Project: Introduction to Quantitative Finance

M1 EEF

Please, provide only one single PDF file where the Python Code is given, the figures, the answers to the question are also provided. I do not accept several files. Please indicate the author names.

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We consider a stochastic basis $(\Omega, (\mathcal{F}_t)_{t\in[0,T]}, P)$. The risk-free return is described by the log return instantaneous return $(r_t)_{t\in[0,T]}$, $T>0$, which is by hypothesis the stochastic process solution to the following S.D.E:

$$dr_t = a(b - r_t)\,dt + \gamma\, dW_t, \qquad (1)$$

where $a, b, \gamma > 0$ and W is a standard Brownian motion. The initial value $r_0 \in [\frac{1}{100}, \frac{10}{100}]$ and the coefficients $a \in [\frac{1}{100}, \frac{20}{100}]$, $b \in [r_0; 2r_0]$, $\gamma \in [\frac{1}{100}; \frac{10}{100}]$ are chosen by yourself.

The risk-free asset price is then given by the stochastic price process $(S^0_t)_{t\in[0,T]}$ solution to the SDE:

$$dS^0_t = r_t S^0_t\, dt, \quad S^0_0 = 1. \qquad (2)$$

We suppose that the risky asset price is modeled by the stochastic process $(S_t)_{t\in[0,T]}$ whose discounted value $(\tilde{S}_t)_{t\in[0,T]}$ is solution to the SDE:

$$d\tilde{S}_t = \sigma(t, \tilde{S}_t)\,\tilde{S}_t\, dB_t, \qquad (3)$$

where B is a Brownian Motion independant of W, $S_0 \in [5, 100]$ is chosen by yourself while σ is the function given by

$$\sigma(t, x) = \alpha\,(1 + f(t) + g(x)) \text{ where :}$$

- $\alpha \in [\frac{5}{100}, \frac{20}{100}]$ is chosen by yourself
- f, g are functions which are differentiable with bounded derivatives.
- $f \in [0, \frac{1}{2}]$ and $g \in [0, \frac{1}{2}]$

Rq f and g may depend on T if needed.

Q1 Give the Euler scheme of (1) to deduce approximated trajectories of r. Provide the Python Code and a graphic.

Q2. Deduce the trajectories of S^0 : Euler scheme, Python Code and explanations, see SDE (2)

Q3 Give the Euler scheme of (3) and deduce the trajectories by simulation (Python Code, graphic, explanations).

Q4 Deduce the trajectories of S (....).

Q5 Recall the principle to define and evaluate the price of a payoff g_T; $\mathcal{F}_T$ measurable and integrable (here $P = Q$...).

Q6 Deduce the numerical computation of the following payoffs (Give the Python Code, explanations..)

$g_T^1 = (K^1 - S_T)^+$ where $K^1 \in [\frac{1}{2} S_0, 2 S_0]$ is chosen by yourself.

$g_T^2 = \left(K^2 - \frac{1}{T}\int_0^T S_u du\right)^+$ with $K^2 \in [S_0, 3S_0]$ is chosen by yourself.