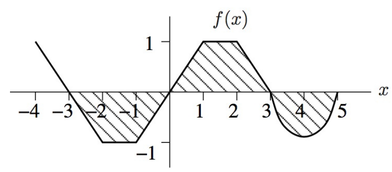
NYU-Polytechnic School of Engineering
MA 1124/1424
Review Problems for Final Exam

(1) Given the following figure:



(a) $\int_{-3}^{2} f(x)dx =$____________

(b) The average value of $f(x)$ over the interval $(0, 3)$ is ____________.

(c) The area of the shaded region is ____________.

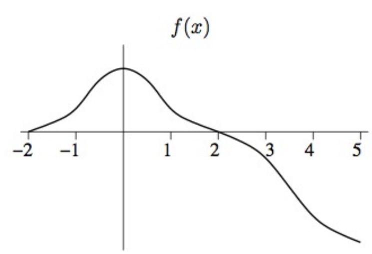
(2) Use geometry or basic properties of integrals to find the exact value of the following integrals.

(a) $\int_{1}^{5} |2x - 7|\, dx =$____________

(b) $\int_{-5}^{5} [5 - \sqrt{25 - x^2}]\, dx =$____________

(c) $\int_{-5}^{5} \cos(x) x^3 e^{-2x^2}\, dx =$____________

(3) For the function f below, consider the average value of f over the following intervals:



I. $0 \le x \le 1$
II. $0 \le x \le 2$
III. $0 \le x \le 5$
IV. $-2 \le x \le 2$

(a) For which interval is the average value of f least?

(b) For which interval is the average value of f greatest?

(4) Let $\int_a^b f(x)dx = -3$, $\int_a^b (f(x))^2 dx = 6$, $\int_a^b g(t)dt = 2$ and $\int_a^b (g(t))^2 dt = 7$. Find the integrals

(a) $\displaystyle\int_a^b (f(x)+g(x))dx$

(b) $\displaystyle\int_b^a ((f(x))^2-(g(x))^2)dx$

(c) $\displaystyle\int_a^b (f(x))^2dx - \left(\int_a^b f(x)dx\right)^2$

(d) $\displaystyle\int_{a+3}^{b+3} f(x-3)dx$

(5) (a) (i) If $f(x)$ is odd and $\displaystyle\int_{-3}^{5} f(x)dx = 20$, find $\displaystyle\int_{5}^{3} f(x)dx$.

(ii) If $f(x)$ is even and $\displaystyle\int_{-6}^{6} (f(x)-5)dx = 44$, find $\displaystyle\int_{0}^{3} f(2x)dx$.

(iii) If $f(x)$ is even, write $\displaystyle\int_{0}^{5} f(x)dx$ in terms of $\displaystyle\int_{-7}^{7} f(x)dx$ and $\displaystyle\int_{5}^{7} f(x)dx$.

(b) The average value of $f(x)$ equals 8 for $4 \le x \le 6$, and equals 7 for $6 \le x \le 12$. What is the average value of $f(x)$ for $4 \le x \le 12$?

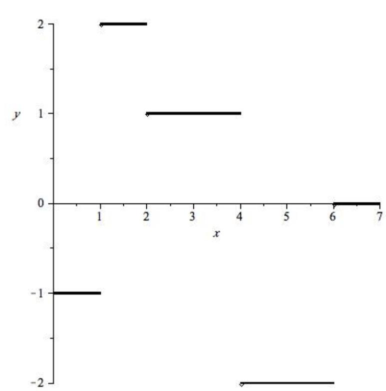
(6) (a) If $F(x) = e^{x^2}$, find $F'(x)$.

(b) Find $\displaystyle\int_0^1 2xe^{x^2}\,dx$ in two ways:

(i) Numerically

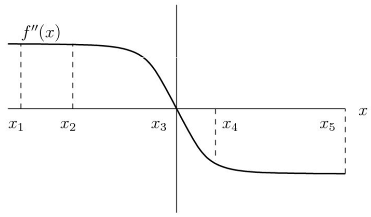
(ii) Using the Fundamental Theorem of Calculus.

(7) Assume $f'(x)$ is given by following graph. Suppose f is continuous and that $f(4) = 2$.



