Quiz 1 Solutions

<u>SECTION B</u>

(1) (a) To approach the sum consider grouping terms together so as to cancel them out:

$$\sum_{n=1}^{50}(-1)^n = \underbrace{(-1+1)+(-1+1)+\cdots+(-1+1)}_{\text{25 such groupings in total}} = 0$$

(b) This is known as a *Telescoping Series* and just like part (a) group the terms and notice the cancellation:

$$\sum_{i=1}^{100}\left[3^i - 3^{i-1}\right] = (\cancel{3}-1)+(\cancel{3^2}-\cancel{3})+(\cancel{3^3}-\cancel{3^2})+\cdots+(3^{100}-\cancel{3^{99}}) = 3^{100}-1$$

(2) (a) To approach evaluation via Riemann Sums first identify:

$$\Delta x = \frac{10}{N}, \quad x_i = -5+\frac{10i}{N}, \quad \text{and} \quad f(x_i) = \frac{30i}{N} - 14$$

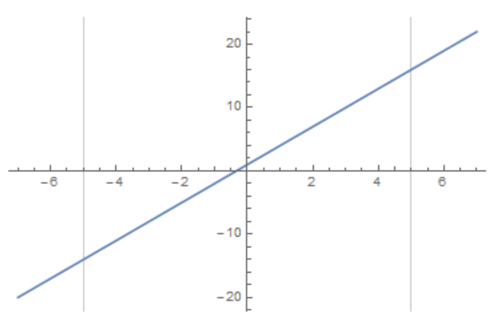
Now by direct evaluation the right hand approach provides:

$$\begin{aligned} R_N &= \Delta x \sum_{i=1}^{N} f(x_i) \\ &= \frac{10}{N}\sum_{i=1}^{N}\left[\frac{30i}{N}-14\right] \\ &= \frac{10}{N}\left[\frac{30}{N}\sum_{i=1}^{N} i - 14\sum_{i=1}^{N} 1\right] \\ &= \frac{300}{N^2}\cdot\frac{N(N+1)}{2} - \frac{140}{N}\cdot N \\ &= \frac{150N+150}{N} - 140 \end{aligned}$$

Now to evaluate the integral:

$$\int_3^7 (2x-10)\,\mathrm{d}x = \lim_{N\to\infty} R_N = 10$$

(b) In order to use the geometry of the function we need the graph:



The area is calculated using the two right triangles:

$$\int_{-5}^{5}(3x+1)\,\mathrm{d}x = \frac{1}{2}\left(\frac{14}{3}\right)(-14)+\frac{1}{2}\left(\frac{16}{3}\right)(16) = 10$$

SECTION C

(1) (a) To approach the sum consider grouping terms together so as to cancel them out:

$$\sum_{n=1}^{50} 2 \cdot (-1)^n = \underbrace{(-2+2)+(-2+2)+\cdots+(-2+2)}_{\text{25 such groupings in total}} = 0$$

(b) This is known as a *Telescoping Series* and just like part (a) group the terms and notice the cancellation:

$$\sum_{i=1}^{100} \left[5^i - 5^{i-1}\right] = (\cancel{5} - 1) + (\cancel{5^2} - \cancel{5}) + (\cancel{5^3} - \cancel{5^2}) + \cdots + (5^{100} - \cancel{5^{99}}) = 5^{100} - 1$$

(2) (a) To approach evaluation via Riemann Sums first identify:

$$\Delta x = \frac{4}{N}, \quad x_i = 3 + \frac{4i}{N}, \quad \text{and} \quad f(x_i) = \frac{8i}{N} - 4$$

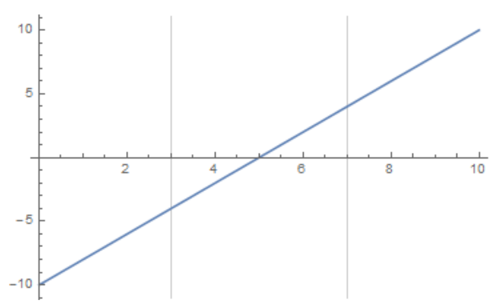
Now by direct evaluation the right hand approach provides:

$$\begin{aligned} R_N &= \Delta x \sum_{i=1}^{N} f(x_i) \\ &= \frac{4}{N} \sum_{i=1}^{N} \left[\frac{8i}{N} - 4\right] \\ &= \frac{4}{N}\left[\frac{8}{N}\sum_{i=1}^{N} i - 4\sum_{i=1}^{N} 1\right] \\ &= \frac{32}{N^2} \cdot \frac{N(N+1)}{2} - \frac{16}{N} \cdot N \\ &= \frac{16N+16}{N} - 16 \end{aligned}$$

Now to evaluate the integral:

$$\int_3^7 (2x - 10)\,\mathrm{d}x = \lim_{N\to\infty} R_N = 0$$

(b) In order to use the geometry of the function we need the graph:



The area is calculated using the two right triangles:

$$\int_3^7 (2x - 10)\,\mathrm{d}x = \frac{1}{2}(2)(-4) + \frac{1}{2}(2)(4) = 0$$