Calculus 1 with Precalculus
Recitation 10

Nathan Marianovsky

Lines and Linear Functions

Definition 1 (Slope of a Line)**.** Given any two points $(x_0, y_0), (x_1, y_1) \in \mathbb{R}^2$, the slope of the line connecting the two points is defined as:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_1 - y_0}{x_1 - x_0}$$

Definition 2 (Slope-Intercept Form of a Line)**.** A line in $\mathbb{R}^2$ can be formally written down as:

$$y = mx + b$$

where m is the slope and b is the y-intercept.

Definition 3 (Point-Slope Form of a Line)**.** Another way to identify a line, is to use the form:

$$y - y_0 = m(x - x_0)$$

where m is the slope and (x_0, y_0) is any point lying on the line.

Definition 4 (Standard Form of a Line)**.** The most general way to write down the equation for a line is given by:

$$Ax + By = C$$

where $A, B, C \in \mathbb{R}$. Even though this form does not provide an easy way to read off the slope or intercept right away, it does provide an easy way to generalize to higher dimensions.

Definition 5 (Some Special Lines)**.** Given $C \in \mathbb{R}$ and the standard xy-plane, two lines of interest include:

(a) A horizontal line which is always in the form of:

$$y = C$$

(b) A vertical line which is always in the form of:

$$x = C$$

Definition 6 (Parallel and Perpendicular Lines)**.** Given $L_1, L_2 \in \mathbb{R}^2$:

(a) $L_1 \parallel L_2$ iff $m_1 = m_2$ where m_i represents the slope of the line L_i $\forall i \in \{1, 2\}$.
(b) $L_1 \perp L_2$ iff $m_1 = -\frac{1}{m_2}$ where m_i represents the slope of the line L_i $\forall i \in \{1, 2\}$.

Example 1. *Determine the slope through the points* $(2,6)$ *and* $(2,-4)$.

Solution 1.

$$m = \frac{\Delta y}{\Delta x} = \frac{6-(-4)}{2-2} = \frac{10}{0} = \boxed{\infty}$$

Since the slope is infinite, this represents that a vertical line connects the two points.

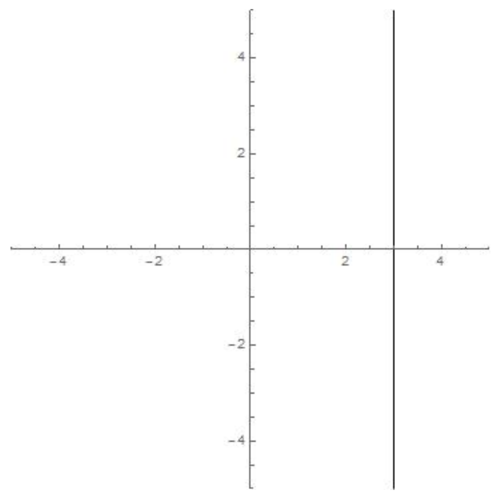
Example 2. *Determine the slope through the points* $(-1,2)$ *and* $(2,5)$.

Solution 2.

$$m = \frac{\Delta y}{\Delta x} = \frac{5-2}{2-(-1)} = \frac{3}{3} = \boxed{1}$$

Example 3. *Graph the equation of the line and determine the slope and intercepts of* $x = 3$.

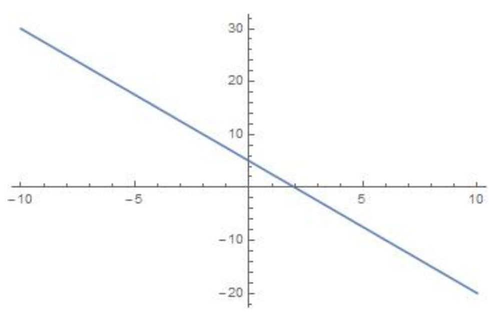
Solution 3. *The graph is*



The slope is $\boxed{m = \infty}$ *and the* x*-intercept is given by* $\boxed{(3,0)}$.

Example 4. *Graph the equation of the line and determine the slope and intercepts of* $\frac{x}{2} + \frac{y}{5} = 1$.

Solution 4. *The graph is*



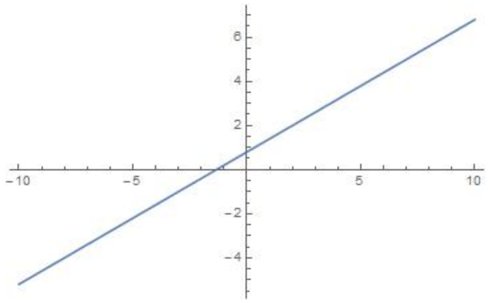
The equation of the line can be rewritten as:

$$y = -\frac{5}{2}x + 5$$

which implies that $\boxed{m = -\frac{5}{2}}$, *the* y*-intercept is* $\boxed{(0,5)}$, *and the* x*-intercept is* $\boxed{(2,0)}$.

Example 5. *Graph the equation of the line and determine the slope and intercepts of* $5y - 3x = 4$.

Solution 5. *The graph is*



The equation of the line can be rewritten as:

$$y = \frac{3}{5}x + \frac{4}{5}$$

which implies that $\boxed{m = \frac{3}{5}}$, *the* y*-intercept is* $\boxed{\left(0, \frac{4}{5}\right)}$, *and the* x*-intercept is* $\boxed{\left(-\frac{4}{3}, 0\right)}$.

Example 6. *Write an equation for the line going through* $(2, 0)$ *with slope* 1.

Solution 6. *Given the slope and a single point, the easiest way to write down an equation is to use the Point-Slope form:*

$$y - 0 = (1)(x - 2) \implies \boxed{y = x - 2}$$

Example 7. *Write an equation for the line going through* $(-1, 2)$ *with slope* $\frac{2}{3}$.

Solution 7. *Given the slope and a single point, the easiest way to write down an equation is to use the Point-Slope form:*

$$\boxed{y - 2 = \frac{2}{3}(x + 1)}$$

Example 8. *Write an equation for the line going through* $(2, 5)$ *and* $(1, -2)$.

Solution 8. *First, find the slope:*

$$m = \frac{\Delta y}{\Delta x} = \frac{5 + 2}{2 - 1} = 7$$

With the slope determined, there are a few choices to go with. Just to switch up, lets use the Slope-Intercept form:

$$y = 7x + b$$

To determine the y*-intercept use any of the given points:*

$$5 = 7(2) + b \implies b = -9$$

With this the final equation is written down as:

$$\boxed{y = 7x - 9}$$

Example 9. *Write an equation for the line going through* $(3, 5)$ *and parallel to the line* $x + y = 4$.

Solution 9. *Since it has to be parallel, the slope must be the same. Therefore,* $m = -1$. *Next to write down the equation of the line, use the Point-Slope form:*

$$\boxed{y - 5 = -(x - 3)}$$

Example 10. *Write an equation for the line going through* $(-2, 3)$ *and perpendicular to the line* $x + 3y = 5$.

Solution 10. *Since it has to be perpendicular, the slope must be the negative reciprocal. Therefore,* $m = 3$. *Next to write down the equation of the line, use the Point-Slope form:*

$$\boxed{y - 3 = 3(x + 2)}$$