Third Semester B.E. Degree Examination, Dec.2017/Jan.2018
Engineering Mathematics – III

Time: 3 hrs. Max. Marks: 80

Note: *Answer FIVE full questions, choosing one full question from each module.*

Module-1

1 a. Express $f(x) = (\pi - x)^2$ as a Fourier series of period 2π in the interval $0 < x < 2\pi$. Hence deduce the sum of the series $1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots$ **(08 Marks)**

b. The turning moment T units of the Crank shaft of a steam engine is a series of values of the crank angle θ in degrees. Find the first four terms in a series of sines to represent T. Also calculate T when $\theta = 75^\circ$. **(08 Marks)**

θ:	0°	30°	60°	90°	120°	150°	180°
T:	0	5224	8097	7850	5499	2626	0

OR

2 a. Find the Fourier Series expansion of the periodic function, $f(x) = \begin{cases} l + x, & -l \le x \le 0 \\ l - x, & 0 \le x \le l \end{cases}$. **(06 Marks)**

b. Obtain a half-range cosine series for $f(x) = x^2$ in $(0, \pi)$. **(05 Marks)**

c. The following table gives the variations of periodic current over a period:

t sec:	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$
A amp:	1.98	1.30	1.05	1.30	-0.88	-0.25

Show that there is a direct current part 0.75 amp in the variable current and obtain the amplitude of the first harmonic. **(05 Marks)**

Module-2

3 a. Find the Fourier transform of $f(x) = \begin{cases} 1 & \text{for } |x| < 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ and evaluate $\int_0^\infty \left(\frac{\sin x}{x}\right) dx$ **(06 Marks)**

b. Find the Fourier cosine transform of, $f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2 - x & \text{for } 1 < x < 2 \\ 0 & \text{for } x > 2 \end{cases}$. **(05 Marks)**

c. Obtain the inverse Z-transform of the following function, $\frac{z}{(z-2)(z-3)}$. **(05 Marks)**

OR

4 a. Find the Z-transform of $\cos\left(\frac{n\pi}{2} + \alpha\right)$. **(06 Marks)**

b. Solve $u_{n+2} - 5u_{n+1} + 6u_n = 36$ with $u_0 = u_1 = 0$, using Z-transforms. **(05 Marks)**

c. If Fourier sine transform of $f(x)$ is $\frac{e^{-a\alpha}}{\alpha}$, $\alpha \ne 0$. Find $f(x)$ and hence obtain the inverse Fourier sine transform of $\frac{1}{\alpha}$. **(05 Marks)**

Module-3

5 a. Calculate the Karl Pearson's co-efficient for the following ages of husbands and wives: **(06 Marks)**

Husband's age x:	23	27	28	28	29	30	31	33	35	36
Wife's age y:	18	20	22	27	21	29	27	29	28	29

b. By the method of least square, find the parabola $y = ax^2 + bx + c$ that best fits the following data: **(05 Marks)**

x:	10	12	15	23	20
y:	14	17	23	25	21

c. Using Newton-Raphson method, find the real root that lies near $x = 4.5$ of the equation $\tan x = x$ correct to four decimal places. (Here x is in radians). **(05 Marks)**

OR

6 a. In a partially destroyed laboratory record, only the lines of regression of y on x and x on y are available as $4x - 5y + 33 = 0$ and $20x - 9y = 107$ respectively. Calculate $\bar{x}$, $\bar{y}$ and the coefficient of correlation between x and y. **(06 Marks)**

b. Find the curve of best fit of the type $y = ae^{bx}$ to the following data by the method of least squares: **(05 Marks)**

x:	1	5	7	9	12
y:	10	15	12	15	21

c. Find the real root of the equation $xe^x - 3 = 0$ by Regula Falsi method, correct to three decimal places. **(05 Marks)**

Module-4

7 a. From the following table of half-yearly premium for policies maturing at different ages, estimate the premium for policies maturing at age of 46: **(06 Marks)**

Age:	45	50	55	60	65
Premium (in Rupees):	114.84	96.16	83.32	74.48	68.48

b. Using Newton's divided difference interpolation, find the polynomial of the given data: **(05 Marks)**

x	3	7	9	10
f(x)	168	120	72	63

c. Using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule to find $\int_0^{0.6} e^{-x^2}dx$ by taking seven ordinates. **(05 Marks)**

OR

8 a. Find the number of men getting wages below ₹ 35 from the following data: **(06 Marks)**

Wages in ₹ :	0 – 10	10 – 20	20 – 30	30 – 40
Frequency :	9	30	35	42

b. Find the polynomial f(x) by using Lagrange's formula from the following data: **(05 Marks)**

x:	0	1	2	5
f(x):	2	3	12	147

c. Compute the value of $\int_{0.2}^{1.4} (\sin x - \log_e x + e^x)dx$ using Simpson's $\left(\frac{3}{8}\right)^{th}$ rule. **(05 Marks)**

Module-5

9 a. A vector field is given by $\vec{F} = \sin y\,\hat{i} + x(1+\cos y)\hat{j}$. Evaluate the line integral over a circular path given by $x^2 + y^2 = a^2$, $z = 0$. **(06 Marks)**

b. If C is a simple closed curve in the xy-plane not enclosing the origin. Show that $\int_C \vec{F}.d\vec{R} = 0$, where $\vec{F} = \dfrac{y\hat{i} - x\hat{j}}{x^2 + y^2}$. **(05 Marks)**

c. Derive Euler's equation in the standard form viz., $\dfrac{\partial f}{\partial y} - \dfrac{d}{dx}\left[\dfrac{\partial f}{\partial y'}\right] = 0$. **(05 Marks)**

OR

10 a. Use Stoke's theorem to evaluate $\int_C \vec{F}.d\vec{R}$ where $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ over the upper half surface of $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy-plane. **(06 Marks)**

b. Show that the geodesics on a plane are straight lines. **(05 Marks)**

c. Find the curves on which the functional $\int_0^1 \left((y')^2 + 12xy\right)dx$ with $y(0) = 0$ and $y(1) = 1$ can be extremized. **(05 Marks)**

* * * * *