Project Euler: Problem 42

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Problem (Coded Triangle Numbers)**.** The n^{th} term of the sequence of triangle numbers is given by, $t_n = \frac{n(n+1)}{2}$; so the first ten triangle numbers are:

$$1, 3, 6, 10, 15, 21, 28, 36, 45, 55, \ldots$$

By converting each letter in a word to a number corresponding to its alphabetical position and adding these values we form a word value. For example, the word value for SKY is 19 + 11 + 25 = 55 = t_{10}. If the word value is a triangle number then we shall call the word a triangle word. Using *words.txt*, a 16K text file containing nearly two-thousand common English word, how many are triangle words?

Solution. This problem overall mainly deals with opening the file and extracting the words and then computing the numerical value of each word. Once the value is obtainted we need a way to tell whether a number is a triangle number or not. To do this observe:

$$\begin{aligned} t_n &= \frac{n(n+1)}{2} \\ t_n &= \frac{1}{2}n^2 + \frac{1}{2}n \\ 0 &= n^2 + n - 2t_n \end{aligned}$$

Using the quadratic formula on n gives:

$$n = \frac{-1 \pm \sqrt{1 + 8t_n}}{2}$$

where the negative radical can be dropped since n is strictly positive:

$$n = \frac{-1 + \sqrt{1 + 8t_n}}{2}$$

Using the above formula given any number t_n, determine n and if it is an integer, then the number is a triangle number.