University of Waterloo

Faculty of Engineering

Department of Electrical and Computer Engineering

ECE 481: Lab 1

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Contents

Lab 1: Modeling of the Motor System

Introduction

In this lab, I will apply various control techniques in order to remove non-linearities, perform system identification on an unknown plant, and limit the output of the system. All the simulations are done in Simulink and Matlab and some calculations are done in Microsoft Excel.

The system being modelled is a motor which will ultimately be used to control the position of a ball on a track. The track angle is dependent on the motor angle so to control the position of the ball, we must first characterize and control the motor properly.

Stiction compensator

In order to characterize the stiction behaviour. I used a ramp source as a reference that started at 0 V and ramped up. This way, I could graph the motor angle vs time (time is equivalent to the reference voltage here since the slope of my ramp was 1 V/s). I could then use the graph to see when the motor started to turn and overcome the stiction. As shown in figure 1, the motor started to turn at 0.371 V at the input to the motor.

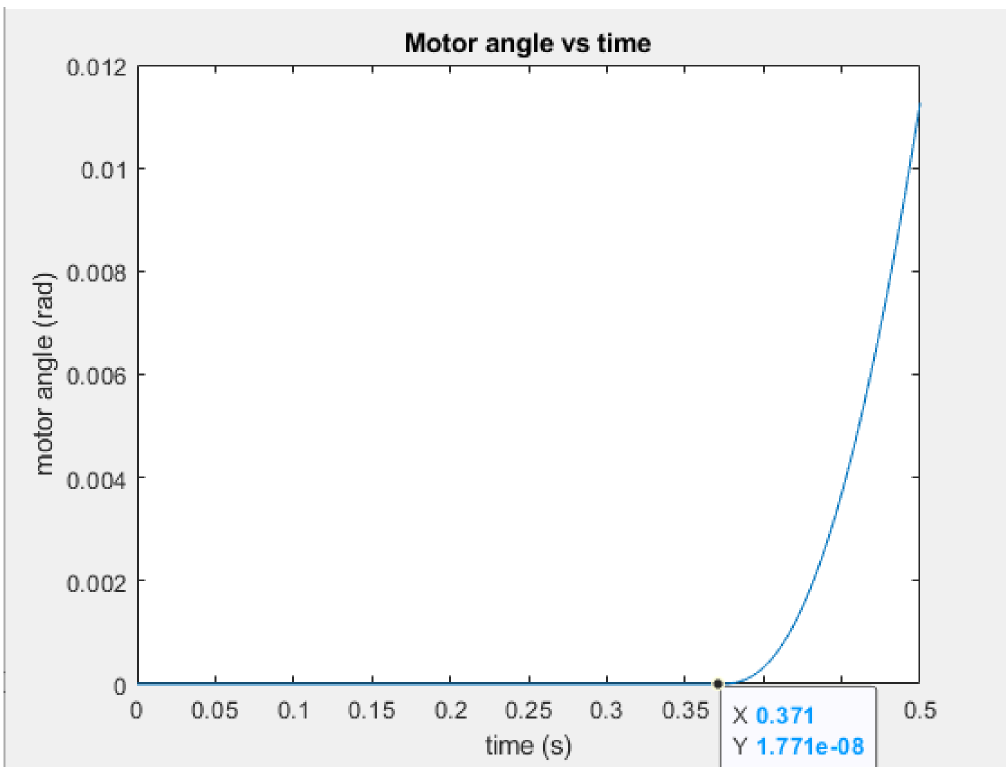

Figure 1. Stiction for counter-clockwise rotation (positive applied voltage)

I applied the same strategy to find the stiction behaviour in the clockwise direction. I had a ramp source reference that started at 0 V and changed its output voltage at a rate of -1 V/s. I then graphed the result as shown in figure 2, and found that the motor began to turn at an input voltage of -0.601 V.

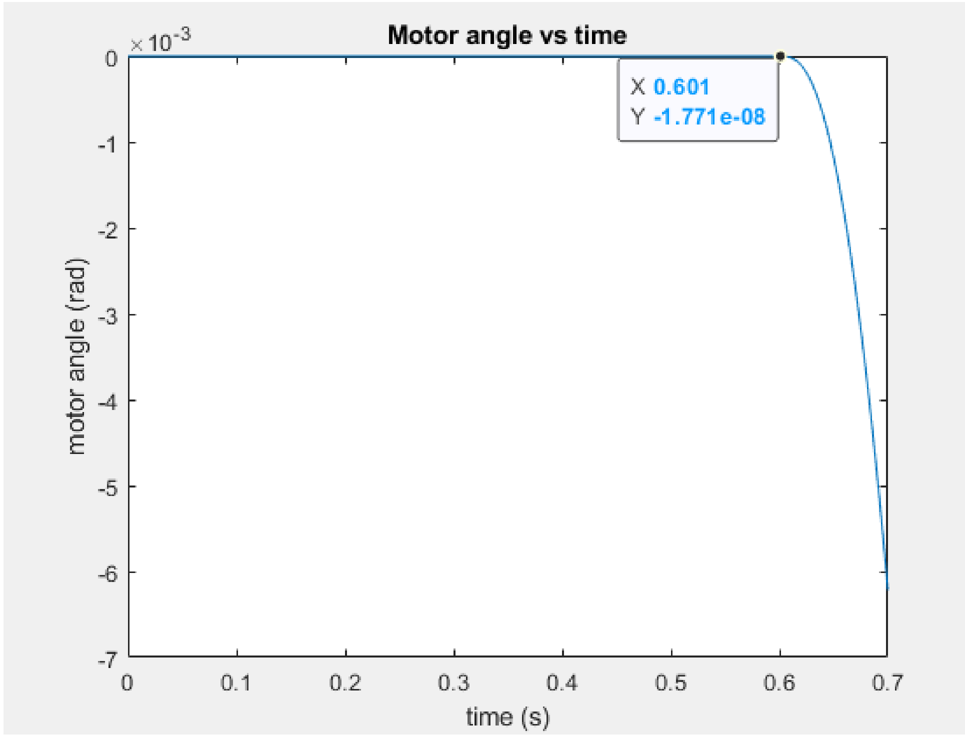

Figure 2. Stiction for clockwise rotation (negative applied voltage)

Thus, in order to avoid stiction, I needed to apply an offset of roughly -0.602 V for negative references and 0.37 V for positive references. This was implemented with a relay with it's on voltage set to 0.37 V and its off voltage set to -0.602 V. The block diagram for the stiction compensator described here is shown in figure 3.

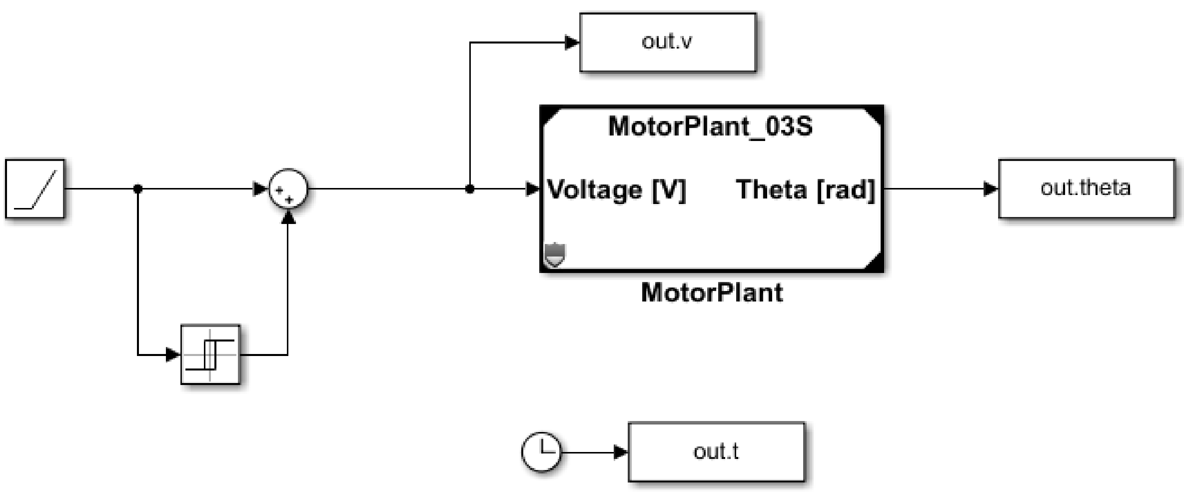

Figure 3. Stiction compensator block diagram

Figures 4 and 5 show the performance of the motor with the stiction compensator in place. As can be seen from the graphs, there is no deadzone and the motor begins turning for any non-zero voltage. Again, I used a ramp as a source starting from 0 and increasing the voltage at a rate of 1 V/s for figure 4 and -1 V/s for figure 5.

