Whinston (2019).

[6]Previous research points out that merger efficiencies reduce the scope for profitable entry (Cabral (2003); Erkal and Piccinin (2010)).

is that entry should be evaluated in conjunction with efficiencies in merger review, because entry alone is not sufficient to preserve consumer surplus. Our results also call into question whether antitrust authorities should bear the burden of enumerating entry barriers, as a merger proposal may reveal a belief that barriers exist.

Our research builds on a number of articles that consider the relationship between mergers and entry. The closest is Anderson et al. (2018), which examines mergers in an oligopolistic market with a competitive fringe, also using an aggregative games framework. Under a free-entry assumption, they show that mergers do not harm consumers in the long run, but also do not occur as they are unprofitable. Our research differs in that it examines profitable mergers and best-case entry. This refocusing of the analysis obtains novel policy insights. Further, we provide numerical results to help quantify the extent to which entry could mitigate adverse merger effects.

Also relevant are Werden and Froeb (1998), Spector (2003), and Miller (2014, 2017), which develop revealed preference arguments similar to ours. The first of these addresses Bertrand competition with logit demand but does not provide analytical results. Rather, the authors conduct a Monte Carlo experiment and find that mergers with entry are profitable only if the entrant is highly inefficient. This accords with our results, as the best-case entrant typically is small. Spector (2003) examines a general Cournot model and proves that profitable mergers are incompatible with entry that preserves consumer surplus. Miller (2014, 2017) examines mergers in second-score auctions; as the auctions are efficient, there is no merger-induced entry.

Considering these articles together, many of the workhorse models most used in merger review (e.g., see Miller and Sheu (2020)) imply that merger-induced entry is not sufficient to offset the consumer harm caused by profitable mergers. Whether this extends to price competition with other demand systems, such as the almost ideal demand system (e.g., Deaton and Muellbauer (1980)) or the random coefficient logit (e.g., Berry et al. (1995)), is an open question which we are investigating.[7]

Finally, our paper also contributes to recent research that applies the aggregative games framework of Nocke and Schutz (2018a) to questions relevant to antitrust policy. Nocke and Schutz (2018b) provide conditions under which the change in the Herfindahl Index approximates the market power effects of a merger, and also examine merger efficiencies. Garrido (2019) explores endogenous product portfolios in a

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[7]The attributes of a firm in these other models cannot be summarized with a single parameter (the type), so a unique best-case entrant may not exist. This complicates any general analysis. Still, it may be possible to develop specific results using numerical techniques.

dynamic game. Parameterizations based on the ready-to-eat cereal industry suggest that allowing for endogenous products amplifies the consumer harm of mergers because the merging firms' portfolio reductions are greater than the portfolio expansions of non-merging competitors. Nocke and Whinston (2019) derive the efficiencies necessary to counterbalance adverse merger effects. The formula requires only the merging firms' market shares and thus relates to our partial calibration routine. Alviarez et al. (2020) examines global beer mergers and the adequacy of the obtained divestitures.

The remainder of the paper is organized into two parts. Section 2 outlines the model and provides the main theoretical result. Section 3 discusses partial calibration and the numerical results.

2   Theoretical Model

We use a logit Bertrand setup, expressed as an aggregative game following Nocke and Schutz (2018a), and embed it in a three-stage entry model. In the first stage of the game, two firms decide whether to merge. After observing the outcome of that decision, in the second stage an additional firm decides whether to enter. Then in the third stage, firms sell to consumers in Bertrand competition.

2.1   Bertrand Competition Stage

We begin by describing the demand and supply setup in the final stage of the game. The profits earned from these sales determine the payoffs for the decisions made in earlier stages. Demand takes the multinomial logit form, and firms set prices simultaneously. Firms have knowledge of the consumer demand function and observe the quality and marginal costs for their own and for rivals' products.

Let there be a finite and nonempty set of differentiated products $\mathcal{J}$ available to consumers. Each consumer purchases a single product or forgoes a purchase by selecting the outside good. Let the indirect utility that consumer i receives from product $j \in \mathcal{J}$ be given by $u_{ij} = \delta_j - \alpha p_j + \epsilon_{ij}$, where δ_j is the quality of product j, and p_j is its price. The α measures sensitivity to price, whereas ϵ_{ij} is a consumer-specific preference shock. The utility of the outside good is normalized such that $u_{i0} = \epsilon_{i0}$.

We assume that the preference shocks are independently and identically distributed with a Type 1 extreme value distribution, and that consumers maximize utility. This

generates the logit market shares:

$$s_j(p) = \frac{\exp(\delta_j - \alpha p_j)}{1 + \sum_{k \in \mathcal{J}} \exp(\delta_k - \alpha p_k)} \quad \forall j \in \mathcal{J}, \tag{1}$$

for a vector of prices, p. The share of the outside good is $s_0(p) = 1/(1 + \sum_{j \in \mathcal{J}} \exp(\delta_j - \alpha p_j))$. Throughout, we normalize the market size to unity, which allows us to treat market shares as synonymous with quantity demanded. In dollar terms, consumer surplus is given by

$$CS = \frac{1}{\alpha} \ln \left(1 + \sum_{j \in \mathcal{J}} \exp(\delta_j - \alpha p_j) \right). \tag{2}$$

For any fixed p, consumers benefit from having additional products in the market due to increased variety.

