The Emergence of Group Identity Topologies in the Generalized Bach or Stravinsky Game

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Abstract

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1 Introduction

As early as the Third Dynasty of Ur in ancient Mesopotamia, we have literary evidence of complex social identities; cities as far apart as Ur and Mari (well over 100 miles apart) evidence a connonical body of literature indicative of a shared Sumerian identity across several cities in the region, but also exhibit literature concerned with a city's patron gods indicative of a localized social identity (ref Delnero 2015). We also have reason to believe that long before the invention of writing, humans used markers of group identity to facilitate coordination within groups that were too large for a group member to know every individual in the group (ref Moffett 2013). This participation in, sometimes complex, group identities continues today (ref Roccas and Brewer). Some social groups follow straightforward rules; all astrophysicists are physicists and all physicists are scientists. The background knowledge that you can assume in conversation with a scientist can also be assumed in conversation with a physicist and, likewise, the background knowledge you can assume in conversation with a physicist can also be assumed in conversation with a astrophysicist. But there are also ways in which social identities break from the contours of simple set membership relations. The mere conjunction dominate narratives in Black liberation and in mainstream feminism does not yield important narratives of Black feminists (ref Combahee River Collective 1977, Crenshaw 1991). This paper provides a formal model of how a variety of social identity topologies can obtain in a population of agents trying to coordinate behavior and preferentially interact with ingroup members.

Social identity signaling is often characteristic of how we form norms and coordinate behavior (ref McElreth Boyd and Richardson 2003, Moffett 2013). A growing body of literature has provided formal models how selection pressures can lead to agents evolving social signals used for coordinating behaviors (reference dump). However, this literature has modeled agents who broadcast a single unitary social signal. It has not yet to incorporated social signals that reflect the multidimensional social identities that real people exhibit.[1] This paper aims to be a first step in integrating multidimensional social identities with formal models of the evolution of social identity signaling for the purpose of coordinating behavior. In this paper's model agents learn to broadcast social signals in multiple dimensions mirroring the multiple dimensions of their social identities. In unison with this, agents learn to condition their actions on observed social signals in a way that reflects the population's group structure.

This paper does not focus on a specific example of coordinated behavior or mode of social signal. The model to be presented is abstracted away from differences between signaling social identity with clothing as opposed to accent. But some intuition pumps for how we benefit from coordinating behavior and how social identity signaling can relate to that coordination might be helpful. To that end, first note that there are many settings in which we benefit from coordinating our behavior such that we all do the same thing, e.g. driving on the right side of the road, or using the same currency. While some norms of coordination might be legislated, many contexts do not have strict rules for how to coordinate. After a success do you high five, handshake or hug your teammates? Do you greet an acquaintance with "hello", "shalom", "as-salamu alaykum", or "que pasa"? Absent strict rules, we often rely on social signals for coordinating behavior. Your teammates' business attire might indicate handshakes are more appropriate than hugs. An acquaintance wearing a yamaka might indicate that "shalom" is an appropriate greeting, particularly if one shares the indicated cultural heritage. Smaldino (2019) gives a more thorough discussion of empirical and theoretical evidence of how social identity signaling relates to coordinating behavior.

With respect to the breadth of types of coordination this paper's model reflects, it should be noted that while simple examples of coordination are employed for the sake of clarity, real world correlates need not be so simple. The positive feeling of a good high five and the negative feeling of going for a hug just as someone reaches out to shake your hand, is a simple and straightforward example getting a positive payoff when two people do the same thing and no payoff when do different things. Mirroring this simplicity, the model to be presented involves two agents getting a positive payoff when they both do the same action and no payoff when they do different actions. However, there are also more nuanced examples of coordination that might fit this paper's model. Consider the problem of coordinating vocabulary choice. If you ask someone to pass the orange soda by saying "please pass the coke" rather than saying "please pass the orange pop", then your failure to coordinate on the hearers' more familiar vocabulary might cause some initial confusion but still eventually

[1]It can also be noted that at least one paper has provided a formal model of how agents occupying the intersection of two marginalized groups can be disadvantaged in a way that is not merely the conjunction of the disadvantage experienced by members of the two marginalized groups who do not occupy the intersection (ref O'Connor and Bruner). But, that paper focused on bargaining rather than coordination and did not involve the evolution of any social identity signals.

lead to their passing the orange soda. There is also no reason to assume that coordinating on vocabulary is a problem unique to the modern world. It is easy to imagine a traveler in ancient Mesopotamia having to choose between a native Sumerian, Akkadian or Hurrian lone word to refer to the horse harness that she needs to replace. In scenarios like these where failure to coordinate might only temporarily inhibit an agent from achieving their desired goal, it makes sense to employ a model in which there is merely a reduced payoff for failures to coordinate rather than no payoff at all. Appendix C shows that results from the model are essentially unchanged (or improved) when using reduced payoffs for failures to coordinate rather than the simplifying assumption of zero payoff when agents fail to coordinate.

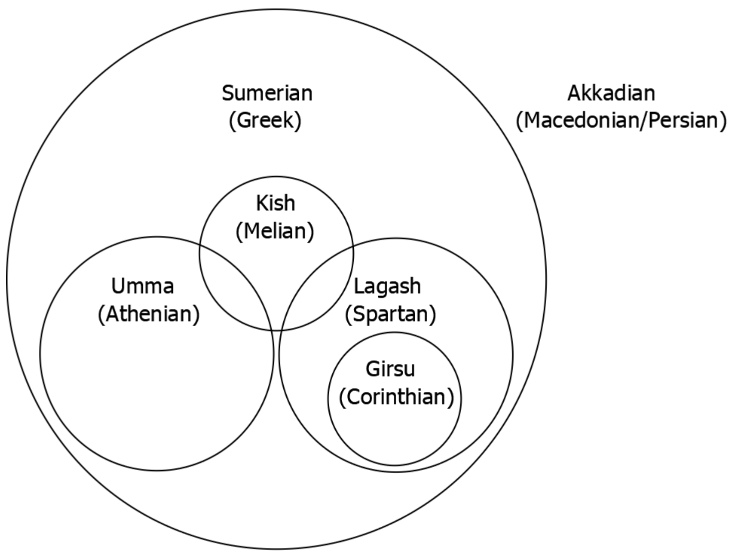

Figure 1: Fictionalized diagram of social identity relationships in Mesopotamia for the purpose of illustrating the group topologies that will be discussed in the paper. As many people are more familiar with the classical Greek city-states, loose correlates to the Sumerian city-states are given in parentheses. But, the overemphasis of Greco-Roman history does not need to be perpetuated.[2]

For the purposes of illustrating the types of social identity group topologies that will be modeled, this paper tells a fictional story about regional greetings

[2]There are also a number of ways in which the Greco-Roman diagram is not analogous to the Mesopotamian version. First, Sparta and Corinth were not as closely related either geographically or culturally as Lagash and Girsu. Second, while Melos tried to maintain neutrality between Sparta and Athens, it was a Spartan colony and ultimately was not treated as being neutral between the two. In contrast with this, Kish seems to have been treated as a neutral third party for determining the border between Umma and Lagash. Finally, Akkadian is a distinct language from Sumerian just as Persian is distinct from Greek (though both are Indo-European) but it was ultimately the Macedonians not the Persians who conquered the two cities after they weakened each other through their rivalry.

in Mesopotamia. In our story, there is an Akkadian, Sumerian, Umma, Lagash, and Girsu greeting. Different types of people have different preferences for greetings, but a person prefers giving the same greeting as a partner over giving her favorite greeting but failing to coordinate because the fictional greetings function like handshakes, high fives, and hugs. If two people fail to give each other the same greeting, then they do not get any payoff from the interaction. As illustrated in Figure 1, Umma, Lagash, and Girsu are all Sumerian cities, and the broader region is dominated by Akkadians. Sumerians who most prefer the Umma, Lagash, or Girsu greeting also find the generic Sumerian greeting preferable to the Akkadian greeting. The patron god of Lagash is Ningirsu, who's temple is located in Girsu. People from Girsu most prefer the Girsu greeting and also prefer the Lagash greeting more than the generic Sumerian greeting. Umma and Lagash are rivals. People from Umma most prefer the Umma greeting and get nothing from coordinating on the Lagash greeting. Likewise, people from Lagash and Girsu most prefer the corresponding greetings and get nothing from coordinating on the Umma greeting. Thus, if a person who likes Umma interacts with one who likes Lagash, then their optimal greeting is the generic Sumerian greeting. Kish is friendly with both Umma and Lagash. People from Kish most prefer the Kish greeting and also prefer the Umma and Lagash greetings over the generic Sumerian greeting. Likewise, people who most prefer the Umma or Lagash greetings prefer the Kish greeting over the generic Sumerian greeting. Finally, people who most prefer the Akkadian greeting get nothing from coordinating on any of the other greetings.[3]

Considering all of these relationships between the city-states of Mesopotamia as a whole is very complicated and at no point does this paper present a model that captures everything in Figure 1. Rather, we start with a base model in which only two types of agents are considered and slowly builds up complexity by adding in social signals, additional types of agents, multiple dimensions to social signals, and agents' ability to choose whether to attend to a dimension of social signaling. Sections 2.1 and 2.2 explain the base model that this paper builds on and the evolutionary dynamics that are employed. Then Section 2.3 gives simulation results for the base model. The simulations have two types of agents, where one type has the Umma greeting as their most preferred greeting and the other type has the Kish greeting as their most preferred greeting. But, since Umma and Kish are friendly with each other, people still get some payoff for coordinating on their less preferred greeting. There is no social signaling in this base model, which means people are not able to use a social signal to determine which greeting the should use. Consequently, there are just two

[3]Furthermore, we might suppose for this example that all of these people are located in Babylon despite being from a variety of different cities. The point of this further supposition is that the model does not presuppose that people assort more frequently with those of their own type, which would be a reasonable assumption to make if we were considering a population that is dispersed among a variety of different cities. So supposing that all of these people are interacting in the same city circumvents that issue. That said, people of different ethnicities can self segregate within a single city. Mirroring this phenomenon, this paper's model does allow a dynamic in which people preferentially interact with those who broadcast the same social signals as themselves.

types of outcomes in the simulations. When preferences are weak (when the difference in payoff for coordinating on an agents preferred greeting verses the alternative greeting is small), then everyone uses the same greeting because the benefit of always coordinating outweighs the benefit of always giving the most preferred greeting but frequently failing to coordinate behavior. When preferences are strong, then each type of person in the population always gives their preferred greeting because the payoff from that greeting outweighs the failures in coordination when people of different types interact. The simulations also show that when the Umma people outnumber the Kish people and preferences are weak, then the population as a whole always settles on the Umma greeting as the norm. This shows that the model coheres with the reality that majority groups often have their norms dominate the broader population; e.g. people often equate being American with being white (ref Devos and Banaji 2005).

Section 2.4 explicitly gives the dynamics for adding signaling and assortment to the model. Then Section 2.5, continuing the prior example, shows simulation results from the new model. The simulation results show that people almost always learn to use the signals to broadcast their social identity for coordination. When Umma people interact with themselves, they give the Umma greeting. Likewise, when Kish people interact with themselves, they give the Kish greeting. But when Umma people interact with Kish people, the population still has to settle on whether to coordinate on the Umma greeting or the Kish greeting. As before, the more numerous population is advantaged. If the Umma people outnumber the Kish people, then the population is more likely to settle on the Kish greeting as the norm when people of different types interact.

Section 2.6 gives dynamics for adding attention and signal costs to the model. With attention, agents can learn whether or not to pay a signal cost that allows them to broadcast a social signal and condition their actions on that signal. This dynamic is useful because it allows agents to abstain from social signaling when the agent has no reason to broadcast a social signal. This is illustrated in Section 2.7, which extends the prior example of coordination between people from Umma and Kish by adding in people from Akkad. Since Akkadians get nothing from any of the Sumerian greetings, they should always give the Akkadian greeting and have no reason to pay the cost of broadcasting a social signal. Simulations show that the presence of the Akkadians can increase the likelihood that the people from Umma and Kish use social signals for coordination. However, the simulations also show that if the people from Umma are sufficiently numerous, they can end up always giving the Umma greeting and never signaling.[4]

At this point, the model is complex enough to produce a very wide range of outcomes. Section 2.7 discusses the most frequent outcomes which are relevant to understanding the model's dynamics. For completeness, further details about possible outcomes from the given parameters are discussed in Appendix B as well

[4]The preferences of people from Umma and Kish are symetric. If people from Kish are sufficiently numerous, they can also end up always giving the Kish greeting and never signaling. Since the two populations preferences are symetric, results are only shown for increasing the proportion of people from Umma.

as a supplemental PDF. Subsequent sections of the paper focus only on outcomes that exhibit the group topology that parameters are aimed at producing. Section 2.8 details the final addtion to the mode, allowing multiple dimension to social signals. What differentiates signals in the same dimension from those in different dimensions is that only one signal can be sent in each dimension; i.e. signals in the same dimension are mutually exclusive, while those in different dimensions are not. Now that the model is fully described, Section 2.9 shows how four different group topologies can be produced. The corresponding examples of Mesopotamian greetings are given in the relevant subsections. Finally, Section 3 discusses how the model can be related to empirical evidence and possible future amendments to the model.

