HW2

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Problem 1

(a)

$\hat{\beta}x_i = \hat{\alpha}z_i$

$\hat{\beta} = c\hat{\alpha}$

$\hat{\alpha}_j = \frac{\hat{\beta}_j}{c}$ for j = 1 to p

(b)

No it is not necessary. The least squares estimates automatically adjust the scale to compensate for the units. The fit of the model will be the same.

(c)

Linearity: The relationship between the independent variables and the dependent variable follows a straight-line pattern.

Independence: The data points are collected independently; one person's data doesn't influence another's.

Homoscedasticity (Constant Variance): The "spread" or "scatter" of the errors is the same across all levels of your predictors.

Normality: For any given value of X, the errors (residuals) should follow a bell-shaped curve.

(d)

I would say this is incorrect because the student forgot the ϵ.

(e)

False. You can make the training model whatever size you want. The training model is usually smaller but this is not required.

(f) TODO

Unbiased means that if you did the experiment a large amount of times and calculate $\hat{\beta}$ for each of those times, the average of all the estimates would exactly equal β.

Problem 2

```r
#install.packages("ISLR2") #you only need to do this one time.
library(ISLR2) #you will need to do this every time you open a new R session.
```

(a)

```r
head(Boston)
```

```
##      crim zn indus chas   nox    rm  age    dis rad tax ptratio lstat medv
## 1 0.00632 18  2.31    0 0.538 6.575 65.2 4.0900   1 296    15.3  4.98 24.0
## 2 0.02731  0  7.07    0 0.469 6.421 78.9 4.9671   2 242    17.8  9.14 21.6
## 3 0.02729  0  7.07    0 0.469 7.185 61.1 4.9671   2 242    17.8  4.03 34.7
## 4 0.03237  0  2.18    0 0.458 6.998 45.8 6.0622   3 222    18.7  2.94 33.4
## 5 0.06905  0  2.18    0 0.458 7.147 54.2 6.0622   3 222    18.7  5.33 36.2
## 6 0.02985  0  2.18    0 0.458 6.430 58.7 6.0622   3 222    18.7  5.21 28.7
```

```r
set.seed(1)

n <- nrow(Boston)

train_index <- sample(1:n, floor(n / 2), replace = FALSE)


train_boston <- Boston[train_index, ]
test_boston <- Boston[-train_index, ]

m1 <- lm(crim ~ ., data = train_boston)

summary(m1)
```

```
##
## Call:
## lm(formula = crim ~ ., data = train_boston)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -10.574  -2.723  -0.566   1.351  57.279
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 18.665277  10.693890   1.745  0.08219 .
## zn           0.046105   0.028198   1.635  0.10336
## indus       -0.127032   0.116665  -1.089  0.27730
## chas        -0.916885   1.769171  -0.518  0.60476
## nox        -11.606805   7.924234  -1.465  0.14431
## rm           0.738859   0.913675   0.809  0.41951
## age         -0.010585   0.026291  -0.403  0.68761
## dis         -1.184115   0.427288  -2.771  0.00602 **
## rad          0.671788   0.130702   5.140  5.7e-07 ***
## tax         -0.004607   0.007552  -0.610  0.54237
## ptratio     -0.515160   0.284824  -1.809  0.07175 .
## lstat        0.296310   0.114591   2.586  0.01031 *
## medv        -0.249594   0.097588  -2.558  0.01115 *
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.693 on 240 degrees of freedom
## Multiple R-squared:  0.5021, Adjusted R-squared:  0.4772
## F-statistic: 20.17 on 12 and 240 DF,  p-value: < 2.2e-16
```

```r
trainMSE <- sum(m1$residuals^2)/nrow(train_boston)

trainMSE
```

```
## [1] 42.49345
```

```r
test_predict <- predict(m1, newdata = test_boston)

test_MSE <- mean((test_predict - test_boston$crim)^2)

test_MSE
```

```
## [1] 41.19923
```

(b)

```r
set.seed(1)

n <- nrow(Boston)

train_index <- sample(1:n, floor(n / 2), replace = FALSE)


train_boston <- Boston[train_index, ]
test_boston <- Boston[-train_index, ]

m1 <- lm(crim ~ zn + indus + nox + dis + rad + ptratio + medv, data = train_boston)

summary(m1)
```

```
##
## Call:
## lm(formula = crim ~ zn + indus + nox + dis + rad + ptratio +
##     medv, data = train_boston)
##
## Residuals:
##    Min      1Q Median      3Q     Max
## -9.145 -2.542 -0.645  1.175 57.028
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  29.29068    8.84292   3.312 0.001065 **
## zn            0.04940    0.02779   1.777 0.076733 .
## indus        -0.13410    0.10956  -1.224 0.222122
## nox         -12.25307    7.66941  -1.598 0.111408
## dis          -1.36389    0.39946  -3.414 0.000748 ***
## rad           0.63065    0.07015   8.990  < 2e-16 ***
## ptratio      -0.57530    0.28209  -2.039 0.042483 *
## medv         -0.35150    0.06458  -5.443 1.27e-07 ***
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6.739 on 245 degrees of freedom
## Multiple R-squared:  0.4847, Adjusted R-squared:   0.47
## F-statistic: 32.93 on 7 and 245 DF,  p-value: < 2.2e-16
```

```r
trainMSE <- sum(m1$residuals^2)/nrow(train_boston)

trainMSE
```

```
## [1] 43.97466
```

```r
test_predict <- predict(m1, newdata = test_boston)

test_MSE <- mean((test_predict - test_boston$crim)^2)

test_MSE
```

```
## [1] 39.62763
```

The training MSE collected from part A was slightly smaller than the one from part B. The test MSE collected from part A was slightly larger than the one from part B.

(c)

I expected part b to have a larger MSE because it is being trained on less variables so it's not likely to be as good as part a that was trained on every variable.

(d)

I expected the test MSE for part b to be larger than part a because removing predictors usually increases bias. However, if some predictors in part a were just noise, part b could potentially have a smaller test MSE. In my case, part b was actually slightly better, suggesting some variables in part (a) weren't helpful for generalization.

Problem 3

(a)

$\beta_0 = 2$ $\beta_1 = 3$ $\beta_2 = 5$

(b)

