Quiz 6 Solutions

MATH 100
November 19, 2018

(1) (Q) With $a, b, c \in \mathbb{Z}$ prove or disprove that at least one of the numbers $a + b$, $a + c$, and $b + c$ is even.

(A) Given the fact that there are two parities for any given integer, by the Pigeonhole Principle two of the given values have to be either even or odd. Out of the three possible sums, one is guaranteed to be even because:

$$2k + 2l = 2(k + l) \in 2\mathbb{Z}$$
$$(2k + 1) + (2l + 1) = 2(k + l + 1) \in 2\mathbb{Z}$$

for any $k, l \in \mathbb{Z}$.

(2) (Q) Prove that $n! > 2^n$ for $n \in \mathbb{N}$ satisfying $n \geq 4$.

(A) We approach via a proof by induction:

* For the base case take $n = 4$ and note that $4! = 24 > 16 = 2^4$.
* Now assuming that $n! > 2^n$ we aim to show the $n + 1$ case:

$$(n + 1)! = (n + 1) \cdot n! > (n + 1) \cdot 2^n > n \cdot 2^n > 2 \cdot 2^n = 2^{n+1}$$