2 Model Description and Results

This section iterates between model description and simulation results for the model. It begins with a description of the traditional Bach or Stravinsky game and then extends the game to a more general framework and adds complexity in agents ability to broadcast identity signals and condition their actions on those signals. At each step in extending the game, simulation outcomes from the resulting model are presented. When appropriate motivations for the subsequent extension of the model are also included.

2.1 Generalizing the Bach or Stravinsky Game

The traditional Bach or Stravinsky (BoS) game is a one shot coordination game between two players. It was introduced to the literature Luce and Raiffa 1957 (NEED ACTUAL CITATION) as the “battle of the sexes” game, in which two players have differing preferences over whether to attend a prize fight or ballet, but get no payoff if attending alone:

		Player 2	
		Bach (Prize Fight)	Stravinsky (Ballet)
Player 1	Bach (Prize Fight)	t, 1	0, 0
	Stravinsky (Ballet)	0, 0	1, t

Table 1: Traditional Bach or Stravinsky Game: $t > 1$.

More recently, authors have called this the “Bach or Stravinsky” game to dissociate it from various undesirable prejudices and stereotypes frequently attached to sex and gender while maintaining the same abbreviation, BoS.

What the game is intended to capture, rather than gender prejudices, is the dynamics of differing preferences or norms in contexts of coordination. While the choice between meeting at a prize fight or ballet can be a coordination problem, the class of contexts in which we have to coordinate is much broader than determining where to meet. We coordinate when deciding to shake hands or high five, or when choosing music for a party. Even in communication we can face a coordination problem in selecting the right name for something given both our own preferences and our audience's; e.g. the same thing might be called "battle of the sexes" or "Bach or Stravinsky", it might be called "the morning star", "the evening star", or "Venus", it might be called "the electric slide" or "the wobble".

As a first step in extending the BoS game to better capture its target phenomenon, we consider a model in which the game is played repeatedly among intermixing agents in a population. In this model, agents still interact pairwise, but who an agent is paired with varies as the game is repeatedly played. Agents payoffs in the game are given by their type. Thus, replacing the two players in the traditional BoS game we get two types in the generalized BoS game:

		Type 0	
		Bach	Stravinsky
Type 0	Bach	t, t	0, 0
	Stravinsky	0, 0	1, 1
		Type 1	
		Bach	Stravinsky
Type 0	Bach	t, 1	0, 0
	Stravinsky	0, 0	1, t
		Type 1	
		Bach	Stravinsky
Type 1	Bach	1, 1	0, 0
	Stravinsky	0, 0	t, t

Table 2: Generalized Bach or Stravinsky for a population of two types: $t > 1$.

Since agents always get a payoff of 0 when they fail to coordinate, we can equivalently present agents' payoffs as their coordination preferences:

Coordination Preferences	Bach	Stravinsky
type 0	t	1
type 1	1	t

Table 3: Coordination Preferences: generalized BoS for a population of two types, $t > 1$.

Table 3 expresses all of the information in Table 2, but does so much more concisely. This concise format will be particularly useful when we consider populations with more than just two types.

2.2 Learning Dynamics

The generalization of the BoS game just given is independent of learning dynamics. This paper presents results modeling agents as evolving their dispositions through replicator dynamics. The dynamics was selected because it has substantial established literature and is computationally tractable.

In a simulation of the model, agents begin with a randomly selected strategy profile. On each timestep, the prevalence of each strategy profile among a type is adjusted according to a discrete replicator equation:

$$N_{t+1}(x) = N_t(x) + N_t(x) \times [U(x) - Avg(U(i))_{i \in X}]$$

Where, for a given type, X is the set of all strategy profiles, $N_t(x)$ is the number of agents of the given type with strategy profile x at timestep t, $U(x)$ is the utility of strategy profile x, and $Avg(U(i))_{i \in X}$ is the average utility of all the strategy profiles present among the given type. A strategy profile's utility for a given type is calculated as:

$$\begin{aligned} U(x) &= \sum_{i \in Y} [M(i) \times p_{xi}] & \text{if } x \neq i \\ U(x) &= \sum_{i \in Y} [(M(i) - 1) \times p_{xi}] & \text{if } x = i \end{aligned}$$

Where Y is the set of all (for any type) strategy profiles present in the population, $M(i)$ is the number of agents (of any type) in the population who play strategy i, and p_{xi} is the payoff an agent of the given type gets for playing strategy x when paired with an agent who plays strategy i. After adjusting the prevalence of strategy profiles, their quantity is normalized so that the number of agents of a given type remains constant throughout a simulation.

Finally, the possibility of an agent's strategy profile mutating is also included. This is governed by two parameters, m_0 and m_1. m_0 is the probability that an agent is selected for mutation. m_1 is the probability that an element in a string expression of the agents strategy profile changes to a random value. Presently, agents' strategy profiles are expressed as strings of length one, which specify which of the two actions an agent plays. An agents strategy profile can be $< B >$, play Bach, or $< S >$ play Stravinsky. So with probability m_0 an agent is selected for mutation, and if selected with probability m_1 a random value of B or S is used to replace the only element in the string expression of the agent's strategy profile. As the model is extended to allow agents to broadcast social signals and to choose their action based on the signal of a paired agent, the string expression of agents' strategy profiles will have length greater than one and the rational for using two mutation parameters will become more clear. Consequently, the mutation dynamic will be revisited at that point.

2.3 Simulation Results for BoS Generalized to a Population with Repeated Plays

To illustrate this simple version of the model, we consider a population with two types of agents, people from Umma and people from Kish. In addition to the parameters listed in Table 5, people from Umma (type 0) make up $0.5 + \beta$

proportion of the population (and, accordingly people from Kish are a $0.5 - \beta$ proportion). Coordination preferences are:

Coordination Preferences	Umma greeting	Kish greeting
people from Umma (type 0)	$1 + \alpha$	1
people from Kish (type 1)	1	$1 + \alpha$

Table 4: Coordination Preferences: generalized BoS for a population of two types, $\alpha \geq 0$.

This model produces three possible outcomes: (i) each type plays their preferred action all of the time (shown in Figure 2a), (ii) everyone plays type 0's preferred action, Umma (shown in Figure 2b), and (iii) everyone plays type 1's preferred action, Kish (deducible as 1 - value in Figure 2a - value in Figure 2b). Figure 2a illustrates that as payoffs for preferred action increase, agents are more likely to always play their preference leading to coordination failures when agents of different types are paired. Figure 2b illustrates that as the proportion of type 0 agents increases, the outcome in which everyone plays type 0's preference, the Umma greeting, becomes more likely. This is because, even though type 1 agents have a higher payoff when they successfully coordinate on the Kish greeting, they have more opportunities for successful coordination with type 0 Umma agents, since the Umma agents are more prevelant. Thus type 1 agents (people from Kish) can have a higher net payoff from playing type 0's preference (the Umma greeting). Appendix A shows this analytically. The only time when agents all play type 1's preference (outcome iii), the Kish greeting, is when $\alpha = 0$, but this just means that agents do not actually have any preference between the two actions. Accordingly see that outcome (iii) occured in half of the simulations with $\alpha = 0$ (recall that this is deducible as 1 - value in Figure 2a - value in Figure 2b).

	Number of simulations[5]	Simulation length[6]	Population size[7]	m_0	m_1	Signal cost, c
Section 2.3 Figure 2	10,000	4×10^4	10,000	0.01	0.1	n/a
Section 2.5 Figure 3	10,000	4×10^4	10,000	0.01	0.1	n/a
Section 2.7 Figures 4 & 5	1,000	8×10^4	10,000	0.01	0.1	-0.01
Section 2.9.1 Figure 7 & 8	1,000	4×10^4	10,000	0.01	0.1	-0.0002
Section 2.9.2 Figure 10	1,000	8×10^4	10,000	0.01	0.1	-0.0005
Section 2.9.3 Figure 12	100	2×10^4	5,000	0.01	0.1	-0.0005
Section 2.9.4 Figure 14a & c	100	4×10^4	5,000	0.01	0.1	-0.0002
Section 2.9.4 Figure 14b	100	4×10^4	10,000	0.01	0.1	-0.0002

Table 5

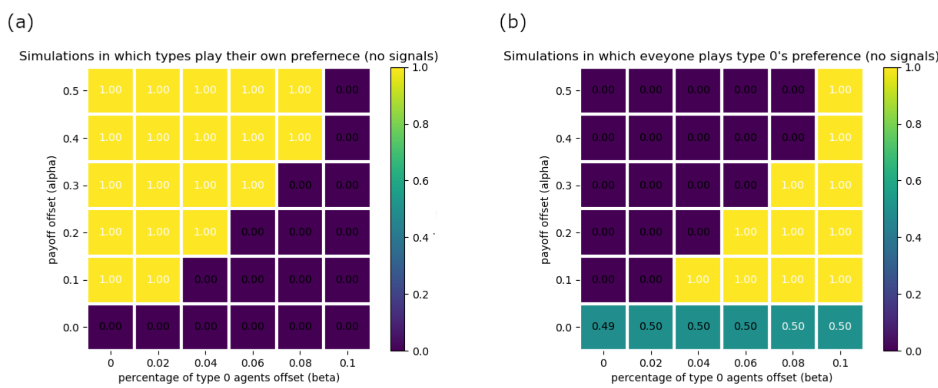

Figure 2: (a) Proportion of simulations that resulted in outcome (i) in which agents play their preference; this means that when agents of different types fail to coordinate. (b) Proportion of simulations that resulted in outcome (ii)in which everyone plays type 0's preferred action.

2.4 Adding Signals and Assortment to the Generalized BoS Game

In our next extension of the model, we allow agents to broadcast social signals and to use their partner's signal to determine their action in the game. Additionally, a homophily parameter, h, is included to model contexts in which people interact more with those who broadcast the same social signals as themselves.

Now that agents broadcast social signals, their strategy profiles can be expressed as strings of length 1+# of signals, where the first element is occupied by the social signal that an agent broadcasts and the subsequent elements are occupied by the action that is played when paired with an agent who broadcasts the corresponding signal. For example, suppose the there are two signals 1 and 2, with everything else remaining as before. Then there are 2^3 strategy profiles: $< 1BB >$, $< 1BS >$, $< 1SB >$, $< 1SS >$, $< 2BB >$, $< 2BS >$, $< 2SB >$, and $< 2SS >$. The strategy profile $< 1BB >$ corresponds to an agent broadcasting social signal 1, playing Bach when paired with an agent who broadcasts 1, and playing Bach when paired with an agent who broadcasts 2; the strategy profile $< 1BS >$ corresponds to an agent broadcasting social signal 1, playing Bach when paired with an agent who broadcasts 1, and playing Stravinsky when paired with an agent who broadcasts 2; etc.. Now we can see the rational for the two parameters for mutation. Suppose $m_0 = 0.01$ and $m_1 = 0.1$, then an agent has a 1 in 100 chance of being selected for mutation, and if selected each element

[5] As the model becomes more complex, simulations take longer to run. Consequently, there are fewer simulations for the most complex systems. In particular, the hierarchical double embedding topology from Section 2.9.3 is produced with a system in which there are 524,288 different possible strategy profiles. This computational difficulty also caused concessions in population size, which is the reason Section 2.9.4 gives results for both the smaller and larger population sizes.

[6] In these simulations, every 100 timesteps it was checked whether the distribution of agents' strategy profiles was unchanged. If so, the simualtion was halted prior to reaching the number of timesteps listed here.

