Quiz 6

MATH 19B - Discussion Section C
December 1, 2016

Name & ID # : _____

Directions: Leave your final answer in exact form and box it in.
Formulas: You may find the following useful:

$$\int_a^b u\,\mathrm{d}v = uv\Big|_a^b - \int_a^b v\,\mathrm{d}u, \quad \sum_{k=i}^{n} r^k = \frac{r^i(1-r^{n-i+1})}{1-r}, \quad \text{and} \quad \sum_{k=i}^{\infty} r^k = \frac{r^i}{1-r} \quad \text{for} \quad |r|<1$$

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(1) The *gamma function* can be defined as the smooth curve that connects the points (x, y) given by $y = (x-1)!$ at the positive integer values of x. In fact, the function can be extended for all complex numbers with a positive real part:

$$\Gamma(z) = \int_0^\infty x^{z-1}e^{-x}\,\mathrm{d}x$$

We want to restrict our interest to the integers and show that it matches the factorial function via proof by induction.

(a) Calculate $\Gamma(1)$ through the improper integral (*This proves that* $0! = 1$).

(b) Calculate $\Gamma(n+1)$ with the improper integral. You are allowed to use the fact that $\Gamma(n) = (n-1)!$ (*Hint: Use integration by parts once and identify the new integral in terms of the gamma function*).

(2) The *golden ratio* is defined as $\phi = \frac{1+\sqrt{5}}{2}$. It is considered to be the 10^{th} most important number in mathematics and is the driving force behind the Fibonacci numbers. Letting ϕ be the above, evaluate the following sums: (*Hint: Use a geometric series*)

(a)

$$\sum_{n=1}^{\infty} \frac{1}{\phi^n}$$

a) 1 b) $\frac{1}{\phi}$ c) ϕ d) ∞

(b)

$$\sum_{n=1}^{\infty} \frac{1}{\phi^{2n}}$$

a) 1 b) $\frac{1}{\phi}$ c) ϕ d) ∞