Nathaniel Hobbs - Convex Optimization - HW 3

1 Problem 1

The guitar image has been corrupted (presumably) by an affine function of the form $a \cdot i + b \cdot j + c$, where i and j correspond to pixels in the image. For ease of notation, I will use im_c for the corrupted image and im for the ideal image

That is, $im_c(i, j) = im(i, j) \cdot [a \cdot i + b \cdot j + c]$

But we are given information that in the upperhand corner of the image, it should be pure white (i.e. have value 255). For that region, then, we know

$$im_c(i, j) = 255 \cdot [a \cdot i + b \cdot j + c]$$

$$\implies a \cdot i + b \cdot j + c = im_c/255$$

The objective function, therefore is to minimize

$$f_0 = ||a \cdot i + b \cdot j + c - im_c/255||$$

applied in the upper corner. That is, for each pixel in the upper left corner, this must be true. I implement this by putting everything in a matrix to make for less code.

$$f_0 = ||R - IM\_C_{corner}/255||$$

, where R is a matrix of $a \cdot i + b \cdot j + c$ and IM_C_{corner} is a matrix of the corner of the image we know about.

Only a, b, c are the optimization variables, and we have constraints that

$$0 \leq a \cdot i + b \cdot j + c \leq 1$$

.

2 Problem 2

We are given an image with a river, and are to do spline fitting with cubic polynomials to be able to have a line trace the path of the river.

We take a spline to be composed of M (I choose M=24) piece-wise cubic polynomials P_i which is the parametric representation of a cubic polynomial, i.e.:

$$x_i(t) = a_3t^3 + a_2t^2 + a_1t + a_0$$
$$y_i(t) = b_3t^3 + b_2t^2 + b_1t + b_0$$
$$P_i(t) = (x_i(t), y_i(t))$$

Here, the a_is and b_is are the optimization variables.

We have 3 constraints applied to each spline, the C0, C1, and C2 constraints, i.e.:

$$P_i(x_i(t), y_i(t)) = P_{i+1}(x_i(t), y_i(t))$$
$$P_i'(x_i(t), y_i(t)) = P_{i+1}'(x_i(t), y_i(t))$$

$$P_i''(x_i(t), y_i(t)) = P_{i+1}''(x_i(t), y_i(t))$$

The objective function is to minimize

$$f_0 = ||At - x||_2^2 + ||Bt - y||_2^2$$

where A are the $x(t)$ coefficients, B are the $y(t)$ coefficients, and x and y correspond to the (x, y) points taken from using ginput on an image of the river.