[7] In Section 2.9.4 population sizes of both 5,000 and 10,000 are investigated. In that section we see that changing the population size does effect the optimal α value for producing the topology, but the general structure of coordination preferences does not need to change. As stated in footnote5 the smaller population size was used to make simulations more computationally tractable. However, there is a reason all other topologies were investigated using the larger population size:

Originally, group structures were investigated using a population size of just 1,000 agents. This produced a statistical anomaly that I was unable to identify the origin of. The anomaly was as follows. For the model from Section 2.7, when $\beta = 0$ the size of the type 0 and type 1 populations is equal. Consequently, the number of optimal outcomes favoring type 0 and favoring type 1 should also be close to equal. However, simulations consistently resulted in the optimal outcome favoring type 1 agents being 2-7% (depending on α value) more than than those favoring type 0. This anomaly was observable but nominal at a population size of 3,000 and not observable for populations equal to or larger than 4,000 agents. It seems most likely that the anomaly is a consequence of some idiosyncrasy in the random number generator used or in how the compiler utilizes it. However the cause could not be identified. To be safe, simulations used a population of 10,000 except for the hierarchical double embedding for which the larger population was not computationally tractable. Given the anomaly was unobservable for a population of 4,000, the simulations with a population of 5,000 should still be sound.

in the string expression of the agent's strategy profile has a 1 in 10 chance being changed to a random value. This results in mutations to a new strategy profile that is similar to the prior strategy profile being more likely than mutations to a strategy profile that is maximally dissimilar. This improves learning when there is a large set of possible strategy profiles.[8]

Intuitively, the homophily parameter captures random assortment when $h = 0$ and assortment where agents who broadcast the same signal are twice as likely to interact as agents who broadcast differing signals when $h = 1$. Mathematically, this is captured by using the following utility function:

$$U(x) = \sum_{i \in Y} [H(h, x, i) \times M(i) \times p_{xi}] \qquad \text{if } x \neq i$$

$$U(x) = \sum_{i \in Y} [H(h, x, i) \times (M(i) - 1) \times p_{xi}] \qquad \text{if } x = i$$

where $H(h, x, i)$ is defined as:

$$H(h, x, i) = \frac{N}{\sum_{j \in Y} [S(h, x, j)]} \times S(h, x, i)$$

where N is the total number of agents in the population and $S(h, x, j)$ is:

$$S(h, x, j) = 2^h \qquad \text{if profile x entails broadcasting the same social signal as j}$$

$$S(h, x, j) = 1 \qquad \text{if x does not entail broadcasting the same social signal as j}$$

It is easy to see that this amounts to the same utility function as before when $h = 0$. Likewise, it is easy to see how $S(h, x, i)$ reflects agents with profile x having twice as much utility (when $h = 1$) from interactions with agents with profile i, due to more frequent interactions, when profiles x and i entail broadcasting the same social signal. The component of the equation requiring some explanation is $\frac{N}{\sum_{j \in Y} [S(h,x,j)]}$. This factor normalizes $S(h, x, i)$ relative to the signaling dispositions of the entire population.

To see why this normalization is necessary, consider counterfactually what would happen if we just defined $H(h, x, i)$ as equal to $S(h, x, i)$. Suppose further that for a population with the preferences of Table 4, every agent is broadcasting the same signal and playing their preferred action irrespective of the signal observed. If each type makes up half of the population, then agents are successfully coordinating in about half of their interactions. The signal being broadcast is meaningless for assortment since everyone is broadcasting the same signal. But suppose one agent mutates and starts broadcasting a different signal. Given the

[8] Functionally similar behavior could be obtained with a single parameter rather than two by using just the m_1 parameter with a smaller value (say $m_0 \times m_1$) and always selecting all agents for mutation. However, the two parameter version is also computationally more tractable.

uniformity of everyone else in the population, the mutated agent is still just as likely to interact with a type 0 agent as a type 1 agent. Consequently, it will still be the cast that about half of the agent's interactions will result in successful coordination. However, with our counterfactual condition that $H(h, x, i)$ as equal to $S(h, x, i)$. The mutated agent's utility would suddenly be cut in half even though the agent is just as likely to successfully coordinate on her preferred action as before. But this makes no sense. By including the normalization factor $\frac{N}{\sum_{j \in Y}[S(h,x,j)]}$ in our definition of $H(h, x, i)$, we ensure that agents only see an increase in utility from broadcasting a particular signal in proportion to how much that signal impacts the frequency of their interactions.[9]

2.5 Simulation Results for Generalized BoS with Signals and Assortment

Type 0 agents make up $0.5 + \beta$ proportion of the population (and type 1 agents are a $0.5 - \beta$ proportion). Coordination preferences are:

Coordination Preferences	Umma greeting	Kish greeting
type 0 (people from Umma)	$1 + \alpha$	1
type 1 (people from Kish)	1	$1 + \alpha$

Table 6: Coordination Preferences: generalized BoS for a population of two types, $\alpha \geq 0$.

This model produces many different outcomes. The outcome (i) in which each type plays their preferred action all of the time is possible, but never occurs under the parameters shown in this section. The outcomes in which (ii) everyone plays type 0's preferred action, the Umma greeting, and (iii) everyone plays type 1's preferred action, the Kish greeting, are outcomes in which agents fail to make use of the social signals available to them.[10] The outcomes in which (iv) type 0 agents always play their preferred action and type 1 agents play 0's preference with type 0's and play their own preference among themselves, and (v) type 1 agents always play their preferred action and type 0 agents play 1's preference

[9] If one wished to model a scenario in which an agent, in virtue of adopting an unused signal, simply ceased to interact with all agents since no other agents shared that signal, then it still would not make sense to let $H(h, x, i)$ equal $S(h, x, i)$. For that scenario, $S(h, x, i)$ should also be modified to be 0 when profile x does not entail broadcasting the same social signal as i. However, such antisocial behavior seems out of place given that this paper is modeling agents in coordination games where there is some common ground between agents of different types; i.e. an agents who prefers Bach still gets some benefit for coordinating on Stravinsky. In adopting the normalized equation, this paper reflects agents having the same number of interactions regardless of their signal, but preferentially interacting with agents who share their signal.

[10] The frequency of this outcome is close to 1- the frequency of (iv) - the frequency fo (v), which can be deduced from the figures given in this section. However, it is more precisely 1- the frequency of (iv) - the frequency of (v) - the frequency of (vi); and in a few limited cases (vi) made up close 0.06 proportion of outcomes though in most cases the proportion was closer to 0.

with type 1's and play their own preference among themselves are optimal outcomes which depend on the agents evolving meaningful signals. Finally, though infrequent, (vi) there are a variety of suboptimal outcomes in which meaningful signals evolve but at least one type does not play their preferred action among themselves.

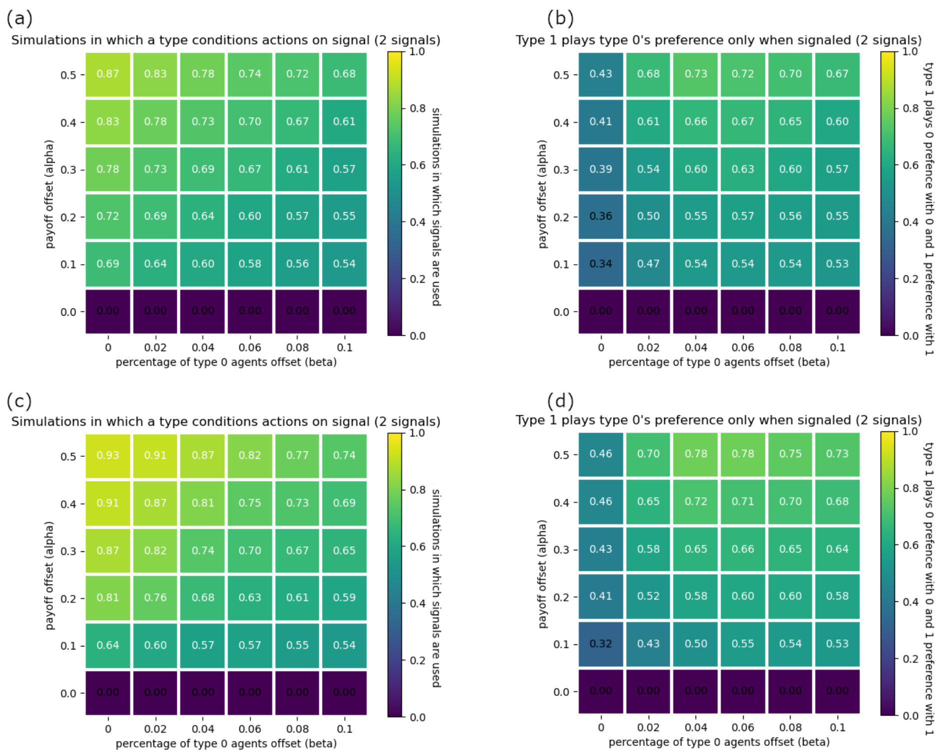

Figure 3: (a) Proportion of outcomes that are (iv) or (v) for $h = 0$. (b) Proportion of outcomes that are (iv) for $h = 0$. (c) Proportion of outcomes that are (iv) or (v) for $h = 1$. (d) Proportion of outcomes that are (iv) for $h = 1$.

Figure 3 shows all optimal outcomes (outcomes (iv) and (v)) on the left and just the optimal outcome favoring type 0 (iv) on the right (so it is impossible for a value on the right to exceed the corresponding value on the left). Outcome (iv) favors type 0 because it involves agents coordinating on type 0's preferred action whenever an interaction between agents of different types occurs. Just as before, increasing the proportion of type 0 agents (i.e. increasing β) makes outcomes favoring type 0 more frequent. Thus, as β increases, the numbers on the right hand side converge towards those on the left. Likewise, when the proportion of type 0 and type 1 agents is equal (i.e. $\beta = 0$), the values on the right are roughly half of those on the left. The top of the figure is for simulaltions with no assortment ($h = 0$), and the bottom is for simulations with strong assortment. Comparing the top and bottom shows that, in this simple model, assortment increases the likelihood of optimal outcomes obtaining. This is because the assortment dynamics creates another avenue of incentives for agents to use social signals.

2.6 Adding Attention and Signal Costs to the Generalized BoS Game

It is often the case that social signals require some effort to broadcast or attend to and only those groups for whom the social information is relevant invest the effort in engaging with the signals; among a subpopulation individuals may invest effort in signaling whether they are lefts or rights, while the broader population remains entirely oblivious to the social signals surrounding them. To capture this, the model is extended with a special signal, 0, which indicates an agent is not attending to signals and a signal cost, c, that an agent incurs if she broadcasts any signal other than 0. When an agent does not attend to signals, i.e. when she broadcasts 0, she interacts with all other agents as if they had also broadcast 0. Interacting with all other agents as if they broadcast 0 means that actions cannot be chosen based on the social signal that was broadcast, which is exactly what should be the case for an agent who is not attending to the social signals.

To illustrate this extension of the model, lets consider how the example from Section 2.4 changes under this new extension. While there were previously two signals 1, and 2, there are now three signals 0, 1, and 2. Consequently there are now 3×2^3 strategy profiles: $< 0BBB >$, $< 0BBS >$, $< 0BSB >$, $< 0BSS >$, $< 0SBB >$, $< 0SBS >$, $< 0SSB >$, $< 0SSS >$, $< 1BBB >$, $< 1BBS >$, $< 1BSB >$, $< 1BSS >$, $< 1SBB >$, $< 1SBS >$, $< 1SSB >$, $< 1SSS >$, $< 2BBB >$, $< 2BBS >$, $< 2BSB >$, $< 2BSS >$, $< 2SBB >$, $< 2SBS >$, $< 2SSB >$, and $< 2SSS >$. Now, it might be noted that the strategy profiles $< 0BBB >$, $< 0BBS >$, $< 0BSB >$, and $< 0BSS >$ all involve the same dispositions since an agent who does not attend to signals will treat agents who signal 1 or 2 as if they had signaled 0, i.e. agents with these profiles will always play B. Likewise, the strategy profiles $< 0SBB >$, $< 0SBS >$, $< 0SSB >$, and $< 0SSS >$ all involve the same dispositions since an agent who does not attend to signals will treat agents who signal 1 or 2 as if they had signaled 0, i.e. agents with these profiles will always play S. However, if an agent with one of these strategy profiles mutates in just the signaling position of the strategy profile the resulting strategy profile differs depending on what the agents prior profile was even when comparing two profiles that entailed the same dispositions. E.g. while $< 0SBS >$ and $< 0SSB >$ entail the same dispositions, a mutation to signaling 1 instead of 0 results in either $< 1SBS >$ and $< 1SSB >$ which entail very different dispositions. For this reason, distinct string representations of strategy profiles are considered distinct strategy profiles even if those distinct strings entail the same dispositions.[11]

[11] It is also computationally convenient to have all string representations of strategy profiles

At the macro level, this extension of the model changes nothing in our formula for the utility of a strategy profile:

$$\begin{aligned} U(x) &= \sum_{i \in Y} [H(h, x, i) \times M(i) \times p_{xi}] && \text{if } x \neq i \\ U(x) &= \sum_{i \in Y} [H(h, x, i) \times (M(i) - 1) \times p_{xi}] && \text{if } x = i \end{aligned}$$

However, the p_{xi} component of the formula does change. Previously, p_{xi} was simply the payoff specified in the agent's coordination preferences for an action if strategy profiles x and i lead to coordination on the same action and zero otherwise. Now, if strategy profile x entails broadcasting a social signal that is not zero, then p_{xi} is the payoff specified in the agent's coordination preferences for an action plus c if strategy profiles x and i lead to coordination on the same action and c otherwise (c is added to the prior p_{xi} value because this paper's convention is to use negative values for signal costs). If strategy profile x entails broadcasting 0, then p_{xi} is unchanged since no signaling cost is incurred. For example, if two agents have a coordination preference of 1 for coordinating on Bach and $c = -0.1$ then they will have a payoff of $p_{xx} = 1$ if both agents use strategy profile $x =< 0BBS >$; but, if both agents use strategy profile $x =< 1SBS >$ then they will have a payoff of $p_{xx} = 1 - 0.1 = 0.9$.

