Group Identity in the Generalized Bach or Stravinsky Game

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  - Signals and Assortment
  - Attention and Signal Costs
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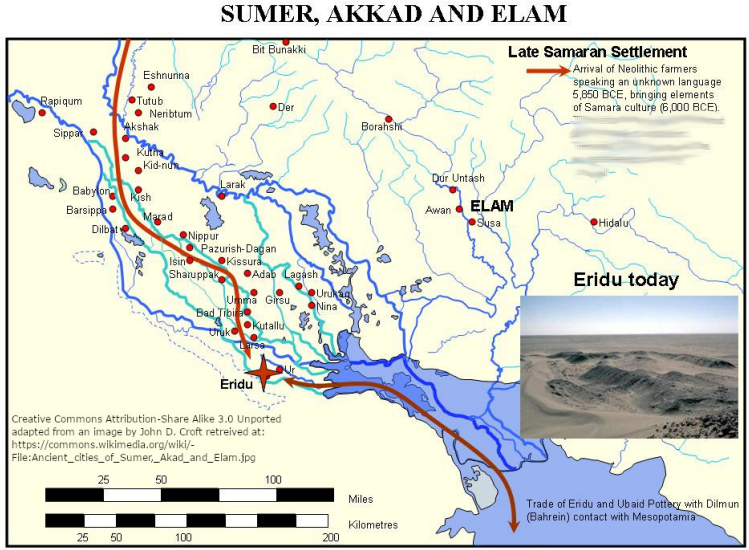
- Simulations with Multidimensional Signaling
  - Single Embedding
  - Hierarchical Double Embedding
  - Disjoint Double Embedding
  - Intersection

Ancient Mesopotamia



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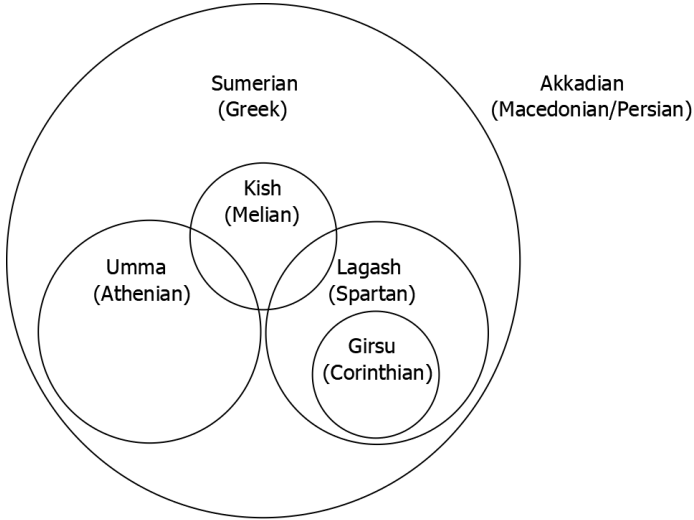


The Bach or Stravinsky Game

- Coordination problem: Handshakes, Hugs, and High Fives
- Modeling coordination:

		Player 1	
		Bach	Stravinsky
Player 0	Bach	3, 1	0, 0
	Stravinsky	0, 0	1, 3

Generalized Bach or Stravinsky

		Type 0	
		Bach	Stravinsky
Type 0	Bach	3, 3	0, 0
	Stravinsky	0, 0	1, 1

		Type 1	
		Bach	Stravinsky
Type 0	Bach	3, 1	0, 0
	Stravinsky	0, 0	1, 3

		Type 1	
		Bach	Stravinsky
Type 1	Bach	1, 1	0, 0
	Stravinsky	0, 0	3, 3

.

This could alternatively be expressed as:

Coordination Preferences

	Bach	Stravinsky
Type 0	3	1
Type 1	1	3

.

Coordination preferences can be further generalized to allow arbitrarily many types and arbitrarily many actions over which they have preferences.

