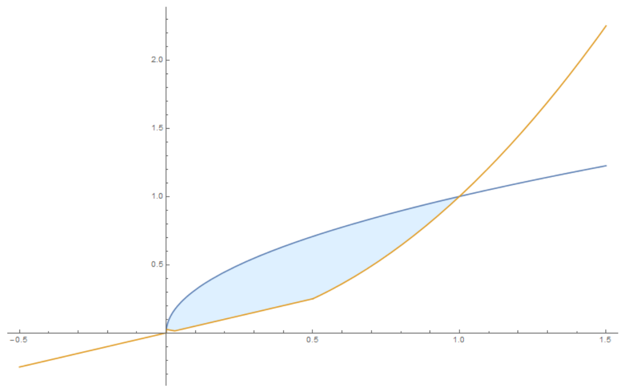
Quiz 3 Solutions

Section B

(1) Our region takes the form:



(2) To calculate the area there is a need to split up the integrals:

$$\begin{aligned}
\text{Area} &= \int_0^{\frac{1}{2}} \left(x^{\frac{1}{2}} - \frac{x}{2}\right) \mathrm{d}x + \int_{\frac{1}{2}}^{1} \left(x^{\frac{1}{2}} - x^2\right) \mathrm{d}x \\
&= \left(\frac{2}{3}x^{\frac{3}{2}} - \frac{x^2}{4}\right)\Bigg|_0^{\frac{1}{2}} + \left(\frac{2}{3}x^{\frac{3}{2}} - \frac{x^3}{3}\right)\Bigg|_{\frac{1}{2}}^{1} \\
&= \left(\frac{2}{3}\cdot\frac{1}{8^{\frac{1}{2}}} - \frac{1}{16}\right) + \left(\frac{2}{3} - \frac{1}{3} - \frac{2}{3}\cdot\frac{1}{8^{\frac{1}{2}}} + \frac{1}{24}\right) \\
&= \frac{5}{16}
\end{aligned}$$

(3) Similar to the area the volume requires a split:

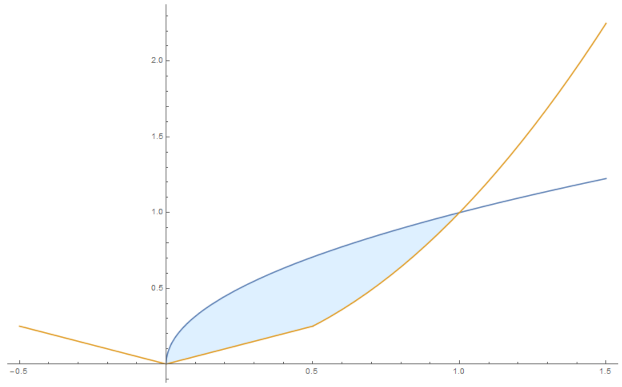
$$\text{Volume} = \pi \int_0^{\frac{1}{2}} \left(x - \frac{x^2}{4}\right) \mathrm{d}x + \pi \int_{\frac{1}{2}}^{1} \left(x - x^4\right) \mathrm{d}x$$

(4) Once again a split is required, but it will be split along the y interval:

$$\text{Volume} = 2\pi \int_0^{\frac{1}{4}} y\left(2y - y^2\right) \mathrm{d}y + 2\pi \int_{\frac{1}{4}}^{1} y\left(y^{\frac{1}{2}} - y^2\right) \mathrm{d}y$$

<u>SECTION C</u>

(1) The region takes the form:



(2) Exactly the same as Section B.

(3) Exactly the same as Section B.

(4) Exactly the same as Section B.