The Emergence of Group Identity Topologies in the Generalized Bach or Stravinsky Game

Nathan Gabriel & Paul E. Smaldino

University of California Merced

January 30, 2025

Abstract

Over the past 300,000 years humans have been evolving increasing complex cultures. Today, our social identities cut across multiple dimensions such as religion, education, ethnicity, gender, and many more (Roccas and Brewer, 2002). Complex social identities are associated with (i) group normative preferences, (ii) social signals that indicate group membership, and (iii) the propensity to condition our actions on the social signals of both ingroup and outgroup members. Various empirical studies have provided some insights on how these different aspects of social identity are related to each other, but they fall short of capturing all of this in the context of complex identities. This paper provides a model that exhibits these three aspects of social identity in the context of complex identities. Prior cultural evolution models have modeled groups with different preferences evolving social signals for coordination, but have only done so with very simple social identities. This paper begins with a base model and then iteratively building up complexity by adding in social signals, attention to signals, assortment by signals, and then multiple dimensions in which signaling can occur. Simulation results from the model are consistent with empirical evidence of minority-majority group dynamics. As the first agent based model of multidimensional social identities, the model can serve as a basis for future research. In its role as this basis, the paper shows how a variety of different social group topologies can obtain. One of these topologies exposes flaws in naive measures of group structures.

1 Introduction

As early as the Third Dynasty of Ur (2100 BCE) in ancient Mesopotamia, we have literary evidence of complex social identities; cities as far apart as Ur and Mari (well over 100 miles apart) evidence a canonical body of literature indicative of a shared Sumerian identity across several cities in the region, but also exhibit literature concerned with a city's patron gods indicative of a localized social identity (Delnero, 2016). We also have reason to believe that long before the invention of writing, humans used markers of group identity to facilitate coordination within groups that were too large for a group member to know every individual in the group (Moffett, 2013). This participation in, sometimes complex, group identities continues today (Roccas and Brewer, 2002). Some social groups follow straightforward rules; all astrophysicists are physicists and all physicists are scientists. The background knowledge that you can assume in conversation with a scientist can also be assumed in conversation with a physicist and, likewise, the background knowledge you can assume in conversation with a physicist can also be assumed in conversation with a astrophysicist. But there are also ways in which social identities break from the contours of simple set membership relations. The mere conjunction of dominate narratives in Black liberation and in mainstream feminism does not yield important narratives of Black feminists (Combahee River Collective, 1977; Crenshaw, 1991). This paper provides a formal model of how a variety of social identity topologies can obtain in a population of agents trying to coordinate behavior and preferentially interact with ingroup members.

Conceptually, there are three aspects of social group identities that we model: (i) the *preferences* and norms of social groups, (ii) the emergence of social *signals* used to express and identify group membership, and (iii) then the use of those social signals to *coordinate* actions. We can identify this sort of signaling for coordination in everyday life. Perhaps, in school, goths and emos have different music preferences; they use differences in hair style to identify group members; and, they use those hair styles to identify who to join at recess. Swingers might use upside-down pineapples to signal their preference for "coordinating behavior" with other couples (Pelzer, 2023). It is difficult to find robust empirical evidence that captures complex social identities with corresponding preferences, signaling, and coordination all in a single experiment. Berger and Heath (2008) produced a series of studies that examine preferred interaction partners and clear group selection of markers such as wrist bands. Together these studies are suggestive of groups with different preferences using social signals to coordinate action. Arguably, Lin et al. (2024) captures preferences for business transactions with trustworthy partners, signaling trustworthiness with a red face from drinking alcohol, and likelihood to coordinate in future business partnerships. But, these studies still lack complexity that we exhibit in our social identities. There is evidence that social identity in multiple dimensions (nationality, ethnicity, religion, and education) can be indicative of differneces in preferences about openness, conservatism, universality, and power Roccas and Brewer (2002). Bunce (2020) shows that an education system dominated by a majority ethnicity can lead to a minority group learning norms related to coordination, in the domains of both work and family life, while still remaining competent in norms associated with their own ethnicity. Literature on code switching is also suggestive of how multiple dimensions of social identity relate to coordination. Both Black and female coworkers are judged to be more prefessional when they adopt norms such as dialect and hairstyle associated with white men (McCluney et al., 2021).[1]

This paper's model ties together all of these aspects of social identity. In fact, simply modeling groups with different preferences and the need to coordinate actions will be enough to produce simulation results in which a minority population conforms to the preferences of the majority group in accordance with the empirical studies just cited. Adding in social signals and multidimensional social identities allows even more interesting simulation results. However, substantial groundwork needs to be laid prior to explaining those results. Prior literature has provided formal models how selection pressures can lead to agents evolving social signals used for assortment or coordinating behaviors (McElreath et al., 2003; Smaldino et al., 2018; Smaldino and Turner, 2020; Goodman et al., 2023; Macanovic et al., 2024). However, none of these prior models have incorporated multidimensional social identities. Consequently, there are two important ways in which this paper's model will diverge from prior models. First, we allow agents to broadcast social signals in multiple dimensions rather than as single dimension. Second, we allow a broad spectrum of payoffs that can be associated with successful coordination. For example, it might be the case that goths prefer handshakes as a greeting and emos prefer hugs. In our model, this would be represented as both a goth an an emo getting some payoff when they hug, but the emo getting a higher payoff than the goth. In prior models, agents were either indifferent between two ways in which they might coordinate, shake hands or hug, or agents got no payoff from successful coordination on a less preferred action. Allowing a spectrum of payoffs allows us to model overlapping social identities in which agents may only have a strict subset of their social identities in common. Similar to prior models, agents will not get any payoff when they fail to coordinate their actions, e.g. one agent tries to shake hands while the other tries to hug. This is a convenient simplifying assumption, but not all real world examples are like this. For example, two people trying to go to a movie together can still get some reduced payoff for watching the movie alone in the event that they go to different movie theaters. There will always be simplifying assumptions in models that do not capture all of the complexity of the real world contexts we are interested in. However, Appendix D shows that the model's results persist when this particular simplifying assumption is removed. Finally, stepping away from current social identities and their associated politics, we explain the model using an example of social groups from ancient Mesopotamia accompanied by a fictional story of coordinating on greetings.

[1] Presumably, coworkers judged to be more professional will likewise be judged to be better partners for collaborative work.

Figure 1: Ancient Sumerian cities that will be discussed.

In ancient Mesopotamia there were two rival Sumerian cities, Umma and Lagash. The rivalry between the two cities stemmed from a disagreement over who would control the fertile land between them. The temple of Lagsh's patron diety, Ningirsu, was located in the city of Girsu on the edge of the disputed land. Around 2550 BCE, the king of Kish, a Sumerian city far to the north, negotiated a treaty between the two cities.[2] In this treaty, Umma was allowed to farm some of Lagash's land, but was required to give Lagash a third of the crops grown on this land. Umma regularly tried hold back these tributes and suffered several military defeats by Lagash, who would reinstate treaty. Eventually, in the 2300s BCE, Umma triumphs over Lagash. However, the victory is short lived. In 2334 BCE, Sargon of Akkad conquers Sumeria and establishes the Akkadian Empire, likely the first ever empire, stretching from the Mediterranean Sea to the Persian Gulf (Castor, 2006; Sallaberger and Schrakamp, 2015).[3] From this history, we will be able to discuss complex group structures, such as a Girsu identity being a subset of the Laghash identity and Umma and Lagash being disjoint subsets of a Sumerian identity that is surrounded by a dominant population of Akkadians.

[2]This treaty is thought to be the earliest example of environmental dispute resolution (Sand, 2020).

[3]Interestingly, Sargon claims to have been the cup bearer to the king of Kish.

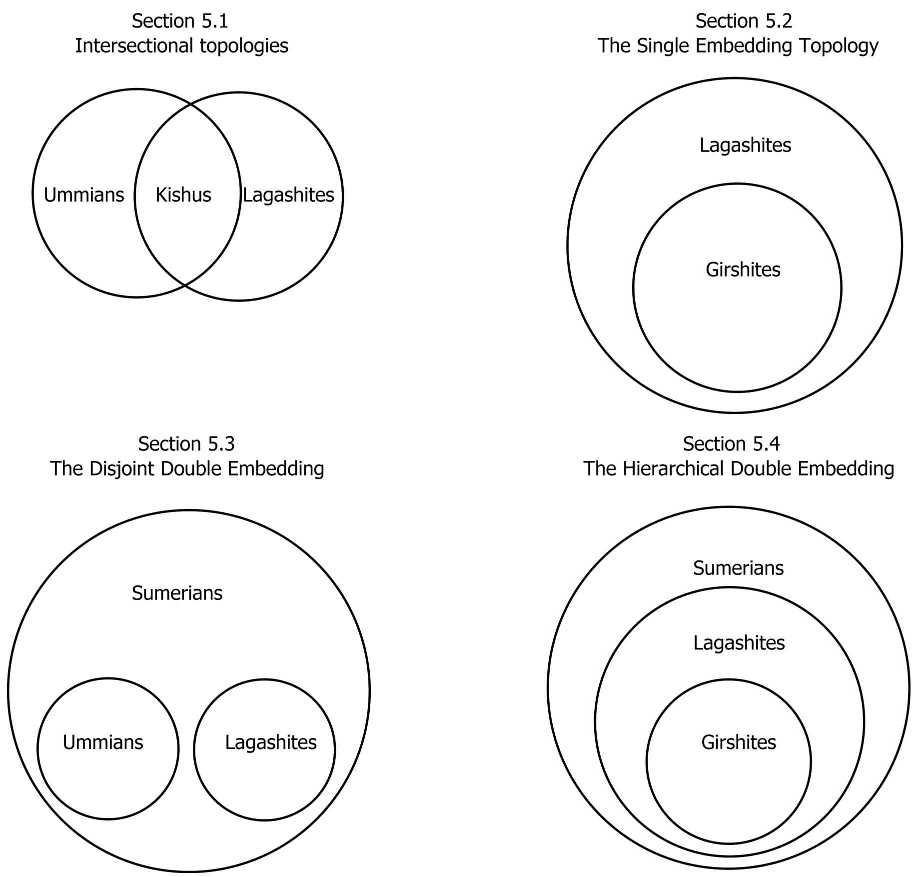


Figure 2: Overview of some group topologies that can be produced with this paper's model.

The way in which complex social identity structures are produced in the model takes some time to elaborate. In this paper, a large population of agents plays a coordination game, the generalized Bach or Stravinsky game, that is representative of how people coordinate on different preferences. Section 2.1 explains how the Bach or Strvinsky game, which was designed for just two players, is generalized to a large population of more than two players and a wide variety of possible preferences. Section 2.2 explains the learning dynamics that agents employ in this paper. At this point, the base model of the paper, which has no social identity signaling, has been fully described and simulation results for it are given in Section 2.3. Section 3 explains how various aspects of social identity signaling are added to the model and gives simulation results along the way. These simulations establish a few properties of the model that are carried forward as more complex signaling is added. First, we see that larger groups are privileged over smaller groups; specifically, the population as a whole is more likely to settle on the preferences of a larger group than a smaller group.

Second, we see that the presence of a larger group incentivizes a smaller group to adopt social identity signals that they might not otherwise use.

The big payoff of the paper then comes in Section 4, which show the emergence of the group structures depicted in Figure 2. The paper begins with a model in which there are two social signals, each of which is associated with a distinct identity group. In this model, there is a third group which broadcasts the social signals of both groups. This can be reflective of two different group topologies, one in which the third group that uses both social signals should not be considered a third group in its own right, and one in which it should be. We have objective reasons to think that people, who occupy the intersection of two marginalized groups, experience disadvantages that are not merely the sum across both groups of the disadvantages experienced by members of just one of the two groups (O'Connor et al., 2019). While this paper is not aimed at modeling intersectional disadvantage, it is important to keep the concept in mind. This is because it highlights a reason why we should not focus only on agents social signals when identifying group topologies. Section 4.1 shows how we can distinguish between two different group topologies that exhibit identical social signaling by looking at agents' actions in addition to their social signaling. Subsequent group topologies are identified by the conjunction of social signaling and actions that are reflective of the topology.

Sumerians	Ummians	Kishus	Lagashites	Girshites	Akkadians
⬡	□	⌂	△	♢	✲

Table 1: In diagrams, the different Mesopotamian identities will be associated with different shapes. These shapes are chosen to reflect the social relations between the different identities. Kish, as the original treaty negotiator between Ummma and Lagash, is associated the a pentagon shape, ⌂, that is a combination of the Umma square, □, and the Lagash triangle, △. Girsu, which houses the temple to Lagash's patron diety and is also the Lagashite settlement closest to the disputed land, is associated with a diamond shape, ♢, and can be thought of as two of the Lagash triangles, △. This might reflect the Girshites being the most zealous of the Lagashites. While these shapes will be useful for identifying different types of agents in diagrams, it is important to note that they do not represent characteristics that are visible to agents in the model.

Section 4.2 shows how a single embedding topology can emerge. In this section one group of agents signals the Lagashite identity while another group signals both the Lagashite and Girshite identities.[4] The Girshites are a strict subgroup of the Lagashites since everyone who signals the Girshite identity also signals the Lagashite identity. Additionally, Girshites adopt Lagashite preferences or norms when interacting with an agent who has only signaled the Lagashite identity while adopting Girshite preferences or norms when interacting

[4]While this topology only involves two groups, simulations involve three different types of agents. This is because having a third type of agent; that is more populous than the two of interest makes those two groups more likely to adopt social signals.

with other Girshites. Similar descriptions can be given for the disjoint double embedding in Section 4.3 and the hierarchical double embedding in Section 4.4. For example, in the disjoint double embedding, Ummians use both a Sumerian social signal and an Ummian social signal; likewise, Lagashites use both a Sumerian social signal and a Lagashite social signal. When Ummians and Lagashites interact with each other, they adopt the Sumerian preferences since they have the Sumerian identity in common. In Section 4.4, the hierarchical double embedding, Girshites broadcast a Sumerian, a Lagashite, and a Girshite social signal with all three social identity groups displaying actions that accord with the Venn diagram of the group topologies in Figure 2.

2 Model Description and Incremental Results

This section iterates between model description and simulation results for the model. It begins with a description of the traditional Bach or Stravinsky game and then extends the game to a more general framework and adds complexity in agents ability to broadcast identity signals and condition their actions on those signals. At each step in extending the game, simulation outcomes from the resulting model are presented. When appropriate motivations for the subsequent extension of the model are also included.

2.1 Generalizing the Bach or Stravinsky Game

The traditional Bach or Stravinsky (BoS) game is a one shot coordination game between two players. It was introduced to the literature by Luce and Raiffa (1957) as the "battle of the sexes" game, in which two players have differing preferences over whether to attend a prize fight or ballet, but get no payoff if attending alone:

		Player 2	
		Bach	Stravinsky
Player 1	Bach	$1+\alpha$, 1	0, 0
	Stravinsky	0, 0	1, $1+\alpha$

Table 2: Traditional Bach or Stravinsky Game: $\alpha \geq 0$.

More recently, authors have called this the "Bach or Stravinsky" game to dissociate it from various undesirable prejudices and stereotypes frequently attached to sex and gender while maintaining the same abbreviation, BoS.

What the game is intended to capture, rather than gender prejudices, is the dynamics of differing preferences or norms in contexts of coordination. While the choice between meeting at a prize fight or ballet can be a coordination problem, the class of contexts in which we have to coordinate is much broader than determining where to meet. We coordinate when deciding to shake hands or high five, or when choosing music for a party. Even in communication we can face a coordination problem in selecting the right name for something given both our own preferences and our audience's; e.g. the same thing might be called "battle of the sexes" or "Bach or Stravinsky", it might be called "the morning star", "the evening star", or "Venus", it might be called "the electric slide" or "the wobble".

As a first step in extending the BoS game to better capture its target phenomenon, we consider a model in which the game is played repeatedly among intermixing agents in a population. In this model, agents still interact pairwise, but who an agent is paired with varies as the game is repeatedly played. Agents payoffs in the game are given by their type. Thus, replacing the two players in the traditional BoS game we get two types in the generalized BoS game:

		Bachites	
		Bach	Stravinsky
Bachites	Bach	$1+\alpha$, $1+\alpha$	0, 0
	Stravinsky	0, 0	1, 1
		Stravinskians	
		Bach	Stravinsky
Bachites	Bach	$1+\alpha$, 1	0, 0
	Stravinsky	0, 0	1, $1+\alpha$
		Stravinskians	
		Bach	Stravinsky
Stravinskians	Bach	1, 1	0, 0
	Stravinsky	0, 0	$1+\alpha$, $1+\alpha$

Table 3: Generalized Bach or Stravinsky for a population of two types: $\alpha \geq 0$.

Since agents always get a payoff of 0 when they fail to coordinate, we can equivalently present agents' payoffs as their coordination preferences:

Coordination Preferences	Bach	Stravinsky
Bachites	$1+\alpha$	1
Stravinskians	1	$1+\alpha$

Table 4: Coordination Preferences: generalized BoS for a population of two types, $\alpha \geq 0$.

Table 4 expresses all of the information in Table 3, but does so much more concisely. This concise format will be particularly useful when we consider populations with more than just two types.

2.2 Learning Dynamics

The generalization of the BoS game just given is independent of learning dynamics. This paper presents results modeling agents as evolving their dispositions through replicator dynamics. The dynamics was selected because it has substantial established literature and is computationally tractable.

In a simulation of the model, agents begin with a randomly selected strategy profile. On each timestep, the prevalence of each strategy profile among a type is adjusted according to a discrete replicator equation:

$$N_{t+1}(x) = N_t(x) + N_t(x) \times [U(x) - Avg(U(i))_{i \in X}]$$

Where, for a given type, X is the set of all strategy profiles, $N_t(x)$ is the number of agents of the given type with strategy profile x at timestep t, $U(x)$ is the utility of strategy profile x, and $Avg(U(i))_{i \in X}$ is the average utility of all the strategy profiles present among the given type. In simple words, the replicator equation increases the prevalence of strategies that allow agents to more frequently succeed at coordinating (where successful coordination on a more preferred greeting is given more weight than successful coordination on a less preferred greeting).

