Review Session Fix

Winter 2019

I provided you with some "fake news" during the review session on Tuesday and need to fix it. For question 15 from homework 5 we have the differential equation:

$$y'' - 6y' + 9y = (12t + 2)e^{3t} + 18t - 3$$

The homogeneous solution has the corresponding characteristic polynomial:

$$m^2 - 6m + 9 = (m - 3)^2 = 0$$

with corresponding solution $m = 3$. It follows that the homogeneous solution is given by:

$$y_h(t) = c_1 e^{3t} + c_2 t e^{3t}$$

Now in order to use the method of undetermined coefficients we guess:

$$\begin{aligned}
y_{nh}(t) &= (At^3 + Bt^2)e^{3t} + (Ct + D) \\
y'_{nh}(t) &= (3At^3 + (3A + 3B)t^2 + 2Bt)e^{3t} + C \\
y''_{nh}(t) &= (9At^3 + (18A + 9B)t^2 + (6A + 12B)t + 2B)e^{3t}
\end{aligned}$$

Plugging all of this in provides:

$$\begin{aligned}
y''_{nh} - 6y'_{nh} + 9y_{nh} &= (12t + 2)e^{3t} + 18t - 3 \\
(6At + 2B)e^{3t} + 9Ct + (9D - 6C) &= (12t + 2)e^{3t} + 18t - 3
\end{aligned}$$

Comparison of both sides forces $A = 2$, $B = 1$, $C = 2$, $D = 1$. What I really want to stress is that the polynomial attached to the exponential in the non-homogeneous guess is specifically of third degree (not second). This occurs because e^{3t} already showed up twice in the homogeneous solution. Thus, if we had a $t^2 e^{3t}$ factor on the right-hand side we would guess $(At^4 + Bt^3 + Ct^2)e^{3t}$ (two degrees higher). Similarly, if we had a $t^3 e^{3t}$ factor on the right-hand side we would guess $(At^5 + Bt^4 + Ct^3 + Dt^2)e^{3t}$. In general if a non-homogeneous component shows up in the homogeneous solution k times, than you guess something k degrees higher.