Learning Dynamics

- Discrete Replicator Equation:

$$N_{t+1}(x) = N_t(x) + N_t(x) \times [U(x) - Avg(U(i))_{i \in X}]$$

$$U(x) = \sum_{i \in Y} [M(i) \times p_{xi}] \qquad \text{if } x \neq i$$

$$U(x) = \sum_{i \in Y} [(M(i) - 1) \times p_{xi}] \qquad \text{if } x = i$$

- Mutation Dynamics: $m_0 = 0.01$ $m_1 = 0.1$

$$x =< 20BBSBSBBSS >$$

Simulation Parameters

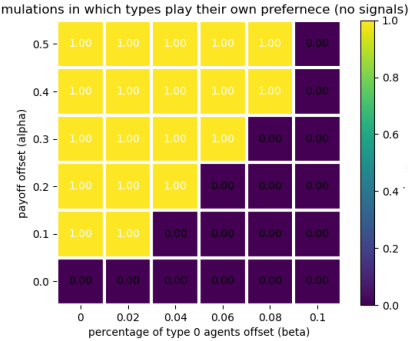
Type 0 agents make up $0.5 + \beta$ proportion of the population (and type 1 agents are a $0.5 - \beta$ proportion). Coordination preferences are:

Coordination Preferences	Umma greeting	Kish greeting
type 0 (people from Umma)	$1 + \alpha$	1
type 1 (people from Kish)	1	$1 + \alpha$

Table: Coordination Preferences: generalized BoS for a population of two types, $\alpha \geq 0$.

Simulation Results

(a)



(b)

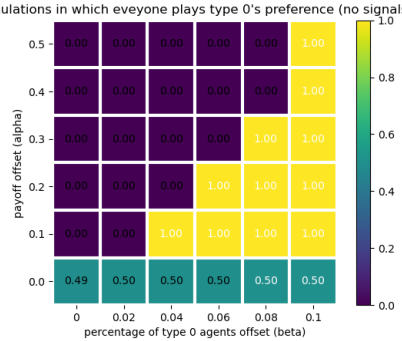


Figure: (a) Proportion of simulations that resulted in outcome (i) in which agents play their preference; this means that when agents of different types fail to coordinate. (b) Proportion of simulations that resulted in outcome (ii) in which everyone plays type 0's preferred action.

Analytically

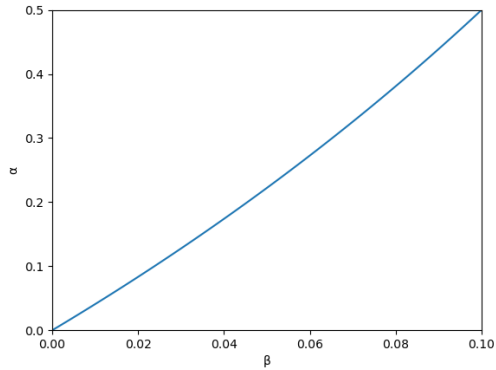


Figure: Type 1 agents should continue playing their preferred action if α and β place them above this line.

Key Takeaway

Larger groups have more power to make their preference the norm.

Signals and Assortment Dynamics

- A strategy profile contains an agents broadcast signal and what action they play for each signal observed. For example, suppose the there are two signals 1 and 2, with everything else remaining as before. Then there are 2^3 strategy profiles: $< 1BB >$, $< 1BS >$, $< 1SB >$, $< 1SS >$, $< 2BB >$, $< 2BS >$, $< 2SB >$, and $< 2SS >$.
- As before, agents learn through the discrete replicator equation:

$$N_{t+1}(x) = N_t(x) + N_t(x) \times [U(x) - Avg(U(i))_{i \in X}]$$

Signals and Assortment Dynamics

- Utility function is modified to reflect assortment:

$$U(x) = \sum_{i \in Y} [H(h, x, i) \times M(i) \times p_{xi}] \qquad \text{if } x \neq i$$

$$U(x) = \sum_{i \in Y} [H(h, x, i) \times (M(i) - 1) \times p_{xi}] \qquad \text{if } x = i$$

where $H(h, x, i)$ is defined as:

$$H(h, x, i) = \frac{N}{\sum_{j \in Y} [S(h, x, j)]} \times S(h, x, i)$$

where N is the total number of agents in the population and $S(h, x, j)$ is:

$S(h, x, j) = 2^h$ if profile x entails broadcasting the same social signal as j

$S(h, x, j) = 1$ if x does not entail broadcasting the same social signal as j

Simulation Parameters

Type 0 agents make up $0.5 + \beta$ proportion of the population (and type 1 agents are a $0.5 - \beta$ proportion). Coordination preferences are:

Coordination Preferences	Umma greeting	Kish greeting
type 0 (people from Umma)	$1 + \alpha$	1
type 1 (people from Kish)	1	$1 + \alpha$

Simulation Results

i. The outcome in which each type plays their preferred action all of the time is possible, but never occurs under the parameters shown in this presentation.
ii. The outcomes in which everyone plays type 0's preferred action, the Umma greeting.
iii. The outcome in which everyone plays type 1's preferred action, the Kish greeting.
iv. The outcome in which type 0 agents always play their preferred action and type 1 agents play 0's preference with type 0's and play their own preference among themselves.
v. The outcome in which type 1 agents always play their preferred action and type 0 agents play 1's preference with type 1's and play their own preference among themselves.
vi. Outcomes in which meaningful signals evolve, but at least one type does not play their preference among themselves.

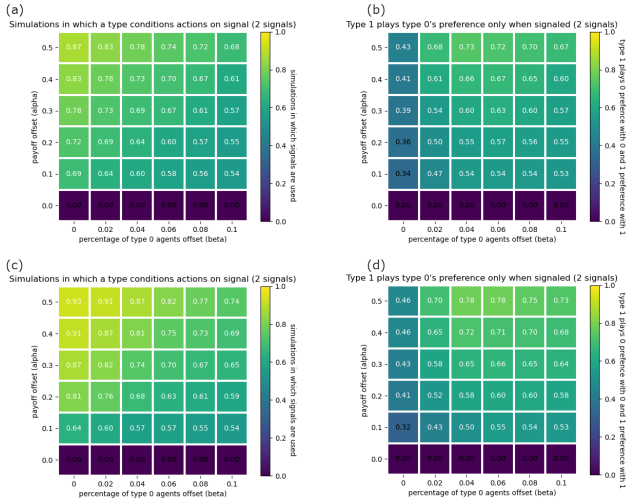


Figure: (a) Proportion of outcomes that are (iv) or (v) for $h = 0$. (b) Proportion of outcomes that are (iv) for $h = 0$. (c) Proportion of outcomes that are (iv) or (v) for $h = 1$. (d) Proportion of outcomes that are (iv) for $h = 1$.

Key Takeaway

Signaling and assortment are beneficial.

Attention and Signal Costs

- The model now incorporates a special signal, the null signal 0, which indicates that an agent neither broadcasts a signal nor do they differentiate their dispositions towards signals (in that dimension).
- Agents now incur a cost for broadcasting a signal that is not the null signal. In our utility function this is just p_{xi} being as before less the signal cost, c.

$$U(x) = \sum_{i \in Y} [H(h, x, i) \times M(i) \times p_{xi}] \qquad \text{if } x \neq i$$

$$U(x) = \sum_{i \in Y} [H(h, x, i) \times (M(i) - 1) \times p_{xi}] \qquad \text{if } x = i$$

Attention and Signal Costs Parameters

Type 0 agents make up $0.33 + \beta$ proportion of the population, type 1 agents are a $0.33 - \beta$ proportion of the population, type 2 agents are a 0.34 proportion of the population, and signal cost $c = -0.01$. Coordination preferences are:

Coordination Preferences	Umma greeting	Kish greeting	Akkadian greeting
type 0 (people from Umma)	$1 + \alpha$	1	0.5
type 1 (people from Kish)	1	$1 + \alpha$	0.5
type 2 (people from Akkad)	0	0	1

Attention and Signal Costs Results

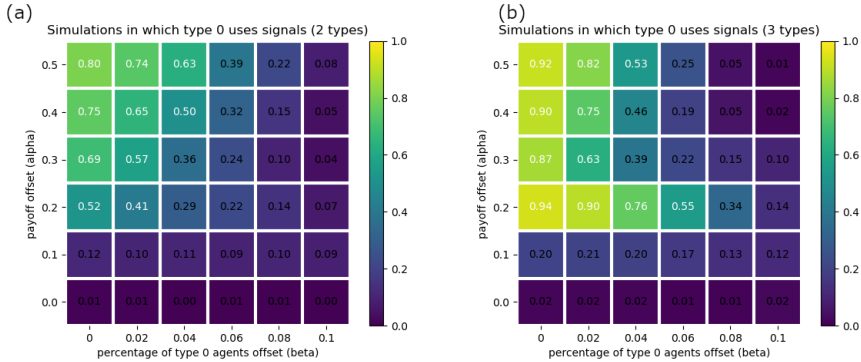


Figure: (a) Proportion of outcomes in which type 0 agents broadcast a social signal, for the model with two types of agents. (b) Proportion of outcomes in which type 0 agents broadcast a social signal, for the model with three types of agents.

