$\sim$

$$10^6$$

$\sim 10'$

$$N_{\text{pix}} \sim 10^7$$

N_{pix}

$N_{\mathrm{side}} =$

$$N_{\mathrm{pix}} = 12 \times N_{\mathrm{side}}^2$$

$$N_{\text{ring}} = 4 \times N_{\text{side}} - 1$$

N_{side}

$$4 \times N_{\text{side}} - 1$$

$$N_{\text{eq}} = 4 \times N_{\text{side}}$$

$$\cos\theta = a \pm b \cdot \phi$$

$$|\cos\theta| \le 2/3$$

$$\cos\theta = a + b/\phi^2$$

$$\cos\theta = a + b/(\pi/2 - \phi)^2$$

θ

ϕ

$$N_{\text{side}} = 2$$

$N_{\rm side} = 4$

×

$$N_{\text{side}} = 2^k$$

k

p

$$[0, 12N_{\rm side}^2 - 1]$$

u

$$u = p + 4N_{\text{side}}^2,$$

$$N_{\mathrm{side}} = 1$$

N_{side}

$$= 2^{\text{floor}(\log_2(u/4)/2)},$$

p

$$= u - 4N_{\mathrm{side}}^2,$$

$$4p,\ 4p+1,\ 4p+2,\ 4p+3$$

$$2N_{\mathrm{side}}$$

$$4u,\ 4u+1,\ 4u+2,\ 4u+3$$

f

$Y_{\ell m}$

$$f(\gamma) = \sum_{\ell=0}^{\ell_{\max}} \sum_{m} a_{\ell m} Y_{\ell m}(\gamma),$$

γ

$$\theta \in [0, \pi]$$

$\phi \in [0, 2\pi)$

$$\ell > \ell_{\max}$$

m

$-\ell_{\max}$

$$\ell_{\max}$$

ℓm

$m > \ell$

$$a_{\ell m} \equiv \int d\gamma Y^*_{\ell m}(\gamma) f(\gamma),$$

$$f(\gamma)$$

γ_p

$$p \in [0, N_{\text{pix}} - 1]$$

f_p

$a_{\ell m}$

$$\hat{a}_{\ell m} = \frac{4\pi}{N_{\mathrm{pix}}} \sum_{p=0}^{N_{\mathrm{pix}}-1} Y_{\ell m}^{*}(\gamma_p) f(\gamma_p),$$

$$\hat{a}_{\ell m}$$

$$\hat{C}_\ell$$

$$\hat{C}_\ell = \frac{1}{2\ell+1} \sum_m |\hat{a}_{\ell m}|^2.$$

$$f(\gamma_p) \longrightarrow f(\gamma_p)w(\gamma_p)$$

C_ℓ

$$f(\gamma_p)$$

ℓ

a_{00}

$$f(\gamma)/\sqrt{4\pi}$$

$$C_0$$

$$1/(4\pi)$$

$$f(\gamma)$$

4π

$$(2\ell + 1)C_\ell$$

$$2\ell + 1$$

$$\Delta T/T$$

$$\Delta T$$

μK

K

$$C_{X,\ell}$$

$$D_{X,\ell} = \frac{\ell(\ell+1)}{(2\pi)T_{CMB}^2} C_{X,\ell},$$

$$T_{CMB} = 2.726K$$

X

$$C_X(\ell)$$

2

$$\frac{\ell(\ell+1)}{2\pi} C_X(\ell)$$

μ

$$a_{21}^{TEMP} = -\ a_{2-1}^{TEMP} = 1$$

$$a_{21}^{GRAD} = -\, a_{2-1}^{GRAD} = 1$$

2×2

I_{ij}

Q

U

$$Q = (I_{11} - I_{22})/4$$

$$U = I_{12}/2$$

$$T = (I_{11} + I_{22})/4$$

V

n

$$(\mathbf{e}_1, \mathbf{e}_2)$$

ψ

$$\mathbf{e}_1{}' = \cos\psi \ \mathbf{e}_1 + \sin\psi \ \mathbf{e}_2$$

$$\mathbf{e}_2{}' = -\sin\psi \ \mathbf{e}_1 + \cos\psi \ \mathbf{e}_2$$

Q'

