

# Group Exercises 4

MATH 11A - Discussion Section F  
February 14, 2017

(1) Determine the derivative of the following functions using the definition of the derivative.

(a)  $f(x) = x^2 + 5$

(b)  $g(x) = \frac{1}{\sqrt{x+1}}$

(c)  $h(x) = \frac{1}{x} + x$

(d)  $j(x) = \frac{1}{x^2+1}$

(e)  $p(x) = \frac{x+1}{\sqrt{x}}$

(2) Evaluate the following expressions:

(a)  $\frac{d}{dx} \frac{x}{x+1}$

(b)  $\frac{d^2}{dx^2}(ax^2 + bx + c)$  for  $a, b, c \in \mathbb{R}$

(c)  $\frac{d}{dx}(x-2)(x+3)$

(d)  $\frac{d^2}{dx^2}\sqrt{x}(x-14)$

(e)  $\frac{d}{dx}(x-a)(x-b)(x-c)$  for  $a, b, c \in \mathbb{R}$

(3) Using the power rule, evaluate the following expressions:

(a)  $\frac{d}{dx}(3x^2)$

(b)  $\frac{d}{dx}\left(2x^{\frac{1}{2}} + \frac{4}{x^2}\right)$

(c)  $\frac{d}{dx}x^{\pi+1}$

(d)  $\frac{d}{dx}\ln(e^{2x})$

(e)  $\frac{d}{dx}\frac{1}{\pi x}$

(4) Suppose the position of a particle is modeled by the function:

$$f(x) = 2x^2 - 16x + 30$$

At what position is the particle not moving? On what intervals is the distance increasing and decreasing?

(5) Explain why a function that is differentiable at a point is also continuous at that point. Does the reverse hold true?

(6) Using the power rule, prove the following:

$$\frac{d^p}{dx^p}x^p = p! \quad \text{where } p \in \mathbb{N}$$

and  $p! = p(p-1)(p-2)\dots(3)(2)(1)$  is the factorial of  $p$ .

(7) Using the power rule, prove the following:

$$\frac{d^{p+1}}{dx^{p+1}}x^p = 0 \quad \text{where } p \in \mathbb{N}$$

(8) You are given the following power function:

$$g(x) = \sum_{i=0}^n a_i x^i \quad \text{where } a_i \in \mathbb{R}$$

Use this information to determine a formula for  $g'(x)$  using the power rule.

- (9) Using the derivative prove that the vertex of a quadratic function,  $h(x) = ax^2 + bx + c$ , is always the maximal/minimal value attained by the function depending on whether  $a$  is positive or negative.
- (10) Using the derivative find the conditions on  $a, b, c \in \mathbb{R}$  s.t.  $h(x) = ax^3 + bx^2 + cx + d$  attains two real critical numbers (A *critical number* is the value of  $x$  s.t.  $h'(x) = 0$ ).