Attention and Signal Costs Results

(viii) The outcome in which type 0 and type 2 agents always played their respective preference, and type 1 agents played U with type 0, K with type 1, and A with type 2.

(xii–xiii) Agents condition their actions optimally, in the sense that there were never failures of coordination and agents played their most prefered greeting among themselves.

Attention and Signal Costs Results

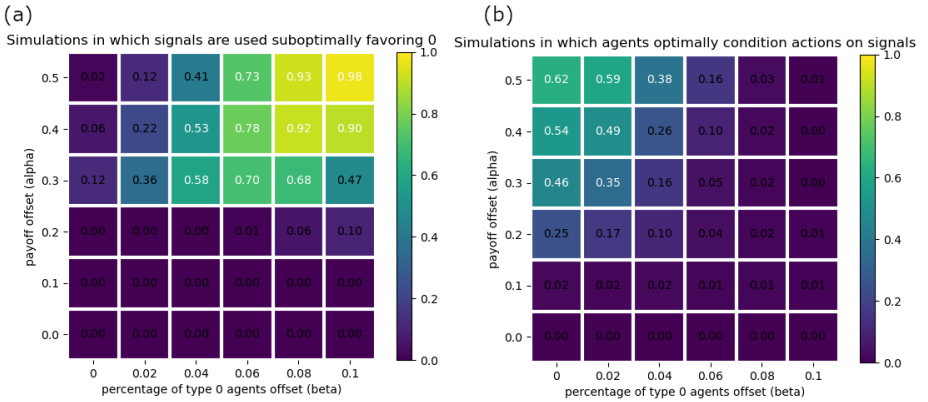


Figure: (a) Proportion of outcomes that are (viii). (b) Proportion of outcomes that are (xii-xiii).

Key Takeaway

The presence of a dominant majority group incentivises identity signaling among the minority groups.

Multidimensional Signaling Dynamics

- Now have multiple dimensions in which agents can signal.
  - So strategy profiles look like:

$$x =< 20BBSBSBBSS >$$

- Signal costs are incurred for each dimension attended to.
  - For example, if strategy profile x leads to successful coordination with agents employing strategy profile i by performing an action with coordination preferences value of 1, then if $x =< 00BBSBSBBSS >$ this results in $p_{xi} = 1$, if $x =< 20BBSBSBBSS >$ this results in $p_{xi} = 1 + c$, and if $x =< 14BBSBSBBSS >$ this results in $p_{xi} = 1 + 2c$.

Single Embedding Signaling

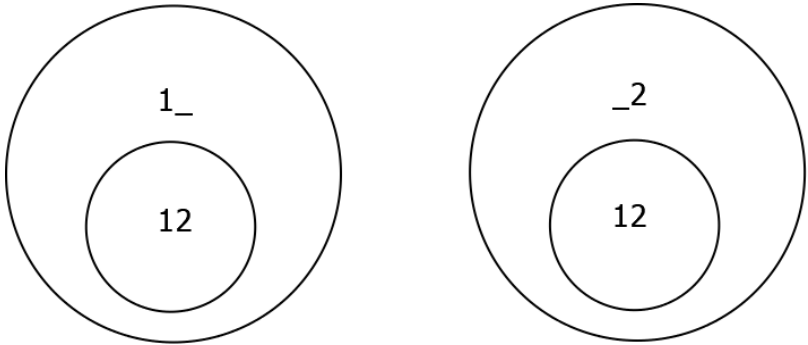


Figure: The two possible single embedding signaling systems when given one signal in each of two dimensions. The "_" can be read as 0, the null signal. The "_" is used to provide a visually apparent articulation of the signaling that highlights in the left figure all the 12 signalers are also 1 signalers and on the right all the 12 signalers are also 2 signalers.

