Forward Contracts, Market Structure, and the Welfare Effects of Mergers*

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Abstract

We nest the two-stage oligopoly model of Allaz and Vila (1993) within a richer industry structure to explore how the presence of a forward market affects the welfare consequences of mergers as well as the incentive to merge. Our key insight is that while the presence of a forward market is welfare enhancing, the incentive to sell in the forward market decreases with the level of concentration. As a consequence, firms have a greater incentive to merge and the reduction in consumer surplus due to mergers may be exacerbated. We explore the implications of these results for antitrust policy.

Keywords: forward contracting; hedging; mergers; antitrust policy
JEL classification: L13; L41; L44

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1 Introduction

Many imperfectly competitive industries are characterized by forward markets in which firms contract to supply output at a locked-in price upon the opening of a subsequent spot market. Among such industries are crude oil, tin, aluminum, copper, coffee, cocoa, and deregulated wholesale electricity. A long-standing result in the theoretical literature is that the presence of forward markets can result in greater output and lower prices (Allaz and Vila (1993)). Little attention has been paid, however, to the role of competition in determining the magnitudes of these effects. One practical implication is that the theoretical literature provides scant guidance to antitrust authorities seeking to evaluate how to incorporate forward markets into merger analysis.

In the current study, we model an oligopolistic industry in which firms sell a homogeneous product and compete through their choices of quantities. Competition happens in two stages: first in a contract market and second in a spot market. Following Allaz and Vila (1993), we model the level of forward contracting as a strategic variable of the firm. The forward market allows firms to make strategic commitments that increase aggregate output in equilibrium. These commitments discipline the exercise of market power in the spot market because firms are less sensitive to the price effect of increasing or decreasing output. Following Perry and Porter (1985), we model firms as having heterogeneous capital stocks that reflect their respective capacities. The model is rich enough to incorporate any number of firms as well as firms of varying sizes (i.e. capital stocks), thereby facilitating the analysis of any arbitrary change in structure. In this way, our analysis brings together two established theoretical literatures: one on the strategic incentive for forward contracting (e.g., Allaz and Vila (1993)), and the other on the effects of horizontal mergers with homogeneous products (e.g., Perry and Porter (1985); Farrell and Shapiro (1990)).

We develop three main sets of results. The first set of results establishes the relationship between market structure and the effect of forward contracting on welfare. We find that while the presence of a forward market always leads to weakly greater surplus relative to Cournot, the magnitude of this effect varies non-monotonically with competition. While surplus increases in the rates at which firms contract forward (their "hedge rates"), which are themselves larger when market structure is more conducive to competition, competition also leads to greater surplus in the absence of forward contracting. In the extreme cases of monopoly and perfect competition, respectively, forward contracting has no effect on welfare. It follows that the welfare-enhancing impact of a forward market is greatest at intermediate levels of competition.

The second set of results prove that forward contracts exacerbate the loss of consumer surplus caused by mergers if the merging firms are sufficiently large, but mitigate consumer surplus loss otherwise. This can be understood as the combination of two forces. As stated above, forward contracts discipline the exercise of market power. This would be sufficient to mitigate consumer surplus loss if hedge rates were to remain constant. However, firms have a greater incentive to enter forward contracts if the industry is relatively competitive. It follows that mergers reduce forward contracting and thereby soften an important constraint on the exercise of market power. The latter effect dominates if the merging firms are sufficiently large, such that the presence of forward market amplifies consumer surplus loss.

The third set of results relates to the profitability of mergers. We prove that the presence of a forward market makes all mergers privately profitable. To motivate this result, we point out that mergers are not profitable in Cournot models with constant marginal costs except in the case of merger to monopoly (Salant, Switzer, and Reynolds (1983)). With marginal costs increasing due to capital stocks, more mergers are profitable, but many still are not (Perry and Porter (1985)). Forward markets create a *prisoners' dilemma* for firms: contracts are individually rational but collectively damaging. A merger reduces the incentive for the merging firms and their competitors to contract forward, which in turn causes price to increase by a greater amount than it would with hedge rates fixed at the pre-merger level. The strength of this effect is sufficient that even the smallest mergers are profitable.

One limitation of our model is that it does not incorporate that forward contracts may reduce firms' exposure to price volatility in the spot market. However, Allaz (1992) shows that the risk-hedging and strategic motives can coexist in equilibrium, with each contributing to an expansion of output relative to the Cournot benchmark. The mechanisms that we identify extend to that setting cleanly. Further, we anticipate that many of our results also would extend to models in which forward contracts exist only to hedge risk; the basis being that if mergers make the exercise of market power more profitable, but for some limiting constraint, then they also introduce incentives to relax the constraint. This principle applies well beyond models of forward contracting; the dynamic price signaling game of Sweeting and Tao (2016) is one recent example that shares a core intuition with our own research.

This study blends the literatures on horizontal mergers and strategic forward contracting. In the former literature, Perry and Porter (1985) introduce the concept of capital stocks to model mergers among Cournot competitors as making the combined firm larger instead of merely reducing the number of firms. McAfee and Williams (1992) solve for the equilibrium strategies under any arbitrary allocation of capital stocks. Farrell and Shapiro (1990) allow for fully general cost functions which incorporate the possibility of merger-specific cost efficiencies, and also develop the usefulness of examining "first-order" impact of mergers. Jaffe and Wyle (2013) apply the first-order approach to study merger effects under a general model of competition that nests conjectural variations, Cournot, and Bertrand as special cases. The solution techniques that we employ extend the methodologies developed in these articles. We also supplement our theoretical results with Monte Carlo analyses, as in Miller, Remer, Ryan, and Sheu (2017a) and Miller, Remer, Ryan, and Sheu (2017b).

The seminal article on strategic forward contracting is Allaz and Vila (1993). The main result developed is that as the number of contracting stages increases in a model of duopoly, total output approaches the perfectly competitive level. The subsequent literature has gone in a number of directions. Hughes and Kao (1997) and Ferreira (2006) consider the importance of the assumption that contracts are observable to the market. Green (1999) extends the model to markets in which firms submit supply schedules. Ferreira (2003) explores equilibria of the game as number of contracting rounds approaches infinity. Mahenc and Salanie (2004) analyze the impact of forward contracting when firms compete via differentiated products Bertrand in the spot market. Liski and Montero (2006) consider the role of forward contracting in sustaining collusive outcomes. All of these studies suggest that the extent to which our results are applicable in real-world settings will depend on a number of features of the market. Empirical evidence on the importance of forward contracting is presented in Wolak (2000), Bushnell (2007), Bushnell, Mansur, and Saravia (2008), Hortacsu and Puller (2008) and Brown and Eckert (2016).

Among the aforementioned studies, the closest to our research are Bushnell (2007) and Brown and Eckert (2016). Bushnell (2007) examines the welfare impact of a forward market for a symmetric N-firm oligopoly with a single round of forward contracting. The model is calibrated to a number of deregulated electricity markets in order to ascertain the impact of forward markets on prices and output. Mergers are not examined. Brown and Eckert (2016) allow firms to have heterogeneous capital stocks as we do, but the focus is primarily empirical and as a result, they do not address the same questions. Ours is the only study we are aware of that solves for the subgame perfect equilibrium of the game with fully differentiated firms.

The paper proceeds as follows. Section 2 describes the model of two-stage quantity competition, solves for equilibrium strategies using backward induction and analyzes the welfare impact of forward contracting. The key insight is that welfare is increasing in forward contracting which itself is increasing in the competitiveness of the industry. Section 3 formally models the welfare impacts of mergers highlighting how the results differ from the baseline model of Cournot competition. Section 4 concludes with a discussion of the applicability of our results.

