Franco-Sicilian Abstraction

transpositions, transitivity and transfer learning with Lewis-Skyrms signaling games

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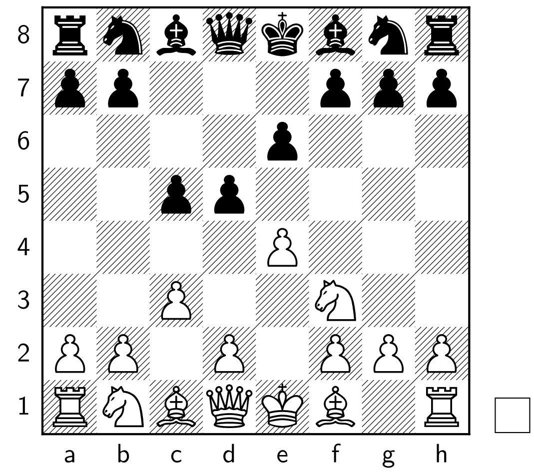
Abstract

This paper shows how a rudimentary type of abstraction can obtain in Lewis-Skyrms signaling games. Here, abstraction is understood as occurring when different particulars take on the same functional role. Some abstraction may be guided by innate biases, and the paper develops an analogy of reasoning about strategic thinking in chess to highlight some epistemic concerns that are raised by the presentation of abstraction. Concretely, the signaling game models of this paper are developed in the context of two tasks that are much simpler than strategizing in chess. The first task is a transitive inference task that has been substantially studied in both humans and non-human animals. General features of the models developed for the transitive inference task are then carried over to a model for the second task of learning rudimentary grammatical structures. This second task is based on studies of human infants and non-human primates' ability to learn "nonsense grammars". Closing discussion highlights some strengths of the abstraction exhibited in the Lewis-Skyrms signaling game models.

1 Introduction



1 e4 c5 2 ♘f3 e6 3 c3 d5

In the above chess position, computer engines suggest that capturing the d pawn, **4 e×d5**, is the best move. However, about 25% of the time, chess masters instead advance the e pawn, **4 e5**. Why? “White is hoping a **1. . . c5** [Sicilian Defense] player won’t be comfortable in a French [Defense]. And that’s also why Black often avoids the objectively best **4. . . ♘c6 5 d4** and prefers a gambit with **4. . . d4!? 5 c×d4 c×d4 6 ♗b5+ ♘c6 7 ♗c6+ b×c6 8 ♕a4** or **6. . . ♗d7 7 ♘×d4!?**” (Soltis, 2007). In chess, a transposition occurs when a sequence of moves begins with the moves of a particular opening, say a Sicilian Defense, but results in a board position typical of a different opening, say a French defense. When White sub-optimally advances her e pawn, she attempts to transpose the game into a French Defense. If Black sub-optimally advances her d pawn, she avoids the transposition at the cost of a pawn.

This story of chess transpositions serves dual analogical roles for this paper. First, it demonstrates how we might think about the type of abstraction to be described. While this paper explicates a specific and relatively simple example of abstraction using transitive inference, the chess story highlights our ability to use this type of abstraction in more complicated contexts. Consider Frege’s conception of abstraction. According to him, we know the abstract form of a sum by knowing that a particular formula is a sum, say 2+3=5, and seeing that replacing any subset of the numbers in the formula with different numbers will also yield a formula that is a sum (though, it is certainly not guaranteed that the resulting formula’s reference is True). In this conception of abstraction we see different particulars, numbers, can satisfy the same roles, an addend or result, to make a summation formula. It is also the case that in Frege’s conception of abstraction, we know exactly what particulars can be substituted into the formula, any number and nothing that is not a number. While this may be a good concept of abstraction in the context of formal languages, this paper allows that there may be vagueness in the boundaries around what particulars can satisfy the same role in an abstract category.

Chess masters use the abstract category of an opening to narrow the calculations that they have to make. While a particular board position might be novel, their familiarity with the abstract category of positions associated with a particular opening guides what strategic ideas they consider (de Groot, 1978; Gobet et al., 2004). The fact that transpositions are possible entails that some board positions might occupy a vague boundary between different categories of opening and associated strategic ideas. So when not discussing the concrete Lewis-Skyrms signaling game to be presented, this paper's discussion of abstraction simply refers to contexts where different particulars take on the same functional role. That is, just as two different numbers can play the role of addends, two different board positions can play the role of reminding a chess master of strategic ideas in the French Defense; but while we know exactly what can and ought to play the role of an addend, there may be board positions occupying a vague boundary around the set of positions that are or ought to be associated with strategic ideas in the French Defense.[1]

Second, the story points us towards a philosophical problem that accompanies abstraction. Let's suppose that we think beliefs that lead to success in action are true beliefs. The chess story immediately shows why we might doubt this. A player may believe **4 e5** is a stronger move than **4 e×d5** due to her having more success, i.e. more wins, with that continuation. However, this may merely be a consequence of her having more successfully abstracted the strategic dynamics of board positions associated with the French Defense than the Sicilian; i.e. if your only tool is a hammer, then every problem looks like a nail. As another example, one might think that we come to represent the world as having three dimensional Euclidean structure by abstracting over experiences and that since we successfully interact with the world using this representation, then it is true that the world has this structure. But just because we successfully navigate the world representing it as Euclidean does not mean there is not alternative representation with which we would have more success. Worse, sections 3.3 and 3.4 will suggest that some abstractions may be a consequence of (possibly arbitrary) innate dispositions rather than being wholly empirical.[2]

Some research has investigated how humans store abstract information about chess positions (Chassy, 2013). However, this paper focuses on simpler cognitive tasks. This allows us to examine correspondingly simple models that are more easily interpreted; in turn, this allows us to develop a clearer understanding of the philosophical implications of the models rather then getting lost in the details of continually evolving complex models. Accordingly, this paper

[1] See De Marzo and Servedio for a statistical clustering of board positions that coheres with theoretical openings (De Marzo and Servedio, 2023).

[2] Explicitly, the worry is that if our representation of something in the world is reflective of an arbitrary disposition, then this may entail that in some relevant way we understand the world as having a particular structure when that structure is merely a consequence of our arbitrary innate disposition. Some have, perhaps wrongly, interpreted Kant as having a concern along these lines (Janiak, 2022; Warren, 1998).

models two tasks taken from the comparative psychology literature since we take nonhuman animals to have less sophisticated cognitive resources available for learning and inference. Of course, these tasks have also been studied with human subjects who behave similarly to the nonhuman animal subjects that succeed at the tasks.

The first task that is modeled is a transitive inference task, which involves testing whether a subject trained on contiguous pairs of linearly ordered stimuli can abstract information about that relation to apply it two a novel non-contiguous pair of stimuli. Psychology experiments have shown children as well as a variety of nonhuman animals succeed at the transitive inference task. Later discussion of this task, in accordance with the second analogical role of the chess transpositions story, notes that expected behavior for the task should not strictly be called "success" as the inference is not warranted.

