

# A Price Leadership Model for Merger Analysis\*

Ryan Mansley  
Georgetown University<sup>†</sup>

Nathan H. Miller  
Georgetown University<sup>‡</sup>

Gloria Sheu  
Federal Reserve Board<sup>§</sup>

Matthew C. Weinberg  
The Ohio State University<sup>¶</sup>

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## Abstract

We provide a methodology to simulate the coordinated effects of a proposed merger using data commonly available to antitrust authorities. The model follows the price leadership structure in Miller, Sheu, and Weinberg (2021) in an environment with logit or nested logit demand. The model calibration leverages profit margin data to separately identify the extent of coordinated pricing from marginal costs. Using this framework, we demonstrate how mergers can shift incentive compatibility constraints and thereby lead to adverse competitive effects. The incentive compatibility constraints also affect the extent to which cost efficiencies and divestitures mitigate competitive harms.

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<sup>†</sup>Georgetown University, Department of Economics, 37th and O Streets NW, Washington DC 20057. Email: rem133@georgetown.edu.

<sup>‡</sup>Georgetown University, McDonough School of Business, 37th and O Streets NW, Washington DC 20057. Email: nathan.miller@georgetown.edu.

<sup>§</sup>Board of Governors of the Federal Reserve System, 20th Street and Constitution Avenue NW, Washington DC 20551. Email: gloria.sheu@frb.gov.

<sup>¶</sup>The Ohio State University, 410 Arps Hall, 1945 N High Street, Columbus OH 43210. Email: weinberg.133@osu.edu.

# 1 Introduction

Antitrust authorities often use model-based simulations to quantify the competitive effects of mergers and to evaluate the possible tradeoffs between adverse competitive effects and efficiencies such as marginal cost reductions (Davis and Garcés, 2009; Miller and Sheu, 2021). This approach was developed in the academic literature under the Bertrand assumption that each firm sets its prices to maximize its current profit, conditional on the prices of competitors (e.g., Berry and Pakes, 1993; Hausman et al., 1994; Werden and Froeb, 1994; Nevo, 2000). As such, the simulations are intended to capture the changes in unilateral pricing incentives that are created by a merger.<sup>1</sup>

However, it is well understood that mergers also can facilitate or enhance collusion and thereby have “coordinated effects” on market outcomes. The 2010 *Horizontal Merger Guidelines* of the Department of Justice (DOJ) and Federal Trade Commission (FTC) describe coordinated effects as follows:

A merger may diminish competition by enabling or encouraging post-merger coordinated interaction among firms in the relevant market that harms customers. Coordinated interaction involves conduct by multiple firms that is profitable for each of them only as a result of the accommodating reactions of the others.<sup>2</sup>

The *Horizontal Merger Guidelines* also clarify that coordinated effects could, but need not, involve price-fixing or other explicit agreements:

Coordinated interaction includes a range of conduct. Coordinated interaction can involve the explicit negotiation of a common understanding of how firms will compete or refrain from competing. Such conduct typically would itself violate the antitrust laws. Coordinated interaction also can involve a similar common understanding that is not explicitly negotiated but would be enforced by the detection and punishment of deviations that would undermine the coordinated interaction.<sup>3</sup>

The economics of coordinated effects have received considerable attention in the literature (e.g., Porter, 2020) and, indeed, coordinated effects may manifest differently than unilateral effects. As one of many examples, if a merger occurs in a market that already features monopoly prices due to collusion, then it is unlikely to cause prices to increase (e.g., Verboven, 1995).

Less well-developed are simulation methods for coordinated effects. There are a number of challenges. For example, it can be unclear how to select among multiple equilibria, given that the folk theorems indicate that there can be many different pricing strategies that constitute a subgame

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<sup>1</sup>Unilateral effects merger simulations have been presented as economic evidence in a number of recent merger cases in the United States and Canada. Examples include *H&R Block/TaxACT* (2011), *AT&T/DirectTV* (2015), *Aetna/Humana* (2016), *Wilhelmsen/Drew Marine* (2018), *Parrish & Heimbecker/Louis Dreyfus Company* (2021), and *Secure/Tervita* (2022).

<sup>2</sup>*Horizontal Merger Guidelines*, §7.

<sup>3</sup>*Horizontal Merger Guidelines*, §7.

perfect equilibrium (SPE) in repeated pricing games. Furthermore, many canonical models of collusion allow for the possibility that incentive compatibility (IC) constrains the prices that can be sustained in SPE (e.g., Green and Porter, 1984; Rotemberg and Saloner, 1986). Thus, it can be necessary to model how punishment might unfold and how firms tradeoff current and future profit. In practice, coordinated effects analyses often default to identifying characteristics of an industry that make it more or less susceptible to collusion.<sup>4</sup> This “checklist” approach is grounded in economic theory but does not quantify harm, and also can yield ambiguity if different aspects of an industry or merger have different implications for the viability of collusion.

In this paper, we examine a modeling framework that can overcome these challenges in specific settings: the model of *oligopolistic price leadership* developed in Miller, Sheu, and Weinberg (2021) (henceforth “MSW”). Our contribution is practical, as we seek to facilitate the appropriate use of the model in merger review. To that end, we simplify the MSW framework to a setting with a single market and logit or nested logit demand. We then show how the model can be calibrated with information that often becomes available to antitrust authorities during the course of merger review. We explore how a number of different mergers affect equilibrium outcomes in the model, and examine the conditions under which efficiencies and divestitures mitigate adverse competitive effects. We also discuss the settings for which the modeling framework may be appropriate.

We organize the paper as follows. We start with the model of price leadership in Section 2. The model is a repeated two-stage game of perfect information. In the first stage of each period, the leader announces a *supermarkup* to be applied above Bertrand prices by members of a coalition. In the second stage, all firms set prices simultaneously. Along the equilibrium path, the leader selects the supermarkup that maximizes its profit subject to the IC constraints of the coalition firms, and the coalition firms adopt the supermarkup in the subsequent pricing stage. Thus, although the leader’s announcement is cheap talk, it provides a coordination device that resolves (by assumption) the multiple-equilibrium problem. The model incorporates a set of fringe firms that price to maximize static profit functions. A merger in this context can affect the IC constraints and thus the magnitude of the supermarkup that emerges in SPE.

The model is intended to provide a reasonable representation of a pricing practice that has been observed across a range of settings. In 1946, the United States Supreme Court ruled that price leadership in the tobacco industry violated the Sherman Act, and this motivated the earliest academic articles on the subject (e.g., Stigler, 1947; Markham, 1951; Oxenfeldt, 1952; Bain, 1960). Anecdotal examples of price leadership are discussed in Scherer (1980), Lanzillotti (2017), and Harrington (2017), and evidence of price leadership has been considered in a number of price-fixing lawsuits when courts have weighed whether discovery should be granted to the plaintiffs.<sup>5</sup> Recent

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<sup>4</sup>This approach is typified by the list of factors discussed in §7.2 of the *Horizontal Merger Guidelines*.

<sup>5</sup>Examples include firms involved in flat glass (*Re: Flat Glass Antitrust Litig.*, 385 F.3d 350 (3rd Cir 2004)), text messaging (*Re: Text Messaging Antitrust Litig.*, 782 F.3d 867 (7th Cir 2015)), titanium dioxide (*Re: Titanium Dioxide Antitrust Litig.*, RDB-10-0318 (D. Md. 2013)), and chocolate (*Re: Chocolate Confectionary Antitrust Litig.*, 801 F.3d 383 (3rd Cir 2015)).

empirical applications document follow-the-leader pricing in retail industries including supermarkets, pharmacies, gasoline, and beer (Clark and Houde, 2013; Seaton and Waterson, 2013; Chilet, 2018; Lemus and Luco, 2021; Byrne and de Roos, 2019; Miller et al., 2021). We suspect price leadership may be (at least somewhat) prevalent because it simplifies the process through which coordinating firms select prices. Thus, what is convenient from a modeling standpoint—that the leader’s announcement effectively selects among the SPE that are generally available under the folk theorems—may also be convenient for the firms involved.

In Section 3, we develop how the primitives of the model can be calibrated to match information that often becomes available to antitrust authorities during merger review. Here we depart from MSW, which pairs demand estimates for the beer market with a supply-side orthogonality condition.<sup>6</sup> Our focus reflects that what is possible in merger review can differ from what is possible for academic research. Specifically, in merger review, the time and data necessary for sophisticated demand estimation may be unavailable whereas, subject to the usual caveats, data on equilibrium objects such as margins and diversion may be possible to obtain from proprietary data or confidential business documents (e.g., Davis and Garcés, 2009; Miller and Sheu, 2021). With logit demand, calibration can be accomplished with as little as market shares, prices, two margins, and a diversion ratio. With somewhat more information, a nested logit demand system can be calibrated to allow for richer consumer substitution patterns. We illustrate by calibrating a version of the model using statistics on the beer industry presented in MSW; our merger simulation results are similar to what is obtained using the estimation-based approach of MSW.

We end the paper by exploring the antitrust implications of horizontal mergers in industries characterized by price leadership (Section 4). Using simulations, we first show how prices change as concentration increases in a market with symmetric firms. Under price leadership, prices can rise more quickly with concentration than under a standard Bertrand equilibrium, and near-monopoly prices can be attained at lower levels of concentration. This points to the additional observation that, to the extent price leadership allows firms to realize monopoly prices, mergers can lead to smaller price increases than would arise with Bertrand equilibrium (for a given market configuration). We then examine a variety of horizontal mergers in a market with asymmetric firms. Mergers involving the coalition firm with the binding IC constraint can have outsized effects on prices and welfare, as these mergers tend to relax the IC constraint and thereby result in higher equilibrium supermarkups. Another class of mergers that can have large effects involves the acquisition of a fringe firm by a coalition firm—the merged firm internalizes that the price of the fringe product affects IC constraints within the coalition, and this can amplify price changes.

We also show that price leadership can impact the ability of efficiencies and divestitures to mitigate the effects of a merger. For example, a merger that creates efficiencies for the firm with the binding IC constraint also creates slack in that firm’s IC constraint, and this can lead to *higher*

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<sup>6</sup>The specific supply-side orthogonality assumption used is that the marginal costs of Anheuser-Busch did not change with the Miller/Coors joint venture differently from the costs of Modelo and Heineken.

supermarkups in equilibrium. Such an effect does not arise if the merger does not involve the firm with the binding constraint. With regard to divestitures, we provide examples in which they can amplify, diminish, be neutral for the coordinated effects of mergers. What appears to matter is who has the binding IC constraint—one of the merging firms, the divestiture recipient, or some other firm. Putting these results together, the intuitions for efficiencies and divestitures that arise under Bertrand competition do not always extend cleanly to models of pricing coordination.