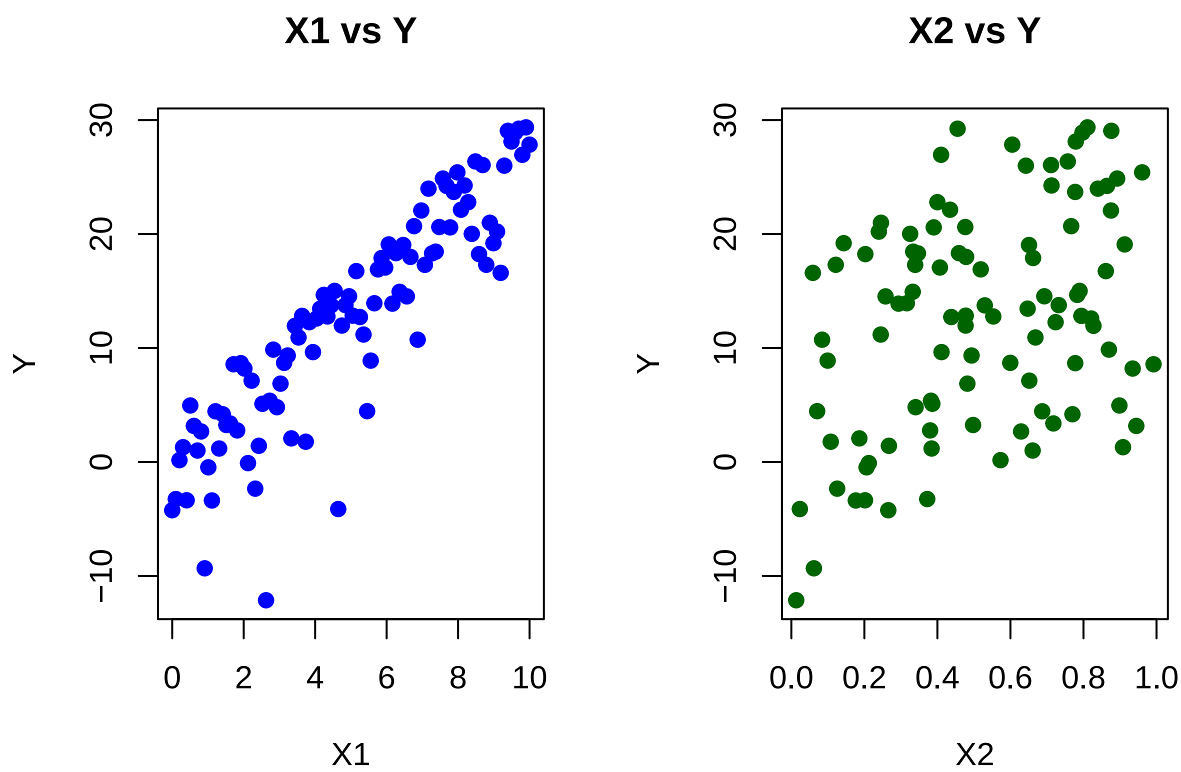
```r
set.seed(1) # For reproducibility
X1 = seq(0, 10, length.out = 100)
X2 = runif(100)
epsilon = rnorm(100, mean = 0, sd = 1)

Y = 2 + 3*X1 + 5*log(X2) + epsilon
Y
```

```
##   [1]  -4.23243317  -3.25163817   0.16155277   1.29831740  -3.36017219
##   [6]   4.95979463   3.16639013   1.00554051   2.67674841  -9.32815238
##  [11]  -0.46809191  -3.37647122   4.44916434   1.18318278   4.19129737
##  [16]   3.24545027   3.38444034   8.57643582   2.77034153   8.67147697
##  [21]   8.19849529  -0.09879487   7.13633701  -2.33945369   1.42069144
```

```
## [26]   5.10909193 -12.13061519   5.37632558   9.86110241   4.80943921
## [31]   6.87401554   8.70101323   9.34431309   2.07223625  11.94948013
## [36]  10.92516778  12.82034189   1.77720644  12.26835426   9.64280773
## [41]  12.59220416  13.45553046  14.66413305  12.76885860  13.74312926
## [46]  15.01216225  -4.12701656  11.97037589  13.76311067  14.53952062
## [51]  12.83644531  16.74957378  12.72008067  11.18201057   4.46102116
## [56]   8.89426498  13.93063644  16.90011981  17.89753266  17.06416690
## [61]  19.09030520  13.89557628  18.32735160  14.93305155  19.03933549
## [66]  14.53050849  17.99498458  20.69307892  10.73023226  22.06593930
## [71]  17.29846356  23.98309105  18.30688716  18.45521373  20.61605275
## [76]  24.86960493  24.22779022  20.58752240  23.69519179  25.41423139
## [81]  22.13662294  24.26178623  22.79845699  20.01888537  26.36971877
## [86]  18.24079376  26.05506809  17.30404313  20.99204629  19.19887827
## [91]  20.21505958  16.59568737  26.00022507  29.05787821  28.11965954
## [96]  28.90449384  29.24380140  26.95436970  29.36243300  27.84620901
```

(c)

```r
par(mfrow=c(1,2))
plot(X1, Y, main="X1 vs Y", col="blue", pch=19)
plot(X2, Y, main="X2 vs Y", col="darkgreen", pch=19)
```