2 Model

2.1 Overview

We consider a modified Cournot model that features two stages. The model is a variant of Allaz and Vila (1993) but we allow for an arbitrary number of firms with heterogeneous production technologies as in McAfee and Williams (1992). In the first stage ($t = 0$), firms can contract at a set price to buy or sell output to be delivered at time $t = 1$. We assume that contracts are observable. In the second stage ($t = 1$), production occurs, contracts are settled, and firms compete via Cournot to sell any residual output in the spot market. We refer to the first and second stages as the "contract" and "spot" markets, respectively. The solution concept is subgame perfection.

Formally, suppose each firm $i \in \{1, \ldots, N\}$, sells q_i^s in the spot market given the vector of contracted quantities $\mathbf{q}^f = \left\{ q_1^f, ..., q_N^f \right\}$ and the other firms' spot market supply. This determines the firm's output, q_i, as the sum of its contracted and spot quantities. Total output is the sum of all firms' output and is defined $Q = \sum_i q_i$. The spot market price is given by the linear schedule $P(Q) = a - bQ$, for $a, b > 0$. The total costs of firm i's output is $C_i(q_i) = cq_i + dq_i^2/2k_i$, so that the marginal cost is $C_i'(q_i) = c + dq_i/k_i$. We assume $a > c \geq 0$ to ensure that gains to trade exist. The parameter d is binary ($d \in \{0, 1\}$) and allows the model to nest constant marginal costs as a special case.

Forward contracts are entered into taking as given the equilibrium quantities bought and sold by other firms, and with knowledge of corresponding subgame equilibrium that arises in the spot market. We assume rational expectations and risk neutrality among all players and that the contract market is efficient. It follows that in the subgame perfect equilibrium (SPE) the forward prices equal the subsequently realized spot market price, which eliminates arbitrage between the contract and spot markets and makes it so that firms are net suppliers (as opposed to net demanders) in the contract market. Though Allaz and Vila (1993) do offer an extended-form-game representation of a forward market whose equilibrium satisfies these properties, the result is better thought of as axiomatic: If all agents are risk neutral and entry into the forward market is free, then any arbitrage opportunity should be competed away.

2.2 Spot market subgame

Solutions are obtained by first considering the output decisions of firms in the spot market, given any vector of contracted quantities, and then considering the contract market. Firm i chooses its total output, q_i (the sum of forward and spot market quantities), taking as given its contracted quantity, q_i^f as well as the vector of other firms' outputs, $\mathbf{q_{-i}}$, to maximize the profit function

$$\pi_i^s = \left(q_i; q_i^f, \mathbf{q_{-i}}\right) = P\left(Q\right)\left(q_i - q_i^f\right) - C_i\left(q_i\right).$$

The first-order condition implies that

$$P\left(Q\right) + \left(q_i - q_i^f\right) P'\left(Q\right) = C_i'\left(q_i\right). \tag{1}$$

The inclusion of q_i^f in (1) says that the firm is less concerned about the effect of output on price since selling an additional unit has no effect on the revenue from forward sales. We derive closed-form expressions for equilibrium price and quantities using the following steps. First, for a given price, P, we can express equation (1) as

$$q_i = \left(\frac{k_i}{bk_i + d}\right)(P - c) + \left(\frac{bk_i}{bk_i + d}\right) q_i^f$$

Next, define the following values:

$$\beta_i = \frac{bk_i}{bk_i + d},\ B = \sum_i \beta_i,\ B_{-i} = \sum_{j \neq i} \beta_j,\ F = \sum_i \beta_i q_i^f,\ F_{-i} = \sum_{j \neq i} \beta_j q_j^f,$$

so that

$$q_i = \frac{\beta_i}{b}(P - c) + \beta_i q_i^f$$

and

$$Q = \sum_i q_i = \frac{B}{b}(P - c) + F$$

Finally, substituting the identity $Q = (a - P)/b$ into the left-hand side of the above expression yields

$$\frac{a - P}{b} = \frac{B}{b}(P - c) + F$$

It is now straightforward to derive subgame equilibrium price and quantities as functions of $\mathbf{q^f}$, the vector of contracted quantities:

$$\begin{aligned} P\left(\mathbf{q^f}\right) &= \frac{a + Bc}{1 + B} - \frac{bF}{1 + B} \\ Q\left(\mathbf{q^f}\right) &= \left(\frac{a - c}{b}\right)\frac{B}{1 + B} + \frac{F}{1 + B} \\ q_i\left(\mathbf{q^f}\right) &= \left(\frac{a - c}{b}\right)\frac{\beta_i}{1 + B} + \frac{\beta_i}{1 + B}\left[(1 + B_{-i})\, q_i^f - F_{-i}\right] \end{aligned} \tag{2}$$

The above values have been expressed so as to highlight the differences between the two-stage model of competition considered here and a baseline model of Cournot competition without forward contracts in which $q_i^f = F_{-i} = F = 0$. In the baseline model, total output increases in B and price decreases in B. A larger value for B corresponds to conditions typically associated with a more competitive industry: a larger number of firms, holding fixed capital stock per firm; greater capacity (i.e. capital stock) per firm, holding fixed the number of firms; and a more symmetric distribution of capacity among firms.

If F, a weighted average of firms' contracted quantities, is positive then price is lower and total quantity is higher than under the baseline model. This foreshadows many of the results obtained below. A given firm's quantity may be higher or lower than the Cournot baseline, depending on how its contracted quantity compares to that of other firms. One could imagine a firm would want to contract a large share of its productive capacity to grab a larger share of the overall market. However, since other firms are employing the same strategy, each firm must adjust its spot-market quantity to the contracted quantities of its competitors. We will be able to say more about which of these forces dominates after deriving the equilibrium in the contract market.

2.3 Contract market

Our formulation of the contract market follows Allaz and Vila (1993). We consider a contract market where firms face a price P^f and decide how much to supply or demand. Assume there are at least two speculators willing to take the opposite side of any long or short position if an arbitrage opportunity presents itself. Absent uncertainty, the spot price is known, conditional on equilibrium (pure) strategies. Perfect foresight along with competition among speculators rules out any price other than the resulting spot price. Therefore, we require $P^f = P(Q)$, where Q is total output, given the amount producers demand or supply at price P^f.[1]

Consider then firm i's decision of how much to supply (or demand) in the contract market. Firm i chooses q_i^f to maximize the following profit function:

$$\pi_i\left(q_i^f; \mathbf{q_{-i}^f}\right) = P^f q_i^f + P(Q)\left(q_i - q_i^f\right) - C_i(q_i) = P(Q)\, q_i - C_i(q_i)$$

The first-order condition implies that

$$P(Q) + (1 + R_i)\, q_i P'(Q) = C_i'(q_i), \tag{3}$$

where $R_i \equiv \sum_{j \neq i} \frac{\partial q_j}{\partial q_i^f} / \frac{\partial q_i}{\partial q_i^f} = \sum_{j \neq i} \frac{\partial q_j}{\partial q_i}$ is the quantity response from all of firm i's competitors to a marginal increase in firm i's output. In a Cournot game with Nash conjectures (McAfee and Williams (1992)), this term is zero. But when competition spills across multiple periods as in the current setting, each firm recognizes that by increasing the quantity it contracts in advance of the spot market, it reduces the quantity that competing firms will choose to sell in the spot market. This creates an incentive for each firm to expand output beyond its Cournot level. From equation (2), we can express the response as

$$R_i = -\frac{B_{-i}}{1 + B_{-i}}. \tag{4}$$

Because $1 + R_i < 1$, the marginal revenue curve facing firm i in the contract market as expressed in equation (3) is flatter in own quantity than it would be under Cournot (i.e., when $R_i = 0$). A marginal increase in firm i's contracted quantity lowers the price as it would under Cournot, but not by as much since

[1] The issue of commitment arises in that once contracts are signed and made public, a firm would want to increase its contracted quantity prior to $t = 1$ which it would then reveal to its competitors. Our results are qualitatively unchanged if additional rounds of forward contracting are allowed provided contracting frictions limit it to a finite number of rounds.

other firms respond by reducing their own quantities. Holding all other firms' quantities fixed at their Cournot levels, the inclusion of $1 + R_i$ in equation (3) pivots firm i's marginal revenue curve up from the vertical axis, which suggests firm i will increase output relative to Cournot. However, if other firms increase their output, firm i's marginal revenue curve shifts down because quantities are strategic substitutes. This shift curbs firm i's incentive to increase output relative to Cournot and may even decrease it if other firms increase their output by a large enough amount.