On the supply-side of the model, let firms be indexed by f and the set of firms active in the market be denoted by $\mathcal{F}$. The products in $\mathcal{J}$ are partitioned into a series of sets, where the set $\mathcal{J}^f$ indicates the products sold by firm f. The profit of firm $f \in \mathcal{F}$ is

$$\pi^f(p) = \sum_{j \in \mathcal{J}^f} (p_j - c_j) s_j(p), \tag{3}$$

where c_j is the marginal cost of product j. We assume that each firm maximizes its profit conditional on the prices of other firms. The first order conditions for profit maximization take the form

$$\sum_{k \in \mathcal{J}^f} (p_k - c_k) \frac{\partial s_k(p)}{\partial p_j} + s_j(p) = 0 \quad \forall j \in \mathcal{J}. \tag{4}$$

A price vector that satisfies these first order conditions defines a Bertrand equilibrium.

2.2 Type-Aggregation Representation

We now reformulate the Bertrand logit stage as an aggregative game, following Nocke and Schutz (2018a). The primitives of the firm-level model are the vector of firm-specific types, $\{T^f\} \; \forall f \in \mathcal{F}$, and the price parameter, α. The type of firm f is defined as $T^f \equiv \sum_{j \in \mathcal{J}^f} \exp(\delta_j - \alpha c_j)$, which represents the firm's contribution to consumer surplus if prices equal marginal costs.

From these primitives, the Bertrand equilibrium can be characterized as a vector of firm-level market shares, $\{s^f\} \; \forall f \in \mathcal{F}$, a vector of "$\iota$-markups," $\{\mu^f\} \; \forall f \in \mathcal{F}$,

and a market aggregator, H. We define markups below, and let $s^f = \sum_{j \in \mathcal{J}^f} s_j$. The aggregator is defined as $H \equiv 1 + \sum_{j \in \mathcal{J}} \exp(\delta_j - \alpha p_j)$, which also is the denominator from the market share formula of the product-level model (equation (1)).

To characterize equilibrium, we first derive a relationship between the ι-markups and firm-level market shares. Recall that the product-specific price derivatives for logit demand are

$$\frac{\partial s_j}{\partial p_k} = \begin{cases} -\alpha s_j(1 - s_j) & \text{if } k = j \\ \alpha s_j s_k & \text{if } k \neq j. \end{cases} \tag{5}$$

Substituting these demand derivatives into the first order conditions of equation (4) for some product j and rearranging gives

$$\alpha(p_j - c_j) = 1 + \alpha \sum_{k \in \mathcal{J}^f} (p_k - c_k)s_k. \tag{6}$$

The right-hand-side of this equation does not depend on the which product $j \in \mathcal{J}^f$ is referenced. This implies that the left-hand-side is equivalent for all products sold by firm f, meaning each firm imposes a common markup. Define the ι-markup of firm f as $\mu^f \equiv \alpha(p_j - c_j) \; \forall j \in \mathcal{J}^f$. Substituting back into equation (6) obtains the relationship

$$\mu^f = \frac{1}{1 - s^f}. \tag{7}$$

Turning to the shares, we have $s^f = (1/H) \sum_{j \in \mathcal{J}^f} \exp(\delta_j - \alpha p_j)$ from equation (1), after substituting in for the definition of the aggregator, H. We add and subtract αc_j inside the exponential and apply the definitions of μ^f and T^f to obtain

$$s^f = \frac{T^f}{H} \exp\left(-\mu_f\right). \tag{8}$$

Plugging equation (8) into (7), we obtain that ι-markups satisfy

$$\mu_f \left(1 - \frac{T^f}{H} \exp(-\mu_f)\right) = 1 \tag{9}$$

Let the unique solution in μ_f be $m(T^f/H)$. The *markup fitting-in function*, $m(\cdot)$, has the properties $m(0) = 1$ and $m'(\cdot) > 0$. To close the system, the aggregator satisfies an adding-up constraint,

$$\frac{1}{H} + \sum_{f \in \mathcal{F}} s^f = 1, \tag{10}$$

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which applies because market shares sum to one. Equilibrium is defined by the shares, markups, and aggregator that satisfy equations (8)-(10).

Nocke and Schutz (2018a) show that a unique equilibrium exists. Thus, there is a unique mapping from the primitives to firm-level shares and markups, and vice-versa. Without knowledge of marginal costs, there is no mapping to product-level prices and shares, though clearly we have that $p_j = \frac{1}{\alpha}\mu^f + c_j$ for $j \in \mathcal{J}^f$. The following equations characterize profit, consumer surplus, welfare, and dead-weight loss:

$$\pi^f = \frac{1}{\alpha}\left(\mu^f - 1\right), \tag{11}$$

$$CS = \frac{1}{\alpha}\ln(H), \tag{12}$$

$$W = \frac{1}{\alpha}\left(\ln(H) + \sum_{f\in\mathcal{F}}\left(\mu^f - 1\right)\right), \tag{13}$$

$$DWL = \frac{1}{\alpha}\ln\left(1 + \sum_{f\in\mathcal{F}}T^f\right) - \frac{1}{\alpha}\left(\ln(H) + \sum_{f\in\mathcal{F}}\left(\mu^f - 1\right)\right). \tag{14}$$

Equation (11) is obtained by rearranging equation (6). As share is a function of the ratio of a firm's type to the aggregator, T^f/H, so too are markups and profit. Equation (12) is obtained by substituting H into equation (2). Equation (13) is simply the sum of consumer surplus and profit. Finally, the first term on the right-hand-side of equation (14) is welfare under marginal cost pricing and the second term is realized welfare.

Before turning to the analysis of mergers and entry, we characterize some helpful properties of the ι-markup, profits, and prices. First, each firm's markup, profit, and share increase with its type and decrease with the types of competitors:

Proposition 1. *For every firm $f \in \mathcal{F}$, the ι-markup, μ^f; profit, π^f; and market share, $s_j \ \forall j \in \mathcal{J}^f$ are all increasing in the ratio T^f/H. Furthermore, these objects are also increasing in own-type, T^f, and decreasing in rivals' types, $T^g \ \forall g \neq f$.*

Proof. See the appendix, which follows Nocke and Schutz (2018a, Proposition 6).

