Calculus 2
Review Answer Key for Final

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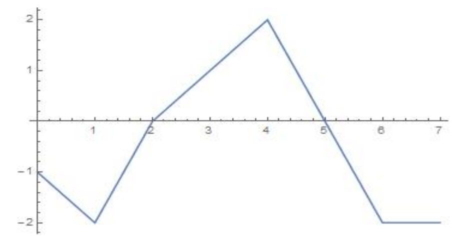
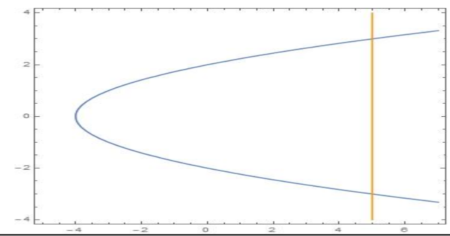
Problem #	Hint	Answer
1.a	Find the respective areas and add	$-\frac{1}{2}$
1.b	$f_{avg} = \frac{1}{b-a}\int_a^b f(x)dx$	$\frac{2}{3}$
1.c	**Area** $= \int_a^b \lvert f(x)\rvert dx$	$4 + \frac{\pi}{2}$
2.a	Find the areas of the two triangles	$\frac{17}{2}$
2.b	Separate the integrals and evaluate	$\frac{25}{2}(4-\pi)$
2.c	Show that the function is odd	0
3.a	$f_{avg} = \frac{1}{b-a}\int_a^b f(x)dx$	III
3.b	$f_{avg} = \frac{1}{b-a}\int_a^b f(x)dx$	I
4.a	Separate and evaluate	-1
4.b	Separate and evaluate	-1
4.c		-3
4.d	Substitute: $u = x - 3$	-3
5.a	$\int_{-a}^a f(x)dx = 0$ if $f(x)$ is odd	-20
5.b	$\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$ if $f(x)$ is even	26
5.c	$\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$ if $f(x)$ is even	$\frac{1}{2}\int_{-7}^7 f(x)dx - \int_5^7 f(x)dx$
6.a	Differentiate by chain rule	$F'(x) = 2xe^{x^2}$
6.b.i	Using $n = 5$	**MID** $= 1.6948$ and **TRAP** $= 1.7656$
6.b.ii	Use 6.a to evaluate	$e - 1$

7.a	$f(a) = f(b) - \int_a^b f'(x)dx$ and $f(b) = f(a) + \int_a^b f'(x)dx$	
7.b	$f_{avg} = \frac{1}{b-a}\int_a^b f(x)dx$	$-\frac{5}{14}$
8.a		x_1
8.b		x_5
8.c		x_3
8.d		Either x_1 or x_5
8.e		x_1
8.f		x_5
9	$H(1) = 16, H(3) = 22$, and $H(5) = 6$	Critical Points: $x = 0, 1, 3, 5$
10	Flip the rate to make sense of the units	**L** $= 0.0147$ min and **R** $= 0.0159$ min
11.a	$\frac{d}{dx}\int_{f(x)}^{g(x)} h(t)dt = h(g(x))g'(x) - h(f(x))f'(x)$	$(1+x^3)^{299}$
11.b	$\frac{d}{dx}\int_{f(x)}^{g(x)} h(t)dt = h(g(x))g'(x) - h(f(x))f'(x)$	$2x(1+x^6)^{299}$
12	$\frac{d}{dx}\int_{f(x)}^{g(x)} h(t)dt = h(g(x))g'(x) - h(f(x))f'(x)$	$\frac{40}{3}$
13.a	Substitution: $u = x^4 + 12x + 100$	$\frac{1}{4}\ln\lvert x^4 + 12x + 100\rvert + C$
13.b	Foil out and evaluate separately	$t - \frac{9}{t} - 6\ln\lvert t\rvert + C$
14.a	Substitution: $u = t - 2$	$\frac{2}{105}(t-2)^{\frac{3}{2}}(15t^2 + 24t + 32)$
14.b	Substitution: $u = -\lambda x$	$-e^{-\lambda x} + C$
15.a	$\sin^2(2x) = 1 - \cos^2(2x)$	$\frac{1}{140}\cos^5(2x)(10\cos^2(2x) - 14)$
15.b	Substitution: $u = -\cos(x)$	$\frac{e-1}{e}$
16.a	Substitution: $u = x + 1$	$2\sqrt{3}$
16.b	Substitution: $u = 1 + x^2$	$\frac{6}{25}$