The second task is a nonsense grammar task. It is a variation of experiments habituating human infants and nonhuman primate infants to sound patterns, called nonsense grammars, and testing whether the habituation persists for novel sounds presented in the same patterns. While this is an entirely distinct task from the transitive inference task, it can be similarly described: it tests whether a subject trained on a relation between triplets of stimuli can abstract information about that relation to apply it to a novel triplet of stimuli. This second task is important for showing that the themes abstracted from the transitive inference task are not merely consequences of particular details of that task.

2 Transitive Inference

Abstractly, transitive inference task is as follows: Five objects/stimuli are arranged in a serial order: A, B, C, D, E. The agent to be tested is conditioned on adjacent pairs in the serial ordering through rewards for selecting the first object in a presented pair. E.g. If presented with A-B the agent is rewarded for choosing the item on the left. If presented with D-C the agent is rewarded for choosing the item on the right. After the agent has learned to respond correctly to pairs of objects that are adjacent in the serial order, she is then tested on the non adjacent pairs B-D and D-B. (A and E are not tested since choosing A is always rewarded and choosing E is never rewarded; thus, choosing the left object in the pair A-C might reflect choosing A always having been rewarded rather than reflect the transitive ordering having been represented.) Choosing the correct item in the non-adjacent pairing is then understood as the agent representing the transitive ordering.

Concretely, Bryant and Trabasso showed that children ages 4-6 succeeded at the transitive inference task. The stimuli used were rods of different colors and varying lengths. The children were shown adjacent adjacent pairs, in a serial ordering by length, were shown to the children, with the rods protruding through holes such that one inch of the rod was visible. Thus the children could observe the color of the rods, but not the length. After being asked which rod in a pair was taller or shorter, the rods were removed and laid flat on a table in front of the child so that they could examine whether they had answered correctly. After the children learned to answer correctly for pairs that were adjacent in the ordering by length, they were then test on non adjacent pairs and tended to answer correctly. This experiment was taken to disprove earlier work by Piaget who claimed that children did not understand or represent transitive relations (Bryant and Trabasso, 1971).

Concretely, Gillan used colored plastic containers to show that chimpanzees succeed at the transitive inference task. The colored containers were assigned an arbitrary serial ordering and then adjacent pairs were presented to the chimps with a food reward in whichever container was first in the serial ordering. After the chimps were conditioned to almost always choose the container with the food, they were then tested on the nonadjacent pairs. The result was that the chimpanzees tended to chose the container that had food in it, i.e. the one that was first in the serial ordering (Gillan, 1981). In a similar experiment, von Fersen et al. showed Pigeons succeed at the transitive inference task using black and white ink blots as the stimuli and and using a contraption that rewarded the pigeons with food when they pecked at the correct ink blot in a pair (von Fersen et al., 1991). Likewise, Davis showed that rats succeed at the transitive inference task using the stimuli of distinct odors to mark pairs of doors that the rats could open to retrieve a food reward (Davis, 1992). Bond et al. showed that scrub-jays succeed at the transitive inference task using a mechanism similar to von Feresen et al., but with projections of different colors of light rather than ink blots as the stimuli (Bond et al., 2003).

It should be noted that, in the nonhuman animal experiments just listed, the transitive inference is not warranted; e.g. food always being in the green container when a green and yellow pair is presented and always being the the yellow container when a yellow and red pair is presented does not entail that the food will be in the green container when a green and red pair is presented. This contrasts with Bryant and Trabasso's study involving different colored rods of varying lengths because transitivity does hold for length: i.e. if a green rod is longer than a yellow rod and that yellow rod is longer than a red rod, then this entails that the green rod is longer than the red rod. That the transitive inference is warranted in the human experiment and unwarranted in the nonhuman animal experiments is merely a coincidence of the experiments that happened to comprise the most pertinent background information. There have been experiments showing successful transitive inference in nonhuman animals for which transitivity does hold for the stimuli, such as physical dominance, and there have been experiments showing "successful" transitive inference in human participants for which transitivity does not hold for the stimuli, such as characters from a written language that the participants do not know (Grosenick et al., 2007; Frank et al., 2005).

There have also been many computation models of transitive inference. Barrett's (Barrett, 2014) model is the most transparent. In Barrett's model, agents are first conditioned on a full serial ordering of stimuli including non-adjacent pairs. The agents are then presented with a new set of serially ordered stimuli, this time conditioned on only adjacent pairs. Modeling a type of transfer learning, the agents are allowed to transfer some of their dispositions from the first set of stimuli to their dispositions for the new stimuli by allowing initial signals transmitted for the new set of stimuli to be processed as if the signals were the old stimuli. That is, the agents learn to map the new stimuli to their evolved dispositions for the old stimuli, which now serve as an intermediary representation in an agent's evolution of dispositions for the new stimuli. This allows the agents to infer the appropriate response to a novel pairing of nonadjacent stimuli from the new ordering most of the time.

Many computational models of transitive inference do not explicitly invoke transfer learning. However, given that the correct action for the test pair is underdetermined by the contiguous pairs that a model is trained on, some type of prior bias must be present to explain successful action for the test pair. For example, Siemann and Delius develop a model that makes use of lateral inhibition, whereby, on the transmission of two signals, only the signal of larger magnitude is acted upon. When presented with a stimulus pair from a serial ordering, an agent might initially transmit both a signal to choose the left object and a signal to choose the right object, but only the signal of greater magnitude will go through (Siemann and Delius, 1998). On reflection, it is clear that this is a type of transfer learning since the lateral inhibition mechanism is itself a presupposed set of dispositions representing the serial ordering of signal magnitudes. Other models are more opaque. De Lillo et al. give a three layer neural network model, which, although not explicitly invoking lateral inhibition, is trained using a cost function that always moves the weights in the direction of hidden layer nodes always associating the rewarded stimulus with a greater magnitude than the magnitude associated with the other stimulus in a pair (or always with a smaller magnitude depending on the randomly determined initial state of the network) (De Lillo et al., 2001). So this model might also be thought of as presupposing a serially ordered representation.[3]

3 Transitive Inference with Transfer Learning

This section presents three models of transitive inference by means of types of learning that can be characterized as transfer learning. The first model is a straightforward example of transfer learning. In this model, an agent first learns to choose the preferred stimuli in a pair, being trained on both adjacent and non-adjacent pairs in a serial ordering. Then, the dispositions from this first phase of training are co-opted in a second phase. In this second phase of training an agent is trained on only adjacent pairs in a new set of serially ordered stimuli. Since this second phase of training involves mapping the new stimuli to the dispositions formed in the first phase of training, agents subsequently succeed

[3]It is possible that this sort of prior bias could have evolved for an organism to navigate an environment in which there is often a linear relation between objects. However, section 3.4 will explain that such a bias could be a result of relatively abstract combinatorial properties. This makes it plausible that the bias facillitating the inference is a spandrel rather than reflecting environmental evolutionary pressures.

when tested on the non-adjacent pair for the new stimuli. This first model is essentially the same as Barrett's 2014 model, albeit with some negligible differences (Barrett, 2014). Call this the "diachronic model".