In this model, it is always the case than an agent not conditioning her actions on signals and not broadcasting a social signal co-occur. However, in the real world, these are independent dispositions. An agent might abstain from broadcasting a social signal, but still use social signals to determine what action to perform. Conversely, an agent might broadcast a social signal, but not pay attention to signals when choosing what action to perform. Combining these two dispositions so that they always co-occur is computationally convenient, plausible for some social contexts, and does not inhibit us from producing some interesting and novel social signaling topologies. So, for now, we maintain this assumption and leave investigation of the model with independence between these two dispositions as a task for future research.

2.7 Simulation Results for Generalized BoS with Attention and Signal Costs

Type 0 agents make up $0.33 + \beta$ proportion of the population, type 1 agents are a $0.33 - \beta$ proportion of the population, and type 2 agents are a 0.34 proportion of the population. Coordination preferences are:

Coordination Preferences	Umma greeting	Kish greeting	Akkadian greeting
type 0 (people from Umma)	$1 + \alpha$	1	0.5
type 1 (people from Kish)	1	$1 + \alpha$	0.5
type 2 (people from Akkad)	0	0	1

be the same length.

The inclusion of a third type of agent, type 2, who has no incentive to condition actions on signals, since the Akkadian greeting is the only action with a payoff, makes signaling more prevalent for the other two types. To illustrate this, Figure 4a keeping all other parameters the same considers the model where type 0 agents make up $0.5 + \beta$ proportion of the population, type 1 agents are a $0.5 - \beta$ proportion of the population; i.e. there are no type 2 agents.

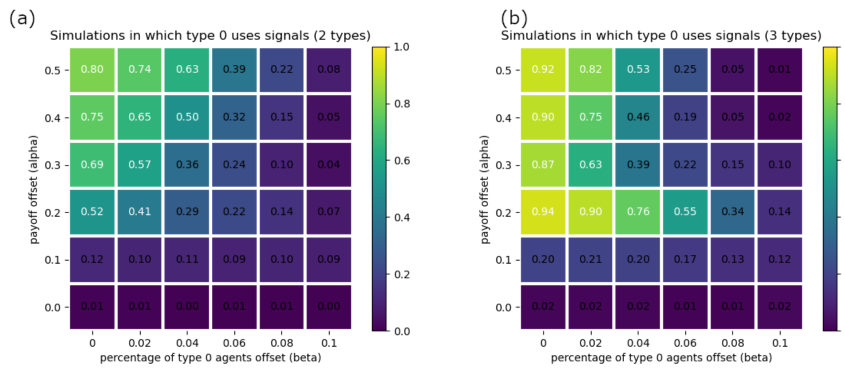

Figure 4: (a) Proportion of outcomes in which type 0 agents broadcast a social signal, for the model with two types of agents. (b) Proportion of outcomes in which type 0 agents broadcast a social signal, for the model with three types of agents.

Adding in the type 2 agents makes type 0 more likely to transmit a signal when β is small (see Figure 4b). When type 0 agents become more prevalent in the population (i.e. when β increases), signaling from type 0 agents decreases since they have leverage to always play their preferred action and type 2 agents frequently adopt one of the two signals (this type of outcome is shown in Figure 5a). But the general trend is clear, minority groups have more incentive to use social signals. In Figure 4b, there is some noticable stratification around $\alpha = 0.2$. This seems to be a consequence of the threshold for type 0 and 1 agents being highly incentivized play their preference among ingroup members occurs around $\alpha = 0.2$; while the threshold for type 0 agents to use their power as the largest group to only play their preference occurs around $\alpha = 0.3$ (see also Figure 5a).

For the parameters investigated, there were thirteen different outcomes that occurred at least 0.5% of the time for some data point. These outcomes are given a detailed description in Appendix B. This section shows three prominent outcomes.

(viii) The outcome in which type 0 and type 2 agents always played their respective preference, and type 1 agents played U with type 0, K with type 1, and A with type 2. The prevalence of this outcome is shown in Figure 5a. This outcome was frequently characterized by type 0 agents signaling

0 and type 1 and 2 agents attending to signals that reliably identified their type. One can check that this is a Nash equilibrium. It is suboptimal in the sense that when type 0 agents are paired with type 2 agents, there is a failure to coordinate. As previously noted, the type 2 agents should always give the Akkadian greeting. So one might wonder why this population ends up signaling? I.e. why can this behavior be a Nash equilibrium? The reason is that if the type 1 agents are giving the Umma greeting when they see the null signal (since this allows coordination with type 0 agents), then they will only coordinate with type 2 agents if the type 2 agents broadcast a social signal. So type 2 agents are not attending to the signals because they are conditioning their actions on the signals. Rather they are attending to the signals because they benefit from those signals allowing type 1 agents to coordinate with them.

(xii–xiii) Agents condition their actions optimally, in the sense that there were never failures of coordination and agents played their most prefered greeting among themselves. This is shown in Figure 5b.

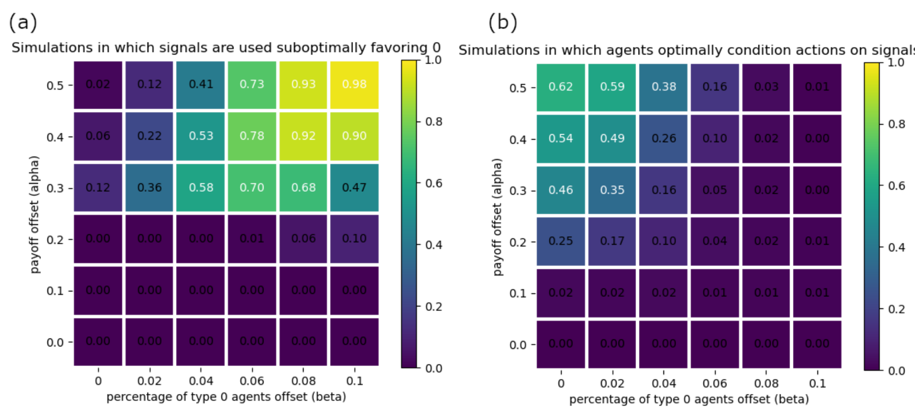

Figure 5: (a) Proportion of outcomes that are (viii). (b) Proportion of outcomes that are (xii-xiii).

2.8 Adding Multiple Dimensions to Signals in the Generalized BoS Game

The final extension this paper makes to the model is allowing agents to broadcast social signals in multiple dimensions. Agents can broadcast up to one social signal in each dimension available. This can be thought of as allowing agents to broadcast independent social signals in correspondence with independent social signals. For example, signaling one is a Baltimore Orioles fan by wearing a corresponding baseball cap is incompatible with wearing a Washington Nationals cap, but entirely independent of signaling one is a San Antonio Spurs fan by wearing a corresponding basketball jersey. In this example signals in the baseball dimension are mutually exclusive, but independent of signals in the basketball dimension. In the model whether or not an agent attends to signals in one dimension is independent of whether another dimension is attended to. Thus, if there are two dimensions for signaling and two signals in each dimension (excluding the null signal), then there are nine different social signals an agent can broadcast, letting signals in the first dimension be 1, 2 and the signals in the second dimension be 3, 4: 00, 03, 04, 10, 20, 13, 14, 23, and 24 are all possible signals.

In the string representation of strategy profiles, there is one place for each dimension of signaling and one place for each possible signal. Thus, if there are two dimensions for signaling and two signals in each dimension (excluding the null signal) and two actions, then there are $3^2 \times 2^9 = 4608$ unique strategy profiles. The signal cost c is incurred fore each dimension attended to. Thus if strategy profile x leads to successful coordination with agents employing strategy profile i by performing an action with coordination preferences value of 1, then if $x =< 00BBSBSBBSS >$ this results in $p_{xi} = 1$, if $x =< 20BBSBSBBSS >$ this results in $p_{xi} = 1 + c$, and if $x =< 14BBSBSBBSS >$ this results in $p_{xi} = 1 + 2c$.

In this extension of the model our utility function is still

$$U(x) = \sum_{i \in Y} [H(h, x, i) \times M(i) \times p_{xi}] \qquad \text{if } x \neq i$$

$$U(x) = \sum_{i \in Y} [H(h, x, i) \times (M(i) - 1) \times p_{xi}] \qquad \text{if } x = i$$

where $H(h, x, i)$ is defined as:

$$H(h, x, i) = \frac{N}{\sum_{j \in Y} [S(h, x, j)]} \times S(h, x, i)$$

However the definition of $S(h, x, j)$ is amended to be:

$$S(h, x, j) = 2^{kh}$$

where k is the number of dimensions in which x entails broadcasting the same social signal as j. It is easy to check that when there is only one dimension of signals this equation is equivalent to our prior equation.

In general this sections amended equations are quite simple and intuitive. p_{xi} is amended to include costs in proportion to the number of dimensions attended to, higher cost for attending to more dimensions. Likewise assortment is modified for agents to have greater preference towards interacting with agents who share their social signal in more dimensions; when $h = 1$ an agent exhibits twice the preference for interacting when an agents who signals identically in two dimensions than those who signal identically in one dimension and exhibits four times the preference for interacting with an agents who signals identically in two dimensions than those who do not signal identically in any dimensions.

2.9 Simulation Results for Generalized BoS with Multidimensional Signals

Now we have the full model. There are no more new dynamics to introduce. The multidimensional signaling allows agents to represent their common interests when their interests only partially overlap. For example, two agents can broadcast the same signal in one dimension and different signals in another. The following subsections show four different contexts in which agents take advantage of this capacity for multidimensional signaling. Given the increased complexity of the model, this paper only discusses the optimal outcomes from simulations.[12] That means an outcome has to have both optimal signaling behavior and corresponding actions to be reported. What is discussed is how agents' preferences produce the relevant topologies. Section 2.9.1 looks at agents who occupy the intersection of two groups. It shows how the same signaling system can reflect different topologies. This section occurs first for the purpose of making it clear why it is important to not focus exclusively on the social signaling systems that evolve, but to also consider the actions that are performed by agents using those signals. Section 2.9.2 shows how a single group can be a embedded in another. That is, one group signals in a single dimension and the other adopts the same signal in that dimension but also uses a signal in another dimension. Finally, Sections 2.9.3 and 2.9.4 each present a different way in which two groups can be embedded in another.

2.9.1 The Intersection

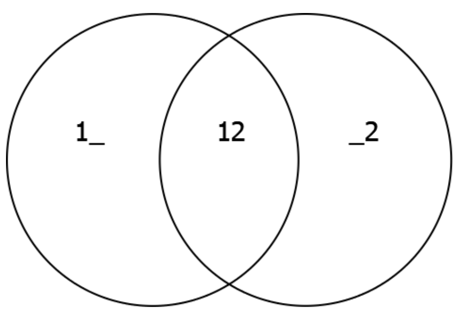

Figure 6: A signaling system in which we can investigate intersectionality.

Type 0 agents make up $0.2 - 0.5 \times \beta$ proportion of the population, type 1 agents are a $0.2 + \beta$ proportion of the population, type 2 agents are a $0.2 - 0.5 \times \beta$ proportion of the population, and type 2 agents are a 0.4 proportion of the population. Coordination preferences are:

[12]Except, in Section 2.9.1.

Coordination preferences	Umma greeting	Kish greeting	Lagash greeting	Akkadian greeting
Type 0 (people from Umma)	1	0	0	0.5
Type 1 (people from Kish)	1	α	1	0.5
Type 2 (people from Lagash)	0	0	1	0.5
Type 3 (Akkadians)	0	0	0	1

Table 7: Coordination preferences for investigating intersectionality. Simulations consider $\alpha \in \{0, 1, 2\}$

Figure 7 shows how frequently we see the signaling system shown in Figure 6. But this signaling system cannot be identified with a single group topology. The proportion of outcomes identified in Figure 8a have the type 1 agents behaving as if their identity is a mere conjunction of the type 0 and 2 social identities; i.e. rather than giving the Kish greeting among themseleves, they settle on giving either the Umma or Lagash greetings (which are types 0 and 2's preference). The proportion of outcomes identified in Figure 8b have the type 1 agents behaving as if their identity is *not* a mere conjunction of the type 0 and 2 social identities; i.e. among themselves they use the Kish greeting which has no payoff for type 0 and 2 agents. These outcomes align with what one should expect from the coordination preferences. When $\alpha \leq 1$, the type 1 agents have no added benefit to coordinating on the Kish greeting over the Umma or Lagash greetings; and this is exactly where we see the type 1 agents behaving as if their identity is a mere conjunction of the Umma and Lagash identities. When $\alpha > 1$, the type 1 agents have an added benefit to coordinating on the Kish greeting over the Umma or Lagash greetings; and this is exactly where we see the type 1 agents behaving as if their identity is a not mere conjunction of the Umma and Lagash identities. That said, these are trends not rules. For $\alpha = 2$ and $\beta = .01$, 17% of simulations resulted in type 1 agents using the Umma or Lagash greetings among themseleves even though they would have a higher payoff if they coordinated on the Kish greeting among themselves.[13] This seems to indicate a blindspot for quantitative research as it is how people signal and coordinate that is observable, not their preferences prior establishing conventions; when sufficiently marginalized, this model suggests that a group of people may interact, even among themselves, in a way that obscures their preference for an alternative.