$$= \cos 2\psi \ Q + \sin 2\psi \ U$$

U'

$$= -\sin 2\psi \; Q + \cos 2\psi \; U$$

$$Q \pm iU$$

± 2

$${}_{\pm 2}Y_l^m$$

$T(\mathbf{n})$

$$= \sum_{lm} a_{T,lm} Y_{lm}(\mathbf{n})$$

$$(Q + iU)(\mathbf{n})$$

$$= \sum_{lm} a_{2,lm} \ {}_2Y_{lm}(\mathbf{n})$$

$$(Q - iU)(\mathbf{n})$$

$$= \sum_{lm} a_{-2,lm} \ {}_{-2}Y_{lm}(\mathbf{n}).$$

$$(\mathbf{e}_1, \mathbf{e}_2) = (\mathbf{e}_\theta, \mathbf{e}_\phi)$$

$\mathbf{e}_\theta$

$\mathbf{e}_{\phi}$

$\pm_2 a_{lm}$

$$a_{E,lm}$$

$$= -(a_{2,lm} + a_{-2,lm})/2$$

$a_{B,lm}$

$$= -(a_{2,lm} - a_{-2,lm})/2i,$$

T

E

B

$$\langle a^*_{X,lm} a_{X,lm'} \rangle$$

$$= \delta_{m,m'} C_{Xl},$$

$$\langle a_{T,lm}^{*} a_{E,lm} \rangle$$

$$= \delta_{m,m'} C_{Cl},$$

$$\left\langle \cdot\ \cdot\ \cdot \right\rangle$$

$\delta_{i,j}$

$Q(\mathbf{n})$

$$= -\sum_{lm} a_{E,lm} X_{1,lm} + i a_{B,lm} X_{2,lm}$$

$U(\mathbf{n})$

$$= -\sum_{lm} a_{B,lm} X_{1,lm} - i a_{E,lm} X_{2,lm}$$

$$X_{1,lm}(\mathbf{n}) = (\,_2Y_{lm} + \,_{-2}Y_{lm})/2$$

$$X_{2,lm}(\mathbf{n}) = ({}_2Y_{lm} - {}_{-2}Y_{lm})/2$$

$$Y_{lm}^* = (-1)^m Y_{l-m}$$

$$X^*_{1,lm} = (-1)^m X_{1,l-m}$$

$$X_{2,lm}^{*} = (-1)^{m+1} X_{2,l-m}$$

$$a_{T,lm} = (-1)^m a^*_{T,l-m}$$

$$a_{E,lm} = (-1)^m a^*_{E,l-m}$$

$$a_{B,lm} = (-1)^m a^*_{B,l-m}$$

$$X_{1,lm}(\mathbf{n})$$

$$X_{2,lm}(\mathbf{n})$$

$$X_{1,lm}(\mathbf{n}) = \sqrt{(2l+1)/4\pi}F_{1,lm}(\theta)\ e^{im\phi}$$

$$X_{2,lm}(\mathbf{n}) = \sqrt{(2l+1)/4\pi} F_{2,lm}(\theta)\ e^{im\phi}$$

$$F_{(1,2),lm}(\theta)$$

$F_{1,lm}(\theta)$

$$= N_{lm}\left[-\left(\frac{l-m^2}{\sin^2\theta}+\frac{1}{2}l(l-1)\right)P_l^m(\cos\theta)+(l+m)\frac{\cos\theta}{\sin^2\theta}P_{l-1}^m(\cos\theta)\right]$$

