Quiz 2 Solutions

Section B

(1) Break up the integral into:

$$\int_{-1001}^{1000} x^3 e^{x^2}\ \mathrm{d}x = \int_{-1001}^{-1000} x^3 e^{x^2}\ \mathrm{d}x + \int_{-1000}^{1000} x^3 e^{x^2}\ \mathrm{d}x$$

and use the fact that the integrand is an odd function to deduce that:

$$\int_{-1001}^{1000} x^3 e^{x^2}\ \mathrm{d}x = \int_{-1001}^{-1000} x^3 e^{x^2}\ \mathrm{d}x$$

Notice that the integrand on the right hand side is strictly negative. We also know that $f(x) < 0$ on $[a, b]$ implies $\int_a^b f(x)\ \mathrm{d}x < 0$. Therefore, the answer is option (b).

(2) To determine the displacement calculate:

$$d = \int_0^{2\pi} \cos(t)\ \mathrm{d}t = \sin(t)\Big|_0^{2\pi} = 0$$

and for the distance traveled:

$$\begin{aligned}\int_0^{2\pi} |\cos(t)|\ \mathrm{d}t &= \int_0^{\frac{\pi}{2}} \cos(t)\ \mathrm{d}t - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos(t)\ \mathrm{d}t + \int_{\frac{3\pi}{2}}^{2\pi} \cos(t)\ \mathrm{d}t \\ &= \sin(t)\Big|_0^{\frac{\pi}{2}} - \sin(t)\Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + \sin(t)\Big|_{\frac{3\pi}{2}}^{2\pi} \\ &= (1) - (-2) + (1) \\ &= 4\end{aligned}$$

Therefore, the correct choices are (g) and (b).

(3) (a) Use the trig identity and identify $f(x) = 1$ as an even function:

$$\int_{-2\pi}^{2\pi} (\cos^2(x) + \sin^2(x))\ \mathrm{d}x = \int_{-2\pi}^{2\pi} \mathrm{d}x = 2x\Big|_0^{2\pi} = 4\pi$$

(b) First separate:

$$\int \frac{e^{ix} + e^{-ix}}{2}\ \mathrm{d}x = \frac{1}{2}\Big[\int e^{ix}\ \mathrm{d}x + \int e^{-ix}\ \mathrm{d}x\Big]$$

Now make the substitutions:

$$\begin{aligned} u = ix \quad &\text{and} \quad \frac{\mathrm{d}u}{i} = \mathrm{d}x \\ v = -ix \quad &\text{and} \quad -\frac{\mathrm{d}u}{i} = \mathrm{d}x \end{aligned}$$

Plug in and evaluate:

$$\frac{1}{2}\Big[\int e^{ix}\ \mathrm{d}x + \int e^{-ix}\ \mathrm{d}x\Big] = \frac{1}{2i}\Big[\int e^{u}\ \mathrm{d}u - \int e^{v}\ \mathrm{d}v\Big] = \frac{e^u - e^v}{2i} = \frac{e^{ix} - e^{-ix}}{2i} = \sin(x)$$

Section C

(1) Break up the integral into:

$$\int_{-1001}^{1000} x^3 \sin^2(x)\, \mathrm{d}x = \int_{-1001}^{-1000} x^3 \sin^2(x)\, \mathrm{d}x + \int_{-1000}^{1000} x^3 \sin^2(x)\, \mathrm{d}x$$

and use the fact that the integrand is an odd function to deduce that:

$$\int_{-1001}^{1000} x^3 \sin^2(x)\, \mathrm{d}x = \int_{-1001}^{-1000} x^3 \sin^2(x)\, \mathrm{d}x$$

Notice that the integrand on the right hand side is strictly negative. We also know that $f(x) < 0$ on $[a, b]$ implies $\int_a^b f(x)\, \mathrm{d}x < 0$. Therefore, the answer is option (b).

(2) To determine the displacement calculate:

$$d = \int_{2\pi}^{4\pi} \cos(t)\, \mathrm{d}t = \sin(t)\Big|_{2\pi}^{4\pi} = 0$$

and for the distance traveled:

$$\begin{aligned}\int_{2\pi}^{4\pi} |\cos(t)|\, \mathrm{d}t &= \int_{2\pi}^{\frac{5\pi}{2}} \cos(t)\, \mathrm{d}t - \int_{\frac{5\pi}{2}}^{\frac{7\pi}{2}} \cos(t)\, \mathrm{d}t + \int_{\frac{7\pi}{2}}^{4\pi} \cos(t)\, \mathrm{d}t \\ &= \sin(t)\Big|_{2\pi}^{\frac{5\pi}{2}} - \sin(t)\Big|_{\frac{5\pi}{2}}^{\frac{7\pi}{2}} + \sin(t)\Big|_{\frac{7\pi}{2}}^{4\pi} \\ &= (1) - (-2) + (1) \\ &= 4\end{aligned}$$

Therefore, the correct choices are (g) and (b).

(3) Exactly the same as Section B.