Building on the result, consider a merger that enables a set of merging firms, $\mathcal{C} \subset \mathcal{F}$, to maximize joint profit but does not affect primitives. In the type-aggregation representation of the pricing stage, this simply replaces the individual contributions of the merging firms to the vector of types with an aggregated type: $T^m = \sum_{f\in\mathcal{C}}T^f$. The effect of such a merger is to increase the markup, profit, and prices of all firms:

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Proposition 2. *For every firm $f \in \mathcal{F}$, the ι-markup, μ^f, and profit, π^f, increase due to a merger among firms in $\mathcal{C} \subset \mathcal{F}$, but the market aggregator, H, and consumer surplus, CS, decrease.*

Proof. See the appendix, which follows Anderson et al (2018, Section 4.3).

2.3   Merger and Entry

Having characterized competition in the third and final stage, we now turn to the entry and merger phases. Without loss of generality, we label firms sequentially according to $f = 1, \ldots, F$, where $F \geq 3$. We refer to the first $F - 1$ firms as incumbents. Firms 1 and 2 are potential merging partners. The timing of the three-stage game follows:

1. Two incumbent firms 1 and 2 decide whether to merge to form a combined firm m. The effect of the merger is to commit these firms to maximize joint profits when setting prices in stage 3. Assume that this merger does not result in any cost or quality efficiencies, meaning that $T^m = T^1 + T^2$.

2. Firm F observes whether a merger occurs in stage 1 and then decides whether to enter the market. If it enters, it incurs an entry cost $g(T^F) > 0$, the value of which is commonly known. We sometimes refer to firm F as the market outsider, and to $g(T^F)$ as an entry barrier.

3. All firms observe whether a merger and/or entry occur in stages 1 and 2. The active firms, which include all incumbents and, if entry occurred, the entrant, form the set $\mathcal{F}$. These firms in $\mathcal{F}$ then set prices simultaneously, consumers make purchasing decisions, and firms earn variable profit according to the Bertrand logit setup described in the previous sections.

Our solution concept is subgame perfect equilibrium (SPE). Firm F enters if it can earn positive profits in the Bertrand pricing stage, taking into account its entry costs and whether a merger has occured. Firms 1 and 2 merge if doing so increases their combined profit in the pricing stage, taking into account the effect of the merger on the decision of the prospective entrant. It follows that a unique SPE of the three-stage game exists, because the merger and entry decisions are made sequentially.

The key case for antitrust analysis is that of *merger-induced entry,* in which entry occurs only with a merger. Applying both Propositions 1 and 2 verifies that a merger does indeed increase the profitability of entry and, as a corollary, there exists some

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entry cost $g(T^F)$ such that merger-induced entry occurs. Thus, we state formally the following assumption and apply it in our subsequent analysis:

Assumption 1 (Merger-Induced Entry): *Entry is profitable if and only if a merger occurs in the first stage, meaning:*

$$\pi^F\left(\frac{T^F}{H^{nm,e}}\right) \leq g(T^F) \leq \pi^F\left(\frac{T^F}{H^{m,e}}\right), \tag{15}$$

where $H^{nm,e}$ is the market aggregator with no merger, but with entry, and $H^{m,e}$ is the market aggregator with both merger and entry.

We examine now the incentive to merge in the first-stage of the game. We assume a merger occurs if and only if it increases joint profits:

$$\pi^m\left(\frac{T^m}{H^{m,*}}\right) \geq \pi^1\left(\frac{T^1}{H^{nm,*}}\right) + \pi^2\left(\frac{T^2}{H^{nm,*}}\right), \tag{16}$$

where we let $H^{m,*}$ and $H^{nm,*}$ be the market aggregators with and without a merger, incorporating the best-response of the market outsider. Entry matters for the merger decision only if Assumption 1 holds, because otherwise firm F has a dominant strategy, and Proposition 2 implies that a merger is always profitable. Further, as entry affects the profit of incumbents through the aggregator, H, under Assumption 1 there exists a cut-off rule such that a merger occurs only if the entrant has a sufficiently small type:

Proposition 3. *Under Assumption 1, there is a cutoff level, T^*, such that a merger occurs in the first stage if and only if $T^F \leq T^*$.*

Proof. If Assumption 1 holds, the right hand side of expression (16) does not depend on T^F. Proposition 1 implies that incumbent profits decrease as T^F increases, assuming firm F is active. Because profits are a monotonic function of a rival's type, this implies the existence of the cutoff. $\qquad\square$

The cutoff type, T^*, characterizes what we refer to as *best-case entry*—an entrant that leaves the merger profit-neutral and thus maximizes consumer surplus conditional on a merger occurring in the first stage. We explore the economic impact of mergers with best-case entry in the next section, using numerical simulations. Here we provide our main theoretical result, which addresses whether merger-induced entry of a firm with type $T^F \leq T^*$ (such that mergers are profitable) is sufficient to preserve the aggregator (such that consumers are unharmed).