(a) Sketch a graph of $f(x)$.
(b) What is the average value of $f(x)$ on $[0, 7]$?

(8) The graph f'' is given in the following figure. Assume that $f(0) = 0$ and $f'(0) = 0$. Decide at which of the labeled x-values:



(a) $f(x)$ is greatest.

(b) $f(x)$ is least.

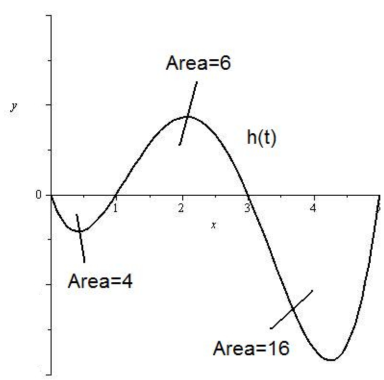
(c) $f'(x)$ is greatest.

(d) $f'(x)$ is least.

(e) $f''(x)$ is greatest.

(f) $f''(x)$ is least.

(9) Sketch a graph of an antiderivative, $H(t)$, of $h(t)$ satisfying $H(0) = 20$. Label each critical point of $H(t)$.



(10) When an aircraft attempts to climb as rapidly as possible, its climb rate decreases with altitude. The table below shows performance data for a certain single-engine aircraft.

Altitude (1000 ft)	0	1	2	3	4	5	6	7	8	9	10
Climb rate (ft/min)	925	875	830	780	730	685	635	585	535	490	440

Calculate upper and lower estimates for the time required for this aircraft to climb from sea level to 10,000 ft.

(11) Use the Fundamental Theorem of Calculus to compute the following:

(a) $\dfrac{d}{dx}\displaystyle\int_{100}^{x}(1+t^3)^{299}dt$

(b) $\dfrac{d}{dx}\displaystyle\int_{100}^{x^2}(1+t^3)^{299}dt$

(12) Find $H'(2)$ given that

$$H(x) = \int_{2x}^{x^3-4} \frac{t}{1+\sqrt{t}} dt.$$

(13) Find each of the following indefinite integrals.

(a) $\displaystyle\int \frac{x^3+3}{x^4+12x+100}\,dx$

(b) $\displaystyle\int \frac{(t-3)^2}{t^2}\,dt$

(14) Find each of the following indefinite integrals.

(a) $\displaystyle\int t^2\sqrt{t-2}\,dt$

(b) $\displaystyle\int \lambda\, e^{-\lambda x}\,dx$ (where λ is a positive constant.)

(15) Find each of the following indefinite integrals.

(a) $\displaystyle\int \cos^4(2x)\sin^3(2x)dx$

(b) $\displaystyle\int_0^{\pi/2} e^{-\cos(x)}\sin(x)\,dx$

(16) Evaluate each of the following definite integrals.

(a) $\displaystyle\int_{-1}^{2} \sqrt{x+1}\,dx$

(b) $\displaystyle\int_0^2 \frac{x}{(1+x^2)^3}\,dx$

(c) $\displaystyle\int_{e^2}^{e^4} \frac{dx}{x\sqrt{\ln x}}$

(17) Evaluate each of the following integrals.

(a) $\displaystyle\int \sqrt{x}\ln(x)\,dx$

(b) $\displaystyle\int x\arctan(x)\,dx$

(18) (a) Let f be twice differentiable with $f(0)=6$, $f(1)=5$, and $f'(1)=2$. Evaluate the integral $\int_0^1 xf''(x)\,dx$.

(b) Let $F(a)$ be the area under the graph of $y=x^2e^{-x}$ between $x=0$ and $x=a$, for $a>0$.

(i) Find a formula for $F(a)$.

(ii) Is F an increasing or decreasing function?

(iii) Is F concave up or concave down for $0 < a < 2$?

(19) Find each of the following integrals.

(a) $\displaystyle\int \frac{4x-1}{(x-1)(x+2)}\,dx$

(b) $\displaystyle\int \frac{x^3+x}{x+1}dx$

(20) Find each of the following integrals.

(a) $\displaystyle\int \frac{dy}{(y^2+16)^{3/2}}$.