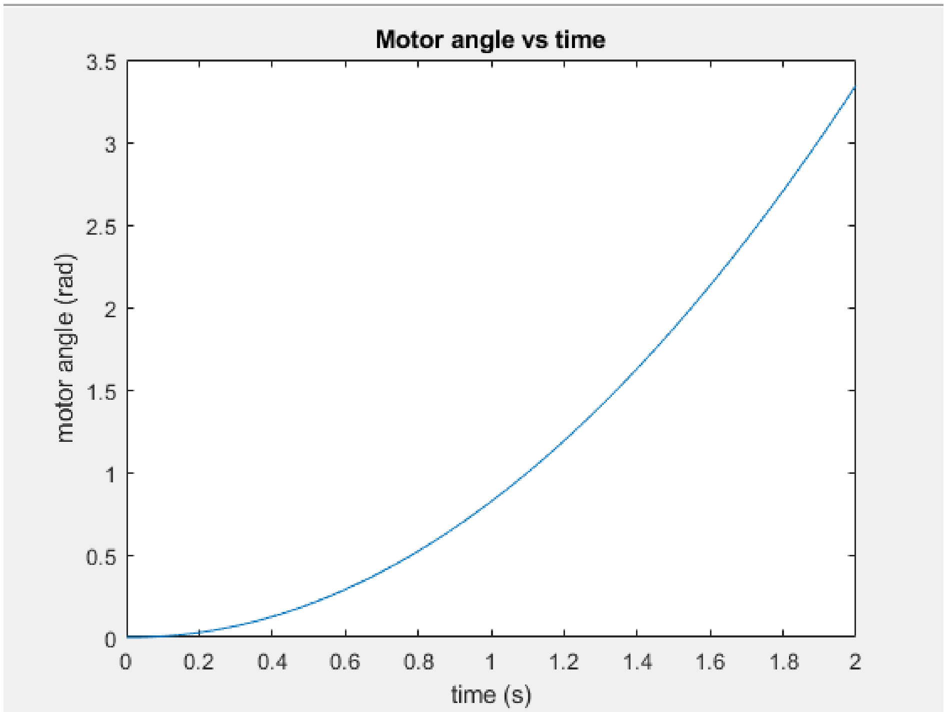

Figure 4. Removal of stiction for counter-clockwise rotation (positive applied voltage)

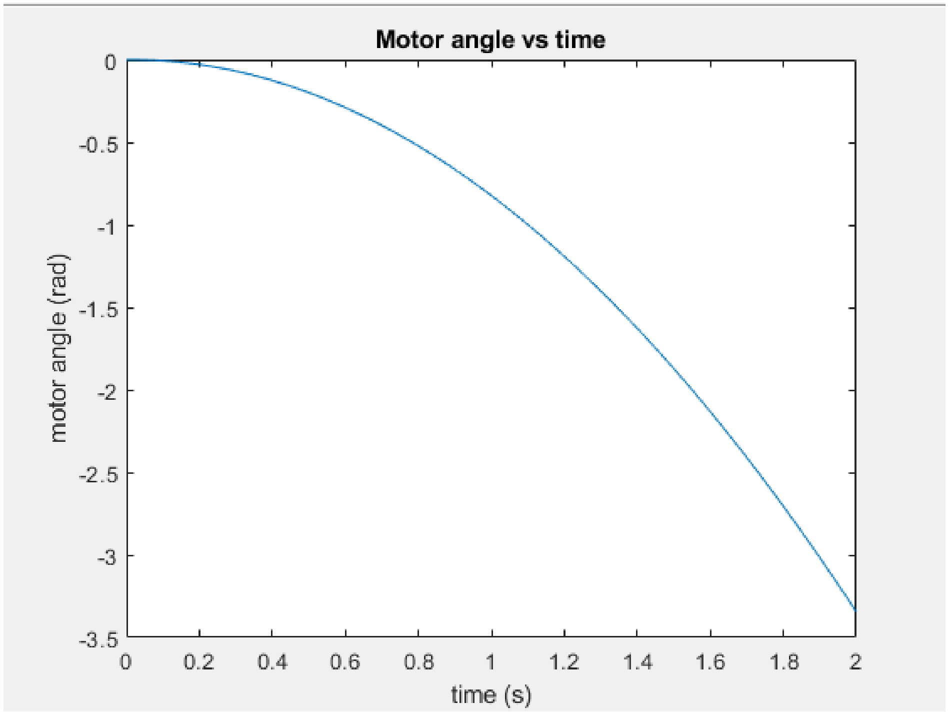

Figure 5. Removal of stiction for clockwise rotation (negative applied voltage)

System identification

The model of the plant takes the form of

$$P(s) = \frac{k_1}{\tau * s^2 + s}$$

In order to find values for k_1 and τ, I used time fitting. In order to do this properly, I had to perform system identification on the closed loop system instead of the plant directly. This is because the plant is open-loop unstable so there was no guarantee that the output of the plant would remain bounded for a given input. The closed loop transfer function takes the following form:

$$G(s) = \frac{C(s) * P(s)}{1 + C(s) * P(s)}$$

$$G(s) = \frac{k * k_1}{\tau s^2 + s + k * k_1}$$

Equation 1:

$$G(s) = \frac{k * k_1 * \frac{1}{\tau}}{s^2 + \frac{s}{\tau} + k * k_1 * \frac{1}{\tau}}$$

I chose C(s) = k, where k is a positive gain greater than one. This works because of the root locus diagram shown in figure 6. The exact location of poles do not really matter, as long as they are in the open left hand plane, which is the case for positive values of τ, k_1, and k. As we can see from figure 6, the closed loop system is always stable, meaning a proportional controller fully stabilizes the system.

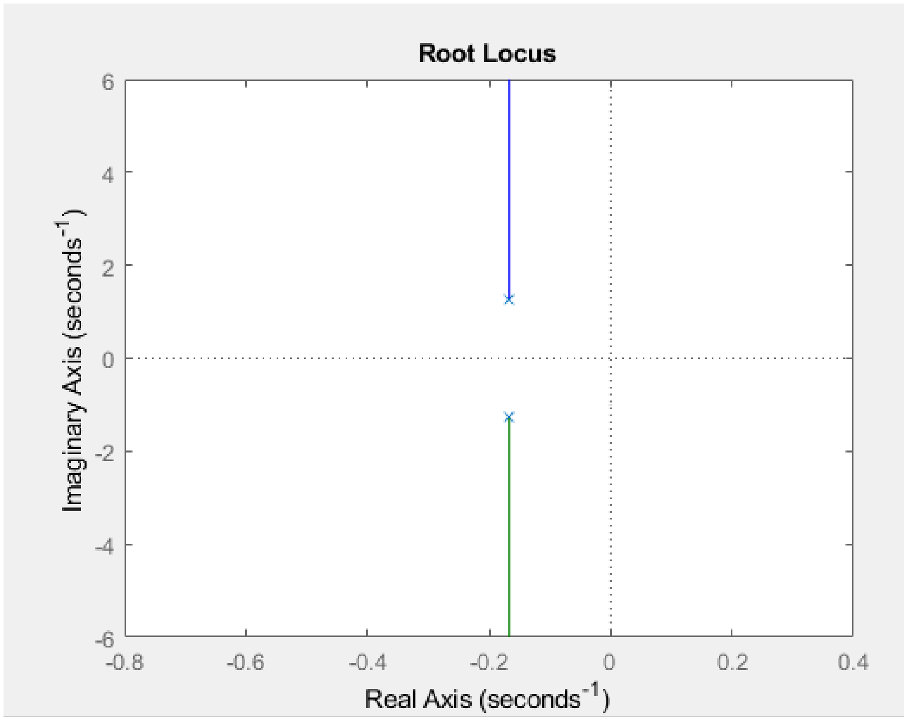

Figure 6. General root locus plot for this closed loop system

Identification by Time Response Fitting

In order to calculate values for τ and k_1, I simply needed to apply a step and design the controller with enough gain to have the system be underdamped. I also needed to ensure the input voltage to the motor was between -6 V and 6 V to ensure the signal did not saturate and the system remained linear. I did this by making the gain of the controller at least 15 and lowering the step voltage such that k*r always remained below 6V, where k is the gain of the controller and r is the step reference voltage level. Then I could make measurements on the response and calculate k_1 and τ.

The measurements I took were the time to peak and percent overshoot. This gave me all the information I needed to characterize the system. Figures 7 through 18 show the step response of the system at various controller gains and step voltage levels as well as the motor input voltages to ensure that the signals are not saturating. The reason multiple steps and gains were used was to help ensure the values for τ and k_1 are as accurate as possible.

For the step response plots, the orange line is the applied step signal and the blue line is the plant output.

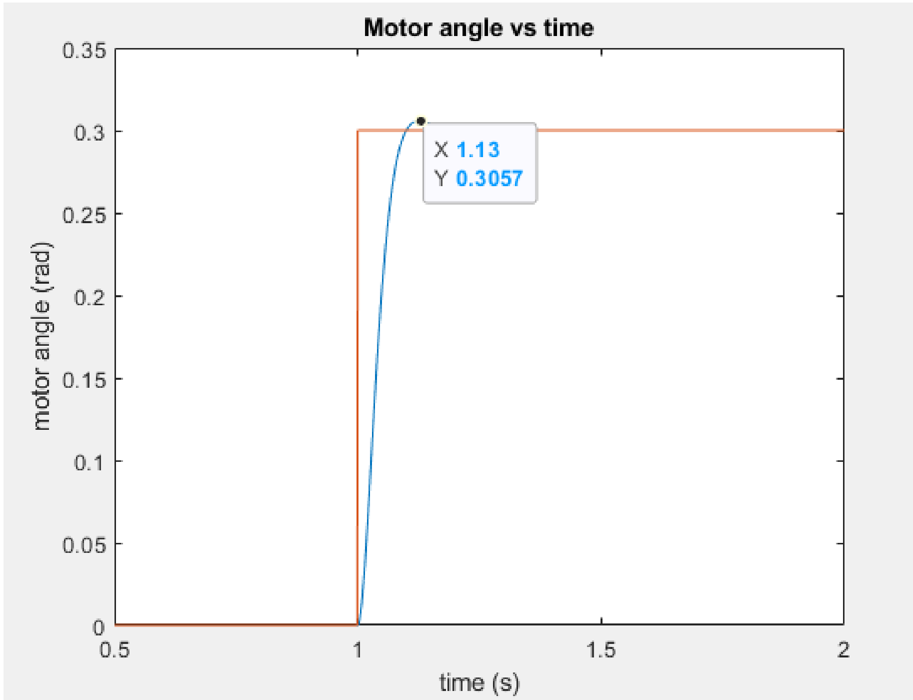

*Figure 7. Step response for time response fitting, C(s) = 15, r(t) = 0.3*u_{step}(t)*

