

Quiz 1 Solutions

MATH 100
October 8, 2018

- (1) (Q) Given $A = \{\emptyset\}$, calculate $\mathcal{P}(A)$ and $\mathcal{P}(\mathcal{P}(A))$ explicitly and use the results to write down $|\mathcal{P}(A)|$ and $|\mathcal{P}(\mathcal{P}(A))|$.

(A) For the first power set we have $|\mathcal{P}(A)| = 2^1 = 2$ with elements:

$$\mathcal{P}(A) = \{\emptyset, \{\emptyset\}\}$$

Using this result we must obtain $|\mathcal{P}(\mathcal{P}(A))| = 2^2 = 4$ with elements:

$$\mathcal{P}(\mathcal{P}(A)) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$$

- (2) (Q) For the following sequences compute $\bigcup_{n \in \mathbb{N}} A_n$ and $\bigcap_{n \in \mathbb{N}} A_n$:

a) $A_n = \left(\frac{1}{n}, e^n\right)$

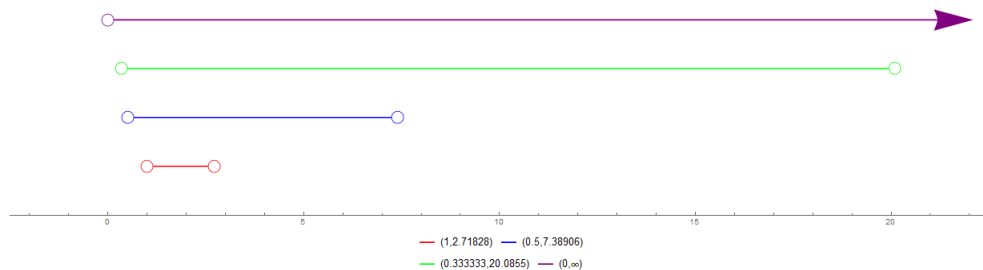
b) $A_n = \left(a - \frac{1}{n}, b\right]$ for any $a, b \in \mathbb{R}$ s.t. $a < b$

(A) To compute the infinite union and intersection it helps to write out the general pattern:

a)

n	Interval
1	$(1, e)$
2	$(\frac{1}{2}, e^2)$
3	$(\frac{1}{3}, e^3)$
\vdots	\vdots
∞	$(0, \infty)$

Since the union will take the biggest collection possible, it has to be that $\bigcup_{n \in \mathbb{N}} A_n = (0, \infty)$. The intersection on the other hand only takes the elements showing up at each step thereby providing $\bigcap_{n \in \mathbb{N}} A_n = (1, e)$. For a visualization you may consider the following image:



b)

n	Interval
1	$(a - 1, b]$
2	$(a - \frac{1}{2}, b]$
3	$(a - \frac{1}{3}, b]$
\vdots	\vdots
∞	$(a, b]$

Since the union will take the biggest collection possible, it has to be that $\bigcup_{n \in \mathbb{N}} A_n = (a - 1, b]$. The intersection on the other hand only takes the elements showing up at each step thereby providing $\bigcap_{n \in \mathbb{N}} A_n = [a, b]$. For a visualization you may consider the following image with $a = 0$ and $b = 1$:

