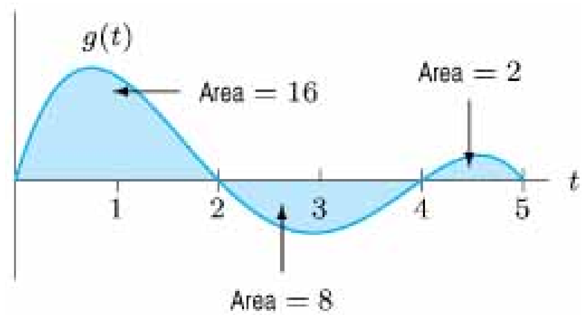
NYU-Polytechnic School of Engineering
MA 1124/1424
Review Problems for Exam 1

(1) Given the graph of the function g as below. Let $G(x) = \int_0^x g(t)\,dt$. Fill in each of the following blanks.



Then $G(5) =$____________, and the average value of g on $[0, 5]$ is ____________.

(2) Let f and g be two continuous functions. Among the following three statements, the correct one(s) is/are ____________. (If all wrong, write NONE.)

(a) $\int_a^b [f(x)g(x)]\,dx = \int_a^b f(x)\,dx \cdot \int_a^b g(x)\,dx$

(b) $\int_a^b f(x^2)\,dx = \int_a^b [f(x)]^2\,dx$

(c) $\int_a^b [f(x)]^2\,dx = \left(\int_a^b f(x)\,dx\right)^2$

(3) Let f and g be two continuous functions. Among the following three statements, the correct one(s) is/are ____________. (If all wrong, write NONE.)

(a) The average value of $f + g$ on the interval $[a, b]$ is the sum of the average value of f on $[a, b]$ and the average value of g on $[a, b]$.

(b) The average value of $f \cdot g$ on the interval $[a, b]$ is the product of the average value of f on $[a, b]$ and the average value of g on $[a, b]$.

(c) The average value of f on the interval $[0, 3]$ is the average of the average value of f on $[0, 1]$ and the average value of f on $[1, 3]$.

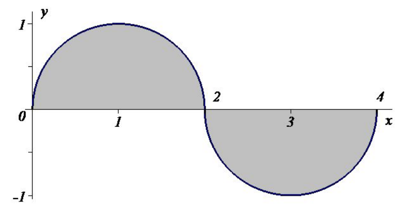
(4) The average value of $f(x)$ equals 8 for $4 \le x \le 6$, and equals 7 for $6 \le x \le 12$. What is the average value of $f(x)$ for $4 \le x \le 12$?

(5) (a) If $f(x)$ is odd and $\int_{-3}^5 f(x)dx = 20$, find $\int_5^3 f(x)dx$.

(b) If $f(x)$ is even and $\int_{-6}^6 (f(x) - 5)dx = 44$, find $\int_0^3 f(2x)dx$.

(c) If $f(x)$ is even, write $\int_0^5 f(x)dx$ in terms of $\int_{-7}^7 f(x)dx$ and $\int_5^7 f(x)dx$.

(6) The graph of function f is shown below. It consists of two semi-circles of radius 1.



Fill in the following blanks.

(a) $\int_0^4 f(x)\,dx =$ ______________;

(b) The average value of f on interval $[0,4]$ is ______________

(c) The area of the shaded region is ______________;

(d) $\int_0^4 |f(x)|\,dx =$ ______________;

(e) $\int_1^5 f(x-1)\,dx =$ ______________;

(f) $\int_0^2 [f(x)]^2\,dx =$ ______________; (Use the fact that f is a semi-circle on $[0,2]$)

(7) Without computing any integrals, explain why the average value of the function $f(x) = \sin(x)$ on $[0,\pi]$ must be between 0.5 to 1.

(8) The velocity $v(t)$, in meters per second, is increasing from $t = 0$ to $t = 10$.

t (sec)	0	2	4	6	8	10
$v(t)$ (meters/sec)	23	37	46	67	91	111

(a) An overestimate of the distance travelled (in meters) from $t = 0$ to $t = 10$ with $\Delta t = 2$ is ______________.

(b) An underestimate of the distance travelled (in meters) from $t = 0$ to $t = 10$ with $\Delta t = 2$ is ______________.

(c) If the velocity was measured every second instead of every two seconds, the difference between upper and lower estimates would be ______________.

(9) If $F(x) = \int_1^x f(t)\,dt$, where $f(t) = \int_1^{t^2} \frac{\sqrt{1+u^2}}{u}\,du$, find $F''(3)$.

(10) Find the interval(s) on which the curve

$$y = \int_0^x \frac{dt}{1+t+t^2}$$

is concave up.