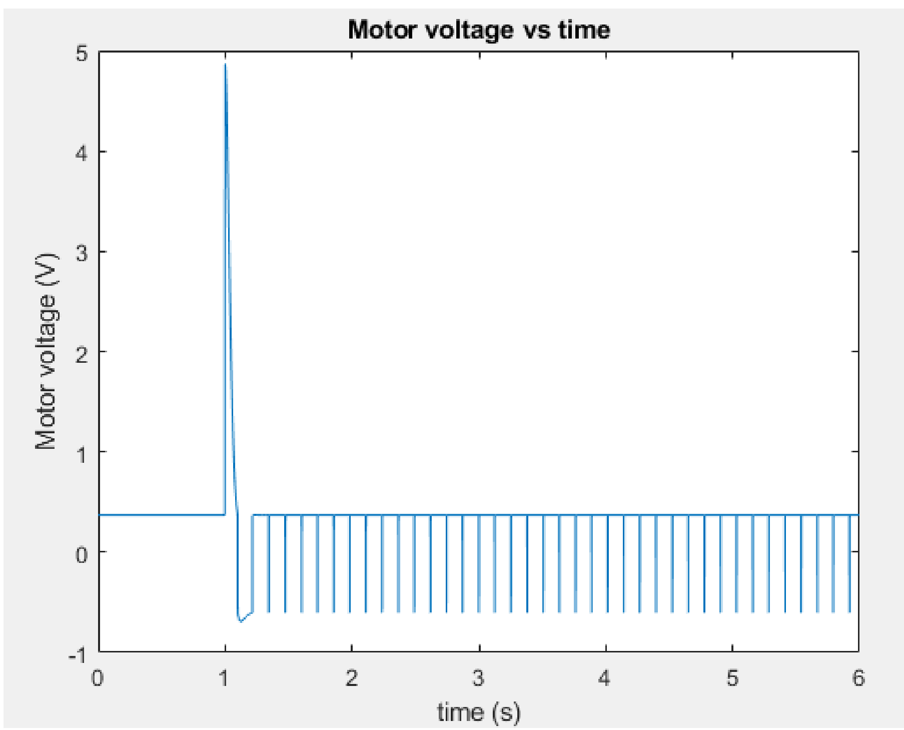

*Figure 8. Motor voltage vs time, C(s) = 15, r(t) = 0.3*ustep(t)*

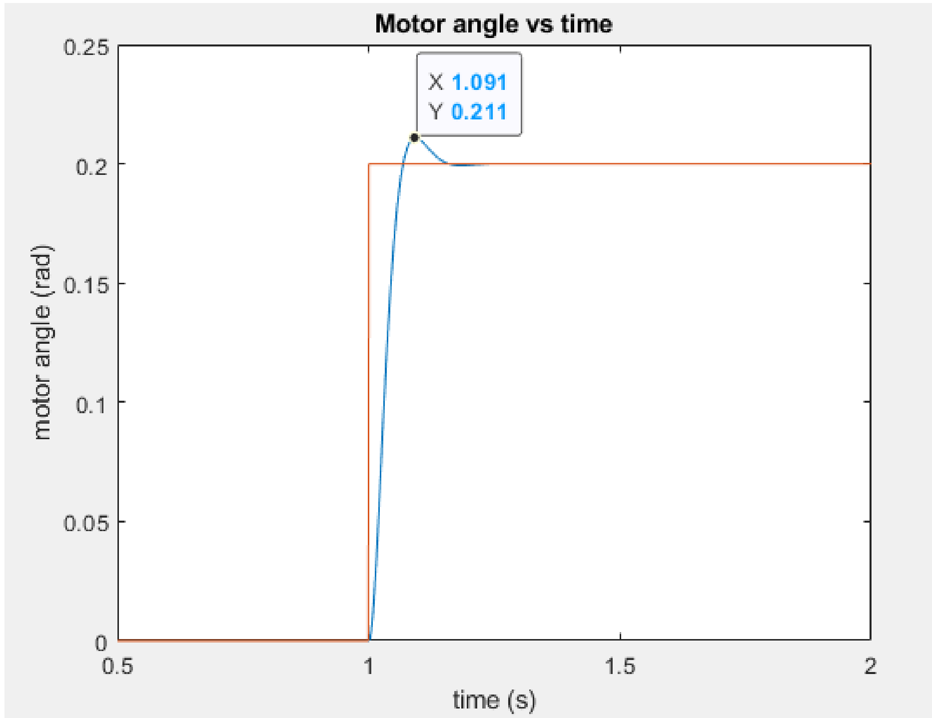

*Figure 9. Step response for time response fitting, C(s) = 20, r(t) = 0.2*u_{step}(t)*

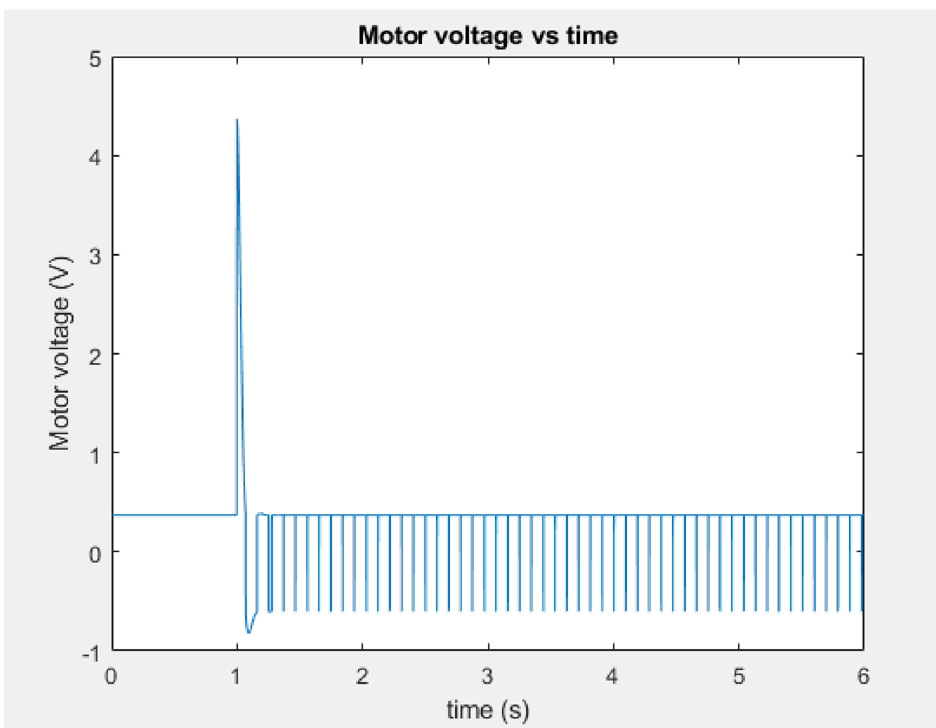

*Figure 10. Motor voltage vs time, C(s) = 20, r(t) = 0.2*u_{step}(t)*

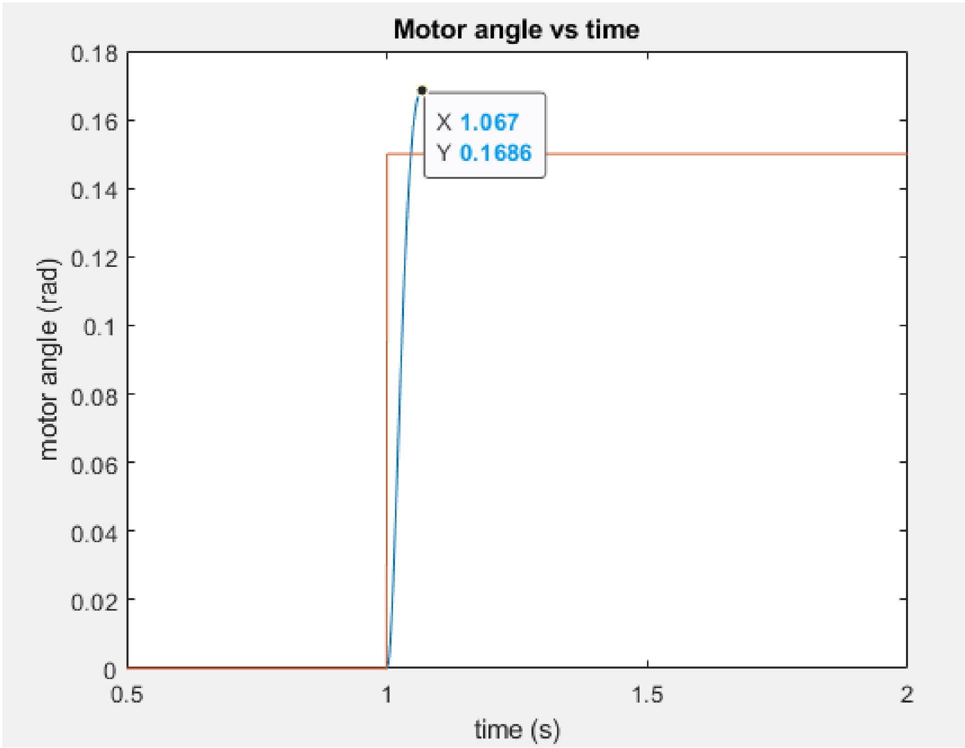

*Figure 11. Step response for time response fitting, C(s) = 30, r(t) = 0.15*u_{step}(t)*

