

# Quiz 1 Solutions

MATH 100  
October 8, 2018

- (1) (Q) Given  $A = \{\emptyset\}$ , calculate  $\mathcal{P}(A)$  and  $\mathcal{P}(\mathcal{P}(A))$  explicitly and use the results to write down  $|\mathcal{P}(A)|$  and  $|\mathcal{P}(\mathcal{P}(A))|$ .

(A) For the first power set we have  $|\mathcal{P}(A)| = 2^1 = 2$  with elements:

$$\mathcal{P}(A) = \{\emptyset, \{\emptyset\}\}$$

Using this result we must obtain  $|\mathcal{P}(\mathcal{P}(A))| = 2^2 = 4$  with elements:

$$\mathcal{P}(\mathcal{P}(A)) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$$

- (2) (Q) For the following sequences compute  $\bigcup_{n \in \mathbb{N}} A_n$  and  $\bigcap_{n \in \mathbb{N}} A_n$ :

a)  $A_n = \left(\frac{1}{n}, e^n\right)$

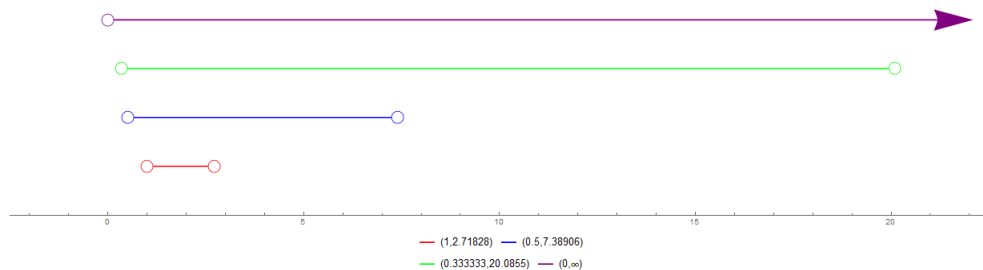
b)  $A_n = \left(a - \frac{1}{n}, b\right]$  for any  $a, b \in \mathbb{R}$  s.t.  $a < b$

(A) To compute the infinite union and intersection it helps to write out the general pattern:

a)

$n$	Interval
1	$(1, e)$
2	$(\frac{1}{2}, e^2)$
3	$(\frac{1}{3}, e^3)$
$\vdots$	$\vdots$
$\infty$	$(0, \infty)$

Since the union will take the biggest collection possible, it has to be that  $\bigcup_{n \in \mathbb{N}} A_n = (0, \infty)$ . The intersection on the other hand only takes the elements showing up at each step thereby providing  $\bigcap_{n \in \mathbb{N}} A_n = (1, e)$ . For a visualization you may consider the following image:



b)

$n$	Interval
1	$(a-1, b]$
2	$(a-\frac{1}{2}, b]$
3	$(a-\frac{1}{3}, b]$
$\vdots$	$\vdots$
$\infty$	$(a, b]$

Since the union will take the biggest collection possible, it has to be that  $\bigcup_{n \in \mathbb{N}} A_n = (a-1, b]$ . The intersection on the other hand only takes the elements showing up at each step thereby providing  $\bigcap_{n \in \mathbb{N}} A_n = [a, b]$ . For a visualization you may consider the following image:

