Week 6 Attendance Solutions

MATH 23A

(1) (Q) Find the equation of the plane that contains the two lines:

$$\overrightarrow{l_1}(t) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} 5 \\ 4 \\ 0 \end{pmatrix} \quad \text{and} \quad \overrightarrow{l_2}(t) = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix} + t \begin{pmatrix} 5 \\ 4 \\ 0 \end{pmatrix}$$

(A) Notice the two given lines are parallel. Thus, we only have one vector on the plane so far. To get another vector on the plane define:

$$\overrightarrow{v_2} = \begin{pmatrix} 5-0 \\ 3-0 \\ 1-1 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \\ 0 \end{pmatrix}$$

Now we construct the normal vector as:

$$\overrightarrow{n} = \overrightarrow{v_1} \times \overrightarrow{v_2} = \det \begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 4 & 0 \\ 5 & 3 & 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -5 \end{pmatrix}$$

This explicitly tells us that the equation takes the form $-5z = d$ where d is still unknown. We now use the fact that the point $(0, 0, 1)$ belongs to the plane forcing $d = -5$ and as a result the plane is given by:

$$z = 1$$

(2) (Q) For the following function calculate the maximal rate of change at the point $(1, 2)$:

$$f(x, y) = x^2 y + y^2$$

(*Calculate the directional derivative in the direction of the function's greatest increase*)

(A) We first calculate the gradient of the function:

$$\nabla f = \begin{pmatrix} 2xy \\ x^2 + 2y \end{pmatrix}$$

The direction of the function's greatest increase is given by:

$$\overrightarrow{u} = \frac{\nabla f}{\|\nabla f\|}\Bigg|_{(1,2)} = \frac{\begin{pmatrix} 4 \\ 5 \end{pmatrix}}{\sqrt{16+25}} = \begin{pmatrix} \frac{4}{\sqrt{41}} \\ \frac{5}{\sqrt{41}} \end{pmatrix}$$

Finally we calculate the maximal rate of change to be:

$$D_{\overrightarrow{u}}(f)\Bigg|_{(1,2)} = \nabla f\Big|_{(1,2)} \cdot \overrightarrow{u} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} \frac{4}{\sqrt{41}} \\ \frac{5}{\sqrt{41}} \end{pmatrix} = \frac{41}{\sqrt{41}} = \sqrt{41}$$