The primary purpose of the diachronic model is to simplify the presentation and explanation of the other two models. Like the diachronic model, both of these models make use of two sets of serially ordered stimuli. However, agents are trained on both sets of stimuli simultaneously. Additionally, agents are trained on pairings between the two sets of stimuli; e.g., let A, B, C, D, E be the first set of stimuli and a, b, c, d, e be the second set of stimuli, then agents are trained to choose B over c in a B-c pair. They are only trained on non-adjacent pairings when both components of a pairing are from the first set of stimuli. One of these models is a simple extension of the Diachronic Model in its first phase of training while the other makes use of signal magnitudes. Call these models the "synchronous model" and "synchronous magnitudes model" respectively.

3.1 Diachronic Transfer

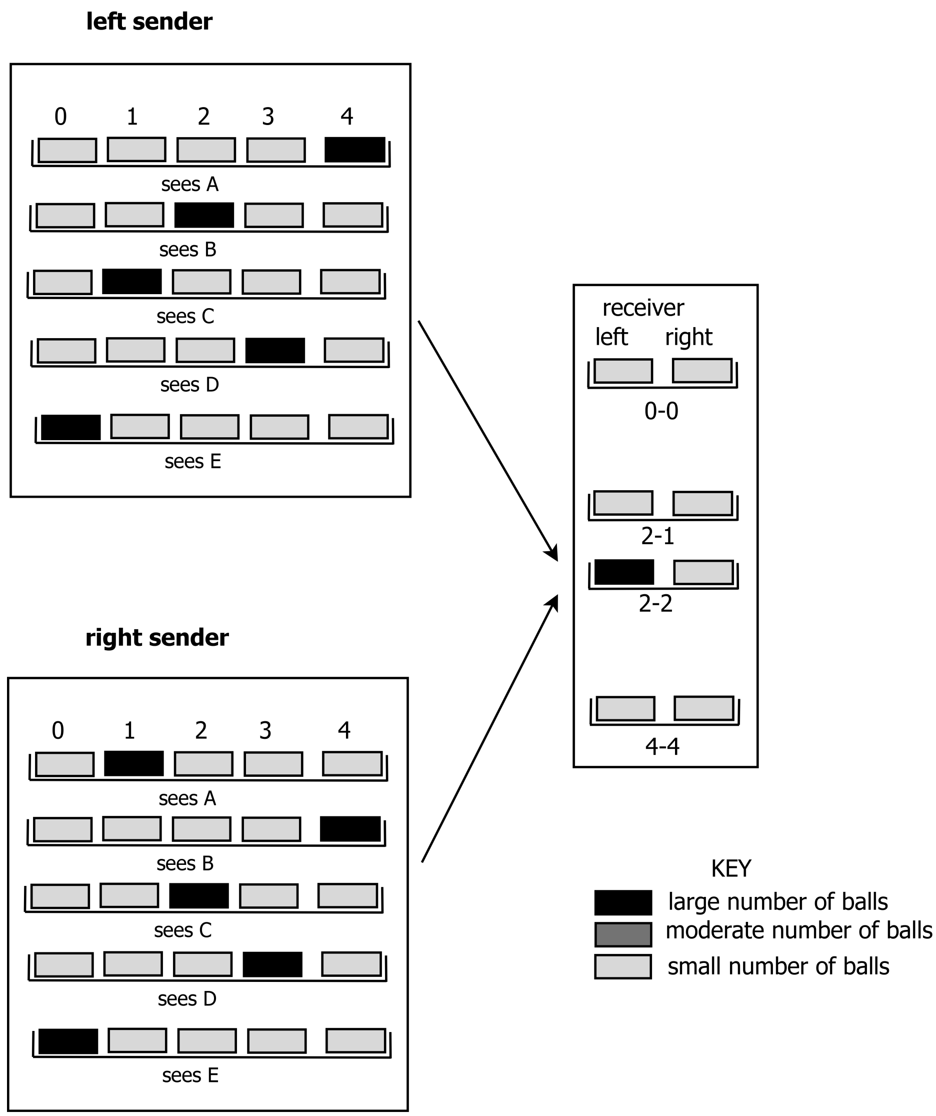


Figure 1: First Training Phase of Diachronic Model

The diachronic transfer learning model is a two sender one receiver game during the first phase of training. Each sender see one of the two stimuli in a pair composed of either adjacent or non-adjacent stimuli in the serial ordering. That is, letting A, B, C, D, E be the stimuli in their serial order. Then every pairing of two distinct stimuli is in the training set. The senders each have one urn for each of the five stimuli that can be observed, and each urn begins the game with a single ball of each type. Figure 1, depicts each sender with five types of balls, but this is parameter in the model that can be varied. The receiver has one urn for each of the possible combinations of signals that can be received from the senders; each of these urns begins the game with two balls in it, a left ball and a right ball. During the training, pairs are selected at random with equal probability. Suppose, the pair B-D is selected. Then the left sender will draw from her B urn and the right from her D urn. Suppose the left sender draws a 2-ball and the right a 1-ball, then the receiver will draw from her 2-1 urn. The receivers draw then determines whether the left or the right stimuli in the pairing will be chosen. If the correct stimmuli is chosen, in this case B which is on the left, then the urns that were drawn from are reinforce with additional balls of the type that were drawn. If the incorrect stimuli is chosen than the urns are punished by removing balls of the type that were drawn, so long as at least one ball of each type remains in each urn. This is the same basic Roth-Erev reinforcement with punishment learning that has been described in detail elsewhere and represented as $(+1, -1)$ (Barrett and Gabriel, 2022). The notation $(+r, -p)$ represents on a success adding r-many balls of the type drawn to each of the urns drawn from, and on a failure removing p-many balls of the type drawn from each of the urns drawn from, so long as at least one ball of each type remains in each urn.