A strategy profile's utility for a given type is calculated as:

$$U(x) = \sum_{i \in Y} [M(i) \times p_{xi}] \qquad \text{if } x \neq i$$

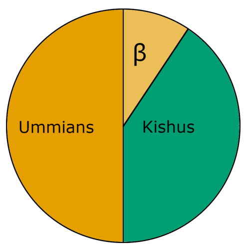
$$U(x) = \sum_{i \in Y} [(M(i) - 1) \times p_{xi}] \qquad \text{if } x = i$$

Where Y is the set of all (for any type) strategy profiles present in the population, $M(i)$ is the number of agents (of any type) in the population who play strategy i, and p_{xi} is the payoff an agent of the given type gets for playing strategy x when paired with an agent who plays strategy i. After adjusting the prevalence of strategy profiles, their quantity is normalized so that the number of agents of a given type remains constant throughout a simulation.

Finally, the possibility of an agent's strategy profile mutating is also included. This is governed by two parameters, m_0 and m_1. m_0 is the probability that an agent is selected for mutation. m_1 is the probability that an element in a string expression of the agents strategy profile changes to a random value. Presently, agents' strategy profiles are expressed as strings of length one, which specify which of the two actions an agent plays. An agents strategy profile can be $< B >$, play Bach, or $< S >$ play Stravinsky. So with probability m_0 an agent is selected for mutation, and if selected with probability m_1 a random value of B or S is used to replace the only element in the string expression of the agent's strategy profile. As the model is extended to allow agents to broadcast social signals and to choose their action based on the signal of a paired agent, the string expression of agents' strategy profiles will have length greater than one and the rational for using two mutation parameters will become more clear. Consequently, the mutation dynamic will be revisited at that point.

2.3 Simulation Results for BoS Generalized to a Population with Repeated Plays

To illustrate this simple version of the model, we consider a population with two types of agents, Ummians (□) and Kishus (⌂). In addition to the parameters listed in Table 6, Ummians (□) make up $0.5 + \beta$ proportion of the population (and, accordingly Kishus (⌂), are a $0.5-\beta$ proportion). Coordination preferences are:



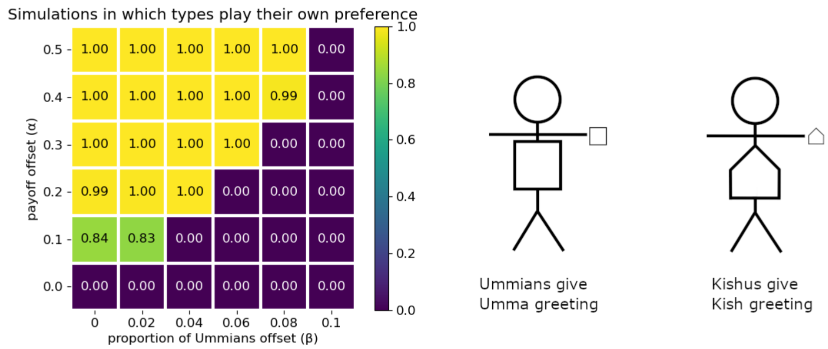
Coordination Preferences	Umma greeting	Kish greeting
Ummians □	$1 + \alpha$	1
Kishus ⌂	1	$1 + \alpha$

Table 5

This represents a population of two types of agents, where one type, Ummians, has the Umma greeting as their most preferred greeting and the other type, Kishus, has the Kish greeting as their most preferred greeting. Since Umma and Kish are friendly with each other, people still get some payoff for coordinating on their less preferred greeting. There is no social signaling in this base model, which means people are not able to use a social signal to determine which greeting the should use. Consequently, there are only two prominent types of outcomes in the simulations. When preferences are weak (when the difference in payoff for coordinating on an agents preferred greeting verses the alternative greeting is small), then everyone uses the same greeting because the benefit of always coordinating outweighs the benefit of always giving the most preferred greeting but frequently failing to coordinate behavior. When preferences are strong, then each type of person in the population always gives their preferred greeting because the payoff from that greeting outweighs the failures in coordination when people of different types interact. The simulations also show that when the Ummians (□) outnumber the Kishus (⌂) (i.e. when β increases) and preferences are weak (i.e. α is small), then the population as a whole always settles on the Umma greeting as the norm.

Figure 3 shows simulation results for populations of 1,000 agents. The models produces three possible outcomes: (i) each type plays their preferred action all of the time (shown in Figures 3a), (ii) everyone plays Ummians's preferred action (shown in Figures 3b), and (iii) everyone plays Kishus' preferred action,(deducible as 1 - value in Figure 3a - value in Figure 3b). Figurs 3a illustrates that as payoffs for preferred action increase, agents are more likely to always play their preference leading to coordination failures when agents of different types are paired.

(a) outcome (i)



(b) outcome (ii)

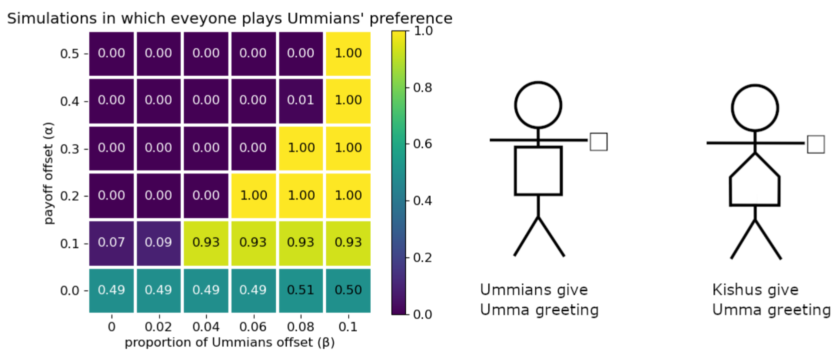


Figure 3: (a) Proportion of simulations that resulted in outcome (i) in which agents always give their preferred greeting; this means that when agents of different types fail to coordinate. (b) Proportion of simulations that resulted in outcome (ii) in which everyone gives Ummians' preferred greeting.

Figure 3b illustrates that as the proportion of Ummians (□) agents increases, the outcome in which everyone plays Ummians's preference, the Umma greeting, becomes more likely. This is because, even though Kishu agents have a higher payoff when they successfully coordinate on the Kish greeting, they have more opportunities for successful coordination with Ummians, since the Ummians (□) are more prevelant. Thus Kishu agents can have a higher net payoff from playing Ummians' preference. Appendix A shows this analytically. The only time when agents all play Kishus' preference (outcome iii), the Kish greeting, is when $\alpha = 0$, but this just means that agents do not actually have any preference between the two actions. Accordingly see that outcome (iii) occured in half of the simulations with $\alpha = 0$ (recall that this is deducible as 1 - value in Figure 3a - value in Figure 3b).

	Number of simulations	Simulation length	Population size	m_0	m_1	Signal cost, c
Section 2.3 Figure 3	10,000	4×10^4	1,000	0.01	0.1	n/a
Section 3.2 Figure 5	10,000	4×10^4	1,000	0.01	0.1	n/a
Section 3.4 Figures 7 & 11	1,000	8×10^4	1,000	0.01	0.1	-0.01
Section 4.1 Figure 14 & 15	1,000	4×10^4	1,000	0.01	0.1	-0.0002
Section 4.2 Figure 17	1,000	4×10^4	1,000	0.01	0.1	-0.0005
Section 4.3 Figure 19a	100	4×10^4	1,000	0.01	0.1	-0.0002
Section 4.3 Figure 19b	100	4×10^4	1,000	0.05	0.2	-0.0002
Section 4.4 Figure 21	100	4×10^3	1,000	0.05	0.2	-0.0005

Table 6: Additional details about simulation parameters in this paper.

3 Extending the Generalized Bach or Stravinsky Game

3.1 Adding Signals and Assortment to the Generalized BoS Game

In our next extension of the model, we allow agents to broadcast social signals and to use their partner's signal to determine their action in the game. Additionally, a homophily parameter, h, is included to model contexts in which people interact more with those who broadcast the same social signals as themselves.

Now that agents broadcast social signals, their strategy profiles can be expressed as strings of length 1+# of signals, where the first element is occupied by the social signal that an agent broadcasts and the subsequent elements are occupied by the action that is played when paired with an agent who broadcasts the corresponding signal. For example, suppose the there are two signals 1 and 2, with everything else remaining as before. Then there are 2^3 strategy profiles: $< 1BB >$, $< 1BS >$, $< 1SB >$, $< 1SS >$, $< 2BB >$, $< 2BS >$, $< 2SB >$, and $< 2SS >$. The strategy profile $< 1BB >$ corresponds to an agent broadcasting social signal 1, playing Bach when paired with an agent who broadcasts 1, and playing Bach when paired with an agent who broadcasts 2; the strategy profile $< 1BS >$ corresponds to an agent broadcasting social signal 1, playing Bach when paired with an agent who broadcasts 1, and playing Stravinsky when paired with an agent who broadcasts 2; etc.. Now we can see the rational for the two parameters for mutation. Suppose $m_0 = 0.01$ and $m_1 = 0.1$, then an agent has a 1 in 100 chance of being selected for mutation, and if selected each element in the string expression of the agent's strategy profile has a 1 in 10 chance being changed to a random value. This results in mutations to a new strategy profile that is similar to the prior strategy profile being more likely than mutations to a strategy profile that is maximally dissimilar. This improves learning when there is a large set of possible strategy profiles.[5]

Intuitively, the homophily parameter captures random assortment when $h = 0$ and assortment where agents who broadcast the same signal are twice as likely to interact as agents who broadcast differing signals when $h = 1$. Mathematically, this is captured by using the following utility function:

$$U(x) = \sum_{i \in Y} [H(h, x, i) \times M(i) \times p_{xi}] \qquad \text{if } x \neq i$$

$$U(x) = \sum_{i \in Y} [H(h, x, i) \times (M(i) - 1) \times p_{xi}] \qquad \text{if } x = i$$

where $H(h, x, i)$ is defined as:

$$H(h, x, i) = \frac{N}{\sum_{j \in Y} [S(h, x, j)]} \times S(h, x, i)$$

where N is the total number of agents in the population and $S(h, x, j)$ is:

$$S(h, x, j) = 2^h \quad \text{if profile x entails broadcasting the same social signal as j}$$

$$S(h, x, j) = 1 \quad \text{if x does not entail broadcasting the same social signal as j}$$

It is easy to see that this amounts to the same utility function as before when $h = 0$. Likewise, it is easy to see how $S(h, x, i)$ reflects agents with profile x having twice as much utility (when $h = 1$) from interactions with agents with profile i, due to more frequent interactions, when profiles x and i entail broadcasting the same social signal. The component of the equation requiring some explanation is $\frac{N}{\sum_{j \in Y} [S(h,x,j)]}$. This factor normalizes $S(h, x, i)$ relative to the signaling dispositions of the entire population.

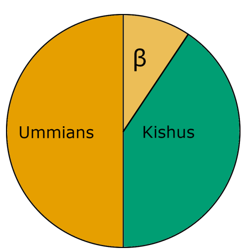
To see why this normalization is necessary, consider counterfactually what would happen if we just defined $H(h, x, i)$ as equal to $S(h, x, i)$. Suppose further that for a population with the preferences of Table 5, every agent is broadcasting the same signal and playing their preferred action irrespective of the signal observed. If each type makes up half of the population, then agents are successfully

[5] Functionally similar behavior could be obtained with a single parameter rather than two by using just the m_1 parameter with a smaller value (say $m_0 \times m_1$) and always selecting all agents for mutation. However, the two parameter version is also computationally more tractable.

coordinating in about half of their interactions. The signal being broadcast is meaningless for assortment since everyone is broadcasting the same signal. But suppose one agent mutates and starts broadcasting a different signal. Given the uniformity of everyone else in the population, the mutated agent is still just as likely to interact with a Bachite agent as a Stravinskian agent. Consequently, it will still be the cast that about half of the agent's interactions will result in successful coordination. However, with our counterfactual condition that $H(h, x, i)$ as equal to $S(h, x, i)$. The mutated agent's utility would suddenly be cut in half even though the agent is just as likely to successfully coordinate on her preferred action as before. But this makes no sense. By including the normalization factor $\frac{N}{\sum_{j \in Y}[S(h,x,j)]}$ in our definition of $H(h, x, i)$, we ensure that agents only see an increase in utility from broadcasting a particular signal in proportion to how much that signal impacts the frequency of their interactions.[6]

3.2 Simulation Results for Generalized BoS with Signals and Assortment

The simulations in this section continue with the example already given in Section 2.3, in which there are just two types of agents, Ummians (□) and Kishus (⌂). The only difference is that now the agents broadcast one of two social signals (● or ●), choose their action based the signal they observe from a partner, and, if $h \neq 0$ assort more frequently with others who broadcast the same social signal. As before, Ummians (□) make up $0.5 + \beta$ proportion of the population (and Kishus (⌂) are a $0.5 - \beta$ proportion). Coordination preferences are:



Coordination Preferences	Umma greeting	Kish greeting
Ummians □	$1 + \alpha$	1
Kishus ⌂	1	$1 + \alpha$

Broadly described, the simulation results show that people almost always learn to use the signals to broadcast their social identity for coordination. When

[6] If one wished to model a scenario in which an agent, in virtue of adopting an unused signal, simply ceased to interact with all agents since no other agents shared that signal, then it still would not make sense to let $H(h, x, i)$ equal $S(h, x, i)$. For that scenario, $S(h, x, i)$ should also be modified to be 0 when profile x does not entail broadcasting the same social signal as i. However, such antisocial behavior seems out of place given that this paper is modeling agents in coordination games where there is some common ground between agents of different types; i.e. an agents who prefers Bach still gets some benefit for coordinating on Stravinsky. In adopting the normalized equation, this paper reflects agents having the same number of interactions regardless of their signal, but preferentially interacting with agents who share their signal.

Umma people interact with themselves, they give the Umma greeting. Likewise, when Kish people interact with themselves, they give the Kish greeting. But when Umma people interact with Kish people, the population still has to settle on whether to coordinate on the Umma greeting or the Kish greeting. The more numerous population is still advantaged. If the Umma people outnumber the Kish people, then the population is more likely to settle on the Umma greeting as the norm when people of different types interact.

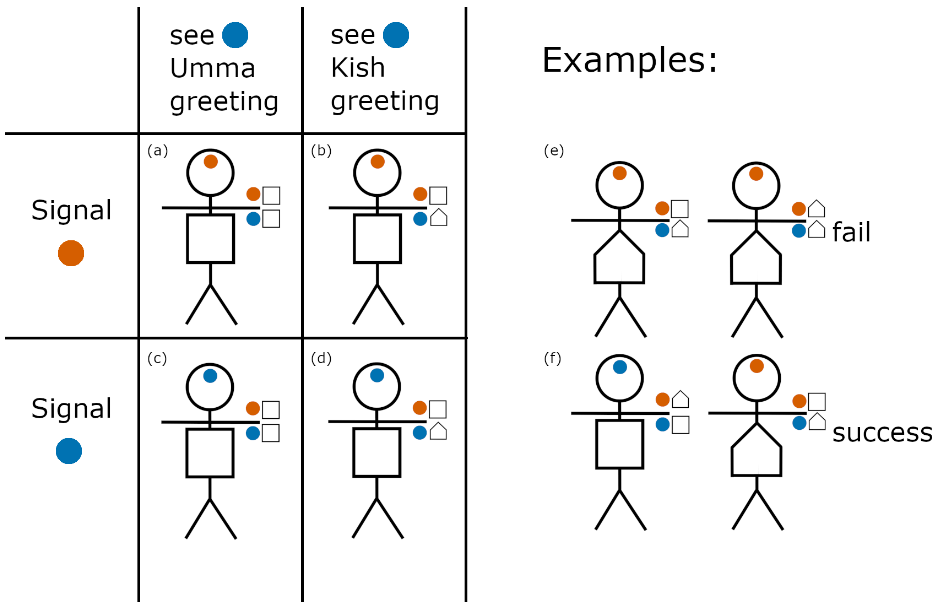


Figure 4: Diagrams explanation: (a) An Ummian who signals ● and gives the Umma greeting in response to both signals. (b) An Ummian who signals ● and gives the Umma greeting in response to ● and the Kish greeting in response to ●. (c) An Ummian who signals ● and gives the Umma greeting in response to both signals. (d) An Ummian who signals ● and gives the Umma greeting in response to ● and the Kish greeting in response to ●. (e) Two agents who fail to coordinate actions. Both agents broadcast ●, but one gives the Umma greeting in response and the other the Kish greeting. (f) Two agents who successfully coordinate. The Ummian □ broadcasts ● and gives the Kish greeting when observing ●. The Kishu ⌂ broadcasts ● and gives the Kish greeting when observing ●. Consequently, the interaction results in successful coordination.

More precisely, there are four different outcomes that frequently obtain in this model. In outcome (i) each type plays their preferred action all of the time is possible, but this outcome never occurs under the parameters shown in this section. Outcomes in which (ii) everyone plays Ummians' preferred action, and (iii) everyone plays Kishus' preferred action, are outcomes in which agents fail to make use of the social signals available to them.[7] The outcomes in which (iv) Ummians (□) always play their preferred action and Kishus (⌂) play Umma greeting with Ummians (□) and play the Kish greeting among themselves, and (v) Kishus (⌂) always play their preferred action and Ummians (□) play the Kish greeting with Kishus (⌂) and play the Umma greeting among themselves are optimal outcomes which depend on the agents evolving meaningful signals. Finally, though infrequent, (vi) there are a variety of suboptimal outcomes in which meaningful signals evolve but at least one type does not play their preferred action among themselves.

Figure 5 shows all optimal outcomes (outcomes (iv) and (v)) on the left and just the optimal outcome favoring Ummians (□) (iv) on the right (so it is impossible for a value on the right to exceed the corresponding value on the left). Outcome (iv) favors Ummians (□) because it involves agents coordinating on the Umma greeting whenever an interaction between agents of different types occurs. Just as before, increasing the proportion of Ummians (□) (i.e. increasing β) makes outcomes favoring Ummians (□) more frequent. Thus, as β increases, the numbers on the right hand side converge towards those on the left. Likewise, when the proportion of Ummians (□) and Kishus (⌂) is equal (i.e. $\beta = 0$), the values on the right (b & d) are roughly half of those on the left (a & c). The top of the figure (a & b) is for simulaltions with no assortment ($h = 0$), and the bottom (c & d) is for simulations with strong assortment ($h = 1$). Comparing the top and bottom shows that, in this simple model, assortment increases the likelihood of optimal outcomes obtaining. This is because the assortment dynamics creates another avenue of incentives for agents to use social signals.