[13]As in previous sections, a suboptimal norm is maintained because a single or small number of type 1 agents deviating from the normal greeting among themselves will lower their payoff due to the failures to coordinate that this will generate.

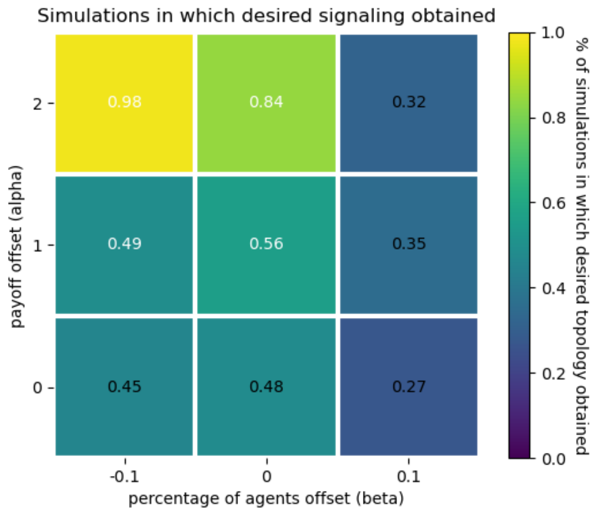

Figure 7: Proportion of outcomes in which type 0s signal in one dimension, type 1s singal in two dimensions, type 2s signal in one dimension that is different than the type 0s dimension, and type 3s do not signal in any dimension.

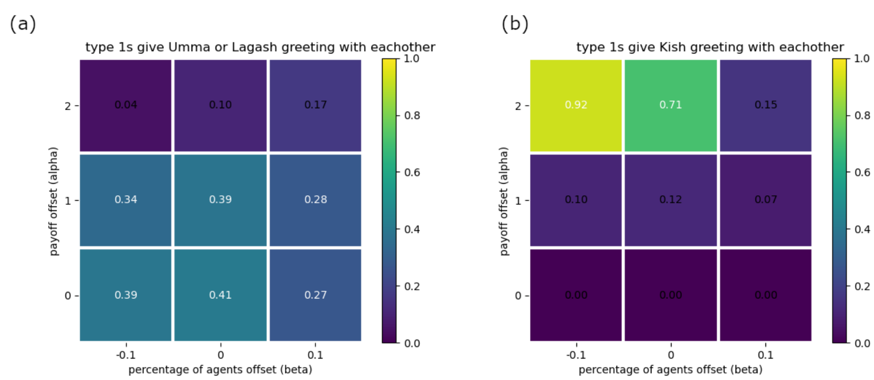

Figure 8: (a) Proportion of outcomes in which type 0s give the Umma or Lagash greeting with eachother in addition to exhibiting the desired signaling behavior; i.e. type 0s signal in one dimension, type 1s singal in two dimensions, type 2s signal in one dimension that is different than the type 0s dimension, and type 3s do not signal in any dimension. (b) Proportion of outcomes in which type 0s give the Kish greeting with eachother in addition to exhibiting the desired signaling behavior; i.e. type 0s signal in one dimension, type 1s singal in two dimensions, type 2s signal in one dimension that is different than the type 0s dimension, and type 3s do not signal in any dimension.

2.9.2 The Single Embedding Topology

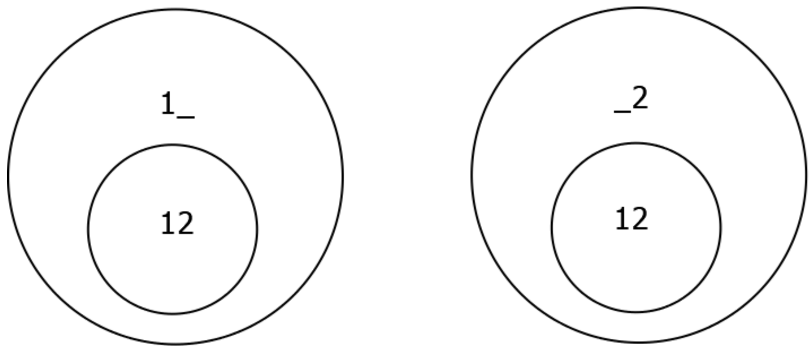

Figure 9: The two possible single embedding signaling systems when given one signal in each of two dimensions. The "_" can be read as 0, the null signal. The "_" is used to provide a visually apparent articulation of the signaling that highlights in the left figure all the 12 signalers are also 1 signalers and on the right all the 12 signalers are also 2 signalers.

Coordination preferences are:

Coordination Preferences	Lagash greeting	Girsu greeting	Akkadian greeting
type 0 (people from Lagash)	1	0	0.5
type 1 (people from Girsu)	1	$1+\alpha$	0.5
type 2 (Akkadians)	0	0	1

Recall from the introduction that Girsu is home to the temple of Ningirsu, the patron god of Lagash. So, in our fictional story of Mesopotamian greetings, people from Girsu most prefer the Girsu greeting and their second preference is the Lagash greeting. As is shown in the coordination preferences table, people from Lagash get nothing from the Girsu greeting and Akkadians get nothing from either the Lagash or Girsu greetings. While people from Lagash and Girsu get some payoff from the Akkadian greeting, it is a smaller payoff than what they get from the Lagash greeting.

Before looking at simulation results, we should expect these coordination preferences to produce the single embedding topology. That is, (i) we should expect the Akkadians to not signal in either dimension because they should always give the Akkadian greeting irrespective of an agents social signal; (ii) we should expect people from Lagash to only signal in a single dimension because this is the minimal number of signals sufficient for coordination with the Akkadians on the Akkadian greeting and coordination with others on the Lagash greeting, if we assume that condition (i) has already been met; (iii) assuming conditions (i) and (ii) are met, we should expect the people from Girsu to signal in both dimensions because signaling in the same dimension as people from

Lagash will result in those people giving the Lagash greeting rather than the Akkadian greeting (which is preferable to people from Girsu), and signaling in the additional dimension will allow people from Girsu to coordinate on the Girsu greeting among themselves. In the heat maps below, the single embedding topology is only counted as having obtained if *both* the signaling and the corresponding dispositions towards greetings stated in (i)-(iii) obtain. Though rare, it is possible for the desired signaling to occur without the corresponding greetings behavior. It is for this reason that Figure 9 is captioned as "the two possible single embedding *signaling systems*" rather than as "the two possible single embedding *topologies*". Figure 9 only depicts the desired signaling behavior. The term "topology" is reserved for situations in which both the desired signaling and behavior occur.

Comparing Figure 10 (a) with Figure 10 (c), we see that, when there is no assortment, increasing the proportion of the population that is Akkadian makes the desired topology more likely to obtain. But when there is significant assortment, Figure 10 (b) and (d), this is no longer the case. This can be understood through the fact that when there is no assortment, $h = 0$, then the Akkadians have no incentive to signal and should always give the Akkadian greeting irrespective of signal. However, when there is assortment, even though it is still the case that Akkadians should always give the Akkadian greeting irrespective of signal, they can benefit from adopting a signal for assortment in early timesteps of a simulation because the other types of agents have not yet learned to use a signal (or lack of signal) to coordinate on the Akkadian greeting when interacting with an Akkadian. But if other agent types only learn to give the Akkadian greeting when observing the signal broadcast by Akkadians, then Akkadians have continuing incentive to continue using their social signal.

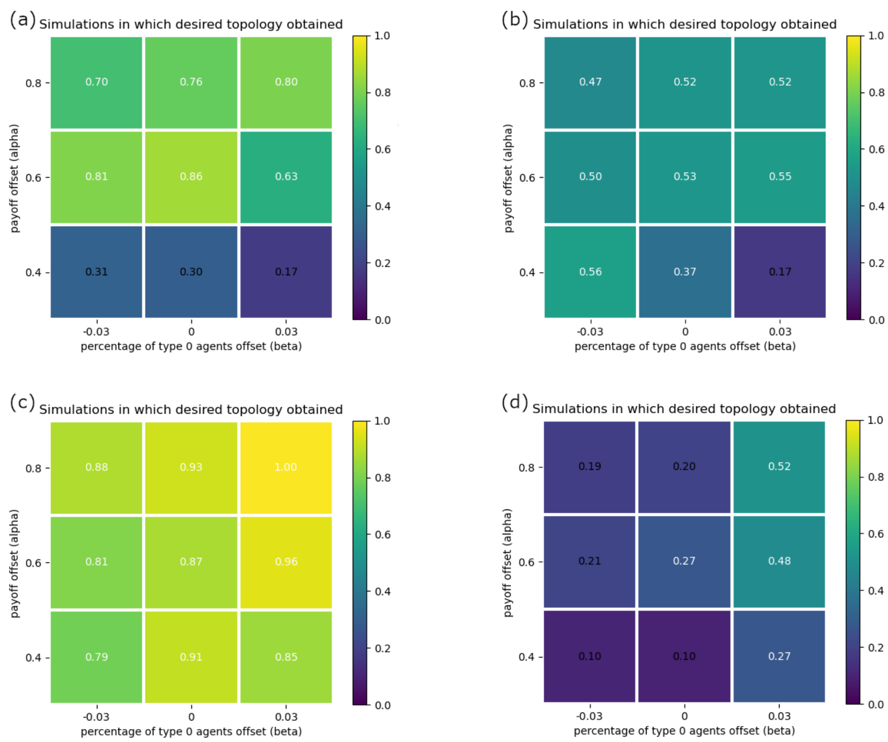

Figure 10: **(a)** Proportion of outcomes that are the single embedding topology, homophily $= 0$.Type 0 agents make up $0.33 + \beta$ proportion of the population, type 1 agents are a $0.33 - \beta$ proportion of the population, and type 2 agents are a 0.34 proportion of the population. **(b)** Proportion of outcomes that are the single embedding topology, homophily $= 1$.Type 0 agents make up $0.33+\beta$ proportion of the population, type 1 agents are a $0.33 - \beta$ proportion of the population, and type 2 agents are a 0.34 proportion of the population. **(c)** Proportion of outcomes that are the single embedding topology, homophily $= 0$. Type 0 agents make up $0.2+\beta$ proportion of the population, type 1 agents are a $0.2 - \beta$ proportion of the population, and type 2 agents are a 0.6 proportion of the population. **(d)** Proportion of outcomes that are the single embedding topology, homophily $= 1$. Type 0 agents make up $0.2 + \beta$ proportion of the population, type 1 agents are a $0.2 - \beta$ proportion of the population, and type 2 agents are a 0.6 proportion of the population.

2.9.3 The Hierarchical Double Embedding

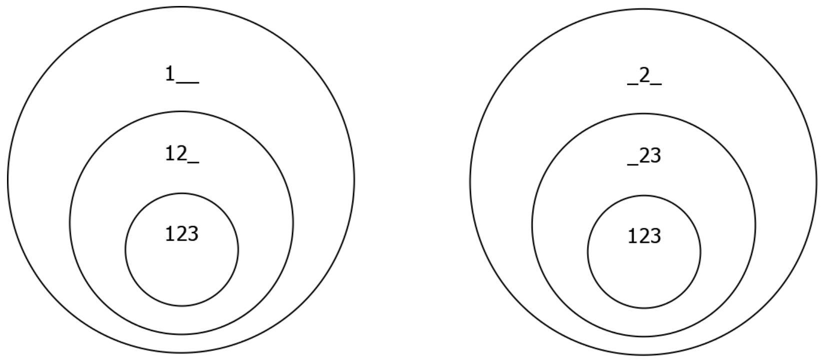

Figure 11: Two of the six hierarchical double embedding signaling systems that are possible when given one signal in each of three dimesions. I.e. there are three possible signals that the largest group can adopt 100, 020, and 003, and for each of those the middle group has two options for a signal in two dimensions that agrees with the signal of the larger group (e.g. if the larger group signals 003, then the middle group can signal in two dimension agreeing with the larger group in one dimension by signaling 103 or 203. The smallest group must signal 123 in all scenarios in which the hierarchical embedding occurs. Thus, there are $3 \times 2 = 6$ possible signaling systems, two of which are shown in this figure. As previously, the "_" can be read as 0, the null signal.

To produce the hierarchical double embedding topology, consider an extension of the example from the prior section. We continue the scenario from the single embedding of the people from Girsu into the people from Lagash, but add in coordination preferences for Sumerians in general. So we have People from Girsu and Lagash having more in common with people each other than Sumerians in general, but also preferring the Sumerian greeting over the Akkadian greeting. Coordination preferences are:

Coordination preferences	Sumerian greeting	Lagash greeting	Girsu greeting	Akkadian greeting
Type 0 (Sumerians)	1	0	0	0.25
Type 1 (people from Lagash)	1	$1 + \alpha$	0	0.25
Type 2 (people from Girsu)	1	$1 + 0.5 \times \alpha$	$1 + \alpha$	0.25
Type 3 (Akkadians)	0	0	0	1

Table 8: Coordination preferences for producing the hierarchical double embedding topology.