Closed-form expressions for equilibrium price and quantities are derived using the same approach as in the spot-market subgame. For the sake of brevity, we omit the steps except to define the following terms:

$$\mu_i = \frac{\beta_i}{1 + \beta_i R_i}, \; M = \sum\nolimits_i \mu_i$$

The equilibrium price and quantities for the two-stage game are:

$$\begin{aligned} P &= \frac{a + Mc}{1 + M} \\ Q &= \left(\frac{a - c}{b}\right) \frac{M}{1 + M} \\ q_i &= \left(\frac{a - c}{b}\right) \frac{\mu_i}{1 + M} \end{aligned} \tag{5}$$

Absent a contract market ($R_i = 0$), μ_i and M reduce to β_i and B, respectively, so that the price and quantities in (5) collapse to their values in the Cournot game of McAfee and Williams (1992).

The impact of forward contracting on each firm's output is related to the fraction of its output that is contracted in advance of the spot market, i.e. its "hedge rate." The following result aids the understanding of this relationship.

Lemma 1 *Given equilibrium strategies within the SPE of the two-stage game, the hedge rate can be expressed as* $h_i \equiv \frac{q_i^f}{q_i} = |R_i| = \frac{B_{-i}}{1+B_{-i}}$.

Proof. Given equilibrium play in the spot market and contract market, the left-hand side of (1) must equal the left-hand side of (3). The result follows immediately. ∎

The result is fairly general in that it does not rely on the shape of the demand or cost functions. It follows from the fact that a firm, when deciding how much to supply on the contract market, takes into account that a marginal increase in supply will be met by a decrease in the amount its competitors supply in the spot market. Thus, while a marginal increase in contracted supply on its own causes the price to decline, the corresponding decrease in competitors' spot market supply partially offsets this. The optimum equates marginal revenue across the two stages much in the way that a third-degree price discriminating monopolist equates marginal revenue across customer segments.

An implication of Lemma 1 is that a firm's hedge rate depends on the amount of capital stock controlled by its competitors as well as the distribution of capital stock among them. Competitors with larger capital stocks produce more irrespective of hedging, so their response to firm i's contracted quantity will be larger. At the same time, because larger firms make less efficient use of their capital stocks, firm i's hedge rate is larger when the capital stocks of its competitors are more symmetrically distributed.[2] The structural conditions which lead a firm to sell a larger fraction of its output in the contract market are the same conditions that lead to greater output in the baseline Cournot model.

As a further illustration, we have that in the perfectly symmetric case (i.e., $\beta_j = \beta$ for all $j \neq i$),

$$h_i = \frac{(N-1)\,\beta}{1+(N-1)\,\beta}. \tag{6}$$

That (6) is larger for larger values of N suggests that from a welfare perspective, a forward market is not a perfect substitute for a competitive industry structure because forward contracting is more prevalent when the industry is more competitive. In the case of monopoly ($N = 1$), the hedge rate is zero as forward contracting has no strategic impact.

Because $h_i > 0$ for all i outside of the monopoly case, we can say,

Proposition 1 *Price is (weakly) lower and total output is (weakly) higher in the SPE of the two-stage game than in the baseline model of Cournot competition. Each inequality is strict outside of the monopoly case. An individual firm's output can nevertheless be lower in the SPE of the two-stage game if its capital stock is sufficiently small relative to that of its competitors.*

Proof. From the first two lines of expression (2), the price and total output results hold if $F \geq 0$. Using Lemma 1, we have that, $F \equiv \sum_i \beta_i q_i^f = \sum_i \beta_i h_i q_i$. This expression is strictly greater than zero in all but the monopoly case.

[2]Each β_i is concave in capital due to increasing marginal costs. Thus, firms with larger capital stocks produce less per unit of capital than smaller firms. Note that if marginal costs are constant ($d = 0$) then $\beta_i = 1\ \forall i$.

From the third line of expression (2), a firm's output is lower in the SPE of the two-stage game relative to Cournot if and only if, $(1 + B_{-1})\, q_i^f < F_{-i}$. Using Lemma 1 and expression (5), this is equivalent to $\mu_i < (M_{-i} - B_{-i})\,/B_{-i}$. Because μ_i is monotonically increasing in k_i, we can define its inverse, μ_i^{-1} such that the desired condition becomes, $k_i < \mu_i^{-1}\left(\frac{M_{-i}-B_{-i}}{B_{-i}}\right)$. ■

The impact of the forward market on output can be substantial. Consider the special case of constant marginal cost ($d = 0$). This implies that $\beta_i = 1$ so that $R_i = -\frac{N-1}{N}$, $\mu_i = N$, and $M = N^2$. We calculate the increase in industry output due to the presence of a forward market by comparing the output in equation (5) to what the value would be if $M = N^2$ were replaced with $B = N$. The presence of a forward market increases output by 140 percent when $N = 2$ and by nearly 600 percent when $N = 6$. These increases would be somewhat smaller if marginal costs were instead increasing.

2.4 Market structure and welfare

We now examine the impact of a forward market on welfare. In the last subsection, we saw that total output is increasing in F, a weighted-average of contracted quantities. A firm's contracted quantity is increasing in its hedge rate, which is a function of market structure. In particular, when the market structure is more *competitive*—e.g., there are more firms or capital is distributed more symmetrically among a given number of firms—hedge rates are higher. This suggests that a forward market creates an additional channel through which market structure affects welfare.

To formalize this point, we first consider the industry-average Lerner Index, which summarizes the degree to which market output diverges from perfect competition and hence is useful as a proxy for consumer and total surplus (Shapiro (1989)). Let $s_i = q_i/Q$ denote firm i's market share and let $\epsilon = -\left(\partial Q/\partial P\right)\left(P/Q\right)$ denote the price elasticity of demand.

Lemma 2 *Given a vector of hedge rates $\{h_i\}$, the Lerner Index equals*

$$LI \equiv \sum_i \left(\frac{P - C_i'}{P}\right) s_i = \sum_i \frac{s_i^2}{\epsilon}\left(1 - h_i\right)$$

Proof. Substituting $-h_i$ for R_i in equation (3) and rearranging terms, we have

$$\frac{P - C_i'}{P} = -\frac{q_i}{P} P'(Q)(1 - h_i)$$

Making the following substitutions $q_i = s_i Q$ and $-(Q/P) P'(Q) = 1/\epsilon$, pre-multiplying by s_i then summing over all i obtains the result. ■

Lemma 2 shows that each firm's price-cost margin is a product of two terms, the typical Cournot term, s_i^2/ϵ, and a term reflecting the importance of forward contracting, $(1 - h_i)$. The LI can be evaluated at the SPE hedge rates, $h_i = B_i/(1+B_i)$, but it also holds for an arbitrary vector of hedge rates, keeping in mind that s_i and ϵ are themselves functions of the hedge rates. As hedge rates increase uniformly from zero to unity, price-cost margins and hence consumer and total surplus, span the Cournot outcome at one extreme and perfect competition at the other. Again, the structural conditions that give rise to larger hedge rates hedge rates close to unity are the same conditions that give rise to competitive outcomes in the absence of forward contracting.

The results already established are sufficient to establish that forward markets have the greatest impact on outcomes in markets characterized by some intermediate level of competition. The two-stage model is equivalent to the baseline model of Cournot competition in the monopoly case (Lemma 1), and both models converge to perfect competition as market shares approach zero (Lemma 2). Thus, if forward markets lower price and increase output (Proposition 1) then the magnitude of these must be maximized in markets with firms that have market shares bounded strictly by zero and unity.