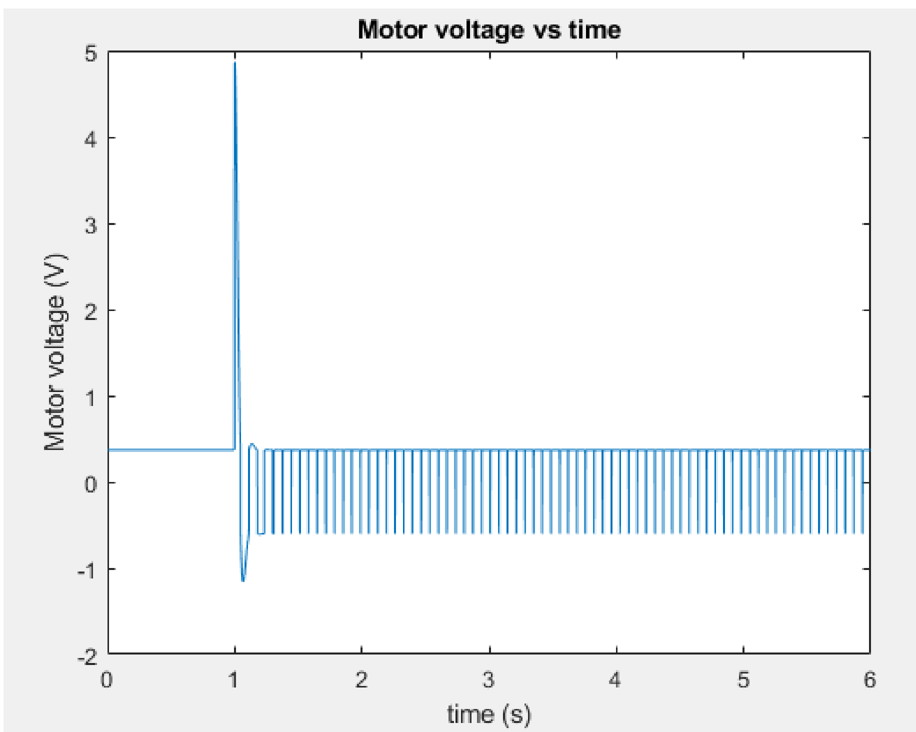

*Figure 12. Motor voltage vs time, C(s) = 30, r(t) = 0.15*u_{step}(t)*

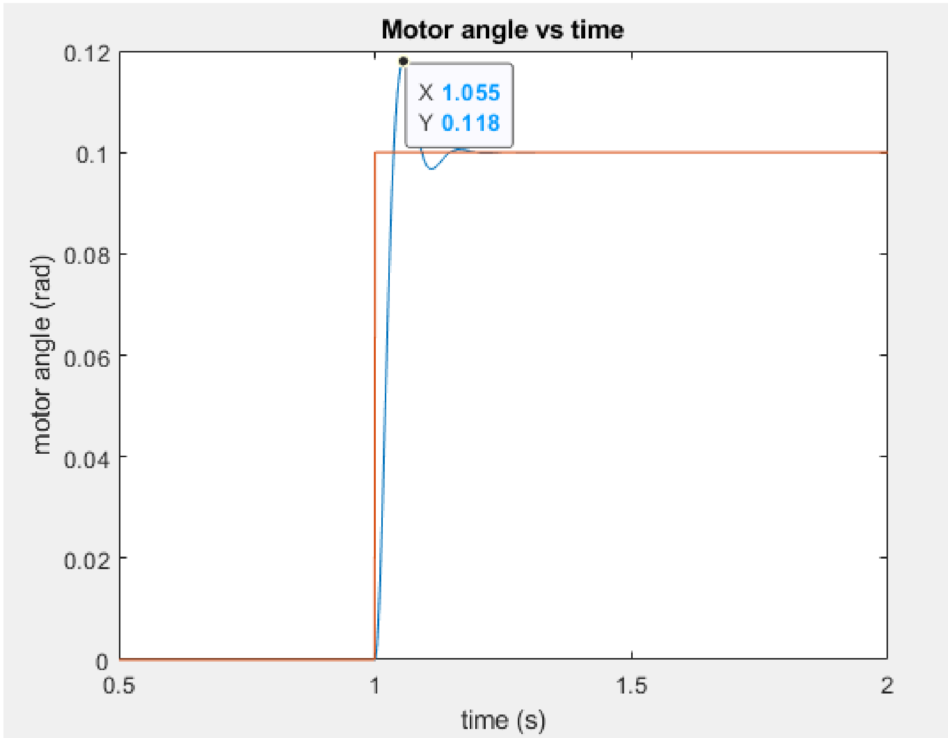

*Figure 13. Step response for time response fitting, C(s) = 40, r(t) = 0.1*u_{step}(t)*

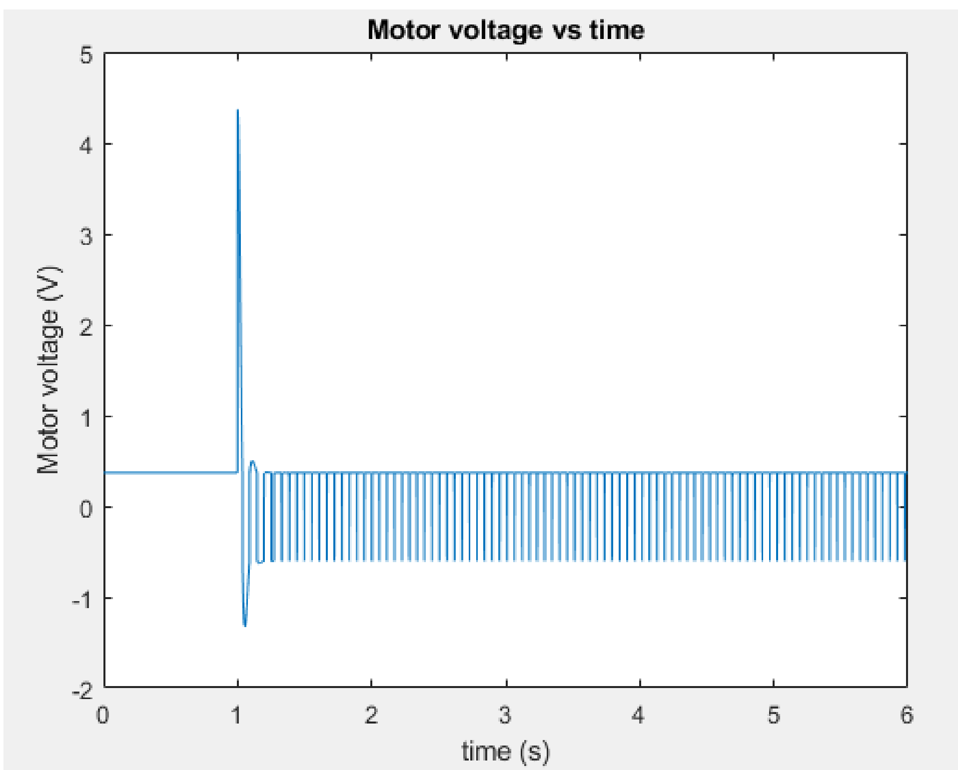

*Figure 14. Figure 12. Motor voltage vs time, C(s) = 40, r(t) = 0.1*u_{step}(t)*

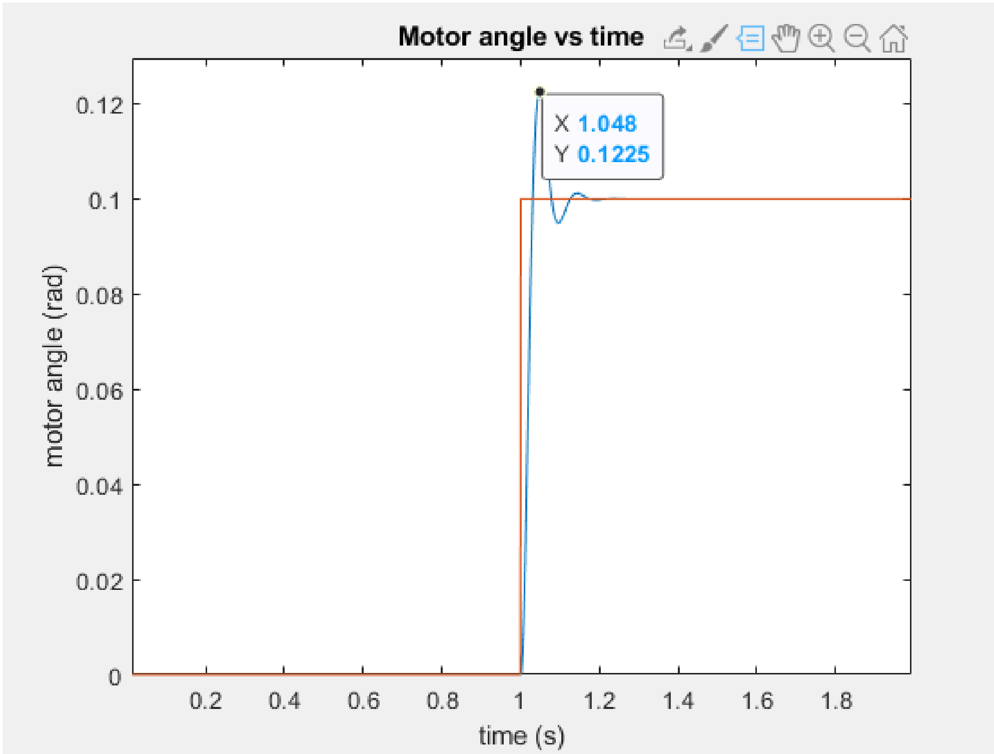

*Figure 15. Step response for time response fitting, C(s) = 50, r(t) = 0.1*u_{step}(t)*

