Project Euler: Problem 124

Nathan Marianovsky

Problem (Ordered Radicals)**.** The radical of n, rad(n), is the product of the distinct prime factors of n. For example, 504 = $2^3 \times 3^2 \times 7$, so rad(504) = $2 \times 3 \times 7$ = 42. If we calculate rad(n) for $1 \leq n \leq 10$, then sort them on rad(n), and sorting on n if the radical values are equal, we get:

n	rad(n)
1	1
2	2
3	3
4	2
5	5
6	6
7	7
8	2
9	3
10	10

$\rightarrow$

n	rad(n)	k
1	1	1
2	2	2
4	2	3
8	2	4
3	3	5
9	3	6
5	5	7
6	6	8
7	7	9
10	10	10

Let $E(k)$ be the kth element in the sorted n column; for example, $E(4) = 8$ and $E(6) = 9$. If rad(n) is sorted for $1 \leq n \leq 100000$, find $E(10000)$.

Solution. It is best to approach this problem by first assuming that the list is of length N, though the problem specifically wants $N = 100000$. For each value of n in the table produced, we need the prime factorization [1]:

$$n = p_1^{m_1} p_2^{m_2} \dots p_l^{m_l}$$

so that:

$$\text{rad}(n) = p_1 p_2 \dots p_l$$

After this it is a simple task of organizing the list based on rad(n) first, then n.

[1]See the personal comments for problem 3 for a proof of the Fundamental Theorem of Arithmetic