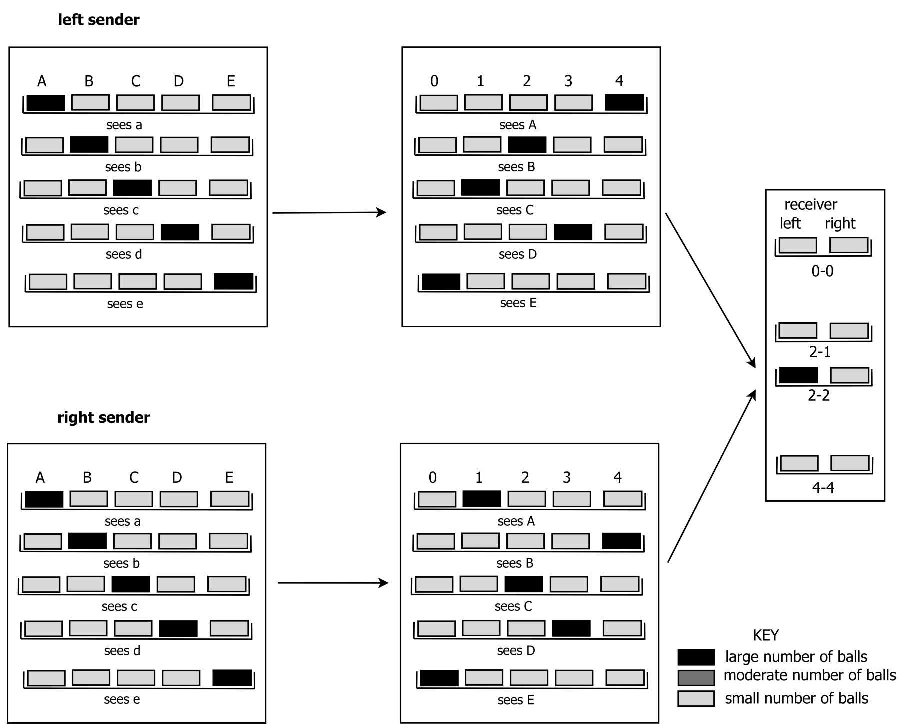


Figure 2: Second Training Phase of Diachronic Model

In the second phase of training (see Figure 2), the left and right senders are given new urns, one for each of the new set of stimuli, a, b, c, d, e. Each of these urns have ball types A, B, C, D, and E which then determine which urn from the first phase of training will be drawn from. Draws from the urns the A, B, C, D, and E urns then determine what is transmitted to the receiver who retains her urns from the first phase of training. It is in this way that dispositions from the first phase of training are co-opted in learning correct actions for the second set of stimuli. Learning during the second phase of training also proceeds according to Roth-Erev reinforcement with punishment learning dynamics. The testing phase is then performed with the b-d and d-b pairs. No learning occurs during the testing phase.

3.2 Synchronous Transfer

In the synchronous transfer model (see Figure 3), there is only one training phase. The dynamics of the model are the same as the model in the first phase of the training for the diachronic model. However, the training consists in pairings from both the A, B, C, D, and E stimulus set and the a, b, c, d, and e stimulus set. Pairings across the stimulus sets are allow, e.g. B-c. But non adjacent pairs are only trained for stimuli from the first set, e.g. C-E. There are no pairings in which both elements are occupy the same position in the serial ordering, e.g. D-d is disallowed. So, A-D, B-A, d-C, and e-d are all members of the training set and neither B-d nor a-d are member of the training set. During training pairs from the training set are selected at random with equal probability and the learning dynamics are the same as in the diachronic model, i.e. Roth-Erev reinforcement with punishment. After the training phase is complete, the testing phase consists of the b-d and d-b pairs and no learning occurs during testing.

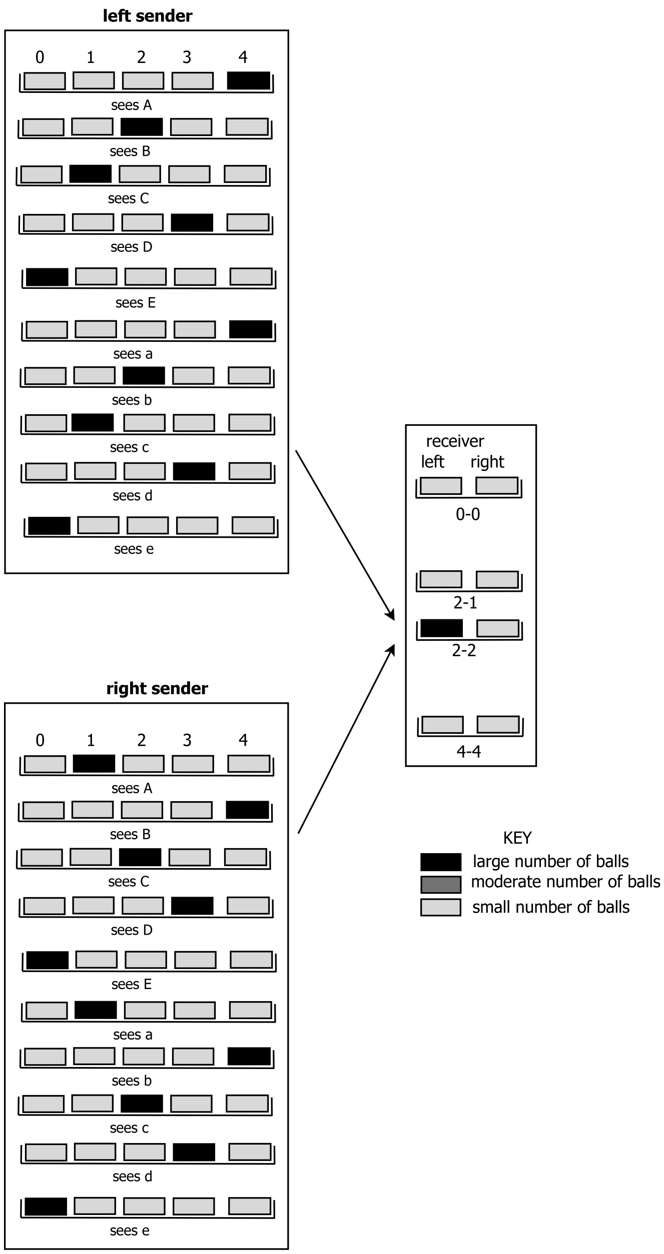


Figure 3: Synchronous Model. For simplicity, this model is depicted with each of a sender's urns containing five ball types. However this is a parameter that can be varied. The results section focuses on simulation results for which each urn contained ten ball types.

3.3 Synchronous Magnitudes

In the synchronous magnitudes model (see Figure 4), the training and testing sets are the same as in the synchronous model. Likewise there is only a single training phase before the testing phase. The only difference in the synchronous magnitudes model is in the senders' urns. Rather than the senders each having a single urn for each of the stimuli that can be observed, each stimuli is associated with M-many urns with each of these urns beginning the game with one 0 ball and one 1 ball. Figure 4 depicts the model with M=6, though later results are reported for M=10. When a stimuli is observed by one of the senders, that sender draws from every urn associated with that stimulus and then transmits the sum of her draws. For example, if the left sender, upon seeing A draws five 1 balls and one 0 ball, then she will transmit 5; if the right sender upon seeing B draws two 1 balls and four 0 balls, she will transmit 2; then the receiver will draw from the 5-2 urn just as she would in the prior models. Again, the learning dynamics is urn based reinforcement with punishment. Thus if, in the example just given, the correct stimuli was chosen, then the left sender would reinforce 1 balls in the five urns that she drew 1 balls from and reinforce 0 balls in the urn that she drew the 0 ball from. Likewise the right sender would reinforce 1 balls in the two urns that she drew 1 balls from and reinforce 0 balls in the four urns that she drew zero balls from.