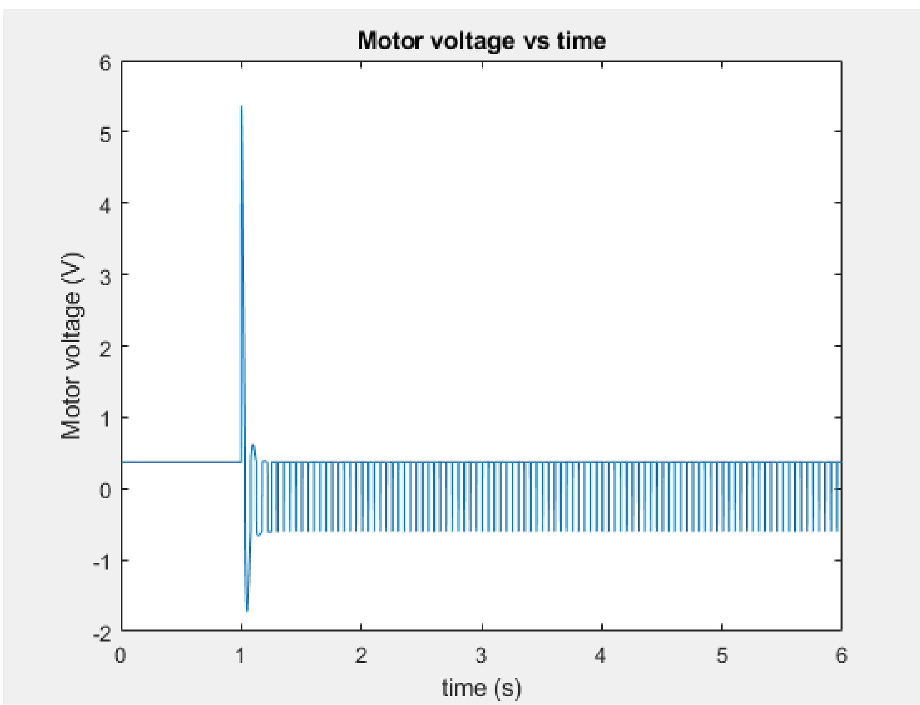

*Figure 16. Motor voltage vs time, C(s) = 50, r(t) = 0.1*u_{step}(t)*

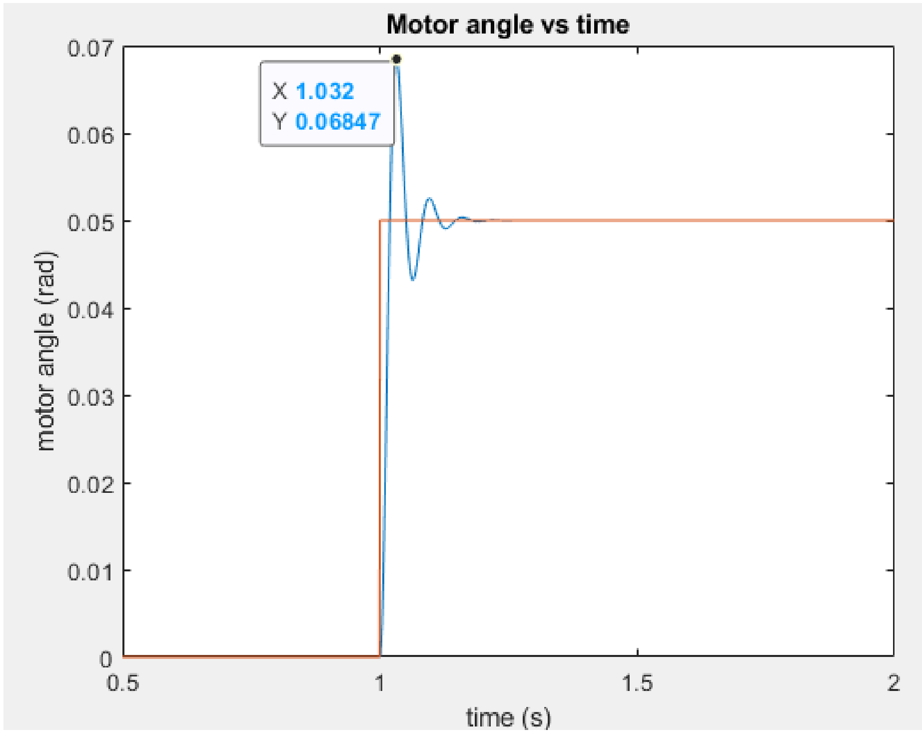

*Figure 17. Step response for time response fitting, C(s) = 100, r(t) = 0.05*u_{step}(t)*

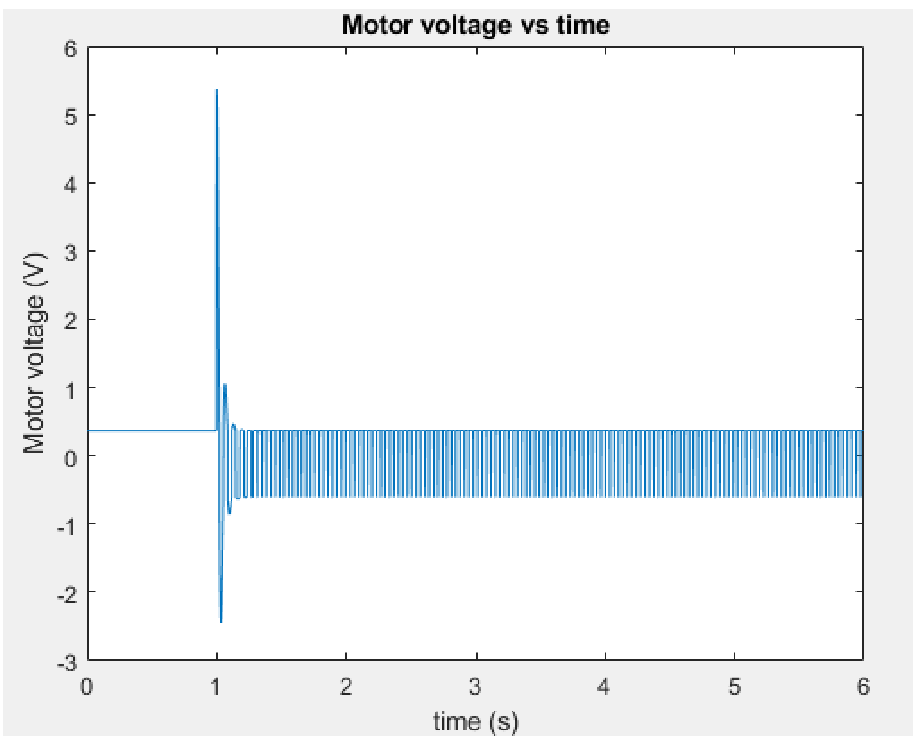

*Figure 18. Motor voltage vs time, C(s) = 100, r(t) = 0.05*ustep(t)*

The closed loop transfer function has the same form as a general 2nd order system.

$$G(s) = \frac{a\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Comparing this general form of a 2nd order system to equation 1, we can see that

Equation 2:

$$\frac{k * k_1}{\tau} = \omega_n^2$$

And

Equation 3:

$$\frac{1}{\tau} = 2\zeta\omega_n$$

To find ζ and ω_n, the following formulas are used.

Equation 4:

$$\zeta = \frac{-\ln(OS)}{\sqrt{\pi^2 + \ln^2(OS)}}$$

Equation 5:

$$\omega_n = \frac{\pi}{T_p\sqrt{1-\zeta^2}}$$

Where OS refers to the overshoot and T_p is the time to peak.

