4. Low Risk Anomaly

The low-risk anomaly is a combination of three effects, with the third a consequence of the first two:[22]

1. Volatility is negatively related to future returns;
2. Realized beta is negatively related to future returns; and
3. Minimum variance portfolios do better than the market.

The risk anomaly is that risk—measured by market beta or volatility—is negatively related to returns. Robin Greenwood, a professor at Harvard Business School and my fellow adviser to Martingale Asset Management, said in 2010, "We keep regurgitating the data to find yet one more variation of the size, value, or momentum anomaly, when the Mother of all inefficiencies may be standing right in front of us—the risk anomaly."

4.1 HISTORY

The negative relation between risk (at least measured by market beta and volatility) and returns has a long history. The first studies showing a negative relation appeared in the late 1960s and early 1970s.[23] Friend and Blume (1970) examined stock portfolio returns in the period 1960–1968 with CAPM beta and volatility risk measures. They concluded (my italics):

> The results are striking. In all cases risk-adjusted performance is dependent on risk. *The relationship is inverse and highly significant*.

Haugen and Heins (1975) use data from 1926 to 1971 and also investigate the relation between beta and volatility risk measures and returns. They report (my italics):

> The results of our empirical effort do not support the conventional hypothesis that risk—systematic or otherwise—generates a special reward. Indeed, our results indicate that, over the long run, *stock portfolios with lesser variance in monthly returns have experienced greater average returns than "riskier" counterparts*.

[22] Some references for the third are Haugen and Baker (1991), Jagannathan and Ma (2003), and Clarke, de Silva, and Thorley (2006). I cover references for the others below.

[23] In addition to the papers in the main text, also see Pratt (1971), Sodolfsky and Miller (1969), and Black (1972).

Most of these results were forgotten. But these old results recently have come roaring back.

4.2 VOLATILITY ANOMALY

I was fortunate to write one paper that helped launch the new "risk anomaly" literature in 2006 with Robert Hodrick, one of my colleagues at Columbia Business School, and two of our former students, Yuhang Xing and Xiaoyan Zhang, who are now professors at Rice University and Purdue University, respectively. We found that the returns of high-volatility stocks were "abysmally low." So low that they had zero average returns. This paper now generates the most cites per year of all my papers and has spawned a follow-up literature attempting to replicate, explain, and refute the results.[24]

First, should there even be a relation between volatility and returns? The whole point of the CAPM and the many multifactor extensions (see chapter 7) was that stock return volatility itself should not matter. Expected returns, according to these models, are determined by how assets covary with factor risks. Idiosyncratic volatility, or tracking error (see equation (10.3)), should definitely *not* have any relation to expected returns under the CAPM. But in markets that are segmented due to clientele effects—where some agents cannot diversify or where some agents prefer to hold some assets over others for exogenous reasons—idiosyncratic volatility should be positively related to returns. Intuitively, agents have to be paid for bearing idiosyncratic risk, resulting in a positive relation between idiosyncratic risk and volatility in equilibrium. In later models with "noise traders," who trade for random reasons unrelated to fundamental valuation, higher volatilities are associated with higher risk premiums.[25]

The Ang et al. (2006) results show exactly the opposite.

Particularly notable is the robustness of the negative relation between both idiosyncratic and total volatility with returns. We employed a large number of controls for size, value, leverage, liquidity risk, volume, turnover, bid–ask spreads, co-skewness risk, dispersion in analysts' forecasts, and momentum. We also did not find that aggregate volatility risk explained our result—even though volatility risk is a pervasive risk factor (see chapter 7). In subsequent work, Ang et al. (2009), we showed that the volatility effect existed in each G7 country and across all developed stock markets. We also controlled for private information, transactions costs, analyst coverage, institutional ownership, and delay measures, which

[24] Volatility makes many appearances, of course, in tests of cross-sectional asset pricing models before Ang et al. (2006), but most of them are negative results or show a slight positive relation. For example, in Fama and MacBeth's (1973) seminal test of the CAPM, volatility is included and carries an insignificant coefficient. Eric Falkenstein (2012) recounts that he uncovered a negative relation between volatility and stock returns in his PhD dissertation in 1994, which was never published.

[25] For clientele models, see Merton (1987). For noise trader models, see DeLong et al. (1990).

