The Emergence of Group Identity Topologies in the Generalized Bach or Stravinsky Game

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Abstract

Over the past 300,000 years humans have been evolving increasing complex cultures. Today, our social identities cut across multiple dimensions such as religion, education, ethnicity, gender, and many more. Complex social identities are associated with (i) group preferences, (ii) social signals that indicate group membership, and (iii) the propensity to condition our actions on the social signals of both ingroup and outgroup members. Various empirical studies have provided some insights on how these different aspects of social identity are related to each other, but they fall short of capturing all of this in the context of complex identities. This paper provides a model that exhibits these three aspects of social identity in the context of complex identities. Prior cultural evolution models have modeled groups with different preferences evolving social signals for coordination, but have only done so with very simple social identities. This paper begins with a base model and then iteratively building up complexity by adding in social signals, attention to signals, assortment by signals, and then multiple dimensions in which signaling can occur. Simulation results provide a quantitative explanation of why multiple dimensions of social signals are ubiquitous in society. Our model also shows how group structures are underdetermined by social signals, lables and ethnic markers; and it explains how observing how people coordinate with each other can resolve this ambiguity in group structure. Finally, it highlights some general themes of how different social signaling systems and group structures evolve.

1 Introduction

As early as the Third Dynasty of Ur (2100 BCE) in ancient Mesopotamia, we have literary evidence of complex social identities; cities as far apart as Ur and Mari (well over 100 miles apart) evidence a canonical body of literature indicative of a shared Sumerian identity across several cities in the region, but also exhibit literature concerned with a city's patron gods indicative of a localized social identity (Delnero, 2016). We also have reason to believe that long before the invention of writing, humans used markers of group identity to facilitate coordination within groups that were too large for a group member to know every individual in the group (Moffett, 2013). This participation in, sometimes complex, group identities continues today (Roccas and Brewer, 2002). Some social groups follow straightforward rules; all astrophysicists are physicists and all physicists are scientists. The background knowledge that you can assume in conversation with a scientist can also be assumed in conversation with a physicist and, likewise, the background knowledge you can assume in conversation with a physicist can also be assumed in conversation with a astrophysicist. But there are also ways in which social identities break from the contours of simple set membership relations. The mere conjunction of dominate narratives in Black liberation and in mainstream feminism does not yield important narratives of Black feminists (Combahee River Collective, 1977; Crenshaw, 1991). This paper provides a formal model of how a variety of social identity topologies can obtain in a population of agents trying to coordinate behavior and preferentially interact with ingroup members.

Conceptually, there are three aspects of social group identities that we model: (i) the *preferences* of social groups, (ii) the emergence of *social signals* used to express and identify group membership, and (iii) then the use of those social signals to *coordinate actions*. We can identify this sort of signaling for coordination in everyday life. Perhaps, in grade school cliques, Goths and Emos have different music preferences; they use differences in hair style to identify group members; and, they use those hair styles to identify who to join at recess. Swingers might use upside-down pineapples to signal their preference for "coordinating behavior" with other couples (Pelzer, 2023). There is a variety of empirical evidence of the relationship between (i) preferences, (ii) social signals, and (iii) norms in the context of complex social identities. Berger and Heath (2008) produced a series of studies that examine preferred interaction partners and clear group selection of markers such as wrist bands. Together, these studies are suggestive of groups with different preferences using social signals to coordinate action. Arguably, Lin et al. (2024) captures preferences for business transactions with trustworthy partners, signaling trustworthiness with a red face from drinking alcohol and the likelihood of coordinating in future business partnerships. But, these studies still lack complexity that we exhibit in our social identities. There is evidence that social identity in multiple dimensions (nationality, ethnicity, religion, and education) can be indicative of differences in preferences about openness, conservatism, universality, and power Roccas and Brewer (2002). Bunce (2020) shows that an education system dominated by a majority ethnicity can lead to a minority group learning norms related to coordination, in the domains of both work and family life, while still remaining competent in norms associated with their own ethnicity. Literature on code switching is also suggestive of how multiple dimensions of social identity relate to coordination. Both Black and female coworkers are judged to be more professional when they adopt norms such as dialect and hairstyle associated with white men (McCluney et al., 2021). We are unaware of any empirical evidence that captures (i) preferences, (ii) social signals, and (iii) norms in the context of complex social identities *all* in a single study. The benefit of working with a model, is that we can integrate *all* of these relationships together.

This paper's model assigns agents different preferences reflective of a complex society and shows how these preferences can lead to social signaling of complex social identities and norms for coordination that are conditioned on the multidimensional social signals that agents adopt. In fact, simply modeling groups with different preferences and the need to coordinate actions will be enough to produce simulation results in which a minority population conforms to the preferences of the majority group in accordance with the empirical studies just cited. Adding in social signals and multidimensional social identities allows even more interesting simulation results. However, substantial groundwork needs to be laid prior to explaining those results. Prior literature has provided formal models how selection pressures can lead to agents evolving social signals used for assortment or coordinating behaviors (McElreath et al., 2003; Smaldino et al., 2018; Smaldino and Turner, 2020; Goodman et al., 2023; Macanovic et al., 2024). However, none of these prior models have incorporated multidimensional social identities. Consequently, there are two important ways in which this paper's model will diverge from prior models. First, we allow agents to broadcast social signals in multiple dimensions rather than as single dimension. Second, we allow a broad spectrum of payoffs that can be associated with successful coordination. This second feature of our model does cohere with a prior model given in Bunce (2021). To illustrate this variance in payoffs, consider a hypothetical example in which Catholics prefer handshakes as a greeting and Protestants prefer hugs. In our model, this would be represented as both a Catholic and a Protestant getting some payoff when they hug, but the Protestant getting a higher payoff than the Catholic. In most prior models, agents were either indifferent between two ways in which they might coordinate, shake hands or hug, or agents got no payoff from successful coordination on a less preferred action. Allowing a spectrum of payoffs allows us to model overlapping social identities in which agents may only have a strict subset of their social identities in common. Similar to prior models, agents will not get any payoff when they fail to coordinate their actions, e.g. one agent tries to shake hands while the other tries to hug. This is a convenient simplifying assumption, but not all real world examples are like this. For example, two people trying to go to a movie together can still get some reduced payoff for watching the movie alone in the event that they go to different movie theaters. There will always be simplifying assumptions in models that do not capture all of the complexity of the real world contexts we are interested in. However, Appendix H shows that the model's results persist when this particular simplifying assumption is removed. Finally, stepping away from current social identities and their associated politics, we explain the model using an example of social groups from ancient Mesopotamia accompanied by a fictional story of coordinating on greetings.

Figure 1: Ancient Sumerian cities that will be discussed.

Stepping away from current social identities and their associated politics, we explain the model using an example of social groups from ancient Mesopotamia accompanied by a fictional story of coordinating on greetings. In ancient Mesopotamia there were two rival Sumerian cities, Umma and Lagash. The rivalry between the two cities stemmed from a disagreement over who would control the fertile land between them. This rivalry will be the motivation for the preferences we assign agents in Section 4.5.1 to produce the disjoint double embedding group structure. Since Umma and Lagash have a deep enmity for each other we will model them as getting no payoff for a greeting associated with the other's city. However, as both are Sumerian speaking cities, both will prefer a generic Sumerian greeting over an Akkadian greeting. The Akkadians were a semetic language speaking people surrounding the northern Sumeria, who eventually conquered the region in 2334 BCE after a centuries long military conflict between Umma and Lagash substantially weakened the Sumerian city states. Prior to that conquest, Lagash established the city of Girsu in the middle of the disputed territory between themselves and Umma. Girsu housed the temple to Lagashes patron diety Ningirsu. So for our purposes, we will think of Girshites as being like the zealots of the broader Lagash community. This will motivate the preferences we assign agents in Section 4.3 to produce the single embedding group structure and in Section 4.5.2 to produce the hierarchical double embedding group structure. The Girshites are not just Lagashites, but Lagashites zealous enough to establish a settlement in the middle of the disputed land between Umma and Lagash. There is one more ancient Sumerian city we will discuss, Kish. Around 2550 BCE, the king of Kish negotiated a treaty between the Umma and Lagash. It is unclear why exactly Kish was considered a neutral third party for negotiating this treaty, but we will use this ambiguity in the Kishus' relationship to other Summerians liberally. In initial sections of the paper establishing the base model and basic dynamics of broadcasting social signals, we will treat Kishus as having some overlapping identity with Ummians. Then in Section 4.4 we will vary their preferences in a way that illustrates an objective difference between two identical signaling systems in with Kishus signal that they are members of both the Ummian and Lagashite groups.

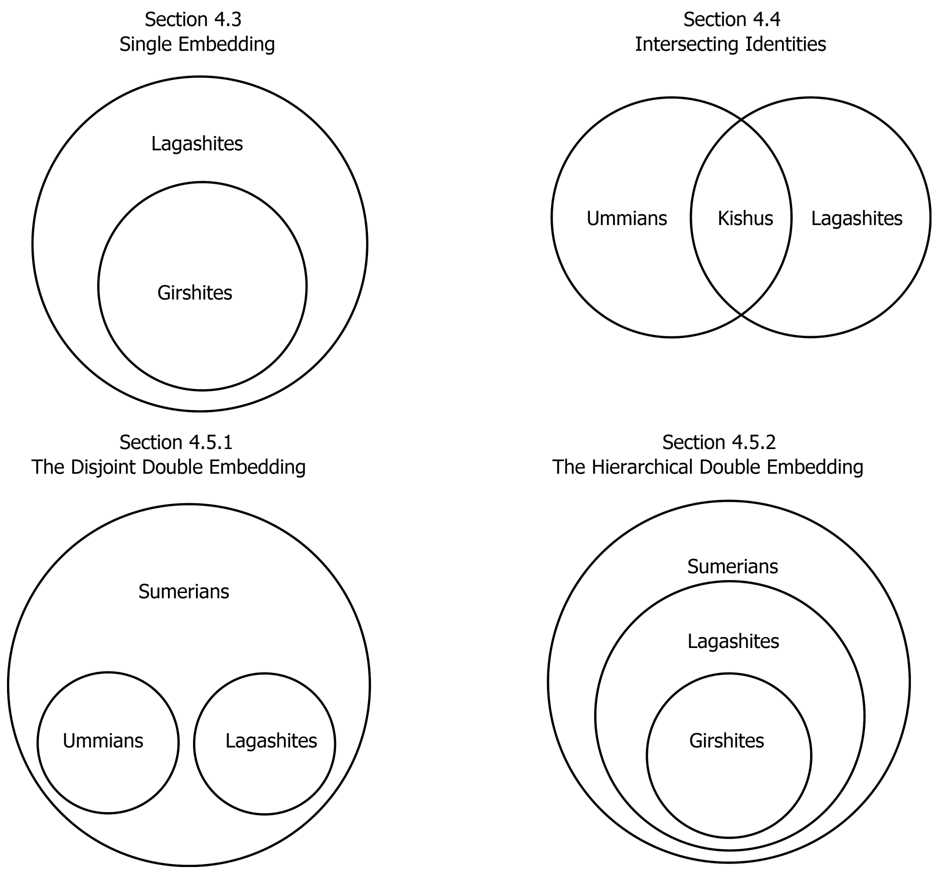


Figure 2: Overview of some group topologies that can be produced with this paper's model.

While the general structure of the preferences we assign to agents is reflective of historical relationships between the various groups of ancient Mesopotamians just discussed, the specific actions that agents have preferences over are not reflective of any historical reality. The specific actions we discuss for the model are fictional "greetings". Nothing in our model is reflective of the nature of greetings, we just need something for agents to be coordinating their behavior on. We call this thing that agents coordinate on a "greeting" as a reminder of the dynamics of the Generalized Bach or Stravinsky game. In the generalized Bach or Stravinsky game players do not get any payoff from doing opposite actions such as one agent trying to shake hands while the another tries to hug. So we talk about agents coordinating on greetings as a reminder of this technical detail of the game. However, it should not be assumed that the story of how people greet each other is an exemplar of the real world scenarios that might be represented by this sort of coordination problem. The generalized Bach or Stravinsky game can be representative of a wide variety of real world contexts in which we have to coordinate our actions. One of many contexts for which the model is well suited is power imbalances. For example, suppose there are two English dialects in America, say a northern dialect and a southern dialect. In a workplace, it may be most productive for everyone to speak the same dialect. Perhaps "the cash register doesn't have no money" in the northern dialect means it does have money, in virtue of the double negative, but in the southern dialect the statement means that it does not have any money. If the workplace does not have a clear norm, then a floor manager might send someone for a fresh till when none is needed or fail to do so when it is needed. Everyone speaking the same dialect eliminates this problem. In the model, giving the same greeting is analogous to speaking the same dialect; it is the action over which agents have preferences but get no payoff if failing to both do the same action. In the model, a majority group often exploits their power as the majority to make their preference the norm, which is analogous to a workplace dominated by northerners making the northern dialect the norm. This is just one example of how the paper's model might be analogous to real world scenarios in which we need to coordinate on the same action.

As already stated, the real world has many complexities and nuances that the model may fail to capture. In this paper's model, power imbalances typically result from one group being more numerous than another. But in the real wold this is just one of many ways in which power imbalances occur. After all, in ancient Sumer, Sargon of Akkad was able to make the Akkadian language the official language for government documents in cities irrespective of whether or not Akkadians were a majority of the population. This paper is a starting point for agent based models in which agents signal complex social identities. One feature of the model is that it has three separable components: (1) the way in which agents use social signals in multiple dimensions to signal their identity, (2) the coordination game that governs the payoff agents get from various combinations of signaling their social identity and choosing actions based on others' social signals, and (3) the learning dynamics that allow agents to adapt their signaling and actions to those combinations that increase their payoffs. When targeting real world phenomena that are not captured by the current model, future researchers can pick and chose among these components as appropriate.

One might retain the multidimensional signals and coordination game, but use artificial neural networks to capture how agents learn. Another might retain the social signaling and learning dynamics, but exchange the coordination game for a Nash demand game. Respectively, these modifications might target contexts in which it is important to represent differences between individuals and how they learn or might target contexts that are more adversarial than contexts captured by a coordination game. There are many paths forward from this starting point.

The way in which complex social identity structures are produced in the model takes some time to elaborate. In this paper, a large population of agents plays a coordination game, the generalized Bach or Stravinsky game, that is representative of how people coordinate on different preferences. Section 2.1 explains how the Bach or Stravinsky game, which was designed for just two players, is generalized to a large population of more than two players and a wide variety of possible preferences. Section 2.2 explains the learning dynamics that agents employ in this paper. At this point, the base model of the paper, which has no social identity signaling, has been fully described and simulation results for it are given in Section 2.3. Section 3 explains how various aspects of social identity signaling are added to the model and gives simulation results along the way. These simulations establish a few properties of the model that are carried forward as more complex signaling is added. First, we see that larger groups are privileged over smaller groups; specifically, the population as a whole is more likely to settle on the preferences of a larger group than a smaller group. Second, we see that the presence of a larger group incentivizes a smaller group to adopt social identity signals that they might not otherwise use. For readers not interested in the technical details of the model, Sections 2.2, 3.1, 3.3, and 4.1 can be skipped. Before showing any simulation results, Section 4.2 will review the diagrams used to explain simulation outcomes prior to discussing the specific group structures that are modeled. The big payoff of the paper comes in Section 4, which shows the emergence of the group structures depicted in Figure 2.

Sumerians	Ummians	Kishus	Lagashites	Girshites	Akkadians
○	□	⌂	△	◇	✲

Table 1: In diagrams, the different Mesopotamian identities will be associated with different shapes. These shapes are chosen to reflect the social relations between the different identities. Kish, as the original treaty negotiator between Ummma and Lagash, is associated the a pentagon shape, ⌂, that is a combination of the Umma square, □, and the Lagash triangle, △. Girsu, which houses the temple to Lagash's patron diety and is also the Lagashite settlement closest to the disputed land, is associated with a diamond shape, ◇, and can be thought of as two of the Lagash triangles, △. This might reflect the Girshites being the most zealous of the Lagashites. While these shapes will be useful for identifying different types of agents in diagrams, it is important to note that they do not represent characteristics that are visible to agents in the model.

2 Model Description and Incremental Results

This section iterates between model description and simulation results for the model. It begins with a description of the traditional Bach or Stravinsky game and then extends the game to a more general framework and adds complexity in agents ability to broadcast identity signals and condition their actions on those signals. At each step of extending the game, simulation outcomes from the resulting model are presented. When appropriate motivations for the subsequent extension of the model are also included.

2.1 Generalizing the Bach or Stravinsky Game

The traditional Bach or Stravinsky (BoS) game is a one shot coordination game between two players. It was introduced to the literature by Luce and Raiffa (1957) as the “battle of the sexes” game, in which two players have differing preferences over whether to attend a prize fight or ballet, but get no payoff if attending alone:

		Player 2	
		Bach	Stravinsky
Player 1	Bach	$1{+}\alpha$, 1	0, 0
	Stravinsky	0, 0	1, $1{+}\alpha$

Table 2: Traditional Bach or Stravinsky Game: $\alpha \geq 0$.

More recently, authors have called this the “Bach or Stravinsky” game to dissociate it from various undesirable prejudices and stereotypes frequently attached to sex and gender while maintaining the same abbreviation, BoS.

What the game is intended to capture, rather than gender prejudices, is the dynamics of differing preferences or norms in contexts of coordination. While the choice between meeting at a prize fight or ballet can be a coordination problem, the class of contexts in which we have to coordinate is much broader than determining where to meet. We coordinate when deciding to shake hands or high five, or when choosing music for a party. Even in communication we can face a coordination problem in selecting the right name for something given both our own preferences and our audience’s; e.g. the same thing might be called “battle of the sexes” or “Bach or Stravinsky”, it might be called “the morning star”, “the evening star”, or “Venus”, it might be called “the electric slide” or “the wobble”.

As a first step in extending the BoS game to better capture its target phenomenon, we consider a model in which the game is played repeatedly among intermixing agents in a population. In this model, agents still interact pairwise, but who an agent is paired with varies as the game is repeatedly played. Agents payoffs in the game are given by their type. Thus, replacing the two players in the traditional BoS game we get two types in the generalized BoS game:

		Bachites	
		Bach	Stravinsky
Bachites	Bach	$1+\alpha$, $1+\alpha$	0, 0
	Stravinsky	0, 0	1, 1
		Stravinskians	
		Bach	Stravinsky
Bachites	Bach	$1+\alpha$, 1	0, 0
	Stravinsky	0, 0	1, $1+\alpha$
		Stravinskians	
		Bach	Stravinsky
Stravinskians	Bach	1, 1	0, 0
	Stravinsky	0, 0	$1+\alpha$, $1+\alpha$

Table 3: Generalized Bach or Stravinsky for a population of two types: $\alpha \geq 0$.

Since agents always get a payoff of 0 when they fail to coordinate, we can equivalently present agents' payoffs as their coordination preferences:

Coordination Preferences	Bach	Stravinsky
Bachites	$1+\alpha$	1
Stravinskians	1	$1+\alpha$

Table 4: Coordination Preferences: generalized BoS for a population of two types, $\alpha \geq 0$.

Table 4 expresses all of the information in Table 3, but does so much more concisely. This concise format will be particularly useful when we consider populations with more than just two types.

2.2 Learning Dynamics

The generalization of the BoS game just given is independent of learning dynamics. This paper presents results modeling agents as evolving their dispositions through replicator dynamics. The dynamics was selected because it has substantial established literature and is computationally tractable.

In a simulation of the model, agents begin with a randomly selected strategy profile. On each timestep, the prevalence of each strategy profile among a type is adjusted according to a discrete replicator equation:

$$N_{t+1}(x) = N_t(x) + N_t(x) \times [U(x) - Avg(U(i))_{i \in X}]$$

Where, for a given type, X is the set of all strategy profiles, $N_t(x)$ is the number of agents of the given type with strategy profile x at timestep t, $U(x)$ is the utility of strategy profile x, and $Avg(U(i))_{i \in X}$ is the average utility of all the strategy profiles present among the given type. In simple words, the replicator equation increases the prevalence of strategies that allow agents to more frequently succeed at coordinating (where successful coordination on a more preferred greeting is given more weight than successful coordination on a less preferred greeting).

A strategy profile's utility for a given type is calculated as:

$$U(x) = \sum_{i \in Y} [M(i) \times p_{xi}] \qquad \text{if } x \neq i$$

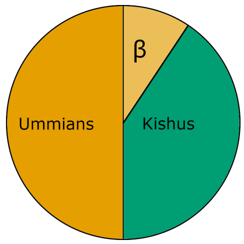
$$U(x) = \sum_{i \in Y} [(M(i) - 1) \times p_{xi}] \qquad \text{if } x = i$$

Where Y is the set of all (for any type) strategy profiles present in the population, $M(i)$ is the number of agents (of any type) in the population who play strategy i, and p_{xi} is the payoff an agent of the given type gets for playing strategy x when paired with an agent who plays strategy i. After adjusting the prevalence of strategy profiles, their quantity is normalized so that the number of agents of a given type remains constant throughout a simulation.

Finally, the possibility of an agent's strategy profile mutating is also included. This is governed by two parameters, m_0 and m_1. m_0 is the probability that an agent is selected for mutation. m_1 is the probability that an element in a string expression of the agent's strategy profile changes to a random value. Presently, agents' strategy profiles are expressed as strings of length one, which specify which of the two actions an agent plays. An agent's strategy profile can be $< B >$, play Bach, or $< S >$ play Stravinsky. So with probability m_0 an agent is selected for mutation, and if selected with probability m_1 a random value of B or S is used to replace the only element in the string expression of the agent's strategy profile. As the model is extended to allow agents to broadcast social signals and to choose their action based on the signal of a paired agent, the string expression of agents' strategy profiles will have length greater than one and the rational for using two mutation parameters will become more clear. Consequently, the mutation dynamic will be revisited at that point.

2.3 Simulation Results: Majority Groups Have Power to Make Their Preference the Norm

To illustrate this simple version of the model, we consider a population with two types of agents, Ummians (□) and Kishus (⌂). In addition to the parameters listed in Table 6, Ummians (□) make up $0.5 + \beta$ proportion of the population (and, accordingly Kishus (⌂), are a $0.5-\beta$ proportion). Coordination preferences are:



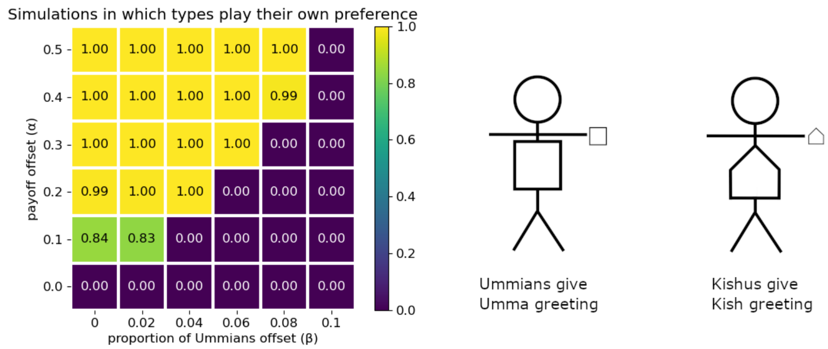
Coordination Preferences	Umma greeting	Kish greeting
Ummians □	$1+\alpha$	1
Kishus ⌂	1	$1+\alpha$

Table 5

This represents a population of two types of agents, where one type, Ummians, has the Umma greeting as their most preferred greeting and the other type, Kishus, has the Kish greeting as their most preferred greeting. Since Umma and Kish are friendly with each other, people still get some payoff for coordinating on their less preferred greeting. There is no social signaling in this base model, which means people are not able to use a social signal to determine which greeting the should use. Consequently, there are only two prominent types of outcomes in the simulations. When preferences are weak (when the difference in payoff for coordinating on an agents preferred greeting verses the alternative greeting is small), then everyone uses the same greeting because the benefit of always coordinating outweighs the benefit of always giving the most preferred greeting but frequently failing to coordinate behavior. When preferences are strong, then each type of person in the population always gives their preferred greeting because the payoff from that greeting outweighs the failures in coordination when people of different types interact. The simulations also show that when the Ummians (□) outnumber the Kishus (⌂) (i.e. when β increases) and preferences are weak (i.e. α is small), then the population as a whole always settles on the Umma greeting as the norm.

Figure 3 shows simulation results for populations of 1,000 agents. Stick figure diagrams are placed next to the heat maps that illustrate a simulation outcome. The shape in a stick figure's body indicates its type, either Ummian or Kishu. The shape next to its hand indicates the action, which is either the Umma or Kish greeting. Since simulations resulted in all agents of a given type adopting the same behavior, only one stick figure is given for each type of agent. The model produces three possible outcomes: (i) each type plays their preferred action all of the time (shown in Figures 3a), (ii) everyone plays Ummians's preferred action (shown in Figures 3b), and (iii) everyone plays Kishus' preferred action,(deducible as 1 - value in Figure 3a - value in Figure 3b). Figure 3a illustrates that as payoffs for preferred action increase, agents are more likely to always play their preference leading to coordination failures when agents of different types are paired.

(a) outcome (i)



(b) outcome (ii)

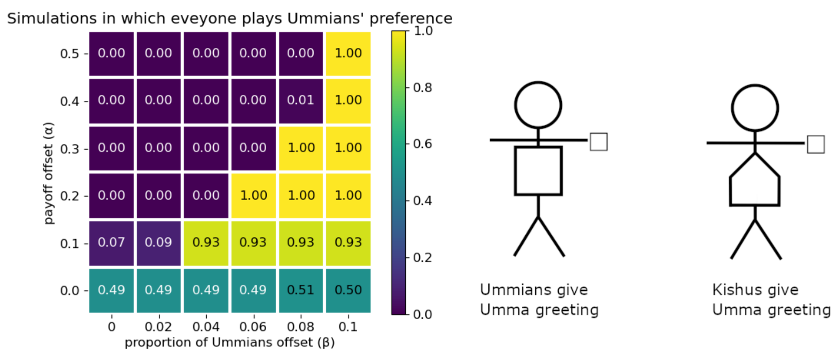


Figure 3: (a) Proportion of simulations that resulted in outcome (i) in which agents always give their preferred greeting; this means that when agents of different types fail to coordinate. (b) Proportion of simulations that resulted in outcome (ii) in which everyone gives Ummians' preferred greeting.