We conclude in Section 5 with a summary, a discussion about the applicability of the model, and directions for future research. Our work contributes to a broad literature that develops and evaluates methodologies with which to analyze the competitive effects of mergers. Two contributions that are close to our own are Moresi et al. (2015) and Igami and Sugaya (2021), which focus on understanding coordinated effects outside the price leadership context. More research analyzes unilateral effects. Among the topics that have been examined are upward pricing pressure (e.g., Farrell and Shapiro, 2010; Jaffe and Weyl, 2013; Moresi and Salop, 2013; Miller et al., 2016, 2017; Greenfield and Sandford, 2021) and auction models and bargaining (e.g., Waehrer and Perry, 2003; Miller, 2014; Loertscher and Marx, 2019; Sheu and Taragin, 2021). Other research explores the accuracy of modeling or the extent to which specific modeling choices affect inferences (e.g., Peters, 2006; Weinberg and Hosken, 2013; Garmon, 2017; Slade, 2021; Panhans and Taragin, 2022).

## 2 Model of Price Leadership

The model assumes there is a market composed of firms selling horizontally differentiated products, where a subset of these firms are coordinating their pricing. We start with a standard logit-based discrete-choice demand system, where each consumer chooses to purchase at most one unit of one product. The logit has the benefit of being able to accommodate numerous products with relatively few parameters, which is particularly important for antitrust practitioners who are often dealing with small amounts of available data. However, one could also use other types of demand functions featuring horizontal differentiation, should one have access to the data required to recover their parameters.

The supply-side closely follows that of the MSW price leadership model. We depart from MSW mainly in assuming a single market and in assuming that demand and cost conditions do not change over time, which simplifies the notation and analysis. As in MSW, firms in a coalition together raise their prices by a fixed increment that has been dictated by the coalition leader.<sup>7</sup> Firms outside the coalition are free to set their own prices. Given the assumed punishment strategies, the equilibrium is constructed to be incentive compatible for every firm. Products are produced with constant marginal costs, and there are no capacity constraints.

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<sup>7</sup>We specify the supermarkup in levels for simplicity and because the logit demand function results in fixed Bertrand markups in levels for each firm. One could switch to using percentages, which may be convenient if using a different demand function.

We assume that the identity of the coalition firms is known, both to the market participants and to the researcher. Therefore, the model is best suited to instances where there is a set of firms that would naturally form the coalition, perhaps because they are much larger than other competitors or share some other observable commonality. Alternatively, antitrust authorities may have access to confidential business documents that could indicate the likely coalition.<sup>8</sup>

## 2.1 Price Leadership Structure

Let there be a set  $\mathbb{F}$  of firms indexed by  $f = 1, \dots, F$  and a set  $\mathbb{J}$  of products indexed by  $j = 0, \dots, J$ . Each firm sells a subset  $\mathbb{J}_f$  of the available products. Time periods are discrete and a timing parameter,  $\delta \in (0, 1)$ , plays the role of a discount factor. However, its value encompasses more than just the patience of firms, a matter that we discuss in greater detail below. Without loss of generality, we label firm 1 the *leader*. Firms are separated into two groups, a pricing coalition that includes the leader and at least one other firm, and a fringe comprising all firms not contained in the coalition. We collect the coalition firms in the set  $\mathbb{C}$ , such that the firm  $f$  is in the coalition if  $f \in \mathbb{C}$ .

In each time period, play proceeds as follows:

1. The leader announces a supermarkup,  $m \geq 0$ , to all firms.
2. All firms (including the leader) set prices simultaneously and receive payoffs given by the profit function  $\pi_f(\mathbf{p})$ , where  $\mathbf{p}$  is the vector of prices.

We assume that the economic environment, including demand conditions and marginal costs, is common knowledge.<sup>9</sup> The equilibrium concept is subgame perfection.

We focus on a specific SPE that MSW refers to as the *price leadership equilibrium* (PLE) of the game. In the PLE, all coalition firms set prices in the second stage that equal static Bertrand prices plus the supermarkup, so long as there has been no prior deviation from that practice. The supermarkup is cheap talk that affects firm beliefs and thus guides the prices that obtain in the second stage. Fringe firms maximize their profit function given the prices of other firms, taking into account the supermarkup.

The profit function of each firm  $f$  is given by

$$\pi_f(\mathbf{p}) = \sum_{j \in \mathbb{J}_f} (p_j - c_j) s_j(\mathbf{p}) M, \quad (1)$$

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<sup>8</sup>If there is some uncertainty about the coalition members, one could potentially calibrate and simulate the model under different options to see what impact that has on the results. Such an exercise could be paired with some of the exercises discussed in Section 3 for cases where the model is overidentified. See also footnote 27.

<sup>9</sup>The assumption that the economic environment is common knowledge is often maintained in the literature, but there are settings for which an alternative assumption is more appropriate. Two recent articles that explore how private information can matter for the competitive effects of mergers are Harrington (2021) and Sweeting et al. (2022).

where  $c_j$  is the marginal cost of product  $j$ ,  $s_j(\cdot)$  is a demand function that characterizes the fraction of consumers that purchase product  $j$  as determined by a vector of prices, and  $M$  is the total number of consumers. The realized value of the demand function may differ from an antitrust market share for reasons that we discuss later. Fixed costs are excluded from the model because they do not affect pricing decisions.

Bertrand prices obtain if each firm maximizes its current profit function given the prices of other firms. Denote the Bertrand prices and the associated profits as  $\mathbf{p}^B$  and  $\pi_f^B$  for all  $f$ , respectively. If instead coalition firms adopt the supermarkup  $m$ , their prices are given by  $p_j^{PL} = p_j^B + m$ . We denote the resulting vector of prices (including entries for both coalition and fringe firms' products) by  $\mathbf{p}^{PL}(m)$  and the associated profits by  $\pi_f^{PL}(m)$  for all  $f$ .

Any deviations from the prices in  $\mathbf{p}^{PL}(m)$  by a coalition firm are punished with reversion to Bertrand equilibrium forevermore. Thus, the coalition firms employ Grim Trigger strategies. At first blush, this may appear to be a strong assumption, as in many applications it can be difficult to ascertain how punishment would occur in the event of a deviation. Furthermore, a period defines the length of time over which a firm earns deviation profit before punishment, and it might not be clear in practice whether this corresponds to a month, year, or something else. However, the interpretation of  $\delta$  as a timing parameter, not just a discount factor, weakens these assumptions substantially, as the model is isomorphic to alternatives with finite punishment or with different durations of deviation profit, provided the timing parameter is allowed to summarize both the patience of firms and the timing of the game.<sup>10</sup> A benefit of the Grim Trigger formulation is notational tractability.

We assume that firms believe that the supermarkup will be adopted in the second stage if the present value of adoption weakly exceeds the present value of deviation for all coalition firms. This is the case if  $g_f(m) \geq 0$  for all  $f \in \mathbb{C}$ , where

$$g_f(m) = \underbrace{\pi_f^{PL}(m) - \pi_f^D(m)}_{\text{(negative) current foregone profits if do not deviate}} + \underbrace{\frac{\delta}{1-\delta} (\pi_f^{PL}(m) - \pi_f^B)}_{\text{future additional profits if do not deviate}}. \quad (2)$$

We refer to  $g_f(m)$  as the *slack function* of firm  $f$ . In this expression,  $\pi_f^D(m)$  is the profit obtained by deviating: any firm that deviates selects prices that maximize its profit function given other prices are at their values in  $\mathbf{p}^{PL}(m)$ .<sup>11</sup> As demand and cost conditions are common knowledge, if  $g_f(m) < 0$  for any  $f \in \mathbb{C}$  then all firms believe that a deviation will occur, and this too is common knowledge, so play collapses to Bertrand immediately.

<sup>10</sup>This equivalence is recognized in Rotemberg and Saloner (1986), which argues that infinite punishment with a low discount factor is equivalent to finite punishment with a high discount factor. Formal proofs are provided in the Appendix of MSW.

<sup>11</sup>Equation (2) highlights that in our version of the price leadership model the profit functions do not change over time. MSW discusses the challenges of allowing for time-varying profit functions given that data are inevitably finite, and takes one approach to partially incorporate some intertemporal changes.

In considering its announcement of the supermarkup, the leader assesses its profit function and the supermarkups to which the coalition  $\mathbb{C}$  would adhere. The leader selects the supermarkup to solve the following constrained maximization problem:

$$\max_{m \geq 0} \pi_1^{PL}(m) \quad \text{s.t. } g_f(m) \geq 0, \quad \forall f \in \mathbb{C}, \quad (3)$$

conditional on the coalition  $\mathbb{C}$ . Thus, the slack functions characterize the IC constraints of the coalition firms. Under mild regularity conditions, there is always some positive supermarkup that increases the profit of the leader above its Bertrand level and does not violate incentive compatibility. Therefore, deviations do not occur along the equilibrium path. The solution to the leader's maximization problem may or may not be constrained by the IC constraints. We refer to these scenarios as being associated with the constrained PLE and unconstrained PLE, respectively.

## 2.2 Numerical Example

We now present a numerical example to help build intuition. We assume a multinomial logit demand system. The fraction of consumers that purchase product  $j$  is given by

$$s_j(\mathbf{p}) = \frac{\exp(\beta_j - \alpha p_j)}{\sum_{k \in \mathbb{J}} \exp(\beta_k - \alpha p_k)}, \quad (4)$$

where  $\beta_j$  is the quality parameter of product  $j$  and  $\alpha$  is the price coefficient for all products. This formulation incorporates an “outside good” that is not owned by any of the firms. We index the outside good as  $j = 0$  and normalize its quality and price to zero. We assume that there are three single-product firms, meaning the firm and product indices are synonymous. Firm qualities are  $\beta_1 = \beta_2 = 3$  and  $\beta_3 = 1$ , and their marginal costs are  $c_1 = c_2 = 0$  and  $c_3 = 1.25$ . Thus, the first two firms have higher qualities and lower costs than the third firm. We assume a price parameter of  $\alpha = 1.5$ . All three firms are in the coalition.

Figure 1 shows how price leadership selects among the many possible collusive outcomes that may constitute an SPE under the folk theorems. This figure depicts the reaction functions of Firms 1 and 2, which are identical. To start, the Bertrand equilibrium is identifiable as the intersection of these reaction functions. Any collusive equilibrium featuring prices above Bertrand levels would be in the northeast region from that intersection. However, under price leadership, the leader considers only symmetric price increases above the Bertrand equilibrium, which we represent as the dotted 45-degree line. Furthermore, under price leadership, the leader selects where along this line prices will fall. If the timing parameter is sufficiently close to one, then the unconstrained PLE is selected. Otherwise, a constrained PLE is selected; the one that we show in the figure corresponds to  $\delta = 0.40$ . Regardless, the structure of the model collapses a continuum of possible SPE to a unique solution.