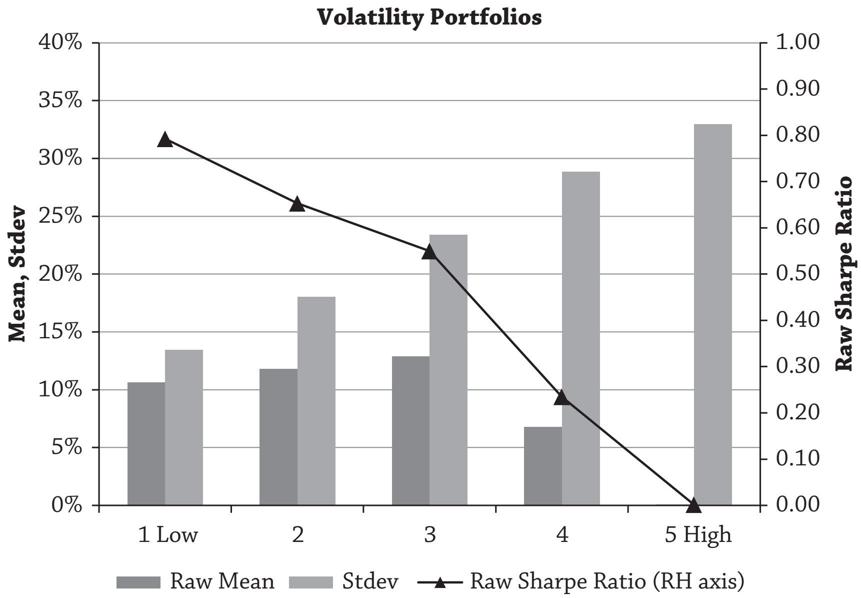

Figure 10.8

recorded how fast information is impounded into stock prices. Skewness did not explain the puzzle.

Lagged Volatility and Future Returns

To see the volatility anomaly, I take U.S. stocks, rebalance quarterly from September 1963 to December 2011, and form quintile portfolios. I construct monthly frequency returns. I sort on idiosyncratic volatility using the Fama–French (1993) factors with daily data over the past quarter. (Ranking on total volatility produces very similar results.) I market weight within each quintile similar to Ang et al. (2006, 2009).

In Figure 10.8, I report the mean and standard deviations of the quintile portfolios on the left-hand axis in the two bars. The volatilities increase going from the low- to high-volatility quintiles, by construction. The average returns are above 10% for the first three quintiles, fall to 6.8% for quintile 4, and then plummet to 0.1% for the highest volatility stocks. High volatility stocks certainly do have "abysmally low" returns. The right-hand axis reports raw Sharpe ratios, which are the ratios of the means to the standard deviations. These monotonically decline from 0.8 to 0.0 going from the low- to high-volatility quintiles.

Contemporaneous Volatility and Returns

Do stocks with high volatilities also have high returns over the same period used to measure those volatilities?

I examine this question in Figure 10.9 by forming portfolios at the end of the period based on realized idiosyncratic volatilities. I then measure realized returns over the same period. Note that these are not tradeable portfolios. Figure 10.9

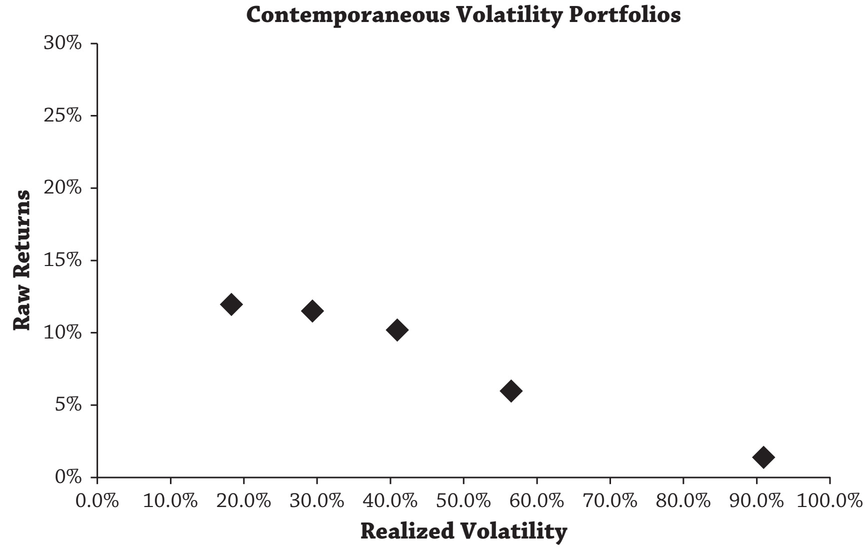

Figure 10.9

plots the average realized volatility and average realized returns of these quintile portfolios; there is still a negative relation between contemporaneous volatility and returns. Thus, the most volatile stocks *currently* lose money (which we cannot forecast), and they also tend to lose money in *the future* as well (which is predictable).

