Quiz 3 Solutions

MATH 100
October 22, 2018

(1) (Q) Find all $n \in \mathbb{Z}$ such that $3n+7$ is divisible by 11.

(A) The given condition can be rewritten as $3n + 7 \equiv 0 \pmod{11}$. It now follows that:

$$\begin{aligned} 3n+7 &\equiv 0 \pmod{11} \\ 3n &\equiv -7 \pmod{11} \\ 3n &\equiv 4 \pmod{11} \\ 12n &\equiv 16 \pmod{11} \\ n &\equiv 5 \pmod{11} \end{aligned}$$

Reading off the result tells us that $n = 11k + 5$ for any $k \in \mathbb{Z}$.

(2) (Q) Let $\varepsilon \in \mathbb{R}_{>0}$ and $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n \in \mathbb{R}$. Given $|x_1 - y_1| < \varepsilon$, $|x_2 - y_2| < \varepsilon^2$, $\ldots$, and $|x_n - y_n| < \varepsilon^n$ prove that:

$$\left| \sum_{i=1}^{n} x_i - \sum_{j=1}^{n} y_j \right| < \frac{\varepsilon(1-\varepsilon^n)}{1-\varepsilon}$$

(A) The given difference of sums can be rewritten as:

$$\left| \sum_{i=1}^{n} x_i - \sum_{j=1}^{n} y_j \right| = \left| \sum_{i=1}^{n} (x_i - y_i) \right|$$

Using the triangle inequality in conjunction with the geometric series we arrive at:

$$\begin{aligned} \left| \sum_{i=1}^{n} (x_i - y_i) \right| &\leq \sum_{i=1}^{n} |x_i - y_i| \\ &< \sum_{i=1}^{n} \varepsilon^i \\ &= \varepsilon \sum_{i=0}^{n-1} \varepsilon^i \\ &= \frac{\varepsilon(1-\varepsilon^n)}{1-\varepsilon} \end{aligned}$$