Figure 2 shows the slack functions that arise with  $\delta = 0.40$  for the leader (Panel A) and



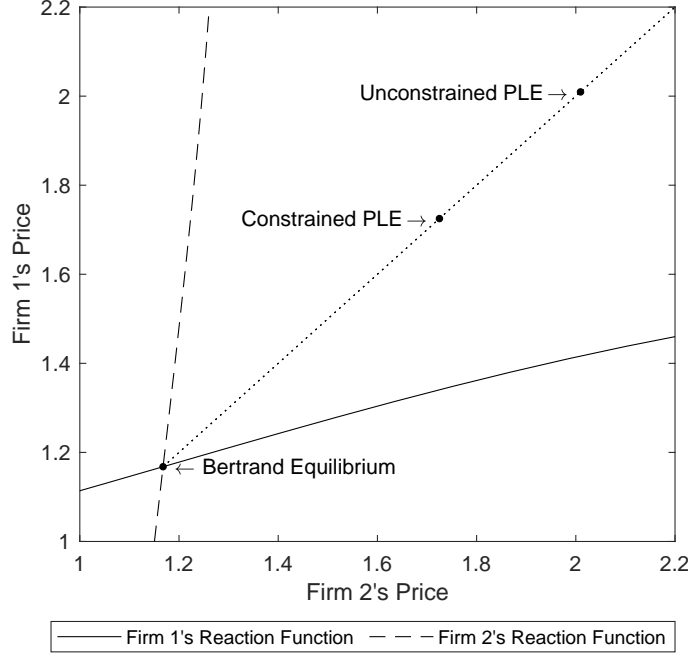


Figure 1: Illustration of the Price Leadership Equilibrium

for Firm 3, the smaller coalition member (Panel B). The slack functions are positive for small supermarkups, and negative for larger supermarkups. That is, the present value of price leadership exceeds the present value of deviation for lower price elevations, but this flips as prices increase. The slack function of Firm 3 crosses zero at  $m = 0.56$ , marked in both panels by the vertical blue line. As the slack function for the other firms is positive at this point, it is Firm 3 that constrains equilibrium prices. The higher supermarkups preferred by the leader would not be accepted because the smaller coalition member would deviate. Being the higher cost, lower quality producer, Firm 3 loses profit at a high relative rate as prices rise across all products in the market. As a result, it would deviate sooner as the supermarkup increases. Recognizing this, the leader chooses the supermarkup where Firm 3's slack function just crosses zero.

In order to further provide intuition about the PLE, it is helpful to compare it to a Bertrand equilibrium. Given the firm profit function in equation (1) and logit demand from equation (4), the Bertrand first order condition for price  $p_j$  is

$$p_j^B - c_j = \frac{1}{\alpha(1 - \sum_{k \in \mathbb{J}_{f(j)}} s_k(\mathbf{p}^B))}, \quad (5)$$

where  $\mathbb{J}_{f(j)}$  is the set of products sold by the firm who owns product  $j$ , labeled firm  $f(j)$ . As the right-hand side of this expression does not depend on which  $j \in \mathbb{J}_{f(j)}$  is chosen, this condition

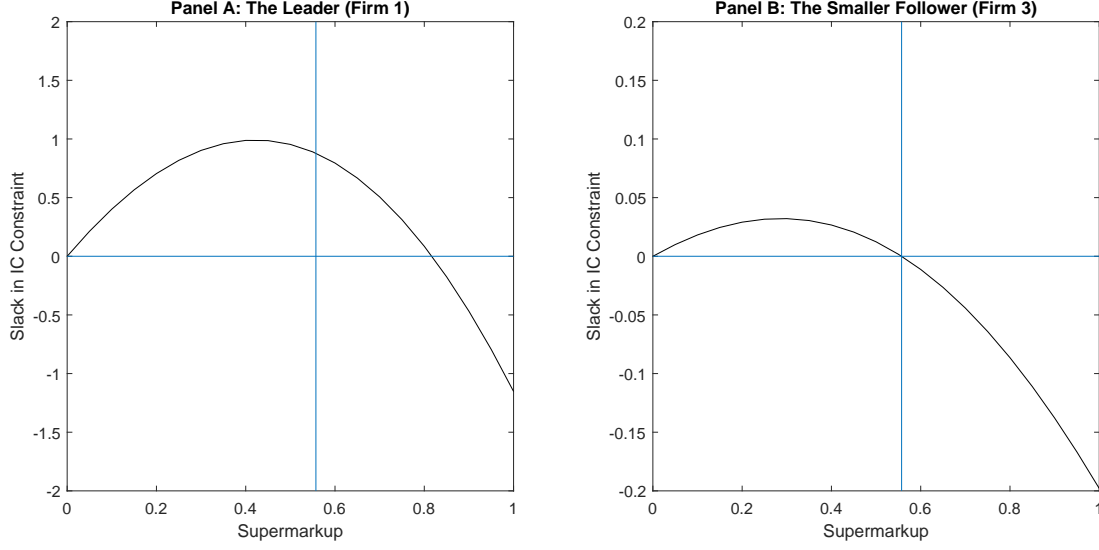


Figure 2: Slack Functions in the Numerical Illustration

Notes: The figure provides the slack functions for the leader (Panel A) and one of the other coalition members (Panel B) with supermarkups  $m \in [0, 1]$ . Incentive compatibility is satisfied for supermarkup  $m$  if the slack functions are positive (i.e., above the horizontal blue line). The vertical blue line shows the equilibrium supermarkup of 0.56, with  $\delta = 0.40$ .

implies that any multi-product firm applies the same markup to each of its products. The markup increases as consumers become less price sensitive according to  $\alpha$  and decreases as competitors grow in relative size. The latter effect embeds that consumers substitute to rivals as a firm raises its price. Taking our example above, if the leader were to price according to Bertrand, it would set its optimal markup by accounting for competition from other firms as measured via  $(1 - s_1)$ .

If one or more IC constraints bind, the PLE does not have an analogous first order condition. In order to derive such a relationship, we must focus on an unconstrained equilibrium. In our example with three single-product firms in a coalition, now assume that  $\delta$  is high enough to generate an unconstrained equilibrium. The market would shift to the “Unconstrained PLE” point in Figure 1. In order to solve for this point, we use the leader’s first order condition for the supermarkup,

$$p_1^B + m - c_1 = \frac{1}{\alpha(1 - s_1(\mathbf{p}^{PL}(m)) - s_2(\mathbf{p}^{PL}(m)) - s_3(\mathbf{p}^{PL}(m)))}. \quad (6)$$

Similar to in the Bertrand first order condition, the supermarkup increases as consumers become less price sensitive, based on  $\alpha$ . As for the impact of competitors, what matters for the supermarkup is the purchase probability outside the coalition, dictated by  $(1 - s_1 - s_2 - s_3)$ , as opposed to  $(1 - s_1)$ .<sup>12</sup> This difference occurs because the leader knows, assuming no firm in the coalition deviates, that increasing the supermarkup will raise the prices of all coalition products together, which limits

<sup>12</sup>Since the coalition in this example comprises all inside products, one minus the coalition probability is also the share of the outside good,  $s_0$ .

consumer substitution. In this way, the leader’s problem captures coordinated interactions with other firms.

### 3 Calibration of the Model

Empirical articles often estimate the structural parameters of a model using cross-sectional or panel data (e.g., Berry et al., 1995; Nevo, 2000, 2001). Indeed, this is the approach of MSW for the price leadership model. Methodologies for structural estimation are well-covered elsewhere in the literature (e.g. Berry and Haile, 2021; Gandhi and Nevo, 2021) and they are not our focus. Instead, we develop how the structural parameters of the model can be calibrated to match a set of empirical objects, namely market shares, diversion, and markups. Calibration is widely used in antitrust investigations, in part because such empirical objects are more readily available in that context and in part because of time and data constraints.<sup>13</sup> In this section, we discuss the data that may be available in antitrust investigations, develop intuition in the context of the Bertrand/logit model, and then turn to the calibration of the price leadership model.

#### 3.1 Data

We now discuss the data that often are used in calibration. We start with market shares, which here we define as the volume share of firms within a relevant antitrust market.<sup>14</sup> The *Horizontal Merger Guidelines* state that a relevant market for antitrust merger review comprises a set of products for which a hypothetical monopolist would find it profitable to impose a small but significant price increase on at least one of the merging firms’ products. It follows that there is a distinction between an antitrust market share, which is calculated using only those products deemed to be in the relevant market, and a logit choice probability (e.g., equation (4)), which reflects all the choices that are available to consumers.

To build on this discussion, suppose that the relevant antitrust market used for the calculation of market shares is deemed to be the set of all inside goods. Consumer demand follows the logit, with choice probabilities  $\{s_j\}_{j \in \mathbb{J}}$ . Then there is a mapping from a product’s market share to its choice probability,

$$x_j = \frac{s_j}{1 - s_0}, \quad (7)$$

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<sup>13</sup>Early research on merger simulation contemplated using calibrated models (Werden and Froeb, 1994). Background on estimation and calibration in the merger context is provided in Davis and Garcés (2009) and Miller and Sheu (2021). Calibrated models were used in the merger trials of *Penguin Random House/Simon & Schuster* (Demonstrative Exhibits for the Direct Testimony of Dr. Nicholas Hill, 8/8/22, slides 52-54), *Wilhelm Wilhelmsen/Drew Marine* (Memorandum Opinion, 9/28/18, pages 44-45), and *Anthem/Cigna* (Demonstrative Exhibit Used in the Testimony of David Dranove, Ph.D., 11/29/16, slide 48). Calibration is also used in the empirical industrial organization literature (e.g., Smith, 2004).

<sup>14</sup>Market shares based on revenue rather than volume sometimes are used instead, particularly for products that vary greatly in price.

where we denote market shares as  $\{x_j\}_{j \in \mathbb{J}, j \neq 0}$ . This mapping depends on the choice probability of the outside good, which reflects, for example, that some consumers may substitute away from the market in response to price increases.

Thus, additional information on demand for the outside good is required to identify the choice probability from the market share. Diversion can provide this information. The definition of the “diversion ratio” from product  $k$  to product  $j$  is

$$div_{k \rightarrow j} \equiv \frac{\frac{\partial s_j}{\partial p_k}}{\left| \frac{\partial s_k}{\partial p_k} \right|} = \frac{s_j}{1 - s_k} \quad (8)$$

where the second equality holds with logit demand. This diversion ratio can be conceptualized as the fraction of consumers who select product  $j$  among those consumers who no longer purchase product  $k$  due to an increase in the price of that product.<sup>15</sup> It can be possible to measure diversion without estimating a full demand system. For example, one may observe how consumers respond to price or quality changes, or to competitive events such as previous mergers. Many companies analyze consumer switching behavior in the normal course of business, and this also can inform diversion, with the caveat that in some settings consumers may switch products for reasons unrelated to changes in relative product attractiveness.

A bit of algebra on equation (8) obtains the choice probability of the outside good as a function of market shares and a diversion ratio:

$$s_0 = \frac{div_{k \rightarrow j} \times (x_k - 1) + x_j}{div_{k \rightarrow j} \times x_k + x_j}. \quad (9)$$

A higher diversion ratio implies less substitution from products that are inside the market to outside or, equivalently, that the choice probability of the outside good is smaller.<sup>16</sup> Together, equations (7) and (9) provide a mapping from market shares to choice probabilities.

Another object often used in calibration is a markup. The literature defines markups in a variety of different ways. For our purposes, let the markup of firm  $j$  be the difference between its price and marginal cost:  $p_j - c_j$ . The markup is useful in calibration in part because it informs the price elasticity of demand, as firms tend to find it profit maximizing to set a low (high) markup if consumers are (are not) price sensitive. Equation (5) shows that with Bertrand competition and logit demand, equilibrium markups depend only on the price parameter,  $\alpha$ , and choice probabilities. Thus, the price parameter is identified from the choice probabilities and a markup.

