Group Exercises 2

MATH 11A - Discussion Section F
January 24, 2017

(1) Find the exact value of each expression without the use of a calculator:
   (a) $\log_3\left(\frac{1}{27}\right)$
   (b) $\ln\left(\frac{1}{e^k}\right)$ where $k \in \mathbb{R}$
   (c) $e^{-2\ln(5)}$
   (d) $\ln\left(\ln\left(e^{e^{10}}\right)\right)$
   (e) $2\log_2(6) - \log_2(15) + \log_2(20)$

(2) Solve the following for x exactly:
   (a) $e^{7-4x} = 6$
   (b) $\ln(3x - 10) = 2$
   (c) $\ln(x^2 - 1) = 3$
   (d) $e^{2x} - 3e^x + 2 = 0$
   (e) $\ln(\ln(x)) = 1$

(3) Solve the following inequalities for x:
   (a) $\ln(x^2 - 1) > e$
   (b) $e^{(x-1)^2} > 5$
   (c) $1 - 5\ln(x) < 7$
   (d) $1 < e^{3x-1} < 2$
   (e) $\ln|x - 3| \geq 5$

(4) Determine the inverse function of the following, if it exists:
   (a) $f(x) = 1 + \sqrt{2 + 3x}$ where $x \in \left[-\frac{2}{3}, \infty\right)$
   (b) $g(x) = \frac{4x-1}{2x+3}$ where $x \in \left(-\infty, -\frac{3}{2}\right) \cup \left(-\frac{3}{2}, \infty\right)$
   (c) $h(x) = e^{2x-1}$ where $x \in \mathbb{R}$
   (d) $j(x) = \frac{e^x}{1+2e^x}$ where $x \in \mathbb{R}$
   (e) $m(x) = \begin{cases} \sqrt{|x|}, & x \leq 0 \\ -x, & x > 0 \end{cases}$

(5) Find the domain and range of the following functions:
   (a) $f(x) = \frac{2}{3x-1}$
   (b) $g(x) = \sqrt{16 - x^4}$
   (c) $h(x) = \ln(x^2 - 1)$
   (d) $j(x) = 3 + \cos(2x)$
   (e) $m(x) = \left|\frac{x+3}{x^2-1}\right|$

(6) Determine whether the following sequences converge or diverge. If they converge determine the exact value they converge to.

(a) $a_n = \frac{1}{3n^4}$

(b) $a_n = \frac{n^3-1}{n^3+1}$

(c) $a_n = \frac{10^n}{1+9^n}$

(d) $a_n = \ln(2n^2+1) - \ln(n^2+1)$

(e) $a_n = \frac{\sqrt[3]{n}}{\sqrt[4]{n}+\sqrt{n}}$

(7) For the following assume that $a_n \to \mathcal{L}$ and determine the value of $\mathcal{L}$ exactly:

(a) $a_{n+1} = \frac{1}{2}a_n + 1$ where $a_1 = 1$

(b) $a_{n+1} = 2a_n - 1$ where $a_1 = 2$

(c) $a_{n+1} = \sqrt{5a_n}$ where $a_1 = 1$

(d) $a_{n+1} = \frac{6}{1+a_n}$ where $a_1 = 1$

(e) $a_{n+1} = \frac{1}{2}\left(a_n + \frac{25}{a_n}\right)$ where $a_1 = 100$

(8) Calculate the following:

(a) $\lim_{n\to\infty} \frac{\sqrt{n}+n^2}{2n-n^2}$

(b) $\lim_{x\to\infty} \frac{(2x^2+1)^2}{(x-1)^2(x^2+x)}$

(c) $\lim_{t\to\infty}(\sqrt{9t^2+t} - 3t)$

(d) $\lim_{x\to-\infty}[\ln(x^2) - \ln(x^2+1)]$

(e) $\lim_{x\to\infty} \frac{e^{3x}-e^{-3x}}{e^{3x}+e^{-3x}}$

(9) Evaluate the following limits, if they exist:

(a) $\lim_{x\to5} \frac{x^2-6x+5}{x-5}$

(b) $\lim_{h\to0} \frac{(4+h)^2-16}{h}$

(c) $\lim_{x\to0} \left(\frac{1}{x} - \frac{1}{x^2+x}\right)$

(d) $\lim_{x\to-2} \frac{x+2}{x^3+8}$

(e) $\lim_{x\to0} \frac{\sqrt{1+x}-1}{x}$

(10) In the context of *Classical Mechanics* as first described by Sir Isaac Newton, *momentum* is defined as $p = mv$ where m and v are the mass and velocity respectively of the object of interest. In the context of *Special Relativity*, momentum is defined as:

$$p^* = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$$

where c is the speed of light. What happens as $v \to c^-$? [1]

[1]This example lays out the very difference between classical mechanics and special relativity. Sir Isaac Newton never accounted for a maximal speed, but Albert Einstein defined it to be the speed of light.