We measure competition and its converse, concentration, by the Herfindahl-Hirschman Index ("HHI"). The HHI is the sum of squared market shares, attaining a maximum of unity in the monopoly case and asymptotically approaching zero as the market approaches perfect competition. The HHI is an appealing statistic due to the theoretical link between HHI and welfare within the baseline Cournot model.[3] Let ρ^{CS} denote the ratio of consumer surplus in the SPE of two-stage model to consumer surplus in Cournot, holding constant all model parameters. Let ρ^{TS} denote the analogous ratio with respect to total surplus. We have that,

Corollary 1 *An incremental decrease in HHI from unity leads to an increase in* $\rho \in \{\rho^{CS}, \rho^{TS}\}$*. As HHI declines further, there exists for each* $\rho \in \{\rho^{CS}, \rho^{TS}\}$*,*

[3] Notice that when all $h_i = 0$, $LI = HHI/\epsilon$.

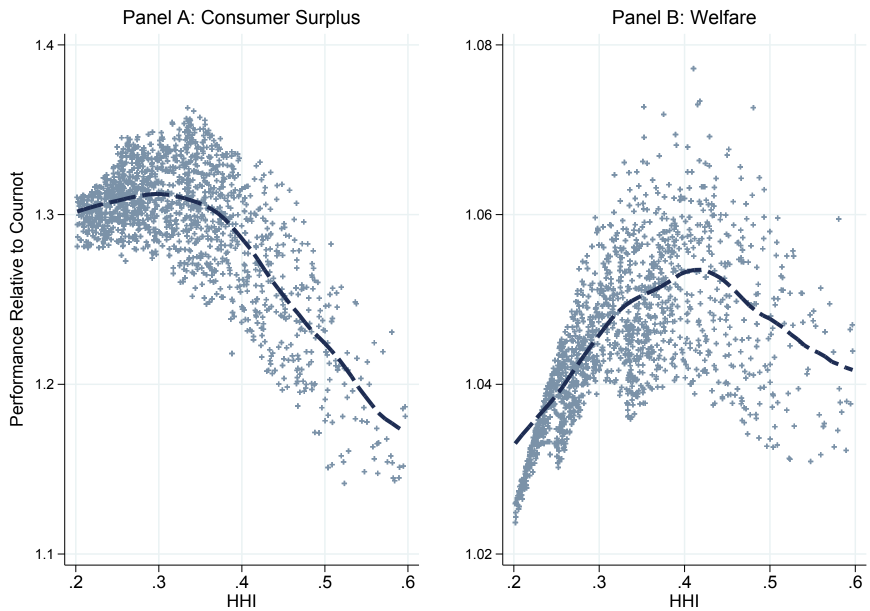

Figure 1: Consumer Surplus and Welfare with Heterogeneous Capital Stocks

a $\tilde{HHI}(\rho)$ *such that* ρ *is decreasing for all* $HHI < \tilde{HHI}(\rho)$.

We develop numerical evidence using Monte Carlo techniques to illustrate the nonmonotic relationship suggested by Corollary 1. For each of many "industries", we obtain the structural parameters of the two-stage model (a, b, c, $\mathbf{k}$), for $\mathbf{k} = \{k_1, ..., k_N\}$, based on randomly-drawn market shares and average margins plus normalizations on price and total output. We obtain consumer and total surplus. We also calculate what these surplus statistics would be under Cournot competition ($h = 0$) with heterogeneous capital stocks. Details on the calibration and simulation techniques are provided in Appendix B.

Figure 1 summarizes the results. The vertical axes provides the ratio of surplus in the two-stage model to surplus in the Cournot model. The horizontal axes shows the HHI. Each dot represents a single randomly-drawn industry, and the lines provide nonparametric fits of the data. All values are above unity because forward markets enhance consumer surplus and welfare at least weakly. Further, consistent with the corollary, forward markets increase surplus the most at intermediate levels of competition. The gain in consumer surplus is maximized at an HHI around 0.30, which corresponds roughly to a symmetric three firm oligopoly. The gain in total surplus is maximized at an HHI around 0.40, between the symmetric triopoly and duopoly levels.

To push intuition further, we consider the special case of symmetric firms and constant marginal costs ($d = 0$). The following expressions for consumer and total surplus can be obtained:

$$
\begin{aligned}
CS(h) &= \frac{(a-c)^2}{2}\left(\frac{N}{N+1-h}\right)^2 \\
W(h) &= \frac{(a-c)^2}{2}\left(\frac{N}{N+1-h} - \frac{1}{2}\left(\frac{N}{N+1-h}\right)^2\right)
\end{aligned}
$$

The analogous expressions with perfect competition are $CS(1) = W(1) = \frac{(a-c)^2}{2}$. Thus, the levels of consumer surplus and of total surplus in the two-stage model, *relative to perfect competition*, are free of the demand and cost parameters and depend only on the number of firms and the hedge rate. This holds for any given hedge rate, including the SPE rate.

Figure 2 plots the ratios $CS(h)/CS(1)$ and $W(h)/W(1)$ for the SPE hedge rate in the two-stage model as well as for a number of fixed-hedge-rate regimes including Cournot ($h = 0$) with $N = 1, ..., 6$. We select fixed hedge rates of 50%, 67%, and 75% because the SPE hedge rates align with these values for $N = 2$, $N = 3$, and $N = 4$, respectively, by equation (6).[4] Thus the surplus values in the two-stage model equal those of Cournot with $N = 1$, those of the 50% hedge rate with $N = 2$, and so on. A value of 0.60, for example, indicates that the model under consideration generates 60% of the perfect competition level. A number of patterns are evident: (i) higher hedge rates are associated with greater consumer and total surplus for any given N; (ii) increasing N leads to greater consumer and total surplus; and (iii) the two-stage model adds "curvature" to the Cournot surplus-concentration relationships, such that both ratios increase faster with N when N is small but slower when N is large.

3 Mergers

In this section, we analyze the welfare impacts of consolidation, which we treat as the transfer of capital stock from small to large firms. Mergers are inherently consolidating regardless of whether the larger or smaller firm is the acquirer since

[4]With constant marginal costs $\beta = 1$, so $h = \frac{N-1}{N}$ in SPE.

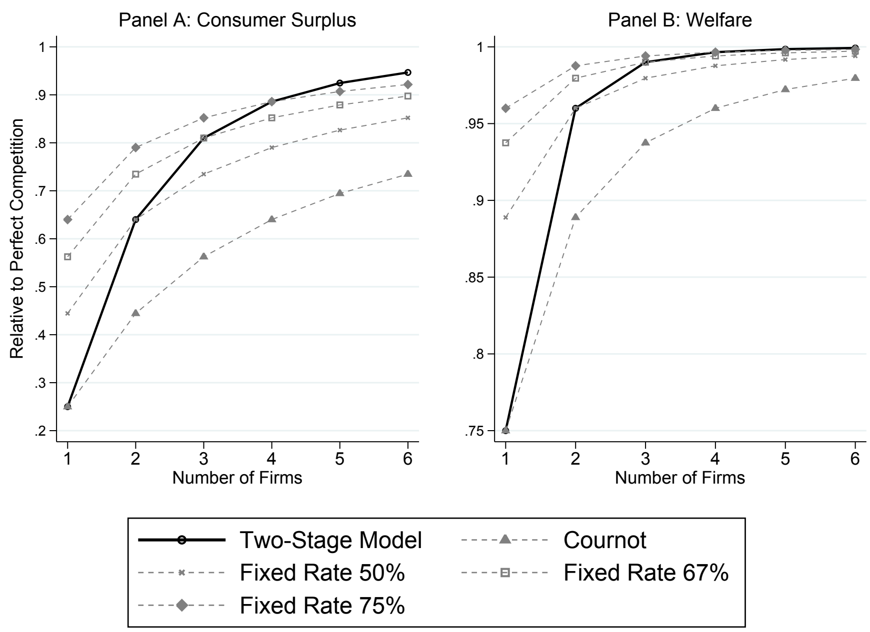

Figure 2: Consumer Surplus and Welfare with Constant Marginal Costs

the merged firm's capital stock will be larger than either of the merging parties', leaving the capital stocks of other firms unchanged. Our interest extends beyond mergers to partial acquisitions as many real-world applications involve the sale of individual plants. Even when evaluating full mergers, antitrust authorities must often consider whether and to what extent a partial divestiture might offset the anticompetitive harm from the merger.