$F_{2,lm}(\theta)$

$$= N_{lm} \frac{m}{\sin^2 \theta} [-(l-1) \cos \theta P_l^m(\cos \theta) + (l+m) P_{l-1}^m(\cos \theta)],$$

$$N_{lm}(\theta) = 2\sqrt{\frac{(l-2)!(l-m)!}{(l+2)!(l+m)!}}.$$

$$F_{2,lm}(\theta) = 0$$

$$m = 0$$

β

$\mathbf{e}_1$

$\mathbf{e}_2$

$$\sum_m {}_{s_1}Y^*_{lm}(\mathbf{n}_1)\,{}_{s_2}Y_{lm}(\mathbf{n}_2) = \sqrt{\frac{2l+1}{4\pi}}\,{}_{s_2}Y_{l-s_1}(\beta,\psi_1)e^{-is_2\psi_2}$$

$$\langle T_1 T_2 \rangle$$

$$= \sum_l \frac{2l+1}{4\pi} C_{Tl} P_l(\cos\beta)$$

$$\langle Q_r(1) Q_r(2) \rangle$$

$$= \sum_l \frac{2l+1}{4\pi}[C_{El}F_{1,l2}(\beta) - C_{Bl}F_{2,l2}(\beta)]$$

$$\langle U_r(1) U_r(2) \rangle$$

$$= \sum_{l} \frac{2l+1}{4\pi}[C_{Bl}F_{1,l2}(\beta) - C_{El}F_{2,l2}(\beta)]$$

$$\langle T(1) Q_r(2) \rangle$$

$$= -\sum_{l} \frac{2l+1}{4\pi} C_{Cl} F_{1,l0}(\beta)$$

$$\langle T(1) U_r(2) \rangle$$

$$= 0.$$

r

$$(Q, U)$$

$$(Q_r, U_r)$$

$$\beta \to 0$$

$$P_\ell(\cos\beta) \to 1$$

$$P_\ell^2(\cos\beta) \to \sin^2\beta \, \frac{(\ell+2)!}{8(\ell-2)!}$$

$$\langle TT \rangle$$

$$= \sum_{\ell} \frac{2\ell + 1}{4\pi} C_{T\ell}$$

$$\langle QQ \rangle + \langle UU \rangle$$

$$= \sum_{l} \frac{2\ell + 1}{4\pi} \left(C_{E\ell} + C_{B\ell} \right)$$

$$\langle TQ \rangle = \langle TU \rangle$$

$$C_{C\ell}$$

$$\begin{pmatrix} Q' \\ U' \end{pmatrix} = \begin{pmatrix} \cos 2\psi & \sin 2\psi \\ -\sin 2\psi & \cos 2\psi \end{pmatrix} \begin{pmatrix} Q \\ U \end{pmatrix},$$

$$\begin{pmatrix} a'_{E,\ell m} \\ a'_{B,\ell m} \end{pmatrix} = \begin{pmatrix} \cos 2\psi & \sin 2\psi \\ -\sin 2\psi & \cos 2\psi \end{pmatrix} \begin{pmatrix} a_{E,\ell m} \\ a_{B,\ell m} \end{pmatrix}.$$

$$Q > 0, U = 0$$

$Q = 0, U > 0$

$\geq$

$\leq$

$$C_\ell^{\mathrm{TEMP}}$$

$$C_{\mathrm{T},\ell}$$

$$C_\ell^{\mathrm{T}}$$

$$C_\ell^{\mathrm{GRAD}}$$

$$C_{\mathrm{E},\ell}$$

$$2C_\ell^{\mathrm{G}}$$

$$2C_\ell^{\mathrm{GRAD}}$$

$$C_\ell^{\mathrm{CURL}}$$

$$C_{\mathrm{B},\ell}$$

$$2C_{\ell}^{\mathrm{C}}$$

$$2C_\ell^{\mathrm{CURL}}$$

$$C_\ell^{\mathrm{T-GRAD}}$$

$$C_{\mathrm{C},\ell}$$

$$-\sqrt{2}$$

$$C_{\ell}^{\mathrm{TG}}$$

$$\sqrt{2}C_{\ell}^{\mathrm{T-GRAD}}$$

$$M_{\ell m} = \begin{pmatrix} X_{1,\ell m} & iX_{2,\ell m} \\ -iX_{2,\ell m} & X_{1,\ell m} \end{pmatrix}$$

$$X_1$$

X_2

$$\begin{pmatrix} Q \\ U \end{pmatrix} = \sum_{\ell m} M_{\ell m} \begin{pmatrix} -a_{\ell m}^{\mathrm{GRAD}} \\ -a_{\ell m}^{\mathrm{CURL}} \end{pmatrix}.$$