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(11) The error function $g(x)$ is defined as:

$$g(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$

Then $\dfrac{d}{dx}(x^2 g(x))$ is:

(a) $\dfrac{2x}{\sqrt{\pi}}\left(2\displaystyle\int_0^x e^{-t^2}\, dt + xe^{-x^2}\right)$

(b) $\dfrac{4x}{\sqrt{\pi}} \cdot \displaystyle\int_0^x e^{-t^2} dt$

(c) $\dfrac{4x}{\sqrt{\pi}}\left(\displaystyle\int_0^x e^{-t^2}\, dt + xe^{-x^2}\right)$

(d) $x^2 \cdot \dfrac{2}{\sqrt{\pi}} e^{-x^2}$

(e) None of the above

(12) Use the Fundamental Theorem of Calculus to find:

(a) $\dfrac{d}{dx}\left[\displaystyle\int_0^x \ln(t)\, dt\right] =$ ______________________

(b) $\dfrac{d}{dx}\left[\displaystyle\int_{-x^2}^{x^2} e^{t^2}\, dt\right] =$ ______________________

(c) $\dfrac{d}{dx}\left[\displaystyle\int_{\pi}^{x} t^2 \sin(t^2)\, dt\right] =$ ______________________

(d) $\dfrac{d}{dx}\left[\displaystyle\int_{\pi}^{x} x^2 \sin(t^2)\, dt\right] =$ ______________________

(13) If $f(x) = \begin{cases} \sqrt{9-x^2}, & 0 \le x \le 3 \\ 3x - 9, & 3 < x \le 5 \end{cases}$.

Then the average value of $f(x)$ on the interval $[0, 5]$ is ____________.

(14) $\displaystyle\int (t^2\sqrt{t} + \frac{1}{\sqrt{t}} + 2e)\, dt$

(15) $\displaystyle\int_0^3 \left(\frac{x^3}{2} - 4e^x\right)\, dx$

(16) $\displaystyle\int_0^5 |2x-7| + \sqrt{25-x^2}\, dx$

(17) $\displaystyle\int (x+2)^3\, dx$

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(18) $\int \left[\sqrt{x} \cdot (\sqrt[3]{x^2} + \sqrt[3]{x} - 2)\right] dx$

(19) $\int x^3 e^{x^4+5}\, dx$

(20) $\int_0^{\pi/2} e^{\sin(\theta)} \cos(\theta) d\theta$

(21) $\int \frac{t}{\sqrt{t+1}}\, dt$

(22) $\int \frac{3x-2}{\sqrt{2x+1}}\, dx$

(23) $\int \frac{(\ln(x))^2}{x}\, dx$

(24) $\int \frac{\ln(x)}{x^2}\, dx$

(25) $\int (x^2+x)\ln(x)\, dx$

(26) $\int \ln(5x+8)\, dx$

(27) $\int (\ln(x))^2\, dx$

(28) $\int \frac{(t+1)^2}{t^2}\, dt$

(29) $\int_1^2 x e^{x^2}\, dx$

(30) $\int_1^3 \frac{dt}{(t+7)^2}$

(31) $\int \frac{e^x}{1+e^x}\, dx$

(32) $\int \frac{e^x}{1+e^{2x}}\, dx$

(33) $\int \frac{e^x}{a^2+e^{2x}}\, dx$ (a is a non-zero constant)

(34) $\int z^2 e^z\, dz$

(35) $\int_0^1 \frac{z}{e^{2z}}\, dz$

(36) $\int e^{2\theta} \sin(3\theta)\, d\theta$

(37) $\int \arctan(3\theta)\, d\theta$

(38) $\displaystyle\int_0^1 (x^2+1)e^{-x}\,dx$

(39) Let $G(x) = \displaystyle\int_0^x f(t)\,dt$, where f is a continuous function. Some of the values of G and its derivatives are given in the table below.

x	$G(x)$	$G'(x)$	$G''(x)$
-1	1	-3	-2.25
1	-2.25	3	1.5

Evaluate the following definite integral. Write "NEI" if there is not enough information to give the answer and show all your work.

$$\int_{-1}^{1} x^2 f''(x)\,dx.$$

(40) Let f be a twice differentiable function. Some of the values of f and its derivatives are given in the table below.

x	$f(x)$	$f'(x)$	$f''(x)$
0	0	3	-1
1	3.5	2	-0.5
2	4	1.6	2.1

(a) Evaluate the integral $\displaystyle\int_1^2 (2x-1)f''(x)dx$.

(b) Evaluate the integral $\displaystyle\int_0^{\pi/2} \cos(x)f'\big(2\sin(x)\big)\,dx$.