

# Group Exercises 1

MATH 11A - Discussion Section F  
January 19, 2017

- (1) For each function transformation graph the result when acted on  $f(x)$  and write down its corresponding formula given  $f(x) = e^x$ :
- (a) Shift down by 3
  - (b) Shift right by 7
  - (c) Reflect about the  $x$ -axis
  - (d) Reflect about the  $y$ -axis
  - (e) Reflect about the origin
- (2) Find the domain of the following functions:
- (a)  $f(x) = \frac{7-x}{e^x-1}$
  - (b)  $g(x) = \frac{e^x}{x^2-x-1}$
  - (c)  $h(x) = \frac{x^\pi-x+1}{x^3+x^2-x}$
  - (d)  $j(x) = \frac{1}{e^{x^2+2x-1}}$
  - (e)  $m(x) = \frac{x+\pi}{e^{\sin(x)}}$
- (3) The half-life of Uranium-238 is  $4.51 \times 10^9$  years. Given that we start with a sample of 500mg, answer the following:
- (a) Write down a formula that models how much of the sample is left after  $t$  years.
  - (b) How much of the sample remains after  $t = 10^{20}$  years?
  - (c) At what value of  $t$  does the sample have 200mg left?
- (4) Determine whether the following functions are one-to-one (injective) on the provided domains:
- (a)  $f(x) = \cos(x)$  where  $x \in \mathbb{R}$
  - (b)  $g(x) = \cos(x)$  where  $x \in [0, \pi]$
  - (c)  $h(x) = \frac{x^2-x+7}{x-1}$  where  $x \in (-\infty, 1) \cup (1, \infty)$
  - (d)  $j(x) = \ln(x)$  where  $x \in (0, \infty)$
  - (e)  $m(x) = \ln|x|$  where  $x \in \mathbb{R}$
  - (f)  $n(x) = \frac{x}{|x|}$  where  $x \in (-\infty, 0) \cup (0, \infty)$
- (5) Determine a formula for the inverse, if it exists:
- (a)  $f(x) = \frac{2x-1}{5x+3}$  where  $x \in \left(-\infty, -\frac{3}{5}\right) \cup \left(-\frac{3}{5}, \infty\right)$
  - (b)  $g(x) = e^{x-3}$  where  $x \in \mathbb{R}$
  - (c)  $h(x) = e^{(7-x)^3+10}$  where  $x \in \mathbb{R}$
  - (d)  $j(x) = \ln(x^3+4)$  where  $x \in \mathbb{R}$
  - (e)  $m(x) = \ln(x^2)$  where  $x \in \mathbb{R}$
  - (f)  $n(x) = x^2$  where  $x \in \mathbb{R}$
  - (g)  $p(x) = x^2$  where  $x \in [0, \infty)$
- (6) Graph the following piecewise functions and determine if they are continuous:

a)  $f(x) = \begin{cases} |x|, & x < 0 \\ x^2, & x \geq 0 \end{cases}$

b)  $g(x) = \begin{cases} x+1, & x < 0 \\ x+10, & x \geq 0 \end{cases}$

c)  $h(x) = \begin{cases} x+1, & x < 0 \\ 0, & x = 0 \\ 1-x, & x > 0 \end{cases}$

- (7) Simplify the following expressions in terms of a single logarithm:
- (a)  $7 \log_{12}(x) + 21 \log_{12}(y)$
  - (b)  $3 \log(x) - 6 \log(y)$
  - (c)  $5 \ln(x + y) - 21 \ln(x) - 8 \ln(y)$
  - (d)  $\log_7(x) + \ln(y)$
  - (e)  $\ln(y^2) - \ln(x) + 2 \log(y)$
- (8) Determine whether the following are true or false:
- (a) If  $x \in \mathbb{R}$ , then  $\sqrt{x^2} = x$ .
  - (b) If  $f$  and  $g$  are two continuous functions, then  $f \circ g = g \circ f$ .
  - (c) For any given function  $f$  we have  $f(x + y) = f(x) + f(y)$ .
  - (d) Given that  $f$  is a linear function, then we must have  $f(\alpha x + \beta y) = \alpha f(x) + \beta f(y)$ .
  - (e) If  $x < y$  and  $f$  is a strictly decreasing function, then  $f(x) > f(y)$ .
  - (f) If  $\alpha, x \in \mathbb{R}$ , then  $\ln(x^\alpha) = \alpha \ln(x)$ .
- (9) Determine a formula for the general term  $a_n$  given the following sequences:
- (a)  $\{1, -1, 1, -1, 1, -1, \dots\}$
  - (b)  $\{2, 3, 2, 3, 2, 3, \dots\}$
  - (c)  $\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \dots\}$
  - (d)  $\{1, -\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}, \frac{1}{81}, \dots\}$
  - (e)  $\{1, 1.9, 1.99, 1.999, 1.9999, 1.99999, \dots\}$
  - (f)  $\{\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \dots\}$
- (10) Determine a formula for each situation:
- (a) Linear function that passes through  $(-2, 4)$  and  $(2, 0)$ .
  - (b) Exponential function that passes through  $(-1, 10)$  and  $(2, 30)$ .
  - (c) Linear function that has slope  $m = 5$  and passes through  $(0, 10)$ .
  - (d) Quadratic function that has roots at  $x = -3$  and  $x = 3$  while passing through  $(10, 10)$ .
  - (e) Exponential function that has a horizontal asymptote of  $y = -1$  and passes through  $(1, 1)$  and  $(5, 10)$ .