Quiz 2 Solutions

MATH 100
October 15, 2018

(1) (Q) Determine the truth value of each of the following statements:

a) $\forall y \in \mathbb{R}, \exists x \in \mathbb{R}$ s.t. $y = x^2 + 1$

b) $\exists x \in [0, 2]$ s.t. $x^5 - 2x^3 - 2 = 0$

(A) a) We can see that for $y = 0$ there does not exist a $x \in \mathbb{R}$ s.t. $x^2 + 1 = 0$ because $x = \pm i \notin \mathbb{R}$. It follows that the statement has a truth value of **F**.

b) Let $f(x) = x^5 - 2x^3 - 2$. By direct evaluation we see that $f(0) = -2$ and $f(2) = 14$ and so by the Intermediate Value Theorem there exists a $x \in [0, 2]$ s.t. $f(x) = 0$. It follows that the statement has a truth value of **T**.

(2) (Q) For $A = \{1, 2, \ldots, 6\}$ and $B = \{1, 2, \ldots, 7\}$ let $P(x) : 7x + 4$ is odd and $Q(y) : 5y + 9$ is odd where $x \in A$ and $y \in B$. Now define:

$$\mathcal{S} = \{(P(x), Q(y)) \mid x \in A, y \in B, \text{ and } P(x) \implies Q(y) \text{ is false}\}$$

What is $|\mathcal{S}|$?

(A) For the implication to be **F** we want $P(x)$ and $Q(y)$ to be **T** and **F** respectively. By directly plugging in we can see that $P(x)$ takes on the truth value **T** exactly when $x = 1, 3$, and 5. Moreover, for $y = 1, 3, 5$, and 7 we see that $Q(y)$ takes on the truth value **F**. Any pair of the x and y values just mentioned will produce $(\mathbf{T}, \mathbf{F}) \in \mathcal{S}$ which is in fact the only element inside the set as no other pair of truth values will make the implication **F**. Therefore, $|\mathcal{S}| = 1$.