Project Euler: Problem 5

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Problem (Smallest Multiple)**.** 2520 is the smallest number that can be divided by each of the numbers from 1 to 10 without any remainder. What is the smallest positive number that is evenly divisible by all of the numbers from 1 to 20?

Solution. On a general scale this problem deals with finding the least common multiple of a list of N positive integers, specifically the first 20 positive integers is of interest to us. So lets assume that there is a list of these divisors:

$$d = \{d_1, d_2, \dots, d_N\} \quad \text{where} \quad N \in \{2, 3, 4, \dots\}$$

Now to compute the LCM(m, n), least common multiple, use:

$$\text{LCM}(m, n) = \frac{mn}{\text{GCD}(m, n)}$$

where GCD(m, n) is the greatest common divisor. With this a recursive relationship for finding the least common multiple of 3 factors is defined as:

$$\text{LCM}(d_N, d_{N-1}, d_{N-2}) = \text{LCM}(d_{N-1}, d_{N-2}) \frac{d_N}{\text{GCD}(\text{LCM}(d_{N-1}, d_{N-2}), d_N)}$$

With this, iterate through all of the divisors in question using the above recursive relationship will obtain LCD$(d_1, d_2, \dots, d_N)$.