Calculus 1 with Precalculus
Recitation 4

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FUNCTIONS

Definition 1 (Notation)**.** The motivation behind functions is a way to relate an input to an output. Given some input x, the output is denoted as $f(x)$. The sets of all possible values for x and $f(x)$ are called the **domain** and **range**, respectively. Another common notation is the case where $y = f(x)$. With this format each x and y denote the **independent** and **dependent** variables, respectively.

Definition 2 (Piecewise Functions)**.** In some cases sets of data cannot be correlated by a singular formula. To handle this, there exist piecewise functions that have different formulas for different parts of the domain.

$$f(x) = \begin{cases} x & \text{if } -\infty < x < 0 \\ x^2 & \text{if } 0 \leq x < \infty \end{cases}$$

In the example above, the formula for any negative x is $f(x) = x$, otherwise for positive x the formula is $f(x) = x^2$.

Example 1. *Compute $h(-1)$, $h(0)$, and $h(1)$ where:*

$$h(t) = (2t+1)^3$$

Solution 1.

$$\boxed{\begin{aligned} h(-1) &= (2(-1)+1)^3 = (-1)^3 = -1 \\ h(0) &= (2(0)+1)^3 = (1)^3 = 1 \\ h(1) &= (2(1)+1)^3 = (3)^3 = 27 \end{aligned}}$$

Example 2. *Compute $f(-3)$, $f(0)$, and $f(1)$ where:*

$$f(t) = \frac{1}{\sqrt{3-2t}}$$

Solution 2.

$$\boxed{\begin{aligned} f(-3) &= \frac{1}{\sqrt{3-2(-3)}} = \frac{1}{\sqrt{9}} = \frac{1}{3} \\ f(0) &= \frac{1}{\sqrt{3-2(0)}} = \frac{1}{\sqrt{3}} \\ f(1) &= \frac{1}{\sqrt{3-2(1)}} = \frac{1}{\sqrt{1}} = 1 \end{aligned}}$$

Example 3. *Compute $f(1)$, $f(2)$, and $f(3)$ where:*

$$f(x) = x - |x-2|$$

Solution 3.

$$\boxed{\begin{aligned} f(1) &= (1) - |(1)-2| = 1 - |-1| = 0 \\ f(2) &= (2) - |(2)-2| = 2 - |0| = 2 \\ f(3) &= (3) - |(3)-2| = 3 - |1| = 2 \end{aligned}}$$

Example 4. *Compute $h(-3)$, $h(0)$, $h(1)$, and $h(3)$ where:*

$$h(x) = \begin{cases} -2x+4 & \textit{if } x \leq 1 \\ x^2+1 & \textit{if } x > 1 \end{cases}$$

Solution 4.

$$\boxed{\begin{aligned} h(-3) &= -2(-3)+4 = 10 \\ h(0) &= -2(0)+4 = 4 \\ h(1) &= -2(1)+4 = 2 \\ h(3) &= (3)^2+1 = 10 \end{aligned}}$$

Example 5. *Determine the domain of:*

$$f(t) = \sqrt{1-t}$$

Solution 5. *For the domain we have to make sure that the inside of the square root never reaches a negative value:*

$$\begin{aligned} 1-t &\geq 0 \\ 1 &\geq t \end{aligned}$$

giving the final solution as:

$$\boxed{(-\infty, 1]}$$

Example 6. *Determine the domain of:*

$$g(x) = \frac{x}{1+x^2}$$

Solution 6. *For the domain we have to make sure that the denominator does not evaluate to zero:*

$$\begin{aligned} 1+x^2 &= 0 \\ x^2 &= -1 \\ x &= \pm i \end{aligned}$$

Since we only care about real values, the denominator never evaluates out to zero. Therefore, the domain is:

$$\boxed{(-\infty, \infty)}$$

Example 7. *Determine the domain of:*

$$g(t) = \frac{t+2}{\sqrt{9-t^2}}$$

Solution 7. *For the domain we have to make sure that the denominator does not evaluate to zero and the inside of the square root does not become negative:*

$$\begin{aligned} 9-t^2 &> 0 \\ 9 &> t^2 \\ \sqrt{9} &> \sqrt{t^2} \\ 3 &> |t| \end{aligned}$$

Therefore, the domain is:

$$\boxed{(-3, 3)}$$

Example 8. *Determine the domain of:*

$$f(t) = \frac{t+1}{t^2 - t - 2}$$

Solution 8. *For the domain we have to make sure that the denominator does not evaluate to zero:*

$$\begin{aligned} t^2 - t - 2 &= 0 \\ (t-2)(t+1) &= 0 \\ t &= -1, 2 \end{aligned}$$

Therefore, the domain is:

$$\boxed{(-\infty, -1) \cup (-1, 2) \cup (2, \infty)}$$

Example 9. *Determine the domain of:*

$$h(s) = \sqrt{s^2 - 4}$$

Solution 9. *For the domain we have to make sure that the inside of the square root does not evaluate to a negative value:*

$$\begin{aligned} s^2 - 4 &\geq 0 \\ s^2 &\geq 4 \\ \sqrt{s^2} &\geq \sqrt{4} \\ |s| &\geq 2 \end{aligned}$$

Therefore, the domain is:

$$\boxed{(-\infty, -2] \cup [2, \infty)}$$

Example 10. *Determine the domain of:*

$$f(x) = x^3 - 3x^2 + 2x + 5$$

Solution 10. *There is not a single value of x that will cause the function to become undefined. Therefore, the domain is:*

$$\boxed{(-\infty, \infty)}$$

Definition 3 (Functions in Economics)**.** Here is a list of commonly used functions in economics:

(a) The **Demand Function**, $D(x)$, for the commodity is the price $p = D(x)$ that must be charged for each unit of the commodity if x units are to be sold.

(b) The **Supply Function**, $S(x)$, for the commodity is the unit price $p = S(x)$ at which producers are willing to supply x units to the market.

(c) The **Revenue Function**, $R(x)$, obtained from selling x units of the commodity is given by the product:

$$\begin{aligned} R(x) &= (\text{number of items sold})(\text{price per item}) \\ &= xp(x) \end{aligned}$$

(d) The **Cost Function**, $C(x)$, is the cost of producing x units of the commodity.

(e) The **Profit Function**, $P(x)$, is the profit obtained from selling x units of the commodity and is given by the difference:

$$\begin{aligned} P(x) &= \text{revenue} - \text{cost} \\ &= R(x) - C(x) \\ &= xp(x) - C(x) \end{aligned}$$

(f) The **Average Cost Function** is $AC(x) = \frac{C(x)}{x}$.

(g) The **Average Revenue Function** is $AR(x) = \frac{R(x)}{x}$.

(g) The **Average Profit Function** is $AP(x) = \frac{P(x)}{x}$.