Single Embedding Parameters

Simulation parameters are $m_0 = .01$, $m_1 = .1$, one signal in each of two dimensions (excluding the 0 signal), signal cost $c = -0.0005$, and run for 8×10^4 timesteps. Coordination preferences are:

Coordination Preferences	Lagash greeting	Girsu greeting	Akkadian greeting
type 0 (people from Lagash)	1	0	0.5
type 1 (people from Girsu)	1	$1 + \alpha$	0.5
type 2 (Akkadians)	0	0	1

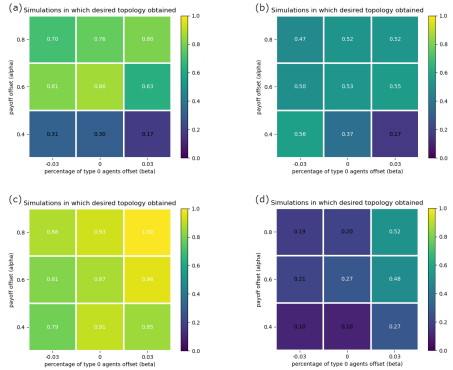


Figure: **(a)** Proportion of outcomes that are the single embedding topology, homophily $= 0$.Type 0 agents make up $0.33 + \beta$ proportion of the population, type 1 agents are a $0.33 - \beta$ proportion of the population, and type 2 agents are a 0.34 proportion of the population. **(b)** Proportion of outcomes that are the single embedding topology, homophily $= 1$.Type 0 agents make up $0.33 + \beta$ proportion of the population, type 1 agents are a $0.33 - \beta$ proportion of the population, and type 2 agents are a 0.34 proportion of the population. **(c)** Proportion of outcomes that are the single embedding topology, homophily $= 0$. Type 0 agents make up $0.2 + \beta$ proportion of the population, type 1 agents are a $0.2 - \beta$ proportion of the population, and type 2 agents are a 0.6 proportion of the population. **(d)** Proportion of outcomes that are the single embedding topology, homophily $= 1$. Type 0 agents make up $0.2 + \beta$ proportion of the population, type 1 agents are a $0.2 - \beta$ proportion of the population, and type 2 agents are a 0.6 proportion of the population.

Hierarchical Double Embedding Signals

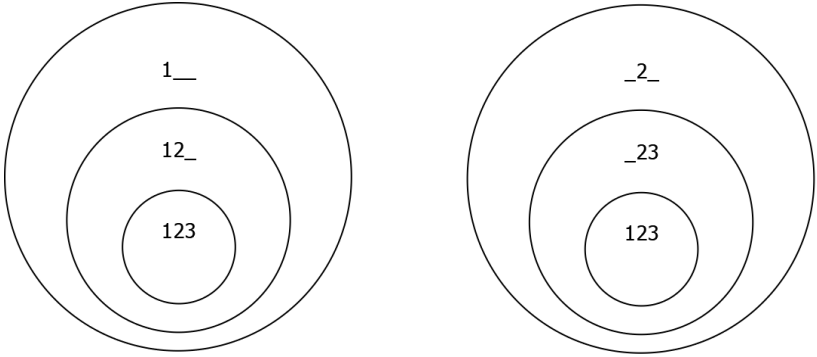


Figure: Two of the six hierarchical double embedding signaling systems that are possible when given one signal in each of three dimesions. I.e. there are three possible signals that the largest group can adopt 100, 020, and 003, and for each of those the middle group has two options for a signal in two dimensions that agrees with the signal of the larger group (e.g. if the larger group signals 003, then the middle group can signal in two dimension agreeing with the larger group in one dimension by signaling 103 or 203. The smallest group must signal 123 in all scenarios in which the hierarchical embedding occurs. Thus, there are $3 \times 2 = 6$ possible signaling systems, two of which are shown in this figure. As previously, the "_" can be read as 0, the null signal.