Our results derive from an analytic "first-order" approach which we supplement in places with simulations. The analytic approach examines effects of small consolidating transfers, restricting attention to pairwise transfers of capital from any firm 2, say, to any firm 1, where $k_1 \geq k_2$.[5] Holding fixed the total capital stock controlled by the two firms, a consolidation of capital among firms 1 and 2 is a transfer of some amount, Δk, such after the transfer, firm 1 has capital stock $k_1 + \Delta k$ while firm 2 has capital stock $k_2 - \Delta k$, leaving the total unchanged. Our analytical approach illuminates the mechanisms un-

[5]Jaffe and Wyle (2013) and Farrell and Shapiro (1990) employ this approach. It should be noted that they do not analyze how the merger changes firms' conjectural variations as we do.

derlying our results while avoiding the integer problem inherent in the analysis of full mergers. Extrapolating to larger transfers such as full mergers involves integrating over these first-order effects. When first-order effects are insufficient to evaluate larger transfers or otherwise are aided by additional illustration, we provide simulations of full mergers.

3.1 Effects on consumer surplus

We begin by analyzing the effect of consolidation on consumer surplus. To the extent that antitrust agencies review mergers under a consumer surplus standard, our results should be directly applicable to antitrust policy. We can express consumer surplus within the SPE of the two-stage game as,

$$CS = \int_0^Q (a - bt - P)\, dt = \frac{b}{2} Q^2.$$

It follows that any transfer of capital that reduces the equilibrium quantity reduces consumer surplus.

Proposition 2 *Within the SPE of the two-stage game, all consolidating transfers reduce consumer surplus. The loss of consumer surplus due to a consolidating transfer is mitigated if each firm's hedge rate remains fixed at its pre-transfer value.*

All proofs in this section are in Appendix A. That consolidation leads to lower output should not be surprising as the result holds within the baseline model of Cournot competition. What it interesting is that the reduction is output is magnified when firms adjust their hedge rates in response to consolidation as they do in the SPE of the two-stage game. We can deconstruct the output effect into two components, a *structural effect*, which measures the change in output holding each firm's hedge rate fixed, and a *conjectural effect*, which measures the incremental change in output due to changes in how the new structure changes firm's perceived conjectures.

To see this, we have that the change in consumer surplus due to a consolidating transfer of capital is,

$$\Delta CS = b\Delta Q = \frac{a - c}{(1 + M)^2} \sum_i \Delta \mu_i$$

where,

$$\Delta\mu_i = \begin{cases} \left(\frac{\mu_i}{\beta_i}\right)^2 \Delta\beta_i - \mu_i^2 \Delta R_i & \text{if } i = 1,2 \\ -\mu_i^2 \Delta R_i & \text{if } i \neq 1,2 \end{cases}$$

Collecting the $\Delta\beta_i$ terms and the ΔR_i terms, respectively, we can express the change in consumer surplus as, $\Delta CS = SE^{CS} + CE^{CS}$, where,

$$\begin{aligned} SE^{CS} &\equiv \frac{a-c}{(1+M)^2}\left[\left(\frac{\mu_1}{\beta_1}\right)^2 \Delta\beta_1 + \left(\frac{\mu_2}{\beta_2}\right)^2 \Delta\beta_2\right] \\ CE^{CS} &\equiv -\frac{a-c}{(1+M)^2}\sum_i \mu_i^2 \Delta R_i \end{aligned}$$

The structural effect says that a transfer of capital to firm 1 from the smaller firm 2 leads firm 1 to increase and firm 2 to decrease output. The net result is a reduction in the combined output of the parties to the transaction. This is because of increasing marginal costs. As firm 1 increases its output in response to the added capital, its marginal cost rises which curbs firm 1's incentive to expand output. Firm 2's marginal cost declines as it lowers its output, which promotes greater output reduction. Third parties react to this net decline in the parties' output by expanding output. The combined increase across all third parties is not enough to offset the decline in the parties' output.

The change in structure due to a consolidating transfer changes what firms conjecture as to how their competitors will respond to a change in their contracted quantity. Third parties anticipate that the parties to the transaction will be less responsive to their contracted quantities on the basis that the parties are expected to produce less overall. This diminishes the strategic rationale for third parties to contract forward. Firm 1 anticipates that firm 2 will be less responsive to its contracted quantity, so its incentive to contract forward is reduced. Firm 2 anticipates that firm 1 will be more responsive to its contracted quantity, but the increase in firm 2's contracted quantity does not offset the decrease in firm 1's contracted quantity. It is clear that the net effect is a reduction in contracted quantity and a reduction in total output.

Next, we ask how the effect of a consolidating transfer on consumer surplus depends on the level of concentration post- (or pre-) transaction.[6] Using the HHI as our measure of concentration, we treat an increase in concentration as

[6] Pertaining to first-order effects, there is no distinction between pre- and post-transaction concentration.

any perturbation of the market structure such that k_i is increased relative to k_j where $k_i > k_j$, keeping the total capital stock constant.

Proposition 3 *Within the SPE of the two-stage game, the loss of consumer surplus due to a consolidating transfer is increasing in the HHI.*

Again, this result also holds within the baseline model, however, what the proof of this result reveals is that the conjectural effect becomes more pronounced for higher values of the HHI. In this way, the structural effect and conjectural effect are mutually reinforcing. It is also interesting to note that while any perturbation that gives greater amounts of capital to larger firms will increase HHI, a given change in HHI caused by increasing k_1 relative to k_2 has a greater impact on the loss of consumer surplus due to a consolidating transfer.

Given that the structural and conjectural effects are reinforcing, it is natural to ask whether the effect of consolidation on consumer surplus is more pronounced within the two-stage game relative to the baseline Cournot game. Let κ_1 denote the proportion of industry capital stock held by firm 1.

Proposition 4 *There exists a threshold $\bar{\kappa}$ such that for $\kappa_1 > \bar{\kappa}$, the reduction in consumer surplus due to a consolidating transfer is larger in the SPE of the two-stage model than in the baseline model, holding fixed the allocation of capital.*

The reason why the two-stage model doesn't always lead to a greater reduction in consumer surplus is that consumer surplus depends on the pre-transaction hedge rate. Within the two-stage model, the effect of a change in structure on each firm's output is proportional to its pre-transaction output. Recall that each firm's output is larger within the two-stage model than the baseline, holding fixed the allocation of capital. Therefore, the structural effect is muted relative to the baseline model, substantially so when the industry is fairly unconcentrated. Recall from Section 2.3 that hedge rates decline in concentration. It follows that as the capital stock becomes more concentrated, hedge rates decline and each firm's output in the two-stage game converges to its output in the Cournot game. Proposition 4 establishes the existence of a threshold level of concentration where the conjectural effect exactly offsets the greater structural effect within the baseline model.

Since Proposition 4 is a statement about first-order effects, it is ambiguous whether this result extends to large transactions including full mergers. For example, suppose $\kappa_1 < \bar{\kappa} < \kappa_1 + \Delta\kappa$ as might reasonably be the case if the transfer of capital $\Delta\kappa$ is large and κ_1 is close to unity. In this case, the first incremental transfer of capital reduces surplus by less than in the base model while the last incremental transfer reduces surplus by more than in the base model, so that the net effect is ambiguous. We use simulations to get around this issue. Details of the simulations can be found in the Appendix.