The literature highlights some challenges that arise in measuring markups (e.g., Fisher and McGowan, 1983). In accounting data, it can be difficult to distinguish between variable costs and

<sup>15</sup>A broader interpretation is possible, as Conlon and Mortimer (2021) show that diversion can be obtained using *any* marginal change in a product’s attractiveness to consumers.

<sup>16</sup>If the diversion ratio is high enough then it may not be possible to rationalize within the context of logit demand, and a nested logit or random coefficients logit model may provide a better fit.

fixed costs and, furthermore, an accounting measure of average variable cost can diverge from marginal cost if marginal cost is not constant. In antitrust investigations, information sometimes becomes available that mitigates these difficulties. An example would be data or documents that inform the costs that were considered by a firm when it contemplated a price change. The *Horizontal Merger Guidelines* state that “The Agencies often estimate incremental costs, for example, using merging parties’ documents or data the merging parties use to make business decisions.” Data on marginal cost and price immediately identify the markup.

The final object that we discuss in the context of calibration is the market elasticity of demand. With logit demand, the market elasticity is given by

$$\epsilon \equiv - \left( \sum_{j \in \mathbb{J}, j \neq 0} \frac{\partial s_j(\mathbf{p})}{\partial p_j} \right) \frac{\bar{p}}{\sum_{j \in \mathbb{J}, j \neq 0} s_j(\mathbf{p})} = \alpha s_0 \bar{p} \quad (10)$$

where  $\bar{p}$  is the weighted average price among the inside products. So long as prices are data, the market elasticity informs the multiplicative product of the price parameter and the outside good choice probability. This reflects that the market elasticity depends both on the degree to which consumers substitute from inside the market to outside and on the responsiveness of consumers to price. If choice probabilities are recovered from data on market shares and diversion, then the market elasticity identifies the price coefficient. Alternatively, the choice probabilities and price parameter can together be identified from market shares, a markup, and the market elasticity.<sup>17</sup>

### 3.2 Bertrand with Logit Demand

We now cover the calibration of a Bertrand/logit model more explicitly. Our goal is to provide a set of intuitions about how the data connects to the models that we can draw on later in this section. The logit demand system is defined in equation (4). We assume that the inside goods constitute a relevant antitrust market, and that the outside good represents products that are outside this market. Our calibration of the model allows for flexible substitution between the inside goods and the outside good because we select the choice probability of the outside good to match diversion or the market elasticity.<sup>18</sup>

We start with the exactly identified case. Let the data include market shares,  $\{x_j\}_{j \in \mathbb{J}, j \neq 0}$ , prices,  $\{p_j\}_{j \in \mathbb{J}, j \neq 0}$ , the marginal cost of one product (let it be product  $l$ ), and diversion between two products. The structural parameters to be calibrated are the price parameter,  $\alpha$ , the product qualities,  $\{\beta_j\}_{j \in \mathbb{J}, j \neq 0}$ , and the marginal costs of each product,  $\{c_j\}_{j \in \mathbb{J} \setminus \{l\}}$ . Recall that equations

<sup>17</sup>A market elasticity was used in economic expert testimony for the *Anthem/Cigna* trial (Demonstrative Exhibit Used in the Testimony of David Dranove, Ph.D., 11/29/16, slide 16). Similarly, a market elasticity was used to calibrate demand in regulatory filings related to the *T-Mobile/Sprint* merger (Appendix F of the 2018 Joint Opposition Filing by T-Mobile and Sprint in FCC WT Docket No. 18-197).

<sup>18</sup>In this way, the model provides the flexibility of a one-level nested logit model in which the inside goods are in one nest and the outside good is in another. Our formulation, however, is mathematically more tractable.

(7) and (9) allow for the choice probabilities,  $\{s_j\}_{j \in \mathbb{J}}$ , to be recovered from the share and diversion data, as discussed above. Then the first order condition of equation (5) can be rearranged such that, for product  $l$  sold by firm  $f(l)$ ,

$$\alpha = \frac{1}{p_l - c_l} \frac{1}{1 - \sum_{j \in \mathbb{J}_{f(l)}} s_j} \quad (11)$$

This obtains the price parameter from the marginal cost (or markup) of product  $l$ . The remaining demand parameters satisfy

$$\beta_j = \log(s_j) - \log(s_0) + \alpha p_j, \quad (12)$$

which obtains by taking logs and differencing equation (4) with respect to  $s_0$ . Higher quality is inferred if the product has a higher choice probability or a higher price, all else equal. Finally, the remaining marginal costs can be recovered, as the first order conditions can be rearranged again to yield

$$c_j = p_j - \frac{1}{\alpha} \frac{1}{1 - \sum_{k \in \mathbb{J}_{f(j)}} s_k} \quad (13)$$

for any product  $j$ . Higher costs are inferred, all else equal, if price is higher or the markup is smaller. Putting all this together, the model is exactly identified from market shares, prices, one diversion ratio, and one marginal cost. Furthermore, the relationships involved have simple intuitions.

We now explore over-identification, which occurs if more data are available to calibrate the model than is strictly necessary. In this situation, there are several options for the practitioner to consider. One could assess whether some data points are better measured than others. In making this determination, it can be helpful to gather evidence on whether any firms have used certain data sources to drive important decisions about things like pricing, advertising, or investment. If one believes that all the data points are well measured, one could implement a “best fit” calibration, which chooses the parameter values in order to minimize some criterion function, such as the sum of squared errors between the model predictions and the data.<sup>19</sup> Another option could be to consider richer model specifications. The extra data could perhaps be used to assess the reasonableness of the logit substitution parameters or to support the calibration of a richer two-level nested logit model or a random coefficients logit model. We return to the nested logit possibility in the context of the price leadership model (Section 3.4).<sup>20</sup>

A particularly interesting situation for our present purposes is when multiple markups are observed. Specifically, suppose that two markups are available, one each for products  $l$  and  $k$  that

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<sup>19</sup>This idea is similar to that used for method-of-moments estimation. However, unlike with econometric estimation, with calibration there may not be sufficient data for formal statistical tests of model validity.

<sup>20</sup>Parameterized random coefficients models can be calibrated, for example, by using “micro-moments” that link the popularity of different brands to specific consumer demographic characteristics such as income or age. This is a straight-forward extension of the calibration methods that we discuss in this article.

are sold by different firms. Taking the ratios of the first order conditions of equation (5), we obtain

$$\frac{p_l - c_l}{p_k - c_k} = \frac{1 - \sum_{j \in \mathbb{J}_{f(k)}} s_j}{1 - \sum_{j \in \mathbb{J}_{f(l)}} s_j} \quad (14)$$

Thus, the Bertrand/logit model connects the relative markups of the two products to the relative choice probabilities of their firms (i.e., summing across each firm's products). If the products of firm  $f(l)$  are purchased more than those of firm  $f(k)$ , then firm  $f(l)$  also has higher markups in equilibrium. Furthermore, the model provides a quantitative link between the gap in the choice probabilities and the gap in the markups.

Equation (14) may not hold exactly in empirical settings simply due to the practical reality that the empirical objects (e.g., markups or shares) are unlikely to be perfectly measured. However, there are also *economic* reasons that the equation may not hold. Among these is that the assumption of Bertrand equilibrium can be inappropriate in the presence of coordination. Suppose that one firm sets prices to maximize its own (static) profit function whereas another firm competes less aggressively, for example due to price coordination with a subset of its competitors. Then the markup of the second firm would be high relative to Bertrand, just as its choice probability would be small relative to Bertrand. This combination of higher markups and smaller choice probabilities is consistent with coordination, and can be identified from equation (14) by comparing the two firms. Thus, modeling can be used to inform the model of competition. With this intuition in hand, we now pivot to the calibration of the price leadership model.

### 3.3 Price Leadership with Logit Demand

As with the Bertrand/logit model, the structural parameters to be recovered include the price parameter,  $\alpha$ , the qualities,  $\{\beta_j\}_{j \in \mathbb{J}}$ , and marginal costs of the products,  $\{c_j\}_{j \in \mathbb{J}}$ . The price leadership model adds to these two additional unknowns: the supermarkup,  $m$ , and the timing parameter,  $\delta$ . Let the data include market shares,  $\{x_j\}_{j \in \mathbb{J}, j \neq 0}$ , prices,  $\{p_j\}_{j \in \mathbb{J}, j \neq 0}$ , the marginal costs of two products, one sold by a firm in the coalition and the other by a firm in the fringe, and the diversion ratio between two products. Relative to the Bertrand/logit calibration, this list differs only in that *two* costs are included, rather than one. This additional cost is key for identifying the supermarkup. The timing parameter is identified from the structure of the profit and slack functions.

The demand parameters and the choice probability of the outside good can be recovered as described in the previous subsections. The only change is that the price parameter must be recovered using the first order condition of a fringe firm, as only fringe firms price to maximize their static profit. For the same reason, once the calibrated price parameter is in hand, only the marginal costs of fringe firms are identified from static first order conditions. In this way, observing marginal cost for a fringe product allows us to identify the demand parameters solely from price setting among non-coalition firms. If one does not observe a marginal cost for a fringe product, we

have found that calibration can perform poorly, and in particular it can be difficult to separately identify the supermarkup and consumers' price sensitivity.

We rely on the structure of the price leadership model to recover the supermarkup and the marginal costs of coalition firms. Here, the second markup plays an important role.

The calibration procedure is as follows. Fix the supermarkup at a candidate value  $\hat{m}$ . The Bertrand coalition prices that would have prevailed in the absence of price leadership can be recovered by subtracting  $\hat{m}$  from the observed coalition prices,

$$p_j^B(\hat{m}) = p_j^{PL} - \hat{m} \quad (15)$$

for any product  $j$  sold by a coalition firm. The Bertrand prices of fringe products then can be computed from their static first order conditions, given the demand parameters and the fringe marginal costs, and holding coalition prices fixed at the Bertrand prices implied by the candidate supermarkup. The mechanics of this step are the same as what one does to calculate a Bertrand equilibrium with logit demand in a typical counterfactual exercise, except some prices have already been specified. The resulting vector of Bertrand prices,  $\mathbf{p}^B(\hat{m})$ , can be combined with the logit demand function in (4) to derive the Bertrand choice probabilities.

Plugging these prices and choice probabilities into the static first order conditions of coalition firms obtains an expression for the marginal costs that are implied by the model given the candidate supermarkup:

$$c_k(\hat{m}) = p_k^B(\hat{m}) - \frac{1}{\alpha} \frac{1}{1 - \sum_{j \in \mathbb{J}_{f(k)}} s_j(\mathbf{p}^B(\hat{m}))} \quad (16)$$

We can then choose the supermarkup that equates the model-implied and observed marginal costs,

$$\{m \mid c_k - c_k(m) = 0\}. \quad (17)$$

There is a unique solution because the model-implied marginal costs,  $c_k(m)$ , are monotonically decreasing in the candidate supermarkup.