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Theorem 1. *No SPE exists in which a merger is profitable and consumer surplus is unharmed, relative to a scenario in which mergers are prohibited.*

Proof. If entry costs are low enough or high enough that the market outsider has a dominant strategy in the second stage, then the theorem is a straight-forward extension of Proposition 2. In those cases, a merger is profitable and decreases consumer surplus. Thus it remains only to consider the case of merger-induced entry. Suppose, for purposes of contradiction, there exists an SPE in which firms 1 and 2 merge, and consumers are unharmed. Thus the merger must increase joint profits:

$$\pi^m \left(\frac{T^1 + T^2}{H^{m,e}} \right) \geq \pi^1 \left(\frac{T^1}{H^{nm,ne}} \right) + \pi^2 \left(\frac{T^2}{H^{nm,ne}} \right),$$

where $H^{nm,ne}$ denotes the aggregator with no merger and no entry. By hypothesis, consumers are unharmed, hence by equation (12), we have $H^{nm,e} \leq H^{m,e}$. Similarly, by Proposition 1, $\pi^1(\cdot)$ and $\pi^2(\cdot)$ are increasing, so equation (16) implies:

$$\pi^m \left(\frac{T^1 + T2}{H^{nm,ne}} \right) \geq \pi^1 \left(\frac{T^1}{H^{nm,ne}} \right) + \pi^2 \left(\frac{T^2}{H^{nm,ne}} \right). \tag{17}$$

Define $x^i = T^i / H^{nm,ne}$ for $i \in \{1,2\}$, and let $\phi(\mu) = \mu e^{-\mu}$. Then equation (17) can be expressed as:

$$(x^1 + x^2)\phi\big(m(x^1 + x^2)\big) \geq x^1 \phi\big(m(x^1)\big) + x^2 \phi\big(m(x^2)\big),$$

where $m(\cdot)$ denotes the markup fitting-in function. This expression is equivalent to:

$$\sum_{i \in \{1,2\}} x^i \big[\phi\big(m(x^i)\big) - \phi\big(m(x^1 + x^2)\big) \big] \leq 0,$$

which is an impossibility. The function $\phi(\cdot)$ is decreasing on $[1, \infty)$, and for all i, $m(t^1 + t^2) > m(t^i) > 1$, forcing the sum to be component-wise strictly positive. $\square$

In words, no two incumbents would merge in the first stage if they expect entry in the second stage to remedy all harms to consumers in the final stage. Additionally, given that a profitable merger lowers the market aggregator, and no firms experience a decrease in types, Proposition 1 implies that all incumbents' markups are higher in the presence of a profitable merger, even with entry. As marginal costs are held fixed, incumbents' prices must also increase with merger.

Finally, we have focused the analysis on a single market outsider in order to simplify notation and exposition. Theorem 1, however, extends more broadly. Consider an augmented model in which there are multiple outsiders who can enter the market in the second stage, subject to some entry cost. Also assume the incumbents have the option to "reposition" by increasing their type, subject to an investment cost. If a merger is to leave consumer surplus unchanged, then it must be that the market aggregator is unchanged. However, we have established that if the aggregator remains the same, then the merger is unprofitable, regardless of the type of entry or repositioning by other firms. Thus, as a corollary, the main result covers multiple entrants and incumbent repositioning.

3 Numerical Analysis

3.1 Model calibration and merger simulation

We obtain numerical results using a partial calibration of the model in which only the types are recovered from data on market shares, $\{s^f\}\ \forall f \in \mathcal{F}$. In the calibration routine, we first obtain the market aggregator from equation (10), and the ι-markups from equation (7). We then recover firm types from a rearranged equation (8):

$$T^f = \frac{s^f H}{\exp\left(-\mu^f\right)}.$$

With this information, profit, consumer surplus, and welfare are identified up to a multiplicative constant (see equations (11)-(14)).[8]

These outcomes can be compared to alternative scenarios in which (i) two incumbent firms merge and (ii) some new firm enters the market. Each of these alternatives is characterized by a new vector of firm types and, given these types, new equilibrium outcomes can be obtained. The main step in simulation involves using a nonlinear equation solver to recover the shares and the market aggregator. There are $F + 1$ nonlinear equations that must be solved simultaneously. One of these is the adding-up constraint of equation (10), and the others are obtained by plugging equation (7) into

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[8]A full calibration of the model can be accomplished with data on shares and one markup. This allows the price parameter, α, to be recovered from the definition of μ^f. See also the Nocke and Schutz (2018a) appendix. Then markups, profit, consumer surplus, and welfare also are obtained (not just up to a multiplicative constant). However, these objects are not necessary for present purposes.

equation (8), which yields

$$s^f = \frac{T^f}{H} \exp\left(-\frac{1}{1-s^f}\right).$$

With shares in hand, markups, profit, consumer surplus, and welfare are identified up to a multiplicative constant, using the same steps enumerated above for calibration.

3.2 Data Generating Process

In the data generating process, we assume that the merging firms have the same pre-merger market share; we use shares of 0.01, 0.02, ..., 0.40. We assume the outside good has a share of 0.20, and assign the remaining share evenly to non-merging incumbents. Thus, for example, when we allow for five incumbents, one pre-merger equilibrium we examine features shares of 0.10 for each merging firm and shares of 0.20 for non-merging firms. There are 40 share vectors for a given number of incumbents; we consider cases with three, five, six, seven, and nine incumbents.

Given the pre-merger shares, we calibrate the model and simulate the post-merger equilibrium under the alternative assumptions that (i) no entry occurs or (ii) a firm enters the market with a type that makes the merger profit-neutral. The latter case is one of best-case entry.[9] We recover the percentage changes in markups, profit, consumer surplus, and welfare for each of these scenarios; these are identified because the levels are known up to a multiplicative constant.

The main results address the extent to which best-case entry mitigates the adverse consequences of the merger for consumers. To that end, we calculate the percentage of consumer surplus loss that is mitigated by entry:

$$1 - \frac{\Delta CS^{\text{entry}}}{\Delta CS^{\text{no entry}}} = 1 - \frac{\alpha \Delta CS^{\text{entry}}}{\alpha \Delta CS^{\text{no entry}}}, \tag{18}$$

where the left-hand-side of the equation is the object of interest and both the numerator and denominator on the right-hand-side can be recovered with simulation. We also calculate the percentage of the merging firms' price increase that is mitigated by entry:

$$1 - \frac{\Delta p^{\text{entry}}}{\Delta p^{\text{no entry}}} = 1 - \frac{\Delta \mu^{\text{entry}}}{\Delta \mu^{\text{no entry}}}, \tag{19}$$

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[9]We search numerically for the type of the best-case entrant by simulating the merger with candidate entrants of increasing type until the merger becomes profit-neutral.

where again the left-hand-side is the object of interest, and both the numerator and denominator on the right-hand-side can be recovered with simulation.