Figure 3 examines how forward contracts affect consumer surplus losses due to mergers among the set of randomly drawn industries. Again, each industry represents a single parameterization obtained from randomly-drawn market shares and average margins. Within each industry we examine every pairwise merger. We calculate the ratio of consumer surplus loss with forward contracts to the corresponding loss without forward contracts (i.e., setting $h = 0$). This appears as the vertical axis on the graph. Values above unity represent mergers for which forward contracts amplify consumer surplus loss. The horizontal axis is the post-merger HHI. As shown, the relative consumer surplus loss with forward contracts increases with the post-merger HHI. Forward contracts tend to amplify loss for HHIs above 0.50, which is equivalent to symmetric duopoly.

3.2 Profitability

We now consider how the presence of a forward market affects the incentive to consolidate.

Proposition 5 *All consolidating transfers increase the combined profit achieved by the parties in the SPE of the two-stage game.*

This result is quite astounding given that in the baseline model, many consolidating transfers are not profitable. As before, we can deconstruct the profit-effect of consolidation into a structural effect and a conjectural effect. Whereas the structural effect is ambiguous, the conjectural effect is always positive. As the proof of Proposition 5 shows, the conjectural effect always dominates the structural effect. This suggests an added motive for consolidating: given the presence of a forward market constrains a firm's ability to exercise market power, the firm has an incentive to loosen that constraint through consolidation. It is reasonable that this logic would extend to activities like advertising that are individually rational but collectively (to firms) damaging.

The result of Proposition 5 helps to offer a more complete response to the "merger paradox." Salant, Switzer, and Reynolds (1983) examined the incentive

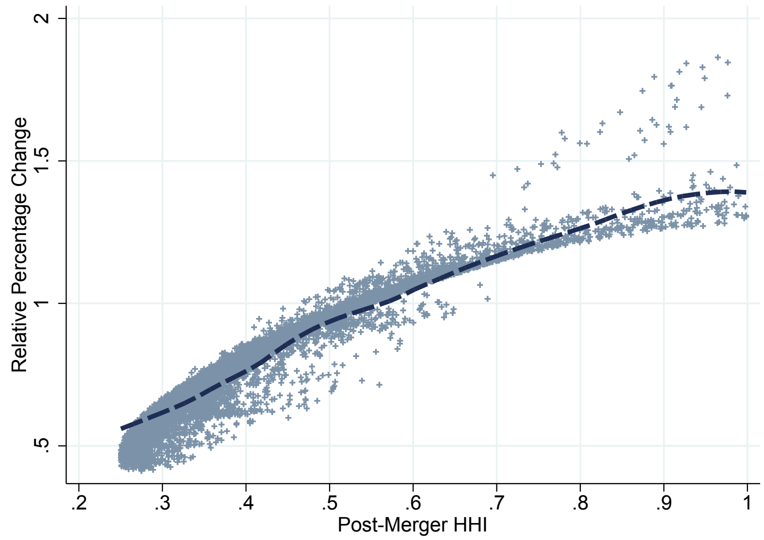

Figure 3: Relative Consumer Surplus Loss with Forward Markets
Notes: The vertical axis provides the percentage change in consumer surplus with forward markets divided by the percentage change without forward markets, given the same parameterization. Values above unity represent the effects of mergers for which forward markets amplify consumer surplus loss. The horizontal axis provides the post-merger HHI. The line provides a nonparametric fit of the data.

to merge within a symmetric model where firms compete on quantity which they produce at a constant marginal cost. They find that mergers are not profitable unless when combining at least 80 percent of firms. It is difficult to explain the prevalence of mergers in light of this result, hence the paradox. Deneckere and Davidson (1985) alter the assumption that firms compete on quantity and show that mergers are always profitable when firms offering differentiated products compete on price.[7] But the assumption that products are differentiated may not be applicable in many settings such as the sale of commodities or wholesale electricity.[8] Perry and Porter (1985) argue that the failure to explain the profitability of mergers is actually a misconception since the mergers are not well-defined conceptually when firms can produce seemingly infinite quantities

[7]Because prices are strategic complements, an increase in the merging firms' prices is met by an increase in the prices of third-party goods, hence mergers are profitable. When firms instead compete on quantities, a decrease in the merging parties' quantities is met with an increase in third parties' quantities, so that mergers are only profitable if the third-party response is sufficiently muted.

[8]With undifferentiated products, Bertrand competition forces price to marginal cost, so that only mergers to monopoly are profitable.

at a constant marginal cost. They propose a model of capital stocks, the same model we have adopted, and find that smaller mergers can indeed be profitable even when firms compete on quantities. Yet many mergers within their framework are unprofitable. Proposition 5 shows that supplementing Perry and Porter (1985) with a single round of forward contracting is sufficient for all mergers to be profitable.[9]

Figure 4 plots the merging parties' profits over the post-merger HHI for the two-stage model (Panel A) and Cournot (Panel B). It is evident that all mergers within the two-stage model are profitable whereas in the baseline model, many are not. In both models, merger profitability is increasing in HHI. This reflects two potential forces, the level of concentration pre-merger and the size of the merger. Mergers of all sizes are more profitable in more highly concentrated industries, while large mergers may serve to significantly increase concentration within an industry.

3.3 Implications for policy and industry structure

The results of this section have direct implications for antitrust policy. Antitrust authorities serve to uphold the relevant antitrust statutes and in doing so, challenge mergers and other such transactions that violate them. Though we have not modeled merger-specific efficiencies and other factors antitrust authorities may consider, our results point to situations where consolidating transactions may be especially problematic and where considering the role of a forward market may change their determination. In theory, all of the consolidating transactions we have modeled are problematic as all reduce consumer surplus whether or not a forward market is present. In practice, however, one might expect consolidating transactions to be challenged only when the consumer surplus preserve in doing so exceeds the resource cost of litigation. In that sense, magnitudes matter.

An implication of Proposition 4 is that when evaluating a merger in an industry with a robust forward market, antitrust authorities may arrive at the incorrect conclusion if their analysis does not explicitly model the impact of a forward market. In particular, the greater the share of the industry's productive

[9] One could also argue that there is no paradox at all even within the framework of Salant, Switzer, and Reynolds (1983). Suppose it is costly to simultaneously negotiate mergers among multiple firms, so that each merger must occur sequentially and involve no more than, say, two firms. In such a world, mergers that are unprofitable in the immediate term would be tolerated by forward looking firms if they lead in the long run to an industry structure where the terminal merger is profitable.

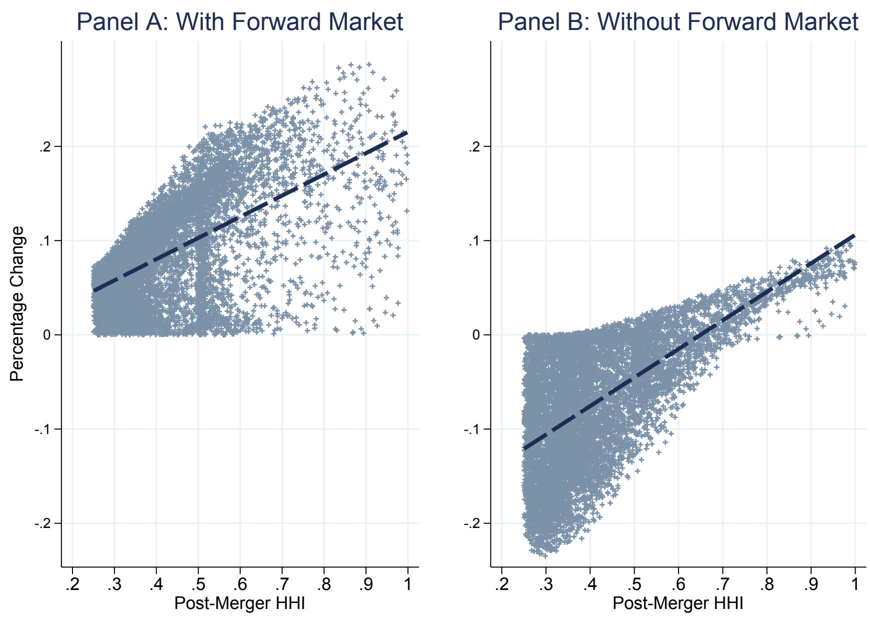

Figure 4: The Effect of Mergers on Producer Surplus

Notes: The figure provides the percentage change in producer surplus captured by the two merging firms for varying levels of post-merger HHI. Also provided are lines of best fit.

assets held by the acquiring firm, the more likely it is than an analysis incorrectly based on the baseline model will fail to identify a merger that should be challenged. This is a type-2 error in the parlance of statistical inference.[10] If mergers in industries with robust forward markets have a higher type-2 error rate, then all else equal, these industries should see more consolidating transactions. This is because not only are highly concentrating transactions more likely to go unchallenged in industries with forward markets, but such transactions are more likely to be profitable in industries with forward markets.