4.3 BETA ANOMALY

The first tests of the CAPM done in the 1970s did find positive relations between beta and expected returns, but they did not find that pure forms of the CAPM worked. Black, Jensen, and Scholes (1972), for example, found the relation between beta and returns to be "too flat" compared with what the CAPM predicted, but at least the relation was positive.

Fama and French wrote a major paper in 1992 that struck at the heart of the CAPM. While their main results showed that size and value effects dominated beta in individual stocks, they noted that "beta shows no power to explain average returns." In fact, their estimated relation between beta and returns was statistically insignificant. Worse, the point estimates indicated that the relation between beta and returns was negative.

Lagged Beta and Future Returns

In Figure 10.10, I form quintile portfolios rebalancing every quarter based on betas estimated over the previous quarter using daily returns. The portfolios are equal weighted so as to form the largest differences in returns and Sharpe ratios, and returns are at the monthly frequency.

The beta anomaly is that stocks with high betas tend to have lower risk-adjusted returns. Panel A of Figure 10.10 shows that the average returns across

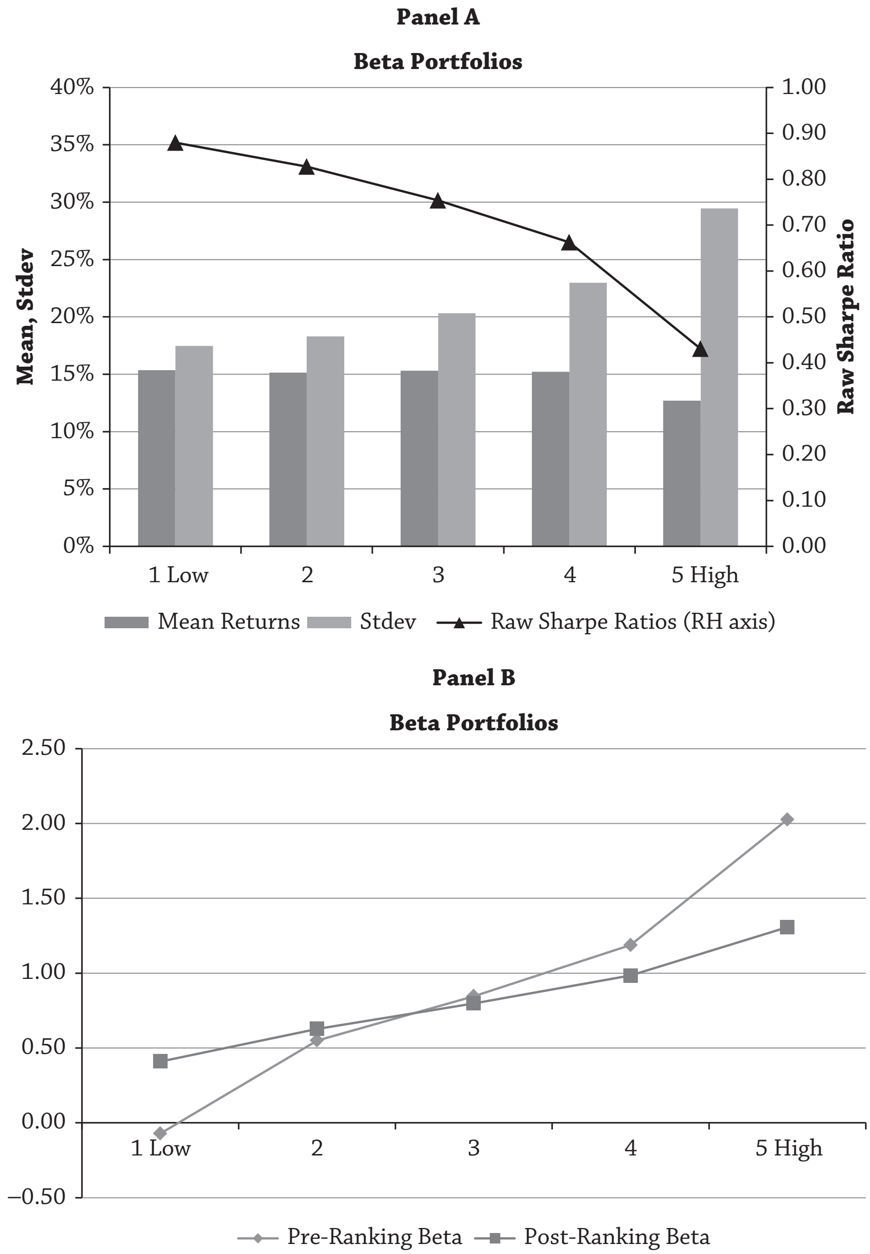

Figure 10.10

the beta quintiles are approximately flat, around 15% for the first four quintiles and slightly lower at 12.7% for quintile 5. The beta anomaly is not that stocks with high betas have low returns—they don't. Stocks with high betas have high volatilities. This causes the Sharpe ratios of high beta stocks to be lower than the Sharpe ratios of low-beta stocks. The right-hand axis of Panel A shows that the raw Sharpe ratios drop from 0.9 to 0.4 moving from the low- to the high-beta quintile portfolios.