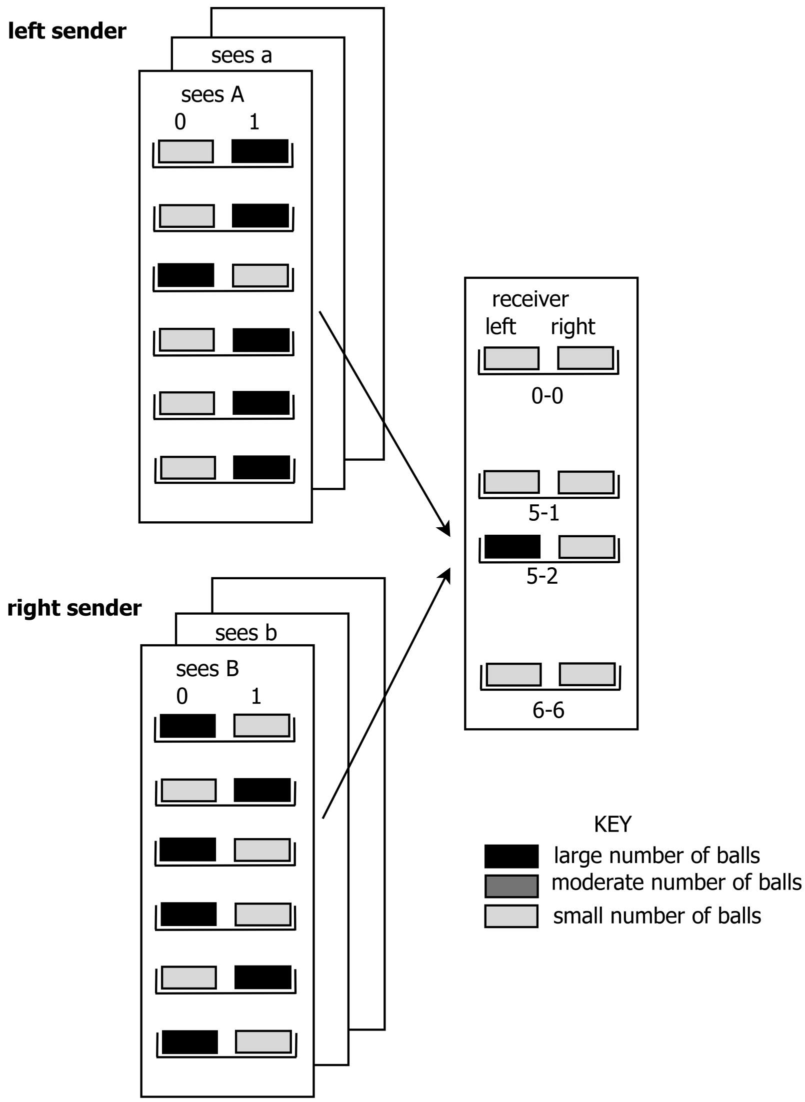


Figure 4: Synchronous Magnitudes Model. For simplicity, this model is depicted with each sender having six urns for each stimuli that can be observed. However, this is a parameter that can be varied. The results section focuses on simulation results for which senders have ten urns for each stimuli that can be observed.

3.4 Transitive Inference Simulation Results

The diachronic model performs very poorly. Running simulations for a variety of parameters never yielded a significantly high mean success rate during just the first phase of training. As stated earlier, the primary purpose of the diachronic model is as a stepping stone towards the synchronous and synchronous magnitudes models. Barrett (Barrett, 2014) achieves significantly high success rates with a similar model by utilizing invention in the learning dynamics. This limits the terms available to the senders to only those that have led to success in action, and this limitation makes learning much easier. As will be seen, the synchronous magnitudes model will yield a similar increase in success rates by making the middle magnitude the most likely magnitude to be transmitted during the early stages of a simulation.

For the synchronous model, with ten ball types in each of the senders' urns, using (+4, -11) reinforcement with punishment, and 2×10^9 training plays before testing yeilded a mean success rate of 0.7228 during training and a mean success rate of 0.66293 during testing. However, the simulations also yeilded a very odd result when training was restricted to just the second set of stimuli and only training on adjacent pairs. The model still performed above chance during the testing phase. Simulations had a mean success rate of 0.7256 during training and 0.6217 for the testing phase.[4]

What is even more confusing is that, prima face, it seems that combinatorially we should expect performance during testing to be worse than chance when restricting training to just the second set of stimuli with only adjacent pairings in the training set. For the five stimuli in the second training set and ten terms in each of the senders urns there are essentially 831,847,680 strategy profiles for which every pairing in the training set gets mapped to the correct action and pairings in the test set tend to get mapped to the *incorrect* action, but there are only 766,177,920 strategy profiles for which every pairing in the training set gets mapped to the correct action and pairings in the test set tend to get mapped to the *correct* action.[5] Thus, it seems like mean success rates for the test pairs should be less than 0.5 when the training phase only consists of the second set of stimuli and the training set only consists of adjacent pairings. The resolution to this problem lies in observing that for strategy profiles that map at least two distinct pairings to the same receiver urn, there are only 78,943,680 strategy profiles for which every pairing in the training set gets mapped to the correct action and pairings in the test set tend to get mapped to the *incorrect* action, there are 137,649,600 strategy profiles for which every pairing in the training set

[4] This is averaging over just 300 simulations as 2×10^9 training plays takes a decent amount of time to simulate.

[5] This is brushing over some details. By using the term 'essentially', what is indicated that this count is of pure strategy profiles rather than mixed strategy profiles. So whats really being counted are the pure strategy profiles that a simulation could converge to rather than the mixed profiles that necessarily obtain (since the learning dynamics require that there is always at least one ball of each type in each urn). The count of 831,847,680 is of pure strategy profiles such that every adjacent pairing is mapped to the correct action and where at least one of the b-d or d-b pair is mapped to the incorrect action and neither is mapped to the correct action. Likewise, the count of 766,177,920 is of pure strategy profiles such that every adjacent pairing is mapped to the correct action and where at least one of the b-d or d-b pair is mapped to the correct action and neither is mapped to the incorrect action. There are 203,400,360 pure strategy profiles such that every adjacent pairing is mapped to the correct action and exactly one of the b-d or d-b pair is mapped to the correct action and the other is mapped to the incorrect action.

gets mapped to the correct action and pairings in the test set tend to get mapped to the *correct* action, and the learning dynamics makes these strategy profiles much more likely to obtain (since it is easy for the senders dispositions towards a pairing, for which successful dispositions have not yet been reinforced, to evolve a mapping to a receiver urn for which successful dispositions have already been reinforced for a distinct pairing).[6]

For the synchronous magnitudes model, with senders having ten urns for each of the stimuli, using (+4, -11) reinforcement with punishment, and 10^7 training plays before testing yielded a mean success rate of 0.999 during training and a mean success rate of 0.845 during testing; 635 simulations of 1000 had a success rate greater than 0.9 for the testing phase. When restricting training to only the second set of stimuli and only training on adjacent pairs, simulations had a mean success rate of 0.999 during training and 0.734 during testing. Again, the explanation for this better than chance performance when only training adjacent pairings from the second set of stimuli seems to be the learning dynamics tendency to produce strategy profiles in which multiple pairings get mapped to the same receiver urn.[7]