[7]The frequency of this outcome is close to 1- the frequency of (iv) - the frequency of (v), which can be deduced from the figures given in this section. However, it is more precisely 1- the frequency of (iv) - the frequency of (v) - the frequency of (vi); and in a few limited cases (vi) made up close 0.06 proportion of outcomes though in most cases the proportion was closer to 0.

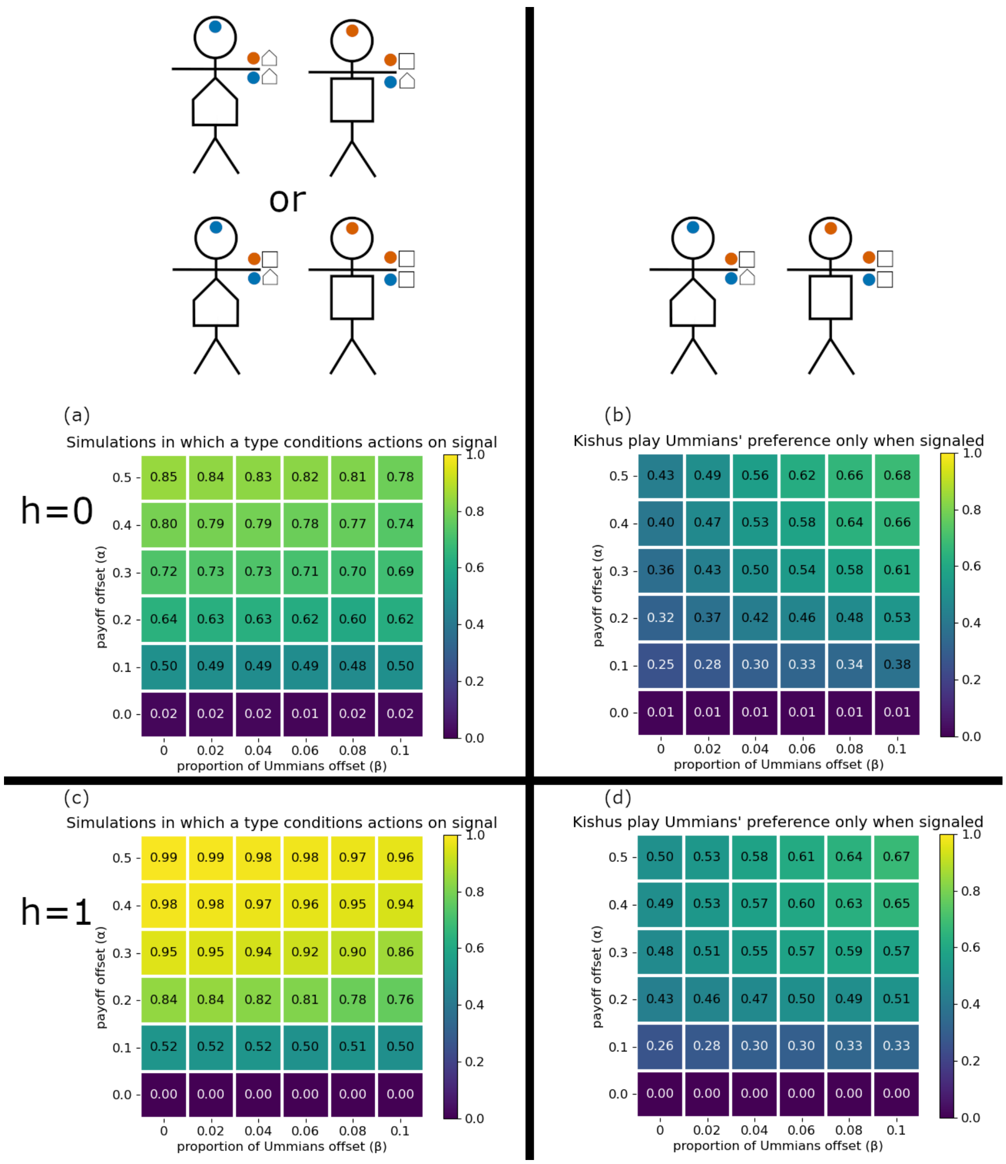


Figure 5: (a) Proportion of outcomes that are (iv) or (v) for $h = 0$. (b) Proportion of outcomes that are (iv) for $h = 0$. (c) Proportion of outcomes that are (iv) or (v) for $h = 1$. (d) Proportion of outcomes that are (iv) for $h = 1$.

3.3 Adding Attention and Signal Costs to the Generalized BoS Game

It is often the case that social signals require some effort to broadcast or attend to and only those groups for whom the social information is relevant invest the effort in engaging with the signals; among a subpopulation individuals may invest effort in signaling whether they are lefts or rights, while the broader population remains entirely oblivious to the social signals surrounding them. To capture this, the model is extended with a special signal, 0, which indicates an agent is not attending to signals and a signal cost, c, that an agent incurs if she broadcasts any signal other than 0. When an agent does not attend to signals, i.e. when she broadcasts 0, she interacts with all other agents as if they had also broadcast 0. Interacting with all other agents as if they broadcast 0 means that actions cannot be chosen based on the social signal that was broadcast, which is exactly what should be the case for an agent who is not attending to the social signals.

To illustrate this extension of the model, lets consider how the example from Section 3.1 changes under this new extension. While there were previously two signals 1, and 2, there are now three signals 0, 1, and 2. Consequently there are now 3×2^3 strategy profiles: $< 0BBB >$, $< 0BBS >$, $< 0BSB >$, $< 0BSS >$, $< 0SBB >$, $< 0SBS >$, $< 0SSB >$, $< 0SSS >$, $< 1BBB >$, $< 1BBS >$, $< 1BSB >$, $< 1BSS >$, $< 1SBB >$, $< 1SBS >$, $< 1SSB >$, $< 1SSS >$, $< 2BBB >$, $< 2BBS >$, $< 2BSB >$, $< 2BSS >$, $< 2SBB >$, $< 2SBS >$, $< 2SSB >$, and $< 2SSS >$. Now, it might be noted that the strategy profiles $< 0BBB >$, $< 0BBS >$, $< 0BSB >$, and $< 0BSS >$ all involve the same dispositions since an agent who does not attend to signals will treat agents who signal 1 or 2 as if they had signaled 0, i.e. agents with these profiles will always play B. Likewise, the strategy profiles $< 0SBB >$, $< 0SBS >$, $< 0SSB >$, and $< 0SSS >$ all involve the same dispositions since an agent who does not attend to signals will treat agents who signal 1 or 2 as if they had signaled 0, i.e. agents with these profiles will always play S. However, if an agent with one of these strategy profiles mutates in just the signaling position of the strategy profile the resulting strategy profile differs depending on what the agents prior profile was even when comparing two profiles that entailed the same dispositions. E.g. while $< 0SBS >$ and $< 0SSB >$ entail the same dispositions, a mutation to signaling 1 instead of 0 results in either $< 1SBS >$ and $< 1SSB >$ which entail very different dispositions. For this reason, distinct string representations of strategy profiles are considered distinct strategy profiles even if those distinct strings entail the same dispositions.[8]

At the macro level, this extension of the model changes nothing in our for-

[8] It is also computationally convenient to have all string representations of strategy profiles be the same length.

mula for the utility of a strategy profile:

$$U(x) = \sum_{i \in Y} [H(h, x, i) \times M(i) \times p_{xi}] \qquad \text{if } x \neq i$$

$$U(x) = \sum_{i \in Y} [H(h, x, i) \times (M(i) - 1) \times p_{xi}] \qquad \text{if } x = i$$

However, the p_{xi} component of the formula does change. Previously, p_{xi} was simply the payoff specified in the agent's coordination preferences for an action if strategy profiles x and i lead to coordination on the same action and zero otherwise. Now, if strategy profile x entails broadcasting a social signal that is not zero, then p_{xi} is the payoff specified in the agent's coordination preferences for an action plus c if strategy profiles x and i lead to coordination on the same action and c otherwise (c is added to the prior p_{xi} value because this paper's convention is to use negative values for signal costs). If strategy profile x entails broadcasting 0, then p_{xi} is unchanged since no signaling cost is incurred. For example, if two agents have a coordination preference of 1 for coordinating on Bach and $c = -0.1$ then they will have a payoff of $p_{xx} = 1$ if both agents use strategy profile $x =< 0BBS >$; but, if both agents use strategy profile $x =< 1SBS >$ then they will have a payoff of $p_{xx} = 1 - 0.1 = 0.9$.

In this model, it is always the case than an agent not conditioning her actions on signals and not broadcasting a social signal co-occur. However, in the real world, these are independent dispositions. An agent might abstain from broadcasting a social signal, but still use social signals to determine what action to perform. Conversely, an agent might broadcast a social signal, but not pay attention to signals when choosing what action to perform. Combining these two dispositions so that they always co-occur is computationally convenient, plausible for some social contexts, and does not inhibit us from producing some interesting and novel social signaling topologies. So, for now, we maintain this assumption and leave investigation of the model with independence between these two dispositions as a task for future research.

3.4 Simulation Results for Generalized BoS with Attention and Signal Costs

This section extends the prior example of coordination between Ummians (□) and Kishus (⌂) by adding in Akkadians (✲). Since Akkadians (✲) get nothing from any of the Sumerian greetings, they should always give the Akkadian greeting and have no reason to pay the cost of broadcasting a social signal. Simulations show that the presence of the Akkadians (✲) can increase the likelihood that the Ummians (□) and Kishus (⌂) use social signals for coordination. However, the simulations also show that if the Ummians (□) are sufficiently numerous, they can end up always giving the Umma greeting and never signaling.[9]

[9] The preferences of Ummians (□) and Kish are symetric. If Kishus (⌂) are sufficiently numerous, they can also end up always giving the Kish greeting and never signaling. Since the two populations preferences are symetric, results are only shown for increasing the proportion of Ummians.

Diagrams of the agents are extended to show an agents disposition towards the null signal, which is represented as and underscore:

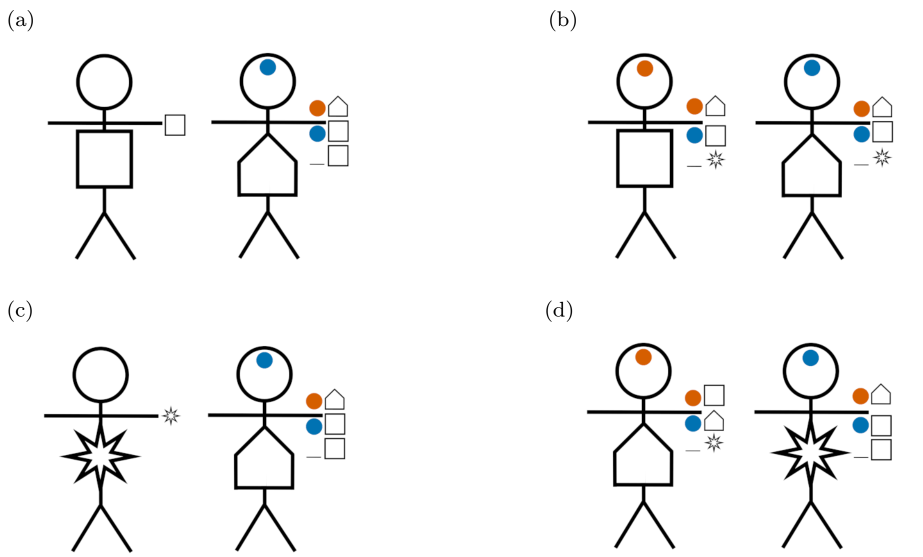
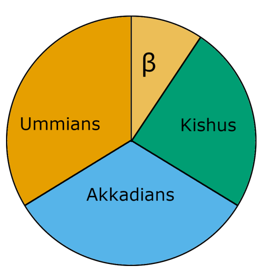


Figure 6: Diagram examples of interactions with the attention dynamic: (a) An Ummian (□) who does not attend to signals and always gives the Umma greeting. A Kishu (⌂) who gives the Umma greeting when she observes the null signal. This results in successful coordination. (b) These agents fail to successfully coordinate. The Ummian (□) sees ● and gives the Umma greeting. The Kishu (⌂) sees ● and gives the Kish greeting. (c) These agents fail to successfully coordinate. The Akkadian (✵) does not attend to social signals and always gives the Akkadian greeting. The Kishu (⌂) gives the Umma greeting when no social signal is observed. (d) These agents successfully coordinate. The Kishu (⌂) broadcasts ● sees ● and gives the Kish greeting in response. The Akkadian (✵) broadcasts ● sees ● and gives the Kish greeting in response.

Ummians (□) make up $0.33+\beta$ proportion of the population, Kishus (⌂) are a $0.33-\beta$ proportion of the population, and Akkadians (✵) are a 0.34 proportion of the population. Coordination preferences are:



Coordination Preferences	Umma greeting	Kish greeting	Akkadian greeting
Ummians (□)	$1+\alpha$	1	0.5
Kishus (⌂)	1	$1+\alpha$	0.5
Akkadians (✵)	0	0	1

The inclusion of a third type of agent, Akkadians (✲), who have no incentive to condition actions on signals, since the Akkadian greeting is the only action with a payoff, makes signaling more prevalent for the other two types. To illustrate this, Figure 7b keeping all other parameters the same considers the model where Ummians (□) make up $0.5+\beta$ proportion of the population, Kishus (⌂) are a $0.5-\beta$ proportion of the population; i.e. there are no Akkadians.

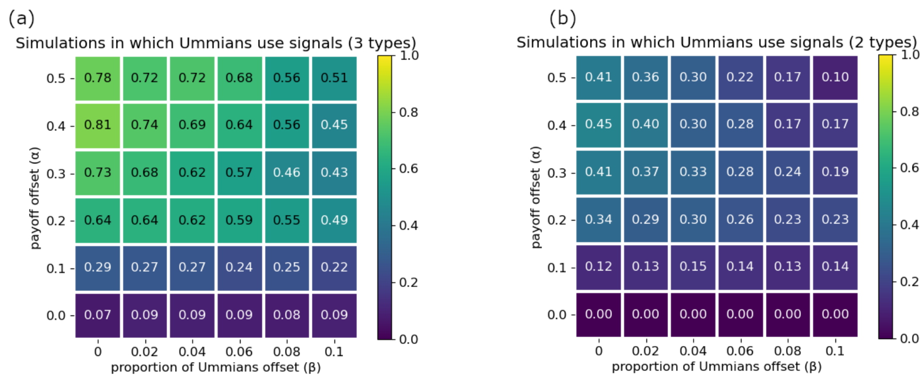


Figure 7: (a) Proportion of outcomes in which Ummians (□) broadcast a social signal, for the model with three types of agents. (b) Proportion of outcomes in which Ummians (□) broadcast a social signal, for the model with two types of agents.

For the parameters investigated, there were thirteen different outcomes that occurred at least 0.5% of the time for some data point. These outcomes are given a detailed description in Appendix C. This section shows three prominent outcomes.

(viii) The outcome in which Ummians (□) and Akkadians (✲) always played their respective preference, and Kishus (⌂) played the Umma greeting with Ummians, the Kish greeting with other Kishus, and the Akkadian greeting with Akkadians. The prevalence of this outcome is shown in Figure 11a. This outcome was frequently characterized by Ummians (□) agents signaling 0 and Kishus (⌂) and Akkadians (✲) attending to signals that reliably identified their type. One can check that this is a Nash equilibrium. It is suboptimal in the sense that when Ummians (□) are paired with Akkadians, there is a failure to coordinate. As previously noted, the Akkadians (✲) should always give the Akkadian greeting. So one might wonder why this population ends up signaling? I.e. why can this behavior be a Nash equilibrium? The reason is that if the Kishus (⌂) are giving the Umma greeting when they see the null signal (since this allows coordination with Ummians), then they will only coordinate with Akkadians (✲) if the Akkadians (✲) broadcast a social signal. So, Akkadians (✲) are not attending to the signals because they are conditioning their actions on the signals. Rather they are attending to the signals because they benefit from those signals allowing Kishus (⌂) to coordinate with them.

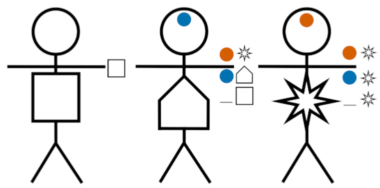


Figure 8: Outcome (viii)

(xii–xiii) Agents condition their actions optimally, in the sense that there were never failures of coordination and agents played their most prefered greeting among themselves. The frequency of this outcome is shown in Figure 11b.

(xii) The optimal outcome favoring Ummians (□).

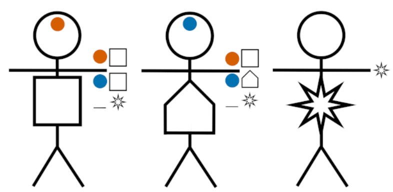


Figure 9: Outcome (xii)

(xiii) The optimal outcome favoring Kishus (⌂).

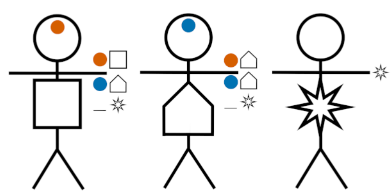


Figure 10: Outcome (xiii)

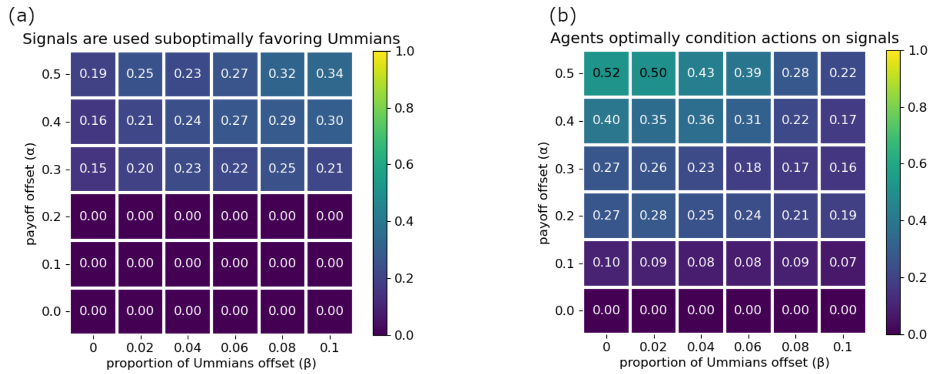


Figure 11: (a) Proportion of outcomes that are (viii). (b) Proportion of outcomes that are (xii-xiii).