16.c	Substitution: $u = \ln(x)$	$4 - 2\sqrt{2}$
17.a	Integration by Parts: $u = \ln(x)$ and $dv = \sqrt{x}dx$	$\frac{2}{9}x^{\frac{3}{2}}(3\ln(x) - 2) + C$
17.b	Integration by Parts: $u = \arctan(x)$ and $dv = xdx$	$\frac{1}{2}\Big((x^2+1)\arctan(x) - x\Big) + C$
18.a	Integration by Parts: $u = x$ and $dv = f''(x)dx$	3
18.b.i		$F(a) = \int_0^a x^2 e^{-x} dx$
18.b.ii	Check the first derivative	Monotonically increasing on $(-\infty, \infty)$
18.b.iii	Check the second derivative	Strictly concave up on $(0, 2)$
19.a	Partial Fractions	$\ln\lvert(x+2)^3(1-x)\rvert + C$
19.b	Long Division and Partial Fractions	$\frac{1}{3}x^3 - \frac{1}{2}x^2 + 2x - 2\ln\lvert x+1\rvert + \frac{17}{6} + C$
20.a	Trigonometric Substitution: $y = 4\tan(\theta)$	$\frac{y}{16\sqrt{y^2+16}} + C$
20.b	Trigonometric Substitution: $x = a\sin(\theta)$	$-\frac{\sqrt{a^2-x^2}}{x} - \arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right) + C$
21.a	Substitution: $u = \sin(x)$	$-\frac{5}{64}$
21.b	Substitution: $u = x + 2$	$\ln\left(\frac{3}{2}\right) - \frac{1}{3}$
22.a		$\mathbf{L} = 120, \mathbf{R} = 248$, **MID** $= 176$, and **TRAP** $= 184$
22.b	The function is strictly increasing and concave up	Underestimate: **L** and **MID** Overestimate: **R** and **TRAP**
23.a	Substitution: $u = x + y$	$\frac{1}{b-a+1}\Big((x+b)^{b-a+1} - (x+a)^{b-a+1}\Big)$
23.b	Separate the exponent	$\frac{(x+y)^b}{\ln(x+y)}\Big((x+y)^{-x} - (x+y)^{-y}\Big)$
24.a	Separate and evaluate	$\frac{3}{2} + \ln(2)$
24.b	Separate and evaluate	π
25.a	$f(x) \approx f(1) + f'(1)(x-1)$	$f(0.9) = 4.8$ and $f(1.1) = 5.2$
25.b	$f(x)$ is strictly increasing and concave down	Overestimate: $f(1.1)$ and Overestimate: $f(0.9)$
26		**Area** $= \frac{1}{2}$

27.a	Top Curve - Bottom Curve	**Area** $= \frac{9}{2}$
27.b	Top Curve - Bottom Curve	$\frac{1}{8}(\pi^2 - 8)$
27.c	Top Curve - Bottom Curve	$\frac{80}{3}$
28.a		
28.b	Top Curve - Bottom Curve	**Area** $= \int_{-4}^{5} 2\sqrt{x+4}dx$
28.c	Right Curve - Bottom Curve	**Area** $= \int_{-3}^{3}(9 - y^2)dy$
28.d		**Area** $= 36$
29	Washer Method: $R = \sqrt{x} + 3$ and $r = x^2 + 3$	**Volume** $= \frac{23\pi}{10}$
30.a	Symmetry and Washer Method: $R = 2 - y^2$ and $r = y$	**Volume** $= \frac{76\pi}{15}$
30.b	Symmetry and Washer Method: $R = 4 - y$ and $r = 2 + y^2$	**Volume** $= \frac{68\pi}{5}$
31.a	$h(y) = y^{\frac{1}{3}}$ and $r(y) = y$	**Volume** $= \frac{768\pi}{7}$
31.b	$h(y) = 4 - y^2$ and $r(y) = 3 - y$	**Volume** $= 24\pi$
32.a	Rewrite the ∞ as a limit	Converges: $\frac{\pi}{2}$
32.b	Break up at $x = 5$	Converges: $\frac{5}{9} + \frac{5}{3^{\frac{1}{5}}}$
32.c	Rewrite the ∞ as a limit	Converges: $\frac{\ln(3)}{4}$
33.a	Rewrite the ∞ and 0 as limits	Diverges
33.b	Break up at $x = 5$	Diverges
34.a	$-1 \leq \cos(x) \leq 1$	Converges
34.b	$-1 \leq \cos(x) \leq 1$	Diverges
34.c	Ignore the b	Diverges
35.a	Compare to $\sum_{n=3}^{\infty} \frac{1}{\sqrt{n+2}\ln(n)}$ for Integral Test	Diverges

