NYU-Polytechnic School of Engineering
Review Problems for Exam 2 (Calculus 2)

(1) Evaluate each of the following definite integrals.

(a) $\displaystyle\int_0^1 y(y^2+4)^6\,dy$

(b) $\displaystyle\int_0^3 \sqrt{x}(x^2+x+1)\,dx$

(c) $\displaystyle\int_0^3 [|3x-7|+5]\,dx$

(2) Evaluate each of the following definite integrals.

(a) $\displaystyle\int_1^2 (\ln(x))^2\,dx$

(b) $\displaystyle\int_0^1 \frac{y}{e^{2y}}\,dy$

(c) $\displaystyle\int_0^{\pi/3} \tan^5(x)\sec^4(x)\,dx$

(d) $\displaystyle\int_0^{\pi/3} \tan^5(x)\sec^5(x)\,dx$

(e) $\displaystyle\int_0^{\pi/2} \frac{\cos(t)}{\sqrt{\sin^2(t)+1}}\,dt$

(3) Find each of the following integrals.

(a) $\displaystyle\int y^2\ln(y)\,dy$

(b) $\displaystyle\int \ln(3x+1)\,dx$

(c) $\displaystyle\int \arctan(2y)\,dy$

For problems 4-19, evaluate the integrals.

(4) $\displaystyle\int t\sqrt{1-t^4}\,dt$

(5) $\displaystyle\int \frac{t}{\sqrt{16-t^2}}\,dt$

(6) $\displaystyle\int \frac{t^2}{\sqrt{(a^2-t^2)^3}}\,dt$ where a is a positive constant.

(7) $\displaystyle\int \frac{1}{x^2\sqrt{x^2+9}}\,dx$

(8) $\displaystyle\int \frac{\sqrt{x^2-25}}{x}\,dx$

(9) $\displaystyle\int \frac{x}{\sqrt{1-x^4}}\,dx$

(10) $\displaystyle\int \sin^3(x)\cos^8(x)\,dx$

(11) $\displaystyle\int \cos^5(\theta)\,d\theta$

(12) $\displaystyle\int \frac{x-9}{(x+5)(x-2)}\,dx$

(13) $\displaystyle\int \frac{ax}{x^2-bx}\,dx$ where a and b are positive constants.

(14) $\displaystyle\int_0^1 \frac{x^3-4x-10}{x^2-x-6}\,dx$

(15) $\displaystyle\int \frac{x^2+2x-1}{x^3-x}\,dx$

(16) $\displaystyle\int \frac{1}{(x+5)^2(x-2)}\,dx$

(17) $\displaystyle\int \frac{5x^2+3x-2}{x^3+2x^2}\,dx$

(18) $\displaystyle\int \frac{\cos(x)}{\sin^2(x)+\sin(x)}\,dx$

(19) $\displaystyle\int_9^{16} \frac{\sqrt{x}}{x-4}\,dx$

For problems 20-33, determine whether each of the improper integrals converges or diverges. Evaluate those that are convergent.

(20) $\displaystyle\int_1^{\infty} \frac{dx}{1+x}$

(21) $\displaystyle\int_0^{\infty} xe^{-x}\,dx$

(22) $\displaystyle\int_0^{5} \frac{dx}{(x-5)^2}$

(23) $\displaystyle\int_0^{5} \frac{dx}{(x-5)^{2/3}}$

(24) $\displaystyle\int_2^{6} \frac{y}{\sqrt{y-2}}\,dy$

(25) $\displaystyle\int_1^{e^8} \frac{1}{x\sqrt[3]{\ln x}}\,dx$

(26) $\displaystyle\int_{-\infty}^{\infty} \frac{x^2}{9+x^6}\,dx$

(27) $\displaystyle\int_{2}^{\infty} \frac{1}{\sqrt{x+3}}dx$

(28) $\displaystyle\int_{1}^{\infty} \frac{dx}{(3x+1)^2}$

(29) $\displaystyle\int_{0}^{\infty} \frac{e^x}{e^{2x}+3}\,dx$

(30) $\displaystyle\int_{0}^{3} \frac{dx}{x^2-x-2}$

(31) $\displaystyle\int_{-\infty}^{-1} e^{-2t}\,dt$

(32) $\displaystyle\int_{0}^{\infty} x^2 e^{-x^3}\,dx$

(33) $\displaystyle\int_{0}^{1} \frac{\ln(x)}{\sqrt{x}}dx$

(34) The integral $\displaystyle\int_0^\infty \frac{dx}{\sqrt{x}(1+x)}$ is improper for two reasons: The interval $[0,\infty)$ is infinite and the integrand has an infinite discontinuity at 0. Evaluate it by expressing it as a sum of improper integrals of Type 2 and Type 1 as follows:

$$\int_0^\infty \frac{dx}{\sqrt{x}(1+x)} = \int_0^1 \frac{dx}{\sqrt{x}(1+x)} + \int_1^\infty \frac{dx}{\sqrt{x}(1+x)}$$

(35) For a continuous function f, you are given the table below. Use left, right, midpoint, trapezoid, and Simpson's rules with $n=2$ to estimate the value of $\displaystyle\int_0^{20} f(x)\,dx$.

x	0	5	10	15	20
$f(x)$	40	30	22	15	10

(36) Use left, right, midpoint, trapezoid, and Simpson's rules with $n=4$ to estimate the value of $\displaystyle\int_0^{2\pi} t\sin(2t)\,dt$.

(37) Use left and right rules with $n=4$ to estimate the value of $\displaystyle\int_0^2 \sqrt{1+x^4}\,dx$. Discuss if each of the estimates is an under or over-estimate of the exact value.

(38) Use midpoint and trapezoid rules with $n=4$ to estimate the value of $\displaystyle\int_0^2 e^{x^2}\,dx$. Discuss if each of the estimates is an under or over-estimate of the exact value.

(39) For each of the following groups of functions, sketch the region bounded by the curves of the respective functions; represent the area of the region in terms of x; represent the area of the region in terms of y, and finally, find the area of the region.

(a) $x + 4 = y^2$ and $x = 5$

(b) $y = 2x$, $x + y = 9$, and $y = x - 1$

(c) $y = -\sqrt{x}$, $y = x - 6$, and $y = 0$

(40) Sketch the region bounded by the following curve(s) and find its area.

(a) $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, where a and b are positive constants.

(b) $x + y^2 - 4 = 0$, $x + y = 2$.

(c) $y = \cos x$, $y = \sec^2 x$, $x \in [-\pi/4, \pi/4]$

(d) $f(x) = 2\cos(x)$, $g(x) = \sin(2x)$, where $x \in [-\pi, \pi]$.