

Project Euler: Problem 2

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Problem (Even Fibonacci Numbers). Each new term in the Fibonacci sequence is generated by adding the previous two terms. By starting with 1 and 2, the first 10 terms will be:

$$1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \dots$$

By considering the terms in the Fibonacci sequence whose values do not exceed four million, find the sum of the even-valued terms.

Solution. Given the Fibonacci sequence:

$$F_n = F_{n-1} + F_{n-2} \quad \forall n \in \{3, 4, 5, \dots\} \quad \text{where } F_1 = 1 \text{ and } F_2 = 2$$

the closed form is given by:

$$F_n = \frac{\phi^n - (-\phi)^{-n}}{\sqrt{5}} \quad \forall n \in \mathbb{N} \quad \text{where } \phi = \frac{1 + \sqrt{5}}{2}$$

Now given some cap N , which in this problem is 4 million, to determine the relevant even Fibonacci numbers define the following set:

$$\mathcal{F} = \left\{ F_n \mid F_n = \frac{\phi^n - (-\phi)^{-n}}{\sqrt{5}}, \quad F_n \pmod{2} \equiv 0, \quad \text{and } F_n < N \quad \forall n \in \mathbb{N} \right\}$$

Now the solution corresponds to:

$$S = \sum_i \mathcal{F}_i$$