Figure 3b illustrates that as the proportion of Ummians (□) agents increases, the outcome in which everyone plays Ummians's preference, the Umma greeting, becomes more likely. This is because, even though Kishu agents have a higher payoff when they successfully coordinate on the Kish greeting, they have more opportunities for successful coordination with Ummians, since the Ummians (□) are more prevalent. Thus, Kishu agents can have a higher net payoff from playing Ummians' preference. Appendix C shows this analytically. The only time when agents all play Kishus' preference (outcome iii), the Kish greeting, is when $\alpha = 0$ or 0.1, but this just means that agents have no preference or only a weak preference for one of the two actions over the other. Accordingly, we see that outcome (iii) occurred in half of the simulations with $\alpha = 0$ (recall that this is deducible as 1 - value in Figure 3a - value in Figure 3b). The general result that, when preferences for one action over the alternative are week (i.e. α is small), the different types of agents fail to evolve different behavioral dispositions replicates an earlier finding by McElreath et al. (2003).[1]

There are two important takeaways from this base model. One was just stated, that the more numerous group, Ummians, is advantaged such that their most preferred greeting often becomes the norm for all agents in the population (Figure 3 ii). The second important thing to see is just how sub-optimal agents behavior is. Either agents are failing to coordinate their actions whenever they interact with those who have different preferences than themselves (Figure 3 i), or one group, typically Kishus, is giving their less preferred greeting even when interacting with others who also prefer that greeting (Figure 3 ii). This is why, in part, cultures have evolved to have social signals and ethnic markers. As we will show in extensions of the base model, social signals allow people to condition their actions on those signals such that they successfully coordinate actions when interacting with out-group members and coordinate on their preferred action when interacting with in-group members. Even when a majority group makes their preference the norm, minority groups can still establish their own norms within their group.

	Number of simulations	Simulation length	Population size	m_0	m_1	Signal cost, c
Section 2.3 Figure 3	10,000	4×10^4	1,000	0.01	0.1	n/a
Section 3.2 Figure 5	10,000	4×10^4	1,000	0.01	0.1	n/a
Section 3.4 Figures 7 & 11	1,000	8×10^4	1,000	0.01	0.1	-0.01
Section 4.3 Figure 15	1,000	4×10^4	1,000	0.01	0.1	-0.0005
Section 4.4 Figure 17 & 18	1,000	4×10^4	1,000	0.01	0.1	-0.0002
Section 4.5.1 Figure 21	100	4×10^4	1,000	0.05	0.2	-0.0002
Section 4.5.2 Figure 23	100	4×10^3	1,000	0.05	0.2	-0.0005

Table 6: Additional details about simulation parameters in this paper.

3 Incremental Extensions of the Model

3.1 Adding Signals to the Generalized BoS Game

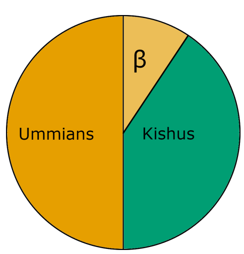
In our next extension of the model, we allow agents to broadcast social signals and to use their partner's signal to determine their action in the game. Now

[1]Specifically, this is equivalent to behavioral differences failing to evolve in their model when δ is small as shown in the grey regions of their Figure 3.

that agents broadcast social signals, their strategy profiles can be expressed as strings of length 1+# of signals, where the first element is occupied by the social signal that an agent broadcasts and the subsequent elements are occupied by the action that is played when paired with an agent who broadcasts the corresponding signal. For example, suppose the there are two signals 1 and 2, with everything else remaining as before. Then there are 2^3 strategy profiles: $< 1BB >$, $< 1BS >$, $< 1SB >$, $< 1SS >$, $< 2BB >$, $< 2BS >$, $< 2SB >$, and $< 2SS >$. The strategy profile $< 1BB >$ corresponds to an agent broadcasting social signal 1, playing Bach when paired with an agent who broadcasts 1, and playing Bach when paired with an agent who broadcasts 2; the strategy profile $< 1BS >$ corresponds to an agent broadcasting social signal 1, playing Bach when paired with an agent who broadcasts 1, and playing Stravinsky when paired with an agent who broadcasts 2; etc.. Now we can see the rational for the two parameters for mutation. Suppose $m_0 = 0.01$ and $m_1 = 0.1$, then an agent has a 1 in 100 chance of being selected for mutation, and if selected each element in the string expression of the agent's strategy profile has a 1 in 10 chance being changed to a random value. This results in mutations to a new strategy profile that is similar to the prior strategy profile being more likely than mutations to a strategy profile that is maximally dissimilar. This improves learning when there is a large set of possible strategy profiles.[2]

3.2 Simulation Results: Social Signals Allow Optimal Behavior and Cross-Cultural Competence

The simulations in this section continue with the example already given in Section 2.3, in which there are just two types of agents, Ummians (□) and Kishus (⌂). The only difference is that now the agents broadcast one of two social signals (● or ●), choose their action based the signal they observe from a partner, and, if $h \neq 0$ assort more frequently with others who broadcast the same social signal. As before, Ummians (□) make up $0.5 + \beta$ proportion of the population (and Kishus (⌂) are a $0.5 - \beta$ proportion). Coordination preferences are:



Coordination Preferences	Umma greeting	Kish greeting
Ummians □	$1 + \alpha$	1
Kishus ⌂	1	$1 + \alpha$

Broadly described, the simulation results show that people almost always

[2]Functionally similar behavior could be obtained with a single parameter rather than two by using just the m_1 parameter with a smaller value (say $m_0 \times m_1$) and always selecting all agents for mutation. However, the two parameter version is also computationally more tractable.

learn to use the signals to broadcast their social identity for coordination. When Umma people interact with themselves, they give the Umma greeting. Likewise, when Kish people interact with themselves, they give the Kish greeting. But when Umma people interact with Kish people, the population still has to settle on whether to coordinate on the Umma greeting or the Kish greeting. The more numerous population is still advantaged. If the Umma people outnumber the Kish people, then the population is more likely to settle on the Umma greeting as the norm when people of different types interact.

Figure 4 shows how the stick figure diagrams are extended to incorporate agents broadcasting social signals and conditioning their actions on social signals. As before, agents' types are given by the shape on their body. But this indicator of an agent's type is just for communicating simulation outcomes. Agents themselves do not see each others' types. What they do see is the social signal that an agent broadcasts, this is depicted on a stick figure's forehead. Finally, an agent's action in response to a given signal is depicted adjacent the given signal near the stick figure's hand.

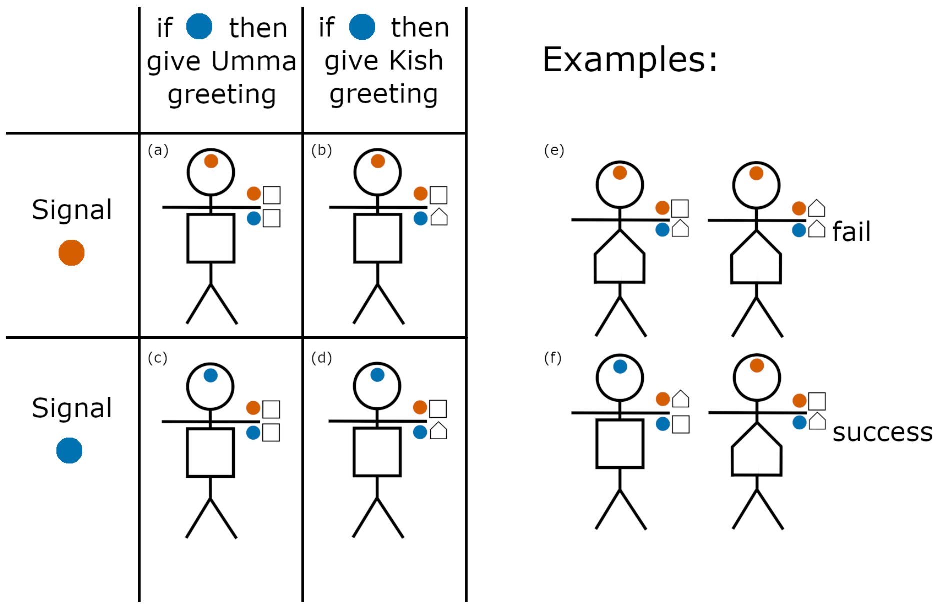


Figure 4: Diagrams explanation: (a) An Ummian who signals ● and gives the Umma greeting in response to both signals. (b) An Ummian who signals ● and gives the Umma greeting in response to ● and the Kish greeting in response to ●. (c) An Ummian who signals ● and gives the Umma greeting in response to both signals. (d) An Ummian who signals ● and gives the Umma greeting in response to ● and the Kish greeting in response to ●. (e) Two agents who fail to coordinate actions. Both agents broadcast ●, but one gives the Umma greeting in response and the other the Kish greeting. (f) Two agents who successfully coordinate. The Ummian □ broadcasts ● and gives the Kish greeting when observing ●. The Kishu ⌂ broadcasts ● and gives the Kish greeting when observing ●. Consequently, the interaction results in successful coordination.

More precisely, there are four different outcomes that frequently obtain in this model. In outcome (i) each type plays their preferred action all of the time is possible, but this outcome never occurs under the parameters shown in this section. Outcomes in which (ii) everyone plays Ummians' preferred action, and (iii) everyone plays Kishus' preferred action, are outcomes in which agents fail to make use of the social signals available to them.[3] The outcomes in which (iv) Ummians (□) always play their preferred action and Kishus (⌂) play Umma greeting with Ummians (□) and play the Kish greeting among themselves, and (v) Kishus (⌂) always play their preferred action and Ummians (□) play the

[3]The frequency of these two outcomes is close to 1- the frequency of (iv) - the frequency of (v), which can be deduced from the figures given in this section. However, it is more precisely 1- the frequency of (iv) - the frequency of (v) - the frequency of (vi); and in a few limited cases (vi) made up close 0.06 proportion of outcomes though in most cases the proportion was closer to 0. That is, approximately: (ii)+(iii)=1-(iv)-(v).

Kish greeting with Kishus (⌂) and play the Umma greeting among themselves are optimal outcomes which depend on the agents evolving meaningful signals. Finally, though infrequent, (vi) there are a variety of suboptimal outcomes in which meaningful signals evolve but at least one type does not play their preferred action among themselves.

Figure 5 shows all optimal outcomes (outcomes (iv) and (v)) on the left and just the optimal outcome favoring Ummians (□) (iv) on the right (so it is impossible for a value on the right to exceed the corresponding value on the left). Outcome (iv) favors Ummians (□) because it involves agents coordinating on the Umma greeting whenever an interaction between agents of different types occurs. Just as before, increasing the proportion of Ummians (□) (i.e. increasing β) makes outcomes favoring Ummians (□) more frequent. Thus, as β increases, the numbers on the right hand side converge towards those on the left. Likewise, when the proportion of Ummians (□) and Kishus (⌂) is equal (i.e. $\beta = 0$), the values on the right (b) are roughly half of those on the left (a).

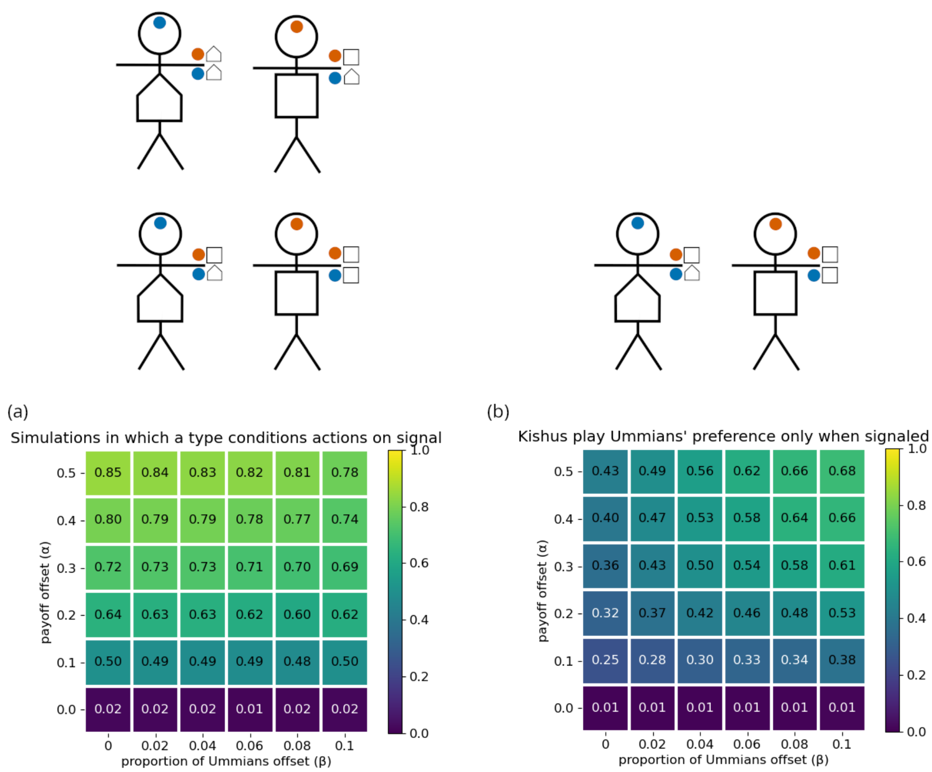


Figure 5: (a) Proportion of outcomes that are (iv) or (v). (b) Proportion of outcomes that are (iv).

Results from this extension of the model recover an earlier result in Bunce

(2021). Bunce is interested in how minority groups maintain their cultural norms. Prior models (Bunce and McElreath, 2018; O'Connor, 2019) suggested that the ability of minority groups to maintain their cultural norms is tenuous at best. We saw this tenuousness in the base model with no signals, Section 2.3. Specifically, Figure 3b shows Kishus (⌂), the minority group, adopting their less preferred greeting, the Ummian greeting (□), because the frequency of their interactions with Ummians giving the Ummian greeting was enough to entail a higher net payoff for Kishus giving the Ummian greeting than Kishus giving the Kish greeting. The Bunce (2021) model shows that a minority group has a much better chance of maintaining their culture if they are able to competently adopt either the majority or minority norms depending on the context. This is called cross-cultural competence. Figure 5b shows exactly this sort of cross-cultural competence with Kishus using the majority's norm, the Ummian greeting (□), when interacting with Ummians, but maintaining their own norm, the Kish greeting (⌂), among themselves.[4]

To see this cross-cultural competence allowing a minority group to maintain their culture, consider specific parameters, say $\alpha = 0.4$ and $\beta = 0.1$, which correspond to agents having relatively strong preferences and Ummians being 60% of the population respectively. Figure 3b shows that under these parameters 100% of simulations resulted in Kishus abandoning their preferred greeting for the majority's preferred greeting. In contrast with this, Figure 5b shows 66% of simulations resulting in Kishus giving the majority group's preferred greeting, the Ummian greeting (□), when interacting with Ummians, but also maintaining their own norm, giving the Kish greeting (⌂), among themselves. It is social signals that enable this cross-cultural competence that facilitate the Kishus being able to maintain their culture, one in which the Kish greeting is the norm, despite the dominant group, Ummians, making their norm the standard for cross-cultural interactions.

Recall that when looking at the simulation results from the base model (Section 2.3), there were two important takeaways; first, that the more numerous group often makes their preference the norm for between group interactions and, second, that agents' behavior was sub-optimal either in virtue of failing to coordinate actions for between group interactions or in virtue of a minority group failing to do their most preferred action when interacting among themselves. In this section, because agents can use social signals, it is no longer the case that agents are always acting sub-optimally. So while it is the case that majority groups, in this case Ummians, frequently make their preference the norm (Figure 5 b), minority groups are able to behave more optimally by at least being able to make their preference the norm among themselves.

[4]While this model recovers a result from Bunce (2021). It is also notable that it does so on fewer assumptions. In this extension of the generalized Bach or Stravinsky model, agents evolve social signals for indicating identity from an initial state in which signals were arbitrary and meaningless. The Bunce (2021) model assumes that agents who are able to act in accordance with either of two norms are simultaneously and immediately able to determine the identity of those they are interacting without learning any social signals.

3.3 Adding Attention and Signal Costs to the Generalized BoS Game

It is often the case that social signals require some effort to broadcast or attend to and only those groups for whom the social information is relevant invest the effort in engaging with the signals; among a subpopulation individuals may invest effort in signaling whether they are lefts or rights, while the broader population remains entirely oblivious to the social signals surrounding them. To capture this, the model is extended with a special signal, 0, which indicates an agent is not attending to signals and a signal cost, c, that an agent incurs if she broadcasts any signal other than 0. When an agent does not attend to signals, i.e. when she broadcasts 0, she interacts with all other agents as if they had also broadcast 0. Interacting with all other agents as if they broadcast 0 means that actions cannot be chosen based on the social signal that was broadcast, which is exactly what should be the case for an agent who is not attending to the social signals.

To illustrate this extension of the model, lets consider how the example from Section 3.1 changes under this new extension. While there were previously two signals 1, and 2, there are now three signals 0, 1, and 2. Consequently there are now 3×2^3 strategy profiles: $< 0BBB >$, $< 0BBS >$, $< 0BSB >$, $< 0BSS >$, $< 0SBB >$, $< 0SBS >$, $< 0SSB >$, $< 0SSS >$, $< 1BBB >$, $< 1BBS >$, $< 1BSB >$, $< 1BSS >$, $< 1SBB >$, $< 1SBS >$, $< 1SSB >$, $< 1SSS >$, $< 2BBB >$, $< 2BBS >$, $< 2BSB >$, $< 2BSS >$, $< 2SBB >$, $< 2SBS >$, $< 2SSB >$, and $< 2SSS >$. Now, it might be noted that the strategy profiles $< 0BBB >$, $< 0BBS >$, $< 0BSB >$, and $< 0BSS >$ all involve the same dispositions since an agent who does not attend to signals will treat agents who signal 1 or 2 as if they had signaled 0, i.e. agents with these profiles will always play B. Likewise, the strategy profiles $< 0SBB >$, $< 0SBS >$, $< 0SSB >$, and $< 0SSS >$ all involve the same dispositions since an agent who does not attend to signals will treat agents who signal 1 or 2 as if they had signaled 0, i.e. agents with these profiles will always play S. However, if an agent with one of these strategy profiles mutates in just the signaling position of the strategy profile the resulting strategy profile differs depending on what the agents prior profile was even when comparing two profiles that entailed the same dispositions. E.g. while $< 0SBS >$ and $< 0SSB >$ entail the same dispositions, a mutation to signaling 1 instead of 0 results in either $< 1SBS >$ and $< 1SSB >$ which entail very different dispositions. For this reason, distinct string representations of strategy profiles are considered distinct strategy profiles even if those distinct strings entail the same dispositions.[5]

At the macro level, this extension of the model changes nothing in our for-

[5] It is also computationally convenient to have all string representations of strategy profiles be the same length.

mula for the utility of a strategy profile:

$$U(x) = \sum_{i \in Y} [M(i) \times p_{xi}] \qquad \text{if } x \neq i$$
$$U(x) = \sum_{i \in Y} [(M(i) - 1) \times p_{xi}] \qquad \text{if } x = i$$

However, the p_{xi} component of the formula does change. Previously, p_{xi} was simply the payoff specified in the agent's coordination preferences for an action if strategy profiles x and i lead to coordination on the same action and zero otherwise. Now, if strategy profile x entails broadcasting a social signal that is not zero, then p_{xi} is the payoff specified in the agent's coordination preferences for an action plus c if strategy profiles x and i lead to coordination on the same action and c otherwise (c is added to the prior p_{xi} value because this paper's convention is to use negative values for signal costs). If strategy profile x entails broadcasting 0, then p_{xi} is unchanged since no signaling cost is incurred. For example, if two agents have a coordination preference of 1 for coordinating on Bach and $c = -0.1$ then they will have a payoff of $p_{xx} = 1$ if both agents use strategy profile $x =< 0BBS >$; but, if both agents use strategy profile $x =< 1SBS >$ then they will have a payoff of $p_{xx} = 1 - 0.1 = 0.9$.

In this model, it is always the case than an agent not conditioning her actions on signals and not broadcasting a social signal co-occur. However, in the real world, these are independent dispositions. An agent might abstain from broadcasting a social signal, but still use social signals to determine what action to perform. Conversely, an agent might broadcast a social signal, but not pay attention to signals when choosing what action to perform. Combining these two dispositions so that they always co-occur is computationally convenient, plausible for some social contexts, and does not inhibit us from producing some interesting and novel social signaling topologies. So, for now, we maintain this assumption and leave investigation of the model with independence between these two dispositions as a task for future research.

3.4 Simulation Results: Majority Groups Ignore Social Signals

This section shows results from the model extension in which agents can chose whether or not to pay the cognitive costs of paying attention to social signals. It builds on the prior example of coordination between Ummians (□) and Kishus (⌂) by adding in Akkadians (✵). Since Akkadians (✵) get nothing from any of the Sumerian greetings, they should always give the Akkadian greeting and have no reason to pay the cost of broadcasting a social signal. Simulations show that the presence of the Akkadians (✵) can increase the likelihood that the Ummians (□) and Kishus (⌂) use social signals for coordination. However, the simulations also show that if the Ummians (□) are sufficiently numerous, they can end up always giving the Umma greeting and never signaling.[6] Diagrams of the agents are extended to show an agents disposition towards the null signal, which is represented as and underscore:

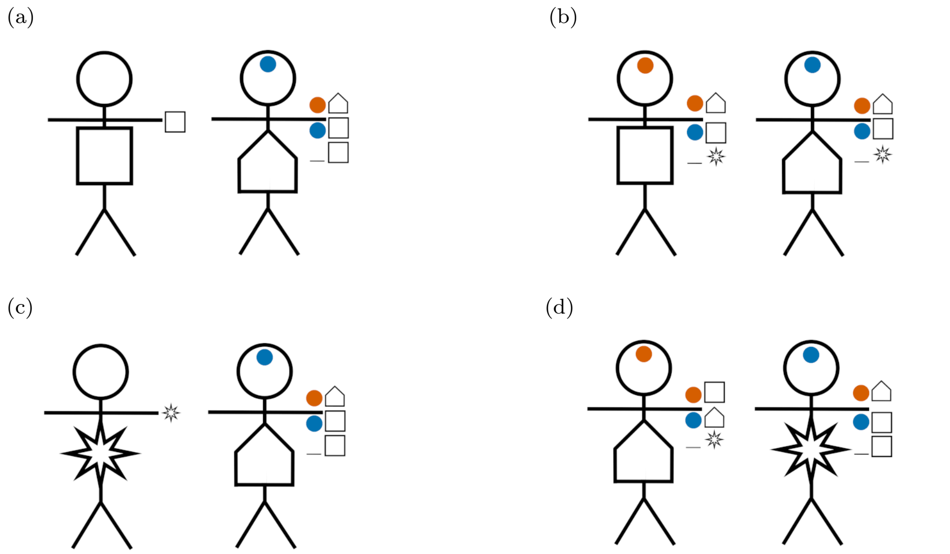
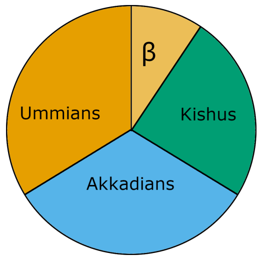


Figure 6: Diagram examples of interactions with the attention dynamic: (a) An Ummian (□) who does not attend to signals and always gives the Umma greeting. A Kishu (⌂) who gives the Umma greeting when she observes the null signal. This results in successful coordination. (b) These agents fail to successfully coordinate. The Ummian (□) sees ● and gives the Umma greeting. The Kishu (⌂) sees ● and gives the Kish greeting. (c) These agents fail to successfully coordinate. The Akkadian (✲) does not attend to social signals and always gives the Akkadian greeting. The Kishu (⌂) gives the Umma greeting when no social signal is observed. (d) These agents successfully coordinate. The Kishu (⌂) broadcasts ● sees ● and gives the Kish greeting in response. The Akkadian (✲) broadcasts ● sees ● and gives the Kish greeting in response.

Ummians (□) make up $0.33 + \beta$ proportion of the population, Kishus (⌂) are a $0.33 - \beta$ proportion of the population, and Akkadians (✲) are a 0.34 proportion of the population. Coordination preferences are:

[6]The preferences of Ummians (□) and Kish are symetric. If Kishus (⌂) are sufficiently numerous, they can also end up always giving the Kish greeting and never signaling. Since the two populations preferences are symetric, results are only shown for increasing the proportion of Ummians.



Coordination Preferences	Umma greeting	Kish greeting	Akkadian greeting
Ummians (□)	$1+\alpha$	1	0.5
Kishus (⌂)	1	$1+\alpha$	0.5
Akkadians (✲)	0	0	1

The inclusion of a third type of agent, Akkadians (✲), who have no incentive to condition actions on signals, since the Akkadian greeting is the only action with a payoff, makes signaling more prevalent for the other two types. To illustrate this, Figure 7b keeping all other parameters the same considers the model where Ummians (□) make up $0.5+\beta$ proportion of the population, Kishus (⌂) are a $0.5-\beta$ proportion of the population; i.e. there are no Akkadians.

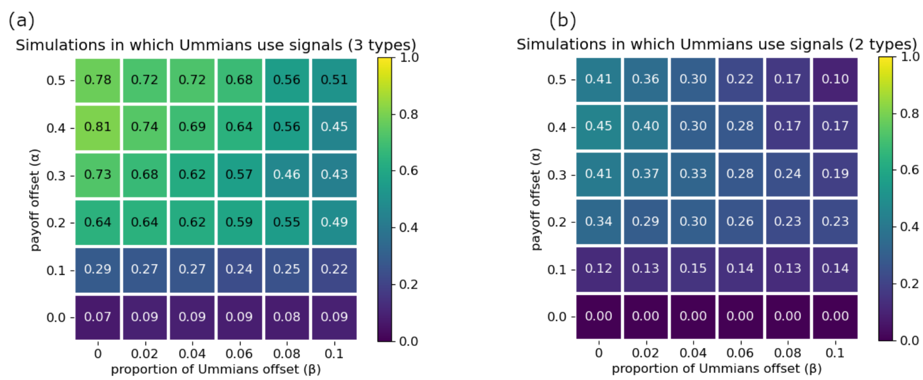


Figure 7: (a) Proportion of outcomes in which Ummians (□) broadcast a social signal, for the model with three types of agents. (b) Proportion of outcomes in which Ummians (□) broadcast a social signal, for the model with two types of agents.