The synchronous magnitudes model can be thought of as baking some ordered structure into the model since there is an ordered relation between the magnitudes that can be transmitted: in the initial state of the model, for any stimuli, if there are ten urns per stimuli, then the most likely magnitude to be transmitted by each sender is 5 and the least likely transmissions are 0 and 10; furthermore, once a disposition to transmit some magnitude, say 7, has been reinforced, then the most likely deviations from this transmission are 6 and 8

[6] Precisely, this explanation needs to be more nuanced for the synchronous model since it was rare for strategy profiles with training phase success rates near 1.0 to obtain. However, this nuance is unnecessary when considering the synchronous magnitudes model for which the learning parameters reported in the next paragraph yielded training phase success rates near 1.0 for every simulation. Here, combinatorially, only 12%, of strategy profiles for which every pairing in the training set gets mapped to the correct action and at least one of the b-d or d-b pairs gets mapped to the same urn as one of the pairs from the training set, are such that at least two pairs from the training set get mapped to the same receiver urn. However, in simulations about 40% of the final strategy profiles have this property. Moreover, for the synchronous model, only 13.6% of strategy profiles for which every pairing in the training set gets mapped to the correct action and at least one of the b-d or d-b pairs gets mapped to the same urn as one of the pairs from the training set, are such that at least two pairs from the training set get mapped to the same receiver urn. However, in simulations 60% of the final strategy profiles have this property.

[7] As stated in a prior footnote, only 12%, of strategy profiles for which every pairing in the training set gets mapped to the correct action and at least one of the b-d or d-b pairs gets mapped to the same urn as one of the pairs from the training set, are such that at least two pairs from the training set get mapped to the same receiver urn. However, in simulations about 40% of the final strategy profiles have this property. This is close, but not entirely sufficient for explaining the 0.734 mean success rate during testing. It could be that some amount of transitivity is baked into the model or simply that further investigation of the combinatorial properties of the game reveals further insight. While the number of strategy profiles that have at least two pairs being mapped to the same receiver urn were counted, it could be that strategy profiles with strictly more than two pairs being mapped to the same receiver urn obtain and that these profiles are even more biased towards the testing pair being mapped to the correct action.

rather than all alternative transmissions be equally likely as in the synchronous model. These properties of magnitudes can be thought of as reflecting neurons having a base firing rate that is then increased or decreased by learning events or can even be thought of as a potential spandrel, a la Gould and Lewontin, simply resulting from the connectivity of neurons (Gould and Lewontin, 1979).

Two variations of the synchronous magnitudes mode were simulated to ensure undue structure was not being baked into the model. First, it was trained on a single set of stimuli, a, b, c, d, and e being conditioned to choose according to the regular ordering for adjacent pairs, but the reverse order for non-adjacent pairs; e.g. for the pair c-b, choosing right was reinforced and choosing left was punished, and for the pair a-d, choosing right was reinforce and choosing left was punished. This variant was successfully learned with a mean success rate of 0.998 across simulations. Second, a variant was explored where for single set of stimuli, a, b, c, d, and e training on only adjacent pairs reinforced choosing according to the serial ordering, but training also included an additional stimuli x, such that choosing left for e-x, right for x-e, right for a-x, and left for x-a were reinforced. That is, the model was trained on adjacent pairs in a circular rather than linear ordering. This yielded a mean success rate of 0.999. So, it seems clear that the model is perfectly capable of representing relations other than linear transitive orderings.

The circular ordering variant of the synchronous magnitudes model is notable because it actually reflects an experiment that was performed with pigeons. Von Fersen et al. in their black and white ink blots version of the transitive inference task hypothesized that the pigeons were relying on a type of analogue magnitude representation, called value transfer, during training and that this explained their success during the testing phase. Thus they predicted that it would not be possible to condition the pigeons on adjacent pairs according to a circular ordering. Contrary to their prediction two of three pigeons did learn the circular ordering (von Fersen et al., 1991). So the synchronous magnitudes model shows how the pigeons may have relied on magnitudes for learning yet still been able to represent a circular ordering.

4 Transferring to Abstraction

As previously stated, this paper's discussion of abstraction simply refers to contexts where different particulars take on the same functional role. In the transitive inference task, we can see a simulation has learned to attribute the same functional role to stimuli from each serial ordering that occupy the same place in their respective ordering if the final dispositions are such that the correct stimuli are chosen in the b-d and d-b test pairs (as already reported, this occurred roughly 63% of the time for the simultaneous magnitudes model). To make this claim concrete, suppose there are five ancient coins each with a head and tail side. Suppose further that the coins are serially ordered by value and that a subject is unfamiliar with the coins, i.e. upon seeing the heads side of a coin she cannot tell you what the tails side looks like and vice versa. In this scenario, the stimuli of a coin's head or tail is equally capable of fulfilling the functional role of indicating where in the serial ordering the coin is. Now, we could describe a deductive process by which the subject upon learning the ordering of the coins according the head sides and having more limited exposure to information about the tail sides deduces which tail stimuli occupies the same place in the ordering as each head stimuli. However, what the models show is how simple associative processes can also lead to different particular stimuli occupying the same functional role. The fact that the model has no prior exposure to the b-d and d-b pairs shows that it is actually dispositions from the first serial ordering to the second rather than merely independently learning the appropriate dispositions for each set of stimuli.

It is also clear that the different particular transitive inference models presented accomplish this abstraction in the same way. The receiver urns are organized in the same way for all three models. This in conjunction with having separate left and right senders who condition their action on just a single stimulus in a pair allows signaling systems to obtain that exhibit already reinforced dispositions towards novel stimulus pairings. As will be seen in the following section, this type of model architecture can accomplish abstraction in tasks other than transitive inference.

5 Nonsense Grammar

In a series of papers documenting experiments on cotton-top tamarins (*Saguinus oedipus*) and human infants, Hauser et al. claimed to have shown that the cognitive mechanisms for processing complex grammatical structures was uniquely human (Hauser et al., 2002b, 2001; Saffran et al., 2008). Critically, the content of these papers was used to support arguments in the prolific "Faculty of Language" paper he co-authored with Chomsky and Fitch (Hauser et al., 2002a). It must be noted that Hauser falsified data in at least one of these papers, making human infants appear to be more competent than the cotton-top monkeys at processing particularly complex grammatical structures. However, data on simpler grammatical structures seems to be accurate and has been independently replicated by Neiworth et al. (Neiworth et al., 2017). The most relevant of these experiments by Neiworth et al. is summarized in the following paragraph.