Type 0 agents make up 0.15 proportion of the population, type 1 agents are a $0.15 + \beta$ proportion of the population, type 2 agents are a $0.15 - \beta$ proportion of the population, and type 2 agents are a 0.55 proportion of the population.

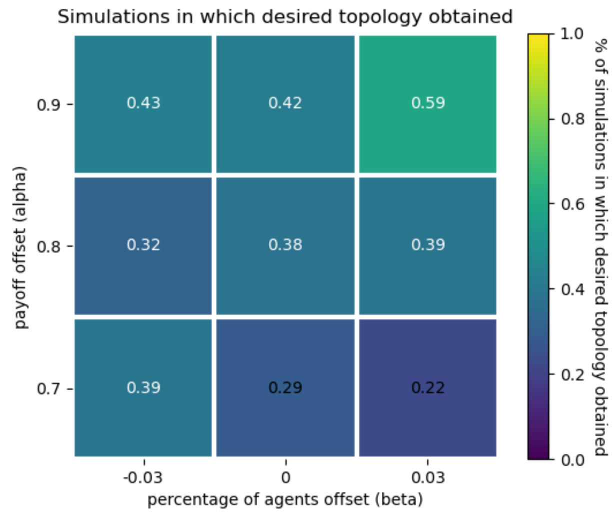

Figure 12: Proportion of outcomes that are the hierarchical double embedding topology.

2.9.4 The Disjoint Double Embedding

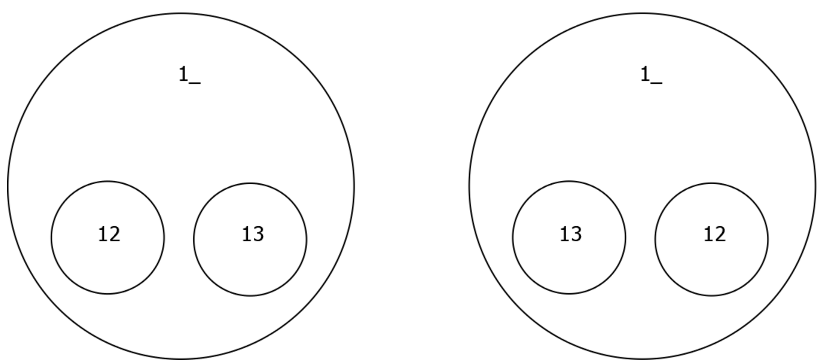

Figure 13: The only two disjoint double embedding signaling systems that are possible when given just two dimensions with one signal in the first dimension and two in the second.

To produce the disjoint double embedding we want two subgroups who have more in common with the larger group than they do with each other. So we consider people from Umma and people from Lagash as our two potential subgroups. Umma and Lagash had a history of conflict with each other stemming from a boarder dispute. In fact, Lugalzagesi's (from Umma) conquest of Lagash is plausibly what weakened the Sumerians enough for Sargon of Akkad to conquer them and impose Akkadian as the requisite language for government. So we assign coordination preferences such that the optimal greeting between a person from Umma and a person from Lagash is the general Sumerian greeting; as always, among themselevs their strongest preference is to use the greeting of their own city. Coordination preferences are:

Coordination preferences	Sumerian greeting	Umma greeting	Lagash greeting	Akkadian greeting
Type 0 (Sumerians)	1	0	0	0.25
Type 1 (people from Umma)	1	$1+\alpha$	0	0.25
Type 2 (people from Lagash)	1	0	$1+\alpha$	0.25
Type 3 (Akkadians)	0	0	0	1

Table 9: Coordination preferences for producing the disjoint double embedding topology.

Type 0 agents make up $0.15+\beta$ proportion of the population, type 1 agents are a $0.15-0.5\times\beta$ proportion of the population, type 2 agents are a $0.15-0.5\times\beta$ proportion of the population, and type 2 agents are a 0.55 proportion of the population.

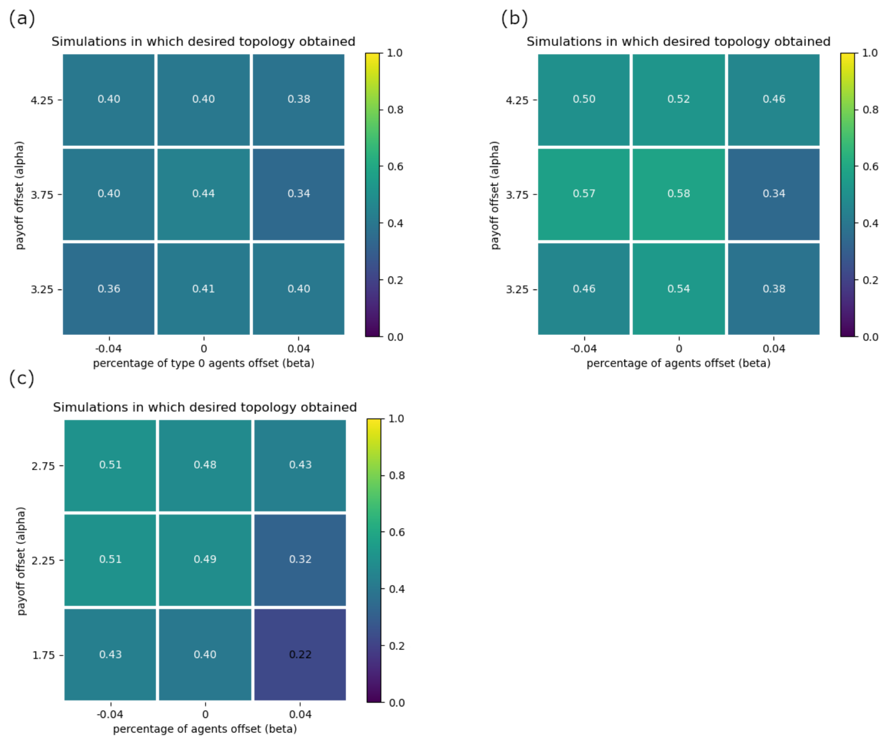

Figure 14: (a)Proportion of outcomes that are the disjoint double embedding topology when the population size is 5,000. The alpha values here are suboptimal for the population size.(b) Proportion of outcomes that are the disjoint double embedding topology for a population size of 10^4 and a simulation length of 4×10^4 timesteps. (c) Proportion of outcomes that are the disjoint double embedding topology. These seem to be optimal values for α when the population size is 5,000.

We can see from the results here that the larger population size results in a higher payoff for agents most preferred action (larger α) being required to produce the disjoint double embedding. This is because in a larger population there is a lower probability of random drift leading to a sufficient proportion of agents having the dispositions needed for optimal behavior to take root; by analogy it is easier to get 7 heads out of 10 coin flips than it is to get 70 heads out of 100 flips. But the higher payoff for agents' preference lowers the proportion of agents, with the right dispositions, needed for optimal behavior to become entrenched. What constitutes the right dispositions for optimal behavior to occur is complex because it requires both a tendency to signal in a particular way and an accompanying tendency to act a particular way when the signal is observed. Appendix D gives a more precise analysis of these interrelations for a simpler system.[14]

3 Discussion

In summary, Section 2.3 considered the Bach or Stravinsky game when generalized to a large population of two types of agents who repeatedly interact with each other rather than being confined to a one off game with a population of two. Simulations showed a clear advantage for the larger of the two types of agents; when both types of agents played the same greeting it was always the greeting preferred by the larger group. This advantage for larger groups persisted throughout subsequent additions to the model. Section 2.5 then looked at simulations that added signals and assortment to the model. Both with and without assortment, agents used social signals to better coordinate behavior. In Section 2.7 looked at the model with the addition of attention and signal costs. Simulations showed that a dominate group could ignore social signals and always use their preferred greeting. They also showed that agents of minority types have increased incentive to broadcast social signals when there is a large dominant group in the population. Finally, Section 2.9 showed how a variety of different group structures could obtain in systems with multidimensional signals. When the preferences of a type align more strongly with a type compared to another, social signals often obtain in a way that reflects this alignment in preferences.

Need to add:

- Discussion of how model can be related to real world data
- Discussion of why intersectional identities might sometimes be unobservable from quantitative data.
- Discussion of how future models might be amended to more reliably produce social identity signaling that mirrors agents preferences.

[14]Specifically comparing Figure 25 with Figure 26 shows how an increase in the payoff for the preferred action increases the area under the curve which correlates with an optimal outcome.

Appendix A Analysis of Section 2.3 Model

Recall that Section 2.3 gave the mean outcome of 10,000 simulations of the generalized BoS game with a population of 10,000 agents, $m_0 = .01$, $m_1 = .1$, and run for more than a sufficiently large number of time steps to reach an equilibrium (4×10^4).[15] Type 0 agents make up $0.5 + \beta$ proportion of the population (and type 1 agents are a $0.5 - \beta$ proportion). Coordination preferences are:

Coordination Preferences	Bach	Stravinsky
type 0	$1 + \alpha$	1
type 1	1	$1 + \alpha$

Table 10: Coordination Preferences: generalized BoS for a population of two types, $\alpha \geq 0$.

Recall that agents begin the simulation with randomly selected strategy profiles. Thus we can assume that roughly half of the agents play Bach and half play Stravinsky on the first timestep of a simulation. This should result in almost all of the agents playing their preferred action on the second time step. We can see this by first calculating the utilities:

$$U(\text{preferred action}) = 5 \times 10^3 \times (1 + \alpha)$$
$$U(\text{undesired action}) = 5 \times 10^3 \times 1$$

Then we can calculate the change in the population $N(\text{preferred action})$ as:

$$= \quad 2.5 \times 10^3 + 2.5 \times 10^3 \times [5 \times 10^3 \times (1 + \alpha) - 0.5 \times (5 \times 10^3 \times (1 + \alpha) + 5 \times 10^3)]$$
$$= \quad 2.5 \times 10^3 + 2.5 \times 10^3 \times [5 \times 10^3 \times (1 + \alpha) - 0.5 \times (5 \times 10^3 \times (1 + \alpha) + 5 \times 10^3 \times \frac{1+\alpha}{1+\alpha})]$$
$$= \quad 2.5 \times 10^3 + 2.5 \times 10^3 \times 2.5 \times 10^3 \times (1 + \alpha) \times [2 - (1 + \frac{1}{1+\alpha})]$$
$$= \quad 2.5 \times 10^3 + 2.5 \times 10^3 \times 2.5 \times 10^3 \times (1 + \alpha) \times (1 - \frac{1}{1+\alpha}$$
$$= \quad 2.5 \times 10^3 \times (1 + 2.5 \times 10^3 (1 + \alpha) - 2.5 \times 10^3)$$
$$= \quad 2.5 \times 10^3 \times (1 + 2.5 \times 10^3 \times \alpha)$$

and $N(\text{preferred action})$ is the same with $\alpha = 0$ therefore the proportion of

[15] In these simulations, every 100 timesteps it was checked whether the distribution of agents' strategy profiles was unchanged. If so, the simualtion was halted prior to reaching 4×10^4 timesteps.

agents of a given type that play their preferred action should be:

$$
\begin{aligned}
& \frac{N(\text{preferred action})}{N(\text{preferred action}) + N(\text{undesired action})} \\
= \quad & \frac{2.5 \times 10^3 \times (1 + 2.5 \times 10^3 \times \alpha)}{2.5 \times 10^3 \times (1 + 2.5 \times 10^3 \times \alpha) + 2.5 \times 10^3} \\
= \quad & \frac{1 + 2.5 \times 10^3 \times \alpha}{1 + 2.5 \times 10^3 \times \alpha + 1}
\end{aligned}
$$

which is close to 1 for the relevant values of α. In other words, we should expect almost all of the agents to be playing their preferred action after the first time step. In accordance with this, when the the mutation rates were set to zero, all simulations resulted in the outcome where all agents play their types preferred action.

Now, if we assume all of the agents are playing their preferred action, then we can calculate when type 1 agents', whose preferred action is Stravinsky, should have a higher utility for playing Stravinsky than Bach:

$$
\begin{aligned}
U(<S>) &> U(<B>) \\
(0.5 - \beta) \times 10^4 \times (1 + \alpha) &> (0.5 + \beta) \times 10^4 \\
0.5 - \beta + \alpha \times (0.5 - \beta) &> (0.5 + \beta) \\
\alpha \times (0.5 - \beta) &> (0.5 + \beta) - (0.5 - \beta) \\
\alpha &> \frac{2\beta}{0.5 - \beta}
\end{aligned}
$$

Figure 15 shows the graph of $\alpha = \frac{2\beta}{0.5-\beta}$ and it is easy to see that this aligns with the simulation results shown in Figure 2.