For the parameters investigated, there were thirteen different outcomes that occurred at least 0.5% of the time for some data point. These outcomes are given a detailed description in Appendix E.[7] This section shows three prominent

[7] The appendix also details four outcomes that occurred in less than 0.5% of simulations. These outcomes are notable not for their prevalence, but in virtue of being what Fulker et al. (2024) call hybrid equilibria, though they now prefer the term bipartite signaling. This is particularly noteworthy because the bipartite signaling produced by this paper's model occurs in a population that is randomly mixing, something that Fulker et al. (2024) seem to have thought impossible. They write "Traditional evolutionary models with large, randomly mixing populations invariably yield one of the two homogeneous signaling systems". It is very reasonable to read Fulker et al. (2024) as merely giving a description of traditional models rather than actually making a claim of impossibility. Either way, the fact that this paper's model can produce bipartite signaling in a system with random mixing is an interesting and

outcomes.

(viii) The outcome in which Ummians (□) and Akkadians (✵) always played their respective preference, and Kishus (⌂) played the Umma greeting with Ummians, the Kish greeting with other Kishus, and the Akkadian greeting with Akkadians. The prevalence of this outcome is shown in Figure 11a. This outcome was frequently characterized by Ummians (□) agents signaling 0 and Kishus (⌂) and Akkadians (✵) attending to signals that reliably identified their type. One can check that this is a Nash equilibrium. It is suboptimal in the sense that when Ummians (□) are paired with Akkadians, there is a failure to coordinate. As previously noted, the Akkadians (✵) should always give the Akkadian greeting. So one might wonder why this population ends up signaling? I.e. why can this behavior be a Nash equilibrium? The reason is that if the Kishus (⌂) are giving the Umma greeting when they see the null signal (since this allows coordination with Ummians), then they will only coordinate with Akkadians (✵) if the Akkadians (✵) broadcast a social signal. So, Akkadians (✵) are not attending to the signals because they are conditioning their actions on the signals. Rather they are attending to the signals because they benefit from those signals allowing Kishus (⌂) to coordinate with them.

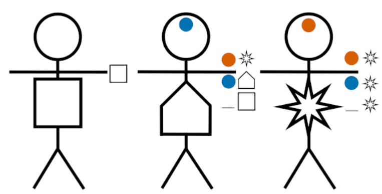


Figure 8: Outcome (viii)

(xii–xiii) Agents condition their actions optimally, in the sense that there were never failures of coordination and agents played their most prefered greeting among themselves. The frequency of this outcome is shown in Figure 11b.

(xii) The optimal outcome favoring Ummians (□).

surprising finding that warrants further research. We have confirmed that all of the equilibria in which bipartite signaling occurs are Nash.

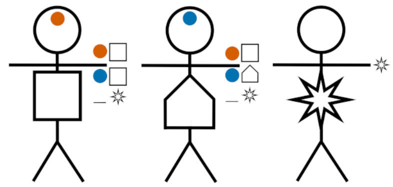


Figure 9: Outcome (xii)

(xiii) The optimal outcome favoring Kishus (⌂).

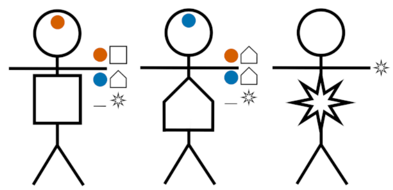


Figure 10: Outcome (xiii)

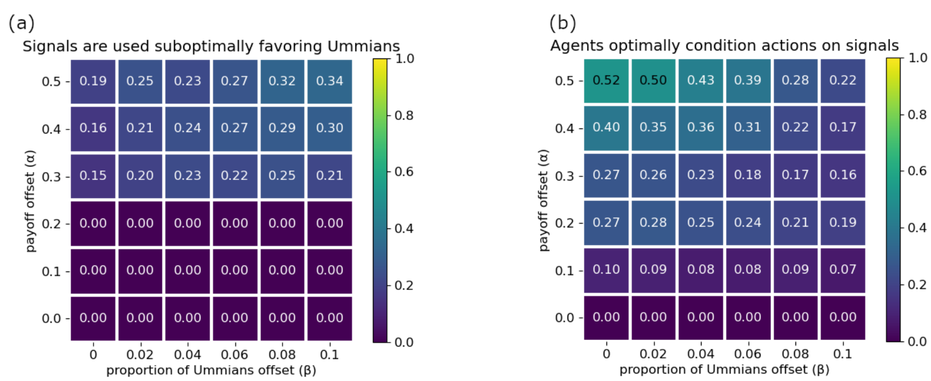


Figure 11: (a) Proportion of outcomes that are (viii). (b) Proportion of outcomes that are (xii-xiii).

Adding in the Akkadians (✵) makes Ummians (□) more likely to transmit a signal when β is small (see Figure 7b). When Ummians (□) become more prevalent in the population (i.e. when β increases), signaling from Ummians (□) decreases since they have leverage to always play their preferred action and Akkadians (✵) frequently adopt one of the two signals (this type of outcome is shown in Figure 11a). But the general trend is clear, minority groups have more incentive to use social signals.[8] In Figure 7a, there is some slight strati-

[8]At first glance, one might object that this does not follow from the results since Ummians

fication around $\alpha = 0.2$ (i.e. what increasing α seems to make signaling more prevelant, there is a decrease in signaling from $\alpha = 0.2$ to $\alpha = 0.3$ for $\beta \geq 0.06$). This seems to be a consequence of the threshold for Ummians (□) and Kishus (⌂) being highly incentivized to play their preference among ingroup members occurs around $\alpha = 0.2$; while the threshold for Ummians (□) to use their power as the largest group to only play their preference occurs around $\alpha = 0.3$ (see also Figure 11a).[9]

In summary, this extension of the model generally maintains and adds to the findings of prior versions of the model. As in previous sections, majority groups typically have the privilege of their most preferred greeting being the norm when interacting with minorities. Likewise, social signals make optimal outcomes possible that are not possible without social signals, ethnic markers, or some other observable feature on which agents can condition their actions. The key result that the attention dynamics adds to the model is that, now, majority groups often do not broadcast a social signal at all. Instead, they simply assume their norm, to give their most preferred greeting, in all interactions. This coheres with the empirical evidence that majority groups, such as whites or heterosexuals, often do not think of themselves as having an identity and taking their norms to be ubiquitous (Simoni and Walters, 2001; Devos and Banaji, 2005).

4 Complex Group Topologies

You can use common sense intuitions for understanding what this section does in expanding our model of social identities. In previous sections the model was restricted to simple one dimensional identity signaling, such as Christian vs. Jewish or Yankees vs. Red Sox fans. Now, we expand the model to allow multiple dimensions of identity signaling, such as signaling both your religion and the sports team you root for. Allowing signals in multiple dimensions also allows signaling an identity that is embedded in a broader group such as signaling that you are Catholic in addition to signaling that you are Christian. Now these identities are not themselves social signals. A cross necklace, a six pointed star necklace, a Yankees baseball cap, and a Red Sox cap are all social signals. Hopefully the distinction between an identity and a signal of that identity is not news to you. Technically, what the model in this section does is allow for multiple dimensions of social signals, such as necklaces and hats. With this affordance of multiple dimensions of signals, agents in the model can, if they see fit to do so, broadcast social signals that indicate multiple dimensions

(□) become the largest group as β increases. However, this response is unsound. Figure 7 clearly shows that Ummians (□) are less likely to signal as they become a larger proportion of the population. Additionally, one can check in supplemental PDF that outcome (viii) (shown here in Figures 11a) typically involves Akkadians (✲) broadcasting a social signal; which is exactly what needs to be the case to support the claim that minority groups have more incentive to use social signals.

[9] This stratification is more pronounced in larger populations (see Appendix D).

of their social identity. That's it. All we do here is allow agents in the model to signal multiple dimensions of their social identities.

It is empirically obvious that people often make use of multiple dimensions of social signals rather than coming up with a single social signal that indicates multiple dimensions of their identity; i.e. there is no widely used social signal that indicates both that a person is Jewish and a Yankees fan. Intuitively this makes sense. If you want to sit on the side of a sports bar that is occupied by other Yankees fans, you do not want to have to know the social signals that mean Jewish and a Yankees fan, Christian and a Yankees fan, Muslim and a Yankees fan, Hindu and a Yankees fan, Buddhist and a Yankees fan, and so on. Instead it is much easier to know just one social signal that indicates someone is a Yankees fan. This intuitive explanation of the utility of having multiple dimensions of social signals can be shown more objectively with our model.[10] The model can also explain some less intuitive aspects of social identities, which is part of the value of having a formal model.

Figure 12: Ancient Sumerian cities that will be discussed.

Before we jump into the details of the model, let's remind ourselves of the ancient Mesopotamian example we use for explaining the model. The benefit of

[10]This explaination also aligns with prior research showing the utility of languages being compositional (Steinert-Threlkeld, 2016).

using an example that we are so temporally distant from is that we can asses the objective features of the model without carrying into it whatever emotions, prejudices, and preconceptions we might associate with modern social identities, such as any divisive attitudes we may have towards Red Sox fans. That said, we will try to make some comparisons to help make the example less abstract.

All of the cities depicted in Figure 12 are Sumerian (○) cities, which are united by a common language and religious pantheon. Umma (□) and Lagash (△) are like the Red Sox and Yankees or Manchester United and Liverpool, they hate each other. Their rivalry stems from a dispute over who controls the fertile farmland between the two cities. While Summerians share a common pantheon, various gods are associated with different cities. Umma's god is Shara and Lagash's god is Ningirsu. Ningirsu's temple is in the city of Girsu. So if Lagashites are Catholics, then Girshites (◇) are like Knights Templar. They are the Lagashites who were zealous enough to build a settlement and temple right in the middle of the disputed territory between Umma and Lagash. Lagash exerted military dominance over Umma for a few hundred years. Kish, which is comparatively distant from both Umma and Lagash, was considered a neutral third party and the king of Kish at some point negotiated a treaty between Umma and Lagash. This is why we will represent Kishus with a pentagon (⌂) in our diagrams, since it is roughly a combination of the Ummian square (□) and the Lagashite triangle (△). Rather than having a fixed role in our simulations we will use the Kishus to illustrate the difference between (1) a group whose members are merely a conjunction of the people who like the Ummian greeting and those who like the Lagashite greeting, and (2) a group whose members share some preferences with both Ummians and Lagashites but are nonetheless a group in their own right. This is a distinction that is best understood in the context of the model that has yet to be elaborated. Finally, all of these cities are eventually conquered by the Akkadians (✲) who make the the Akkadian the official language of the land. 2,000 years later, when Alexander the not so great conquered Mesopotamia, Sumerian had completely died out as a spoken language, though it was still occasionally used in scholarly texts. If Sumerians are like Native Americans, then the Akkadians are like English colonists. Now that we've done violence to all of the social identities we just stereotyped, let's just talk about the model.

Section 4.1 explains the final extension of the formal model to allow the inclusion of multiple dimensions of social signals. You can skip it if uninterested in the formal dynamics. Then Section 4.2 explains how the diagrams are used to depict simulation outcomes. This section should not be skipped as it is essential to interpreting all remaining sections.

4.1 Adding Multiple Dimensions to Signals in the Generalized BoS Game

The final extension this paper makes to the model is allowing agents to broadcast social signals in multiple dimensions. Agents can broadcast up to one social signal in each dimension available. This can be thought of as allowing agents to broadcast independent social signals in correspondence with independent social signals. For example, signaling one is a Baltimore Orioles fan by wearing a corresponding baseball cap is incompatible with wearing a Washington Nationals cap, but entirely independent of signaling one is a San Antonio Spurs fan by wearing a corresponding basketball jersey. In this example signals in the baseball dimension are mutually exclusive, but independent of signals in the basketball dimension. In the model whether or not an agent attends to signals in one dimension is independent of whether another dimension is attended to. Thus, if there are two dimensions for signaling and two signals in each dimension (excluding the null signal), then there are nine different social signals an agent can broadcast, letting signals in the first dimension be 1, 2 and the signals in the second dimension be 3, 4: 00, 03, 04, 10, 20, 13, 14, 23, and 24 are all possible signals.

In the string representation of strategy profiles, there is one place for each dimension of signaling and one place for each possible signal. Thus, if there are two dimensions for signaling and two signals in each dimension (excluding the null signal) and two actions, then there are $3^2 \times 2^9 = 4608$ unique strategy profiles. The signal cost c is incurred fore each dimension attended to. Thus if strategy profile x leads to successful coordination with agents employing strategy profile i by performing an action with coordination preferences value of 1, then if $x =< 00BBSBSBBSS >$ this results in $p_{xi} = 1$, if $x =< 20BBSBSBBSS >$ this results in $p_{xi} = 1 + c$, and if $x =< 14BBSBSBBSS >$ this results in $p_{xi} = 1 + 2c$. The basic replicator dynamics and utility function of the model remain unchanged.

4.2 Explanation of Diagrams

Figure 13 explains how the stick figure diagrams are extended to accommodate multidimensional signals. Agents' types are given by the shape on their body. This indicator of an agent's type is just for communicating simulation outcomes. Agents themselves do not see each others' types. What they do see is the social signal that an agent broadcasts, this is depicted on a stick figure's forehead. Two agents of different types who broadcast the same social signal look the same to other agents in a simulation. For multidimensional signals, the signals in each dimension appear horizontally from left to right on agents forehead. If an agent does not pay attention to a dimension of social signaling, then no signal for that dimension is shown on the agents' forehead. Any particular social signal can only occur in a single dimension. Finally, an agent's action in response to a given signal is depicted adjacent the given signal near the stick figure's hand. Note that if an agent does not pay the cost of attending to a dimension of signals, then she neither signals in that dimension nor does she condition her actions on signals in that dimension. This is illustrated by both stick figures in Figure 13c. The left stick figure does not pay attention to the second dimension. Consequently, she sees ●_ and ● ● as the same social signal. Therefore, her action in response to seeing ●_ and ● ● must be the same. Likewise, _● and __ look the same to her and elicit the same action. In Figure 13c, the right stick figure does not pay attention to the first dimension. Consequently, she sees _● and ● ● as the same social signal. Therefore, her action in response to seeing _● and ● ● must be the same. Likewise, ●_ and __ look the same to her and elicit the same action.

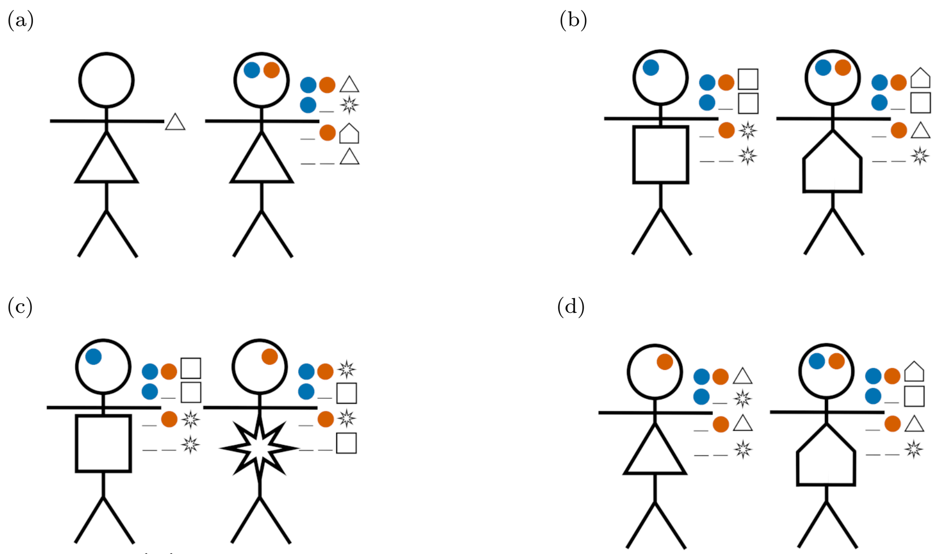


Figure 13: Diagram examples of interactions with multidimensional signals. (a) Successful coordination, both agents give the Lagash greeting (△). (b) Successful coordination, both agents give the Umma greeting (□). (c) Agents fail to coordinate. The Ummian (□) only attends the first dimension and, consequently, does not see any social signal; so she gives the Akkadian greeting. Likewise, the Akkadian (✲) only attends the second dimension and, consequently, does not see any social signal; so, she gives the Umma greeting. (d) Successful coordination, both agents give the Lagash (△) greeting.

4.3 The Single Embedding Topology and the Benefit of Multiple Dimensions

Recall that the Girshites are the zealots of the broader Lagashite community, the Knights Templar of the Catholics, the rioters at the world cup. This section shows how coordination preferences can lead to agents broadcasting social signals and coordinating on actions such that the group structure shows that the Girshites are embedded in the broader Lagashite identity.

We show this in a system that allows agents one signal in each of two dimensions and the following coordination preferences:

Coordination Preferences	Lagash greeting	Girsu greeting	Akkadian greeting
Lagashites ($\triangle$)	1	0	0.5
Girshites ($\diamondsuit$)	1	$1+\alpha$	0.5
Akkadians (✲)	0	0	1

These coordination preferences reflect Girshites sharing with Lagashites the preference for the Lagash greeting over the Akkadian greeting. But additionally, Girshites have a greeting unique to themselves that they most prefer. This might reflect an dialect or accent that is common to the Lagashites generally but is most pronounced among the Girshites. So, the Akkadian greeting might be analogous to speaking in the Akkadian language, the Lagash greeting analogous to speaking the Lagash dialect of Sumerian, and the Girshite greeting being analogous to speaking the Lagash dialect with a heavy Girshite accent. Another story we could tell would be one where the greetings are analogous to how a business deal is sealed. Suppose two merchants are entering a long term agreement to exchange tin for copper at a fixed rate. The Akkadian greeting might be analogous to sealing the deal by going to an Akkadian priestess of Ishtar to bless the agreement. The Lagashite greeting might be analogous to the merchants having their trade agreement blessed by a priest of Ningirsu, and the Girshite greeting analogous to spitting in their hands in shaking while being blessed by a priest of Ningirsu. There are a plethora of different social contexts in which we coordinate our actions and the abstraction of different "greetings" should be very familiar by now. The point of the coordination preferences is that while two interacting agents only get a payoff if they both do the same action, they my differ in their preferences about which action they coordinate on.

Now in many contexts it will be tempting to think that you can identify a group structure simply by observing the labels, social signals, ethnic markers, etc. that a population employs. For example, if all the people who have a tattoo of Ningirsu on their shoulder also wear their robes in the Lagashite style, then you might want to say that the Ningirsu tattoo group is embedded in the Lagashite group. Or, as in Figure 14 (a), if all of the people who signal ${\color{blue}\bullet}$ also signal ${\color{orange}\bullet}$, then you might want to say that the ${\color{blue}\bullet}$ group is embedded in the ${\color{orange}\bullet}$ group. However, in Section 4.4, we show that it is a mistake to identify a group structure using only social signals. Therefore, we distinguish between the single embedding signaling system, 14 (a), and the single embedding topology, 14 (b). The single embedding topology is charcterized by both the signaling system occuring and agents acting in a way that coheres with the embedding. In this case, when Girshites interact with Lagashites, the behave (i.e. give the Lagash greeting) like all of the other Lagashites. It is only among themselves that Girshites behave differently than the other Lagashites.

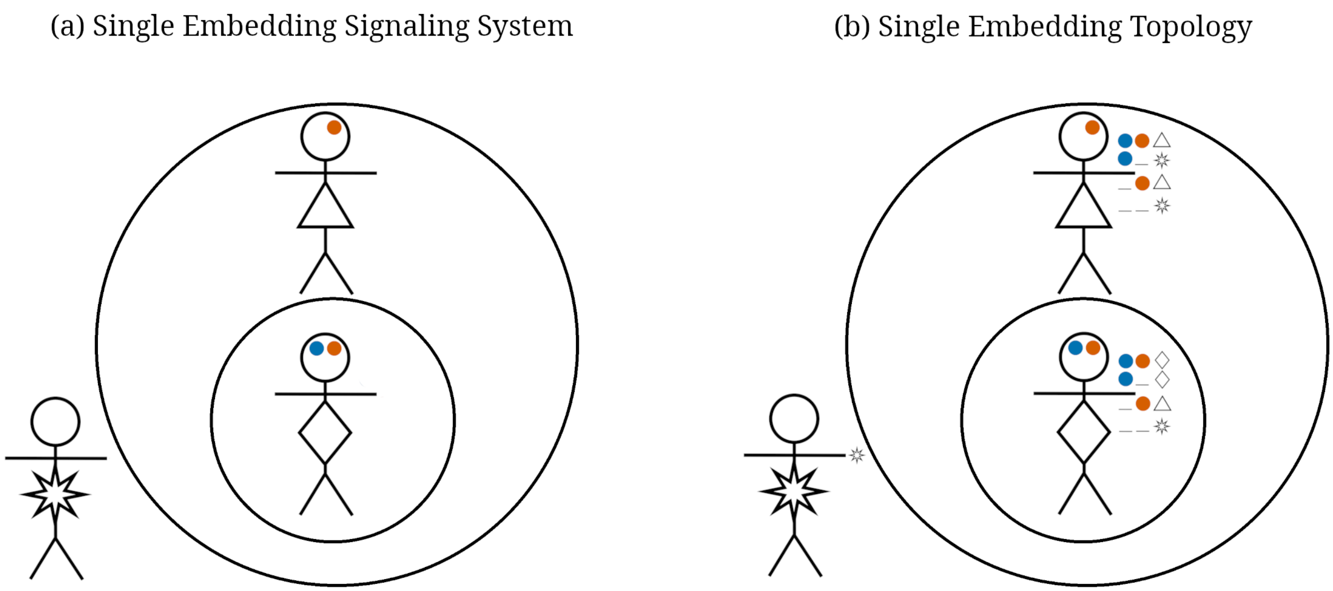


Figure 14: (a) The single embedding signaling system occurs when all agents who broadcast one social signal, say ●, also broadcast a second social signal, say ●. This makes broadcasters of the first social signal a subset of broadcasters of the second signal; in this case all ● signalers are also ● signalers. (b) The single embedding topology occurs when both the signaling system occurs and the embeded group acts like the encompassing group when interacting with the encompasing group while acting distinctly among themselves. In this case, the embedded group, Girshites (♢), coordinates with Lagashites (△) giving the Lagash greeting, but gives a distinct greeting among themselves.

Now this set of dispositions, the single embedding topology, is optimal behavior given agents' preferences. With the dispositions depicted in Figure 14 (b), agents always successfully coordinate on the action that, among those actions which both agents have nonzero payoff for, is the action that collectively provides the most payoff for both actions. For example, when a Lagashite and Girshite interact, they could both get a nonzero payoff from the Akkadian greeting (✵), but the sum of their two payoffs is higher for coordinating on the Lagash greeting (△) than on the Akkadian greeting (✵). It is important to restrict this assesment of what is optimal behavior to coordination on actions for which both agents in an interaction have nonzero payoff because these are the only types of coordination we can reasonably expect agents to arrive at. If an Akkadian merchant this spitting in your hand and shaking is a meaningless nonbinding action (i.e. has zero payoff for the Lagash greeting), then we shouldn't ever expect an Akkadian merchant to use that method of sealing a deal. So, given the above coordination preferences, the optimal action for a Girshite interaction with an Akkadian is coordinating on the Akkadian greeting, even if the sum of the payoffs for successfully coordinating on the Girshite greeting is higher, since Akkadians get no payoff at all from the Girshite greeting. Finally, notice that we can entirely describe the optimal actions for coordination between different types of agents without referencing they type of social signals that they use.

It is entirely possible for agents to produce this optimal behavior without making use of multiple dimensions of social signals. Figure 15 compares simulation outcomes for the given coordination preferences with agents being allowed two dimensions of social signals on the left and allowed only a single dimension of social signals on the right. Here we see that for a variety of parameters, agents using two dimensions of social signals are always more likely to produce optimal behaviors than agents who only have access to a single dimension of social signals. The stick figures above the heatmaps show the dispositions of each type of agent in the optimal outcome (up to isomorphism). Note that, under these parameters, when allowed two dimensions of social signals with just one signal in each dimension, the optimal behavior always co-occurred with the single embedding signaling system. So we do not separately report the prevalence of the single embedding topology in our simulations. Simulation results show that, just as in prior simulations, social signaling is increasingly incentivized in a group as it becomes a smaller proportion of the population.