35.b	Rewrite the ∞ as a limit	Converges
35.c	Ignore the 1	Converges
36.a	$\lim_{n\to\infty} a_n$	Converges: $a_n \to 0$
36.b	$\lim_{n\to\infty} a_n$	Diverges: $a_n \to \infty$
36.c	Squeeze Theorem	Converges: $a_n \to 0$
36.d	Reduce the factorials first	Diverges: $a_n \to \infty$
37.a	p-Series	Converges
37.b	Limit Comparison: $b_n = \frac{1}{n}$	Diverges
37.c	Comparison Test	Diverges
37.d	Limit Comparison: $b_n = \frac{1}{n^{\frac{3}{2}}}$	Converges
38.a	Ratio or Root Test	Converges
38.b	Alternating Series Test	Converges
38.c	Alternating Series Test	Converges
38.d	Root Test	Converges
39.a	Rewrite as a geometric series	$\frac{18}{7}$
39.b	Rewrite as a geometric series	$\frac{1}{2}$
39.c	Rewrite as a geometric series	Diverges
40.a	Ratio Test	$R = 5, [-1, 9)$
40.b	Ratio Test	$R = 1, (0, 2]$
41.a	Use the definition of the taylor series	$\frac{1}{\sqrt{2}} - \frac{x-\frac{\pi}{4}}{\sqrt{2}} - \frac{(x-\frac{\pi}{4})^2}{2\sqrt{2}}$ $+\frac{(x-\frac{\pi}{4})^3}{6\sqrt{2}} + \frac{(x-\frac{\pi}{4})^4}{24\sqrt{2}} - \frac{(x-\frac{\pi}{4})^5}{120\sqrt{2}}$
41.b	Use the definition of the taylor series	$\frac{1}{2} - \frac{x-2}{4} + \frac{(x-2)^2}{8}$ $-\frac{(x-2)^3}{16} + \frac{(x-2)^4}{32} - \frac{(x-2)^5}{64}$
42.a	Evaluate the approximation at 0	$f(0) = 0$
42.b	Differentiate once and evaluate at 0	$f'(0) = 2$

42.c	Differentiate twice and evaluate at 0	$f''(0) = 0$
42.d	Differentiate three times and evaluate at 0	$f^{(3)}(0) = 24$
42.e	Differentiate four times and evaluate at 0	$f^{(4)}(0) = 48$
42.f	Differentiate five times and evaluate at 0	$f^{(5)}(0) = -600$
42.g	Differentiate six times and evaluate at 0	$f^{(6)}(0) = 4320$
43.a	Use the definition of the taylor series	$\sum_{n=0}^{\infty}(-1)^n \frac{(x-\frac{\pi}{2})^{2n}}{(2n)!}$
43.b	Use the definition of the taylor series	$1 + 2\pi\left(x - \frac{1}{4}\right)$ $+2\pi^2\left(x - \frac{1}{4}\right)^2 + \frac{8}{3}\pi^3\left(x - \frac{1}{4}\right)^3$ $+\frac{10}{3}\pi^4\left(x - \frac{1}{4}\right)^4 + \frac{64}{15}\pi^5\left(x - \frac{1}{4}\right)^5 + \dots$
44.a	Expand the $\cos(2x)$	$x^3 - 2x^5 + \frac{2}{3}x^7 - \frac{4}{45}x^9$
44.b	Rewrite the $\cos^2(x)$ using a double angle formula	$1 - x^2 + \frac{1}{3}x^4 - \frac{2}{45}x^6$
45	Expand the sine and exponential terms	-4
46.a	Use the known maclaurin series for $\cos(x)$	$\frac{\sqrt{3}}{2}$
46.b	Use the known maclaurin series for e^x	e^{x^2}
46.c	Use the known maclaurin series for e^x	$\frac{2}{x}(e^{\frac{x}{2}} - 1)$
47.a	Use the known maclaurin series for $\cos(x)$ and e^x	$1 - x - \frac{3}{2}x^2 + \frac{11}{6}x^3$
47.b	Use the known maclaurin series for e^x and $\frac{1}{1-x}$	$1 + 2x + \frac{5}{2}x^2 + \frac{8}{3}x^3$
48.a	Use the known maclaurin series for $\arctan(x)$	$x = \frac{\pi}{4}$
48.b	Use the known maclaurin series for $\cos(x)$	$x = 0, \frac{\pi}{3} + 2\pi n, -\frac{\pi}{3} + 2\pi n$ where $n \in \mathbb{Z}$
49.a	Prove that the function is even	
49.b	Prove that the function is odd	
49.c	Substitution: $u = \frac{x}{\sigma}$	$\sqrt{2\pi}\sigma$
49.d		$\sqrt{2\pi}\sigma^3$