Thus, when combined with assumptions on the structure of demand and information on prices, marginal cost data for the coalition identifies the supermarkup. Intuitively, if one knows the parameters of the demand system and firms' marginal costs, then one can compute prices under Bertrand competition. If coalition firms have higher prices and lower shares than this baseline, the difference can be attributed to the supermarkup. That is, with logit demand, we know that the markup in a Bertrand setting is a simple function of the price coefficient  $\alpha$  and choice probabilities. As we see in equation (14), observed market shares,  $s_0$ , and knowledge of one markup immediately imply what the markup for any other product should be in a Bertrand framework. If an observed markup is higher then this is consistent with behavior that departs from typical static price-setting. This discussion also makes it clear why we require marginal cost information for a product in the coalition. Otherwise, the calibration may not cleanly distinguish between a case where coalition



firms have high marginal costs versus a case where there is a high supermarkup.

All that remains to calibrate is the timing parameter. If data are to inform its magnitude, then it must be that the leader is constrained in its choice of the supermarkup. That is, an IC constraint must bind.<sup>21</sup> Otherwise, the value of  $\delta$  would not have a direct impact on the equilibrium we observe in the data.<sup>22</sup> In a constrained PLE, the timing parameter is pinned down by the slack functions of equation (2). Namely, with the demand parameters and the marginal costs, it is possible to compute the profit objects,  $\pi_f^{PL}(m)$ ,  $\pi_f^D(m)$ , and  $\pi_f^B$  for all  $f \in \mathbb{C}$ . Then  $\delta$  can be selected according to

$$\{\delta \mid \min_{f \in \mathbb{C}} \{g_f(m)\} = 0\}, \quad (18)$$

which involves identifying the coalition firm that has the most restrictive slack function and then finding the timing parameter that sets its slack function to zero at the equilibrium supermarkup.

### 3.4 Price Leadership with Nested Logit Demand

We now discuss calibration with a two-level nested logit demand model, which allows for more flexible substitution patterns relative to the baseline logit model. Because many of the mechanics are similar to those with the logit model, we provide a high-level description here and leave the details to the Appendix.

We assume there are two groups of products, one with all the inside goods and the other with the outside good. Within the group that contains the inside goods, there are additional subgroups. Formally, let the products be partitioned into two groups,  $g = 0, 1$ . The outside good,  $j = 0$ , is the only member of group 0. Further, let group 1 be partitioned into  $H_g$  subgroups,  $h = 1, \dots, H_g$ . Denote the set of subgroups  $\mathbb{H}$  and the set of products in subgroup  $h$  as  $\mathbb{J}^h$ . The choice probability of product  $j \in \mathbb{J}^h$  is given by

$$s_j(\mathbf{p}) = \bar{s}_{j|hg}(\mathbf{p}) \bar{s}_{h|g}(\mathbf{p}) \bar{s}_g(\mathbf{p}), \quad (19)$$

where  $\bar{s}_{j|hg}(\mathbf{p})$  is choice probability of product  $j$  conditional on its subgroup (and group) being selected,  $\bar{s}_{h|g}(\mathbf{p})$  is the choice probability of product  $j$ 's subgroup conditional on its group being

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<sup>21</sup>One might instead choose to set the timing parameter to a commonly used discount factor, such as 0.95, if one has a belief that this figure is an accurate reflection of the value of the future. Doing so magnifies the economic impact of assumptions about the length of the deviation and punishment phases of the game, as the  $\delta$  multiplier reflects both the timing of the game and firms' valuation of the future.

<sup>22</sup>This is testable, as one can simulate the unconstrained supermarkup using objects already obtained and compare it with the calibrated supermarkups.

selected, and  $\bar{s}_g(\mathbf{p})$  is the choice probability of its group. With the following inclusive values

$$I_{hg} = (1 - \sigma_1) \ln \sum_{k \in \mathbb{J}^h} \exp \left( \frac{\beta_k - \alpha p_k}{1 - \sigma_1} \right) \quad (20)$$

$$I_g = (1 - \sigma_2) \ln \sum_{h \in \mathbb{H}} \exp \left( \frac{I_{hg}}{1 - \sigma_2} \right) \quad (21)$$

$$I = \ln(1 + \exp(I_g)), \quad (22)$$

we obtain expressions for the choice probabilities:

$$\bar{s}_{j|hg} = \frac{\exp \left( \frac{\beta_j - \alpha p_j}{1 - \sigma_1} \right)}{\exp \left( \frac{I_{hg}}{1 - \sigma_1} \right)}, \quad \bar{s}_{h|g} = \frac{\exp \left( \frac{I_{hg}}{1 - \sigma_2} \right)}{\exp \left( \frac{I_g}{1 - \sigma_2} \right)}, \quad \bar{s}_g = \frac{\exp(I_g)}{\exp(I)}. \quad (23)$$

The demand parameters are the product qualities,  $\{\beta_j\}_{j \in \mathbb{J}}$ , the price parameter,  $\alpha$ , and two nesting parameters,  $\sigma_1$  and  $\sigma_2$ .

The model is consistent with random utility maximization if  $1 \geq \sigma_1 \geq \sigma_2 \geq 0$ . The parameter  $\sigma_1$  captures the correlation of utilities that consumers experience among products in the same subgroup, and the parameter  $\sigma_2$  captures the same concept for the groups. The model collapses to a one-level nested logit with groups as nests if  $\sigma_1 = \sigma_2$ . The model collapses to a one-level nested logit with subgroups as nests if  $\sigma_2 = 0$ . The model collapses to the baseline logit if  $\sigma_1 = \sigma_2 = 0$ .

Compared to the baseline logit model, here the outside good choice probability is less important because the nesting parameter  $\sigma_2$  also governs substitution between the inside products and the outside good. That is, the value for  $\sigma_2$  can be calibrated to fit data on diversions so long as  $s_0$  is not so small as to swamp the impact of the nesting parameter. In this way, calibrating  $\sigma_2$  and assuming a value for  $s_0$  has the same implications for substitution outside the market as calibrating  $s_0$  does in the logit. Thus, we proceed assuming that we have chosen a value for  $s_0$ . As a consequence, our nested logit model effectively adds only one additional parameter,  $\sigma_1$ , compared to the baseline logit. An implication of a known value of  $s_0$  is that the unconditional and conditional choice probabilities of the inside products can be calculated directly from market shares.

With this model setup, we now turn to calibration. As in the logit model, the demand-side unknowns still include the price parameter,  $\alpha$ , and the qualities,  $\{\beta_j\}_{j \in \mathbb{J}}$ . Information on substitution outside the defined market comes via  $\sigma_2$ . The  $\sigma_1$  parameter governs substitution inside the market. The supply-side unknowns, meaning the marginal costs,  $\{c_j\}_{j \in \mathbb{J}}$ , the supermarkup,  $m$ , and the timing parameter,  $\delta$ , remain the same as under logit. We assume the available data are as follows: (1) the market shares of each product,  $\{x_j\}_{j \in \mathbb{J}}$ , (2) the prices of each product,  $\{p_j\}_{j \in \mathbb{J}}$ , (3) the marginal costs of two products, one sold by a firm in the coalition and the other by a firm in the fringe, and (4) two diversion ratios between products in the market. With the addition of one more demand parameter, we now require *two* diversion ratios, instead of one.

These diversion ratios identify the values of  $\sigma_1$  and  $\sigma_2$  in much the same manner diversion identifies  $s_0$  under logit. The nested logit demand model implies that the diversion ratio is a function of the choice probabilities, analogous to what obtains with logit. However, with nested logit,  $\sigma_1$  and  $\sigma_2$  also enter, allowing diversion between products to vary based on if they are in the same group or not. Therefore, knowledge of two diversion ratios allows us to form a system of two equations to solve for  $\{\sigma_1, \sigma_2\}$ . The specific forms that these equations take, depending on whether the ratios are observed for firms in the same or different nests, are available in the Appendix.

Once the nesting parameters have been recovered, calibration proceeds along lines similar to with the logit price leadership model. The first order condition for the fringe product whose marginal cost we observe is used to identify the price coefficient,  $\alpha$ . For example, if the firm that owns this fringe product has all of its products in one nest, the first order condition is

$$p_k - c_k = \frac{1 - \sigma_1}{\alpha \left( 1 - (1 - \sigma_1) \sum_{j \in \mathbb{J}_{f(k)}} s_j - \frac{(1 - \sigma_1)\sigma_2}{1 - \sigma_2} \sum_{j \in \mathbb{J}_{f(k)}} s_j |h_g s_{h(j)|g} - \frac{\sigma_1 - \sigma_2}{1 - \sigma_2} \sum_{j \in \mathbb{J}_{f(k)}} s_j |h_g \right)}. \quad (24)$$

The general expression appears in the Appendix. The quality intercepts,  $\{\beta_j\}_{j \in \mathbb{J}}$ , are recovered via the expression for  $\ln(s_j) - \ln(s_0)$ , in the nested logit equivalent to equation (12).

As for the supply side, calibration of the unobserved cost terms and the supermarkup proceeds as in the logit case, albeit with the nested logit first order conditions. That is, for a given  $\hat{m}$ , we can find the implied Bertrand prices  $\mathbf{p}^B(\hat{m})$  and choice probabilities. Plugging these into the first order condition for the coalition product whose marginal costs we observe obtains the supermarkup. The timing parameter can also be recovered, just as in the logit case.

### 3.5 Application to Beer Markets

In order to provide an example, we calibrate a model with nested logit demand to the United States beer industry.<sup>23</sup> We show that despite the simplifications we have made to the demand side, we can obtain merger simulation results comparable to those in MSW. We assume that the coalition members are Anheuser-Busch Inbev, SABMiller, and Molson Coors, and the fringe competitors are Grupo Modelo and Heineken. We specify a two-level nested logit with all brewers in one group and the outside good in a second group. The group containing all brewers has two subgroups, with domestic brewers in one and imports in the other. Prices, shares, the market elasticity, and the diversion to fringe suppliers for year 2007 come from MSW. We choose costs that lead to a calibrated supermarkup similar to that found in MSW.<sup>24</sup> A practitioner might instead rely on confidential data on costs, but that is not available to us here in a research context. The calibration

<sup>23</sup>See Miller and Weinberg (2017) and MSW for background on the industry.

<sup>24</sup>The amount of consumer heterogeneity in the price parameter—or equivalently, the amount of demand curvature—can affect how relative markups and shares inform the supermarkup. The MSW demand model incorporates greater consumer heterogeneity than the nested logit and, as a result, if we apply the MSW marginal costs directly we obtain a supermarkup that is somewhat smaller than what is estimated in MSW.

Table 1: Data Used in Calibration Example

Firm	Subgroup	In Coalition?	Price	Share	Cost
ABI	Domestic	Yes	9.11	0.444	3.61
SABMiller	Domestic	Yes	8.38	0.258	3.93
Molson Coors	Domestic	Yes	8.82	0.138	4.81
Grupo Modelo	Foreign	No	14.87	0.100	11.41
Heineken	Foreign	No	14.41	0.060	11.46
Domestic to Foreign Diversion					0.107
Market Elasticity					0.500
Outside Share					0.500

*Notes:* The price, share, diversion, and elasticity are for year 2007 from MSW. The costs shown are calculated during an initial calibration step in order to be consistent with our nesting structure. “ABI” is Anheuser-Busch Inbev.

inputs appear in Table 1.