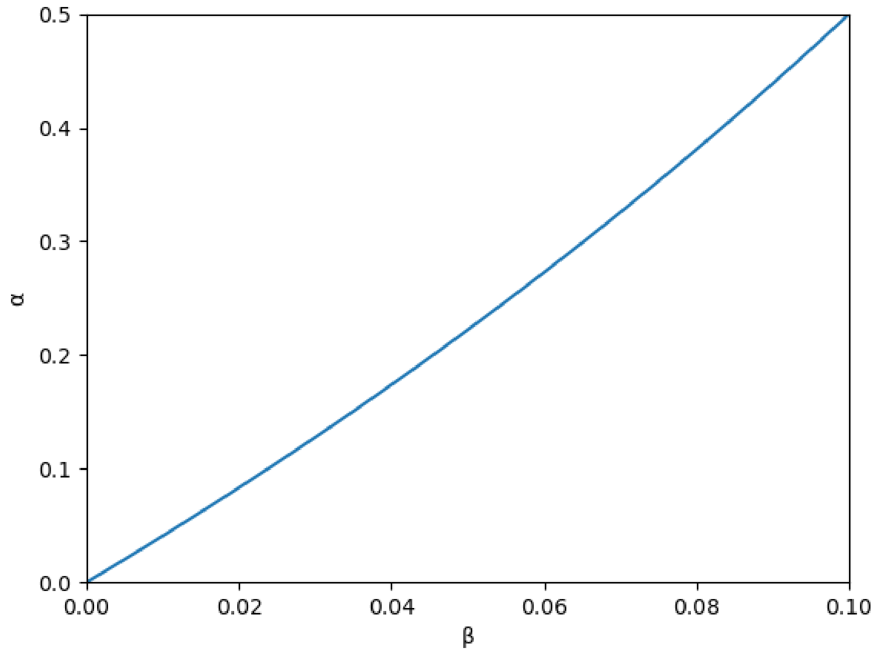

Figure 15: Type 1 agents should continue playing their preferred action if α and β place them above this line.

Appendix B Additional Details on Simulation Results for Generalized BoS with Attention and Signal Costs

Note: this appendix uses data that was generated independently of the data points given in Section 2.7. Therefore, there may be nominal differences in the values given in corresponding heatmaps though the general trends remains the same.

Additionally, note that a supplemental PDF shows the evolution of signaling and behavioral dispositions of individual simulation runs. For each of the seventeen outcomes here, ten different simulations resulting in the given outcome are detailed except when fewer than ten simulations resulted in the outcome. The supplemental PDF was composed prior to developing the Mesopotamian greetings example. It was written in terms of agents deciding between Bach, Stravinsky, or EDM rather than being in terms of the Umma greeting, the Kish greeting, and the Akkadian greeting. As I am disinclined to redo the 116 pages of the supplemental PDF, this appendix is written in terms of Bach, Stravinsky, and EDM. My appologies for the change. Bellow, the table of coordination pref-

erences is restated in those terms with the corresponding Mesopotamian greetings given in parentheses.

Coordination Preferences	Bach (Umma)	Stravinsky (Kish)	EDM (Akkadian)
type 0	$1+\alpha$	1	0.5
type 1	1	$1+\alpha$	0.5
type 2	0	0	1

For the parameters investigated, there were thirteen different outcomes that occurred at least 0.5% of the time for some data point.

(i) Outcomes in which there was no coordination between agents of different types, i.e. agents always play their preference irrespective of the signal transmitted. This is shown in Figure 16.

(ii–ix) Agents exhibit suboptimal coordination, but in an intuitive way. This is shown in Figure 17.

(viii) A notably common outcome was one in which type 0 and type 2 agents always played their respective preference, and type 1 agents played B with type 0, S with type 1, and EDM with type 2. The prevalence of this outcome is shown in Figure 18. This outcome was frequently characterized by type 0 agents signaling 0 and type 1 and 2 agents attending to signals that reliably identified their type. One can check that this is a Nash equilibrium. It is suboptimal in the sense that when type 0 agents are paired with type 2 agents, there is a failure to coordinate.

(x–xi) Agents condition their actions suboptimally, and in a counterintuitive way. This is shown in Figure 19. Specifically what was counterintuitive about these outcomes is that they involved a type playing their preference when paired with a type other than themselves, but not playing their preference among themselves.

(xii–xiii) Agents condition their actions optimally, in the sense that there were never failures of coordination. This is shown in Figure 20.

(xii) A notably common outcome was one in which type 0 played their preference when paired with type 1. The prevalence of this outcome is shown in Figure 21.

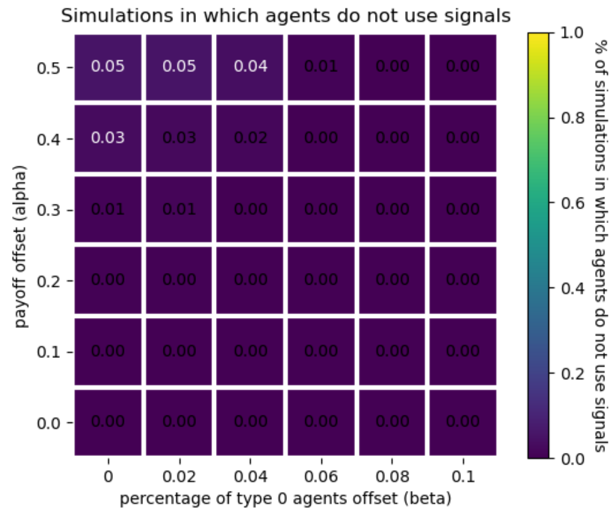

Figure 16: Proportion of outcomes that are (i).

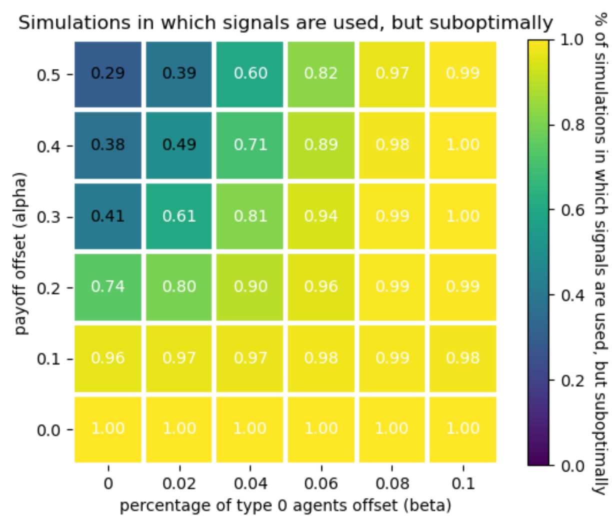

Figure 17: Proportion of outcomes that are (ii-ix).

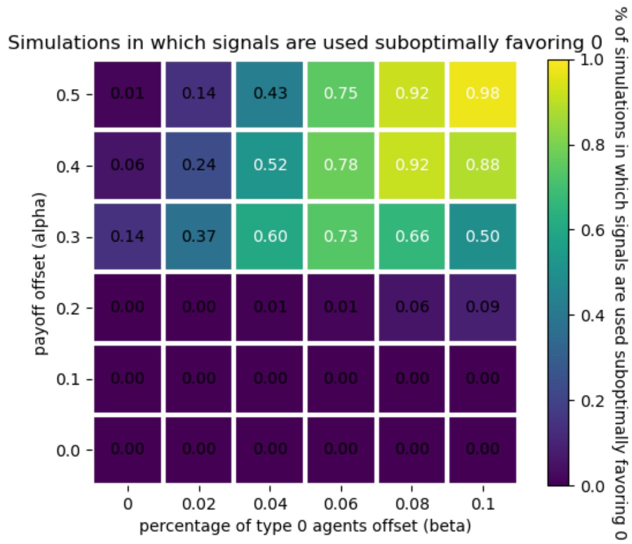

Figure 18: Proportion of outcomes that are (viii).

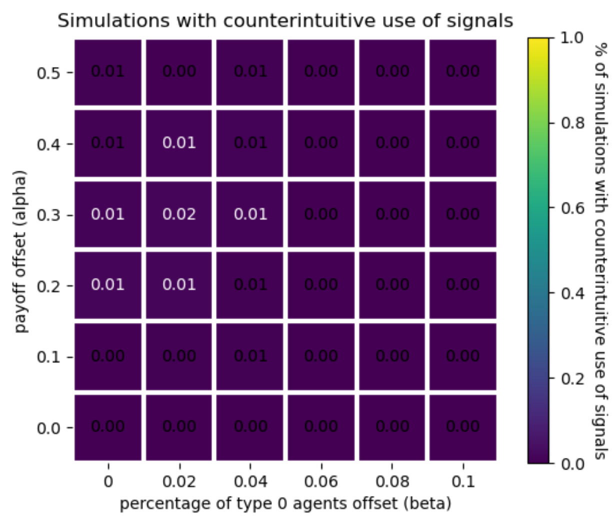

Figure 19: Proportion of outcomes that are (x-xi).

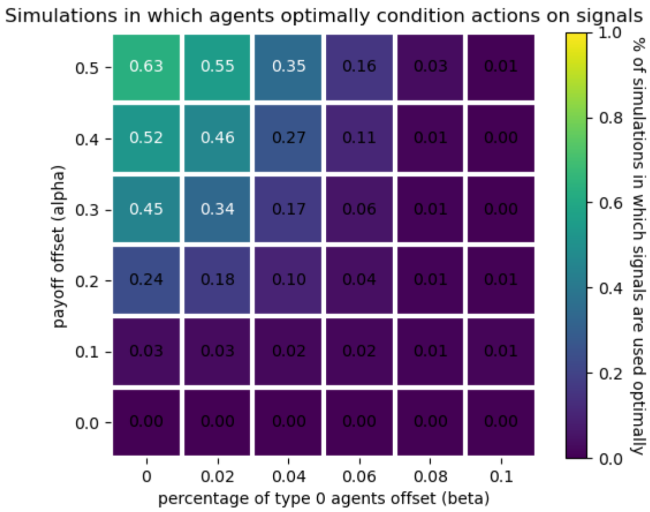

Figure 20: Proportion of outcomes that are (xii-xiii).

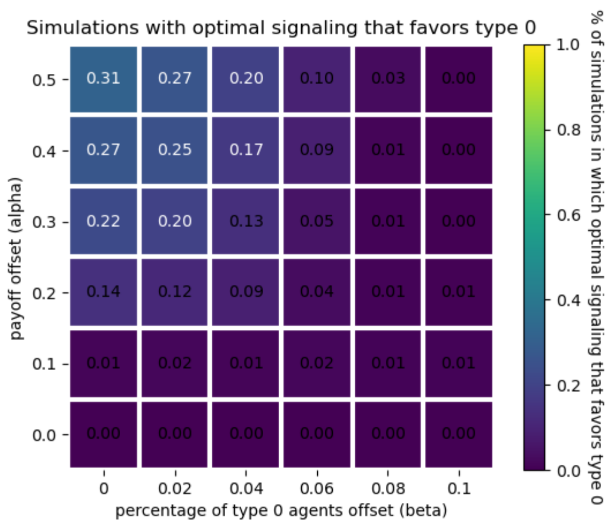

Figure 21: Proportion of outcomes that are (xii).

outcome (i)

y plays this with x	type 0	type 1	type 2
type 0	B	B	B
type 1	S	S	S
type 2	E	E	E

outcome (ii)

y plays this with x	type 0	type 1	type 2
type 0	B	B	B
type 1	B	B	B
type 2	E	E	E

outcome (iii)

y plays this with x	type 0	type 1	type 2
type 0	S	S	S
type 1	S	S	S
type 2	E	E	E

outcome (iv)

y plays this with x	type 0	type 1	type 2
type 0	B	B	E
type 1	B	B	E
type 2	E	E	E

outcome (v)

y plays this with x	type 0	type 1	type 2
type 0	S	S	E
type 1	S	S	E
type 2	E	E	E

outcome (vi)

y plays this with x	type 0	type 1	type 2
type 0	B	B	B
type 1	B	S	B
type 2	E	E	E

outcome (vii)

y plays this with x	type 0	type 1	type 2
type 0	B	S	S
type 1	S	S	S
type 2	E	E	E

outcome (viii)

y plays this with x	type 0	type 1	type 2
type 0	B	B	B
type 1	B	S	E
type 2	E	E	E

outcome (ix)

y plays this with x	type 0	type 1	type 2
type 0	B	S	E
type 1	S	S	S
type 2	E	E	E

outcome (x)

y plays this with x	type 0	type 1	type 2
type 0	S	B	E
type 1	B	S	E
type 2	E	E	E

outcome (xi)

y plays this with x	type 0	type 1	type 2
type 0	B	S	E
type 1	S	B	E
type 2	E	E	E

outcome (xii)

y plays this with x	type 0	type 1	type 2
type 0	B	B	E
type 1	B	S	E
type 2	E	E	E

outcome (xiii)

y plays this with x	type 0	type 1	type 2
type 0	B	S	E
type 1	S	S	E
type 2	E	E	E

outcome (xiv)

y plays this with x	type 0	type 1	type 2
type 0	B	B	B
type 1	B	E	E
type 2	E	E	E

This occurred:
1 time(s) when alpha = 0.1 and beta = 0.08
2 time(s) when alpha = 0.2 and beta = 0.08
1 time(s) when alpha = 0.2 and beta = 0.1

outcome (xv)

y plays this with x	type 0	type 1	type 2
type 0	E	B	E
type 1	B	S	E
type 2	E	E	E

This occurred:
1 time(s) when alpha = 0.3 and beta = 0
1 time(s) when alpha = 0.4 and beta = 0

outcome (xvi)

y plays this with x	type 0	type 1	type 2
type 0	B	S	E
type 1	S	E	E
type 2	E	E	E

This occurred:
1 time(s) when alpha = 0.3 and beta = 0.04
2 time(s) when alpha = 0.4 and beta = 0.02
1 time(s) when alpha = 0.5 and beta = 0
1 time(s) when alpha = 0.5 and beta = 0.02
1 time(s) when alpha = 0.5 and beta = 0.04

outcome (xvii)

y plays this with x	type 0	type 1	type 2
type 0	B	E	E
type 1	E	S	E
type 2	E	E	E

This occurred:
1 time(s) when alpha = 0.3 and beta = 0.06
1 time(s) when alpha = 0.4 and beta = 0.08
1 time(s) when alpha = 0.5 and beta = 0.06
2 time(s) when alpha = 0.5 and beta = 0.1