M

$$\begin{pmatrix} Q \\ -U \end{pmatrix} = \sum_{\ell m} M_{\ell m} \begin{pmatrix} \sqrt{2} a_{\mathrm{E},\ell m} \\ \sqrt{2} a_{\mathrm{B},\ell m} \end{pmatrix},$$

$$\begin{pmatrix} Q \\ U \end{pmatrix} = \sum_{\ell m} M_{\ell m} \begin{pmatrix} -\sqrt{2} a_{\ell m}^{\mathrm{GRAD}} \\ \sqrt{2} a_{\ell m}^{\mathrm{CURL}} \end{pmatrix}.$$

x

y

$+Q$

$+x$

$-Q$

$+y$

$$+U$$

Z

I

$$Y_{\ell m}(\theta, \phi) = \lambda_{\ell m}(\cos\theta)e^{im\phi}$$

$\lambda_{\ell m}(x)$

$$= \sqrt{\frac{2\ell+1}{4\pi}\frac{(\ell-m)!}{(\ell+m)!}}P_{\ell m}(x), \quad \text{for } m \geq 0$$

$$\lambda_{\ell m}$$

$$= (-1)^m \lambda_{\ell|m|}, \quad \text{for } m < 0,$$

$$= 0, \quad \text{for } |m| > \ell.$$

$$x \equiv \cos\theta$$

$P_{\ell m}$

$$(1-x^2)\frac{d^2}{dx^2}P_{\ell m}-2x\frac{d}{dx}P_{\ell m}+\left(\ell(\ell+1)-\frac{m^2}{1-x^2}\right)P_{\ell m}=0.$$

$$P_\ell$$

$$P_{\ell m} = (-1)^m (1 - x^2)^{m/2} \frac{d^m}{dx^m} P_\ell(x),$$

$$P_\ell(x) = \frac{1}{2^\ell \ell!} \frac{d^\ell}{dx^\ell}(x^2 - 1)^\ell.$$

$$(-1)^m$$

$$f(p)$$

Ω_{pix}

$$f(p) = \int d\mathbf{u} w_p(\mathbf{u}) f(\mathbf{u})$$

w_p

$$1/\Omega_{\mathrm{pix}}$$

$$\int d\mathbf{u} w_p(\mathbf{u}) = 1$$

$$f(p) = \sum_{\ell=0}^{\ell_{\max}} \sum_{m} a_{\ell m} w_{\ell m}(p),$$

$$w_{\ell m}(p) = \int d\mathbf{u} w_p(\mathbf{u}) Y_{\ell m}(\mathbf{u}),$$

$$w_{\ell m}(p)$$

$$w_{\ell m}(p) = w_\ell(p) Y_{\ell m}(p)$$

$$w_\ell(p) = \left(\frac{4\pi}{2\ell + 1} \sum_{m=-\ell}^{\ell} |w_{\ell m}(p)|^2 \right)^{1/2},$$

$$C_{\ell}^{\mathrm{pix}}$$

$$C_\ell^{\mathrm{unpix}}$$

$$C_\ell^{\mathrm{pix}} = w_\ell^2 C_\ell^{\mathrm{unpix}}$$

w_ℓ

$$w_\ell = \left(\frac{1}{N_{\rm pix}} \sum_{p=0}^{N_{\rm pix}-1} w_\ell^2(p) \right)^{1/2} .$$

$$\ell \leq 4N_{\mathrm{side}}$$

$$N_{\mathrm{side}} \leq 128$$

$N_{\text{side}} > 128$

$$N_{\text{side}} = 128$$

$$\Delta w/w < 7\ 10^{-4}$$

$$\ell = 2N_{\mathrm{side}}$$

$$\Delta w/w < 1.7\ 10^{-3}$$

$$\ell = 4N_{\text{side}}$$

$x \in]0, 1[$

$$2^{128} - 1 \approx 3.410^{38}$$

$$10^{-15}$$