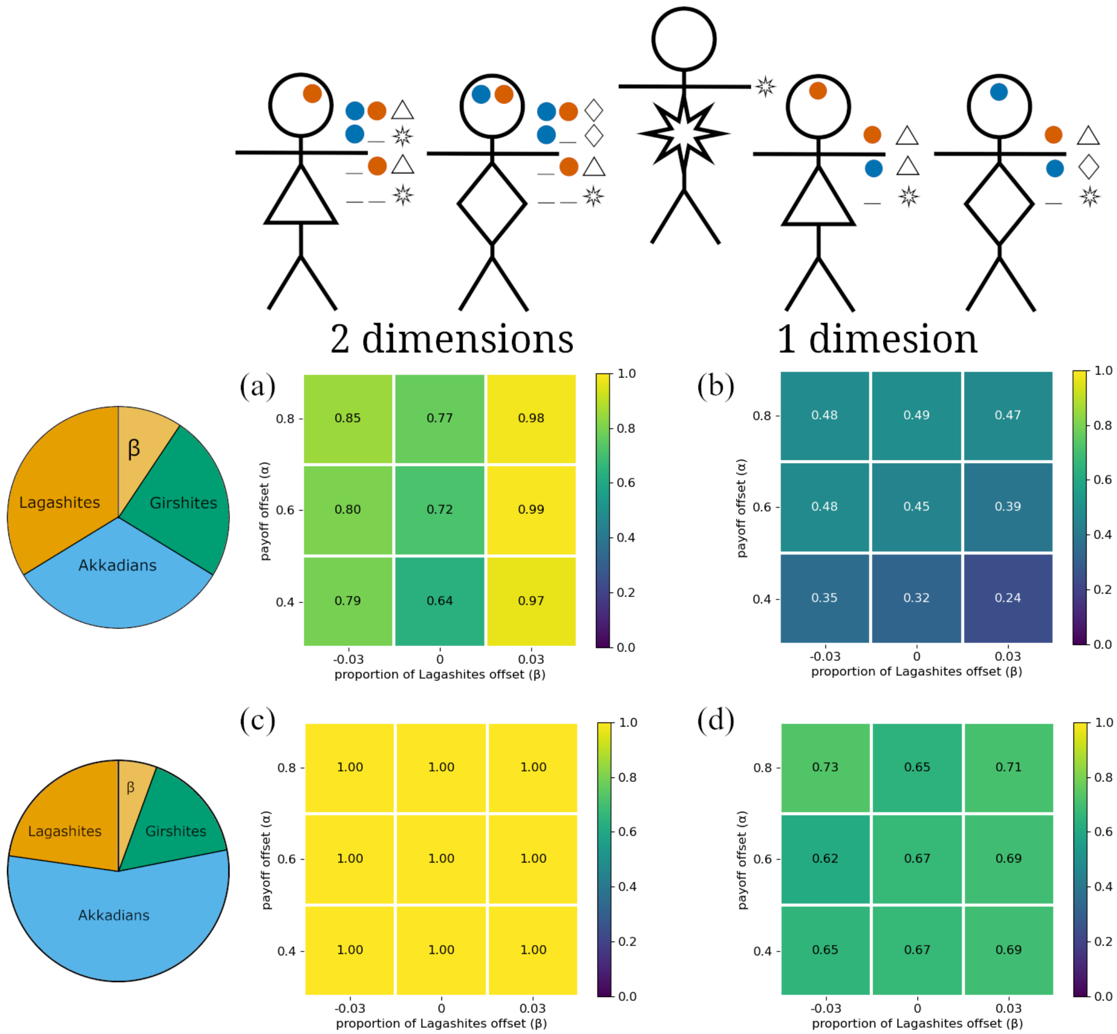


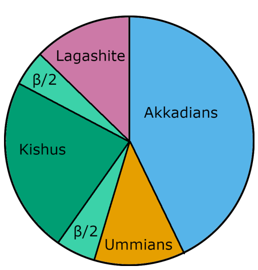
Figure 15: This figure shows how frequently the optimal outcome, relative to agents' preferences, occurred. An outcome is optimal relative to agents' preferences when Lagashites (△) and Girshites (♢) give the Akkadian (✵) greeting with Akkadians, the Lagash greeting with each other, and the Girshites give the Girshite greeting among themselves. **(a) & (c)** show simulation results for agents who have access to signals in two dimensions, once signal in each dimension. **(b) & (d)** show results of simulations for agents who only have access to a single dimension of social signals, but two signals in that dimension. **(a) & (b)** show results of simulations where Lagashites (△) make up $0.33 + \beta$ proportion of the population, Girshites (♢) are a $0.33 - \beta$ proportion of the population, and Akkadians (✵) are a 0.34 proportion of the population. **(c) & (d)** show results of simulations where Lagahites make up $0.2 + \beta$ proportion of the population, Girshites (♢) are a $0.2 - \beta$ proportion of the population, and Akkadians (✵) are a 0.6 proportion of the population.

The key takeaway from this section is that allowing agents multiple dimensions of social signals makes it much easier for to find optimal methods of coordination. This coheres with our prior intuitions. If you are trying to find other Yankees fans, it is easier to have a single symbol associated with that group rather than many different signals that mean Jewish and a Yankees fan, Christian and a Yankees fan, Muslim and a Yankees fan, Hindu and a Yankees fan, Buddhist and a Yankees fan, and so on. This is exactly what is happening in our model. In the simulations with two dimensions of social signals, Lagashites (△) only have to learn to give the Lagash greeting when they see ● because the Girshites (◇) use that same social signal to indicate that aspect of their identity. In the optimal outcome for simulations with a single dimension of social signals, Lagashites (△) have to learn to give the Lagash greeting in response to both the ● signal and the ●. This is because the Girshites (◇) are using the ● signal to coordinate on the Lagash greeting with Lagashites and the Girsu greeting with Girshites. If the Girshites were to adopt the same social signal as the Lagashites, then optimal behavior would be impossible since they would have no way of knowing with they were interacting with someone who most prefers the Lagash greeting or someone who most prefers the Girsu greeting. While the formal model is made more complex with multiple dimensions of social signals, life in the real world is made much simpler.

4.4 The Intersectional Topology and the Importance of Actions

In the previous section, the single embedding signaling system and topology always co-occurred. However, that does not need to be the case for all group structures. In this section we show the importance of looking beyond mere social signals, labels, or ethnic markers for identifying group structures. Specifically we show two very different topologies can be associated with the same signaling system.

To accomplish this, we are going to explore the Kishus (⌂) social identity relative to the Ummians (□) and Lagashites (△). We will again look at simulations in which agents are afforded two dimensions of social signals with one signal in each dimension. However, we arrange their coordination preferences in a very specific way:



Coordination Preferences	Umma greeting	Kish greeting	Lagash greeting	Akkadian greeting
Ummians (□)	1	0	0	0.5
Kishus (⌂)	1	α	1	0.5
Lagashites (△)	0	0	1	0.5
Akkadians (✵)	0	0	0	1

Recall that Ummians and Lagashites have no respect for each other. We see this reflected in their having zero payoff for each other's preferred greeting. The Kishus on the other hand, do get a payoff from both the Umma and Lagash greetings. Importantly for this section, the coordination preferences for Kishus' payoff for the Kish greeting is α not $1 + \alpha$. This is because we will consider $\alpha \in \{0, 1, 2\}$. When $\alpha = 0$ this reflects Kishus having no preference for any greeting over and above the conjunction of those used by the Ummians and Lagashites. When $\alpha = 1$, there is another greeting that Kishus get some payoff for, but it doesn't exceed their payoff for either the Umma greeting or the Lagash greeting. When $\alpha = 2$, not only do Kishus get some payoff from the Kish greeting, the payoff is significantly more than what they get for successfully coordinating on either the Umma or Lagash greeting.

The point of exploring this sort of difference is that there is a substantive difference in identity between Kishus who have no greeting unique to themseves that they prefer (i.e. $\alpha = 0$) and Kishus who do strongly prefer their own greeting (i.e. $\alpha = 2$). Consider our hypothetical story of merchants having their trade agreement made official by a blessing from a priest. For $\alpha = 0$, this is like the Kishu merchants taking an agreement blessed by a Lagashite priest to be just as binding as one blessed by an Ummian priest, but having no special priests of their own that they most prefer. For $\alpha = 2$, this is like the Kishu merchants being willing to make an agreement blessed by a Lagashite or Ummian priest, but taking agreements blessed by a Kishu priest most seriously. Or in the real world, for $\alpha = 0$, this might be like a people who are happy to speak either Fon or French and have no particular language of their own. For $\alpha = 2$, this might be like those people being happy to speak Fon or French, but having a stronger preference for their own language which uses French vocabulary with Fon grammar (i.e. Haitian Creole). The point being that there is a difference between people who are merely members of two groups and people who are a group in their own right but signal that identity with the markers of two groups that they are members of.

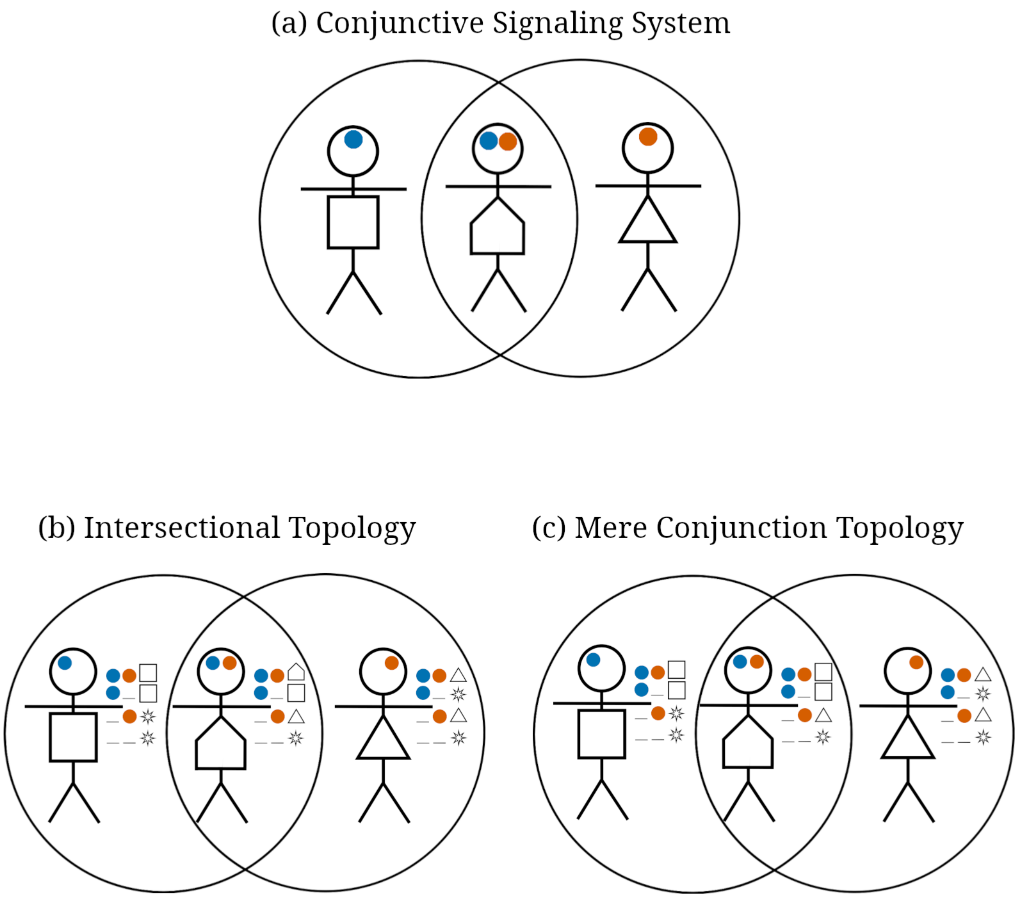


Figure 16: The same signaling system (a) can be used in two distinct topologies (b) & (c).

Figure 16 makes explicit the difference in behavior that we would identify with these two types of identity. In all scenarios depicted in the figure, agents exhibit the conjunctive signaling system whith Ummians signaling ●, Lagashites signaling ●, and Kishus signaling ●●. However, when Kishus have a unique action that they coordinate on among themseves, we call this the intersectional topology. When, among themselves, Kishus simply adopt one of the greetings from the two groups that they identify with, Ummians or Lagashites, then we call this the mere conjunction topology. At this point, the reference to intersectionality should be clear. When the Combahee River Collective formed, it was because there were issues faced by black women that were not being addressed by mainstream feminism nor by Black civil rights movements. People who were members of the two groups wanted to coordinate in a way that was not reducible to the conjunction of the opportunities for coordination afforded them by either group in isolation. This is clearly a different type of group structure than, for example, a group of people who are both rock climbers and sociologist, who may coordinate on activities related to either group but do not have any additional opportunities for coordination afforded them in virtue of being members of both groups.

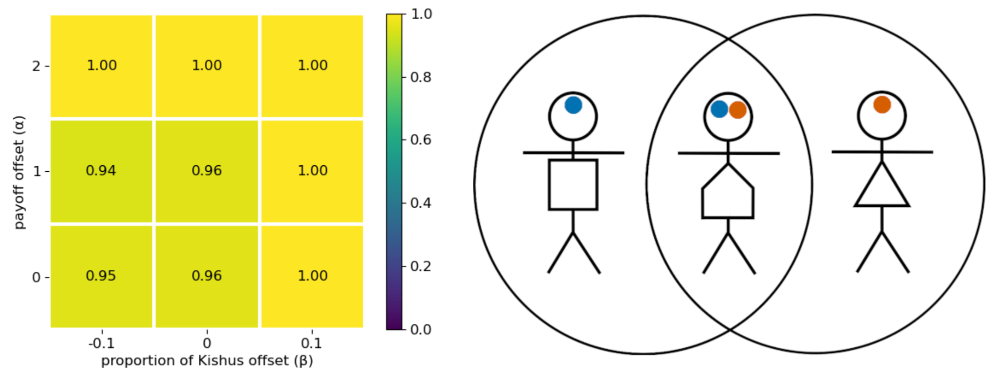


Figure 17: Proportion of simulations that resulted in the conjunctive signaling system.

Now, as Figure 17 shows, under our simulation parameters, the conjunctive signaling system almost always obtained. But we should expect the intersectional topology to occur when $\alpha = 2$ and the mere conjunction topology to occur when $\alpha = 0$. Indeed, this is what we see mostly born out in Figure 18. The intersectional topology never occurs when $\alpha = 0$ and is the most prevalent outcome when $\alpha = 2$. But we do still see some simulations resulting in the mere conjunction topology when $\alpha = 2$. This highlights our first key takeaway from this section.

There are limits to how much decreasing the proportion of the population that is a minority group can promote signaling and optimal outcomes. When $\alpha = 2$ the intersectional topology is the optimal outcome. Kishus should want to coordinate on the Kish greeting when they interact with other Kishus. But as they become a smaller proportion of the population ($\beta = -0.1$), they have fewer opportunities to learn the relevant social signaling and response behaviors to arrive at this optimal outcome. It is only when Kishus are a smaller proportion of the population that the conjunctive signaling system fails to obtain (which is an optimal signaling system for all α values). Additionally, for $\alpha = 2$ and corresponding with having fewer opportunities to interact with other Kishus ($\beta = -0.1$), the probability of Kishus giving the Kish greeting among themselves decreases as they become a smaller proportion of the total population. What this means for empirical research is that we might fail to observe actions indicative of an intersectional identity even if there are people who would have exhibited that identity had they been a larger proportion of the population.

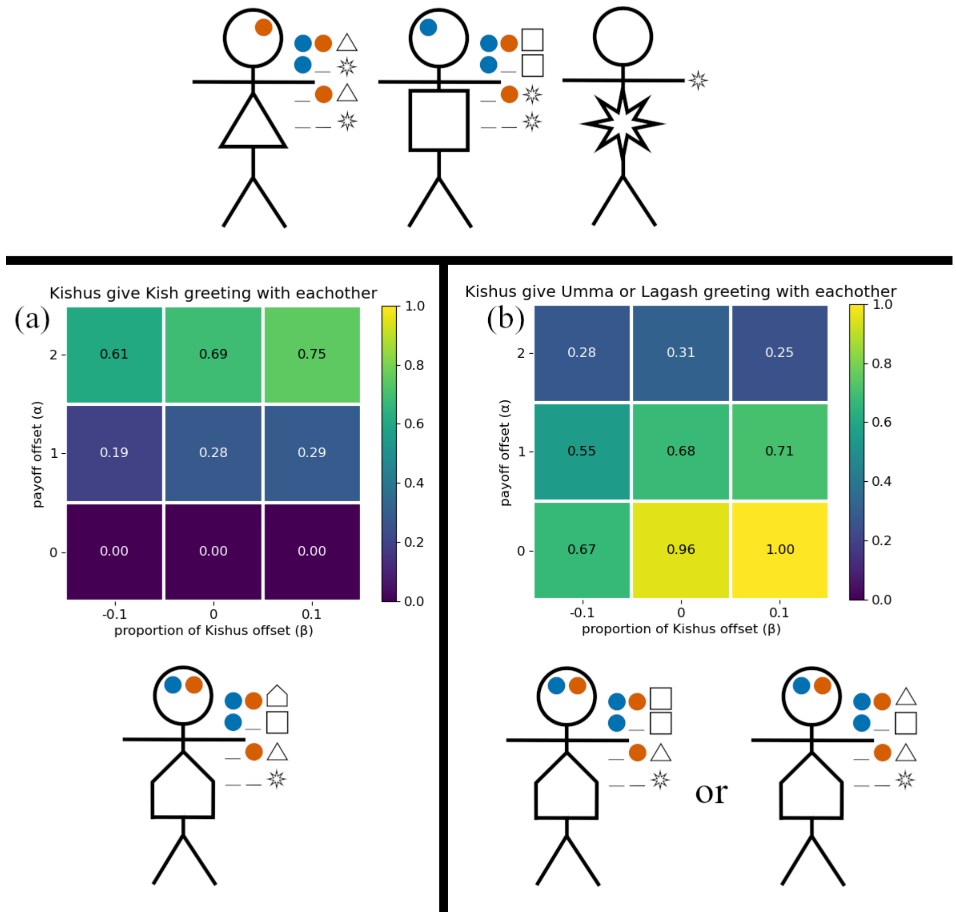


Figure 18: (a) Proportion of outcomes in which Kishus (⌂) give the Kish greeting with eachother in addition to exhibiting the desired signaling behavior. (b) Proportion of outcomes in which Kishus (⌂) give the Umma (□) or Lagash (△) greeting with eachother in addition to exhibiting the desired signaling behavior; i.e. Ummians (□) signal in one dimension, Kishus (⌂) signal in two dimensions, Lagahites (△) signal in one dimension that is different than the Ummian dimension, and Akkadians (✵) do not signal in any dimension.

Finally, most important takeaways from this section are that (1) social signals underdetermine group structures and (2) observing how people act can potentially resolve this underdetermination. We've shown how two different group structures, the mere conjunction and intersectional topologies, can exhibit the same social signaling. But to make the potential for ambiguities exceedingly clear, we can find the same sort of ambiguities in a one dimensional signaling system as depicted in Figure 19. In the one dimensional system, it is people who are merely members of two groups that adopt an intuitively misleading social signal. Imagine a world where all people who were both rock climbers and sociologists weren't given those labels but were instead called "sociorockists". Even if there was no activity they engaged in that was different from what either rock climbers or sociologist did, it would be tempting to think of sociorockists as a unique group. Conversely, even when there have been social issues unique to Black women. Lacking a unitary term for their identity has made it easy to overlook those social issues.

We cannot observe people's internal preferences. But, what we have shown is that the two different types of preferences that lead to the conjunctive signaling system do typically lead to different topologies. In our case, a topology in which Kishus give the Kish greeting among themselves and a topology in which either the Umma or Lagash greeting becomes their norm for coordinating among themselves. So by looking at what ways people are coordinating with each other, we can settle the underdetermination in group structure when faced with conjunctive social signals.

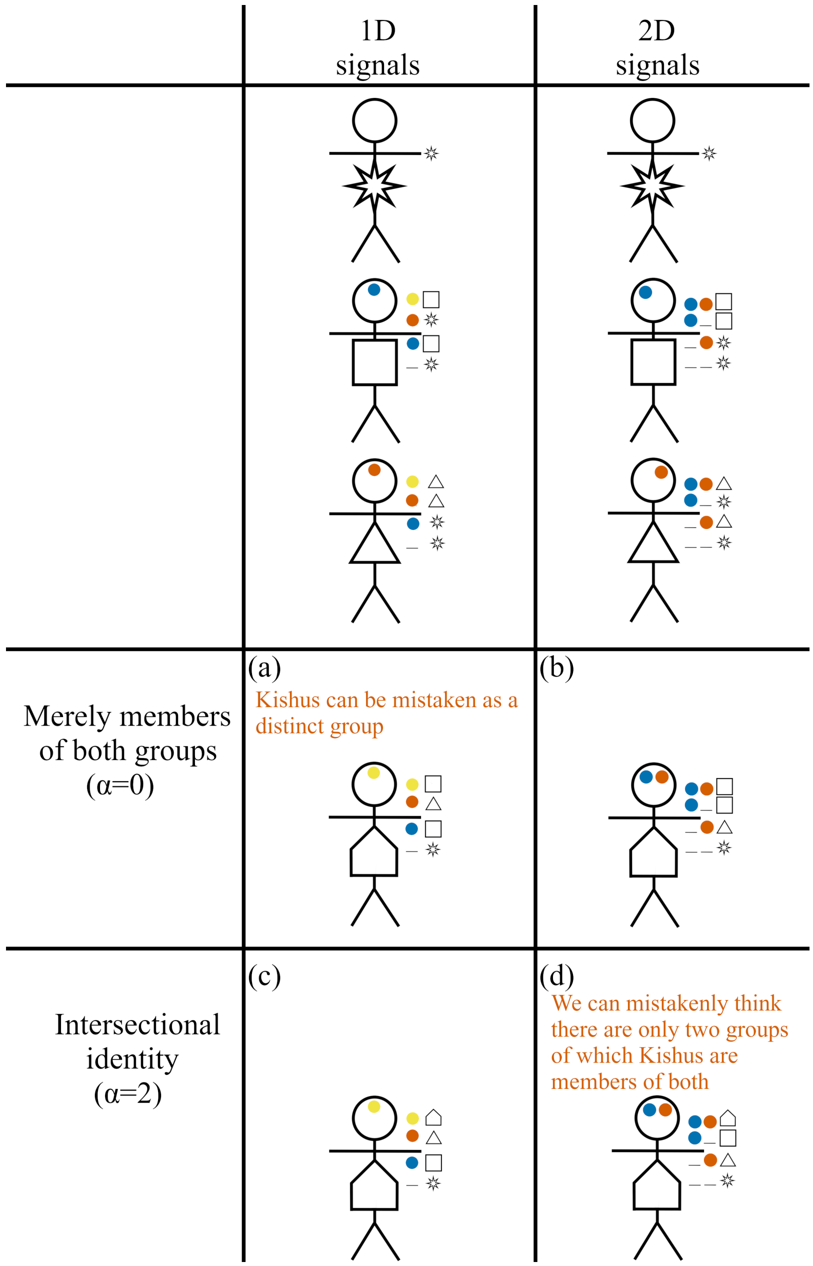
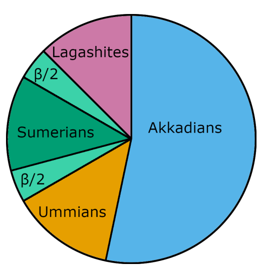


Figure 19: An illustration of two scenarios in which social signals are misleading if agents actions are not also considered.

4.5 Double Embedding Topologies

This section does not thoroghly develop any intuition pumps for the coordinating preferences that produce the double embedding topologies. By now the narratives should be vary familiar along with the relevant relations between the different types of agents. For the disjoint double embedding, there are four types of agents, Sumerians, Ummians, Lagashites, and Akkadians. As previously discussed, Ummians and Lagashites dislike each other and will get no payoff from each other's greeting. But they are still both Sumerian and prefer that over the Akkadian greeting. For the hierarchical double embedding, Ummians are replaced with Girshites who not only like the Lagashite greeting, but are analogous to the zealots of the Lagashite community. As always, the Akkadians are indifferent to the various Sumerian greetings. Subsections 4.5.1 and 4.5.2, simply give the coordination preferences, a diagram of the relevant signaling system and topology, and the simulation results showing the prevalence of the topology given the parameters. After this is done, Subsection 4.5.3 discusses the simulation results and how they compare to each other. Though it is not necessary, when we see these signaling systems occurring we should expect to find the corresponding types of coordination that are exhibited in the relevant topology.

4.5.1 The Disjoint Double Embedding



Coordination Preferences	Sumerian greeting	Umma greeting	Lagash greeting	Akkadian greeting
Sumerians (○)	1	0	0	0.25
Ummians (□)	1	$1+\alpha$	0	0.25
Lagashites (△)	1	0	$1+\alpha$	0.25
Akkadians (✵)	0	0	0	1

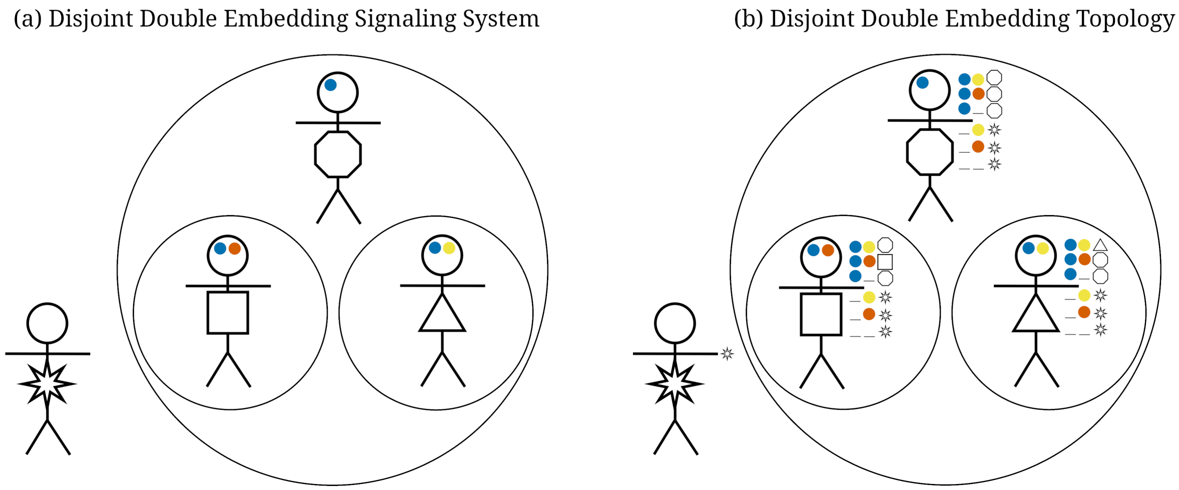


Figure 20: A disjoint double embedding signaling systems and corresponding topology.

