

# Project Euler: Problem 1

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**Problem** (Multiples of 3 and 5). If we list all the natural numbers below 10 that are multiples of 3 or 5, we get 3, 5, 6, and 9. The sum of these multiples is 23. Find the sum of all multiples of 3 or 5 below 1000.

**Solution.** In this problem specifically we want to find the sum below 1000, but this can be generalized to some cap  $N$ . There are two ways that this problem can be approached. First a list can be made of all the natural numbers below  $N$  that follow the requirements where the sum of all the elements corresponds to the solution. The second approach is to find the values of  $k_1$  and  $k_2$  such that:

$$3k_1 = N$$

$$5k_2 = N$$

where the  $k$ 's correspond to how high the multiples of 3 and 5 can go before they go above the cap  $N$ . Their integer solutions are modeled by:

$$k_1 = \left\lfloor \frac{N}{3} \right\rfloor$$

$$k_2 = \left\lfloor \frac{N}{5} \right\rfloor$$

So now instead of taking every natural number starting from 1 and onwards checking whether it fits the requirements, the numbers can be generated into the following list:

$$v = \{3, 6, 9, \dots, 3k_1, 5, 10, 15, \dots, 5k_2\}$$

where the generated list should not contain any repetitions of numbers that are divisible by both 3 and 5. So now the solution corresponds to:

$$V = \sum_i v_i$$