Our calibration exercise proceeds in two steps. First, given prices and market shares of each product and the market share of the outside good, we choose the supermarkup,  $m$ , and demand parameters  $\alpha, \sigma_1$ , and  $\sigma_2$  so the model matches features of the inputs. In this case those features include the market elasticity,  $\epsilon$ , domestic to foreign diversion,  $div_{dom \rightarrow for}$ , and the marginal costs of each of the five firms,  $\mathbf{c}$ . Thus, the model is overidentified because we are calibrating four parameters with seven data points. Let  $\theta = (\alpha, \sigma_1, \sigma_2, m)$ , and let  $\mathbf{g} = (\epsilon, div_{dom \rightarrow for}, \mathbf{c})$ . The calibrated parameters are given by:

$$\hat{\theta} = \arg \min_{\theta} (\mathbf{g} - \hat{\mathbf{g}}(\mathbf{p}, \mathbf{s}, \theta))' W (\mathbf{g} - \hat{\mathbf{g}}(\mathbf{p}, \mathbf{s}, \theta)) \quad (25)$$

where  $\mathbf{g}$  denotes the data calibration targets,  $\hat{\mathbf{g}}(\mathbf{p}, \mathbf{s}, \theta)$  represents the values of  $\mathbf{g}$  implied by the model, and  $W$  is a weighting matrix.<sup>25</sup> In the second step, we find the value of the timing parameter,  $\delta$ , such that one firm’s incentive compatibility constraint binds.

Table 2 shows the results of calibration. The supermarkup and the timing parameter are close to the values obtained in MSW. Their preferred model results in a supermarkup of 1.20 and a timing parameter of 0.26, whereas our calibration places these at 1.20 and 0.21, respectively. We find that Molson Coors’s IC constraint is binding in equilibrium, which also matches MSW.

Table 3 shows the elasticity of the calibrated parameters with respect to the data points used in calibration, in order to help build intuition about how the data inform the parameters. Many of the relationships are intuitive, following our discussion of calibration. For example, a higher market share for one firm implies a higher quality parameter for that firm and lower quality parameters for other firms. A higher cost (lower markup) for coalition firms implies a lower supermarkup and

<sup>25</sup>The weighting matrix used is a diagonal matrix constructed such that the weight on matching market elasticity and diversion are 100 times larger than the weights on matching firm costs. We apply these weights for the purposes of unit conversion, as the elasticity and diversion are an order of magnitude smaller than costs.

Table 2: Calibrated Parameter Values

Quality, $\beta_j$	ABI	0.701
	SABMiller	0.491
	Molson Coors	0.381
	Grupo Modelo	0.645
	Heineken	0.471
Price Parameter, $\alpha$		0.102
Subgroup Nesting Parameter, $\sigma_1$		0.751
Group Nesting Parameter, $\sigma_2$		0.586
Supermarkup, $m$		1.201
Timing Parameter, $\delta$		0.212

*Notes:* “ABI” is Anheuser-Busch Inbev.

a smaller timing parameter, with the opposite relationships obtaining for fringe firms. The market elasticity helps pin down the price coefficient and nesting parameters.

Table 4 shows simulation results for a merger between Miller and Coors (forming MillerCoors). Under an assumption that there are no efficiencies, the supermarkup increases from 1.20 to 1.73, a difference of 0.53, similar to what is obtained in MSW. This coordinated effect represents more than half of the total price effect of the merger.<sup>26</sup> Also, as in MSW, we find that the supermarkup increases by more if efficiencies are introduced, a phenomenon that we explain in the next section. Finally, both here and in MSW, the IC constraint of MillerCoors binds after the merger. Our results demonstrate that calibration based on a handful of data points can obtain results that are similar to those that obtain with estimation that exploits variation in cross-sectional or panel data.

## 4 Implications for Competition Policy

This section analyzes the antitrust implications of horizontal mergers in industries characterized by price leadership. We contrast how market outcomes depend on market structure under price leadership versus under static Bertrand equilibrium. We then examine mergers in industries with price leadership, the pass-through of merger-specific efficiencies, and the consequences of divestitures in mergers of multi-product firms.

### 4.1 Market Structure and Monopoly Outcomes

We first study how prices change when firms play a price leadership equilibrium and as an industry consolidates towards monopoly. The exercise provides some intuition for how mergers under price leadership may impact prices. In setting up the numerical exercise, we posit 100 differentiated

<sup>26</sup>The unilateral effect of the merger is shown in Table 4 by the increase in Bertrand average price from 8.52 to 8.96, an increase of 0.44.

Table 3: Elasticity of Calibrated Parameters to Data

Input		Calibrated Parameters									
		$\alpha$	$\sigma_1$	$\sigma_2$	$m$	$\delta$	$\beta_{ABI}$	$\beta_M$	$\beta_{MC}$	$\beta_{GM}$	$\beta_H$
Market Elasticity		1.00	-0.37	-0.60	-0.11	0.04	0.99	0.95	0.89	1.15	1.17
Diversion		0.00	-0.09	0.35	-0.64	-0.61	-0.01	-0.09	-0.23	0.53	0.65
Share	ABI	0.08	-0.05	0.14	-0.66	-0.43	0.48	-0.29	-0.43	-0.12	-0.22
	SABMiller	0.04	-0.07	0.09	-0.29	-0.25	-0.09	0.49	-0.33	-0.02	-0.09
	Molson Coors	0.01	-0.04	0.06	-0.13	-0.39	-0.05	-0.11	0.58	0.00	-0.04
	Grupo Modelo	-0.05	0.06	-0.02	0.39	0.45	-0.10	-0.08	-0.03	0.42	0.04
	Heineken	-0.03	0.06	-0.19	0.37	0.38	-0.07	-0.05	0.01	-0.31	0.19
Cost	ABI	0.02	0.04	-0.02	-0.66	-0.49	0.05	0.10	0.18	0.05	0.09
	SABMiller	-0.01	0.00	-0.01	-1.09	-0.94	-0.02	-0.02	-0.03	-0.04	-0.05
	Molson Coors	-0.02	-0.05	0.03	-1.67	-1.51	-0.06	-0.12	-0.21	-0.02	-0.07
	Grupo Modelo	0.03	0.38	2.09	3.26	3.86	0.61	1.18	1.99	3.76	5.46
	Heineken	-0.06	0.92	-0.16	6.85	7.25	0.53	1.54	3.12	0.11	0.90

*Notes:* Numbers displayed are the change, in percent, of each parameter value in response to a 1% change in the relevant input, holding all other inputs constant. The following abbreviations are used in this table: “ABI” is Anheuser-Busch Inbev, “M” is SABMiller, “MC” is Molson Coors, “GM” is Grupo Modelo, and “H” is Heineken.

products with identical (logit) demand and the same marginal costs. We assign the products to some number of firms,  $N \in \{100, 50, 25, 20, 10, 5, 4, 2, 1\}$ , where we have selected the numbers so that the product allocations can be made symmetrically. For each value of  $N$ , we compute the PLE using three distinct timing parameters,  $\delta \in \{0.2, 0.4, 0.7\}$ , and we maintain the assumption that all firms are in the coalition. We also compute the Bertrand equilibrium. Of interest is how prices vary with market structure under the different equilibrium assumptions and timing parameters.

Figure 3 plots the equilibrium prices that obtain as a function of the Herfindahl-Hirschman Index (HHI), measured on a scale from 0 to 10,000. As one would expect, equilibrium prices increase at least weakly in the HHI for each model, just as for a given HHI equilibrium prices are higher for larger timing parameters. (Bertrand can be conceptualized as PLE with  $\delta = 0$ .)

What is more interesting is the *curvature* of prices as HHI approaches the monopoly level. With Bertrand, price differentials are relatively modest at the lower levels of HHI, and they become more pronounced at the higher levels of HHI. The greatest price differential is between duopoly (HHH=5000) and monopoly (HHI= 10,000), which also is associated with the largest jump, 5000 points, in HHI. This conforms to the standard intuition that arises for the unilateral effects of mergers: the potential for anti-competitive effects often is greater, the greater the change in market concentration that the merger would induce, all else equal (e.g., Nocke and Whinston, 2022). A different pattern is apparent with PLE and a timing parameter of 0.70. In that model, price differentials are largest at lower levels of HHI. With ten firms (HHI=1000), prices are 90% of the monopoly price, and the monopoly price is achieved if there are five or fewer firms (HHI above 5000). Thus, prices are flat at the higher levels of HHI. The patterns for PLE with  $\delta = 0.2$  and

Table 4: MillerCoors Merger Counterfactual Results

		Baseline	Merger	Merger with Efficiencies
Bertrand Average Price		8.52	8.96	8.65
Supermarkup		1.20	1.73	1.83
ICC Slack	ABI	$2.04 \times 10^{-2}$	$8.90 \times 10^{-3}$	$5.91 \times 10^{-3}$
	SABMiller	$6.70 \times 10^{-3}$		
	Molson Coors	$1.07 \times 10^{-10}$		
	MillerCoors		$1.41 \times 10^{-9}$	$1.53 \times 10^{-9}$

*Notes:* “ABI” is Anheuser-Busch Inbev. In the merger with efficiencies, MillerCoors is given the same cost as ABI after the merger. “Bertrand Average Price” is the share weighted average price that would be observed in a Bertrand equilibrium.

$\delta = 0.4$  fall in between these two cases.

The figure illustrates that price leadership affects the relationship between market structure and prices. This has implications for the price effects of mergers. In particular, it indicates that price leadership can generate significant merger price effects at lower levels of market concentration than would obtain under Bertrand competition. The flip side is if price leadership allows firms to set monopoly prices, then further consolidation would not cause prices to increase further (though it may lock in coordination or have other adverse competitive effects). These patterns are not unique to price leadership, as they have been explored using other models of coordination (e.g., Verboven, 1995). Furthermore, other considerations affect the relationship between market structure and prices, including forward contracts and imperfect information about rivals’ marginal costs (Miller and Podwol, 2020; Sweeting et al., 2022). Still, our results highlight that the model of competition has implications for the price effects of mergers, and we explore these in greater detail next.

## 4.2 IC Constraints and Mergers

In the price leadership model, mergers affect the equilibrium supermarkup if they shift or change the binding IC constraint.<sup>27</sup> The two scenarios for which this effect tends to be relatively large arise if (i) the merger involves the firm with the binding IC constraint, or (ii) the merger involves a fringe firm whose presence in the market limits the supermarkup that can be set without inducing deviation. Some other mergers that can present significant concerns about adverse unilateral effects tend to result in smaller changes in the equilibrium supermarkup. An example is a merger that involves two coalition firms that both have slack in their IC constraints—although such a merger affects the binding IC constraint (e.g., by changing punishment profit), changes to the equilibrium

<sup>27</sup>This assumes that the coalition is exogenously determined. A merger also could change the set of firms that engage in coordination, and a number of different assumptions could be incorporated to endogenize any such changes within the model (e.g., d’Aspremont et al., 1983; Donsimoni et al., 1986; Pastine and Pastine, 2004; Ishibashi, 2008; Bos and Harrington, 2010; Mouraviev and Rey, 2011).

