Quiz 5 Solutions

MATH 100
November 5, 2018

(1) (Q) Prove that there exist four distinct real numbers a, b, c, and d such that exactly four of the numbers ab, ac, ad, bc, bd, and cd are irrational.

(A) Consider $a = \sqrt{2}$, $b = \frac{1}{\sqrt{2}}$, $c = \sqrt{3}$, and $d = \frac{1}{\sqrt{3}}$ which provide:

$$ab = cd = 1,\ ac = \sqrt{6},\ ad = \sqrt{\frac{2}{3}},\ bc = \sqrt{\frac{3}{2}},\ \text{and}\ bd = \frac{1}{\sqrt{6}}$$

(2) (Q) For $n \in \mathbb{N}$ prove that:

$$\frac{\mathrm{d}}{\mathrm{d}x}x^n = nx^{n-1}$$

(If using proof by induction you have to prove the base case using the formal definition of the derivative)

(A) We proceed via a proof by induction. For the base case consider $n = 1$ giving:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}x &= \lim_{h\to 0}\frac{(x+h)-x}{h}\\ &= \lim_{h\to 0} 1\\ &= 1\\ &= 1\cdot x^{1-1}\end{aligned}$$

Now we assume the induction hypothesis for n and go on to check if it holds true for $n+1$:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}x^{n+1} &= \frac{\mathrm{d}}{\mathrm{d}x}(x\cdot x^n)\\ &= \Big(\frac{\mathrm{d}}{\mathrm{d}x}x\Big)x^n + x\Big(\frac{\mathrm{d}}{\mathrm{d}x}x^n\Big)\\ &= 1\cdot x^n + x\cdot nx^{n-1}\\ &= (n+1)x^n\end{aligned}$$