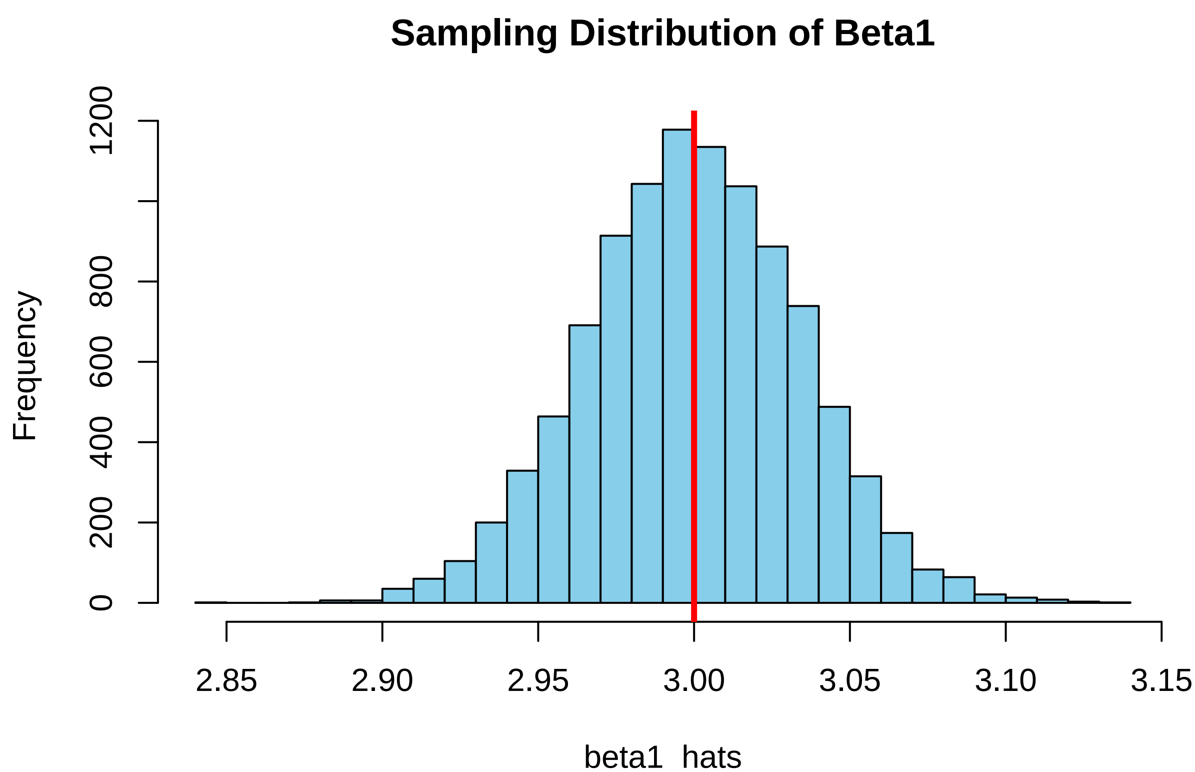
(d)

```r
n_sim <- 10000
beta1_hats <- rep(NA, n_sim)

for(i in 1:n_sim) {
  # We generate new noise each time to simulate a new sample
  eps_sim <- rnorm(100, 0, 1)
  Y_sim <- 2 + 3*X1 + 5*log(X2) + eps_sim

  # Fit the model (using log(X2) since that is the true relationship)
  fit <- lm(Y_sim ~ X1 + log(X2))
  beta1_hats[i] <- coef(fit)[2]
}

mean(beta1_hats)
```

```
## [1] 2.999792
```

(e)

```r
hist(beta1_hats, breaks=30, col="skyblue", main="Sampling Distribution of Beta1")
abline(v = 3, col = "red", lwd = 3) # The true Beta1
```



(f)

```r
beta2_hats <- rep(NA, n_sim)

for(i in 1:n_sim) {
```

```
  eps_sim <- rnorm(100, 0, 1)
  Y_sim <- 2 + 3*X1 + 5*log(X2) + eps_sim
  fit <- lm(Y_sim ~ X1 + log(X2))
  beta2_hats[i] <- coef(fit)[3]
}

mean(beta2_hats)
```

```
## [1] 4.999968
```

(g)

```
hist(beta2_hats, breaks=30, col="lightgreen", main="Sampling Distribution of Beta2")
abline(v = 5, col = "red", lwd = 3) # The true Beta2
```