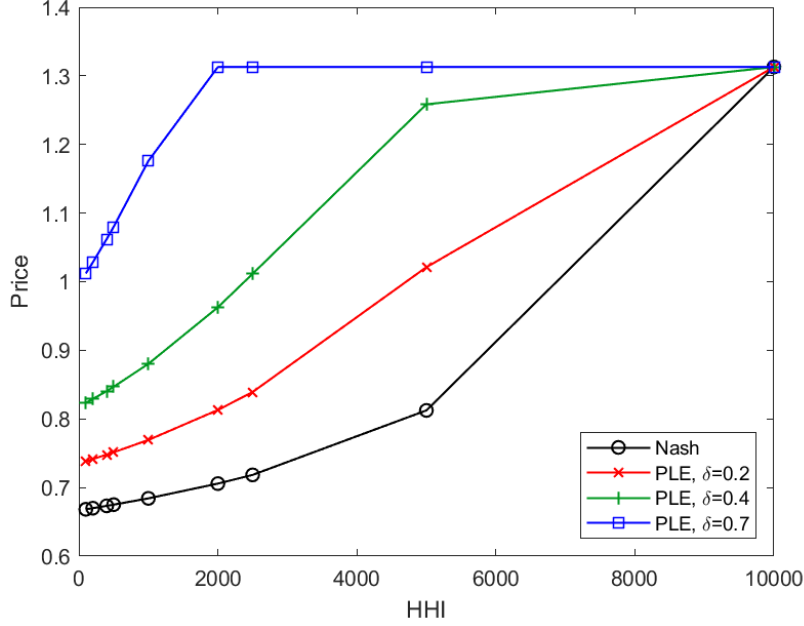


Figure 3: Market Structure and Prices

supermarkup tend to be relatively small in the calibrations that we have considered.

We illustrate this numerically using a six-firm industry in which three firms are in the coalition and three firms are in the fringe. We assume a nested logit demand system in which the coalition's products are in a different nest from the products of the fringe. Table 5 shows the parameter values that we use along with selected outcomes from the pre-merger equilibrium. The leader, Firm 1, has the highest market share because, with our calibration, it has relatively high quality and low cost. Firm 3 has relatively lower quality and higher costs. Among the coalition firms, it has the most elastic demand, and this contributes to it being the firm that has the binding IC constraint.

Table 6 shows how equilibrium outcomes change due to a number of different mergers. The first column provides the pre-merger outcomes to facilitate comparisons. The second and third columns consider mergers among coalition firms. We find that a merger between Firms 1 and 3 increases the supermarkup by 32 percent (column (ii)). The reason is that the merger combines the economic interests of the firm with the binding IC constraint with those of another coalition firm, and thereby allows a higher supermarkup to be set without inducing deviation. By contrast, a merger between Firms 1 and 2 does not change the equilibrium supermarkup (at least not enough to survive rounding to the second decimal point). With this second merger, the presence of Firm 3 in the coalition limits the supermarkup, and its IC constraint is largely unaffected.

We next consider two mergers that involve the largest fringe firm, Firm 4, and one coalition firm. In column (iv) the merging partner is Firm 2 and in column (v) it is Firm 3. In simulating the post-merger equilibrium, we allow the coalition to set different supermarkups across the nests.



Table 5: Baseline Price Leadership Equilibrium

Firm	Nest	Quality	Marginal Cost	Coalition Member	Pre-Merger Price	Pre-Merger Share	Binding ICC
1	1	2.00	0.40	Yes	1.00	0.28	
2	1	1.75	0.50	Yes	1.00	0.17	
3	1	1.50	0.70	Yes	1.13	0.06	X
4	2	2.00	1.30	No	1.66	0.07	
5	2	2.00	1.50	No	1.80	0.04	
6	2	2.00	1.70	No	1.97	0.02	

*Notes:* The price coefficient is  $\alpha = -2$ , the nested logit parameter is  $\sigma = 0.5$ , and the timing parameter is  $\delta = 0.3$ . The outside good has a pre-merger share of 0.37, total variable profit is 0.32, the pre-merger supermarkup is  $m = 0.16$ , and consumer surplus (net of a constant) is 0.50.

Although alternative assumptions are possible, our approach tracks how MSW model an acquisition of Modelo by ABI in beer markets, and it makes economic sense insofar as it increases the profit of the coalition and does not undermine the ability of the coalition firms to implement coordination.<sup>28</sup> In antitrust parlance, it may be reasonable to interpret these mergers as eliminating a “maverick” (Firm 4), where the term is defined in the *Horizontal Merger Guidelines* as a firm that “plays a disruptive role in the market for the benefit of consumers.”<sup>29</sup>

We find that in the merger between Firms 2 and 4, the supermarkup for Group 1 remains unchanged at 0.16, and the (new) supermarkup for Group 2 is 0.05. In the merger between Firms 3 and 4, the supermarkup for Group 1 increases to 0.21, and the (new) supermarkup for Group 2 is 0.04. Thus, while both mergers result in higher prices, the merger involving the coalition firm with the binding IC constraint (Firm 3) matters more for the supermarkup of the coalition. The reason is that the merger allows the binding firm to recapture some of the sales that go from the coalition to Group 2, and this softens its IC constraint to a greater degree.

### 4.3 Efficiencies and Pass-Through

In some cases, mergers may result in marginal cost reductions that offset the incentives to raise prices. The pass-through of cost reductions into prices depends on how firms compete with one another. We explore the pass-through of marginal cost reductions under price leadership using a numerical example where we reduce the marginal costs of each of the merged firms’ products by varying percentages. Our results suggest a more nuanced and conditional role for efficiencies in assessments of consumer welfare than obtains with Bertrand competition.<sup>30</sup>

<sup>28</sup>The supermarkup in Group 2 is set by one of the coalition firms that owns a product in that nest. This is the leader if the leader owns a product in Group 2, or it is another firm if the leader does not. After the Group 2 supermarkup is set, the leader chooses the Group 1 supermarkup.

<sup>29</sup>*Horizontal Merger Guidelines* §2.1.5.

<sup>30</sup>Another modeling framework in which the merged firm does not pass-through marginal cost savings is provided in Harrington (2021). In that model, the merged firms have private information on the magnitude of (realized) merger

Table 6: Mergers in PLE

	(i)	(ii)	(iii)	(iv)	(v)
Ownership Structure	1*	1*	1*	1*	1*
	2*	2*	1*	2*	2*
	3*	1*	3*	3*	3*
	4	4	4	2*	3*
	5	5	5	5	5
	6	6	6	6	6
Normalized Prices	1.00	1.09	1.16	1.01	1.05
	1.00	1.07	1.26	1.01	1.05
	1.00	1.23	1.02	1.01	1.06
	1.00	1.00	1.00	1.05	1.03
	1.00	1.00	1.00	1.01	1.01
	1.00	1.00	1.00	1.01	1.01
Outside Share, $s_0$	0.37	0.40	0.43	0.38	0.39
Consumer Surplus	0.50	0.45	0.42	0.49	0.47
Total Profit	0.32	0.34	0.34	0.32	0.33
Group 1 Supermarkup, $m_1$	0.16	0.21	0.16	0.16	0.21
Group 2 Supermarkup, $m_2$				0.05	0.04

*Notes:* Firm's with a \* are in the coalition. Firm 3 has the binding IC constraint in the first and fourth columns. Firm 2 has the binding IC constraint in the second and fifth columns. Firm 1 has the binding IC constraint in the third column. Prices are normalized by dividing by pre-merger prices.

Consider the hypothetical market that we use in the preceding subsection. Figure 4 shows how the equilibrium supermarkup changes as marginal costs are reduced for each of the coalition firms. When costs are reduced at Firms 1 or 2 (holding the other firms' costs constant) the equilibrium supermarkup exhibits little change. This is because the supermarkup remains constrained by Firm 3's IC constraint. However, if Firm 3's costs fall, the supermarkup exhibits significant change. The reduction in Firm 3's cost creates slack in its IC constraint, which allows the coalition to achieve a higher supermarkup. The supermarkup rises until the reduction in Firm 3's cost reaches 47 percent (denoted with the vertical dashed line), and then falls slowly as Firm 3's costs are reduced further. The reason for this kink is illustrated by Figure 5, which shows the slack in each firm's IC constraint as Firm 3's costs are reduced. For cost reductions below 47 percent, Firm 3 remains the firm with the binding IC constraint. In this range, reducing Firm 3's costs creates slack in its IC constraint, leading to a higher equilibrium supermarkup. For larger cost reductions, Firm 2 becomes the firm with the binding IC constraint. In this range, further reducing Firm 3's cost still creates slack in its IC constraint but this no longer leads to a higher equilibrium supermarkup because the constraint is not binding.

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efficiencies and has an incentive to price as if efficiencies have not been realized in order to avoid tougher pricing by its competitors.

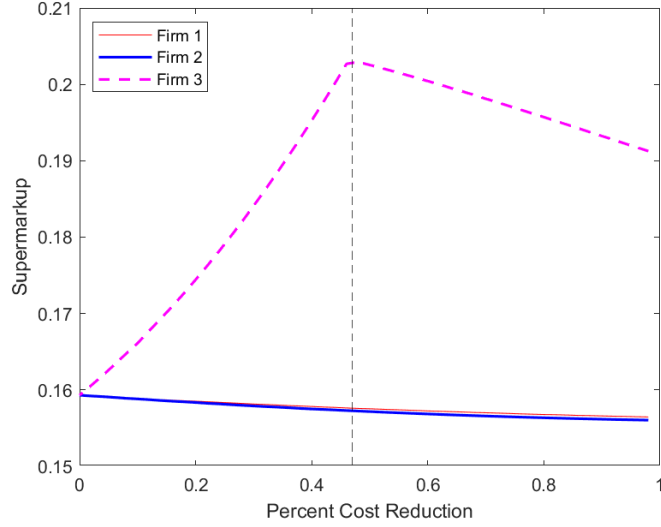


Figure 4: Supermarkup Changes with Varying Marginal Cost Reductions

Figure 6 extends this logic to a scenario in which the cost reductions are the result of a merger. We assume that Firms 2 and 3 merge, and we examine how varying levels of cost reductions at the merged firm affect the equilibrium supermarkup.<sup>31</sup> In the left panel, the black horizontal line denotes the pre-merger supermarkup. For all levels of cost reduction, the post-merger supermarkup is greater than the pre-merger supermarkup. This occurs because Firm 3 has the binding IC constraint pre-merger, and as discussed previously, mergers that involve a firm with a binding IC constraint tend to increase the supermarkup. After the merger, the merged firm has the binding IC constraint when there are no efficiencies, as shown in the right panel. Accordingly, reducing the cost of the merged firm increases the supermarkup until the cost reduction reaches 30 percent. At this point—which we denote using the vertical dashed line in both panels—Firm 1’s IC constraint becomes binding. As the merged firm’s cost is reduced further, the equilibrium supermarkup does not change significantly because the cost reductions no longer affect the firm with the binding IC constraint. This exercise also illustrates that, under the right circumstances, efficiencies can exacerbate the coordinated effects of a merger.

#### 4.4 Divestitures

A common remedy to mergers deemed to be anti-competitive involves the divestiture of some products or brands to a third party. Here, we explore the effects of divestitures in a scenario where firms are engaged in a price leadership equilibrium. As with efficiencies, the implications of divestitures can differ from what would arise under a model of Bertrand competition.