Adding in the Akkadians (✲) makes Ummians (□) more likely to transmit a signal when β is small (see Figure 7b). When Ummians (□) become more prevalent in the population (i.e. when β increases), signaling from Ummians (□) decreases since they have leverage to always play their preferred action and Akkadians (✲) frequently adopt one of the two signals (this type of outcome is shown in Figure 11a). But the general trend is clear, minority groups have more incentive to use social signals.[10] In Figure 7a, there is some slight stratification around $\alpha = 0.2$ (i.e. what increasing α seems to make signaling more prevelant, there is a decrease in signaling from $\alpha = 0.2$ to $\alpha = 0.3$ for $\beta \geq 0.06$). This seems to be a consequence of the threshold for Ummians (□) and Kishus (⌂) being highly incentivized to play their preference among ingroup members occurs around $\alpha = 0.2$; while the threshold for Ummians (□) to use their power as the largest group to only play their preference occurs around $\alpha = 0.3$ (see also Figure 11a).[11]

3.5 Adding Multiple Dimensions to Signals in the Generalized BoS Game

The final extension this paper makes to the model is allowing agents to broadcast social signals in multiple dimensions. Agents can broadcast up to one social signal in each dimension available. This can be thought of as allowing agents to broadcast independent social signals in correspondence with independent social signals. For example, signaling one is a Baltimore Orioles fan by wearing

[10] At first glance, one might object that this does not follow from the results since Ummians (□) become the largest group as β increases. However, this response is unsound. Figure 7 clearly shows that Ummians (□) are less likely to signal as they become a larger proportion of the population. Additionally, one can check in supplemental PDF that outcome (viii) (shown here in Figures 11a) typically involves Akkadians (✲) broadcasting a social signal; which is exactly what needs to be the case to support the claim that minority groups have more incentive to use social signals.

[11] This stratification is more pronounced in larger populations (see Appendix B).

a corresponding baseball cap is incompatible with wearing a Washington Nationals cap, but entirely independent of signaling one is a San Antonio Spurs fan by wearing a corresponding basketball jersey. In this example signals in the baseball dimension are mutually exclusive, but independent of signals in the basketball dimension. In the model whether or not an agent attends to signals in one dimension is independent of whether another dimension is attended to. Thus, if there are two dimensions for signaling and two signals in each dimension (excluding the null signal), then there are nine different social signals an agent can broadcast, letting signals in the first dimension be 1, 2 and the signals in the second dimension be 3, 4: 00, 03, 04, 10, 20, 13, 14, 23, and 24 are all possible signals.

In the string representation of strategy profiles, there is one place for each dimension of signaling and one place for each possible signal. Thus, if there are two dimensions for signaling and two signals in each dimension (excluding the null signal) and two actions, then there are $3^2 \times 2^9 = 4608$ unique strategy profiles. The signal cost c is incurred fore each dimension attended to. Thus if strategy profile x leads to successful coordination with agents employing strategy profile i by performing an action with coordination preferences value of 1, then if $x =< 00BBSBSBBSS >$ this results in $p_{xi} = 1$, if $x =< 20BBSBSBBSS >$ this results in $p_{xi} = 1 + c$, and if $x =< 14BBSBSBBSS >$ this results in $p_{xi} = 1 + 2c$.

In this extension of the model our utility function is still

$$U(x) = \sum_{i \in Y} [H(h, x, i) \times M(i) \times p_{xi}] \qquad \text{if } x \neq i$$

$$U(x) = \sum_{i \in Y} [H(h, x, i) \times (M(i) - 1) \times p_{xi}] \qquad \text{if } x = i$$

where $H(h, x, i)$ is defined as:

$$H(h, x, i) = \frac{N}{\sum_{j \in Y} [S(h, x, j)]} \times S(h, x, i)$$

However the definition of $S(h, x, j)$ is amended to be:

$$S(h, x, j) = 2^{kh}$$

where k is the number of dimensions in which x entails broadcasting the same social signal as j. It is easy to check that when there is only one dimension of signals this equation is equivalent to our prior equation.

In general this sections' amended equations are quite simple and intuitive. p_{xi} is amended to include costs in proportion to the number of dimensions attended to, higher cost for attending to more dimensions. Likewise assortment is modified for agents to have greater preference towards interacting with agents who share their social signal in more dimensions; when $h = 1$ an agent exhibits twice the preference for interacting when an agents who signals identically in two dimensions than those who signal identically in one dimension and exhibits four times the preference for interacting with an agents who signals identically in two dimensions than those who do not signal identically in any dimensions.

4 Complex Group Topologies

Now we have the full model. There are no more new dynamics to introduce. The multidimensional signaling allows agents to represent their common interests when their interests only partially overlap. For example, two agents can broadcast the same signal in one dimension and different signals in another. The following subsections show four different contexts in which agents take advantage of this capacity for multidimensional signaling. Given the increased complexity of the model, this paper only discusses the optimal outcomes from simulations.[12] That means an outcome has to have both optimal signaling behavior and corresponding actions to be reported. What is discussed is how agents' preferences produce the relevant topologies. Section 4.1 looks at agents who occupy the intersection of two groups. It shows how the same signaling system can reflect different topologies. This section occurs first for the purpose of making it clear why it is important to not focus exclusively on the social signaling systems that evolve, but to also consider the actions that are performed by agents using those signals. Section 4.2 shows how a single group can be a embedded in another. That is, one group signals in a single dimension and the other adopts the same signal in that dimension but also uses a signal in another dimension. Finally, Sections 4.3 and 4.4 each present a different way in which two groups can be embedded in another.

[12]Except, in Section 4.1.

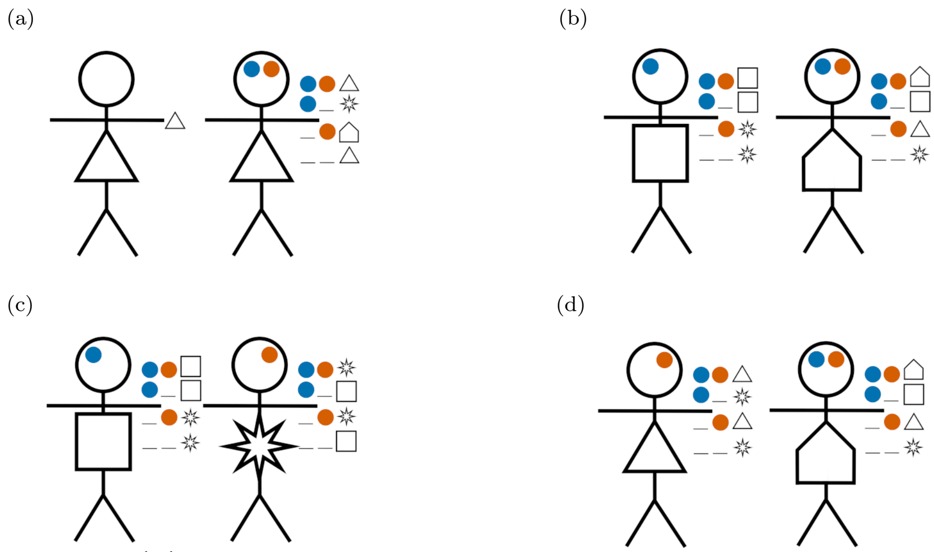


Figure 12: Diagram examples of interactions with multidimensional signals. (a) Successful coordination, both agents give the Lagash greeting (△). (b) Successful coordination, both agents give the Umma greeting (□). (c) Agents fail to coordinate. The Ummian (□) only attends the first dimension and, consequently, does not see any social signal; so she gives the Akkadian greeting. Likewise, the Akkadian (✲) only attends the second dimension and, consequently, does not see any social signal; so, she gives the Umma greeting. (d) Successful coordination, both agents give the Lagash (△) greeting.

4.1 The Intersection

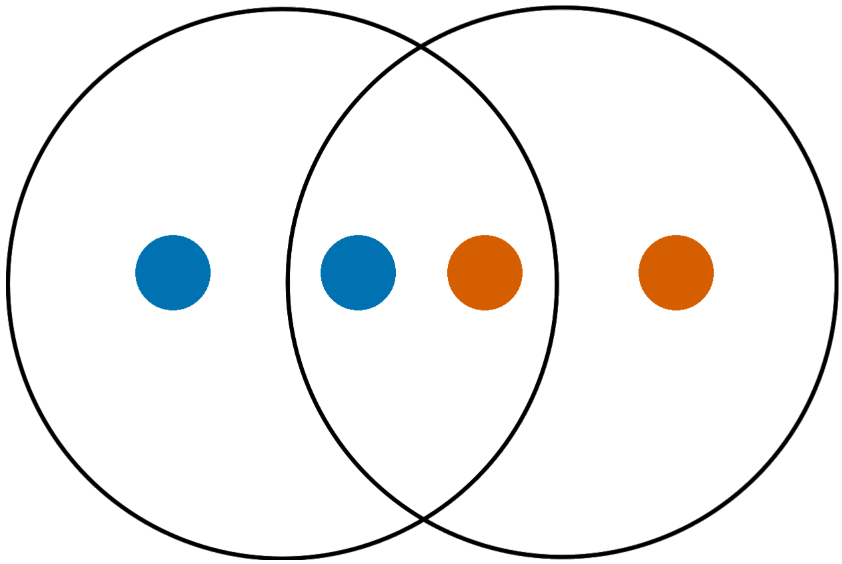
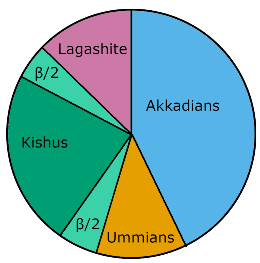


Figure 13: A signaling system in which we can investigate intersectionality.

This section presents a set of model parameters that almost always produce three types of signalers: ●_ signalers, ●● signalers, and _● signalers. The purpose of considering this signaling system is to discuss when people who are members of two or more social identity groups should be considered as composing their own group that is not reducible to the summation of properties of the other groups they belong to; e.g. when should ●● signalers be considered their own distinct group rather than being considered people who just happen to be members of the ●_ signalers group and the _● signalers group.

To this end, consider a population with four types of agents: Ummians, Kishus, Lagashites (△) , and Akkadians. Akkadians (✲) are included merely to promote signaling among the other types of agents. Since Umma and Lagash are rivals, they will be our ●_ and _● signalers; of course, the model does not presuppose that Ummians (□) and Lagashites (△) are 10 and 02 signalers (respectively or vice versa). Rather, Ummians (□) and Lagashites (△) agents almost always learn to signal their identity in different dimensions because they are assigned coordination preferences such that the optimal greeting between an Ummian and a Lagashite is the Akkadian greeting. Or, stated more simply, Ummians (□) and Lagashites (△) learn to signal in different dimensions because they have almost nothing in common. Kishus (⌂) are assigned preferences that are the union of Ummians (□) and Lagashites' preferences. Additionally, Kishus, depending on α, can have a payoff of 0, 1, or 2 for coordinating on the Kish greeting. This corresponds to type 1 agents having no additional preferences compared to the union of Ummians (□) and Lagashites' preferences, nominal additional preferences, and substantial preference for a greeting that is unique to themselves. So, we should expect Kishus (⌂) to act as merely being members of both the Ummian and Lagashite groups when $\alpha = 0$ and to act as a distinct group in their own right when $\alpha = 2$.

In the simulations, Ummians (□) make up $0.2 - 0.5 \times \beta$ proportion of the population, Kishus (⌂) are a $0.2 + \beta$ proportion of the population, Lagashites (△) are a $0.2 - 0.5 \times \beta$ proportion of the population, and Akkadians (✲) are a 0.4 proportion of the population. There is one signal in each of two dimensions. Coordination preferences are:



Coordination Preferences	Umma greeting	Kish greeting	Lagash greeting	Akkadian greeting
Ummians (□)	1	0	0	0.5
Kishus (⌂)	1	α	1	0.5
Lagashites (△)	0	0	1	0.5
Akkadians (✲)	0	0	0	1

Figure 14 shows how frequently we see the signaling system shown in Figure 13. But this signaling system cannot be identified with a single group topology. The proportion of outcomes identified in Figure 15a have the Kishus (⌂) behaving as if their identity is a mere conjunction of the Ummian (□) and Lagashite (△) social identities; i.e. rather than giving the Kish greeting among themseleves, they settle on giving either the Umma or Lagash greetings. The outcomes in Figure 15b have the Kishus (⌂) behaving as if their identity is *not* a mere conjunction of the Ummian and Lagashite social identities; i.e. among themselves they use the Kish greeting which has no payoff for Ummian and Lagashite agents. These outcomes align with what one should expect from the coordination preferences. When $\alpha \leq 1$, Kishus (⌂) have no added benefit to coordinating on the Kish greeting over the Umma or Lagash greetings; and this is exactly where we see the Kushus behaving as if their identity is a mere conjunction of the Ummian and Lagashite identities.

When $\alpha > 1$, Kishus (⌂) have an added benefit to coordinating on the Kish greeting over the Umma or Lagash greetings; and this is exactly where we see Kishus (⌂) behaving as if their identity is a not mere conjunction of the Umma and Lagash identities. That said, these are trends not rules. For $\alpha = 2$, around 25-31% of simulations (depending on β) resulted in Kishus (⌂) using the Umma or Lagash greetings among themseleves even though they would have a higher payoff if they coordinated on the Kish greeting among themselves. This seems to indicate a blindspot for quantitative research as it is how people signal and coordinate that is observable, not their preferences prior establishing conventions; when sufficiently marginalized, this model suggests that a group of people may interact, even among themselves, in a way that obscures their preference for an alternative.

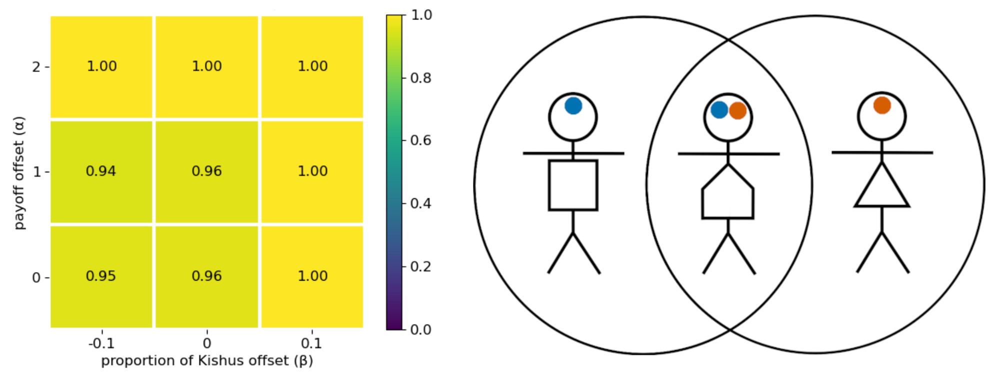


Figure 14: Proportion of outcomes in which Ummians (□) signal in one dimension, Kishus (⌂) singal in two dimensions, Lagahites (△) signal in one dimension that is different than the Ummians (□) dimension, and Akkadians (✵) do not signal in any dimension.

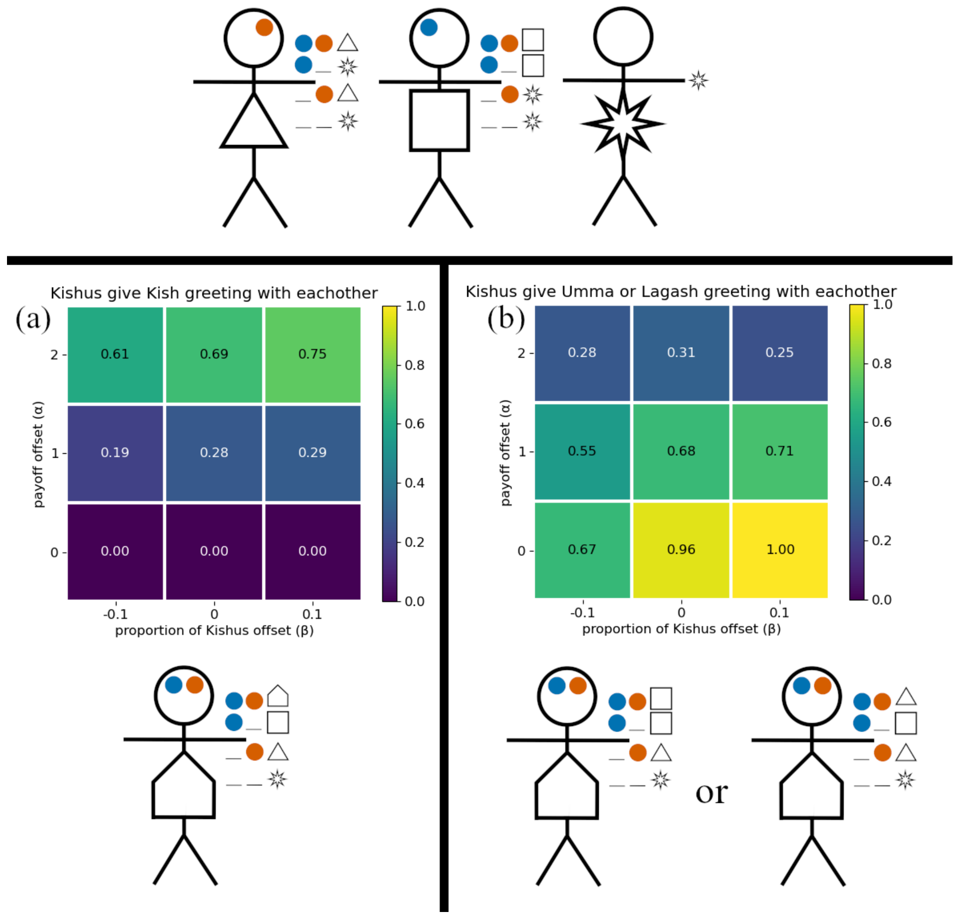


Figure 15: (a) Proportion of outcomes in which Kishus (⌂) give the Kish greeting with eachother in addition to exhibiting the desired signaling behavior. (b) Proportion of outcomes in which Kishus (⌂) give the Umma or Lagash greeting with eachother in addition to exhibiting the desired signaling behavior; i.e. Ummians (□) signal in one dimension, Kishus (⌂) signal in two dimensions, Lagahites signal in one dimension that is different than the Ummian dimension, and Akkadian do not signal in any dimension.