The ability to learn simple grammatical patterns was assessed using a habituation paradigm with 16 adult cotton-tops as subjects. During a habituation phase, cotton-tops were exposed to consonant vowel sequences in an 001 or 011 pattern, e.g. "pupuki" or "pukiki". Seven different consonants (p, b, d, k, t, m, and n) and two vowels (i and u) were used to make the consonant vowel sounds that composed the 001 and 011 sequences. After a group of two or three monkeys was exposed consonant vowel sequence in only the 001 pattern or only the 011 pattern, a testing phase consisted in novel consonant vowel sounds (e.g. "wa", "ji", "la", "ri") arranged in sequences of both the same pattern that had been habituated and the alternative pattern. For example, if the 001 pattern was habituated, the monkeys might be tested on "lalari" as the same pattern and "wajiji" as the alternative pattern. Whether the same or alternative pattern was presented first was varied from group to group. The result was that the monkeys spent significantly more time looking towards the novel sounds in the alternative pattern than the novel sounds in the same pattern. This was interpreted as the monkeys becoming habituated to the pattern from the habituation phase and being capable of detecting the differences between the two nonsense grammars.

The details of the nonsense grammar model that follow differ from the comparative psychology experiment that inspired the model, but follow the experiment in spirit. In the nonsense grammar model, four nonsense grammars are considered: (i) 000, (ii) 001, (iii) 011, and (iv) 010. Sequences are composed of five different stimuli (A, B, C, D, E) arranged in one of the four nonsense grammar patterns. During training, sequences are selected at random with equal probability to be classified as (i), (ii), (iii), or (iv). The training set contains all sequences except those that contain stimuli D or E in pattern (iv). Then, in the testing phase, it is checked whether the sequences DED and EDE are successfully classified as pattern (iv).

The model's architecture, depicted in Figure 5, is essentially the same as the synchronous model. The only real difference is that there are three senders rather than two, the receiver correspondingly has urns indexed by three terms rather than two, and the receiver's urns begin the game with one ball of each of four types (one for each grammar type that a sequence can be classified as). While depicted with each sender's urns containing five different ball types, results are reported for each sender's urns containing ten different ball types.

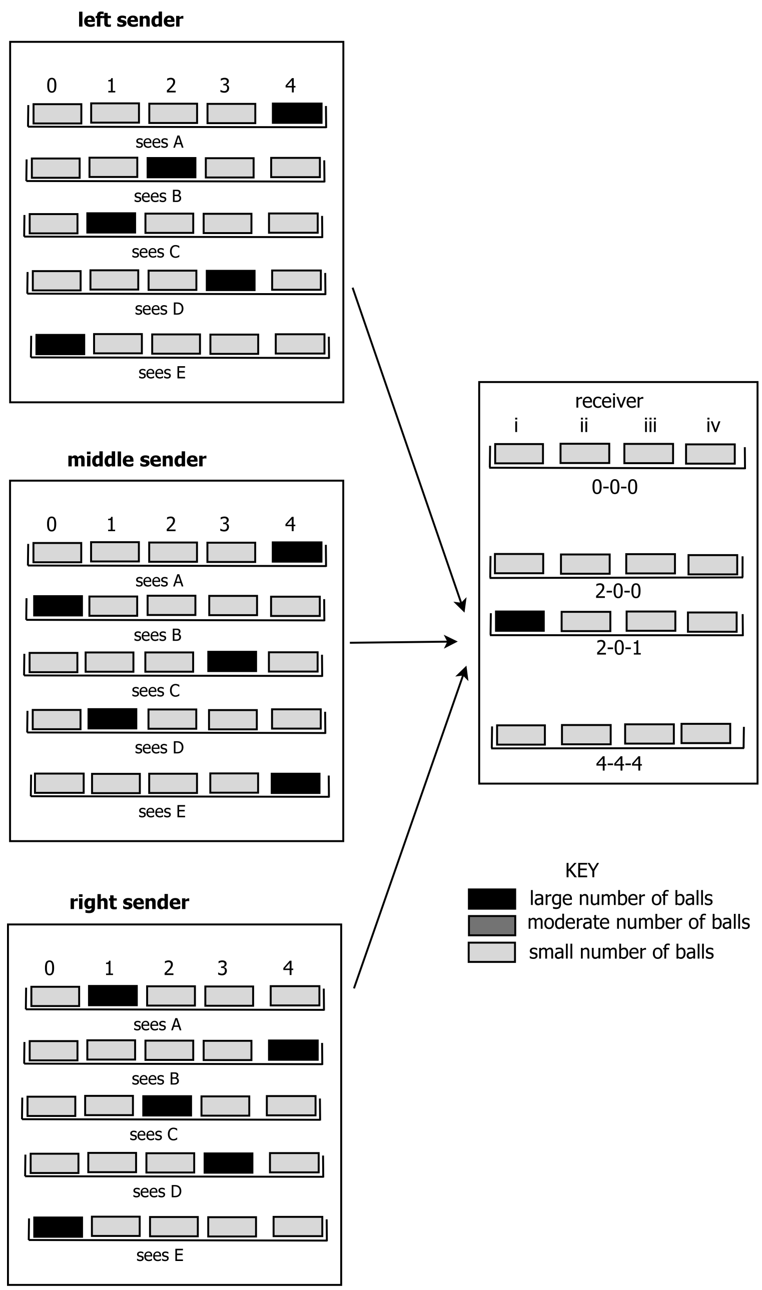


Figure 5: Synchronous Nonsense Grammar Model. For simplicity, this model is depicted with each of a sender's urns containing five ball types. However this is a parameter that can be varied. The results section focuses on simulation results for which each urn contained ten ball types.

For the synchronous nonsense grammars model, simulations were run using [+4, -10] reinforcement with iterated punishment and 10^7 training plays before testing (see Barrett and Gabriel for an explanation of reinforcement with iterated punishment (Barrett and Gabriel, 2022)). During the training phase, all simulations achieved a success rate above 0.85 and 91.1% of runs had a success rate greater than 0.99. During testing, nearly all simulations (96.9%) had a success rate better than chance (0.25), 58.1% of simulations had a success rate greater than 0.75, and 33.5% of simulations had a success rate greater than 0.9. So, a significant number of simulations correctly classified the test sequences.

6 Discussion

The models presented in this paper show how a low level associative learning dynamics, specifically Roth-Erev reinforcement with punishment and reinforcement with iterated punishment, can be employed to succeed at both the transitive inference task and the nonsense grammar task. In all of the models, success involved different stimuli getting mapped to the same intermediary variable, i.e. the same receiver urn.[8] For example, the stimuli pairs B-D and b-d can be mapped to the same receiver urn, transferring the successfully conditioned dispositions towards B-D to b-d. This only happens if the left sender transmits the same signal for B as she does for b, and likewise the right sender sends the same signal for D and d. Concretely, this may look like an agent learning to associate the head side of a coin, call it B, with the same dispositions as the tail side of the same coin, b. In this way, a rudimentary type of abstraction is realized in the models when they evolve successful dispositions; different particular stimuli play the same functional role of eliciting the appropriate action. This is analogous to different particular board positions guiding chess masters to the same strategic ideas in virtue of the, potentially novel, board positions being associated with the same opening.