In this subsection, we consider the same set of products described in the previous subsections,

<sup>31</sup>In this exercise, the costs of the two products owned by the merged firm are being reduced by the same percentage.

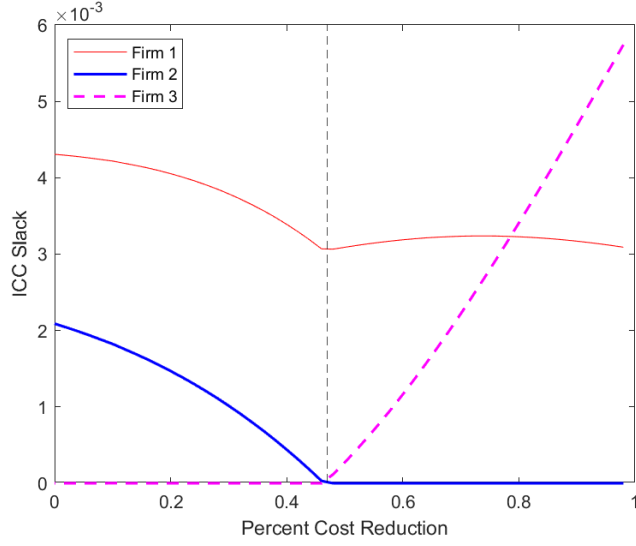


Figure 5: IC Constraint Slack with Varying Marginal Cost Reductions at Firm 3

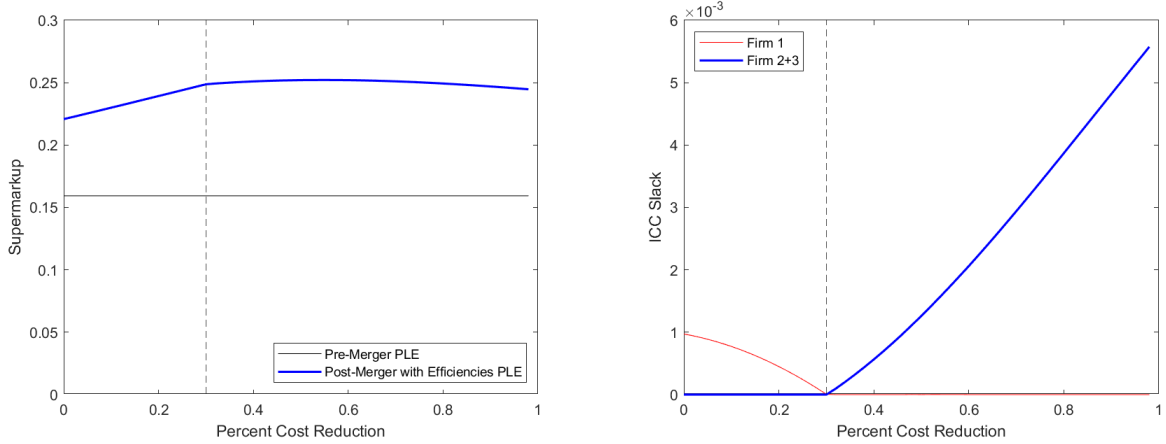


Figure 6: Supermarkup Changes from a Merger with Varying Marginal Cost Reductions

but we now assume all six inside products are in the same nest. Additionally, we consider a different ownership structure, where the six products are owned by four firms (two single product firms and two multiproduct firms). Firms 1, 2, and 3 are in the coalition, while Firm 4 is in the fringe. Column (i) of Table 7 shows the baseline ownership structure, equilibrium supermarkup, and welfare.

Columns (ii)-(iv) of Table 7 consider a merger between Firm 1 and Firm 2, and potential divestitures to remedy the effects of this merger. Column (ii) shows the effect of this merger without divestitures. In the baseline scenario, Firm 3 has the binding IC constraint, and this remains true after the merger of Firm 1 and Firm 2. As a result, this merger does not cause a large change in the equilibrium supermarkup, although welfare falls due to the unilateral effects of the merger. Column (iii) considers the impact of this merger if Firm 1 is required to divest Product

Table 7: Mergers and Divestitures

	(i)	(ii)	(iii)	(iv)	(v)	(vi)
Ownership Structure	1*	1*	1*	1*	1*	1*
	2*	1*	3*	5*	2*	2*
	3*	3*	3*	3*	1*	5*
	3*	3*	3*	3*	1*	1*
	2*	1*	1*	1*	2*	2*
	4	4	4	4	4	4
Consumer Surplus	0.42	0.33	0.37	0.42	0.36	0.42
Total Profit	0.32	0.34	0.35	0.32	0.34	0.32
Supermarkup	0.17	0.18	0.24	0.17	0.23	0.17

*Notes:* Firm's with a \* are in the coalition. Firm 3 has the binding IC constraint in columns (i-iv). Firm 2 has the binding IC constraint in column (v). Firm 5 has the binding IC constraint in column (vi).

2 to Firm 3. Firm 3's acquisition of Product 2 creates slack in their IC constraint, which allows the equilibrium supermarkup to increase. This increase in supermarkup offsets the reduction in Bertrand prices caused by the divestiture, with the result being that the divestiture only partially remedies the effects of the merger. Welfare is higher than with no divestiture, but lower still lower than before the merger. Column (iv) considers a divestiture of Product 2 to Firm 5, a firm that has not previously participated in the market. The supermarkup does not increase as a result of this divestiture, and welfare returns to nearly pre-merger levels.

Columns (v)-(vi) consider a merger between Firm 1 and Firm 3 and a potential divestiture. In column (v), the merger is completed without any divestiture. Because Firm 3 had the binding IC constraint prior to the merger, this merger causes the supermarkup to increase. Column (vi) considers the merger with a divestiture of Product 3 to Firm 5, a firm that has not previously participated in the market. This divestiture remedies the coordinated effects of the merger, causing the supermarkup and welfare to return to nearly pre-merger levels. Thus, across the scenarios presented in Table 7, there are instances in which a divestiture amplifies, diminishes, and has little effect on coordination in the market. What appears to drive these differences is who has the binding IC constraint—one of the merging firms, the divestiture recipient, or some other firm.

## 5 Conclusion

Our purpose has been to provide an accessible methodology for simulating the coordinated effects of mergers. Starting from the price leadership model of MSW, we show how a simplified version of the framework can be calibrated with data that are often available to antitrust authorities. By introducing a supermarkup into the typical Bertrand setup, we demonstrate how the methods that practitioners are already familiar with in the context of unilateral effects simulations can be

extended to quantify the impact of collusion. Our numerical results highlight the role that IC constraints play in determining the equilibrium effects of mergers.

The model that we explore remains somewhat specialized. For example, coordination need not take the form of price leadership. As a general matter, consideration should be given to the applicability of the model to a given empirical setting. Furthermore, the paths to calibration that appear most feasible to us involve a specific set of facts about the market, including that some but not all firms engage in pre-merger coordination. In part because the broad application of our model may not be warranted, we continue to view the modeling of coordinated effects to be a promising area for future research. Adapting recent advancements in the structural empirical literature on cartels and collusion (e.g., Igami and Sugaya, 2021) for use by practitioners, keeping in mind the data and time limitations of merger investigations and litigation, may be helpful to antitrust authorities.

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## A Appendix

### A.1 Details of the Nested Logit Calibration

If products  $k$  and  $j$  are in the same subgroup  $h$  then the diversion ratio that characterizes substitution from  $k$  to  $j$  is given by

$$div_{k \rightarrow j} = \frac{s_j(1 - \sigma_1)(1 - \sigma_2) + \bar{s}_{j|h} \bar{s}_{h|g} \sigma_2(1 - \sigma_1) + \bar{s}_{j|h}(\sigma_1 - \sigma_2)}{(1 - \sigma_2) - s_k(1 - \sigma_1)(1 - \sigma_2) - \bar{s}_{k|h} \bar{s}_{h|g} \sigma_2(1 - \sigma_1) - \bar{s}_{k|h}(\sigma_1 - \sigma_2)}. \quad (\text{A.1})$$

If instead products  $k$  and  $j$  are in different subgroups then the diversion ratio is

$$div_{k \rightarrow j} = \frac{s_j(1 - \sigma_1)(1 - \sigma_2) + \bar{s}_{j|h(j)g} \bar{s}_{h(j)|g} \sigma_2(1 - \sigma_1)}{(1 - \sigma_2) - s_k(1 - \sigma_1)(1 - \sigma_2) - \bar{s}_{k|h(k)g} \bar{s}_{h(k)|g} \sigma_2(1 - \sigma_1) - \bar{s}_{k|h(k)g}(\sigma_1 - \sigma_2)}, \quad (\text{A.2})$$

where we use  $h(k)$  and  $h(j)$  to denote the subgroups of the two products, respectively. The diversion ratio from product  $k$  to the outside good is

$$div_{k \rightarrow 0} = \frac{s_0(1 - \sigma_1)(1 - \sigma_2)}{(1 - \sigma_2) - s_k(1 - \sigma_1)(1 - \sigma_2) - \bar{s}_{k|h} \bar{s}_{h|g} \sigma_2(1 - \sigma_1) - \bar{s}_{k|h}(\sigma_1 - \sigma_2)}. \quad (\text{A.3})$$

The first order condition that characterizes the profit-maximizing price for product  $k$  is given by

$$\begin{aligned} p_k - c_k = & \frac{1 - \sigma_1}{\alpha} + (1 - \sigma_1) \sum_{j \in \mathbb{J}_{f(k)}} (p_j - c_j) s_j \\ & + \frac{(1 - \sigma_1) \sigma_2}{1 - \sigma_2} \sum_{j \in \mathbb{J}_{f(k)} \cap \mathbb{J}_{g(k)}} (p_j - c_j) s_{j|h} s_{h(j)|g} \\ & + \frac{\sigma_1 - \sigma_2}{1 - \sigma_2} \sum_{j \in \mathbb{J}_{f(k)} \cap \mathbb{J}_{h(k)}} (p_j - c_j) s_{j|h}, \end{aligned} \quad (\text{A.4})$$

which simplifies to equation (24) if product  $k$  is sold by a firm operating in only one nest.

Depending on the data available, some combination of equations (A.1)-(A.4) can be solved for the demand parameters  $(\alpha, \sigma_1, \sigma_2)$ .<sup>32</sup> For example, with data on the markup for a fringe firm only selling in one subgroup, along with diversion between two inside products in the same subgroup, and between two inside products in different subgroups, we could use equations (A.1), (A.2), and (24). With  $(\alpha, \sigma_1, \sigma_2)$  in hand, the Berry (1994) inversion then can be used to recover  $\{\beta_j\}_{j \in \mathbb{J}, j \neq 0}$ ,

$$\ln(s_j) - \ln(s_0) = \beta_j - \alpha p_j + \sigma_1 \ln(\bar{s}_{j|h}) + \sigma_2 \ln(\bar{s}_{h|g}), \quad (\text{A.5})$$

which extends equation (12) to the nested logit case.

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<sup>32</sup>Alternatively, one could substitute data on the market elasticity in order to recover  $\alpha$  from equation (10).