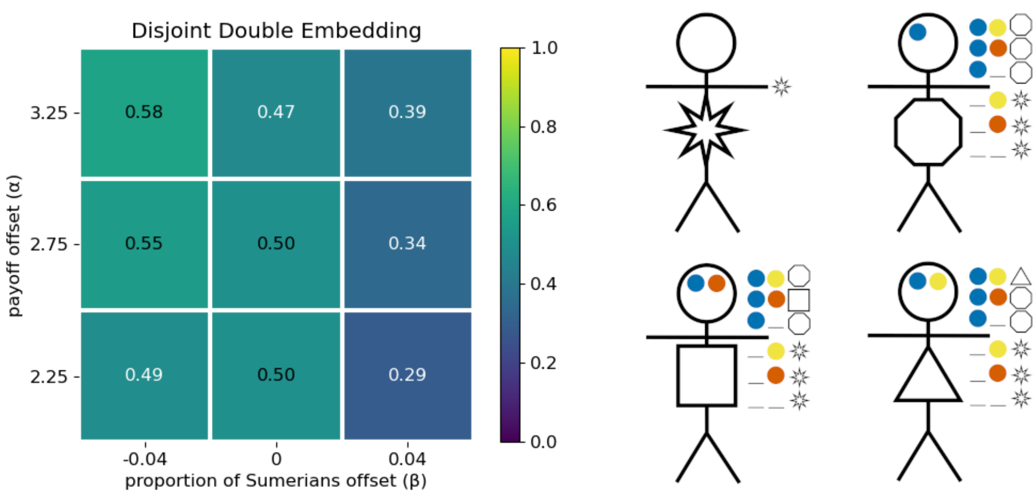
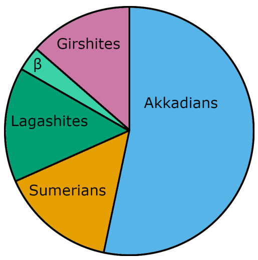


Figure 21: Proportion of outcomes that were the disjoint double embedding topology. Sumerians (⬡) make up $0.15 + \beta$ proportion of the population, Ummians (□) are a $0.15 - 0.5 \times \beta$ proportion of the population, Lagashites (△) are a $0.15 - 0.5 \times \beta$ proportion of the population, and Akkadians (✵) are a 0.55 proportion of the population. For these simulation agents were allowed one signal in the first dimension and two in the second.

4.5.2 The Hierarchical Double Embedding



Coordination Preferences	Sumerian greeting	Lagash greeting	Girsu greeting	Akkadian greeting
Sumerians (⬡)	1	0	0	0.25
Lagashites (△)	1	$1 + 0.5 \times \alpha$	0	0.25
Girshites (♢)	1	$1 + 0.5 \times \alpha$	$1 + \alpha$	0.25
Akkadians (✲)	0	0	0	1

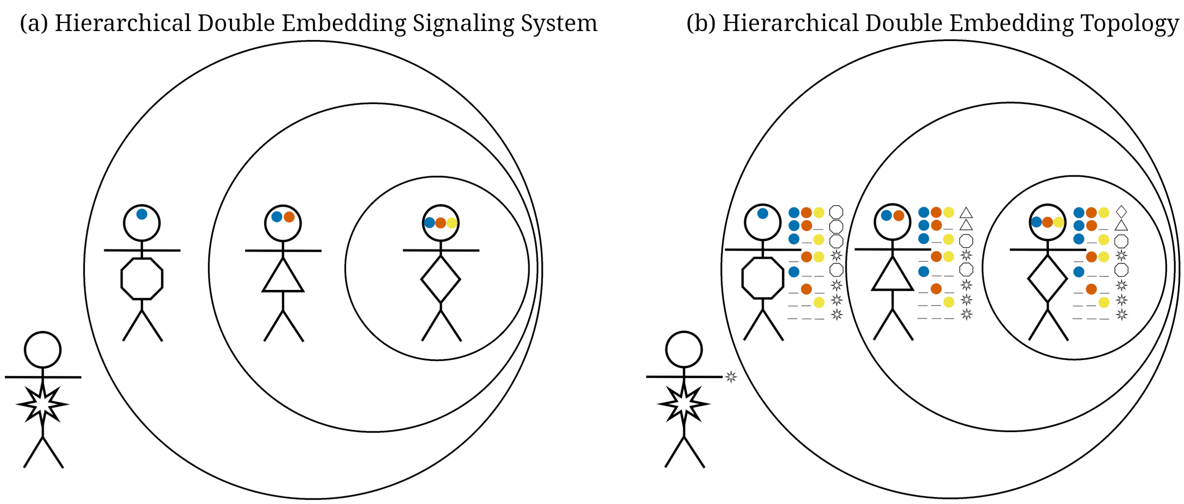


Figure 22: A hierarchical double embedding signaling system and corresponding topology.

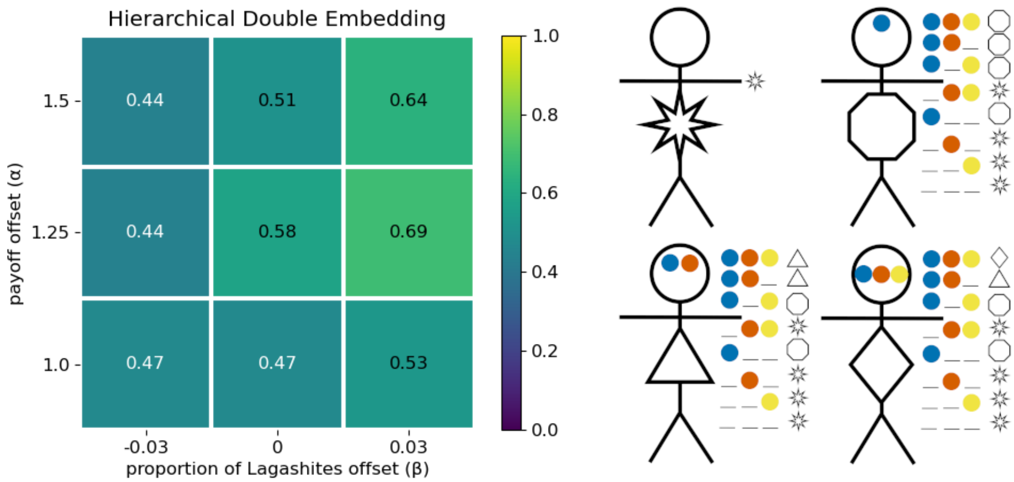


Figure 23: Proportion of outcomes that are the hierarchical double embedding topology. Sumerians (○) make up 0.15 proportion of the population, Lagashites (△) are a $0.15 + \beta$ proportion of the population, Girshites (◇) are a $0.15 - \beta$ proportion of the population, and Akkadians (✵) agents are a 0.55 proportion of the population. For these simulations agents were allowed one signal in each of three dimensions.

4.5.3 General Comparison of Double Embeddings

The key difference in what produces a disjoint double embedding from a hierarchical double embedding is agents' coordination preferences. Hierarchical embeddings are produced by the embedded groups sharing nested preferences with the groups they are embedded in. Disjoint embeddings are produced by the disjoint groups only having a common preference with the larger containing group. Combined with the results from the previous section on intersectionality, we have a good foundation for establishing a general taxonomy of group structures, where group structures are determined by both social signals and actions. Hierarchical groups are characterized by nested levels of coordinated behavior, while disjoint groups do not have mutually exclusive signals and do not coordinate with each other. When there is a population that exhibits conjunctive social signals we can differentiate them by the ways in which they act.

This seems to exhaust the obvious pairwise relationships that groups can have with each other, and we expect these general themes to persist as the complexity of a population grows. For example, both the single embedding and hierarchical double embedding exhibit nested norms for coordination with embedded groups sharing in the social signals of the group they are embedded in. As noted in Section 4.3, this is beneficial for optimizing behavior because members of the containing group do not need to learn how to respond to social signals that they are not already using. A containing group can essentially act as if the embedded group does not exist because they do not have to condition their actions on any signals unique to the embedded group. So were we to simulate a population with five different types of agents with nested preferences, we would expect the agents to, just as in the single embedding and hierarchical double embedding, gravitate towards social signals such that each successive containing group ignores the social signals unique to any groups embedded within them. We can furthermore expect this sort of social signaling to have evolved to facilitate the embedded groups adopting the norms of the group they are embedded in when interacting with members of the larger group.

Thus the model provides some guidance for what sort of empirical questions might be fruitful for better understanding group structures. It is certainly a logical fallacy to assume that if people are using a hierarchical social signaling system then they must also be coordinating their behavior in an analogous way. But our model does suggest that if a hierarchical social signaling system is observed, then a good next step is to check whether the corresponding coordination behavior can be observed. Likewise, it will make sense to anticipate and look for an absence of coordinating behavior between people using mutually exclusive social signals.

5 Discussion

To review what has been shown, Section 2.3 considered the Bach or Stravinsky game when generalized to a large population of two types of agents who repeatedly interact with each other rather than being confined to a one off game with a population of two. Simulations showed a clear advantage for the larger of the two types of agents; when both types of agents played the same greeting it was always the greeting preferred by the larger group. This advantage for larger groups persisted throughout subsequent additions to the model. Section 3.2 then looked at simulations that added signals and assortment to the model. Both with and without assortment, agents used social signals to better coordinate behavior. In Section 3.4 looked at the model with the addition of attention and signal costs. Simulations showed that a dominate group could ignore social signals and always use their preferred greeting. They also showed that agents of minority types have increased incentive to broadcast social signals when there is a large dominant group in the population. Section 4 illustrated a few key points with multidimensional signals. First, that there is a signifcant benefit to using multiple dimensions of social signals. It is much easier to find optimal ways of coordinating our actions. Second, social signals underdetermin group structure. But, third, the underdetermination in group structure can be resolved by looking at how people coordinate their actions. Finally, we highlighted some general patterns in what sort of preferences lead to different types of social signaling systems and corresponding topologies.

Our model yields findings that are broadly consistent with prior modeling and empirical research. Barth (1998) highlighted the importance of ethnic markers at the boundaries of cultures when preferences are strong. While our model does not include spatial boundaries, the fact that signaling is particularly incentivzed in the presence of large majority groups seems consistent with the general themes of Barth's theory. We also noted that an early model of Barth's theory, McElreath et al. (2003), yielded similar results to ours in that people with only weak preferences failed to evolve behavioral differences in the base model. We also saw, in the base model, that minority groups often end up abandoning their preferences for the norms of a majority group, consistent with Bunce and McElreath (2018); O'Connor (2019). Adding in social signals allowed a type of cross-cultural competence which facilitated minority cultures being able to maintain their norms, cohering with Bunce (2021)'s modeling results. Finally, in keeping with straightforward empirical observations (Simoni and Walters, 2001; Devos and Banaji, 2005), we saw that the attention dynamics allowed majority groups to ignore social signals and act as if their norms were the norms of the entire population. While empirically it seems clear that people have complex multidimensional identities, we are unaware of any prior agent based models that attempt to capture this. So we believe we have substantively added to the literature.

There are at least two key directions that we believe are worth pursuing in future research. Our model captures the three step process of (i) differences in preferences (ii) contributing to the evolution of social signals such that (iii) people can optimally condition their actions based on identifying ingroup and outgroup members through their social signals. But our model does not capture how people come to have different preferences in the first place. So, one key direction we would like to take future research is incorporating dynamics that drive the divergence of preferences into the model. A simple starting point for this might simply be capturing this sort of divergence through drift when a population is split apart by a geographic barrier; e.g. when some members of a population migrate North and others South, their accents can naturally drift apart over time, particularly when there is limited contact between members of the two migrations. The second future direction we see for research is expanding the multidimensional signaling component of the model to contexts beyond what is captured by the Bach or Stravinsky game. For example, Bednar and Page (2007) have advocating the importance of exploring contexts in which defecting from a cooperative strategy is incentivized and we may want to explore contexts in which such defection are conditioned on complex group memberships. While we have yet to explore these future research directions, we believe we have laid a solid foundation for these and other possible continuations of modeling social identities.

References

Barth, F. (1998). *Ethnic groups and boundaries: The social organization of culture difference*. Waveland Press.

Bednar, J. and Page, S. (2007). Can game(s) theory explain culture?: The emergence of cultural behavior within multiple games. *Rationality and Society*, 19(1):65–97.

Berger, J. and Heath, C. (2008). Who drives divergence? identity signaling, outgroup dissimilarity, and the abandonment of cultural tastes. *Journal of personality and social psychology*, 95(3):593.

Bunce, J. A. (2020). Field evidence for two paths to cross-cultural competence: implications for cultural dynamics. *Evolutionary Human Sciences*, 2:e3.

Bunce, J. A. (2021). Cultural diversity in unequal societies sustained through cross-cultural competence and identity valuation. *Humanities and Social Sciences Communications*, 8(238):1–9.

Bunce, J. A. and McElreath, R. (2018). Sustainability of minority culture when inter-ethnic interaction is profitable. *Nature Human Behaviour*, 2(3):205–212.

Combahee River Collective (1977). The combahee river collective statement.

Crenshaw, K. (1991). Mapping the margins: Intersectionality, identity politics, and violence against women of color. *Stanford Law Review*, 43(6):1241–1299.

Delnero, P. (2016). Literature and identity in mesopotamia during the old babylonian period. In Ryholt, K. and Barjamovic, G., editors, *Problems of canonicity and identity formation in ancient Egypt and Mesopotamia*, pages 19–50. Museum Tusculanum Press, Copenhagen, Denmark.

Devos, T. and Banaji, M. R. (2005). American = white? *Journal of personality and social psychology*, 88 3:447–66.

Fulker, Z., Forber, P., Smead, R., and Riedl, C. (2024). Spontaneous emergence of groups and signaling diversity in dynamic networks. *Phys. Rev. E*, 109:014309.

Goodman, J. R., Caines, A., and Foley, R. A. (2023). Shibboleth: An agent-based model of signalling mimicry. *PLoS ONE*, 18(7).

Lin, W., Kang, S., Zhu, J., and Ding, L. (2024). Till we have red faces: drinking to signal trustworthiness in contemporary china. *Public Choice*, pages 1–20.

Luce, R. D. and Raiffa, H. (1957). *Games and Decisions: Introduction and Critical Survey*. Wiley, New York.

Macanovic, A., Tsvetkova, M., Przepiorka, W., and Buskens, V. (2024). Signals of belonging: emergence of signalling norms as facilitators of trust and parochial cooperation. *Philosophical Transactions B*, 379(20230029).

McCluney, C. L., Durkee, M. I., Smith II, R. E., Robotham, K. J., and Lee, S. S.-L. (2021). To be, or not to be... black: The effects of racial codeswitching on perceived professionalism in the workplace. *Journal of experimental social psychology*, 97:104199.

McElreath, R., Boyd, R., and Richerson, P. J. (2003). Shared norms and the evolution of ethnic markers. *Current Anthropology*, 44(1):122–130.

Moffett, M. W. (2013). Human identity and the evolution of societies. *Human Nature*, 68:219–267.

O'Connor, C. (2019). *The Origins of Unfairness: Social Categories and Cultural Evolution*. Oxford University Press.

Pelzer, K. (2023). What does an upside-down pineapple mean? the hidden message behind the symbol. *Parade*.

Roccas, S. and Brewer, M. B. (2002). Social identity complexity. *Personality and Social Psychology Review*, 6(2):88–106.

Simoni, J. M. and Walters, K. L. (2001). Heterosexual identity and heterosexism. *Journal of Homosexuality*, 41(1):157–172.

Smaldino, P. E., Flamson, T. J., and McElreath, R. (2018). The evolution of covert signaling. *Scientific Reports*, 8(4905).

Smaldino, P. E. and Turner, M. A. (2020). Covert signaling is an adaptive communication strategy in diverse populations. *Center for Open Science*.

Steinert-Threlkeld, S. (2016). Compositional signaling in a complex world. *Journal of Logic, Language and Information*, 25(3-4):379–397.

Appendix A Assortment Dynamics

Additionally, a homophily parameter, h, can be included to model contexts in which people interact more with those who broadcast the same social signals as themselves. Intuitively, the homophily parameter captures random assortment when $h = 0$ and assortment where agents who broadcast the same signal are twice as likely to interact as agents who broadcast differing signals when $h = 1$. Mathematically, this is captured by using the following utility function:

$$
\begin{aligned}
U(x) &= \sum_{i \in Y} [H(h, x, i) \times M(i) \times p_{xi}] && \text{if } x \neq i \\
U(x) &= \sum_{i \in Y} [H(h, x, i) \times (M(i) - 1) \times p_{xi}] && \text{if } x = i
\end{aligned}
$$

where $H(h, x, i)$ is defined as:

$$H(h, x, i) = \frac{N}{\sum_{j \in Y} [S(h, x, j)]} \times S(h, x, i)$$

where N is the total number of agents in the population and $S(h, x, j)$ is:

$$
\begin{aligned}
S(h, x, j) = 2^h \quad & \text{if profile x entails broadcasting the same social signal as j} \\
S(h, x, j) = 1 \quad & \text{if x does not entail broadcasting the same social signal as j}
\end{aligned}
$$

It is easy to see that this amounts to the same utility function as before when $h = 0$. Likewise, it is easy to see how $S(h, x, i)$ reflects agents with profile x having twice as much utility (when $h = 1$) from interactions with agents with profile i, due to more frequent interactions, when profiles x and i entail broadcasting the same social signal. The component of the equation requiring some explanation is $\frac{N}{\sum_{j \in Y} [S(h,x,j)]}$. This factor normalizes $S(h, x, i)$ relative to the signaling dispositions of the entire population.

To see why this normalization is necessary, consider counterfactually what would happen if we just defined $H(h, x, i)$ as equal to $S(h, x, i)$. Suppose further that for a population with the preferences of Table 5, every agent is broadcasting the same signal and playing their preferred action irrespective of the signal observed. If each type makes up half of the population, then agents are successfully coordinating in about half of their interactions. The signal being broadcast is meaningless for assortment since everyone is broadcasting the same signal. But suppose one agent mutates and starts broadcasting a different signal. Given the uniformity of everyone else in the population, the mutated agent is still just as likely to interact with a Bachite agent as a Stravinskian agent. Consequently, it will still be the cast that about half of the agent's interactions will result in successful coordination. However, with our counterfactual condition that $H(h, x, i)$ as equal to $S(h, x, i)$. The mutated agent's utility would suddenly be cut in half even though the agent is just as likely to successfully coordinate on her preferred action as before. But this makes no sense. By including the normalization factor $\frac{N}{\sum_{j\in Y}[S(h,x,j)]}$ in our definition of $H(h,x,i)$, we ensure that agents only see an increase in utility from broadcasting a particular signal in proportion to how much that signal impacts the frequency of their interactions.[11]

A.1 Simulating Assortment for One Dimensional Singals without the Attention Dynamics

Adding in assortment generally increases the probability of optimal outcomes in the model. This subsection shows simulation results for the same parameters as Section 3.2 in the main body of the paper.

Recall that, there are four different outcomes that frequently obtain in this model. In outcome (i) each type plays their preferred action all of the time is possible, but this outcome never occurs under the parameters shown in this section. Outcomes in which (ii) everyone plays Ummians' preferred action, and (iii) everyone plays Kishus' preferred action, are outcomes in which agents fail to make use of the social signals available to them. The outcomes in which (iv) Ummians (□) always play their preferred action and Kishus (⌂) play Umma greeting with Ummians (□) and play the Kish greeting among themselves, and (v) Kishus (⌂) always play their preferred action and Ummians (□) play the Kish greeting with Kishus (⌂) and play the Umma greeting among themselves are optimal outcomes which depend on the agents evolving meaningful signals. Finally, though infrequent, (vi) there are a variety of suboptimal outcomes in which meaningful signals evolve but at least one type does not play their preferred action among themselves.

[11] If one wished to model a scenario in which an agent, in virtue of adopting an unused signal, simply ceased to interact with all agents since no other agents shared that signal, then it still would not make sense to let $H(h,x,i)$ equal $S(h,x,i)$. For that scenario, $S(h,x,i)$ should also be modified to be 0 when profile x does not entail broadcasting the same social signal as i. However, such antisocial behavior seems out of place given that this paper is modeling agents in coordination games where there is some common ground between agents of different types; i.e. an agents who prefers Bach still gets some benefit for coordinating on Stravinsky. In adopting the normalized equation, this paper reflects agents having the same number of interactions regardless of their signal, but preferentially interacting with agents who share their signal.

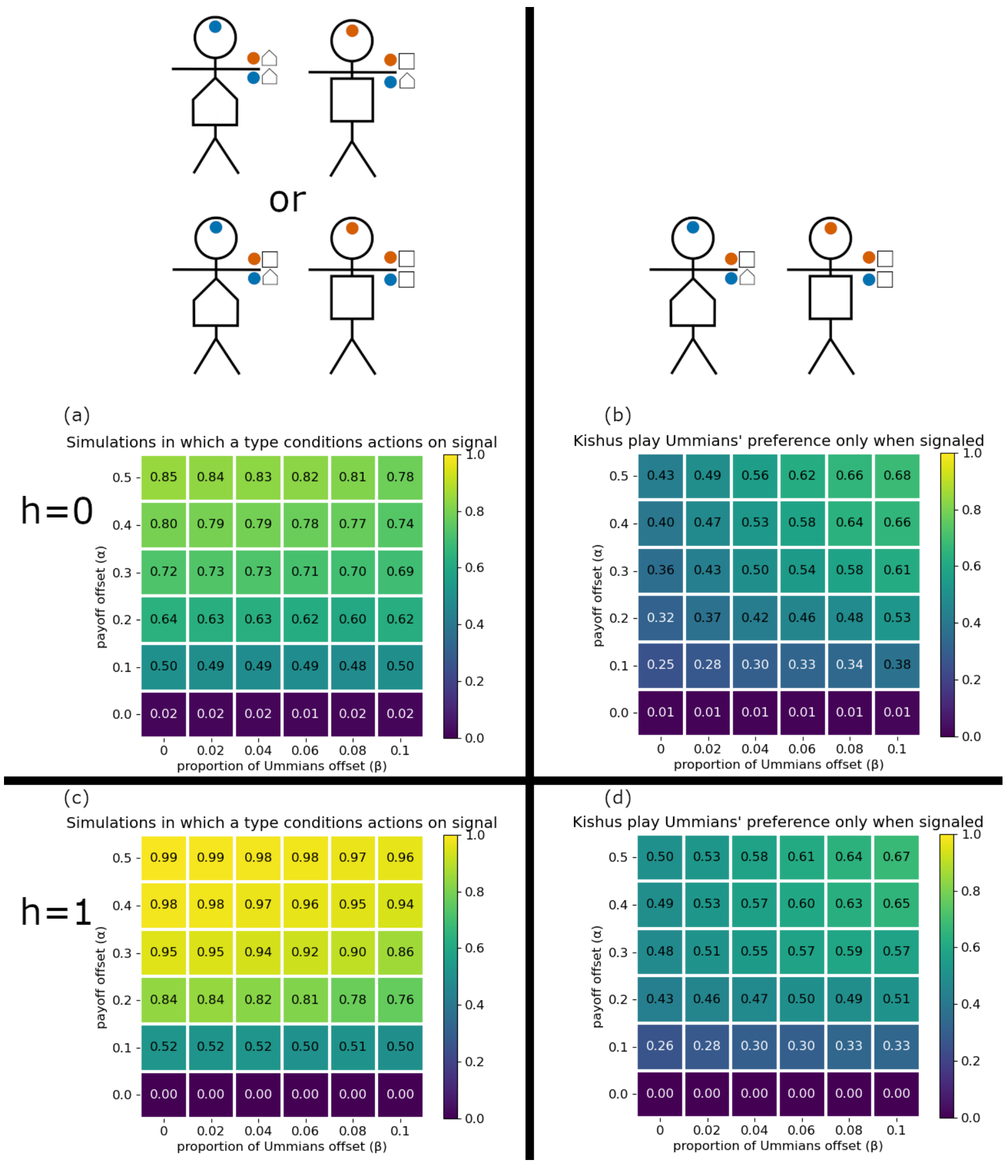


Figure 24: (a) Proportion of outcomes that are (iv) or (v) for $h = 0$. (b) Proportion of outcomes that are (iv) for $h = 0$. (c) Proportion of outcomes that are (iv) or (v) for $h = 1$. (d) Proportion of outcomes that are (iv) for $h = 1$.

A.2 Simulating Assortment for One Dimensional Singals with the Attention Dynamics

At first glance, assortment appears to no longer promote optimal outcomes as the model becomes more complex. We are currently unsure of how to understand this. To some extent, this result seems to reflect it being the case that the assortment dynamics can incentivize signaling for a group that would otherwise avoid signaling (the Akkadians) during early stages of a simulation. When running a simulation with only as many signals as necessary for the optimal outcome to occur (2 signals), this results in agents getting stuck in a local optimum because an agent type that should adopt signals (the Ummians) has no signal available for them to adopt since both of the other agent types are already making use of the two available signals. While allowing agents more signals (Figure 25 c & d) does mitigate the difference between outcomes from simulations with and with out assortment, it is still the case that optimal outcomes are more frequent without assortment (Figure 25 c) than with assortment (Figure 25 d).

Another explanation of could be related to transient diversity. In simulations where agents were afforded 3 signals, runs with no assortment (Figure 25 c) took roughly 20% longer to reach stable equilibria than those with strong assortment (Figure 25 d). We can also see that adding in assortment to the model dynamics does not immediately decrease the likelihood of the optimal outcome obtaining. Figure 25 e shows simulations for weak assortment ($h = 0.5$) producing the optimal outcome within error of the results for the simulations with no assortment (25 c).

