Project Euler: Problem 6

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Problem (Sum Square Difference)**.** The sum of the squares of the first ten natural numbers is,

$$1^2 + 2^2 + \cdots + 10^2 = 385$$

The square of the sum of the first ten natural numbers is,

$$(1 + 2 + \cdots + 10)^2 = 3025$$

Hence the difference between the sum of the squares of the first ten natural numbers and the square of the sum is 3025 - 385 = 2640. Find the difference between the sum of the squares of the first one hundred natural numbers and the square of the sum.

Solution. To approach this problem, the following two formulas are needed to make life easier:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$
$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

where the first equation represents the sum of the first n natural numbers and the second the sum of the squares of the first n natural numbers. This allows for a generalization of n terms, even though this problem specifically wants $n = 100$. Using this the difference between the square of the sum and the sum of the squares of the first n natural numbers is:

$$D = \Big(\sum_{i=1}^{n} i\Big)^2 - \sum_{i=1}^{n} i^2 = \Big(\frac{n(n+1)}{2}\Big)^2 - \frac{n(n+1)(2n+1)}{6} = \frac{n^4}{4} + \frac{n^3}{6} - \frac{n^2}{4} - \frac{n}{6}$$