Project Euler: Problem 12

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Problem (Highly Divisible Triangular Number)**.** The sequence of triangle numbers is generated by adding the natural numbers. So the 7th triangle number would be 1 + 2 + 3 + 4 + 5 + 6 + 7 = 28. The first ten terms would be:

$$1, 3, 6, 10, 15, 21, 28, 36, 45, 55, \ldots$$

Let us list the factors of the first seven triangle numbers:

$$\begin{aligned}
\mathbf{1} &: 1 \\
\mathbf{3} &: 1, 3 \\
\mathbf{6} &: 1, 2, 3, 6 \\
\mathbf{10} &: 1, 2, 5, 10 \\
\mathbf{15} &: 1, 3, 5, 15 \\
\mathbf{21} &: 1, 3, 7, 21 \\
\mathbf{28} &: 1, 2, 4, 7, 14, 28
\end{aligned}$$

We can see that 28 is the first triangle number to have over five divisors. What is the value of the first triangle number to have over five hundred divisors?

Solution. To approach this problem generalize first and assume that we are being asked for the first tirangular number to have at least N divisors. Now generate the triangular numbers by adding sequential natural numbers:

$$T = \left\{ x \middle| x = \sum_{i=1}^{n} i \quad \forall n \in \mathbb{N} \right\}$$

and break each number down into its prime factorization:

$$x = p_1^{m_1} p_2^{m_2} \ldots p_k^{m_k}$$

At each x check the number of distinct prime factors versus the required value N. If this number is at least N, we have found our solution, if not, continue the cycle until the requirement is met.