4.2 The Single Embedding Topology

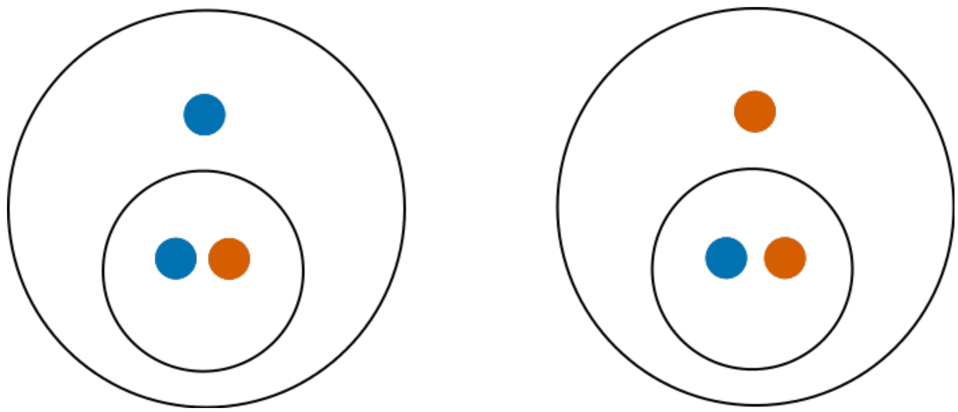


Figure 16: The two possible single embedding signaling systems when given one signal in each of two dimensions.

Recall from the introduction that Girsu is home to the temple of Ningirsu, the patron god of Lagash. So, in our fictional story of Mesopotamian greetings, Girshites ($\diamondsuit$) most prefer the Girsu greeting and their second preference is the Lagash greeting. Continuing the story, Lagashites ($\triangle$) get nothing from the Girsu greeting and Akkadians (✲) get nothing from either the Lagash or Girsu greetings. While Lagashites ($\triangle$) and Girsu get some payoff from the Akkadian greeting, it is a smaller payoff than what they get from the Lagash greeting. These relations are modeled with the coordination preferences table below. To allow Girshites ($\diamondsuit$) to signal their connection with Lagash while also signaling their unique identity, we allow agents one signal in each of two dimensions. Thus, if Lagashites ($\triangle$) adopt a social identity signal, Girshites ($\diamondsuit$) can broadcast that same signal to indicate their shared identity with Lagashites ($\triangle$) while also broadcasting a signal in the other dimension to indicate the component of their identity that is unique to Girsu.

Coordination Preferences	Lagash greeting	Girsu greeting	Akkadian greeting
Lagashites ($\triangle$)	1	0	0.5
Girshites ($\diamondsuit$)	1	$1+\alpha$	0.5
Akkadians (✲)	0	0	1

Before looking at simulation results, we should expect these coordination preferences to produce the single embedding topology. That is, (i) we should expect the Akkadians (✲) to not signal in either dimension because they should always give the Akkadian greeting irrespective of an agents social signal; (ii) we should expect Lagashites ($\triangle$) to only signal in a single dimension because this is the minimal number of signals sufficient for coordination with the Akkadians (✵) on the Akkadian greeting and coordination with others on the Lagash greeting, if we assume that condition (i) has already been met; (iii) assuming conditions (i) and (ii) are met, we should expect the Girshites ($\Diamond$) to signal in both dimensions because signaling in the same dimension as Lagashites ($\triangle$) will result in those people giving the Lagash greeting rather than the Akkadian greeting (which is preferable to Girshites), and signaling in the additional dimension will allow Girshites ($\Diamond$) to coordinate on the Girsu greeting among themselves. In the heat maps below, the single embedding topology is only counted as having obtained if *both* the signaling and the corresponding dispositions towards greetings stated in (i)-(iii) obtain. Though rare, it is possible for the desired signaling to occur without the corresponding greetings behavior. It is for this reason that Figure 16 is captioned as "the two possible single embedding *signaling systems*" rather than as "the two possible single embedding *topologies*". Figure 16 only depicts the desired signaling behavior. The term "topology" is reserved for situations in which both the desired signaling and behavior occur.

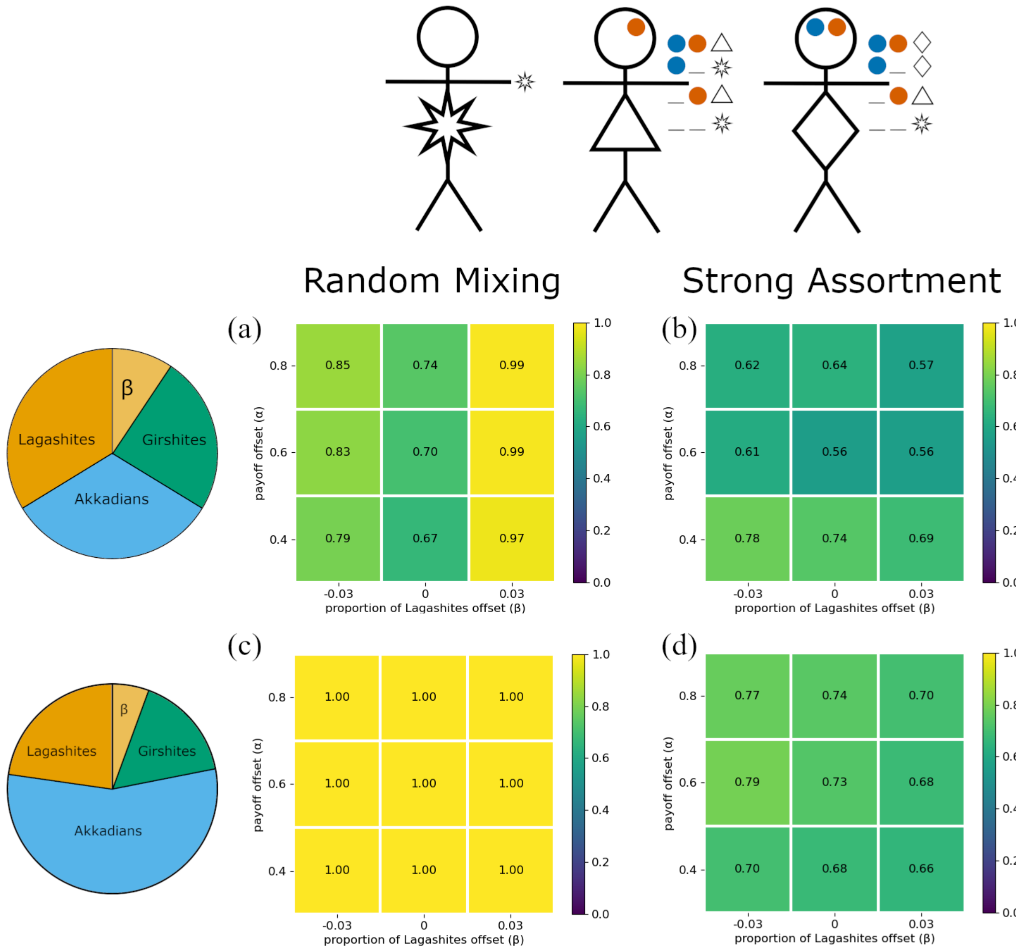


Figure 17: **(a)** Proportion of outcomes that are the single embedding topology, homophily = 0. Lagashites (△) make up $0.33+\beta$ proportion of the population, Girshites (◇) are a $0.33-\beta$ proportion of the population, and Akkadians (✵) are a 0.34 proportion of the population. **(b)** Proportion of outcomes that are the single embedding topology, homophily = 1. Lagashites (△) make up $0.33+\beta$ proportion of the population, Girshites (◇) are a $0.33-\beta$ proportion of the population, and Akkadians (✵) are a 0.34 proportion of the population. **(c)** Proportion of outcomes that are the single embedding topology, homophily = 0. Lagahites make up $0.2+\beta$ proportion of the population, Girshites (◇) are a $0.2-\beta$ proportion of the population, and Akkadians (✵) are a 0.6 proportion of the population. **(d)** Proportion of outcomes that are the single embedding topology, homophily = 1. Lagahites make up $0.2+\beta$ proportion of the population, Girshites (◇) are a $0.2-\beta$ proportion of the population, and Akkadians (✵) are a 0.6 proportion of the population.

Comparing Figure 17 (a) with Figure 17 (c), we see that, when there is no assortment, increasing the proportion of the population that is Akkadian makes the desired topology more likely to obtain. But when there is significant assortment, Figure 17 (b) and (d), this is no longer the case. This can be understood through the fact that when there is no assortment, $h = 0$, then the Akkadians (✲) have no incentive to signal and should always give the Akkadian greeting irrespective of signal. However, when there is assortment, even though it is still the case that Akkadians (✲) should always give the Akkadian greeting irrespective of signal, they can benefit from adopting a signal for assortment in early timesteps of a simulation because the other types of agents have not yet learned to use a signal (or lack of signal) to coordinate on the Akkadian greeting when interacting with an Akkadian. But if other agent types only learn to give the Akkadian greeting when observing the signal broadcast by Akkadians, then Akkadians (✲) have continuing incentive to continue using their social signal.

4.3 The Disjoint Double Embedding

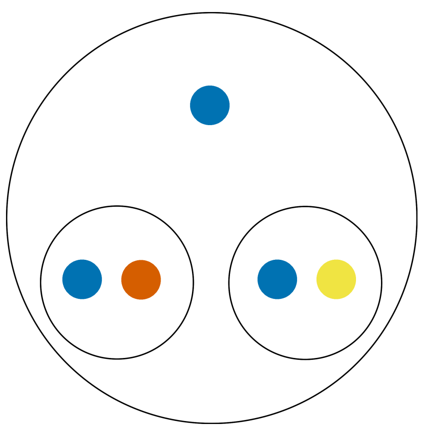
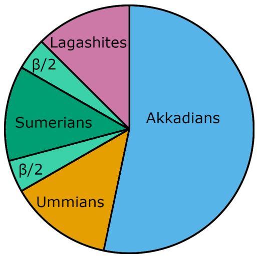


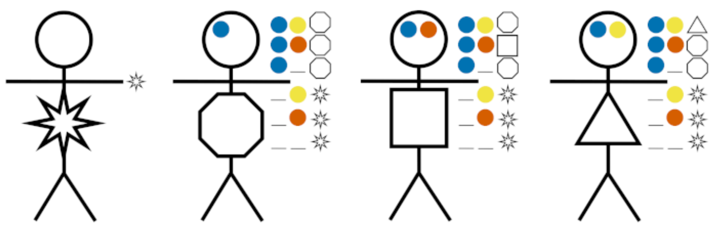
Figure 18: A disjoint double embedding signaling systems that are possible when given just two dimensions with one signal in the first dimension and two in the second.

To produce the disjoint double embedding we want two subgroups who have more in common with the larger group than they do with each other. So we consider Ummians (□) and Lagashites (△) as our two potential subgroups. Umma and Lagash had a history of conflict with each other stemming from a boarder dispute. In fact, Lugalzagesi's (from Umma) conquest of Lagash is plausibly what weakened the Sumerians (○) enough for Sargon of Akkad to conquer them and impose Akkadian as the requisite language for government. So we assign coordination preferences such that the optimal greeting between an Ummian and a Lagashite is the general Sumerian greeting; as always, among themselves their strongest preference is to use the greeting of their own city. Coordination preferences are:



Coordination Preferences	Sumerian greeting	Umma greeting	Lagash greeting	Akkadian greeting
Sumerians (○)	1	0	0	0.25
Ummians (□)	1	$1+\alpha$	0	0.25
Lagashites (△)	1	0	$1+\alpha$	0.25
Akkadians (✵)	0	0	0	1

Sumerians (○) make up $0.15+\beta$ proportion of the population, Ummians (□) are a $0.15-0.5\times\beta$ proportion of the population, Lagashites (△) are a $0.15-0.5\times\beta$ proportion of the population, and Akkadians (✵) are a 0.55 proportion of the population. There is one signal in the first dimension and two in the second.



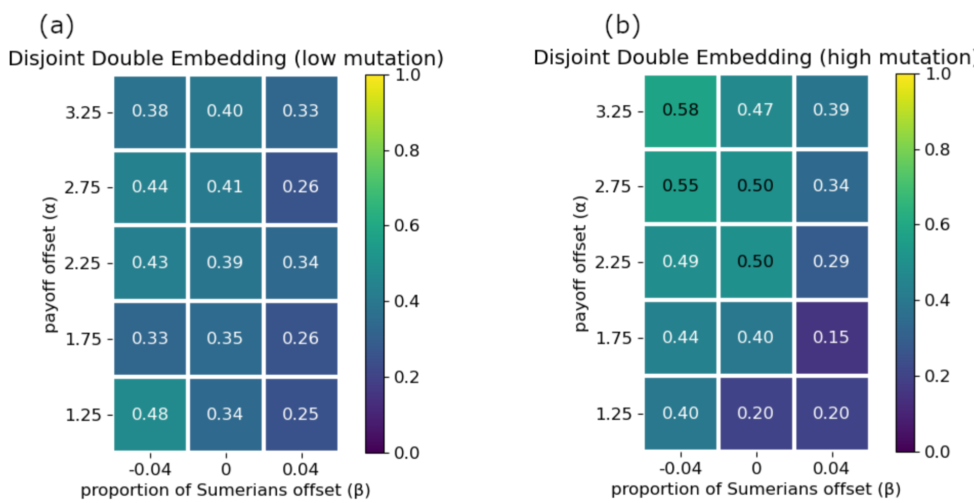


Figure 19: Proportion of outcomes that are the disjoint double embedding topology when (a) the mutation rate is $m_0 = 0.01$ and $m_1 = 0.1$, or (b) the mutation rate is $m_0 = 0.05$ and $m_1 = 0.2$.

Figure 19 shows results using the same mutation rate as prior sections, $m_0 = 0.01$ and $m_1 = 0.1$, as well as results with a higher mutation rate, $m_0 = 0.05$ and

$m_1 = 0.2$. In general, it seems learning is more effective with higher mutation rates as the learning context become more complex. This also requires using higher payoffs to cut through the noise of the increased mutation rate. However, it is difficult to make these claims with certainty as these more complex systems are less computationally tractable, which precludes the more robust parameter sweeps that were done with simpler models. Still there is a rationale for why higher mutation rates might be helpful for learning in the more complex systems. The system that produces the single embedding topology in Section 4.2 only has 324 different possible strategy profiles. Thus, we can expect a large portion of the different strategy profiles to be present on the first timestep of a simulation when they are assigned. Mutation is most likely not needed to produce an agent with any given strategy profile. However, the system in this section allows 24,576 different possible strategy profiles. This means that for a population of 1,000 agents, on any given timestep, there is at most around 4% of the different strategy profiles present in the population. Consequently, a higher mutation rate is likely beneficial in virtue of allowing agents to explore a broader swath of the possible strategy profiles.

4.4 The Hierarchical Double Embedding

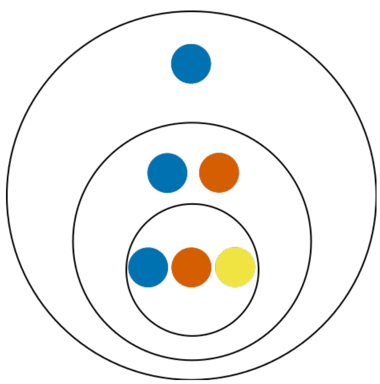
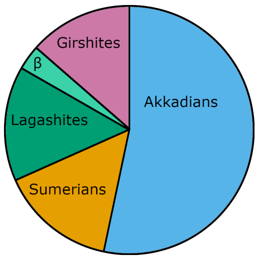


Figure 20: A hierarchical double embedding signaling system when given one signal in each of three dimesions.

To produce the hierarchical double embedding topology, consider an extension of the example from the prior section. We continue the scenario from the single embedding of the Girshites (◊) into the Lagashites, but add in coordination preferences for Sumerians (○) in general. So we have Girshites (◊) and Lagash having more in common with people each other than Sumerians (○) in general, but also preferring the Sumerian greeting over the Akkadian greeting. This can be represented with coordination preferences:



Coordination Preferences	Sumerian greeting	Lagash greeting	Girsu greeting	Akkadian greeting
Sumerians (○)	1	0	0	0.25
Lagashites ($\triangle$)	1	$1 + 0.5 \times \alpha$	0	0.25
Girshites ($\diamond$)	1	$1 + 0.5 \times \alpha$	$1 + \alpha$	0.25
Akkadians (✵)	0	0	0	1

Sumerians (○) make up 0.15 proportion of the population, Lagashites ($\triangle$) are a $0.15 + \beta$ proportion of the population, Girshites ($\diamond$) are a $0.15 - \beta$ proportion of the population, and Akkadians (✵) agents are a 0.55 proportion of the population. There is one signal in each of three dimensions.

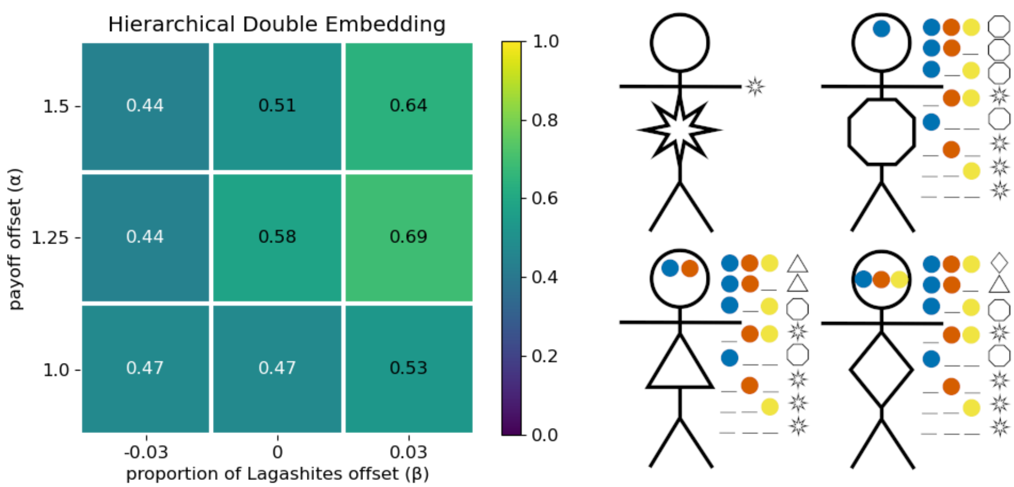


Figure 21: Proportion of outcomes that are the hierarchical double embedding topology.

This set of parameters was the least computationally tractable. Given one signal in each of three dimensions, along with a null signal in each dimension, and four possible greetings for each possible combination of signals, then there are 524,288 different possible strategy profiles. Since the replicator dynamics require tracking all strategy profiles present and quantifying over them to calculate utilities relative to an average, we were unable to find an implementation of the algorithm that did not run into either a computational or memory bottleneck.[13] We can be confident that the coordination preferences given do frequently produce the hierarchical double embedding topology. However, there simply is not enough information to make claims about trends that occur when varying the

[13] Consequently, future research will explore alternative learning dynamics that scale better with increased complexity in the system.

α and β parameters, or even to make the claim that Figure 21 captures the optimal parameters for producing the topology.

5 Discussion

In summary, Section 2.3 considered the Bach or Stravinsky game when generalized to a large population of two types of agents who repeatedly interact with each other rather than being confined to a one off game with a population of two. Simulations showed a clear advantage for the larger of the two types of agents; when both types of agents played the same greeting it was always the greeting preferred by the larger group. This advantage for larger groups persisted throughout subsequent additions to the model. Section 3.2 then looked at simulations that added signals and assortment to the model. Both with and without assortment, agents used social signals to better coordinate behavior. In Section 3.4 looked at the model with the addition of attention and signal costs. Simulations showed that a dominate group could ignore social signals and always use their preferred greeting. They also showed that agents of minority types have increased incentive to broadcast social signals when there is a large dominant group in the population. Finally, Sections 4.1-4.4 showed how a variety of different group structures could obtain in systems with multidimensional signals. When the preferences of a type align more strongly with a type compared to another, social signals often obtain in a way that reflects this alignment in preferences.

The relative population size of different types and the ordering of the types' preferences are the most important variables for understanding the mechanics of the model. As noted in Section 2.2, the generalized Bach or Stravinsky game could be paired with entirely different learning dynamics. Most likely, different learning dynamics would lead to different payoff values being optimal for producing the group structures that this paper has investigated. Sections 4.3 showed that merely changing the mutation rate of the learning dynamics cased different payoffs to be optimal for producing the disjoint double embedding topology. But the same ordering of agents preferences (e.g. types 1 and 2 both still preferred the Summerian greeting over the Akkadian greeting) produced the same topology despite the change in what specific payoff values were optimal for producing the topology. There is a straightforward reason for this; under the given ordering of preferences, irrespective of specific payoff values, the disjoint double embedding topology is an optimal outcome. So it seems reasonable to should expect that under different, but sufficiently intelligent, learning dynamics the same ordering of agents' preferences will produce the same group topologies, because only in these topologies do agents optimally coordinate behavior.

It is good news that we have reason to believe the model can produce the same types of group identity topologies under a variety of learning dynamics. This is because there are both practical and conceptual reasons to look for alternative learning dynamics. Practically speaking, both Sections 4.3 and 4.4 noted computational constraints that stemmed from the replicator dynamics requiring quantifying over all agents' strategy profiles. This is a fast computation for simple systems but quickly becomes bottlenecked as the number of possible strategy profiles grows exponentially. Investigating learning dynamics in which an agent's strategy profile evolves directly as a consequence of her own successes or failures, rather than as a consequence of her successes or failures relative to the population of all agents of her type, might provide a path for overcoming current computational constraints.[14] But there is also a conceptual reason for wanting learning dynamics that do not involve quantifying over all agents of a given type. That is, one can worry that in quantifying over agents of a given type, we are in some sense presupposing the group structures that we are explaining the emergence of. Strictly speaking, this concern is ill-founded. Two different types can certainly evolve the same strategy profiles (e.g. Figure 3b, or if two types are assigned the same preferences). Conversely, two different strategy profiles can be stable within the same type of agents (e.g. supplemental PDF page 89). Still, for the sake of robustness, it is worth showing that the same group topologies can obtain on alternative learning dynamics.

There are other modeling assumptions that we might test the robustness of with a more computationally tractable learning algorithm. To name a few, the model could be modified to relax the assumption that abstaining from broadcasting a social signal in a dimension always co-occurs with not attending to others signals in that dimension, or the assumption that assortment occurs at a fixed rate rather than being a disposition that agents can learn to employ over time. Both of these assumptions limit the number of possible strategy profiles in the current model and are consequently difficult to explore. The model also assumes that coordination occurs exclusively in dyadic interactions rather than in groups of varying size, which is again a simplifying assumption worth investigating in the future. Lastly, the current simulations provide agents with only as many signals as are necessary for a target topology to occur. This can be very limiting. For example, in the system for producing the disjoint double embedding topology, if the type 0 agents are the first to settle on a social signal and that signal happens to be in the dimension in which there are two signals rather than one, then it is very unlikely that the desired topology will obtain since it would require the type 0 agents to abandon the social signal they are benefiting from using. This limitation could be overcome by allowing agents two signals in both of the dimensions, but this causes the number of strategy profiles to go from 24,576 to 2,359,296, which is not a computational tractable system. So there are many directions in which future models might benefit from employing an alternative learning dynamic.

All of that said, there are questions that can be addressed using the gener-

[14] For example, Roth-Erev reinforcement learning (Erev and Roth, 1998) would be a learning dynamic that fits this description. Preliminary investigations showed that basic Roth-Erev reinforcement learning is not powerful enough to solve the learning problem that agents in the BoS game face. However, there are known extensions to Roth-Erev reinforcement learning that are stronger and have yet to be explored; e.g. Barrett and Gabriel (2022)'s reinforcement with iterative punishment. It also seems plausible that Schlag (1998)'s pairwise proportional imitation dynamic could overcome the computational constraints; though it is less clear whether the learning dynamic could be modified to avoid the conceptual problems.

alized BoS model with the learning dynamics already employed. To give one example, consider how sensitive agents signaling behavior is to changes in relative population sizes. This might be relevant for predicting real world changes in how people signal their social identities when moving from a context in which they are members of a majority group into a context where they are now a minority. While this sort of scenario has already been considered in Smaldino (2019), multidimensional social identities opens up a broader range of possibilities. Suppose, hypothetically, that on the island of Fiji, Fijians, pacific islanders in general (who are not Fijian), and Tongans form a single embedding topology like the one described in Section 4.2, where Fijians can abstain from social signaling as they are the dominate majority on the island and can assume their cultural norms without thought, and Tongans are embedded in the broader minority group of pacific islanders (who are not Fijian) in general. While prior models with one dimensional social identities could explain why Fijians who immigrate to the United States might find signaling their identity more important than they did prior to immigrating, these prior models could not say anything about how the overall group structure might change. It could be that if Fijians, pacific islanders in general, and Tongans are all small enough compared to the broader American population, then they might form just a single shared social identity. Or, if Tongans are the most prominent group among pacific islander immigrants, they might come to be the outermost group in a hierarchical embedding. To see how the current generalized BoS model will make predictions about the effects of these types of environmental changes, we simply need to check what happens when fixing a subset of the population at the start of a simulation with the group topology that we take them to have prior to the immigration event. So, this is a simple and straightforward next step that we will take with the model.

There are also a wide variety of extensions to the generalized BoS model worth exploring. The generalized BoS game is a game of pure coordination. But we occupy a variety of contexts many of which do not involve coordination, but might impact our dispositions when we are trying to coordinate our actions (Bednar and Page, 2007). So it would be interesting to integrate the generalized BoS game into the mix of games that Bednar and Page (2007) explore. Worse than mere spillover in dispositions from different games, in some contexts, social identity signals are used to exploit or abuse minority population (Smaldino et al., 2018; Smaldino and Turner, 2020; Bright et al., 2022). Models of these contexts might seem to be in tension with a model of coordination, since exploitation and abuse incentivize agents to use unreliable social signals while coordination promotes reliable identity signals. But, the fact that the Generalized BoS game can produce multidimensional social identity signals might be a substantial asset in oppressive contexts. We know that coalition building between multiple marginalized groups is a powerful tool for combating injustice (Mandela, 2003; Parks and Warren, 2012). Multidimensional identity signaling is exactly the sort of thing that is needed to model agents of various marginalized groups signaling their unique social identities while also signaling their buy-in to a coalition that spans across multiple marginalized groups. In this way, incorporating the generalized BoS game with models of exploitation and oppression might be very fruitful since it can facilitate modeling coalition building.

In summary, this paper provides a first step in modeling agents with complex social identities. It is, as far as we know, among the first papers to show how to model the cultural evolution of multidimensional social identities. As such, it can be seen as complimenting the established literature that only employs one dimensional social signals. This paper has shown how the generalized BoS game can be used to produce a variety of different group topologies. There are some noted ways, related to computational efficiency, in which the model might be improved in the future. However, as it stands, the model is ready to be used for investigating some real world phenomena, such changes in group structures due to population migration. There are also some live prospects for combining the generalized BoS game with other games that have been used to stud cultural evolution.

References

Barrett, J. A. and Gabriel, N. (2022). Reinforcement with iterative punishment. *Journal of Experimental & Theoretical Artificial Intelligence.*

Bednar, J. and Page, S. (2007). Can game(s) theory explain culture?: The emergence of cultural behavior within multiple games. *Rationality and Society*, 19(1):65–97.

Berger, J. and Heath, C. (2008). Who drives divergence? identity signaling, outgroup dissimilarity, and the abandonment of cultural tastes. *Journal of personality and social psychology*, 95(3):593.

Bright, L. K., Gabriel, N., O'Connor, C., and Taiwo, O. (2022). On the stability of racial capitalism. *Ergo.*

Bunce, J. A. (2020). Field evidence for two paths to cross-cultural competence: implications for cultural dynamics. *Evolutionary Human Sciences*, 2:e3.

Castor, A. Q. (2006). *Between the Rivers: The History of Ancient Mesopotamia.* The Teaching Company.

Combahee River Collective (1977). The combahee river collective statement.

Crenshaw, K. (1991). Mapping the margins: Intersectionality, identity politics, and violence against women of color. *Stanford Law Review*, 43(6):1241–1299.

Delnero, P. (2016). Literature and identity in mesopotamia during the old babylonian period. In Ryholt, K. and Barjamovic, G., editors, *Problems of canonicity and identity formation in ancient Egypt and Mesopotamia*, pages 19–50. Museum Tusculanum Press, Copenhagen, Denmark.

Erev, I. and Roth, A. E. (1998). Predicting how people play games: Reinforcement learning in experimental games with unique, mixed strategy equilibria. *The American Economic Review*, 88(4):848–881.

Goodman, J. R., Caines, A., and Foley, R. A. (2023). Shibboleth: An agent-based model of signalling mimicry. *PLoS ONE*, 18(7).

Lin, W., Kang, S., Zhu, J., and Ding, L. (2024). Till we have red faces: drinking to signal trustworthiness in contemporary china. *Public Choice*, pages 1–20.

Luce, R. D. and Raiffa, H. (1957). *Games and Decisions: Introduction and Critical Survey*. Wiley, New York.

Macanovic, A., Tsvetkova, M., Przepiorka, W., and Buskens, V. (2024). Signals of belonging: emergence of signalling norms as facilitators of trust and parochial cooperation. *Philosophical Transactions B*, 379(20230029).

Mandela, N. (2003). *Long Walk to Freedom: The Autobiography of Nelson Mandela*. Number v. 2 in An Abacus Book. Abacus.

McCluney, C. L., Durkee, M. I., Smith II, R. E., Robotham, K. J., and Lee, S. S.-L. (2021). To be, or not to be... black: The effects of racial codeswitching on perceived professionalism in the workplace. *Journal of experimental social psychology*, 97:104199.

McElreath, R., Boyd, R., and Richerson, P. J. (2003). Shared norms and the evolution of ethnic markers. *Current Anthropology*, 44(1):122–130.

Moffett, M. W. (2013). Human identity and the evolution of societies. *Human Nature*, 68:219–267.

O'Connor, C., Kofi Bright, L., and Bruner, J. P. (2019). The emergence of intersectional disadvantage. *Social Epistemology*, 33(1):23–41.

Parks, V. and Warren, D. T. (2012). Contesting the racial division of labor from below: Representation and union organizing among african american and immigrant workers. *Du Bois Review: Social Science Research on Race*, 9(2):395–417.

Pelzer, K. (2023). What does an upside-down pineapple mean? the hidden message behind the symbol. *Parade*.

Roccas, S. and Brewer, M. B. (2002). Social identity complexity. *Personality and Social Psychology Review*, 6(2):88–106.

Sallaberger, W. and Schrakamp, I., editors (2015). *Arcane: Associated regional chronologies for the ancient near east and the Eastern Mediterranean. III, history and philology*. Brepols.

Sand, P. H. (2020). Environmental dispute resolution 4,500 years ago: The case of lagash v umma. *Yearbook of International Environmental Law*, 30(1):137–142.

Schlag, K. H. (1998). Why imitate, and if so, how?: A boundedly rational approach to multi-armed bandits. *Journal of economic theory*, 78(1):130–156.

Smaldino, P. E. (2019). Social identity and cooperation in cultural evolution. *Behavioural Processes*, 161:108–116. Behavioral Evolution.

Smaldino, P. E., Flamson, T. J., and McElreath, R. (2018). The evolution of covert signaling. *Scientific Reports*, 8(4905).

Smaldino, P. E. and Turner, M. A. (2020). Covert signaling is an adaptive communication strategy in diverse populations. *Center for Open Science*.

Appendix A Analysis of Section 2.3 Model

Recall that Section 2.3 gave the mean outcome of 1,000 simulations of the generalized BoS game with a population of 1,000 agents, $m_0 = .01$, $m_1 = .1$, and run for more than a sufficiently large number of time steps to reach an equilibrium (4×10^4).[15] Type 0 agents make up $0.5 + \beta$ proportion of the population (and type 1 agents are a $0.5 - \beta$ proportion). Coordination preferences are:

Coordination Preferences	Bach	Stravinsky
type 0	$1 + \alpha$	1
type 1	1	$1 + \alpha$

Table 7: Coordination Preferences: generalized BoS for a population of two types, $\alpha \geq 0$.

Recall that agents begin the simulation with randomly selected strategy profiles. Thus we can assume that roughly half of the agents play Bach and half play Stravinsky on the first timestep of a simulation. This should result in almost all of the agents playing their preferred action on the second time step. We can see this by first calculating the utilities:

$$U(\text{preferred action}) = 5 \times 10^2 \times (1 + \alpha)$$
$$U(\text{undesired action}) = 5 \times 10^2 \times 1$$

Then we can calculate the change in the population N(preferred action) as:

$$
\begin{aligned}
= &\quad 2.5 \times 10^2 + 2.5 \times 10^2 \times [5 \times 10^2 \times (1 + \alpha) - 0.5 \times (5 \times 10^2 \times (1 + \alpha) + 5 \times 10^2)] \\
= &\quad 2.5 \times 10^2 + 2.5 \times 10^2 \times [5 \times 10^2 \times (1 + \alpha) - 0.5 \times (5 \times 10^2 \times (1 + \alpha) + 5 \times 10^2 \times \frac{1+\alpha}{1+\alpha})] \\
= &\quad 2.5 \times 10^2 + 2.5 \times 10^2 \times 2.5 \times 10^2 \times (1 + \alpha) \times [2 - (1 + \frac{1}{1+\alpha})] \\
= &\quad 2.5 \times 10^2 + 2.5 \times 10^2 \times 2.5 \times 10^2 \times (1 + \alpha) \times (1 - \frac{1}{1+\alpha} \\
= &\quad 2.5 \times 10^2 \times (1 + 2.5 \times 10^2 (1 + \alpha) - 2.5 \times 10^2) \\
= &\quad 2.5 \times 10^2 \times (1 + 2.5 \times 10^2 \times \alpha)
\end{aligned}
$$

and N(preferred action) is the same with $\alpha = 0$ therefore the proportion of

[15] In these simulations, every 100 timesteps it was checked whether the distribution of agents' strategy profiles was unchanged. If so, the simualtion was halted prior to reaching 4×10^4 timesteps.

agents of a given type that play their preferred action should be:

$$
\begin{aligned}
& \frac{N(\text{preferred action})}{N(\text{preferred action}) + N(\text{undesired action})} \\
= & \frac{2.5 \times 10^2 \times (1 + 2.5 \times 10^2 \times \alpha)}{2.5 \times 10^2 \times (1 + 2.5 \times 10^2 \times \alpha) + 2.5 \times 10^2} \\
= & \frac{1 + 2.5 \times 10^2 \times \alpha}{1 + 2.5 \times 10^2 \times \alpha + 1}
\end{aligned}
$$

which is close to 1 for the relevant values of α. In other words, we should expect almost all of the agents to be playing their preferred action after the first time step. In accordance with this, when the the mutation rates were set to zero, all simulations resulted in the outcome where all agents play their types preferred action.

Now, if we assume all of the agents are playing their preferred action, then we can calculate when type 1 agents', whose preferred action is Stravinsky, should have a higher utility for playing Stravinsky than Bach:

$$
\begin{aligned}
U(< S >) &> U(< B >) \\
(0.5 - \beta) \times 10^3 \times (1 + \alpha) &> (0.5 + \beta) \times 10^3 \\
0.5 - \beta + \alpha \times (0.5 - \beta) &> (0.5 + \beta) \\
\alpha \times (0.5 - \beta) &> (0.5 + \beta) - (0.5 - \beta) \\
\alpha &> \frac{2\beta}{0.5 - \beta}
\end{aligned}
$$

Figure 22 shows the graph of $\alpha = \frac{2\beta}{0.5-\beta}$ and it is easy to see that this aligns with the simulation results shown in Figure 3.

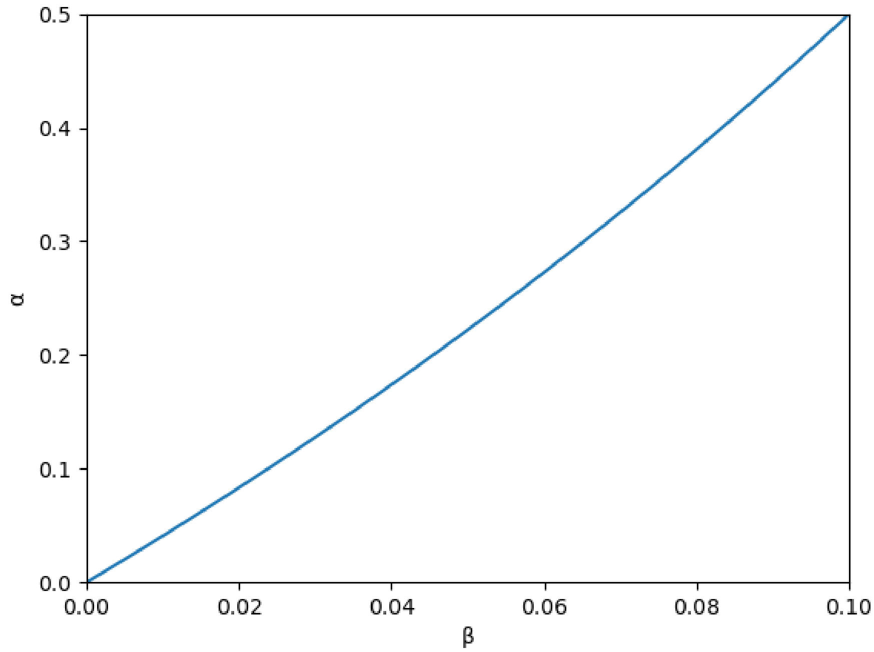


Figure 22: Type 1 agents should continue playing their preferred action if α and β place them above this line.

Appendix B Increasing Population Size Decreases Variance in Outcomes

This appendix compares data points from Sections 2.3 & 3.4 with data points that use the same parameters but increase to population size from 1,000 agents to 10,000 agents. Here we see that there is less variance in outcomes for larger populations; the explanation for this is straightforward and given below.

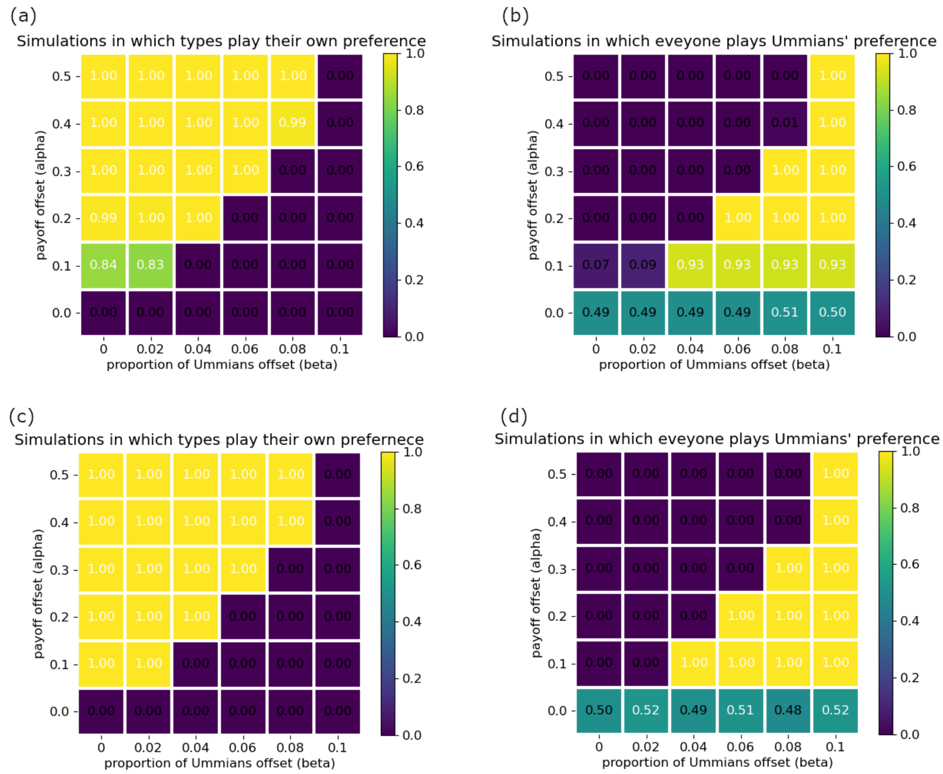


Figure 23: (a)Population size = 1,000 agents. Proportion of simulations that resulted in outcome (i) in which agents always give their preferred greeting; this means that when agents of different types fail to coordinate. (b) Population size = 1,000 agents. Proportion of simulations that resulted in outcome (ii)in which everyone gives Ummians' preferred greeting.(c)Population size = 10,000 agents. Proportion of simulations that resulted in outcome (i) in which agents always give their preferred greeting; this means that when agents of different types fail to coordinate. (d) Population size = 10,000 agents. Proportion of simulations that resulted in outcome (ii)in which everyone gives Ummians' preferred greeting.

Comparing Figures 23a&b with 23c&d we see that there is more variance in the outcomes for simulations of smaller populations than larger populations. This can be understood by considering what must happen to obtain outcome (iii), everyone plays the Kish greeting. This is the same as asking when is it the case that agents' playing the Kish greeting have higher utility from that action than agents playing the Umma greeting. Note that since Kishu agents prefer the Kish greeting over the Umma greeting and vice versa for Ummian agents, then an Ummian agent having higher utility for playing the Kish greeting than the Umma greeting implies that Kishu agents will also have higher utility for playing the Kish greeting than the Umma greeting. So, lets just consider when an Ummian agent has higher utility for playing the Kish greeting than the Umma greeting. By the $U(x)$ equation this happens when:

$$\begin{aligned} U(K) &> U(U) \\ M(K) \times 1 &> M(U) \times (1+\alpha) \end{aligned}$$

where $M(x)$ is the number of agents playing x. Suppose for illustration that $\alpha = 0.1$. Then if the population size is 1,000 (i.e. $M(K)+M(U) = 1000$) agents we get that outcome (iii) is likely to occur if:

$$\begin{aligned} M(K) \times 1 &> M(U) \times 1.1 \\ 1000 - M(U) &> M(U) \times 1.1 \\ \frac{1000}{1.1M(U)} - \frac{M(U)}{1.1M(U)} &> 1 \\ \frac{1000}{1.1M(U)} - \frac{10}{11} &> 1 \\ \frac{10000}{11M(U)} &> \frac{21}{11} \\ 10000 &> 21M(U) \\ 476.19047619 &> M(U) \end{aligned}$$

Similarly, one could calculate that for a population of 10,000 agents the number of agents playing the Umma greeting must be less than 4761.9047619 for Ummian agents to have higher utility from playing the Kish greeting than the Umma greeting. Now if we consider just the first timestep of a simulation in which agents are randomly (with equal probability) assigned strategy profiles, then it is easy to check that it is more likely that (a), a population of 1000 agents will have 476 or fewer agents playing the Umma greeting (and 524 or more playing the Kish greeting), than it is for (b) a population of 10,000 agents to have 4761 or fewer agents playing the Umma greeting (and 5239 or more playing the Kish greeting); i.e. the probability of (a) is:

$$\sum_{k=0}^{476} \binom{1000}{k} \times 0.5^{1000} \approx 0.06858400263532671$$

and the probability of (b) is:

$$\sum_{k=0}^{4761} \binom{10000}{k} \times 0.5^{10000} \approx 0.00000091718051570$$

While these calculations only apply to the probability of Ummian agents having higher utility for playing the Kish greeting than the Umma greeting *on the first* *timestep*. They should provide an intuition for the general case of why low probability outcomes are more likely for smaller populations than larger ones.

We can also see this effect of increasing population size for the model in Section 3.4:

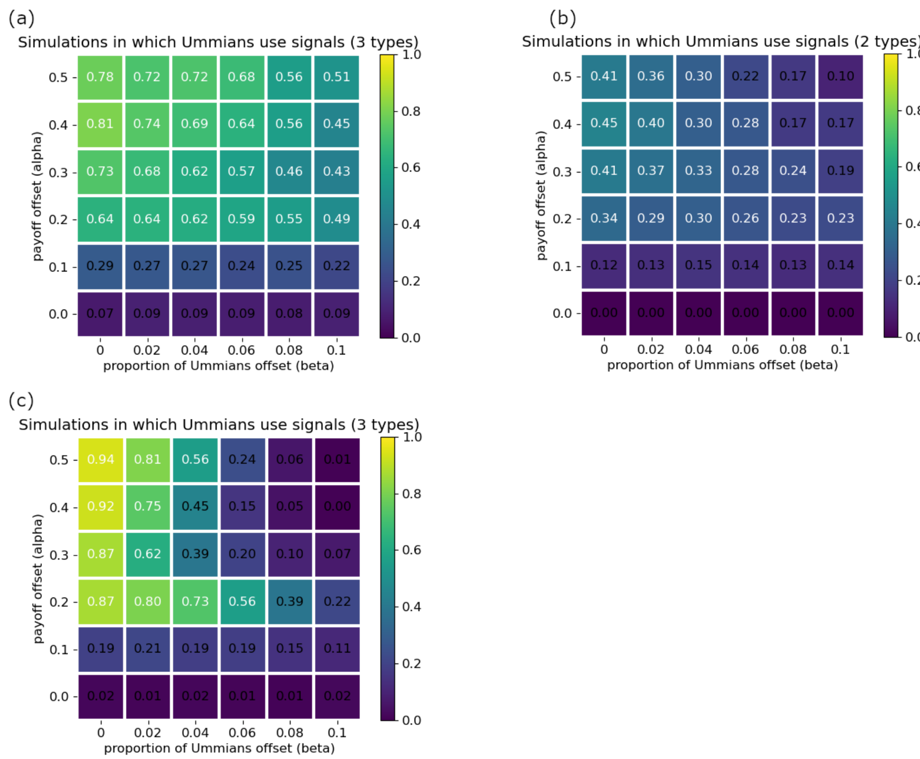


Figure 24: (a) Proportion of outcomes in which type 0 agents broadcast a social signal, for the model with three types of agents, 1,000 total agents. (b) Proportion of outcomes in which type 0 agents broadcast a social signal, for the model with two types of agents, 1,000 total agents. (c) Proportion of outcomes in which type 0 agents broadcast a social signal, for the model with three types of agents, 10,000 total agents.

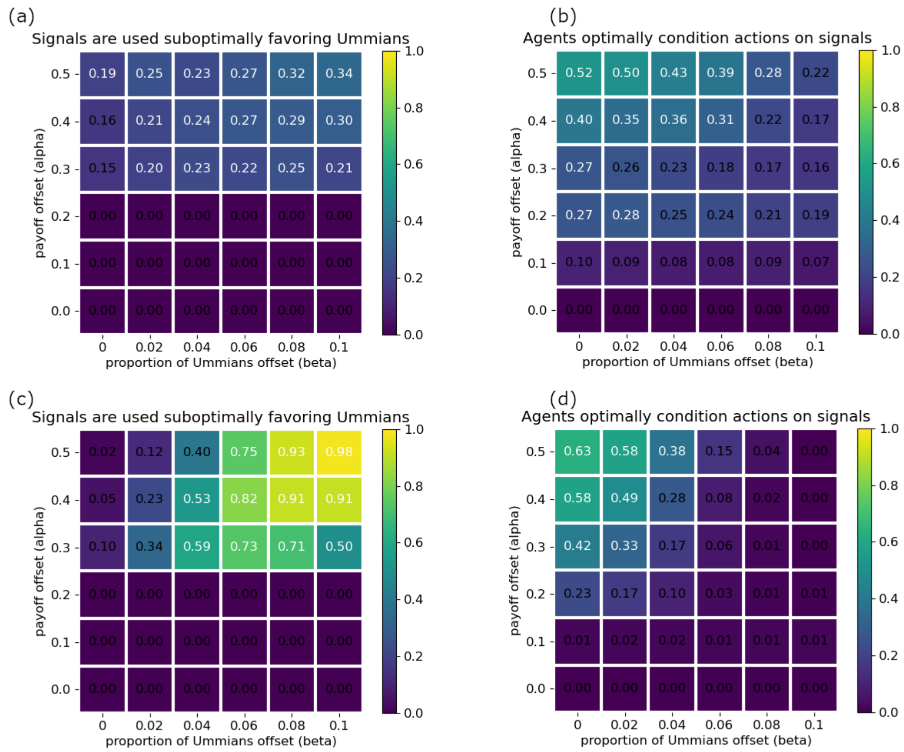


Figure 25: (a) Proportion of outcomes that are (viii), population of 1,000 agents. (b) Proportion of outcomes that are (xii-xiii), population of 1,000 agents. (c) Proportion of outcomes that are (viii), population of 10,000 agents. (d) Proportion of outcomes that are (xii-xiii), population of 10,000 agents.

Appendix C Additional Details on Simulation Results for Generalized BoS with Attention and Signal Costs

Note: this appendix uses data that was generated independently of the data points given in Appendix B. Therefore, there may be nominal differences in the values given in corresponding heatmaps though the general trends remains the same. This appendix's simulations were done with a population size of 10,000 agents; so it can be check that Figures 28 and 30 closely match Figures 24c and 25d from Appendix B.

Additionally, note that a supplemental PDF shows the evolution of signaling and behavioral dispositions of individual simulation runs. For each of the seventeen outcomes here, ten different simulations resulting in the given outcome are detailed except when fewer than ten simulations resulted in the outcome.

The supplemental PDF was composed prior to developing the Mesopotamian greetings example. It was written in terms of agents deciding between Bach, Stravinsky, or EDM rather than being in terms of the Umma greeting, the Kish greeting, and the Akkadian greeting. As I am disinclined to redo the 116 pages of the supplemental PDF, this appendix is written in terms of Bach, Stravinsky, and EDM. My appologies for the change. Bellow, the table of coordination preferences is restated in those terms with the corresponding Mesopotamian greetings given in parentheses.

Coordination Preferences	Bach (Umma)	Stravinsky (Kish)	EDM (Akkadian)
type 0	$1+\alpha$	1	0.5
type 1	1	$1+\alpha$	0.5
type 2	0	0	1

For the parameters investigated, there were thirteen different outcomes that occurred at least 0.5% of the time for some data point.

(i) Outcomes in which there was no coordination between agents of different types, i.e. agents always play their preference irrespective of the signal transmitted. This is shown in Figure 26.

(ii–ix) Agents exhibit suboptimal coordination, but in an intuitive way. This is shown in Figure 27.

- (viii) A notably common outcome was one in which type 0 and type 2 agents always played their respective preference, and type 1 agents played B with type 0, S with type 1, and EDM with type 2. The prevalence of this outcome is shown in Figure 28. This outcome was frequently characterized by type 0 agents signaling 0 and type 1 and 2 agents attending to signals that reliably identified their type. One can check that this is a Nash equilibrium. It is suboptimal in the sense that when type 0 agents are paired with type 2 agents, there is a failure to coordinate.

(x–xi) Agents condition their actions suboptimally, and in a counterintuitive way. This is shown in Figure 29. Specifically what was counterintuitive about these outcomes is that they involved a type playing their preference when paired with a type other than themselves, but not playing their preference among themselves.

(xii–xiii) Agents condition their actions optimally, in the sense that there were never failures of coordination. This is shown in Figure 30.

- (xii) A notably common outcome was one in which type 0 played their preference when paired with type 1. The prevalence of this outcome is shown in Figure 31.

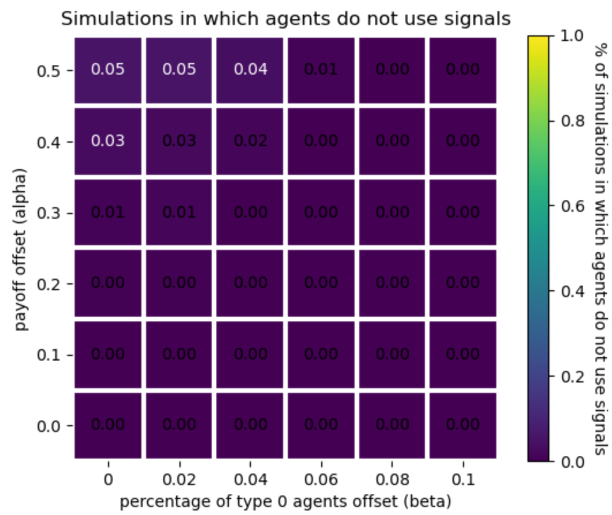


Figure 26: Proportion of outcomes that are (i).

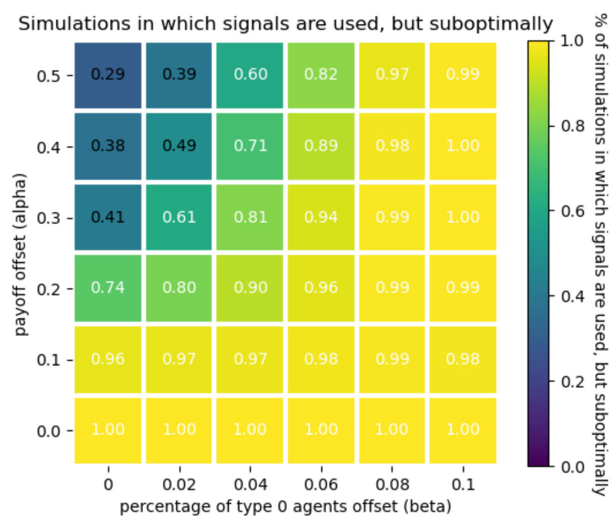


Figure 27: Proportion of outcomes that are (ii-ix).

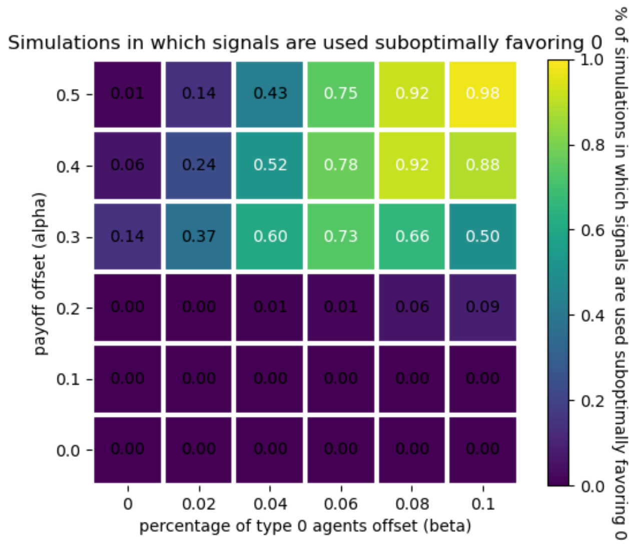


Figure 28: Proportion of outcomes that are (viii).

