CS333 Homework 2

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1.

a. $X_0 = x_0 + x_1 + x_2 = 1 + 0 + 1 = 2$

$X_1 = x_0 cos[\pi/6] + x_1 cos[3\pi/6] + x_2 cos[5\pi/6] = \sqrt{3}/2 + 0 - \sqrt{3}/2 = 0$

$X_2 = x_0 cos[2\pi/6] + x_1 cos[6\pi/6] + x_2 cos[10\pi/6] = 1/2 + 0 + 1/2 = 1$

b. $f(n) = \frac{2}{N}(\frac{1}{2}X_0 + \sum_{k=1}^{N-1} X_k cos[\frac{\pi}{N}k(n + \frac{1}{2})])$

$f(n) = \frac{2}{3}(1 + cos[\frac{2\pi}{3}(n + \frac{1}{2})])$

$f(0) = \frac{2}{3}(1 + cos[\frac{2\pi}{3}(0 + \frac{1}{2})]) = 1$

$f(1) = \frac{2}{3}(1 + cos[\frac{2\pi}{3}(1 + \frac{1}{2})]) = 0$

$f(2) = \frac{2}{3}(1 + cos[\frac{2\pi}{3}(2 + \frac{1}{2})]) = 1$

c. Because of the trigonometric identity that $cos(\theta) = -cos(\pi - \theta)$ along with the fact that we can assume that N is even, every other value would cancel each other out.

d. Considering that every coefficient will be 0 besides X_0, and assuming each number in the sparse matrix representation of arrays is equal to one byte, it will take 3 bytes to store the non-zero number and its position.

2.

a. The NM aspect of the runtime comes from having to go through every 8x8 block in the matrix and then each individual cell would need N+M time to compute the coefficient which comes from the equation above, because each cell would need to traverse through the entire row as well as the column, which would be N+M

b. The runtime of the compression algorithm for W x H image would be O(WH(W+H)) for the same reasons as part a, where we would only have to divide by the quantization table and the quality factor, which takes negligible time.

c. As N and M get larger, the new algorithm is much faster than the original one.

3. See code file

4.

a. For a q factor of 1, we get 8837/28800
In terms of compression, it takes 3 bytes for each non-zero number when using the sparse matrix representation using three lists. Therefore, total number of bytes needed to store sparse compressed coefficients is equal to $8837 * 3 = 26511$.

Total number of bytes to store uncompressed image = 28800
Compression rate $= 28800/26511 = 1.08$

For a q-factor of 2, we get 6611/28800
Total number of bytes to store sparse compressed coefficients $= 6611 * 3 = 19833.$
Compression rate $= 28800/19833 = 1.45$

For a q-factor of 4, we get 4674/28800
Total number of bytes to store sparse compressed coefficients $= 4674 * 3 = 14022.$
Compression rate $= 28800/14022 = 2.05$

b. For quality factor of 1, we get 11552 pairs
$11552 * 1.5 = 17328$
Thus we have a compression rate of $28800/17328 = 1.66$

For quality factor of 2, we get 8960 pairs
$8960 * 1.5 = 13440$
Thus we have a compression rate of $28800/13440 = 2.14$

For quality factor of 4, we get 6525 pairs
$6525 * 1.5 = 9787.5$
Thus we have a compression rate of $9787.5/28800 = 2.94$