In Panel B of Figure 10.10, I plot the pre-ranking and post-ranking betas. The pre-ranking beta is the beta over the previous three months, which is used to rank the stocks into portfolios. The post-ranking beta is the realized beta over the next three months after the portfolios have been formed. Panel B graphs the average pre-ranking betas of each portfolio with the average post-ranking betas. There is considerable noise in estimating betas at both ends, which is why the post-ranking beta line is much flatter than the pre-ranking betas. Betas are noisy! There is, however, still a large spread in post-ranking betas of over 1.0 between the highest and lowest beta portfolios.

Contemporaneous Beta and Returns

The CAPM does *not* predict that lagged betas should lead to higher returns. The CAPM actually states that there should be a *contemporaneous* relation between beta and expected returns. That is, stocks with higher betas should have higher average returns over the same periods used to measure the betas and the returns (see chapter 7 for more on factor theory).

Figure 10.11 examines the contemporaneous relation between betas and average returns by graphing average realized returns and average realized betas of portfolios formed at the end of each three-month period. It shows, perhaps surprisingly, that there is a positive contemporaneous relation between beta and returns.[26] This is exactly what the CAPM predicts!

Can we reconcile the negative relation between past betas and future returns and the positive contemporaneous relation between betas and realized returns? If we could find the future beta, future betas line up with future returns in keeping

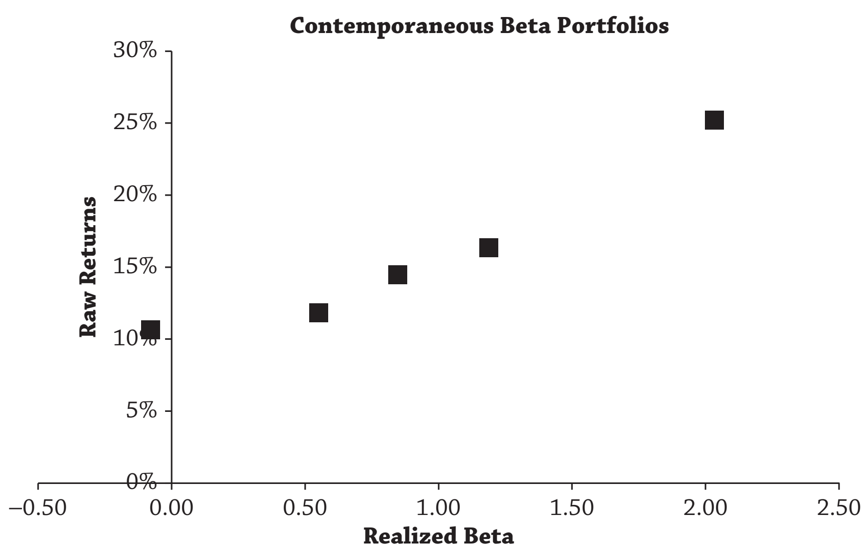

Figure 10.11

[26] See also Ang, Chen, and Xing (2006). Consistent with the early studies like Black, Jensen, and Scholes (1972), Figure 10.11 also shows that the estimated security market line is "too flat," especially near the *y*-axis.

with what the CAPM tells us. But Figure 10.10, Panel B, shows that it is hard to predict future betas. Past betas do a lousy job at predicting future betas. There is large variation in betas, and there is substantial sampling error.[27]

Studies that estimate betas from other information tend to find positive risk relations. Buss and Vilkov (2012) estimate betas from options and find them to be better predictors of future betas than betas estimated from past returns. Their betas estimated from option-implied information yield a positive risk–return relation. Cosemans et al. (2012) use valuation information from accounting balance sheets to compute betas along with past returns. They also estimate a positive relation between betas and returns. Thus, the real mystery in the low-beta anomaly is actually not so much that beta does not work; it is that we have such difficulty in predicting future betas, especially with past betas.

4.4 RISK ANOMALY FACTORS

It is a straightforward extension to use these portfolio results to create a benchmark factor for the risk anomaly.