Hierarchical Double Embedding Parameters

Each data point in the heatmap shown here reflects the mean outcome of 100 simulations of the generalized BoS game with a population of 5,000 agents, $m_0 = .01$, $m_1 = .1$, one signal in each of three dimensions (excluding the 0 signal), signal cost $c = -0.0005$, and run for 2×10^4 timesteps. Type 0 agents make up 0.15 proportion of the population, type 1 agents are a $0.15 + \beta$ proportion of the population, type 2 agents are a $0.15 - \beta$ proportion of the population, and type 2 agents are a 0.55 proportion of the population. Coordination preferences are:

Coordination preferences	Sumerian greeting	Lagash greeting	Girsu greeting	Akkadian greeting
Type 0 (Sumerians)	1	0	0	0.25
Type 1 (people from Lagash)	1	$1 + \alpha$	0	0.25
Type 2 (people from Girsu)	1	$1 + 0.5 \times \alpha$	$1 + \alpha$	0.25
Type 3 (Akkadians)	0	0	0	1

Hierarchical Double Embedding Obtains

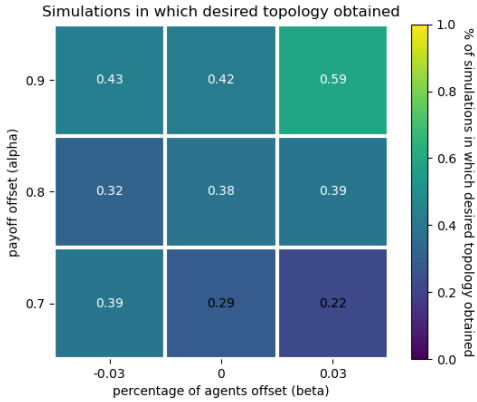


Figure: Proportion of outcomes that are the hierarchical double embedding topology.

Disjoint Double Embedding Signals

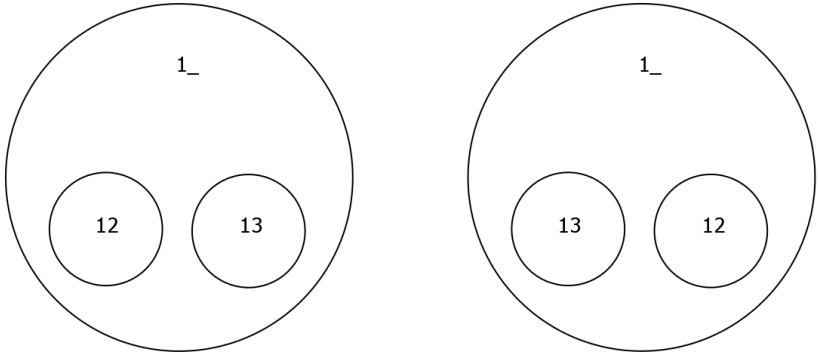


Figure: The only two disjoint double embedding signaling systems that are possible when given just two dimensions with one signal in the first dimension and two in the second.

Disjoint Double Embedding Parameters

Type 0 agents make up $0.15 + \beta$ proportion of the population, type 1 agents are a $0.15 - 0.5 \times \beta$ proportion of the population, type 2 agents are a $0.15 - 0.5 \times \beta$ proportion of the population, and type 2 agents are a 0.55 proportion of the population. Coordination preferences are:

Coordination preferences	Sumerian greeting	Umma greeting	Lagash greeting	Akkadian greeting
Type 0 (Sumerians)	1	0	0	0.25
Type 1 (people from Umma)	1	$1 + \alpha$	0	0.25
Type 2 (people from Lagash)	1	0	$1 + \alpha$	0.25
Type 3 (Akkadians)	0	0	0	1

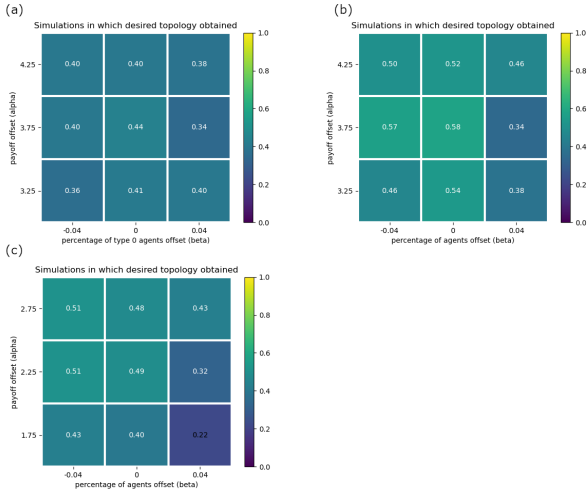


Figure: (a)Proportion of outcomes that are the disjoint double embedding topology when the population size is 5,000. The alpha values here are suboptimal for the population size.(b) Proportion of outcomes that are the disjoint double embedding topology for a population size of 10^4 and a simulation length of 4×10^4 timesteps. (c) Proportion of outcomes that are the disjoint double embedding topology. These seem to be optimal values for α when the population size is 5,000.

