Group Exercises 3

MATH 11A - Discussion Section C
February 13, 2017

(1) Determine the derivative of the following functions using the definition of the derivative.
   (a) $f(x) = x^2 + 5$
   (b) $g(x) = \frac{1}{\sqrt{x+1}}$
   (c) $h(x) = \frac{1}{x} + x$
   (d) $j(x) = \frac{1}{x^2+1}$
   (e) $p(x) = \frac{x+1}{\sqrt{x}}$

(2) Evaluate the following expressions:
   (a) $\frac{d}{dx}\frac{x}{x+1}$
   (b) $\frac{d^2}{dx^2}(ax^2 + bx + c)$ for $a, b, c \in \mathbb{R}$
   (c) $\frac{d}{dx}(x-2)(x+3)$
   (d) $\frac{d^2}{dx^2}\sqrt{x}(x-14)$
   (e) $\frac{d}{dx}(x-a)(x-b)(x-c)$ for $a, b, c \in \mathbb{R}$

(3) Using the power rule, evaluate the following expressions:
   (a) $\frac{d}{dx}(3x^2)$
   (b) $\frac{d}{dx}\left(2x^{\frac{1}{2}} + \frac{4}{x^2}\right)$
   (c) $\frac{d}{dx}x^{\pi+1}$
   (d) $\frac{d}{dx}\ln(e^{2x})$
   (e) $\frac{d}{dx}\frac{1}{\pi x}$

(4) Suppose the position of a particle is modeled by the function:

$$f(x) = 2x^2 - 16x + 30$$

At what position is the particle not moving? On what intervals is the distance increasing and decreasing?

(5) Explain why a function that is differentiable at a point is also continuous at that point. Does the reverse hold true?

(6) Using the power rule, prove the following:

$$\frac{d^p}{dx^p}x^p = p! \quad \text{where} \quad p \in \mathbb{N}$$

and $p! = p(p-1)(p-2)\ldots(3)(2)(1)$ is the factorial of p.

(7) Using the power rule, prove the following:

$$\frac{d^{p+1}}{dx^{p+1}}x^p = 0 \quad \text{where} \quad p \in \mathbb{N}$$

(8) You are given the following power function:

$$g(x) = \sum_{i=0}^{n} a_i x^i \quad \text{where} \quad a_i \in \mathbb{R}$$

Use this information to determine a formula for $g'(x)$ using the power rule.

(9) Using the derivative prove that the vertex of a quadratic function, $h(x) = ax^2+bx+c$, is always the maximal/minimal value attained by the function depending on whether a is positive or negative.

(10) Using the derivative find the conditions on $a, b, c \in \mathbb{R}$ s.t. $h(x) = ax^3+bx^2+cx+d$ attains two real critical numbers (A *critical number* is the value of x s.t. $h'(x) = 0$).