(b) $\displaystyle\int \frac{\sqrt{a^2-x^2}}{x^2}\,dx$, where a is a positive constant.

(21) Evaluate each of the following definite integrals.

(a) $\displaystyle\int_{-\pi/3}^{\pi/4} \sin^3(x)\cos(x)\,dx$

(b) $\displaystyle\int_0^1 \frac{x}{x^2+4x+4}dx$

(22) (a) Find Left(4), Right(4), MID(4) and TRAP(4) for $\displaystyle\int_0^8 (x^2+1)\,dx$.

(b) Illustrate your answer in part (a) graphically. Is each approximation an underestimate or overestimate?

(23) Compute the following integrals:

(a) Assume $-x < a < b$, $\int_a^b (x+y)^{b-a}\,dy$.

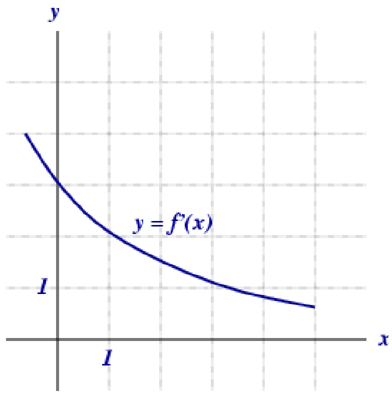
(b) Assume $x+y > 0$, $\int_x^y (x+y)^{b-a}\,da$.

(24) Evaluate each of the following definite integrals.

(a) $\displaystyle\int_1^2 \frac{1+x^2}{x}\,dx$

(b) $\displaystyle\int_0^{\pi/4} \pi(\sin(t)+\cos(t))\,dt$

(25) Suppose that the only information we have about a function is that $f(1) = 5$ and the graph of the *derivative* is as shown.



(a) Use a linear approximation to estimate $f(0.9)$ and $f(1.1)$.

(b) Are your estimates in part (a) larger or smaller than the actual answer? Explain.

(26) Find the area between the graph of f and the x-axis over the given interval.

(a) $f(x) = \sin x$, $x \in [\frac{1}{3}\pi, \frac{1}{2}\pi]$

(27) Find the area of the region bounded by the curves.

(a) $y = 5 - x^2$, $y = 3 - x$
(b) $y = x$, $y = \sin x$, $x = \pi/2$
(c) $y = x^2$, $y = -\sqrt{x}$, $x = 4$

(28) Given the region bounded by the curves $x + 4 = y^2$ and $x = 5$:

(a) Sketch the region.
(b) Represent the area of the region in terms of x.
(c) Represent the area of the region in terms of y.
(d) Find the area of the region.

(29) Given the region bounded by the curves $y = \sqrt{x}$ and $y = x^2$. Find the volume of the solid generated by revolving this region around $y = -3$.

(30) (a) Given the region bounded by the curves $x = |y|$ and $x = 2 - y^2$. Find the volume generated by revolving the region about the y-axis.
(b) Consider the same region. Find the volume generated by revolving the region about the line $x = 4$.

(31) Use the method of cylindrical shells to find the volume of the solid generated by rotating the region bounded by the given curves about the specific axis. Sketch the region and a typical shell might help.

(a) $y = x^3$, $x = 0$, $y = 8$, about x-axis
(b) $y = \sqrt{x - 1}$, $y = 0$, $x = 5$, about $y = 3$

(32) Calculate the improper integrals, if they converge. If diverges, explain why.

(a) $\displaystyle\int_0^\infty \frac{dx}{1+x^2}$

(b) $\displaystyle\int_2^6 \frac{dx}{(x-5)^{4/5}}$

(c) $\displaystyle\int_1^\infty \frac{dx}{(x+5)(x+1)}$

(33) Calculate the improper integrals, if they converge. If diverges, explain why.

(a) $\displaystyle\int_0^\infty \frac{dx}{\sqrt{x}}$

(b) $\displaystyle\int_4^7 \frac{dx}{(5-x)^2}$

(34) Determine if each of the following improper integrals converges or diverges. Use comparison test to justify your conclusion.

(a) $\displaystyle\int_1^\infty \frac{3+\cos(x)}{x^3}\,dx$

(b) $\displaystyle\int_0^1 \frac{3+\cos(x)}{x^3}\,dx$

(c) $\displaystyle\int_1^\infty \frac{dx}{\sqrt{x^2-b}}$, where b is a positive constant.

