Midterm 1 - Review 1

MATH 11A - Discussion Sections C & F

(1) Determine whether the following sequence, a_n, is convergent or divergent. If it is convergent, find the limit.

  a) $a_n = \frac{1}{n} + \frac{1}{n^2}$
  b) $a_n = \frac{e^n + e^{-n}}{e^{2n} - 1}$
  c) $a_n = 2 - \left(\frac{1}{10}\right)^n$
  d) $a_n = 3^{-n} - 4^{-n}$
  e) $a_n = \frac{n^6 + 1}{n^7 - n^5 + 3n^4 + 9}$

(2) Evaluate the following limits:

  a) $\lim\limits_{x \to 1} \frac{5x^2 - 7x + 2}{x^2 - 1}$
  b) $\lim\limits_{x \to 0} \frac{\sqrt{x^2 + 9} - 3}{x^2}$
  c) $\lim\limits_{x \to 0} \frac{\frac{1}{x+3} - \frac{1}{3}}{x}$
  d) $\lim\limits_{x \to \infty} \frac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}}$
  e) $\lim\limits_{x \to 1} \frac{\sqrt{x+6} - x}{x^3 - x^2}$

(3) Evaluate the following limits:

  a) $\lim\limits_{x \to 1} \ln(\cos(x-1))$
  b) $\lim\limits_{x \to 0} \frac{1 - \cos(2x)}{3x}$
  c) $\lim\limits_{x \to 0} \frac{\csc(x) - \cot(x)}{x \csc(x)}$
  d) $\lim\limits_{x \to \frac{\pi}{2}} \frac{\tan^2(x) + 1}{\sec^2(x)}$
  e) $\lim\limits_{x \to \infty} \left(e^{-x} + 3\cos(2x)\right)$

(4) Evaluate the following limits:

  a) $\lim\limits_{x \to 0} x^4 \sin\left(\frac{1}{x}\right)$
  b) $\lim\limits_{x \to \infty} e^{-x} \cos(10x)$
  c) $\lim\limits_{x \to 0} \frac{\sin^2(3x)}{x^2}$
  d) $\lim\limits_{x \to -4} \frac{\sqrt{x^2 + 9} - 5}{x + 4}$
  e) $\lim\limits_{x \to \left(\frac{\pi}{2}\right)^-} \tan(x)$

(5) Evaluate the following limits:

  a) $\lim\limits_{x \to -1} \frac{x^2 - 6x}{x^2 - 5x - 6}$
  b) $\lim\limits_{h \to 0} \frac{(9+h)^3 - 729}{h}$
  c) $\lim\limits_{h \to 0} \frac{\sqrt{64 + h} - 8}{h}$
  d) $\lim\limits_{x \to 0} \left(\frac{2}{x} - \frac{2}{x^2 + x}\right)$
  e) $\lim\limits_{x \to -1} \frac{10x + 10}{|x + 1|}$

(6) Determine the derivative of each function using the definition of the derivative:

  a) $f(x) = x^2 + 1$
  b) $f(x) = x^3 - 2x$
  c) $f(x) = \sqrt{x}$
  d) $f(x) = \frac{4x + 2}{x + 7}$
  e) $f(x) = \sqrt{1 - 3x}$

(7) Determine an equation of the tangent line to the curve at the given point:

  a) $f(x) = x^3 - 2x + 1$ at $(4, 57)$
  b) $f(x) = \sqrt{x}$ at $(1, 1)$
  c) $f(x) = \frac{x+1}{x}$ at $\left(2, \frac{3}{2}\right)$

(8) Determine all values of a s.t. the following function is continuous everywhere:

$$f(x) = \begin{cases} x^2 + 2x, & x < a \\ -1, & x \geq a \end{cases}$$

(9) Given:

$$g(x) = \frac{x^2 + x - 6}{|x - 2|}$$

Determine the following:

  a) $\lim\limits_{x \to 2^+} g(x)$
  b) $\lim\limits_{x \to 2^-} g(x)$
  c) $\lim\limits_{x \to 2} g(x)$

(10) Determine all values of k s.t. each of the following is a continuous function on $\mathbb{R}$:

  a) $f(x) = \begin{cases} x^3 + k, & x \leq 3 \\ kx - 5, & x > 3 \end{cases}$
  b) $h(x) = \begin{cases} \dfrac{3x^2 + 2x - 8}{x + 2}, & x \neq -2 \\ 3x + k, & x = 2 \end{cases}$