Intersection Signals

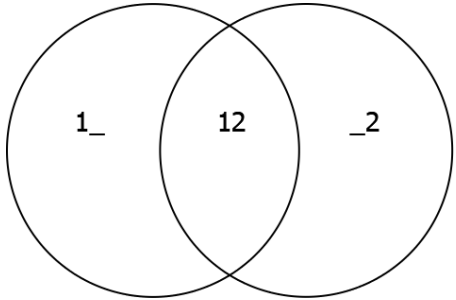


Figure: A signaling system in which we can investigate intersectionality.

Intersection Parameters

Type 0 agents make up $0.2 - 0.5 \times \beta$ proportion of the population, type 1 agents are a $0.2 + \beta$ proportion of the population, type 2 agents are a $0.2 - 0.5 \times \beta$ proportion of the population, and type 2 agents are a 0.4 proportion of the population. Coordination preferences are:

Coordination preferences	Umma greeting	Kish greeting	Lagash greeting	Akkadian greeting
Type 0 (people from Umma)	1	0	0	0.5
Type 1 (people from Kish)	1	α	1	0.5
Type 2 (people from Lagash)	0	0	1	0.5
Type 3 (Akkadians)	0	0	0	1

Table: Coordination preferences for investigating intersectionality. Simulations consider $\alpha \in \{0, 1, 2\}$

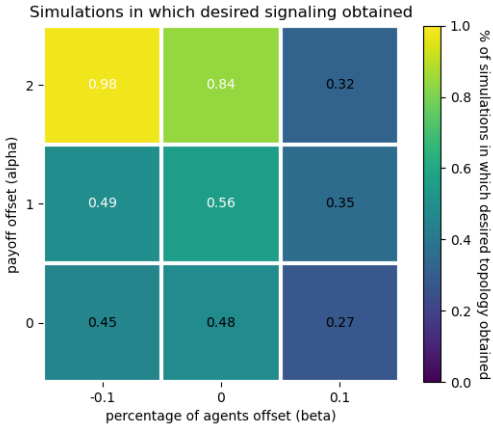


Figure: Proportion of outcomes in which type 0s signal in one dimension, type 1s singal in two dimensions, type 2s signal in one dimension that is different than the type 0s dimension, and type 3s do not signal in any dimension.

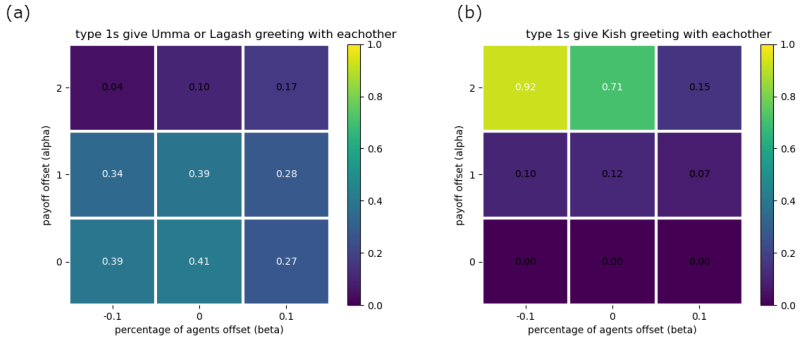


Figure: (a) Proportion of outcomes in which type 0s give the Umma or Lagash greeting with eachother in addition to exhibiting the desired signaling behavior; i.e. type 0s signal in one dimension, type 1s singal in two dimensions, type 2s signal in one dimension that is different than the type 0s dimension, and type 3s do not signal in any dimension. (b) Proportion of outcomes in which type 0s give the Kish greeting with eachother in addition to exhibiting the desired signaling behavior; i.e. type 0s signal in one dimension, type 1s singal in two dimensions, type 2s signal in one dimension that is different than the type 0s dimension, and type 3s do not signal in any dimension.