Since k is known, I now have all the information needed to calculate k_1 and τ. I used 6 different controller gains and different step references to ensure my calculation of k_1 and τ were correct. Using equation 2 through 5, k_1 and τ are easily calculated. The calculations were performed in Microsoft Excel and the values are shown below in figure 19. "Gain" is the controller gain, "p_inf" is the steady state value of the plant output which is the same as the reference voltage, "tp" is the time to peak measured in seconds, "p" is the peak value measured in radians, and "os" is the overshoot.

gain	p_inf	tp	p	os		zeta	omega		tau	k
20	0.2	0.091	0.211	0.055		0.678342	46.986283		0.015687	1.73166
30	0.15	0.067	0.1686	0.124		0.553429	56.296859		0.016048	1.69539
15	0.3	0.127	0.3058	0.019333		0.782332	39.714966		0.016093	1.69216
50	0.1	0.048	0.1225	0.225		0.428916	72.452806		0.01609	1.68921
40	0.1	0.055	0.118	0.18		0.479111	65.075006		0.016037	1.69781
100	0.05	0.032	0.0684	0.368		0.303224	103.025279		0.016005	1.69883
								avg:	0.015993	1.70084

Figure 19. Time fitting excel calculations

I then took the average of the values of k_1 and τ and got roughly 0.016 and 1.7 respectively. As can be seen in figure 19, the values for k_1 and τ are relatively consistent which is expected. To verify that my calculations are correct, I can recreate this closed-loop system in Simulink and see that the response matches with my unknown plant. This is without the stiction compensation because the transfer function of the plant must be LTI so the transfer function I found is without stiction. The closed-loop system used to verify my model is shown in figure 20.

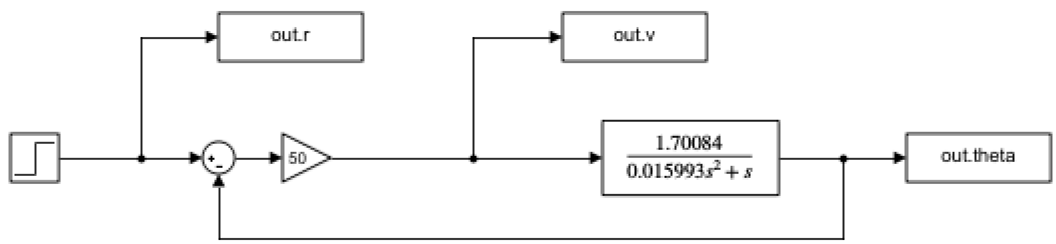

Figure 20. Closed-loop equivalent model for verification (time response fitting)

In figure 21, the step response is shown with a controller gain of 50 and a step size of 0.1 V, and it becomes clear that the time to peak and overshoot is the exact same as with the unknown plant step response from figure 15. The entire response looks incredibly similar, which shows that the transfer function I modelled works well for this data.

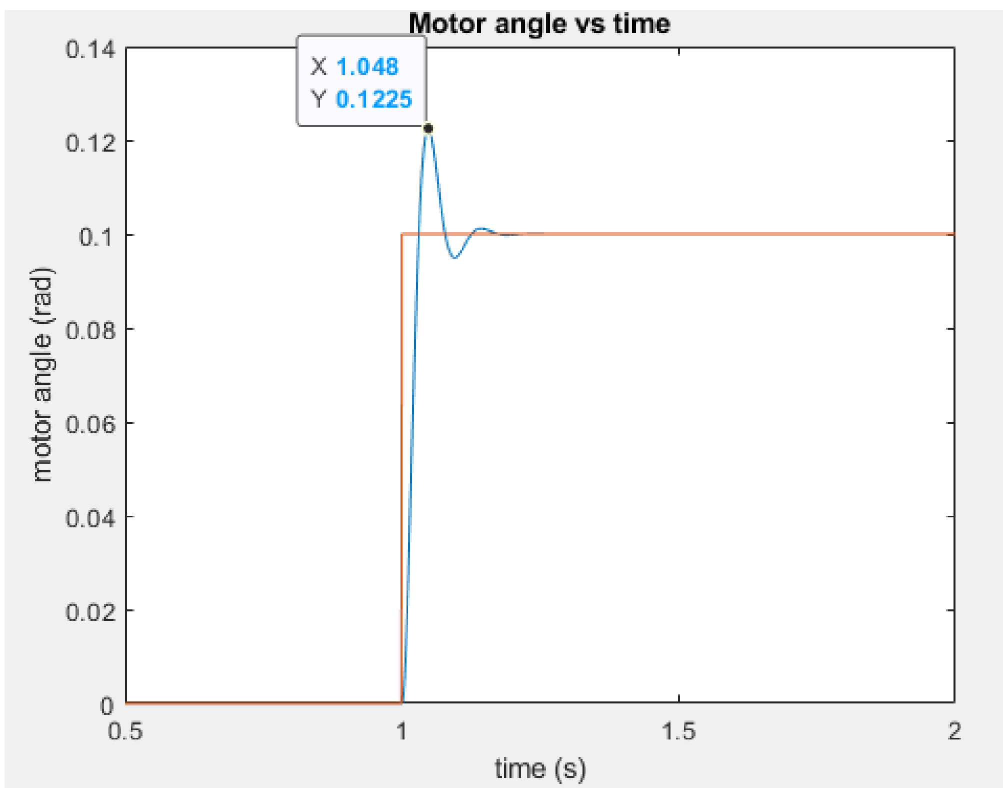

*Figure 21. Step response of modelled transfer function, C(s) = 50, r(t) = 0.1*u_{step}(t)*

Identification by Bode Plot Fitting

Another way to ensure that the model is correct is to use a different method for system identification. This has the benefit of using different data so it reduces the risk of overfitting and it increases the confidence in the model of the unknown plant because multiple methods were used to get the same result.

The second method of system identification used is Bode plot fitting. In order to make this work, I applied a chirp reference signal from 0.01 Hz to 100 Hz. However, the chirp signal amplitude is 1 V which will cause the motor voltage to saturate with the high gain of the controller. So, a gain block was placed after the chirp signal source to lower the amplitude to acceptable levels. To ensure I have enough data points, I ran the simulation at a sampling frequency of 1 kHz for 300 seconds.

To calculate k_1 and τ from the Bode plot, the following equations are useful.

Equation 6

$$f_p = \omega_n \sqrt{1 - 2\zeta^2}$$

Equation 7

$$peak = \frac{1}{2\zeta\sqrt{1 - \zeta^2}}$$

Where f_p is the frequency of where the peak of the magnitude plot occurs, measured in rad/s. Peak refers to the value of the peak in the magnitude, measured as linear gain instead of in dB (so $10^{(p_{dB}/20)}$ where p_{dB} is the peak value in dB). The peak is taken with reference to the DC gain.

Calculating ζ from equation 7 results in two values, however only the smaller of the two values will result in a real number for ω_n, so we will always use the smaller value for ζ. Once ζ and ω_n are found, k_1 and τ can be easily calculated using equations 2 and 3.

The Bode plots and their measurements are shown in figures 22 through 24.