Betting against Beta

Frazzini and Pedersen (2010) construct a betting-against-beta (*BAB*) factor that goes long low-beta stocks and short high-beta stocks. Constructing a factor to trade the beta anomaly cannot be done just by taking differences of the portfolios in Figure 10.10. Remember, the differences in average returns across the beta quintiles are tiny—what's large are the differences in Sharpe ratios across betas. Frazzini and Pedersen form their *BAB* factor by scaling the low- and high-beta portfolios by their betas:

$$BAB_{t+1} = \frac{r_{L,t+1} - r_f}{\beta_{L,t}} - \frac{r_{H,t+1} - r_f}{\beta_{H,t}}, \tag{10.17}$$

where $r_{L,t+1}$ is the return of the low-beta portfolio and $r_{H,t+1}$ is the return of the high-beta portfolio. The betas of the low-beta and high-beta portfolio at the beginning of the period (the pre-ranking betas) are given by $\beta_{L,t}$ and $\beta_{H,t}$, respectively.

Figure 10.12 shows what is going on. The horizontal line labeled "Data" is the empirical pattern of flat average returns with lagged betas in contrast to the upward-sloping line that is predicted by the "Standard CAPM." The long position in the low-beta portfolio is levered. It takes the position where it hits the

[27] Blume (1975) was one of the first to document this. For formal statistics for calculating the paths of time-varying alphas and betas and their standard errors, see Ang and Kristensen (2012).

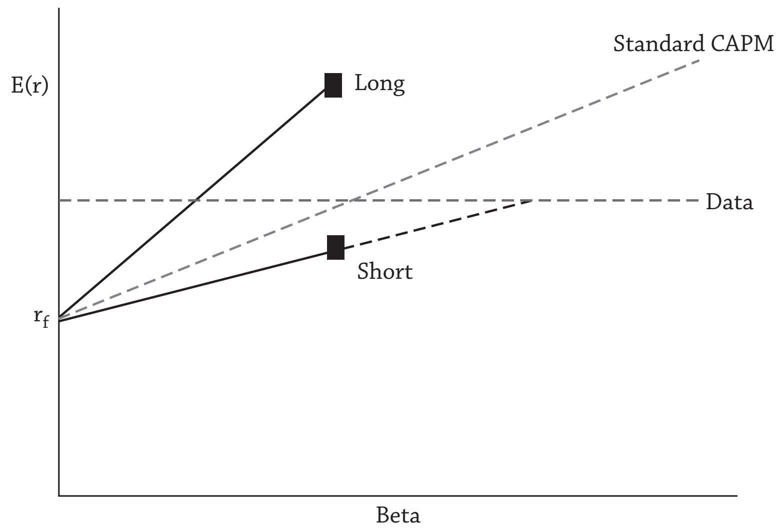

Figure 10.12

"Data" line and levers it up to the square marked "Long." The short position in the high-beta portfolio is marked "Short." The *BAB* portfolio does not take the entire position in the high-beta portfolio; it combines it with the risk-free asset to obtain the position marked Short. In effect, the Long and Short portfolios are unit beta positions in the low- and high-beta portfolios.

Frazzini and Pedersen use just two beta portfolios in creating their *BAB* factor. They have little choice. In Figure 10.10, the first quintile portfolio has a pre-ranking beta close to zero. Levering up this portfolio results in a position close to infinity. Thus, one is forced to create very small numbers of portfolios—two or three at most—in the *BAB* factor. One advantage of the volatility portfolios is that they can be directly traded without using the risk-free asset because there are pronounced differences in expected returns, not only volatilities, across the volatility quintiles.

Volatility Factor

I construct a volatility factor, *VOL*, similar to Frazzini and Pedersen's *BAB*:

$$VOL_{t+1} = \sigma_{\text{target}} \times \left(\frac{r_{L,t+1} - r_f}{\sigma_{L,t}} - \frac{r_{H,t+1} - r_f}{\sigma_{H,t}} \right), \qquad (10.18)$$

where $\sigma_{L,t}$ and $\sigma_{H,t}$ are the pre-ranking volatilities of the low- and high-volatility portfolios. While the *BAB* factor scales to unit betas, the *VOL* factor scales to a target volatility. I use the first and fifth quintile portfolios with returns $r_{L,t}$ and $r_{H,t}$, respectively. I set the target volatility $\sigma_{\text{target}} = 15\%$.

Betting-against-Beta and Volatility Factors

Figure 10.13 compares the *BAB* and *VOL* factors from October 1963 to December 2011.[28] The cumulative returns of the *VOL* factor are higher than *BAB*, and the volatility factor has a slightly higher Sharpe ratio (0.6 vs. 0.5), but the two factors are largely comparable. The main surprising result is that the beta and volatility effects are very lowly correlated; the correlation between *BAB* and *VOL* is –9%. The volatility and beta anomalies, therefore, are distinct.

Running a Fama–French plus momentum factor regression, we obtain

	BAB Factor		**VOL Factor**	
	Coeff	T-stat	Coeff	T-stat
Alpha	0.33%	1.89	0.42%	4.37
MKT Loading	–0.17	–4.13	0.87	38.8
SMB Loading	0.29	5.20	–0.63	–20.3
HML Loading	0.48	7.85	0.20	5.73
UMD Loading	0.09	2.35	0.13	6.00

The alpha for the *BAB* factor is 0.33% per month (4% per year) and the t-statistic of 1.89 corresponds to a p-value of 0.06. So this is borderline statistically significant at the standard 95% level. The *VOL* factor's alpha is slightly

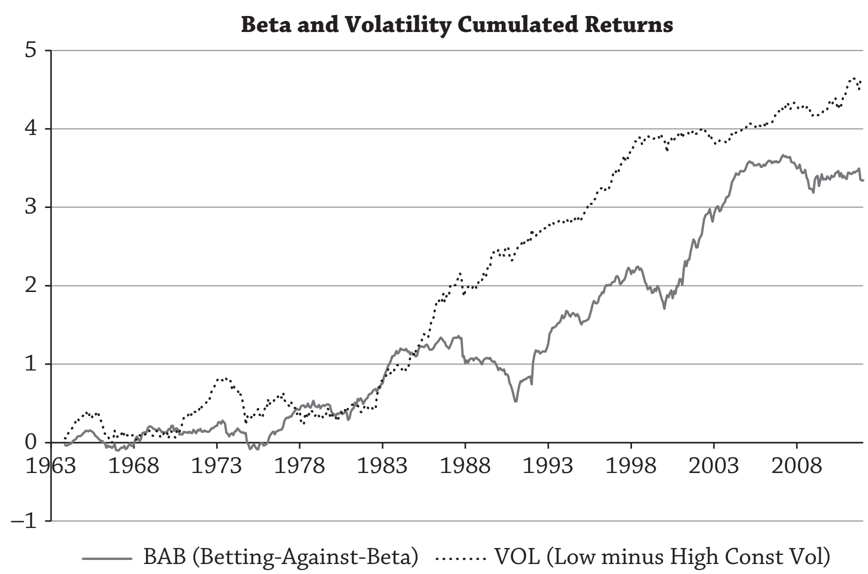

Figure 10.13

[28] I construct a *BAB* factor similar to Frazzini and Pedersen (2012) except I do not follow their step in shrinking the betas. Specifically, betas are computed in one-year rolling regressions using daily frequency returns. There are two beta portfolios created at the end of each month, and the *BAB* factor is constructed using equation (10.14) using the pre-ranking portfolio betas.

higher, at 0.42% per month (5% per year) but is much more statistically significant with a t-statistic of 4.37. Note that both *BAB* and *VOL* have significant value tilts (positive *HML* loadings) and momentum tilts (positive *UMD* loadings). The big difference is that the *BAB* factor carries a negative *SMB* loading, whereas it is positive for the *VOL* factor. That is, the beta anomaly manifests more in small stocks. In contrast, the volatility anomaly is more pervasive in large stocks, which are usually easier to trade because they are more liquid.

So should you do low volatility, or should you do low beta? This is not an either–or choice. You should do both.

4.5 EXPLANATIONS

We are still searching for a comprehensive explanation for the risk anomaly. In my opinion, the true explanation is a combination of all of the explanations listed below, plus potentially others being developed.

Data Mining

Some papers in the literature rightfully point out some data mining concerns with the original results in Ang et al. (2006). There is some sensitivity in the results to different portfolio weighting schemes and illiquidity effects.[29] For the most part, however, the low-risk anomaly is fairly robust. A recent survey article by Chen et al. (2012) argues that "idiosyncratic volatility is a common stock phenomenon" and is not due to microstructure or liquidity biases.

The best argument against data mining is that the low-risk effect is seen in many other contexts. Ang et al. (2006) show that the effect appears during recessions and expansions and during stable and volatile periods. Ang et al. (2009) show that it takes place in international stock markets. Frazzini and Pedersen (2011) show that low-beta portfolios have high Sharpe ratios in U.S. stocks, international stocks, Treasury bonds, corporate bonds cut by different maturities and credit ratings, credit derivative markets, commodities, and foreign exchange. Cao and Han (2013) and Blitz and de Groot (2013) show that the low-risk phenomenon even shows up in option and commodity markets, respectively. Low risk is pervasive.

Leverage Constraints

Many investors are leverage constrained—they wish to take on more risk but are unable to take on more leverage.[30] Since they cannot borrow, they do the next best thing—they hold stocks with "built-in" leverage, like high-beta stocks. Investors bid up the price of high-beta stocks until the shares are overpriced and deliver low

[29] See Bali and Cakici (2008) and Han and Lesmond (2011), respectively.

[30] Black (1972) was the first to develop a theory of the CAPM for when investors cannot lever. Frazzini and Pedersen (2011) apply a leverage-constraint story to explain the low-beta anomaly.

returns—exactly what we see in data. In CAPM parlance, the voracious demand of leverage-constrained investors for high-beta stocks flattens the security market line (see chapter 6). The leverage constraint story, however, does not explain the underpricing of low- beta stocks relative to the market, only the overpricing of high-beta stocks. Thus, it cannot explain why low-beta or low-volatility assets have higher returns than the market portfolio, but it can explain why some low-beta assets have positive alphas. This story also predicts that leverage-constrained institutions should be attracted to high-risk stocks. In reality, though, institutional investors tend to underweight high-risk stocks; stocks with high idiosyncratic volatility are predominantly held and traded by retail investors.[31]

Agency Problems

Many institutional managers can't or won't play the risk anomaly. In particular, the use of market-weighted benchmarks itself may lead to the low volatility anomaly.[32]

Figure 10.14 draws a theoretical relation between beta and expected returns in the diagonal solid line marked "CAPM" (the security market line). The data relation between returns and beta is the horizontal line marked "Data." Consider stock A, which has positive alpha, and B, which has negative alpha. Unconstrained investors simply buy low and sell high. They buy A, which offers a high return relative to the CAPM, and they sell B, whose return is too low relative to the CAPM.

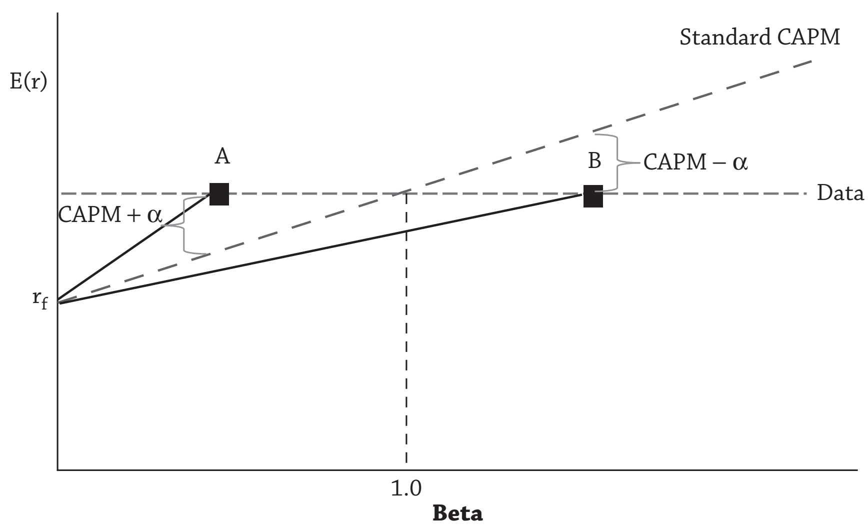

Figure 10.14

[31] For an academic reference, see Han and Kumar (2011). Taking 13-F filings as of June 30, 2012 on Russell 1000 holdings, Martingale calculates that institutions hold 46.5% low-risk stocks and 53.5% high-risk stocks compared to a balanced 50%/50% split.

[32] See Greenwood et al. (2010) and Baker, Bradley, and Wurgler (2011).

In a perfect world, these unconstrained investors would bid up the price of A until it no longer has any excess returns. And they would sell B until its returns reach a fair level relative to the CAPM. In this perfect world, the risk anomaly disappears.

Now consider a long-only investor subject to tracking error constraints that place limits on how much she can deviate from benchmark (see equation (10.3)). This investor cannot short. This investor does *not* invest in A, even though A is offering high returns relative to the CAPM. The returns of A are higher than the CAPM predicts. Stock A would even perform in line with the market. But, by investing in A, she takes on significant tracking error.

What about stock B? Stock B has negative alpha. To make money, she needs to short stock B, and she cannot do so. The best that she can do is to avoid buying stock B, thus making at most a small active bet relative to the market portfolio. The tracking error constraint also limits the underweight position in stock B that she can hold. If the "Data" line is in fact just slightly upward sloping rather than perfectly flat, then she actually has an incentive to buy B rather than sell it short because B could outperform the market.

