Quiz 5

MATH 19B - Discussion Section C
November 10, 2016

Name & ID # : ____________________

Directions: Leave your final answer in exact form and box it in.
Formulas: You may find the following useful:

$$\sin^2(x) + \cos^2(x) = 1, \quad 1 + \tan^2(x) = \sec^2(x), \quad \text{and} \quad 1 + \cot^2(x) = \csc^2(x)$$

and the Binomial Theorem:

$$(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i} \quad \text{where} \quad \binom{n}{i} = \frac{n!}{i!(n-i)!} \quad \text{and} \quad n \in \mathbb{N}$$

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(1) (a) Argue why $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}\, \mathrm{d}x > 0$.

(b) Evaluate: *(Hint: Expand and use long division to simplify)*

$$\int_0^1 \frac{x^4(1-x)^4}{1+x^2}\, \mathrm{d}x$$

(c) Explain using the results of parts (a) and (b) to prove $\pi < \frac{22}{7}$. This approximation for π has been known since antiquity and the first proof of the inequality was provided by Archimedes in the 3rd century BCE.

(2) A charged rod of length $\mathcal{L}$ produces an electric field, along the x direction, at a point $\mathcal{P}(a, b) \in \mathbb{R}^2$ and is given by:

$$\mathbf{E}_x(\mathcal{P}) = \int_{-a}^{\mathcal{L}-a} \frac{x\lambda(x)}{4\pi\epsilon_0(x^2+b^2)^{\frac{3}{2}}}\, \mathrm{d}x$$

where λ is the *charge density* per unit length on the rod and ϵ_0 is the *permittivity of free space*. Evaluate the integral to determine an expression for the electric field assuming that λ is constant.