3.3 Numerical results

Figure 1 shows the effects of a merger without entry for a market with six incumbents, for the purposes of building familiarity with the economic environment. The solid, dashed, and dotted lines, respectively, provide the merger effects on markups, profit, and consumer surplus. The effects depend on the pre-merger shares of the merging firms because, within the context of logit demand, larger shares for the merging firms imply greater diversion, more upward pricing pressure, and, ultimately, larger equilibrium price increases. For the smallest merging firms considered, the effects are barely discernible, whereas for the largest, markups increase by more than 50 percent, profit increases by 20 percent, and consumer surplus falls by 40 percent.[10]

Figure 2 plots the proportion of the merging firms' price increase that is mitigated by the best-case entrant. The four panels correspond to markets with three, five, seven, and nine incumbents, respectively. In each, the amount of mitigation exhibits a similar pattern: entry does little to offset the price effects of smaller mergers, but becomes more important for larger mergers. For example, with the largest mergers, entry offsets almost 30 percent of the merger price increases. In none of the mergers we examine, however, does entry come close to fully offsetting the price increases.

Figure 3 provides the corresponding analysis of consumer surplus. The lines take a ∪-shape in each panel, such that best-case entry mitigates the least amount of consumer surplus loss for moderately-sized mergers. Further, comparing across panels, mitigation is greater if there are fewer incumbents, and this is most pronounced for small mergers. Finally, the mitigation of consumer surplus loss approaches 50 percent from below in each panel, as the pre-merger shares of the merging firms reach their maximum.

These patterns arise because the entrant contributes to consumer surplus both by improving product diversity and by lowering equilibrium prices. For small mergers, the diversity effect is much more important, as our earlier analysis shows negligible mitigation of merger price increases. Further, from a consumer standpoint there are decreasing returns from diversity, so the number of incumbents matters greatly with small

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[10]We show results for the case of six incumbents but this does not matter much because prices exhibit mild strategic complementarity with logit demand (e.g., Werden and Froeb (1994); Miller et al. (2016)). Results developed with more or fewer incumbents are available upon request.
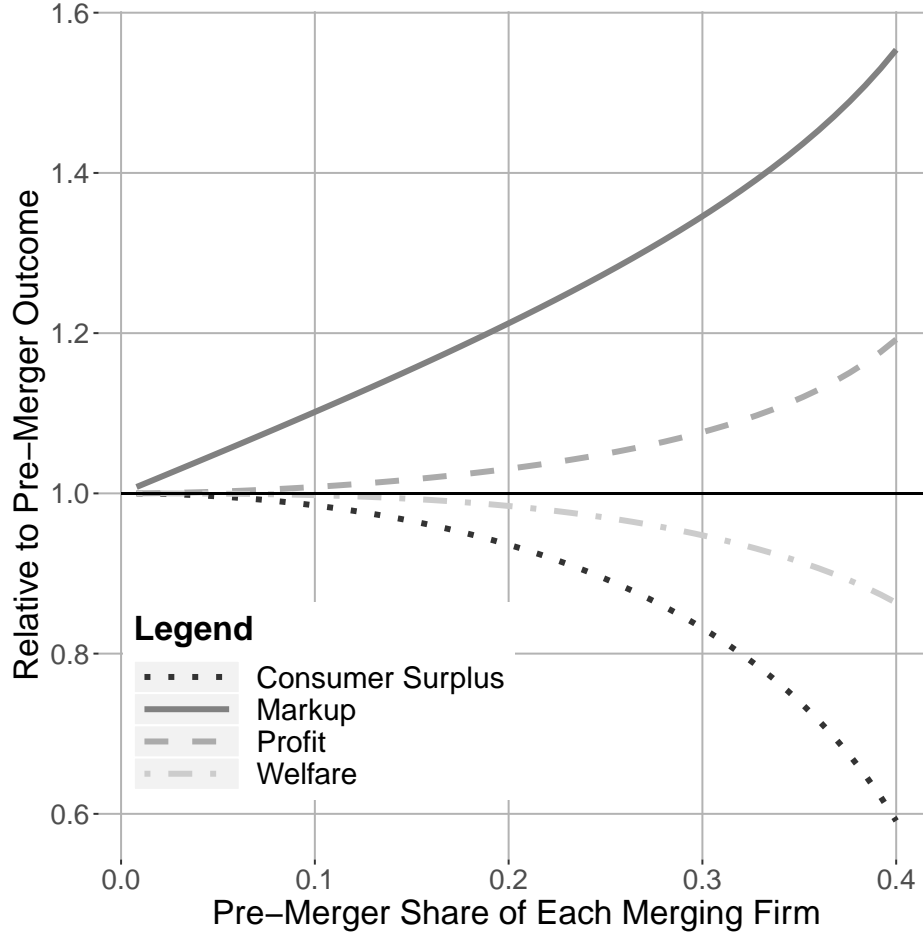
Figure 1: Merger Effects Without Entry

Notes: The figure shows the effects of mergers on consumer surplus, welfare, and the merging firms' markups and profit. These effects are reported as indices relative to pre-merger equilibrium, and depend on the pre-merger shares of the merging firms. There are six incumbents.

mergers.[11] Moderately-sized mergers produce larger price effects that are (mostly) unmitigated, and the value consumers gain from diversity does not rise commensurately, meaning the overall mitigation of consumer surplus loss is less. For the largest mergers, entrants substantially lessen price effects, leading to greater mitigation of consumer surplus loss than with moderately-sized mergers.

