Group Exercises 3

MATH 11A - Discussion Section F
January 31, 2017

(1) Evaluate the following limits:

a) $\lim_{n\to\infty}\left(2^{-n}+3^{-n}\right)$ b) $\lim_{n\to\infty}\frac{6n+5}{n-7}$ c) $\lim_{n\to\infty}\frac{\sqrt{n}+n^2}{100n-3n^2}$ d) $\lim_{n\to\infty}\frac{e^{3n}-e^{-3n}}{10e^{3n}+e^{-2n}}$ e) $\lim_{n\to\infty}\frac{n^4-8n^2+n}{n^3-n+90}$

(2) Determine whether the following approach negative or positive infinity:

a) $\lim_{n\to 3^+}\frac{e^n}{(n-3)^5}$ b) $\lim_{n\to -4^-}\frac{n+3}{n+4}$ c) $\lim_{n\to 5^+}\frac{n^2-5n}{n^2-10n+25}$ d) $\lim_{n\to 3^+}\ln(n^2-9)$ e) $\lim_{x\to(2\pi)^-} x\csc(x)$

(3) Evaluate the following limits:

a) $\lim_{h\to 0}\frac{(4+h)^2-16}{h}$ b) $\lim_{h\to 0}\frac{(2+h)^3-8}{h}$ c) $\lim_{h\to 0}\frac{\sqrt{1+h}-1}{h}$ d) $\lim_{h\to 0}\left(\frac{1}{h}-\frac{1}{h^2+h}\right)$ e) $\lim_{x\to 16}\frac{4-\sqrt{x}}{16x-x^2}$

(4) Evaluate the following limits:

a) $\lim_{x\to 3}(2x-|x-3|)$ b) $\lim_{x\to -6}\frac{2x+12}{|x+6|}$ c) $\lim_{x\to 0^-}\left(\frac{1}{x}-\frac{1}{|x|}\right)$ d) $\lim_{x\to -2}\frac{2-|x|}{2+x}$ e) $\lim_{x\to 0}\frac{|2x-1|-|2x+1|}{x}$

(5) Is it true that $\frac{x^2+x-6}{x-2}=x+3$ for all $x\in\mathbb{R}$? If not, explain why. Next determine whether $\lim_{x\to 2}\frac{x^2+x-6}{x-2}=\lim_{x\to 2}(x+3)$ is true or not.

(6) Assuming $a_i, b_j\in\mathbb{R}$ for $i\le n$ and $j\le m$, prove the following:

$$\lim_{x\to\infty}\frac{a_0+a_1x+a_2x^2+\cdots+a_nx^n}{b_0+b_1x+b_2x^2+\cdots+b_mx^m}=\begin{cases}0 & n<m\\ \frac{a_n}{b_n} & n=m\\ \infty & n>m\end{cases}$$

(7) Evaluate the following using an analytical approach:

$$\lim_{n\to\infty} n^{\frac{1}{n}}$$

(8) Find numbers $a, b\in\mathbb{R}$ s.t.:

$$\lim_{x\to 0}\frac{\sqrt{ax+b}-2}{x}=1$$

(9) Evaluate the following using an analytical approach:

$$\lim_{x\to\infty}\frac{\sin(x)}{x}$$

(10) The gravitational force exerted by the planet Earth on a unit mass at a distance r from the center of the planet is given by:

$$F(r)=\begin{cases}\frac{GMr}{R^3} & r<R\\ \frac{GM}{r^2} & r\ge R\end{cases}$$

where M is the mass of Earth, R is its radius, and G is the gravitational constant. Is F a continuous function of r?