Thus, the use of tracking error with these benchmarks makes it hard to bet against low volatility or low beta. Tracking error is a binding constraint for GM Asset Management. It is also a binding constraint for most institutional asset owners. One obvious solution is to change the benchmarks, and there are certainly more appropriate factor benchmarks available (see section 3 and chapter 14). But changing benchmarks at GM is a lengthy process requiring approval of the investment committee. It opens up a broader issue of how all benchmarks "depend on funded status and on the health of the parent," as Scott explains.

Frazzini, Kabiller, and Pedersen (2012) even argue that low-risk factors play a part in explaining the superior performance of Berkshire Hathaway—a company well known for its ability to go against the crowd and avoid common agency issues. They find that Buffett's alpha declines from 12.5% from 1976 to 2011 using the Fama–French and momentum benchmark we've been using in this chapter to 11.1% when including the *BAB* factor. If they add another factor measuring the underlying quality of companies, Buffett's alpha falls to 7.0%. So some of Buffett's investing prowess is due to Buffett selecting stocks with low risk, but most of Buffett's investment prowess comes from ferreting out gems with high underlying quality—true skill that is unrelated to just holding low-volatility stocks.

Preferences

If asset owners simply have a preference for high-volatility and high-beta stocks, then they bid up these stocks until they have low returns. Conversely, these investors shun safe stocks—stocks with low volatility and low betas—leading to low prices and high returns for these shares. Thus, "hopes and dreams" preferences, where the hopes and dreams are represented by high-volatility and high-beta stocks, could explain the risk anomaly.[33]

Hou and Loh (2012) comprehensively examine many explanations of the low volatility anomaly. They arrange their explanations into three broad groups: (i) lottery preferences, (ii) market frictions including illiquidity, and (iii) "other," which is a broad category that includes uncertainty, short-sales constraints, financial distress, investor inattention, growth options, earnings shocks, and other variables. Hou and Loh find that when individual explanations are taken alone, each explains less than one-tenth of the volatility anomaly. But taken as groups, the most promising explanation is lottery preferences. When individual lottery preference stories are taken together, they explain close to half of the low volatility puzzle. But close to half of the puzzle remains unexplained.

Agents disagreeing with each other (heterogeneous preferences) combined with the inability to short could also account for some of the risk anomaly. Hong and Sraer (2012) show that when disagreement is low and everyone takes long-only positions, the CAPM holds. But when disagreement is high, some agents want to sell short and they cannot. High beta stocks become overpriced. Large enough disagreement causes the relation between beta and returns to be downward sloping.[34]

5. GM Asset Management and Martingale Redux

Martingale's low volatility strategy is attractive compared to the market portfolio. It delivers alpha relative to the Russell 1000 benchmark of 1.50% per year. Adjusting the Russell 1000 for risk increases that alpha to 3.44% per year. Alpha is all about the benchmark. What if we changed the benchmark of the Martingale's strategy to be the low volatility strategy itself? Then, there wouldn't be any alpha of course, as alpha morphs into the benchmark (or beta, as some in industry like to call it). This is not just philosophical—GM Asset Management might be in a position to internally do low volatility strategies. But low-risk strategies appear to have significant alpha relative to standard market capitalization benchmarks and sophisticated factor benchmarks that control for risk using dynamic value-growth and momentum factors along with the market portfolio.

Yet alpha is not the only consideration for GM Asset Management. Martingale's alpha comes with high tracking error relative to the Russell 1000 benchmark. In fact, the ubiquitous tracking error constraints employed in

[33] For stories along these lines, see Boyer, Mitton, and Vorkink (2010), Bali, Cakici, and Whitelaw (2011), and Ilmanen (2012).

[34] See also Jiang, Xu, and Yao (2009) for the relation between earnings uncertainty and low volatility.

the asset management industry may partly give rise to the risk anomaly in the first place.

Will the risk anomaly persist? I am hoping that it goes away as soon as possible, and I have a large academic stake in this debate. As much as I enjoy seeing new explanations being proposed (including some of my own), the risk anomaly is an enigma. If it does disappear, then the low-risk trades already put on by the smart money will pay off handsomely—low-volatility or low-beta stocks have returns that are too high and prices that are too low. Capital should be drawn to these stocks, driving up their prices and removing the anomalous returns. If that happened, current low-risk anomaly investors would enjoy large capital gains.

But I doubt this will happen. Low-volatility strategies are far from predominant, as most institutional investors appear to be underweight low-risk stocks. More fundamentally, the fact that we see the risk anomaly in many markets—U.S. and international, stocks, bonds, commodities, foreign exchange, and derivatives—suggests that the effect is pervasive and requires a deep explanation. As Greenwood says, the low-risk anomaly is the mother of all inefficiencies.