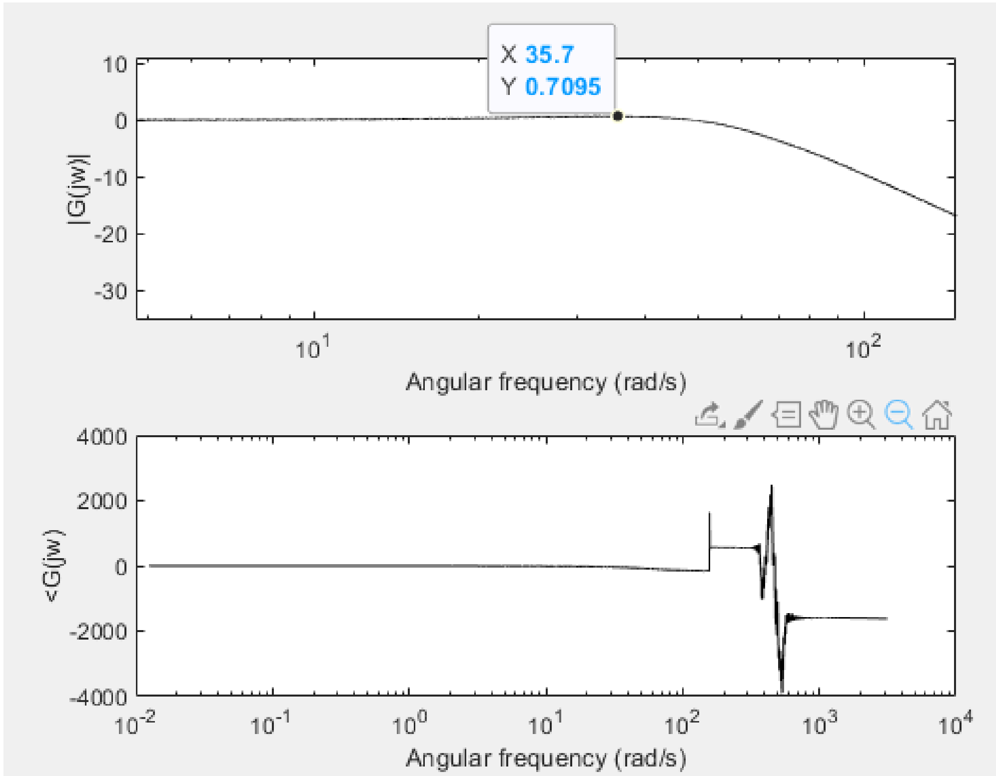

Figure 22. Bode plot, chirp amplitude = 0.15, C(s) = 30

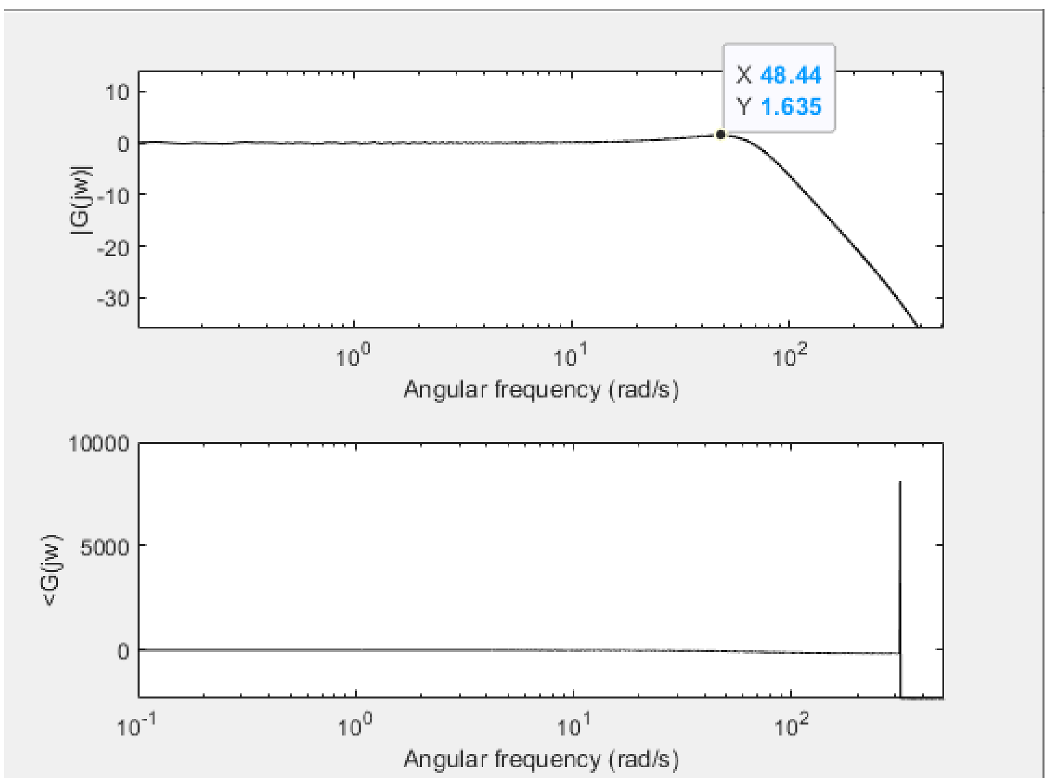

Figure 23. Bode plot, chirp amplitude = 0.1, C(s) = 40

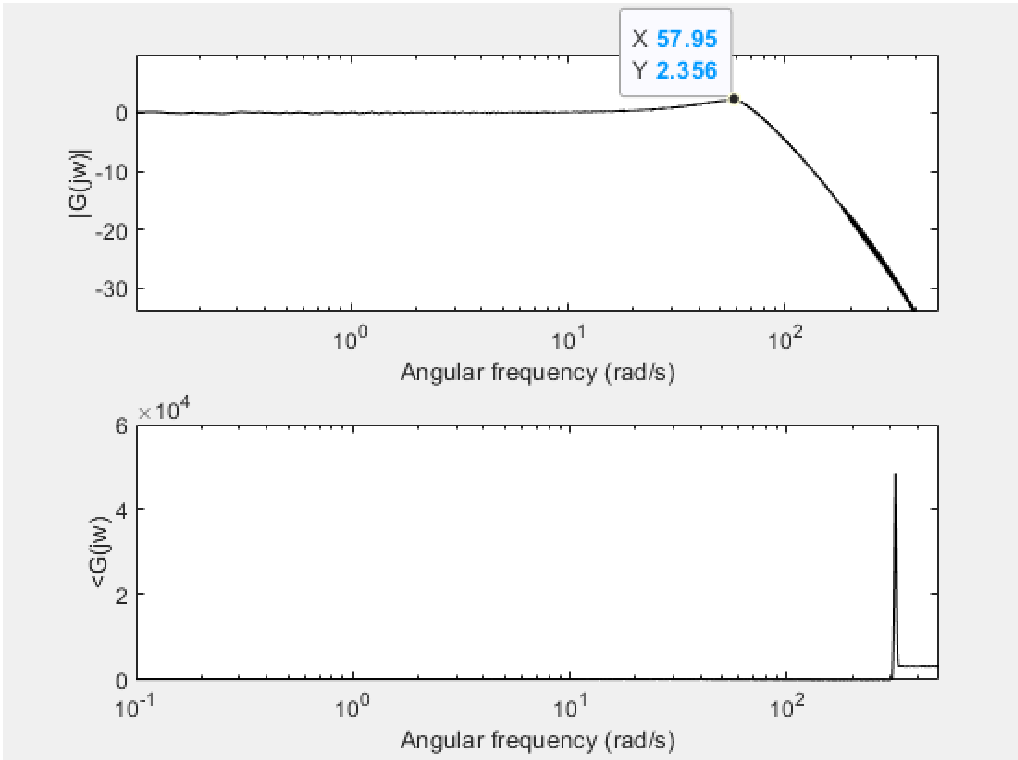

Figure 24. Bode plot, chirp amplitude = 0.1, C(s) = 50

The actual calculations to find k_1 and τ were done in Excel and the results of these calculations are shown in figure 25. The average values of k_1 and τ calculated as shown in the bottom right of the figure. Where "gain" is the controller gain, "DC gain (dB)" is the DC gain of the system in dB, "freq at pe" is the frequency where the peak of the Bode plot occurs measured in rad/s, "peak (dB)" is the peak value of the Bode plot in dB, "peak-DC" is the peak value minus the DC gain and converted from dB to a linear gain.

gain	DC gain (dB)	freq at pe	peak (dB)	peak-DC		zeta	omega		tau	k
50	0.08378	57.95	2.356	1.299006		0.425287	72.536061		0.016208	1.70558
40	0.1047	48.44	1.635	1.192655		0.47699	65.617759		0.015975	1.71958
30	-0.05215	35.7	0.7095	1.091648		0.54723	56.370702		0.016209	1.71685
								avg:	0.016131	1.71400

Figure 25. Excel calculations for Bode plot fitting

The measurements from the Bode plot result in roughly 1.71 and 0.0161 for k_1 and τ respectively. These values are very close to the time response fitting results meaning that these values are likely correct.

Again, this can be verified in Simulink by replacing the motor plant with a transfer function using the values found for k_1 and τ, and by removing the stiction compensator. Figure 26 shows the configuration of this test set-up.

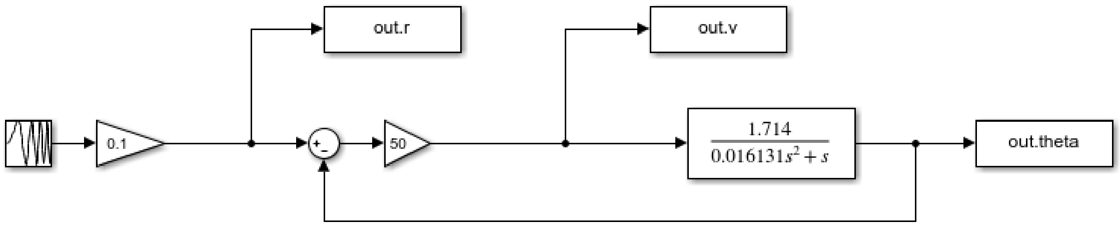

Figure 26. Closed-loop equivalent model for verification (Bode plot fitting)

An example of the resulting Bode plot is shown in figure 27.

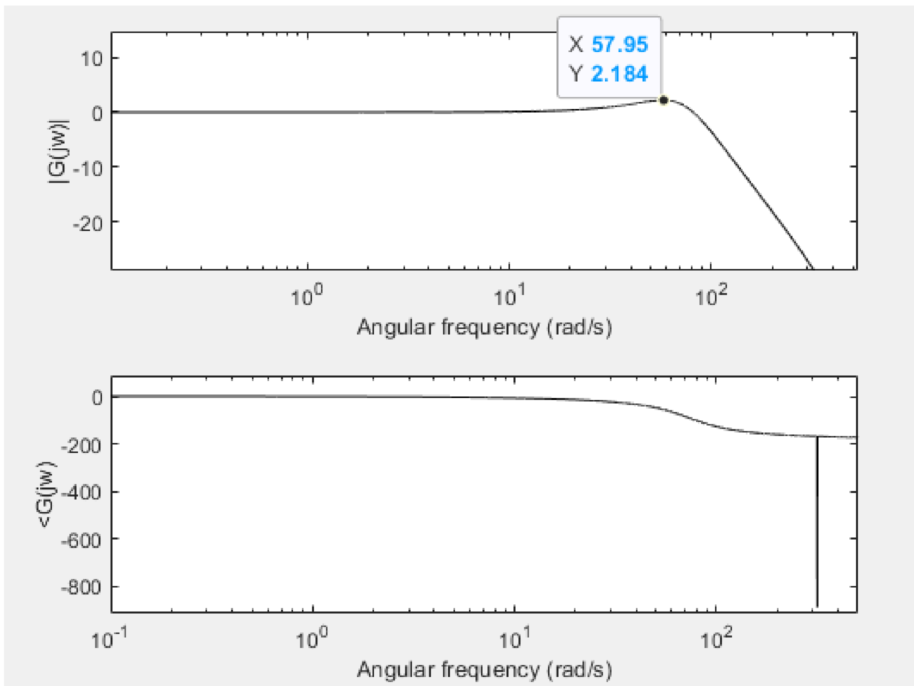

Figure 27. Bode plot of modelled transfer function, C(s) = 50, chirp amplitude = 0.1

The values in figure 27 are similar to the values in figure 24, the discrepancies are likely due to unaccounted for noise in the unknown plant and because of the influence of the other data taken with different controller gains since I used the averaged values of k_1 and τ in the transfer function used for verification. Another thing to note is that the phase plots seem to look different, figure 24 has a large spike around 300 rad/s while figure 27 has a negative spike at the same spot. This is because I am only supplying a maximum frequency of 100 Hz in the chirp signal. When I apply a chirp signal with a higher maximum frequency, these spikes go away and the phase goes from 0 to -180 degrees in the way we would expect for both the motor plant and transfer function I implemented for verification purposes. In fact, this is still the behaviour in figures 22 through 24 and figure 27 if the spikes are ignored as outliers. I left the phase plots the way the are because only the magnitude plot is needed for system identification purposes of a 2nd order system such as this one.