Abstraction provides significant heuristic utility to finite boundedly rational agents. Chess masters would have no use for abstract categories of openings if they had the capacity to calculate every possible continuation of a board position. In fact, reliance on this sort of abstraction and chunking of common piece configurations, while common amongst human players, is virtually non-existent in computer chess engine methods of position evaluation (Gobet et al., 2004). However useful, abstraction's role as a heuristic carries with it epistemic concerns. This paper's introduction gave an example of how a person's familiarity with success playing the against the French Defense may lead to a suboptimal transposition from the Sicilian into a French.[9] If humans had unlimited cognitive resources, a chess master could take into account strategic ideas for all

[8]The term "intermediary variable" is used here in accordance with Marr's account of the algorithmic level of explanation (Marr, 2010).

[9]Specifically, this is objectively a suboptimal transposition for white when black is equally capable of playing a French or Sicilian. It is only in service of illustrating the epistemic concern that this paragraph ignores the possibility that white is making a strategic bet that black is less comfortable in French positions than Sicilian positions.

openings in a given board position rather than only those associated with a particular opening. Masters can also fail to notice a transposition leading to evaluating a position according to a suboptimal set of strategic considerations (Soltis, 2007).[10]

Building on the chess analogy, the synchronous magnitudes model illustrates how abstraction can be facilitated by innate biases and we can worry that such biases lead us to systematically misrepresent the world. Now it could be that an organism is innately (and possibly for arbitrary reasons) biased towards representing things in its environment as linearly ordered because it evolved in an environment where it was adaptive to represent as being linearly ordered. But it seems equally plausible that such a bias could be a spandrel of neuron connectivity.[11] Regardless of whether an innate bias is a result of environment pressures or an accidental consequence of some other adaptive trait, it is an open question whether that bias leads to systematic misrepresentation of the world. If objects in an environment are typically linearly related, an organism biased towards representing things as linearly related might represent those few things that are not linearly related as being linearly related. If an organism is innately biased towards represent things as linearly related for arbitrary reasons, it might still be able to represent things as not being linearly related; e.g. results showed that the simultaneous magnitudes model could learn a non-linear relation between the stimuli when conditioned to choose according to one ordering for contiguous stimuli but according to the reverse ordering for noncontiguous stimuli.

It seems that there are a couple viable responses to a generic worry that abstraction facilitated by innate biases puts us in an epistemically precarious position. First, it is true that the synchronous magnitudes model demonstrates how abstraction can be facilitated by some, perhaps arbitrary, innate bias. However, it is also the case that the model demonstrated an ability to represent a non linear relation despite its bias towards representing things as linearly or-

[10]Similar epistemic concerns can be raised for chunking of piece configurations. While there may be principles that are generally true of some common piece configurations, those principles need not hold for all positions. It is said that "a knight on the rim is dim", referencing the general principle that it is typically a mistake to move a knight to the edge of a board, say a5, because on the edge the knight can only control four squares rather than the eight it can control in the center of the board and also because, compared to a bishop, rook or queen, a knight requires substantially more moves to transition from attacking one side of the board to the other. But this principle might be violated if it allows the knight to be exchanged for a more valuable piece. Nimzovich dedicates the entire second part of his iconic book, *My System*, to analyzing common piece configurations (Nimzovich, 1964). What me might worry is that we are innately disposed only condition our judgments on a strict subset of relevant piece configurations. For example, novice chess players often find it easier to analyze the linear movement of the bishop than the non-linear movement of the knight and consequently systematically miss important strategic considerations involving knights. This might reflect a general bias towards representing things as being linearly related. It is an open empirical question whether chess masters' chunking of piece configurations is such that they systematically make suboptimal judgments when certain piece configurations occur.

[11]In defense of this claim, it might be noted that the architecture of the synchronous magnitudes model was not intended to exhibit a bias towards linearly ordered representations. Upon discovery of this bias, the author's first response was to look for an error in the model's code.

dered. Recall that the model was successfully trained to chose according to the regular ordering when presented with adjacent pairs, but also conditioned to chose according to the reverse ordering when presented with nonadjacent pairs; for example, choosing left was rewarded when presented with the pair a-b or b-c, but choosing right was rewarded when presented with the pair a-c.

By analogy, suppose we found some species that succeeded on the transitive inference task with minimal training, but was so innately biased towards linear representation that the species could not be conditioned to correctly choose the winner in rock paper scissors (i.e. conditioned to chose left for the pairs rock-scissors, scissors-paper, and paper-rock as well as conditioned to chose right for the pairs scissors-rock, paper-scissors, and rock-paper). The species continued survival is indicative of it generally representing the world well enough sustain itself and procreate. But in the context of rock paper scissors, the species is a miserable failure. Upon discovering that some way in which we abstract information is facilitated by some innate bias, we can worry that perhaps we are like this species there is some niche context in which we are miserable failures. Given our generally proficient navigation of the world we might be entirely blind to our failure in this niche context. We might simply represent such contexts as being governed by random chance rather than a recognizable pattern.

What the synchronous magnitudes model shows is that such worries can be too hasty. Abstraction can be facilitated by an innate bias in conjunction with enough flexibility in the learning mechanism to represent the world as contrary to the bias if necessary. Novice chess players often find it easier to understand tactics involving the bishop which moves linearly compared to tactics involving the knight which does not move linearly. It could be that this reflects some innate bias towards linear representation, but it would be wrong to conclude that we are incapable of appropriately representing tactics involving knights.[12]

Beyond highlighting the fact that the synchronous magnitudes model is able to represent things as being contrary to its innate bias given the appropriate training, it is worth considering the power of accomplishing abstraction through low level associative learning. What the models in this paper show is how minimal the preconditions for abstracting from experience can be. If we were to give a linguistic description of how one might abstract a serial ordering from pairs or a grammatical relation from triplets, it would be difficult to do so without using language that appears to presuppose some prior representation of the type of relation that is being learned.[13] But the dynamics of the models previously described need not operate at the coarseness of the language used to describe them. The model does not need to first reliably choose left for an

[12] Anecdotally, it is well known that novices struggle more with the knight than the bishop. As far as academic research goes, Gobet et al. (2004) does make explicit claims that the power of the bishop pair in open positions is more easily understood by humans than queen and knight vs queen and bishop citing Sturman (1996) in support of this claim. But, the evidence is weak, essentially amounting to anecdotes from world champion Capablanca and Grand Master Timoshchenko.

[13] This mirrors an interpretation of one of Kant's arguments that our representation of space cannot be empirical because in order to abstract the concept of space from relations between objects occupying space we first have to represent them as such (Janiak, 2022; Warren, 1998).

A-B pair before those dispositions can be co-opted for learning to choose left in response to observing the a-b pair. Those joint dispositions can grow gradually and simultaneously.

To conclude, this discussion has not argued that our inferences guided by abstraction and biases are always warranted. We can always worry that we are suboptimally advancing a pawn. However, there are some reasons to be optimistic about our use of abstraction. Abstraction can be a useful heuristic tool for navigating a complex world (or complex chess positions). Furthermore, while it may be facilitated by some biases, there is also evidence that those biases can be overcome. Finally, it was suggested that abstraction through associative learning has the benefit of requiring minimal preconditions for learning when compared to explicit symbolic reasoning.

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