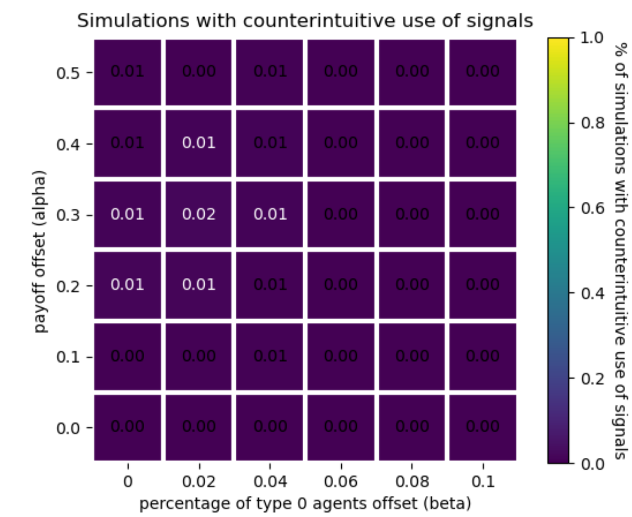


Figure 29: Proportion of outcomes that are (x-xi).

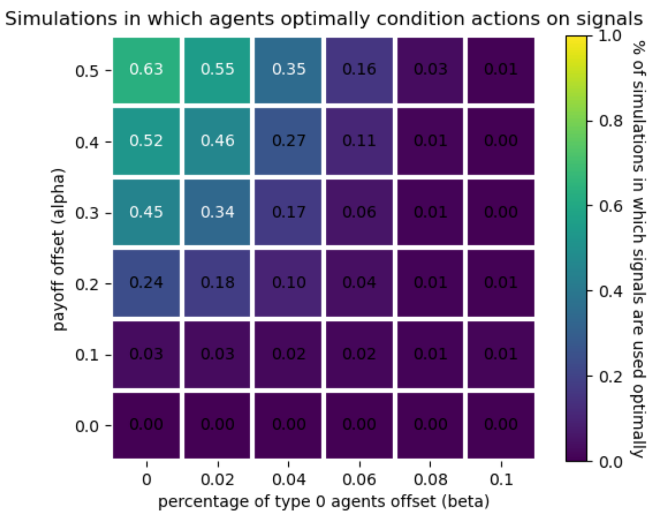


Figure 30: Proportion of outcomes that are (xii-xiii).

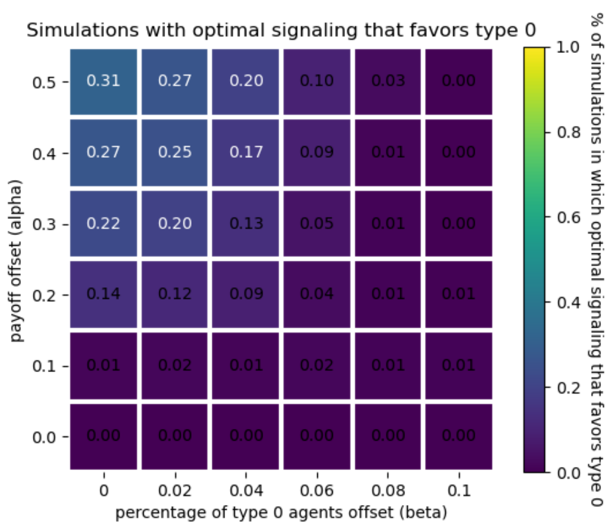


Figure 31: Proportion of outcomes that are (xii).

outcome (i)

y plays this with x	type 0	type 1	type 2
type 0	B	B	B
type 1	S	S	S
type 2	E	E	E

outcome (ii)

y plays this with x	type 0	type 1	type 2
type 0	B	B	B
type 1	B	B	B
type 2	E	E	E

outcome (iii)

y plays this with x	type 0	type 1	type 2
type 0	S	S	S
type 1	S	S	S
type 2	E	E	E

outcome (iv)

y plays this with x	type 0	type 1	type 2
type 0	B	B	E
type 1	B	B	E
type 2	E	E	E

outcome (v)

y plays this with x	type 0	type 1	type 2
type 0	S	S	E
type 1	S	S	E
type 2	E	E	E

outcome (vi)

y plays this with x	type 0	type 1	type 2
type 0	B	B	B
type 1	B	S	B
type 2	E	E	E

outcome (vii)

y plays this with x	type 0	type 1	type 2
type 0	B	S	S
type 1	S	S	S
type 2	E	E	E

outcome (viii)

y plays this with x	type 0	type 1	type 2
type 0	B	B	B
type 1	B	S	E
type 2	E	E	E

outcome (ix)

y plays this with x	type 0	type 1	type 2
type 0	B	S	E
type 1	S	S	S
type 2	E	E	E

outcome (x)

y plays this with x	type 0	type 1	type 2
type 0	S	B	E
type 1	B	S	E
type 2	E	E	E

outcome (xi)

y plays this with x	type 0	type 1	type 2
type 0	B	S	E
type 1	S	B	E
type 2	E	E	E

outcome (xii)

y plays this with x	type 0	type 1	type 2
type 0	B	B	E
type 1	B	S	E
type 2	E	E	E

outcome (xiii)

y plays this with x	type 0	type 1	type 2
type 0	B	S	E
type 1	S	S	E
type 2	E	E	E

outcome (xiv)

y plays this with x	type 0	type 1	type 2
type 0	B	B	B
type 1	B	E	E
type 2	E	E	E

This occurred:
1 time(s) when alpha = 0.1 and beta = 0.08
2 time(s) when alpha = 0.2 and beta = 0.08
1 time(s) when alpha = 0.2 and beta = 0.1

outcome (xv)

y plays this with x	type 0	type 1	type 2
type 0	E	B	E
type 1	B	S	E
type 2	E	E	E

This occurred:
1 time(s) when alpha = 0.3 and beta = 0
1 time(s) when alpha = 0.4 and beta = 0

outcome (xvi)

y plays this with x	type 0	type 1	type 2
type 0	B	S	E
type 1	S	E	E
type 2	E	E	E

This occurred:
1 time(s) when alpha = 0.3 and beta = 0.04
2 time(s) when alpha = 0.4 and beta = 0.02
1 time(s) when alpha = 0.5 and beta = 0
1 time(s) when alpha = 0.5 and beta = 0.02
1 time(s) when alpha = 0.5 and beta = 0.04

outcome (xvii)

y plays this with x	type 0	type 1	type 2
type 0	B	E	E
type 1	E	S	E
type 2	E	E	E

This occurred:
1 time(s) when alpha = 0.3 and beta = 0.06
1 time(s) when alpha = 0.4 and beta = 0.08
1 time(s) when alpha = 0.5 and beta = 0.06
2 time(s) when alpha = 0.5 and beta = 0.1

Appendix D Adding in Partial Payoffs on Failures to Coordinate

This appendix shows simulation results for the model with one change. The change is that instead of having $p_{xi} = 0$ when strategy profiles x and i result in a failure to coordinate p_{xi} is twenty percent of the payoff that would have been received if the action chosen by profile x had lead to successful coordination. For example, if an agent receives a payoff of 1 for coordinating on a particular action, then if that agent performs the same action but fails to coordinate her payoff will be 0.2. Of course this value of twenty percent was not hard coded and is a parameter that can be varied. But, twenty percent is sufficient for illustrating what happens in the model. If the payoff percentage on failures to coordinate is too small, then simulation results are identical to results from the model in which agents get nothing on failures to coordinate; and, if the payoff percentage on failures is too high, then agents will always perform their most preferred action because the payoff on failures for that action exceeds their payoff for successful coordination on a less preferred action.

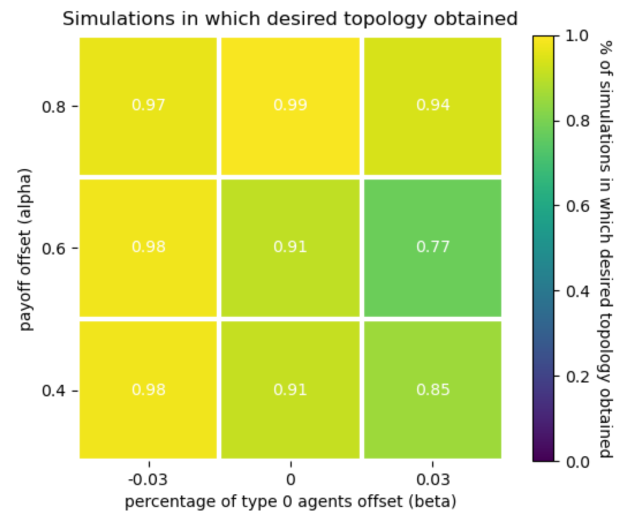


Figure 32: Proportion of outcomes that are the single embedding topology under parameters identical to those used for Figure 17 (c) but with the modified model where agents get twenty percent on failures.

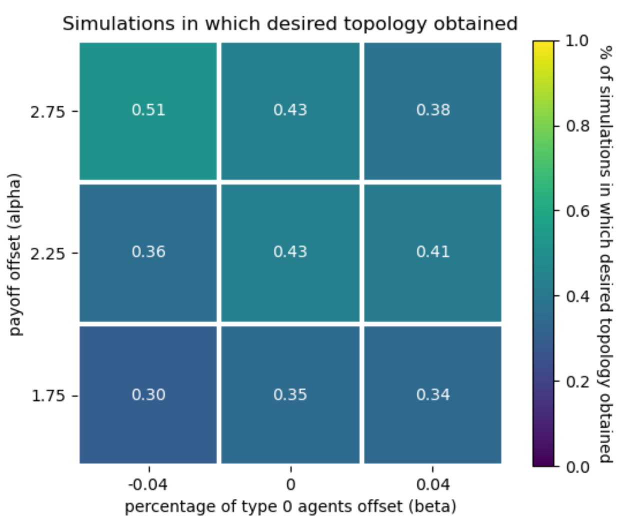


Figure 33: Proportion of outcomes that are the disjoint double embedding topology under parameters identical to those used for Figure 19c but with the modified model where agents get twenty percent on failures.

Appendix E Analytic Analysis for 2x2x2 game

2 types, 2 signals (α, β), and 2 actions (no homophily)

In this simple system, this appendix analyzes when it is the case that an agent should play their less preferred action.

		Type 1	
		Bach	Stravinsky
Type 0	Bach	a, b	c, c
	Stravinsky	c, c	b, a

With $a > b$ and $c < 0$.

- Let $n_i, i \in \{0, 1\}$ the proportion of the population that is type i.
- Let $x_{ijk}, i \in \{0, 1\}, j \in \{\alpha, \beta\}, k \in \{\text{Bach, Stravinsky}\}$ be the probability that a type i agent plays k given signal j is observed.
- Let $y_{ij}, i \in \{0, 1\}, j \in \{\alpha, \beta\}$ be the probability that a type i agent transmits signal j. Note that $y_{i\beta} = 1 - y_{i\alpha}$
- Let A_i be type i's preferred action and B_i be the alternative action.
- Let $p(i|j), i \in \{0, 1\}, j \in \{\alpha, \beta\}$ be the probability that signal j was sent by a type i agent given signal j was observed.
- Let $E_{ij}(k), i \in \{0, 1\}, j \in \{\alpha, \beta\}, k \in \{\text{Bach, Stravinsky}\}$ be the expected utility for a type i agent playing k after having seen signal j.

Then:

$$p(i|j) = \frac{n_i y_{ij}}{n_0 y_{0j} + n_1 y_{1j}}$$

$$\begin{aligned}
E_{ij}(A_i) &= a[y_{i\alpha}(x_{0\alpha A_i} p(0|j) + x_{1\alpha A_i} p(1|j)) + (1 - y_{i\alpha})(x_{0\beta A_i} p(0|j) + x_{1\beta A_i} p(1|j))] \\
&\quad + c[y_{i\alpha}(x_{0\alpha B_i} p(0|j) + x_{1\alpha B_i} p(1|j)) + (1 - y_{i\alpha})(x_{0\beta B_i} p(0|j) + x_{1\beta B_i} p(1|j))] \\
E_{ij}(B_i) &= b[y_{i\alpha}(x_{0\alpha B_i} p(0|j) + x_{1\alpha B_i} p(1|j)) + (1 - y_{i\alpha})(x_{0\beta B_i} p(0|j) + x_{1\beta B_i} p(1|j))] \\
&\quad + c[y_{i\alpha}(x_{0\alpha A_i} p(0|j) + x_{1\alpha A_i} p(1|j)) + (1 - y_{i\alpha})(x_{0\beta A_i} p(0|j) + x_{1\beta A_i} p(1|j))]
\end{aligned}$$

In the initial state of the reinforcement learning model, $x_{ijk} = y_{ij} = 0.5$ for all i, j, and k. Since $a > b$, it follows that in the initial state of the game agents should always play their preferred action; i.e. $E_{ij}(A_i) > E_{ij}(B_i)$ for all i and j.

Given that all agents have more incentive to play their preferred action than the alternative action irrespective of the signal let's assume that $x_{ijA_i} = 0.5 + \epsilon$ for all i and j where $\epsilon \geq 0$. (Note that $x_{ijB_i} = 1 - x_{ijA_i}$). Furthermore, let's assume that due to random drift $y_{0\alpha} = y_{1\beta} = 0.5 + \delta$ for some $\delta > 0$. Without loss of generality, consider expected utility for type 0 agents. We the get that type 0 agents have higher expected utility for playing their less preferred action when observing β (the more frequent signal among type 1 agents) if $E_{0\beta}(B_0) > E_{0\beta}(A_0)$.

Let:

$$Q = \frac{n_0(.5-\delta)}{n_0(.5-\delta)+(1-n_0)(.5+\delta)}$$

$$R = \frac{(1-n_0)(.5+\delta)}{n_0(.5-\delta)+(1-n_0)(.5+\delta)}$$

If $2Qc-2Rc-Qb+Rb-Qa+Ra > 0$, then it can be checked that $E_{0\beta}(B_0) > E_{0\beta}(A_0)$ when:

$$\epsilon > \frac{Qa+Ra-Qb-Rb}{4Qc-4Rc-2Qb+2Rb-2Qa+2Ra}$$

Here are some plots of:

$$\epsilon = \frac{Qa+Ra-Qb-Rb}{4Qc-4Rc-2Qb+2Rb-2Qa+2Ra}$$

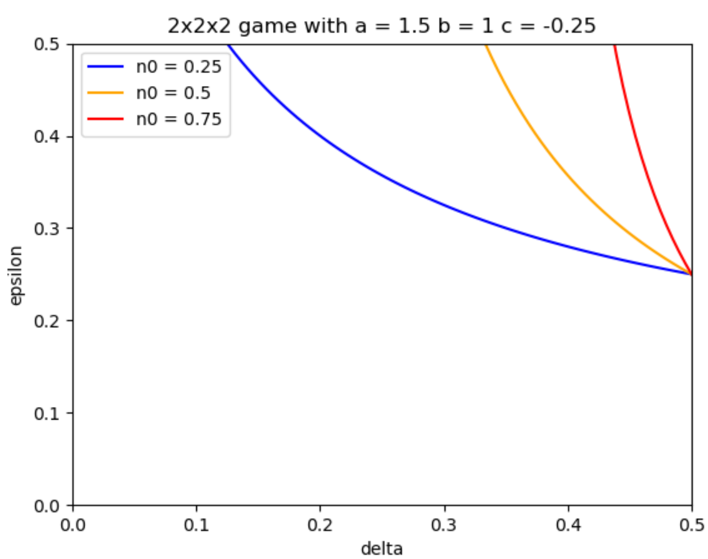


Figure 34

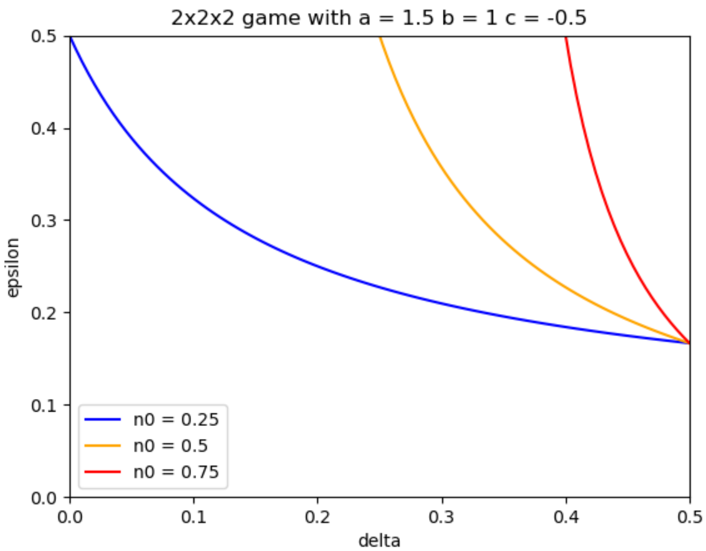


Figure 35

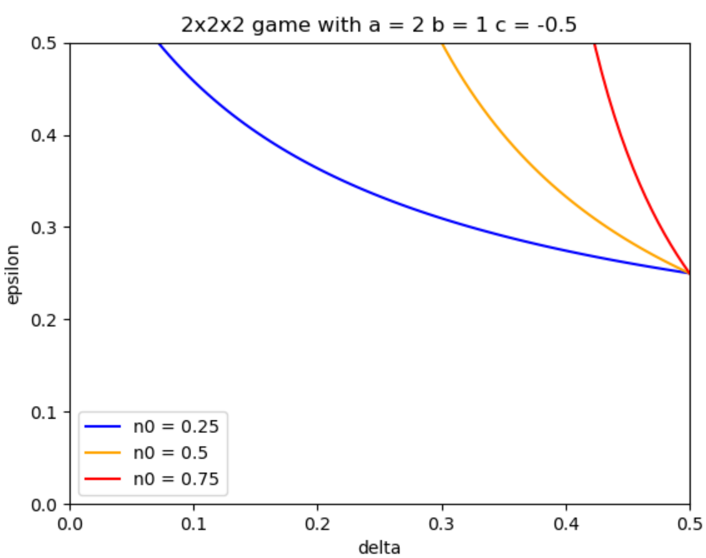


Figure 36