Appendix C Adding in Partial Payoffs on Failures to Coordinate

This appendix shows simulation results for the model with one change. The change is that instead of having $p_{xi} = 0$ when strategy profiles x and i result in a failure to coordinate p_{xi} is twenty percent of the payoff that would have been received if the action chosen by profile x had lead to successful coordination. For example, if an agent receives a payoff of 1 for coordinating on a particular action, then if that agent performs the same action but fails to coordinate her payoff will be 0.2. Of course this value of twenty percent was not hard coded and is a parameter that can be varied. But, twenty percent is sufficient for illustrating what happens in the model. If the payoff percentage on failures to coordinate is too small, then simulation results are identical to results from the model in which agents get nothing on failures to coordinate; and, if the payoff percentage on failures is too high, then agents will always perform their most preferred action because the payoff on failures for that action exceeds their payoff for successful coordination on a less preferred action.

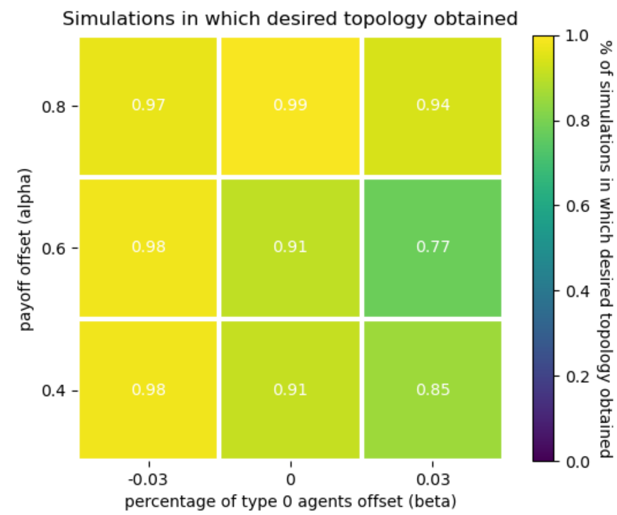

Figure 22: Proportion of outcomes that are the single embedding topology under parameters identical to those used for Figure 10 (c) but with the modified model where agents get twenty percent on failures.

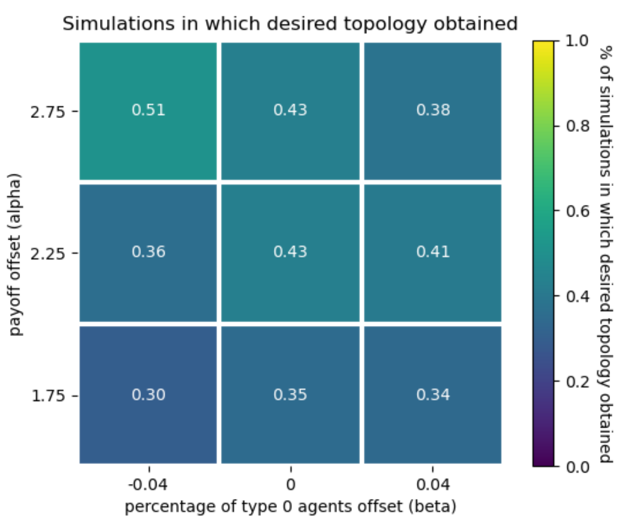

Figure 23: Proportion of outcomes that are the disjoint double embedding topology under parameters identical to those used for Figure 14c but with the modified model where agents get twenty percent on failures.

Appendix D Analytic Analysis for 2x2x2 game

2 types, 2 signals (α, β), and 2 actions (no homophily)

In this simple system, this appendix analyzes when it is the case that an agent should play their less preferred action.

		Type 1	
		Bach	Stravinsky
Type 0	Bach	a, b	c, c
	Stravinsky	c, c	b, a

With $a > b$ and $c < 0$.

- Let $n_i, i \in \{0, 1\}$ the proportion of the population that is type i.
- Let $x_{ijk}, i \in \{0, 1\}, j \in \{\alpha, \beta\}, k \in \{\text{Bach, Stravinsky}\}$ be the probability that a type i agent plays k given signal j is observed.
- Let $y_{ij}, i \in \{0, 1\}, j \in \{\alpha, \beta\}$ be the probability that a type i agent transmits signal j. Note that $y_{i\beta} = 1 - y_{i\alpha}$
- Let A_i be type i's preferred action and B_i be the alternative action.
- Let $p(i|j), i \in \{0, 1\}, j \in \{\alpha, \beta\}$ be the probability that signal j was sent by a type i agent given signal j was observed.
- Let $E_{ij}(k), i \in \{0, 1\}, j \in \{\alpha, \beta\}, k \in \{\text{Bach, Stravinsky}\}$ be the expected utility for a type i agent playing k after having seen signal j.

Then:

$$p(i|j) = \frac{n_i y_{ij}}{n_0 y_{0j} + n_1 y_{1j}}$$

$$\begin{aligned}
E_{ij}(A_i) &= a[y_{i\alpha}(x_{0\alpha A_i}p(0|j) + x_{1\alpha A_i}p(1|j)) + (1 - y_{i\alpha})(x_{0\beta A_i}p(0|j) + x_{1\beta A_i}p(1|j))] \\
&\quad + c[y_{i\alpha}(x_{0\alpha B_i}p(0|j) + x_{1\alpha B_i}p(1|j)) + (1 - y_{i\alpha})(x_{0\beta B_i}p(0|j) + x_{1\beta B_i}p(1|j))] \\
E_{ij}(B_i) &= b[y_{i\alpha}(x_{0\alpha B_i}p(0|j) + x_{1\alpha B_i}p(1|j)) + (1 - y_{i\alpha})(x_{0\beta B_i}p(0|j) + x_{1\beta B_i}p(1|j))] \\
&\quad + c[y_{i\alpha}(x_{0\alpha A_i}p(0|j) + x_{1\alpha A_i}p(1|j)) + (1 - y_{i\alpha})(x_{0\beta A_i}p(0|j) + x_{1\beta A_i}p(1|j))]
\end{aligned}$$

In the initial state of the reinforcement learning model, $x_{ijk} = y_{ij} = 0.5$ for all i, j, and k. Since $a > b$, it follows that in the initial state of the game agents should always play their preferred action; i.e. $E_{ij}(A_i) > E_{ij}(B_i)$ for all i and j.

Given that all agents have more incentive to play their preferred action than the alternative action irrespective of the signal let's assume that $x_{ijA_i} = 0.5 + \epsilon$ for all i and j where $\epsilon \geq 0$. (Note that $x_{ijB_i} = 1 - x_{ijA_i}$). Furthermore, let's assume that due to random drift $y_{0\alpha} = y_{1\beta} = 0.5 + \delta$ for some $\delta > 0$. Without loss of generality, consider expected utility for type 0 agents. We the get that type 0 agents have higher expected utility for playing their less preferred action when observing β (the more frequent signal among type 1 agents) if $E_{0\beta}(B_0) > E_{0\beta}(A_0)$.

Let:

$$Q = \frac{n_0(.5-\delta)}{n_0(.5-\delta)+(1-n_0)(.5+\delta)}$$

$$R = \frac{(1-n_0)(.5+\delta)}{n_0(.5-\delta)+(1-n_0)(.5+\delta)}$$

If $2Qc-2Rc-Qb+Rb-Qa+Ra > 0$, then it can be checked that $E_{0\beta}(B_0) > E_{0\beta}(A_0)$ when:

$$\epsilon > \frac{Qa+Ra-Qb-Rb}{4Qc-4Rc-2Qb+2Rb-2Qa+2Ra}$$

Here are some plots of:

$$\epsilon = \frac{Qa+Ra-Qb-Rb}{4Qc-4Rc-2Qb+2Rb-2Qa+2Ra}$$

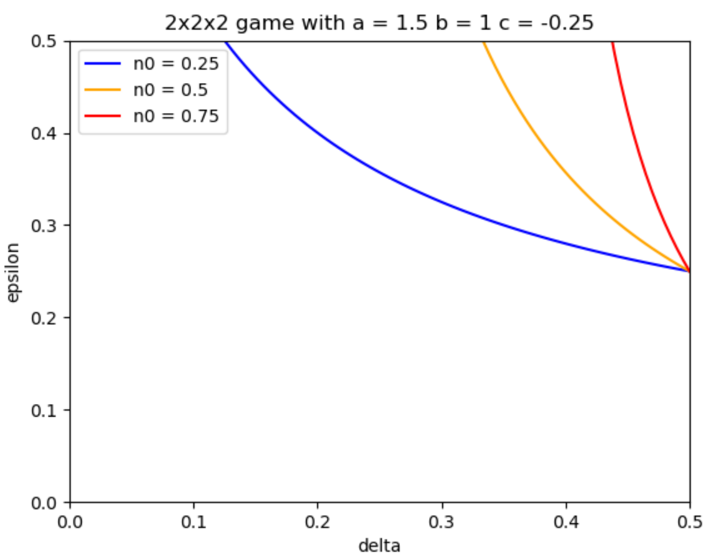

Figure 24

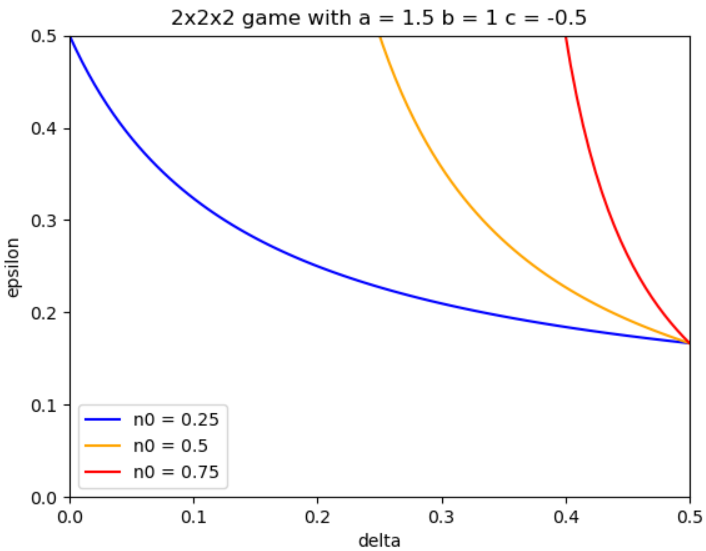

Figure 25

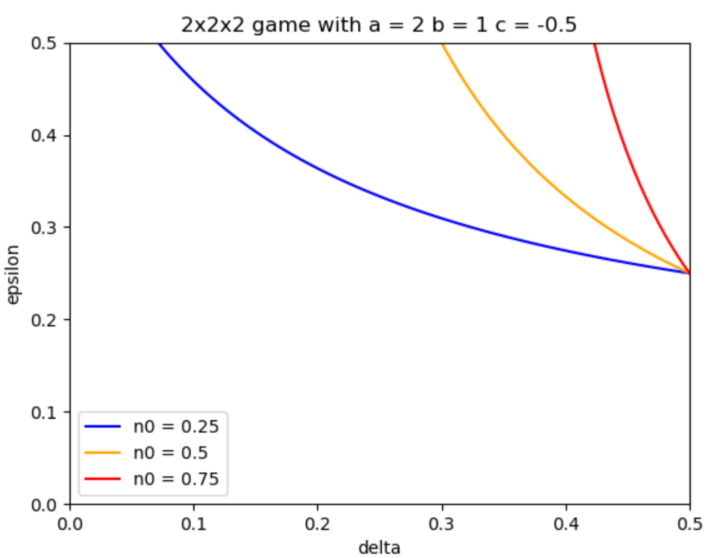

Figure 26