Limiting the Motor Angle

In order to limit the angle of the motor to the range of [-π/4, π/4], I simply use a saturator after the reference signal as shown in figure 28.

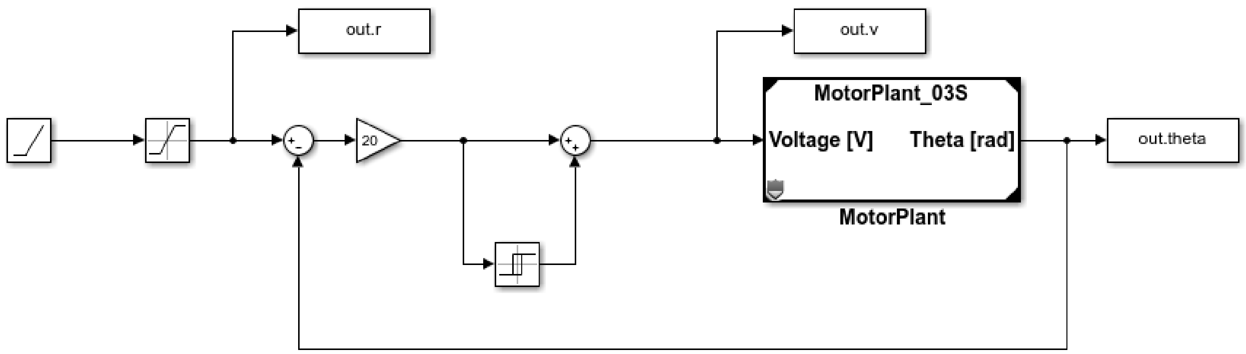

Figure 28. Closed-loop system with saturator to limit motor angle

The saturator is set such that -0.7 is the lower limit and 0.7 is the upper limit. This is below π/4 but this is a safe-guard and leaves a little wiggle room in case there is overshoot. Since in the next lab we will be designing a controller to have an overshoot of no more than 5%, the motor angle range should in theory never go outside of the range of [-0.735, 0.735] radians which is still within the acceptable range.

In order to show that this saturator is sufficient to ensure the motor voltage does not exceed the limits, figures 29 and 30. Show the motor angle (in blue) and the reference voltage (in orange). As is made clear in the figures, there is a little bit of overshoot but the motor angle is still well within the acceptable limits.

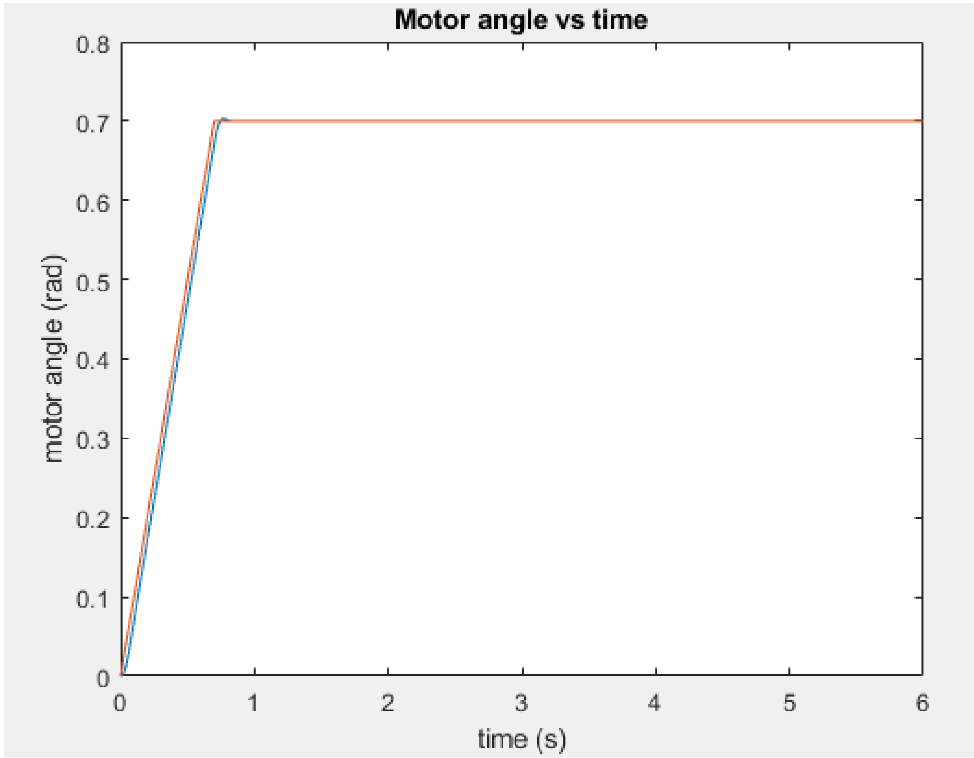

Figure 29. Motor angle with reference saturated at 0.7 V

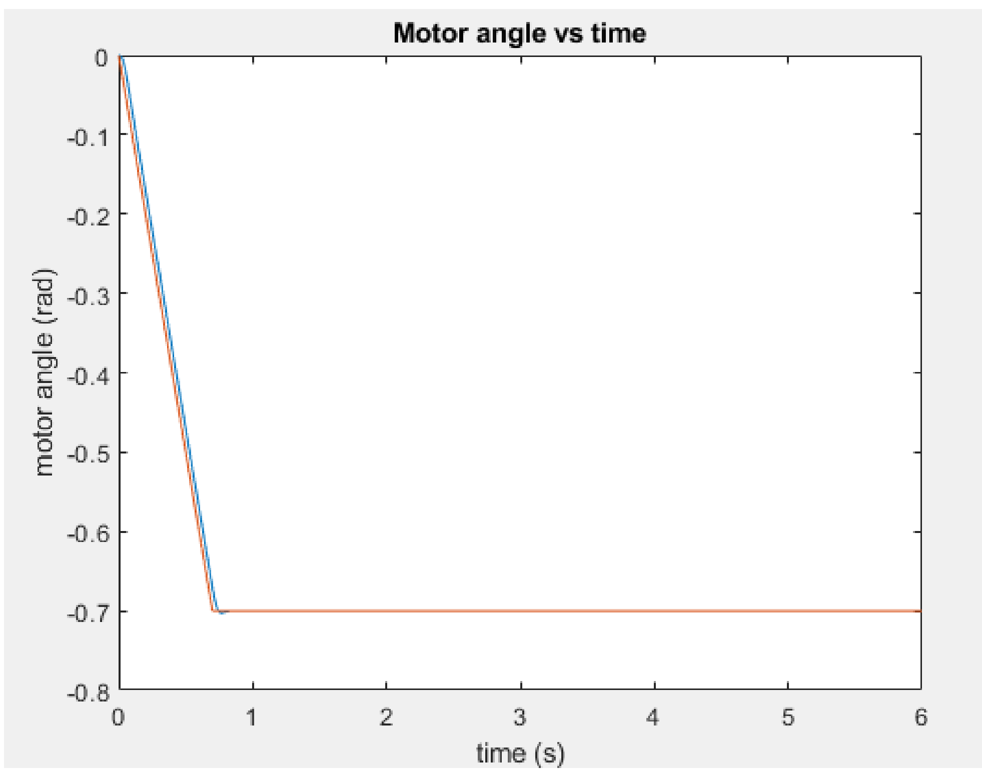

Figure 30. Motor angle with reference saturated at -0.7 V

Conclusions

In this lab, a number of controls techniques were used in order to remove non-linearities, characterize an unknown plant, and limit the operating conditions of the system. First, a deadzone non-linearity representing motor stiction was removed using the offset technique to cancel out the non-linear behaviour. This allowed me to treat the system as LTI and thus find a continuous time transfer function for the unknown plant.

The characterization of this plant was performed using time fitting and Bode plot fitting techniques and yielded and good approximation to the plant being observed. This will allow me to design a controller more effectively in future labs.

Finally, a saturator was added to the reference signal output in order to ensure that the motor angle does not try to exceed the physical limits of the motor and cause damage to the motor. This proved to be an effective method of limiting the motor angle. In addition, I should not need to worry about the non-linearity produced by this saturator in the future as long as I stay within the linear regime. I should only need to operate in this linear regime to accomplish the task of balancing the ball at the correct position, so this non-linearity should not affect my design by too much.

I acknowledge and promise that:

(a) I am the sole author of this lab report and associated simulation files/code.

(b) This work represents my original work.

(c) I have not shared detailed analysis or detailed design results, computer code, or Simulink diagrams with any other student.

(d) I have not obtained or looked at lab reports from any other current or former student of ECE 484/481, and I have not let any other student access any part of our lab work.

(e) I have completely and unambiguously acknowledged and referenced all persons and aids used to help us with our work.

Signed N Johnston