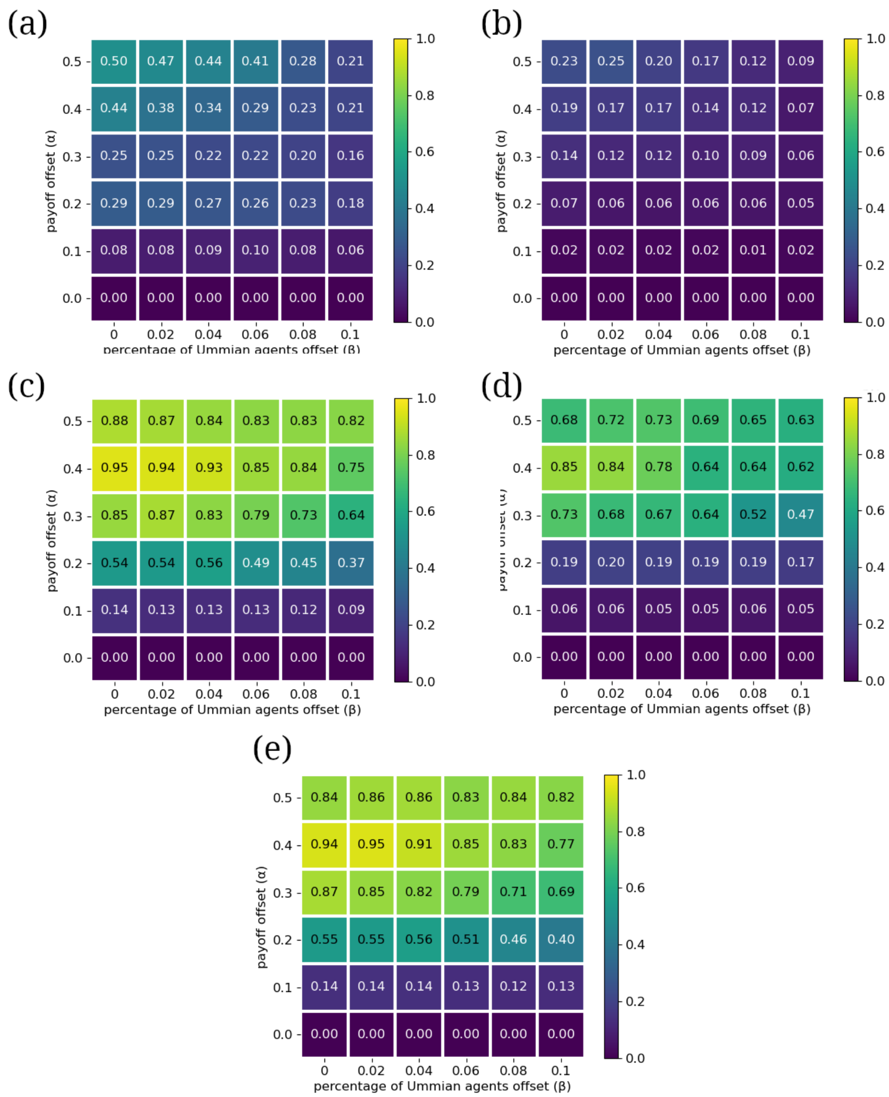


Figure 25: Proportion of outcomes in which agents optimally condition their actions on signals when (a) afforded 2 signals with $h = 0$, (b) afforded 2 signals with $h = 1$, (c) afforded 3 signals with $h = 0$, (d) afforded 3 signals with $h = 1$, and (e) afforded 3 signals with $h = 0.5$.

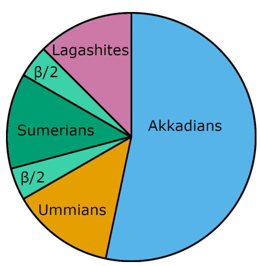
A.3 Difficulties for Simulating Assortment for the Full Model with Multiple Dimensions of Signals

With our current code and the allowance of more signals than is necessary for the optimal outcome, simulating assortment for systems with multiple dimensions of social signals is not computationally tractable. We are working on a future model that uses different learning dynamics and we hope will resolve this issue of computational tractability. For now we have no simulation results to report.

Appendix B The Disjoint Double Embedding

This appendix has a data from a broader range of simulation parameters for producing the disjoint double embedding topology. It's main purpose is to show the rationale behind using stronger mutation values in the learning dynamics for the results that were reported in the main body of the paper. Up until the section on double embeddings, mutation rates were $m_0 = 0.01$ and $m_1 = 0.1$. However the double embedding section shows simulation results for $m_0 = 0.05$ and $m_1 = 0.2$. The short answer explaining this change is just that in the more complex parameter space of the system that produces the disjoint double embedding, having agents explore different possibilities more frequently is very beneficial. At the end of this appendix, we also point out the possibility of a distinct type of disjoint double embedding that could occur in a population. Though the difference between these two types of disjoint double embeddings is narrow.

Recall that for this system, coordination preferences are:



Coordination Preferences	Sumerian greeting	Umma greeting	Lagash greeting	Akkadian greeting
Sumerians (○)	1	0	0	0.25
Ummians (□)	1	$1+\alpha$	0	0.25
Lagashites (△)	1	0	$1+\alpha$	0.25
Akkadians (✲)	0	0	0	1

Sumerians (○) make up $0.15+\beta$ proportion of the population, Ummians (□) are a $0.15-0.5\times\beta$ proportion of the population, Lagashites (△) are a $0.15-0.5\times\beta$ proportion of the population, and Akkadians (✲) are a 0.55 proportion of the population. There is one signal in the first dimension and two in the second.

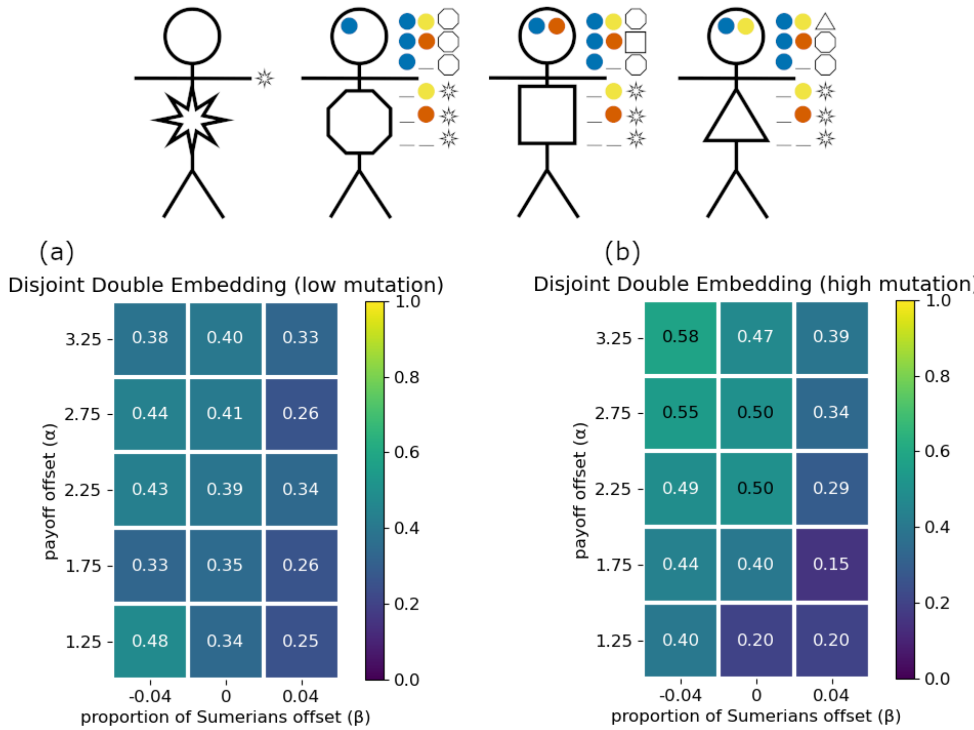


Figure 26: Proportion of outcomes that are the disjoint double embedding topology when (a) the mutation rate is $m_0 = 0.01$ and $m_1 = 0.1$, or (b) the mutation rate is $m_0 = 0.05$ and $m_1 = 0.2$.

Figure 26 shows results using the same mutation rate as prior sections, $m_0 = 0.01$ and $m_1 = 0.1$, as well as results with a higher mutation rate, $m_0 = 0.05$ and $m_1 = 0.2$. In general, it seems learning is more effective with higher mutation rates as the learning context become more complex. This also requires using higher payoffs to cut through the noise of the increased mutation rate. However, it is difficult to make these claims with certainty as these more complex systems are less computationally tractable, which precludes the more robust parameter sweeps that were done with simpler models. Still there is a rationale for why higher mutation rates might be helpful for learning in the more complex systems. The system that produces the single embedding topology in Section 4.3 only has 324 different possible strategy profiles. Thus, we can expect a large portion of the different strategy profiles to be present on the first timestep of a simulation when they are assigned. Mutation is most likely not needed to produce an agent with any given strategy profile. However, the system in this section allows 24,576 different possible strategy profiles. This means that for a population of 1,000 agents, on any given timestep, there is at most around 4% of the different strategy profiles present in the population. Consequently, a higher mutation rate is likely beneficial in virtue of allowing agents to explore a broader swath of the possible strategy profiles.

Just as with the single embedding topology, this section on the disjoint double embedding topology does not illustrate any key point about the model. Rather it is just a further illustration of how the model works. At the beginning of a simulation, as always, agents are randomly assigned social signals to broadcast or not and are randomly assigned greetings to give in response to the social signals. What is not random is the preferences that we assign to the agents. Since Ummians (□) and Lagashites (△) have a preference for coordinating on the Sumerian greeting (○) over the Akkadian greeting they frequently both adopt the social signal that comes to be associated with the Sumerian identity. Their even stronger preference for their own cities' greetings and the fact that they both get no payoff from the other's city greeting produces the incentive for the Ummians and Lagashites to adopt social signals in a second dimension that are distinct from each other and allow them to successfully coordinate on their most preferred greeting among others from their own city. All of this comes together as what we call the disjoint double embedding. The fact that the Ummians and Lagashites had a deep seated rivalry is important to how we've set up this disjoint double embedding topology. Not only is it the justification for Ummians and Lagashites getting no payoff from each other's greetings, but it explains why it is appropriate to allow two signals in the same dimension (which are therefore mutually exclusive) rather than having an additional dimension for each of the signals that the Ummians and Lagashites adopt in the disjoint double embedding topology (see Figure 27).

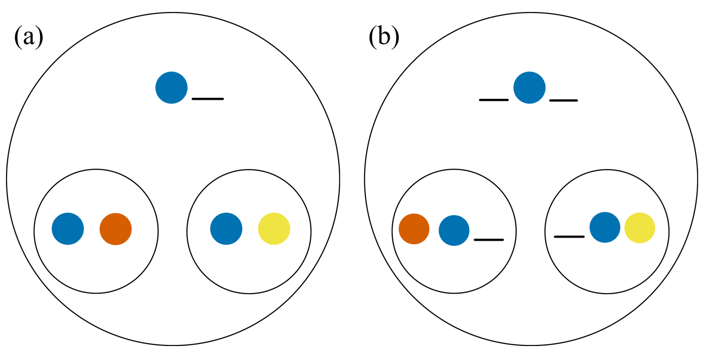


Figure 27: The left side (a) depicts the signaling system that we see showed results for in the disjoint double embedding topology, where there are just two dimensions of social signals with the second dimension having two mutually exclusive social signals. The right side (b) shows an alternative possible double embedding in which the red and yellow signals are in different dimensions and therefore not mutually exclusive.

Appendix C Analysis of Section 2.3 Model

Recall that Section 2.3 gave the mean outcome of 1,000 simulations of the generalized BoS game with a population of 1,000 agents, $m_0 = .01$, $m_1 = .1$, and run for more than a sufficiently large number of time steps to reach an equilibrium (4×10^4).[12] Type 0 agents make up $0.5 + \beta$ proportion of the population (and type 1 agents are a $0.5 - \beta$ proportion). Coordination preferences are:

Coordination Preferences	Bach	Stravinsky
type 0	$1+\alpha$	1
type 1	1	$1+\alpha$

Table 7: Coordination Preferences: generalized BoS for a population of two types, $\alpha \geq 0$.

Recall that agents begin the simulation with randomly selected strategy profiles. Thus we can assume that roughly half of the agents play Bach and half play Stravinsky on the first timestep of a simulation. This should result in almost all of the agents playing their preferred action on the second time step. We can see this by first calculating the utilities:

$$U(\text{preferred action}) = 5 \times 10^2 \times (1+\alpha)$$
$$U(\text{undesired action}) = 5 \times 10^2 \times 1$$

Then we can calculate the change in the population $N(\text{preferred action})$ as:

$$= \quad 2.5 \times 10^2 + 2.5 \times 10^2 \times [5 \times 10^2 \times (1+\alpha) - 0.5 \times (5 \times 10^2 \times (1+\alpha) + 5 \times 10^2)]$$
$$= \quad 2.5 \times 10^2 + 2.5 \times 10^2 \times [5 \times 10^2 \times (1+\alpha) - 0.5 \times (5 \times 10^2 \times (1+\alpha) + 5 \times 10^2 \times \frac{1+\alpha}{1+\alpha})]$$
$$= \quad 2.5 \times 10^2 + 2.5 \times 10^2 \times 2.5 \times 10^2 \times (1+\alpha) \times [2 - (1 + \frac{1}{1+\alpha})]$$
$$= \quad 2.5 \times 10^2 + 2.5 \times 10^2 \times 2.5 \times 10^2 \times (1+\alpha) \times (1 - \frac{1}{1+\alpha}$$
$$= \quad 2.5 \times 10^2 \times (1 + 2.5 \times 10^2 (1+\alpha) - 2.5 \times 10^2)$$
$$= \quad 2.5 \times 10^2 \times (1 + 2.5 \times 10^2 \times \alpha)$$

and $N(\text{preferred action})$ is the same with $\alpha = 0$ therefore the proportion of

[12] In these simulations, every 100 timesteps it was checked whether the distribution of agents' strategy profiles was unchanged. If so, the simualtion was halted prior to reaching 4×10^4 timesteps.

agents of a given type that play their preferred action should be:

$$\begin{aligned}
& \frac{N(\text{preferred action})}{N(\text{preferred action}) + N(\text{undesired action})} \\
= & \frac{2.5 \times 10^2 \times (1 + 2.5 \times 10^2 \times \alpha)}{2.5 \times 10^2 \times (1 + 2.5 \times 10^2 \times \alpha) + 2.5 \times 10^2} \\
= & \frac{1 + 2.5 \times 10^2 \times \alpha}{1 + 2.5 \times 10^2 \times \alpha + 1}
\end{aligned}$$

which is close to 1 for the relevant values of α. In other words, we should expect almost all of the agents to be playing their preferred action after the first time step. In accordance with this, when the the mutation rates were set to zero, all simulations resulted in the outcome where all agents play their types preferred action.

Now, if we assume all of the agents are playing their preferred action, then we can calculate when type 1 agents', whose preferred action is Stravinsky, should have a higher utility for playing Stravinsky than Bach:

$$\begin{aligned}
U(< S >) &> U(< B >) \\
(0.5 - \beta) \times 10^3 \times (1 + \alpha) &> (0.5 + \beta) \times 10^3 \\
0.5 - \beta + \alpha \times (0.5 - \beta) &> (0.5 + \beta) \\
\alpha \times (0.5 - \beta) &> (0.5 + \beta) - (0.5 - \beta) \\
\alpha &> \frac{2\beta}{0.5 - \beta}
\end{aligned}$$

Figure 28 shows the graph of $\alpha = \frac{2\beta}{0.5-\beta}$ and it is easy to see that this aligns with the simulation results shown in Figure 3.

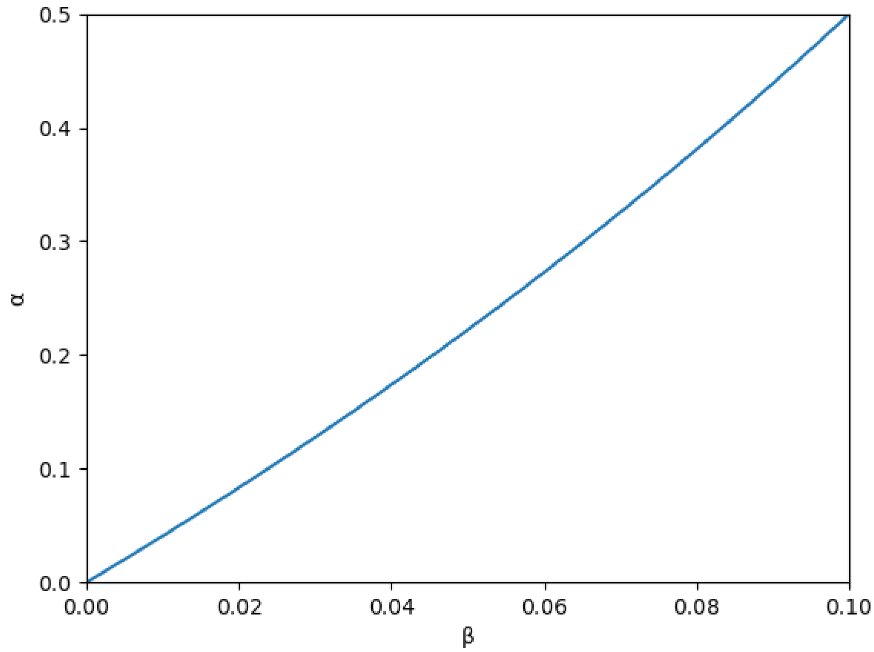


Figure 28: Type 1 agents should continue playing their preferred action if α and β place them above this line.

Appendix D Increasing Population Size Decreases Variance in Outcomes

This appendix compares data points from Sections 2.3 & 3.4 with data points that use the same parameters but increase to population size from 1,000 agents to 10,000 agents. Here we see that there is less variance in outcomes for larger populations; the explanation for this is straightforward and given below.

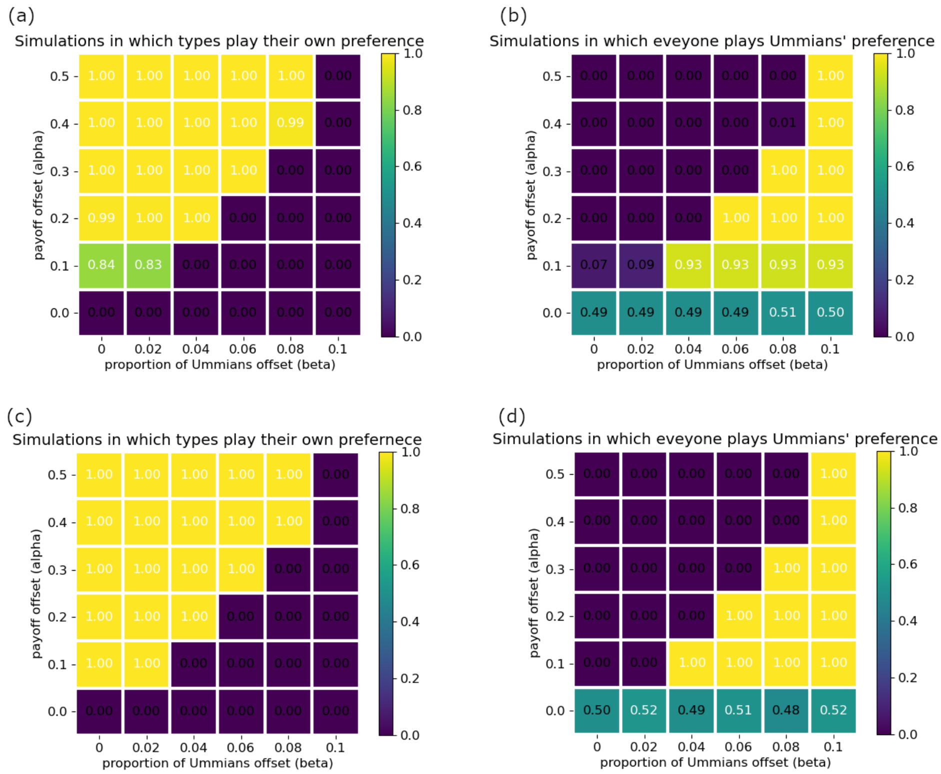


Figure 29: (a)Population size = 1,000 agents. Proportion of simulations that resulted in outcome (i) in which agents always give their preferred greeting; this means that when agents of different types fail to coordinate. (b) Population size = 1,000 agents. Proportion of simulations that resulted in outcome (ii)in which everyone gives Ummians' preferred greeting.(c)Population size = 10,000 agents. Proportion of simulations that resulted in outcome (i) in which agents always give their preferred greeting; this means that when agents of different types fail to coordinate. (d) Population size = 10,000 agents. Proportion of simulations that resulted in outcome (ii)in which everyone gives Ummians' preferred greeting.

Comparing Figures 29a&b with 29c&d we see that there is more variance in the outcomes for simulations of smaller populations than larger populations. This can be understood by considering what must happen to obtain outcome (iii), everyone plays the Kish greeting. This is the same as asking when is it the case that agents' playing the Kish greeting have higher utility from that action than agents playing the Umma greeting. Note that since Kishu agents prefer the Kish greeting over the Umma greeting and vice versa for Ummian agents, then an Ummian agent having higher utility for playing the Kish greeting than the Umma greeting implies that Kishu agents will also have higher utility for playing the Kish greeting than the Umma greeting. So, lets just consider when an Ummian agent has higher utility for playing the Kish greeting than the Umma greeting. By the $U(x)$ equation this happens when:

$$\begin{aligned} U(K) &> U(U) \\ M(K) \times 1 &> M(U) \times (1+\alpha) \end{aligned}$$

where $M(x)$ is the number of agents playing x. Suppose for illustration that $\alpha = 0.1$. Then if the population size is 1,000 (i.e. $M(K)+M(U) = 1000$) agents we get that outcome (iii) is likely to occur if:

$$\begin{aligned} M(K) \times 1 &> M(U) \times 1.1 \\ 1000 - M(U) &> M(U) \times 1.1 \\ \frac{1000}{1.1M(U)} - \frac{M(U)}{1.1M(U)} &> 1 \\ \frac{1000}{1.1M(U)} - \frac{10}{11} &> 1 \\ \frac{10000}{11M(U)} &> \frac{21}{11} \\ 10000 &> 21M(U) \\ 476.19047619 &> M(U) \end{aligned}$$

Similarly, one could calculate that for a population of 10,000 agents the number of agents playing the Umma greeting must be less than 4761.9047619 for Ummian agents to have higher utility from playing the Kish greeting than the Umma greeting. Now if we consider just the first timestep of a simulation in which agents are randomly (with equal probability) assigned strategy profiles, then it is easy to check that it is more likely that (a), a population of 1000 agents will have 476 or fewer agents playing the Umma greeting (and 524 or more playing the Kish greeting), than it is for (b) a population of 10,000 agents to have 4761 or fewer agents playing the Umma greeting (and 5239 or more playing the Kish greeting); i.e. the probability of (a) is:

$$\sum_{k=0}^{476} \binom{1000}{k} \times 0.5^{1000} \approx 0.06858400263532671$$

and the probability of (b) is:

$$\sum_{k=0}^{4761} \binom{10000}{k} \times 0.5^{10000} \approx 0.00000091718051570$$

While these calculations only apply to the probability of Ummian agents having higher utility for playing the Kish greeting than the Umma greeting *on the first* *timestep*. They should provide an intuition for the general case of why low probability outcomes are more likely for smaller populations than larger ones.

We can also see this effect of increasing population size for the model in Section 3.4:

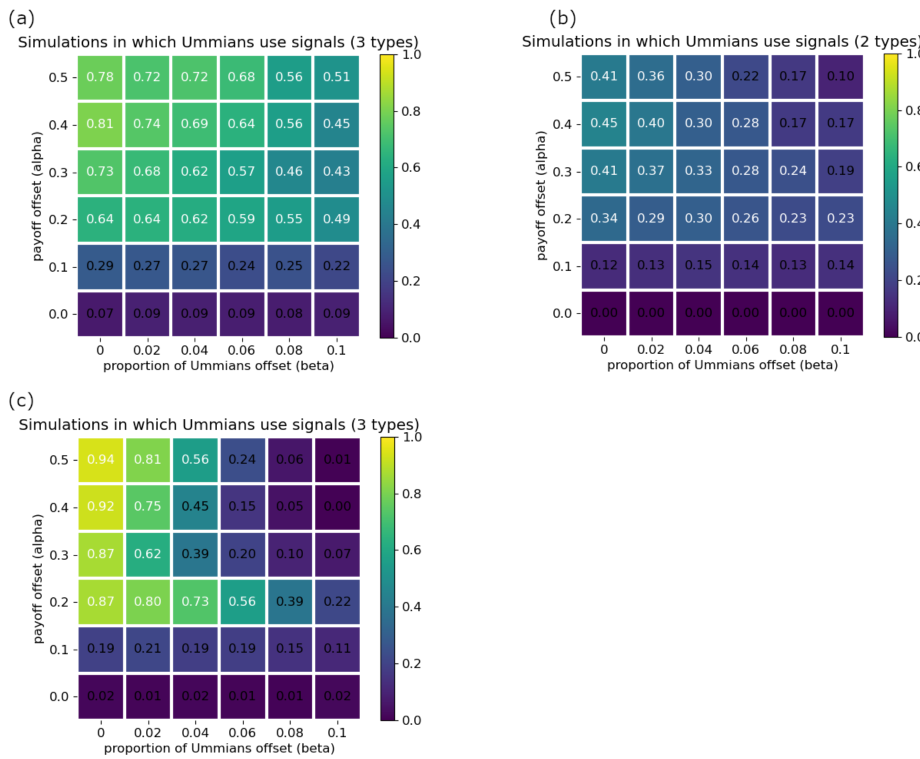


Figure 30: (a) Proportion of outcomes in which type 0 agents broadcast a social signal, for the model with three types of agents, 1,000 total agents. (b) Proportion of outcomes in which type 0 agents broadcast a social signal, for the model with two types of agents, 1,000 total agents. (c) Proportion of outcomes in which type 0 agents broadcast a social signal, for the model with three types of agents, 10,000 total agents.