[10]There is also the possibility for type-1 errors when the acquiring firm has a small share of the industry's productive assets. However, since the loss of consumer surplus under the baseline model is likely to be quite small in these circumstances, such a merger is unlikely to be challenged.

4 Conclusions

We have analyzed consolidation in the presence of a forward market. Our results show that the welfare effects of consolidation are sensitive to the presence of a forward market in important ways. While our model presupposes the existence of a forward market, it is not hard to conceive of forward sales emerging organically. Whenever quantity is the strategic variable and whenever the terms of sale can be revealed to a firm's competitors, a firm will have the strategic incentive to make sales in advance of production. To the extent that such transactions do occur, the applicability of our results may well extend beyond the commodities with established futures markets.

While our results should be relevant for policy makers in the merger review process, we believe an appropriate level of caution should be exercised. The model of capital stocks which we have employed throughout is limiting as it does not reflect firms' actual marginal cost functions. In practice, consolidation may change the shape of firms' marginal cost functions in ways that exacerbate or mitigate harm from mergers.

We have also assumed the strategic variable to be quantity. In wholesale electricity markets, spot prices are determined based on price-quantity schedules submitted by firms. In the supply-function equilibrium model of Klemperer and Meyer (1989), supply functions can be strategic substitutes or complements. Mahenc and Salanie (2004) study strategic complements in the context of differentiated Bertrand spot market competition and find that forward contracting increases spot market prices. We are aware of no studies that analyze the effect of mergers within this context. If consolidation lessens the incentive to contract in advance, then harm from consolidation is mitigated relative to our results.

Finally, we have assumed that all agents have perfect foresight so that the only motive for firms to sell in the contract market is to influence spot market competition. As we do not believe this to be the case in practice, our assumption of perfect foresight was made for the sake of tractability. Allaz (1992) and Hughes and Kao (1997) show that when foresight is imperfect and firms are risk averse, equilibrium hedge rates are higher than in the perfect-foresight case. How hedge rates change in response to a merger in this setting has not been explored to our knowledge. However, it is conceivable that our basic findings would still obtain. Consolidation, by increasing market power, increases the value to the merged firm of withholding output. To the extent that forward contracting even for the sake of hedging risk comes at the expense of exercising market power, mergers may well limit the incentive for firms to forward contract. We leave this issue and the other issues posed in this section to future research.

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Appendices

A Proofs

A.1 Proof of Proposition 2

The proof proceeds in two parts, first showing that the structural effect is negative, then showing that the conjectural effect is negative.

Lemma 3 $SE^{CS} \equiv \frac{a-c}{(1+M)^2}\left[\left(\frac{\mu_1}{\beta_1}\right)^2 \Delta\beta_1 + \left(\frac{\mu_2}{\beta_2}\right)^2 \Delta\beta_2\right] \leq 0$

Proof. Using,

$$\Delta\beta_1 = bd\left(\frac{\beta_1}{bk_1}\right)^2 \Delta k \tag{A.1}$$

and,

$$\Delta\beta_2 = -bd\left(\frac{\beta_2}{bk_2}\right)^2 \Delta k = -\left(\frac{\beta_2}{bk_2}\right)^2\left(\frac{\beta_1}{bk_1}\right)^{-2}\Delta\beta_1 \tag{A.2}$$

SE^{CS} can be expressed as,

$$SE^{CS} = \frac{a-c}{(1+M)^2}\left[\left(\frac{\mu_1}{\beta_1}\right)^2 - \left(\frac{\mu_2}{\beta_2}\right)^2\left(\frac{\beta_2}{bk_2}\right)^2\left(\frac{\beta_1}{bk_1}\right)^{-2}\right]\Delta\beta_1$$

Since $\Delta\beta_1 > 0$, it is sufficient to show that the square-bracketed term is nonpositive. This reduces to,

$$\left(\frac{\mu_1}{bk_1}\right)^2 - \left(\frac{\mu_2}{bk_2}\right)^2 \leq 0$$

Using difference-of-squares (i.e. $x^2 - y^2 = (x+y)(x-y)$), it is sufficient that

$$\frac{\mu_1}{bk_1} - \frac{\mu_2}{bk_2} \leq 0,$$

or equivalently,

$$\mu_1 k_2 - \mu_2 k_1 \leq 0$$

By construction, $k_1 \geq k_2$. We can define $\delta \geq 0$ such that $k_2 \equiv k_1 - \delta$. The above inequality simplifies to,

$$\left(\mu_1-\mu_2\right)k_1-\mu_1\delta\leq 0 \tag{A.3}$$

Using the identity,

$$\mu_i=\frac{\beta_i}{1+\beta_i R_i}=\frac{\beta_i\left(1+B-\beta_i\right)}{\left(1+B\right)\left(1-\beta_i\right)+\beta_i^2} \tag{A.4}$$

we have that,

$$\begin{aligned}\left(\mu_1-\mu_2\right)k_1 &= \frac{\left(1+B\right)\left(1+B-\beta_1-\beta_2\right)\left(\beta_1-\beta_2\right)k_1}{\left[\left(1+B\right)\left(1-\beta_1\right)+\beta_1^2\right]\left[\left(1+B\right)\left(1-\beta_2\right)+\beta_2^2\right]}\\ &= \frac{\left(1+B\right)\left(1+B-\beta_1-\beta_2\right)\beta_1\left(1-\beta_2\right)\delta}{\left[\left(1+B\right)\left(1-\beta_1\right)+\beta_1^2\right]\left[\left(1+B\right)\left(1-\beta_2\right)+\beta_2^2\right]}\end{aligned}$$

If $\delta=0$, then condition (A.3) holds trivially. If $\delta>0$, condition (A.3) reduces to,

$$-\left(1+B\right)\beta_2\left(1-\beta_2\right)\leq\left(1+B-\beta_1\right)\beta_2^2$$

which is true by construction. ■

Lemma 4 $CE^{CS}\equiv-\frac{a-c}{(1+M)^2}\sum_i\mu_i^2\Delta R_i\leq 0$

Proof. We have that,

$$\Delta R_i=-\frac{\Delta B_{-i}}{\left(1+B_{-i}\right)^2},$$

where,

$$\Delta B_{-i}=\begin{cases}\Delta\beta_2 & \text{if } i=1\\ \Delta\beta_1 & \text{if } i=2\\ \Delta\beta_1+\Delta\beta_2 & \text{if } i>2\end{cases}$$

It follows that,

$$CE^{CS}\left(\frac{a-c}{(1+M)^2}\right)^{-1} = [\Delta\beta_1+\Delta\beta_2]\sum_{j\neq 1,2}\left(\frac{\mu_j}{1+B_{-j}}\right)^2 + \left[\left(\frac{\mu_1}{1+B_{-1}}\right)^2\Delta\beta_2 + \left(\frac{\mu_2}{1+B_{-2}}\right)^2\Delta\beta_1\right]$$