Figure 4 plots the welfare effects of merger. Without entry, welfare decreases due to merger by an amount that increases in the size of the merging firms (see also Figure 1). With best-case entry, mergers between small firms do not reduce welfare, and may actually increase it slightly due the entrant's contribution to product diversity.

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[11]From equation (2), with J symmetric firms of type T and marginal cost pricing, we have $CS = (1/\alpha) \ln(1 + JT)$, so consumer surplus grows with the number of firms at a decreasing rate.
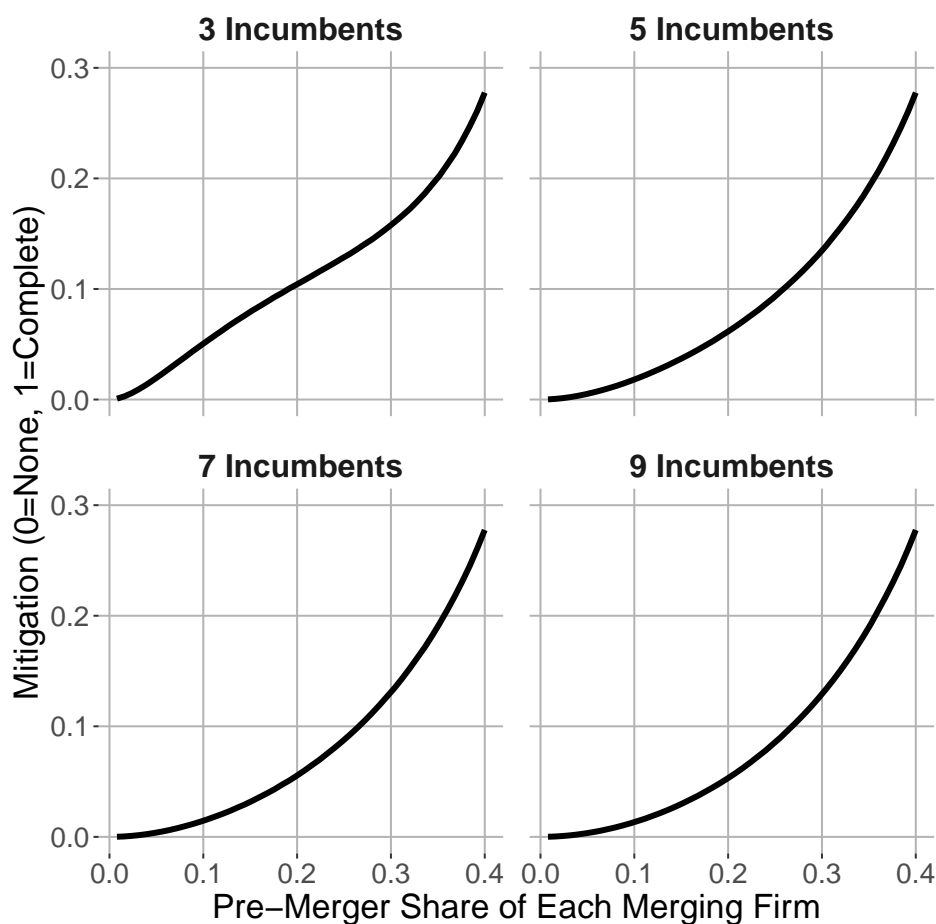
Figure 2: Best-Case Mitigation of Price Increases

Notes: The figure shows the proportion of merging firms' price increases that are mitigated by an entrant that makes the merger profit-neutral. The four panels correspond to models with three, five, seven, and nine incumbents, respectively. Mitigation depends on the pre-merger shares of the merging firms.

These increases are most evident if there are fewer incumbents because there improving product diversity has a greater impact on consumer surplus. For mergers between larger firms, best-case entry mitigates about half of welfare losses.

Figure 5 provides some information on the best case entrant. The dashed line plots the ratio of the entrant type to the type of a merging firm. The entrant type is less than 20 percent of the merging firm type for all mergers considered. The solid line plots the entrant share in the post-merger equilibrium, which is is substantially less than half the pre-merger share of a merging firm for all the mergers considered. We interpret these results as suggesting that modest levels of entry can eliminate the profit that otherwise would be obtained from merger. However, such modest entry does little to offset price increases, and may also leave consumer surplus losses mostly unmitigated.
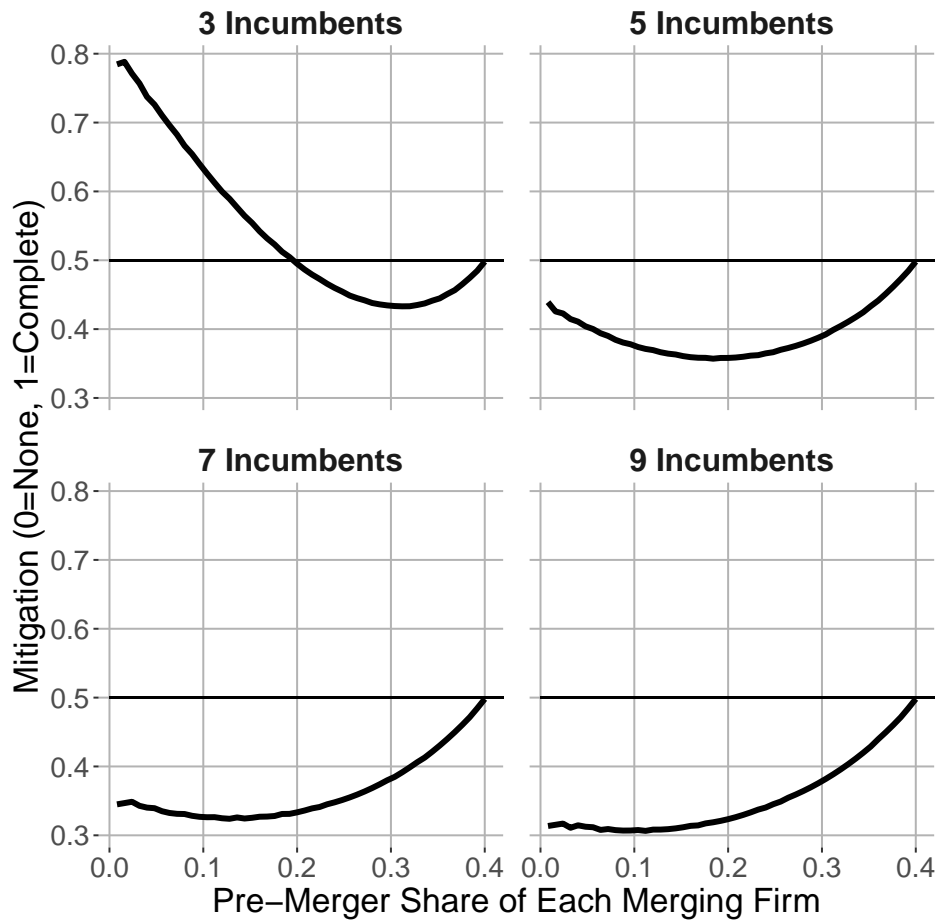
Figure 3: Best-Case Mitigation of Consumer Surplus Loss

Notes: The figure shows the proportion of consumer surplus loss that is mitigated by an entrant that makes the merger profit-neutral. The four panels correspond to models with three, five, seven, and nine incumbents, respectively. Mitigation depends on the pre-merger shares of the merging firms.

Finally, the introduction notes that a combination of efficiencies and merger-induced entry can be compatible with profitable mergers that increase consumer surplus. We have searched for parameterizations that meet the following criteria:

(i) Merger with efficiencies alone reduces consumer surplus.

(ii) Merger with efficiencies and entry increases consumer surplus.

(iii) Merger is profitable.

(iv) Entry is more profitable with merger than without.

We have identified many parameterizations that meet these criteria, and even more that do not. One that does features three incumbents with market shares of 0.30, 0.30,
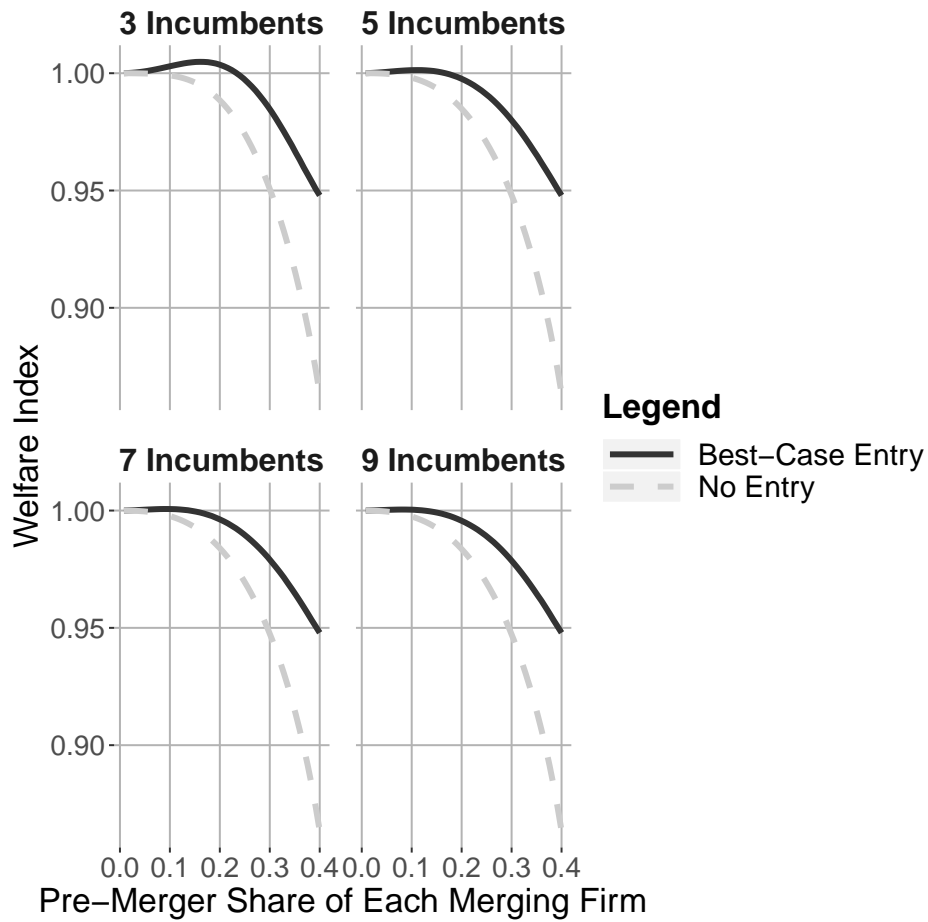
Figure 4: The Welfare Effects of Merger

Notes: The figure shows post-merger welfare with best-case entry and no entry. Welfare is reported as an index relative to the pre-merger value. Results depend on the pre-merger shares of the merging firms.