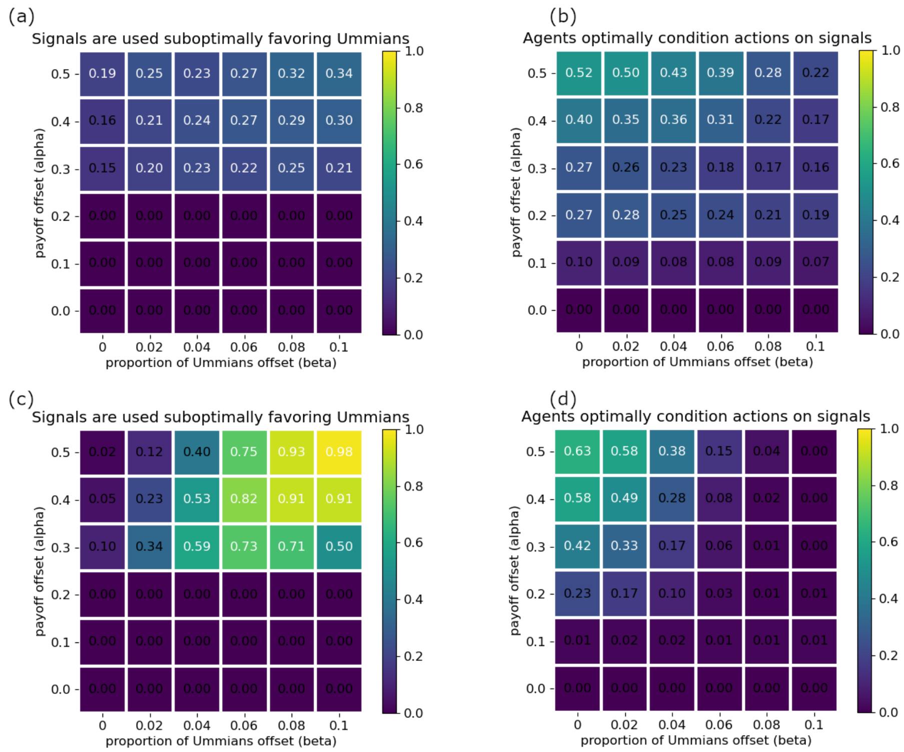


Figure 31: (a) Proportion of outcomes that are (viii), population of 1,000 agents. (b) Proportion of outcomes that are (xii-xiii), population of 1,000 agents. (c) Proportion of outcomes that are (viii), population of 10,000 agents. (d) Proportion of outcomes that are (xii-xiii), population of 10,000 agents.

Appendix E Additional Details on Simulation Results for Generalized BoS with Attention and Signal Costs

Note: this appendix uses data that was generated independently of the data points given in Appendix D. Therefore, there may be nominal differences in the values given in corresponding heatmaps though the general trends remains the same. This appendix's simulations were done with a population size of 10,000 agents; so it can be check that Figures 34 and 36 closely match Figures 30c and 31d from Appendix D.

Additionally, note that a supplemental PDF shows the evolution of signaling and behavioral dispositions of individual simulation runs. For each of the seventeen outcomes here, ten different simulations resulting in the given outcome are detailed except when fewer than ten simulations resulted in the outcome.

The supplemental PDF was composed prior to developing the Mesopotamian greetings example. It was written in terms of agents deciding between Bach, Stravinsky, or EDM rather than being in terms of the Umma greeting, the Kish greeting, and the Akkadian greeting. As I am disinclined to redo the 116 pages of the supplemental PDF, this appendix is written in terms of Bach, Stravinsky, and EDM. My appologies for the change. Bellow, the table of coordination preferences is restated in those terms with the corresponding Mesopotamian greetings given in parentheses.

Finally, it should be noted that the outcomes listed here are relative to the most frequent behavior of the agents rather than the behavior of all of the agents. For outcomes (i)-(xiii) these two measures of an outcome are equivalent. However for outcomes (xiv)-(xvii), the outcomes are what Fulker et al. (2024) call hybrid equilibria, though they now prefer the bipartite signaling equilibria as the term "hybrid equilibria" has previously been used to refer to something else. An example is illustrated here for outcome (xiv), Figure 38, and details of the other outcomes can be found in the supplemental PDF. All of these bipartite signaling equilibria were confirmed to be Nash equilibria.

Coordination Preferences	Bach (Umma)	Stravinsky (Kish)	EDM (Akkadian)
type 0	$1+\alpha$	1	0.5
type 1	1	$1+\alpha$	0.5
type 2	0	0	1

For the parameters investigated, there were thirteen different outcomes that occurred at least 0.5% of the time for some data point.

(i) Outcomes in which there was no coordination between agents of different types, i.e. agents always play their preference irrespective of the signal transmitted. This is shown in Figure 32.

(ii–ix) Agents exhibit suboptimal coordination, but in an intuitive way. This is shown in Figure 33.

- (viii) A notably common outcome was one in which type 0 and type 2 agents always played their respective preference, and type 1 agents played B with type 0, S with type 1, and EDM with type 2. The prevalence of this outcome is shown in Figure 34. This outcome was frequently characterized by type 0 agents signaling 0 and type 1 and 2 agents attending to signals that reliably identified their type. One can check that this is a Nash equilibrium. It is suboptimal in the sense that when type 0 agents are paired with type 2 agents, there is a failure to coordinate.

(x–xi) Agents condition their actions suboptimally, and in a counterintuitive way. This is shown in Figure 35. Specifically what was counterintuitive about these outcomes is that they involved a type playing their preference when paired with a type other than themselves, but not playing their preference among themselves.

(xii–xiii) Agents condition their actions optimally, in the sense that there were never failures of coordination. This is shown in Figure 36.

(xii) A notably common outcome was one in which type 0 played their preference when paired with type 1. The prevalence of this outcome is shown in Figure 37.

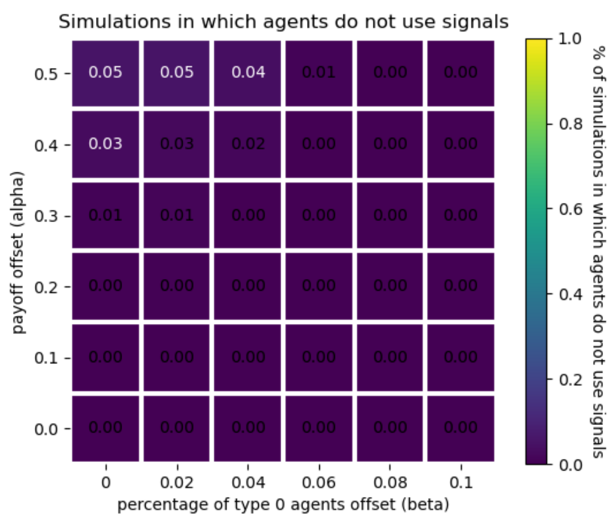


Figure 32: Proportion of outcomes that are (i).

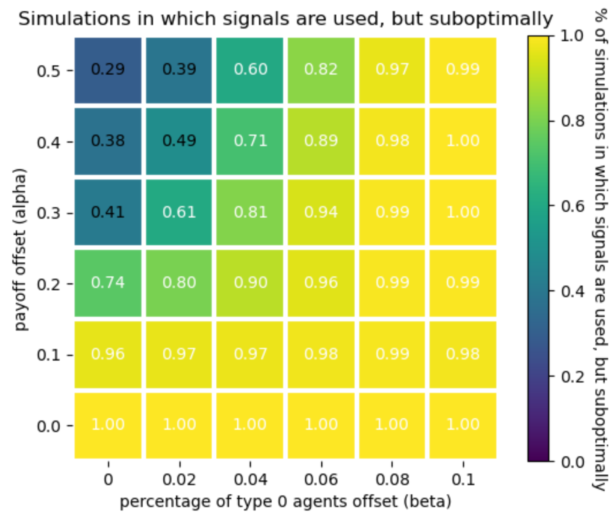


Figure 33: Proportion of outcomes that are (ii-ix).

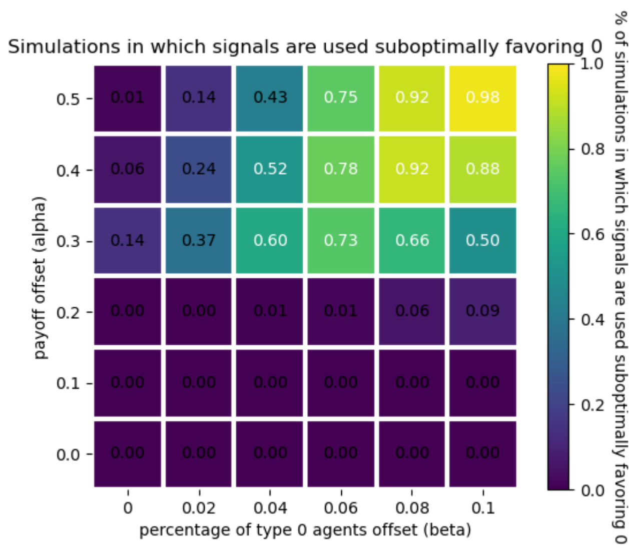


Figure 34: Proportion of outcomes that are (viii).

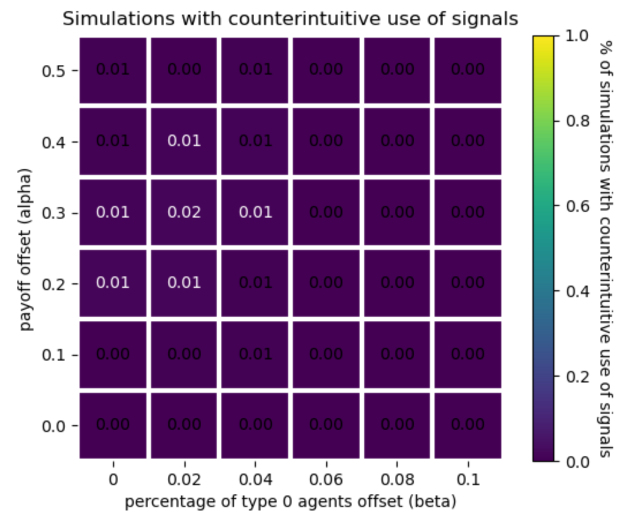


Figure 35: Proportion of outcomes that are (x-xi).

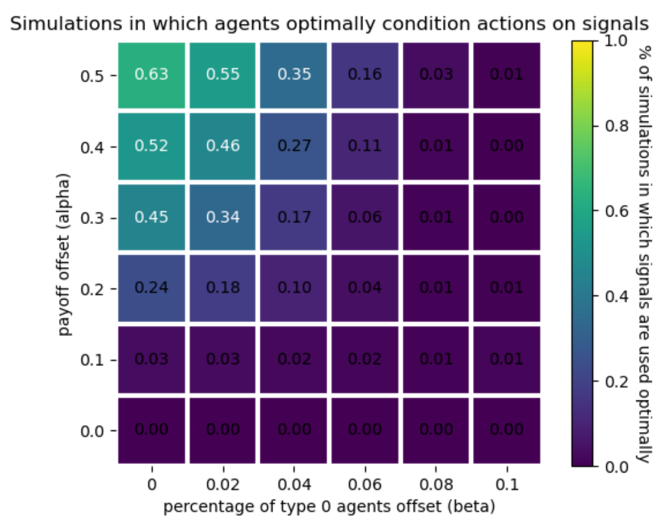


Figure 36: Proportion of outcomes that are (xii-xiii).

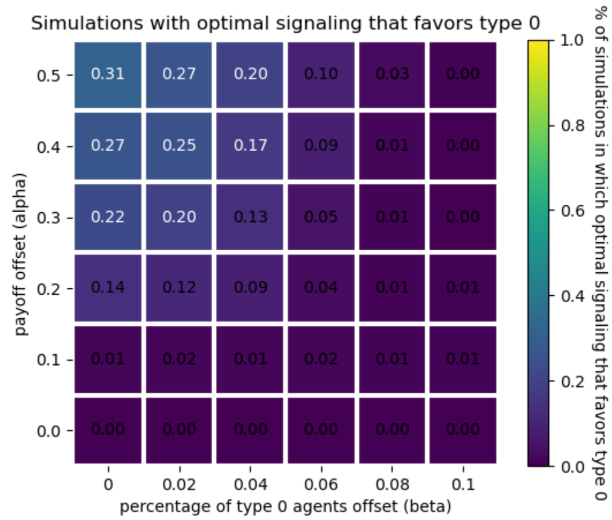


Figure 37: Proportion of outcomes that are (xii).

outcome (i)

y plays this with x	type 0	type 1	type 2
type 0	B	B	B
type 1	S	S	S
type 2	E	E	E

outcome (ii)

y plays this with x	type 0	type 1	type 2
type 0	B	B	B
type 1	B	B	B
type 2	E	E	E

outcome (iii)

y plays this with x	type 0	type 1	type 2
type 0	S	S	S
type 1	S	S	S
type 2	E	E	E

outcome (iv)

y plays this with x	type 0	type 1	type 2
type 0	B	B	E
type 1	B	B	E
type 2	E	E	E

outcome (v)

y plays this with x	type 0	type 1	type 2
type 0	S	S	E
type 1	S	S	E
type 2	E	E	E

outcome (vi)

y plays this with x	type 0	type 1	type 2
type 0	B	B	B
type 1	B	S	B
type 2	E	E	E

outcome (vii)

y plays this with x	type 0	type 1	type 2
type 0	B	S	S
type 1	S	S	S
type 2	E	E	E

outcome (viii)

y plays this with x	type 0	type 1	type 2
type 0	B	B	B
type 1	B	S	E
type 2	E	E	E

outcome (ix)

y plays this with x	type 0	type 1	type 2
type 0	B	S	E
type 1	S	S	S
type 2	E	E	E

outcome (x)

y plays this with x	type 0	type 1	type 2
type 0	S	B	E
type 1	B	S	E
type 2	E	E	E

outcome (xi)

y plays this with x	type 0	type 1	type 2
type 0	B	S	E
type 1	S	B	E
type 2	E	E	E

outcome (xii)

y plays this with x	type 0	type 1	type 2
type 0	B	B	E
type 1	B	S	E
type 2	E	E	E

outcome (xiii)

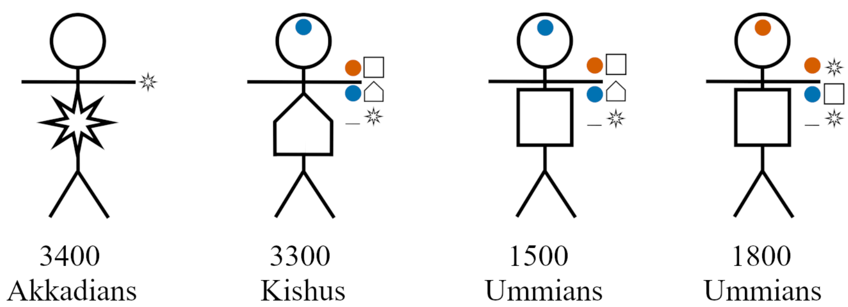
y plays this with x	type 0	type 1	type 2
type 0	B	S	E
type 1	S	S	E
type 2	E	E	E

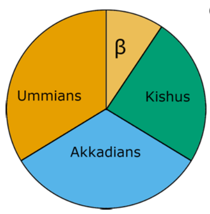
outcome (xiv)

y plays this with x	type 0	type 1	type 2
type 0	B	B	B
type 1	B	E	E
type 2	E	E	E

This occurred:
1 time(s) when alpha = 0.1 and beta = 0.08
2 time(s) when alpha = 0.2 and beta = 0.08
1 time(s) when alpha = 0.2 and beta = 0.1

Here's an example of what one of these outcomes looked like:





outcome (xv) α=0.3 β=0

Coordination Preferences	Umma greeting	Kish greeting	Akkadian greeting
Ummians (□)	$1+\alpha$	1	0.5
Kishus (⌂)	1	$1+\alpha$	0.5
Akkadians (✲)	0	0	1

Figure 38: In this bipartite signaling equilibrium, the population of the Kishus is split between two different ways of signaling and conditioning actions on signals. It can be checked that this is a Nash equilibrium, i.e. no agent can unilaterally change their behavior in a way that increases their net payoff.

outcome (xv)

y plays this with x	type 0	type 1	type 2
type 0	E	B	E
type 1	B	S	E
type 2	E	E	E

This occurred:
1 time(s) when alpha = 0.3 and beta = 0
1 time(s) when alpha = 0.4 and beta = 0

outcome (xvi)

y plays this with x	type 0	type 1	type 2
type 0	B	S	E
type 1	S	E	E
type 2	E	E	E

This occurred:
1 time(s) when alpha = 0.3 and beta = 0.04
2 time(s) when alpha = 0.4 and beta = 0.02
1 time(s) when alpha = 0.5 and beta = 0
1 time(s) when alpha = 0.5 and beta = 0.02
1 time(s) when alpha = 0.5 and beta = 0.04

outcome (xvii)

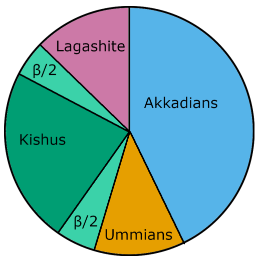
y plays this with x	type 0	type 1	type 2
type 0	B	E	E
type 1	E	S	E
type 2	E	E	E

This occurred:
1 time(s) when alpha = 0.3 and beta = 0.06
1 time(s) when alpha = 0.4 and beta = 0.08
1 time(s) when alpha = 0.5 and beta = 0.06
2 time(s) when alpha = 0.5 and beta = 0.1

Appendix F Intersectional Preferences with One Dimension of Signals

This appendix explores parameter space of the intersectional topologies, but allows agents three signals in one dimension rather than one signal in each of two dimensions (see Figure 19). The one dimensional signaling model is capable of, in a sense, producing the same optimal outcomes as the two dimensional version. However, these optimal outcomes are significantly less frequent than in the model with two dimensions of signaling. Where the two dimensional model produced the appropriate signaling system nearly 100% of the time, the one dimensional model (see Figure 39) achieves the appropriate signaling system much less frequently. The most plausible explanation seems to be that when using multiple dimensions of signals, Lagashites and Ummians do not have to learn anything about the Kishus for the Kishus to successfully coordinate on the optimal action. Kishus can simple use the same signal as either Lagshites and Ummians in corresponding dimensions and to those agents Kishus look like members of their own group. When there is only one dimension of signals, then there is added difficulty for how various types of agents learn optimal behavior because now an optimal outcome requires not only that Lagashites and Ummians learn how to signal their own identities and condition their actions appropriately, but also have to learn what the Kish social signal is and what the optimal action when encountering that signal is. In the one dimensional system, Kishus do not look like Lagashites or Ummians from the perspective of those two groups and it is consequently a more difficult learning problem.

Coordination preferences were equivalent to or close to those given in Section 4.4, i.e.:



Coordination Preferences	Umma greeting	Kish greeting	Lagash greeting	Akkadian greeting
Ummians (□)	1	0	0	0.25 or 0.5
Kishus (⌂)	1	α	1	0.25 or 0.5
Lagashites (△)	0	0	1	0.25 or 0.5
Akkadians (✲)	0	0	0	1

In Section 4.4 Ummians and Lagashites get a payoff of 0.5 for coordinating on the Akkadian greeting and the evolutionary dynamics used $m_0 = 0.01$ and $m_1 = 0.1$. In this appendix we additionally consider, moth those values and a payoff of 0.25 for the Akkadian greeting as well as $m_0 = 0.05$ and $m_1 = 0.2$. The one dimensional signaling system from Figure 19 is a prerequisite for th optimal outcome to obtain. Figure 39 shows the prevelance of that signaling system. This contrasts starkly with the two dimensional model in Section 4.4, for which optimal signaling occurs in close to 100% of simulations. Figures 40 & 41 further illustrate the extent to which Kishus rarely give the Kish greeting among themselves, despite this being an optimal outcome when $\alpha = 2$.

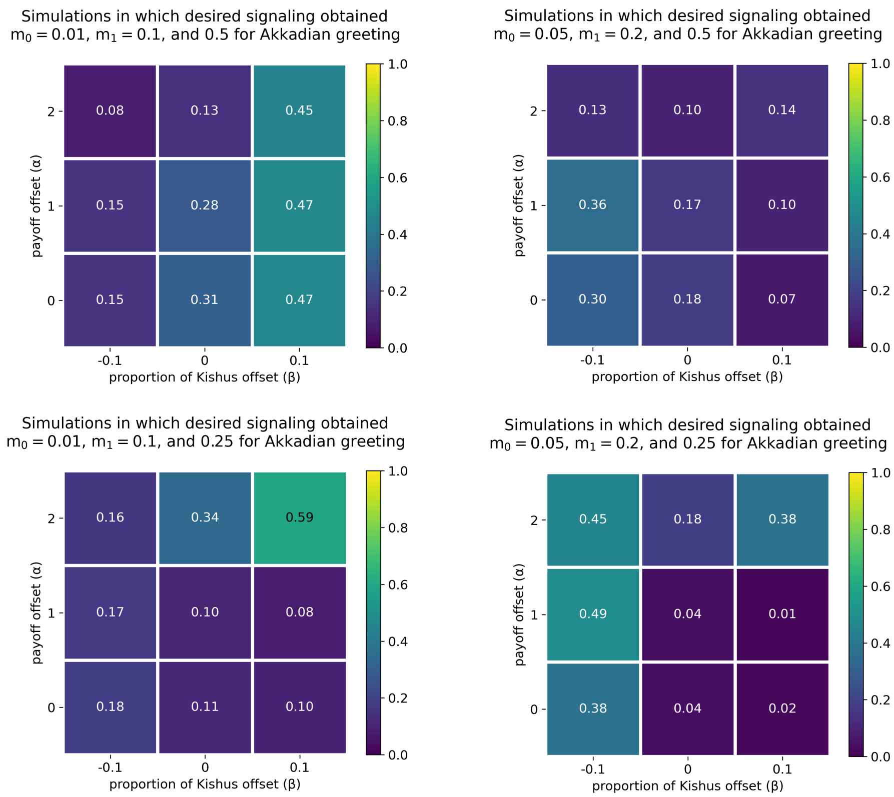


Figure 39: Prevelance of the optimal one dimensional signaling system. This figure is the one dimensional equivalent of Figure 17

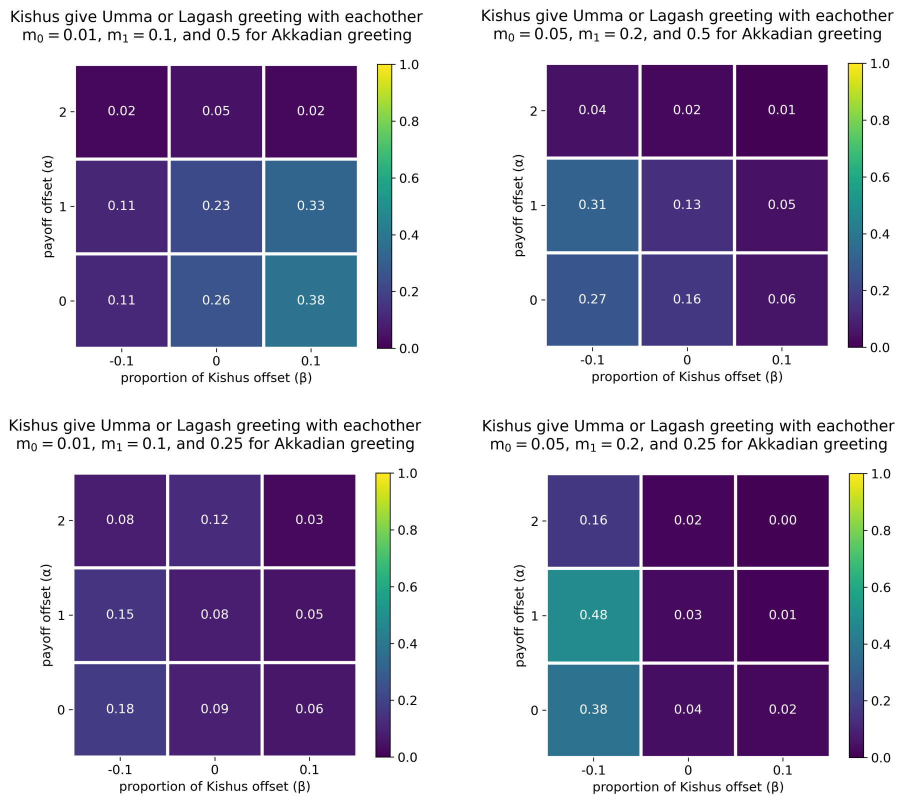


Figure 40: This figure is the one dimensional equivalent of Figure 18(a), the proportion of outcomes in which Kishus (⌂) give the Kish greeting with eachother in addition to exhibiting the desired signaling behavior.

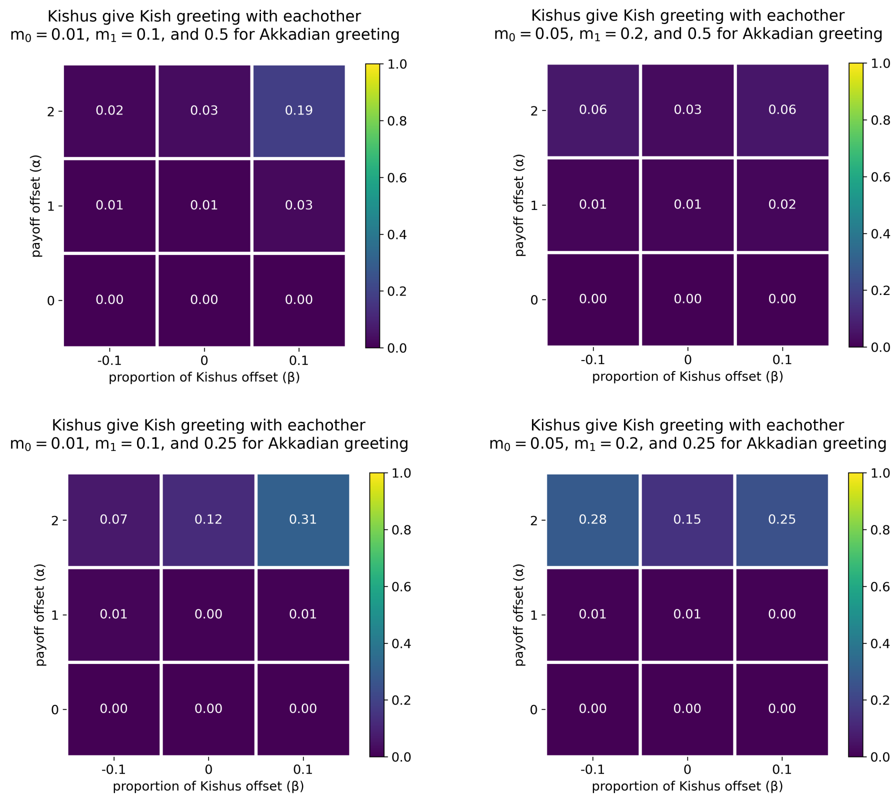


Figure 41: This figure is the one dimensional equivalent of Figure 18(b), the proportion of outcomes in which Kishus (⌂) give the Umma (□) or Lagash (△) greeting with eachother in addition to exhibiting the desired signaling behavior.