It sufficices to show that each of the square-bracketed terms are negative. From equations (A.1) and (A.2) we have that,

$$\begin{aligned}
\Delta\beta_1+\Delta\beta_2 &= \left[1-\left(\frac{\beta_2}{bk_2}\right)^2\left(\frac{\beta_1}{bk_1}\right)^{-2}\right]\Delta\beta_1\\
&= \left[\left(\frac{\beta_1}{bk_1}\right)^2-\left(\frac{\beta_2}{bk_2}\right)^2\right]\left(\frac{\beta_1}{bk_1}\right)^{-2}\Delta\beta_1\\
&= \left[\frac{\beta_1}{bk_1}-\frac{\beta_2}{bk_2}\right]\left(\frac{\beta_1}{bk_1}+\frac{\beta_2}{bk_2}\right)\left(\frac{\beta_1}{bk_1}\right)^{-2}\Delta\beta_1\\
&= -\left(\frac{\beta_2\delta}{bk_1k_2}\right)\left(\frac{\beta_1}{bk_1}+\frac{\beta_2}{bk_2}\right)\left(\frac{\beta_1}{bk_1}\right)^{-2}\Delta\beta_1\\
&\leq 0
\end{aligned}$$

This term is the structural effect in the baseline Cournot model. In the current setting, because the parties are expected to reduce output, they will be less responsive to forward sales of third parties.

Finally, we have that,

$$\begin{aligned}
&\left(\frac{\mu_1}{1+B_{-1}}\right)^2\Delta\beta_2+\left(\frac{\mu_2}{1+B_{-2}}\right)^2\Delta\beta_1\\
&\quad= \left[\left(\frac{\mu_2}{1+B_{-2}}\right)^2\left(\frac{\beta_2}{bk_2}\right)^{-2}-\left(\frac{\mu_1}{1+B_{-1}}\right)^2\left(\frac{\beta_1}{bk_1}\right)^{-2}\right]\left(\frac{\beta_2}{bk_2}\right)^2\Delta\beta_1
\end{aligned}$$

Due to difference-of-squares, it is sufficient that,

$$\left(\frac{\mu_2}{1+B_{-2}}\right)\left(\frac{\beta_2}{bk_2}\right)^{-1}-\left(\frac{\mu_1}{1+B_{-1}}\right)\left(\frac{\beta_1}{bk_1}\right)^{-1}\leq 0$$

Using equation (A.4), this is equivalent to,

$$\left[(1+B)(1-\beta_1)+\beta_1^2\right]k_2+\left[(1+B)(1-\beta_2)+\beta_2^2\right]k_1\leq 0$$

Using the identity, $k_2 = k_1 - \delta$, this reduces to,

$$-(1+B-\beta_1-\beta_2)\,\beta_1(1-\beta_2)\,\delta-\left[(1+B)(1-\beta_1)+\beta_1^2\right]\delta\leq 0,$$

which is true by construction. ■

From Lemmas 3-4, we have that $\Delta CS = SE^{CS} + CE^{CS} < 0$, which establishes the first argument of the proposition. The second argument is that $CE^{CS} < 0$ which is shown by Lemma 4.

A.2 Proof of Proposition 3

First we show that a perturbation that increases firm 1's pre-transaction market share and decreases firm 2's share increases HHI while decreasing ΔCS. The proof of Proposition 2 shows that ΔCS is proportional to $\delta \equiv k_1 - k_2$ such that an increase in δ makes ΔCS more pronounced. Since an increase in δ increases firm 1's market share while reducing firm 2's share, which was already smaller than firm 1's share, HHI is increased.

Next, we show that a perturbation that increases concentration among third parties increases HHI and makes ΔCS more pronounced. From Lemma 5 (below), the structural effect becomes more pronounced as hedge rates decline toward zero. Increasing concentration among third parties has the effect of reducing hedge rates in the same way, which makes the structural effect more pronounced.

It remains to show that the conjectural effect becomes more pronounced as concentration increases among third parties. Let $\left(CE^{CS}\right)'$ denote the change in the conjectural effect due to an increase in concentration among third parties. We have that,

$$\left(CE^{CS}\right)'=-\frac{2\cdot CE^{CS}}{1+M}\left(M\right)'-\frac{a-c}{\left(1+M\right)^2}\sum_i\left(\mu_i^2\Delta R_i\right)'$$

We show that $(M)' < 0$ using the same arguments used to show $\Delta CS < 0$. There is a structural effect that reduces output across all the parties directly affected by the purturbation. There is also a conjectural effect which reduces μ_i for each of the non-affected parties. Since $CE^{CS} < 0$, it follows that,

$-\frac{2 \cdot CE^{CS}}{1+M}(M)' < 0$.

Using equation (A.4), it is easy to show that $\mu_i^2 \Delta R_i$ is increasing in B_{-i}. Since B_{-i} is decreasing in concentration, $-\left(\mu_i^2 \Delta R_i\right)' < 0$ for unaffected parties. For the affected parties, the argument is the same as used in the proof of Lemma 4 for why, on net, the parties to the transaction reduce their forward sales. That is, if we perturb the distribution of capital among a subset of third parties, they will reduced their forward sales so that across the firms affected by the perturbation, $-\sum_i \left(\mu_i^2 \Delta R_i\right)' < 0$.

A.3 Proof of Proposition 4

The loss of consumer surplus in the baseline model due to a consolidating transfer is,

$$\Delta CS^0 = \frac{a-c}{(1+B)^2}\left(\Delta\beta_1 + \Delta\beta_2\right)$$

Lemma 5 $\Delta CS^0 \leq SE^{CS} \leq 0$. *The first inequality is strict in all but the monopoly case. The second inequality is strict as long as $k_1 > k_2$.*

Proof. It is sufficient to show that, $\left(\frac{\mu_1}{\beta_1}\right)^2 > \left(\frac{\mu_2}{\beta_2}\right)^2$. Using equation (A.4), this expression reduces to

$$(1+B)(B-\beta_1-\beta_2) + \beta_1\beta_2 > 0 \tag{A.5}$$

which is true by construction. ■

The proof uses limits to show that as the industry approaches monopoly, the loss of surplus is greater under the two-stage model.

Lemma 6 $\lim_{\kappa\to 1} SE^{CS} = \lim_{\kappa\to 1} \Delta CS^0 < 0$.

Proof. In the limit, $\beta_1 \to B$ and $\beta_2 \to 0$. From equation (A.4), we have that,

$$\lim_{\kappa\to 1}\left(\frac{\mu_1}{\beta_1}\right) = \frac{1}{(1+B)(1-B)+B^2} = 1$$

and

$$\lim_{\kappa\to 1}\left(\frac{\mu_1}{\beta_1}\right) = \frac{1+B}{1+B} = 1.$$

Let K denote industry capital stock. It follows that,

$$
\begin{aligned}
\lim_{\kappa\to 1} SE^{CS} = \lim_{\kappa\to 1} \Delta CS^0 &= \lim_{\kappa\to 1} \left(\Delta\beta_1 + \Delta\beta_2\right) \\
&= \left(\frac{1}{\left(bK+d\right)^2} - \frac{1}{d^2}\right) bd\Delta k < 0
\end{aligned}
$$

■

Lemma 7 $\lim_{\kappa\to 1} CE^{CS} < 0$

Proof. Using equation (A.4), we have that,

$$
\lim_{\kappa\to 1} CE^{CS} = -\frac{bB}{d}\Delta k < 0
$$

■

It follows that,

$$
\begin{aligned}
\lim_{\kappa\to 1} \Delta CS &= \lim_{\kappa\to 1} \Delta CS^0 + \lim_{\kappa\to 1} CE^{CS} \\
&< \lim_{\kappa\to 1} \Delta CS^0
\end{aligned}
$$

A.4 Proof of Proposition 5

To be written.

B Data Generating Process

To be written.