and 0.20, corresponding to types of 2.49, 2.49, and 1.46, respectively. Let the first two firms merge with an efficiency that increases the post-merger type from 4.98=2×2.49 to 6.47 (an increase of 30%). Then, with an entrant of type 0.20, the criteria obtain. It follows that entry and efficiencies together can fully mitigate the adverse consumer surplus effects of profitable mergers, even if neither would suffice in isolation.
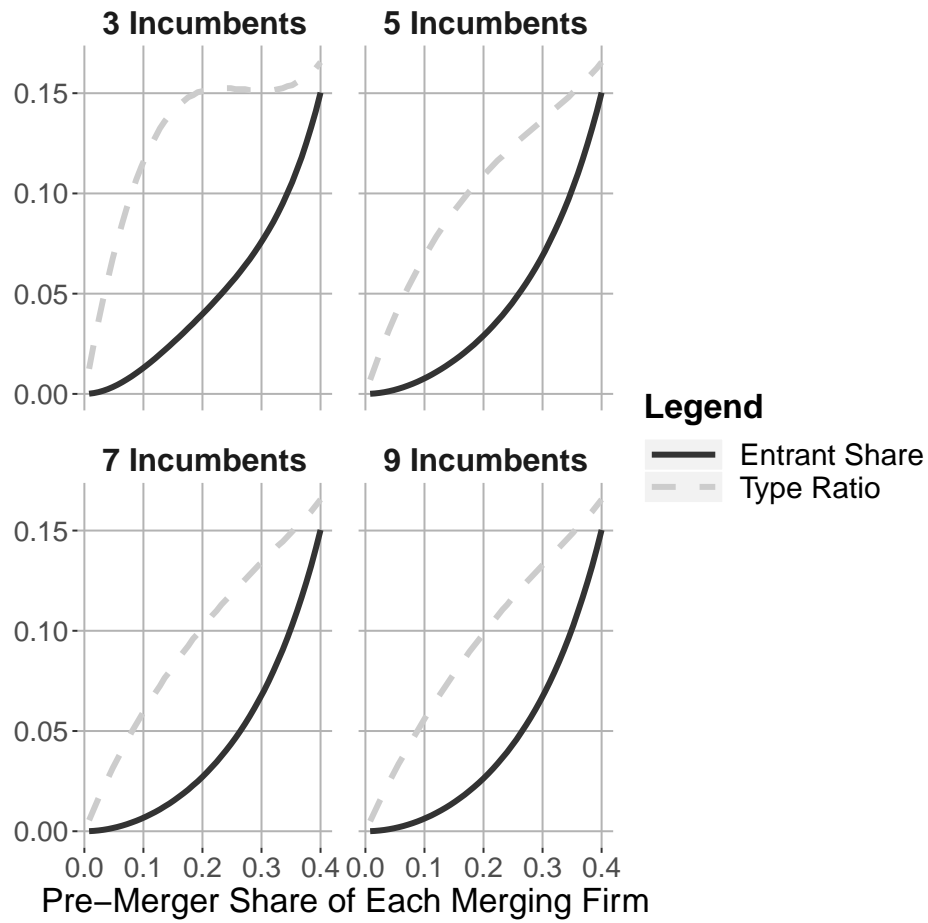
Figure 5: Properties of the Best-Case Entrant

Notes: The figure examines two properties of the entrant that makes a merger revenue-neutral: the entrant share in the post-merger equilibrium and the ratio of the entrant's type to that of a merging firm. The four panels correspond to models with three, five, seven, and nine incumbents, respectively. The properties of the best-case entrant depend on the pre-merger shares of the merging firms.

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A Appendix for Online Publication

A.1 Proof of Proposition 1

Proof. Let $x^f = T^f/H$. After applying the implicit function theorem to equation (9), we find that

$$m'(x^f) = \frac{m(x^f)^2 \exp(-m(x^f))}{1 + m(x^f)^2 x^f \exp(-m(x^f))}.$$

By inspection, this expression is positive. Furthermore, the derivative of π^f with respect to x^f is equal to $(1/\alpha)$ times the derivative of μ^f, meaning it is also positive if $\alpha > 0$. That is, it is positive so long as consumers earn negative utility from price.

By combining equations (7) and (8), solving for s^f, and taking the derivative with respect to x^f, we find that

$$\frac{ds^f}{dx^f} = \frac{m'(x^f)}{m(x^f)^2},$$

which is positive because $m'(x^f)$ is positive. Additionally, by applying the implicit function theorem to the adding-up constraint (10), we find that

$$\frac{dH}{dT^f} = \frac{\frac{ds^f}{dx^f}}{\frac{1}{H} + \sum_{g \in \mathcal{F}} x^g \frac{ds^g}{dx^g}},$$

which is also positive given that $ds^f/dx^f \ \forall f \in \mathcal{F}$ is positive. Computations for types T^f and T^g give

$$\frac{dx^f}{dT^f} = \frac{1}{H}\left(1 - \frac{T^f}{H}\frac{dH}{dT^f}\right),$$

which can be confirmed as positive after substituting in for dH/dT^f, and

$$\frac{dx^f}{dT^g} = -\frac{T^f}{H^2}\frac{dH}{dT^g},$$

which is negative because dH/dT^g is positive. Applying the chain rule gives the desired results.

$\square$

A.2 Proof of Proposition 2

Proof. By Proposition 1, the markups for the merging firms increase because $T^m > \max_{f \in \mathcal{C}}\{T^f\}$. The corresponding prices must also increase as marginal costs are held fixed. Because the aggregator, H, is decreasing in prices, Proposition 1 also implies that the markups (and thus prices) of non-merging incumbents rise as well, and this further decreases the aggregator. Thus, the value T^f/H is higher for all firms with the merger, so Proposition 1 implies that profits increase. Finally, given equation (12), consumer surplus is lower because the aggregator is lower. $\square$