Appendix G Intersectional Preferences with Common Middle Ground

In the main body of the paper, the section on intersectional social identities (Section 4.4) assigned Ummians and Lagashites preferences such that only their own city's greeting and the Akkadian greeting. This was appropriate for illustrating the two very different Kishu identities that can occur for the same signaling system. However, it is a suboptimal choice for thinking about group power dynamics under the same paradigm as prior sections of the paper. In prior sections, a group's power was discussed in terms of group A exerting their power over group B by making group A's preferred action the norm when agents from the two groups interact. If Ummians and Lagashites have zero payoff from the Kish greeting, then it is apriori determined that they will not give the Kish greeting when interacting with Kishus. This appendix shows simulation results from two different amendments to the coordination preferences that give Ummians and Lagashites non-zero payoff for the Kish greeting. It also shows along side these results, comparable simulation results for coordination preferences like the original simulations where they have no payoff for the Kish greeting, though we also vary Ummians and Lagashites' payoff for their most preferred greetings. Note that, unlike Section 4.4, this appendix only considers coordination preferences in which Kishus have a higher payoff for their own greeting than for any others. So, the optimal outcome is always one in which Kishus give the Kish greeting among themselves.

Figures 46 - 49 show that, inline with expectations from prior sections of the paper, decreasing the size of the intersectional group makes it less likely that their preference will be the norm when interacting with agents who are members of just one of the two groups that comprise the intersectional group. Increasing the size of the intersectional group (i.e. increasing β) makes it more likely, though still a small likelihood, that the intersectional (Kish) greeting is the norm for interactions between agents in the intersectional group and agents who are members of just one of the two groups. Figures 43 - 45 are the measures that were used in Sections 4.4; and are included merely for comparability with that section.

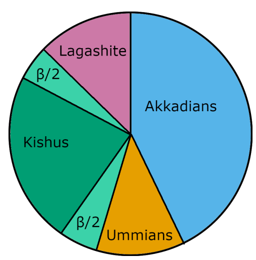


Figure 42: Proportion of agent types

Left (a) Coordination Preferences	Umma greeting	Kish greeting	Lagash greeting	Akkadian greeting
Ummians (□)	1+α	0	0	0.5
Kishus (⌂)	1	1+α	1	0.5
Lagashites (△)	0	0	1+α	0.5
Akkadians (✲)	0	0	0	1

Center (b) Coordination Preferences	Umma greeting	Kish greeting	Lagash greeting	Akkadian greeting
Ummians (□)	1+α	0.5	0	0.5
Kishus (⌂)	1	1+α	1	0.5
Lagashites (△)	0	0.5	1+α	0.5
Akkadians (✲)	0	0	0	1

Right (c) Coordination Preferences	Umma greeting	Kish greeting	Lagash greeting	Akkadian greeting
Ummians (□)	$1{+}\alpha$	0.5	0.5	0.5
Kishus (⌂)	1	$1{+}\alpha$	1	0.5
Lagashites (△)	0.5	0.5	$1{+}\alpha$	0.5
Akkadians (✵)	0	0	0	1

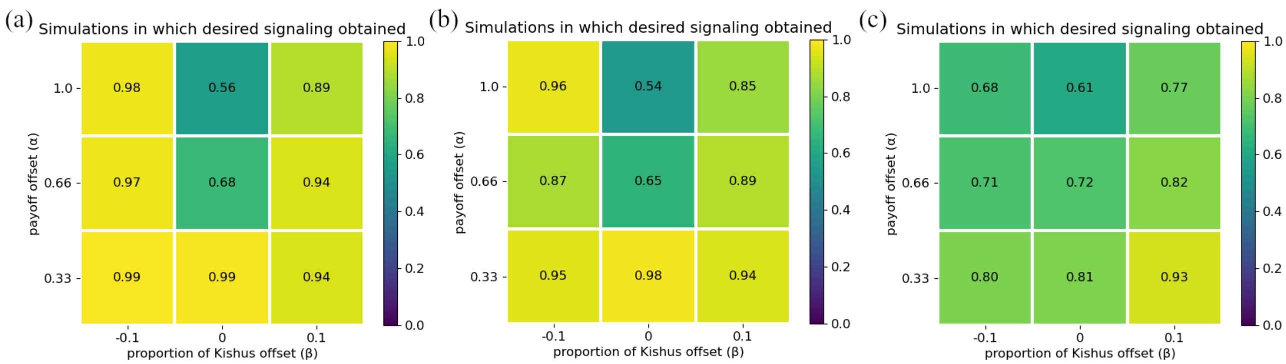


Figure 43: How often the intersectional signaling system obtains.

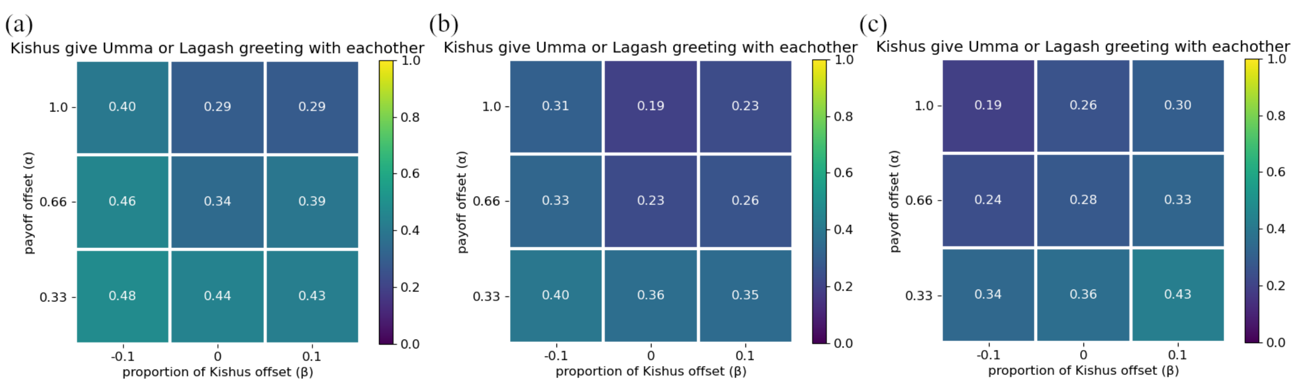


Figure 44: How often Kishus do not give the Kish greeting among themselves among themselves.

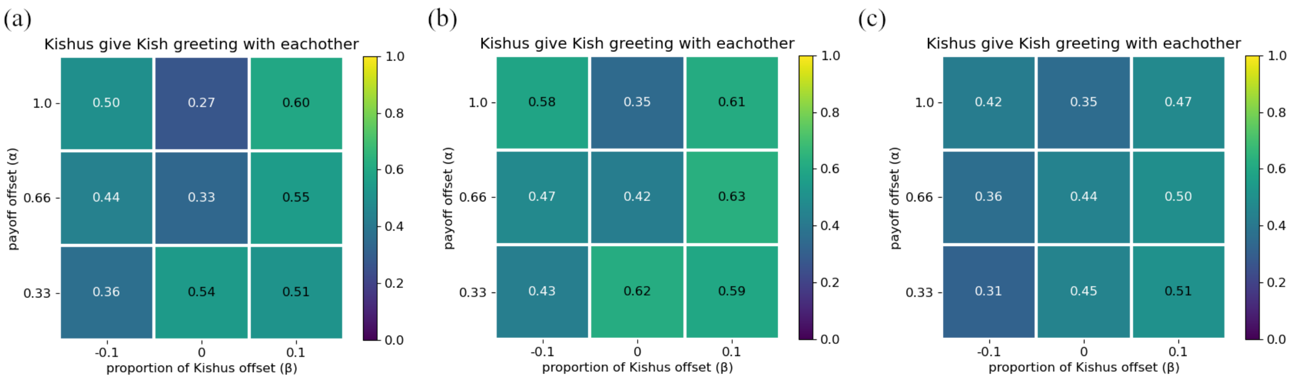


Figure 45: How often Kishus give the Kish greeting among themselves among themselves.

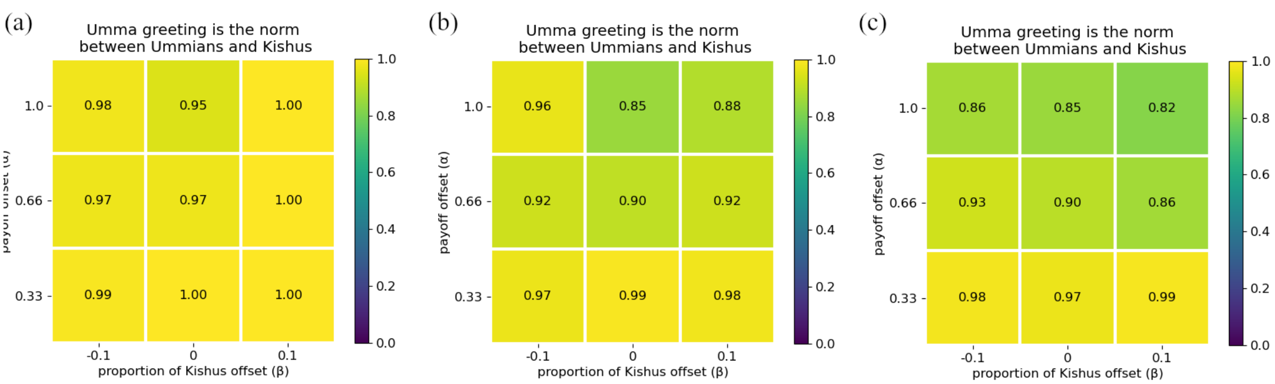


Figure 46: How often Umma greeting is the norm between Ummians and Kishus.

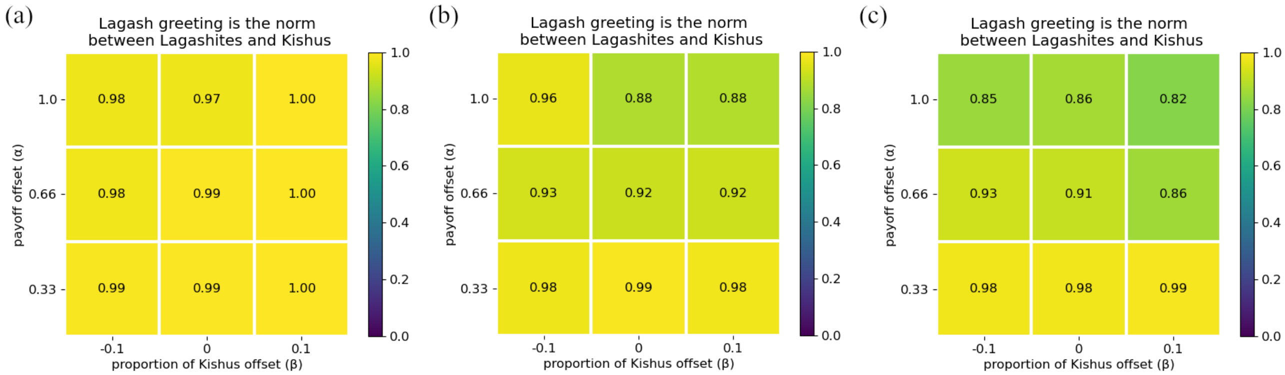


Figure 47: How often Lagash greeting is the norm between Lagashites and Kishus.

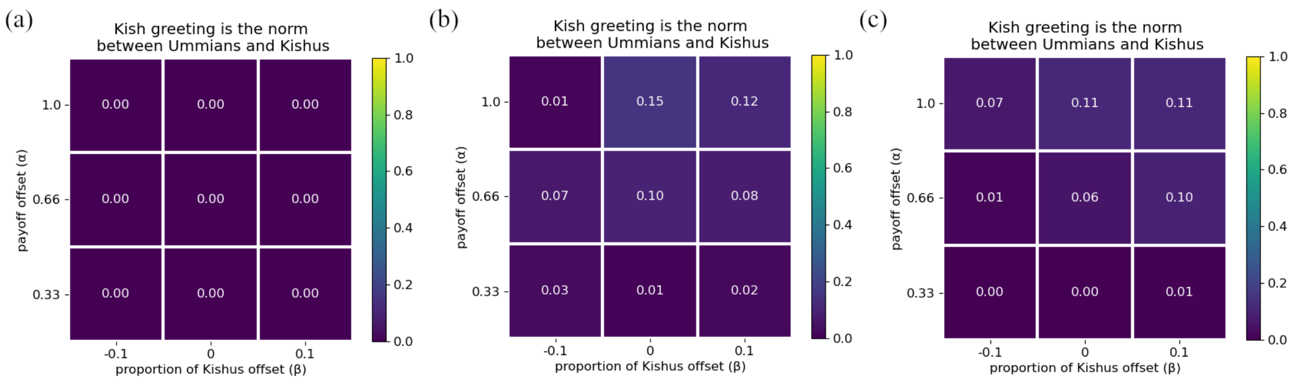


Figure 48: How often Kish greeting is the norm between Ummians and Kishus.

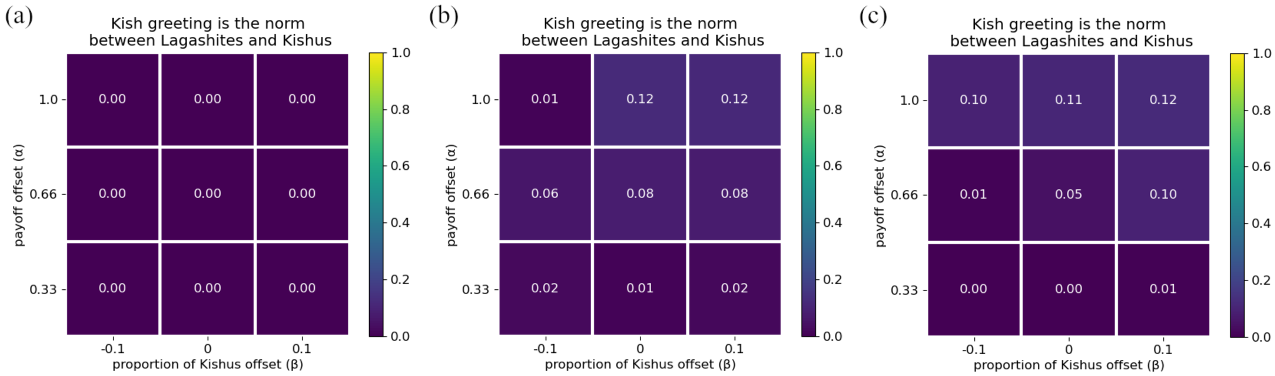


Figure 49: How often Kish greeting is the norm between Lagashites and Kishus.

Appendix H Adding in Partial Payoffs on Failures to Coordinate

This appendix shows simulation results for the model with one change. The change is that instead of having $p_{xi} = 0$ when strategy profiles x and i result in a failure to coordinate p_{xi} is twenty percent of the payoff that would have been received if the action chosen by profile x had lead to successful coordination. For example, if an agent receives a payoff of 1 for coordinating on a particular action, then if that agent performs the same action but fails to coordinate her payoff will be 0.2. Of course this value of twenty percent was not hard coded and is a parameter that can be varied. But, twenty percent is sufficient for illustrating what happens in the model. If the payoff percentage on failures to coordinate is too small, then simulation results are identical to results from the model in which agents get nothing on failures to coordinate; and, if the payoff percentage on failures is too high, then agents will always perform their most preferred action because the payoff on failures for that action exceeds their payoff for successful coordination on a less preferred action.

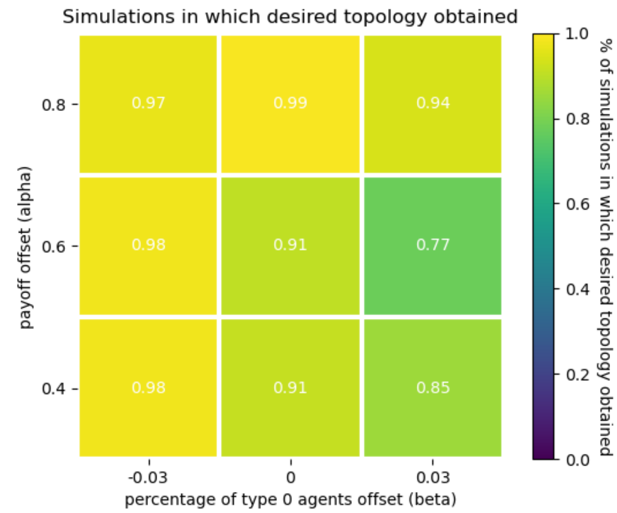


Figure 50: Proportion of outcomes that are the single embedding topology under parameters identical to those used for Figure 15 (c) but with the modified model where agents get twenty percent on failures.

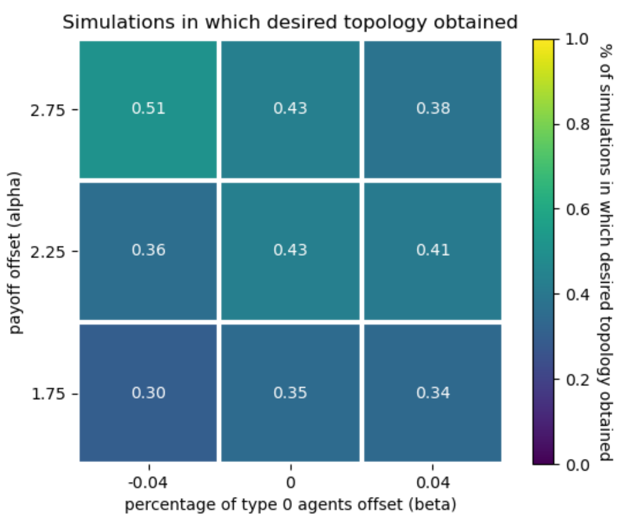


Figure 51: Proportion of outcomes that are the disjoint double embedding topology under parameters identical to those used for Figure 21c but with the modified model where agents get twenty percent on failures.

Appendix I Analytic Analysis for 2x2x2 game

2 types, 2 signals (α, β), and 2 actions (no homophily)

In this simple system, this appendix analyzes when it is the case that an agent should play their less preferred action.

		Type 1	
		Bach	Stravinsky
Type 0	Bach	a, b	c, c
	Stravinsky	c, c	b, a

With $a > b$ and $c < 0$.

- Let $n_i, i \in \{0, 1\}$ the proportion of the population that is type i.
- Let $x_{ijk}, i \in \{0, 1\}, j \in \{\alpha, \beta\}, k \in \{\text{Bach, Stravinsky}\}$ be the probability that a type i agent plays k given signal j is observed.
- Let $y_{ij}, i \in \{0, 1\}, j \in \{\alpha, \beta\}$ be the probability that a type i agent transmits signal j. Note that $y_{i\beta} = 1 - y_{i\alpha}$
- Let A_i be type i's preferred action and B_i be the alternative action.
- Let $p(i|j), i \in \{0, 1\}, j \in \{\alpha, \beta\}$ be the probability that signal j was sent by a type i agent given signal j was observed.
- Let $E_{ij}(k), i \in \{0, 1\}, j \in \{\alpha, \beta\}, k \in \{\text{Bach, Stravinsky}\}$ be the expected utility for a type i agent playing k after having seen signal j.

Then:

$$p(i|j) = \frac{n_i y_{ij}}{n_0 y_{0j} + n_1 y_{1j}}$$

$$\begin{aligned}
E_{ij}(A_i) = {} & a[y_{i\alpha}(x_{0\alpha A_i} p(0|j) + x_{1\alpha A_i} p(1|j)) + (1 - y_{i\alpha})(x_{0\beta A_i} p(0|j) + x_{1\beta A_i} p(1|j))] \\
& + c[y_{i\alpha}(x_{0\alpha B_i} p(0|j) + x_{1\alpha B_i} p(1|j)) + (1 - y_{i\alpha})(x_{0\beta B_i} p(0|j) + x_{1\beta B_i} p(1|j))] \\
E_{ij}(B_i) = {} & b[y_{i\alpha}(x_{0\alpha B_i} p(0|j) + x_{1\alpha B_i} p(1|j)) + (1 - y_{i\alpha})(x_{0\beta B_i} p(0|j) + x_{1\beta B_i} p(1|j))] \\
& + c[y_{i\alpha}(x_{0\alpha A_i} p(0|j) + x_{1\alpha A_i} p(1|j)) + (1 - y_{i\alpha})(x_{0\beta A_i} p(0|j) + x_{1\beta A_i} p(1|j))]
\end{aligned}$$

In the initial state of the reinforcement learning model, $x_{ijk} = y_{ij} = 0.5$ for all i, j, and k. Since $a > b$, it follows that in the initial state of the game agents should always play their preferred action; i.e. $E_{ij}(A_i) > E_{ij}(B_i)$ for all i and j.

Given that all agents have more incentive to play their preferred action than the alternative action irrespective of the signal let's assume that $x_{ijA_i} = 0.5 + \epsilon$ for all i and j where $\epsilon \geq 0$. (Note that $x_{ijB_i} = 1 - x_{ijA_i}$). Furthermore, let's assume that due to random drift $y_{0\alpha} = y_{1\beta} = 0.5 + \delta$ for some $\delta > 0$. Without loss of generality, consider expected utility for type 0 agents. We the get that type 0 agents have higher expected utility for playing their less preferred action when observing β (the more frequent signal among type 1 agents) if $E_{0\beta}(B_0) > E_{0\beta}(A_0)$.

Let:

$$Q = \frac{n_0(.5-\delta)}{n_0(.5-\delta)+(1-n_0)(.5+\delta)}$$

$$R = \frac{(1-n_0)(.5+\delta)}{n_0(.5-\delta)+(1-n_0)(.5+\delta)}$$

If $2Qc-2Rc-Qb+Rb-Qa+Ra > 0$, then it can be checked that $E_{0\beta}(B_0) > E_{0\beta}(A_0)$ when:

$$\epsilon > \frac{Qa+Ra-Qb-Rb}{4Qc-4Rc-2Qb+2Rb-2Qa+2Ra}$$

Here are some plots of:

$$\epsilon = \frac{Qa+Ra-Qb-Rb}{4Qc-4Rc-2Qb+2Rb-2Qa+2Ra}$$

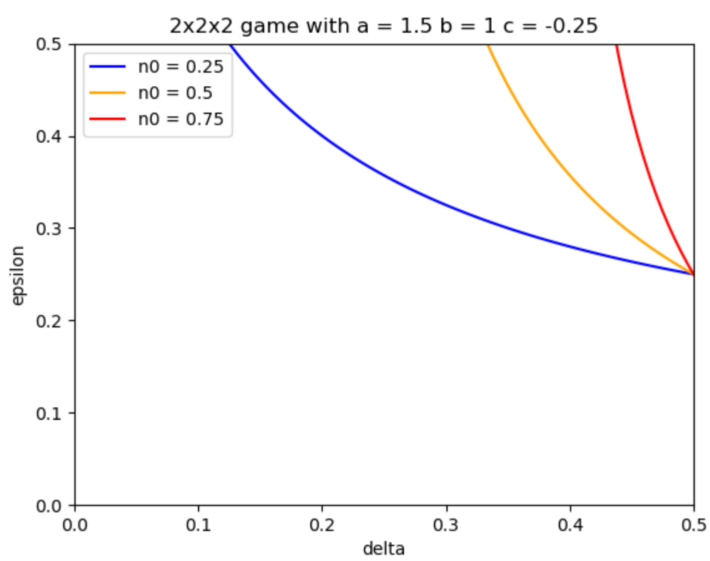


Figure 52

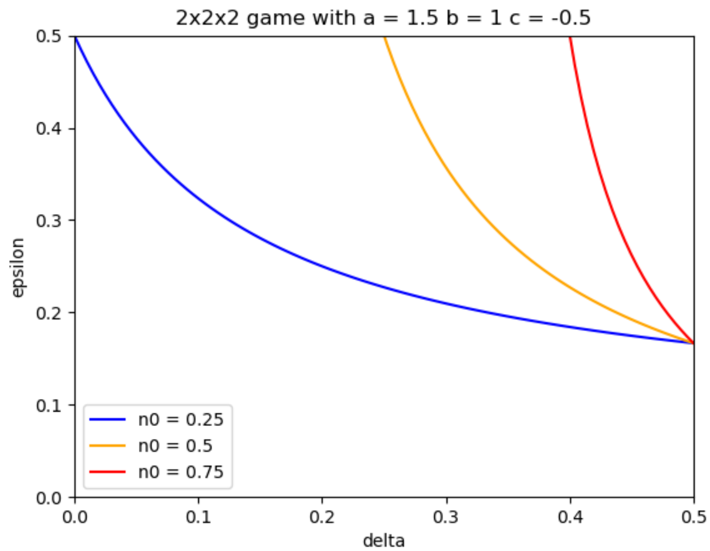


Figure 53

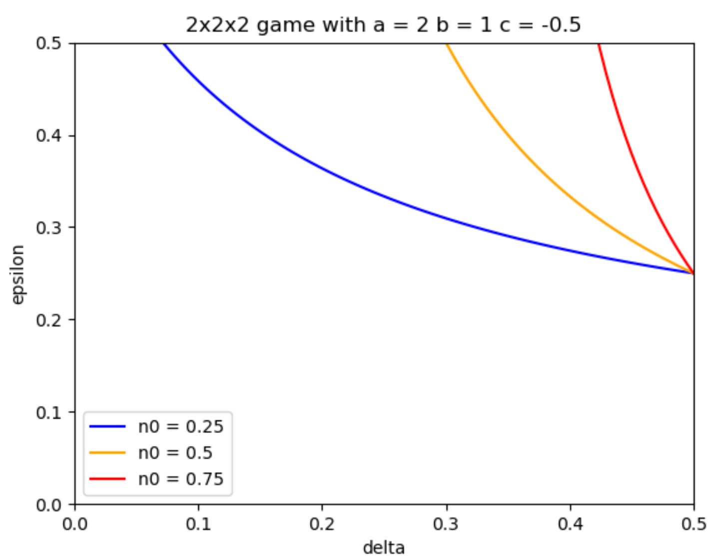


Figure 54