(35) Decide if the improper integral converges or diverges.

(a) $\displaystyle\int_e^\infty \frac{dx}{\sqrt{x+2}\,\ln(x)}$

(b) $\displaystyle\int_1^\infty \frac{\ln x}{x^2}\,dx$

(c) $\displaystyle\int_6^\infty \frac{x}{\sqrt{1+x^5}}dx$

(36) Determine whether each of the following sequences converges or diverges. If it converges, find its limit.

(a) $a_n = \dfrac{6^n+5}{9^n}$

(b) $a_n = \dfrac{6^n+5}{4^n}$

(c) $a_n = \dfrac{\cos^2(n)}{\ln(n)}$

(d) $a_n = \dfrac{(n+1)!}{(n-1)!}$

(37) Determine whether the series converge or diverge.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3+2}{n^4+n^3}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{2+\cos n}{n\ln(n)}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{\sqrt{n^5+1}}$

(38) Determine whether the series converges or diverges.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n2^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{3n+2}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{3n+5}$

(d) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{5n-1}{7n+3}\right)^n$

(39) Find the sum of each of the following series if it converges.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-6)^{n+1}}{8^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(\sqrt{3})^{2n}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{4^{n+1}+2^{n+1}}{3^n}$

(40) Find the radius of convergence and the interval of convergence of the series.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-4)^n}{n5^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{(x-1)^n}{\sqrt{n}}$

(41) Construct the Taylor polynomial of degree 5 for the function about the point a.

(a) $\cos x$, $a = \pi/4$
(b) $1/x$, $x = 2$

(42) The function $f(x)$ is approximated near $x = 0$ by the sixth degree Taylor polynomial

$$P_6(x) = 2x + 4x^3 + 2x^4 - 5x^5 + 6x^6$$

Give the value of

(a) $f(0)$
(b) $f'(0)$
(c) $f''(0)$
(d) $f^{(3)}(0)$
(e) $f^{(4)}(0)$
(f) $f^{(5)}(0)$
(g) $f^{(6)}(0)$

(43) (a) Find Taylor series of $y = \sin(x)$ at $x = \pi/2$.
(b) Find Taylor series of $y = \tan(\pi x)$ at $x = 1/4$.

(44) Use some known Taylor series to find the first 4 nonzero terms of the Taylor series for $f(x)$ at 0.

(a) $f(x) = x^3 \cos(2x)$
(b) $f(x) = \cos^2(x)$

(45) Use series to evaluate the limit.

$$\lim_{x \to 0} \frac{2x \sin(x)}{1 + x - e^x}$$

(46) Use series to find the sum of the series.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n}(2n)!}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{2^n(n+1)!}$

(47) Find the first four nonzero terms in the Taylor series around $a = 0$ for each of the following functions.

(a) $y = e^{-x}\cos(2x)$

(b) $y = \dfrac{e^x}{1-x}$

(48) Solve exactly for the variable.

(a) $x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + ... = 1$

(b) $x - \dfrac{x^3}{2!} + \dfrac{x^5}{4!} - \dfrac{x^7}{6!} + ... = .5x$

(49) It is known that

$$\int_{-\infty}^{\infty} e^{-\frac{z^2}{2}}\, dz = \sqrt{2\pi}. \qquad (I)$$

(a) Explain why

$$\int_{-\infty}^{0} e^{-\frac{z^2}{2}}\, dz = \int_{0}^{\infty} e^{-\frac{z^2}{2}}\, dz = \sqrt{\frac{\pi}{2}}.$$

(b) Explain why

$$\int_{-\infty}^{\infty} z e^{-\frac{z^2}{2}}\, dz = 0.$$

(c) Given $\sigma > 0$, use a change of variables (substitution) and the formula (I) to calculate

$$\int_{-\infty}^{\infty} e^{-\frac{x^2}{2\sigma^2}}\, dx.$$

(d) Use integration by parts and your result from (c) to calculate

$$\int_{-\infty}^{\infty} x^2 e^{-\frac{x^2}{2\sigma^2}}\, dx.$$