Quiz 7 Solutions

MATH 100
November 26, 2018

(1) (Q) Prove or disprove the following: Every even integer can be expressed as the sum of three distinct even integers.

(A) For any even integer $n \in \mathbb{Z}$ it follows that $n+2, n-2$, and $-n$ are distinct even integers. Now we can construct the original even integer out of the following sum:

$$n = (n+2) + (n-2) + (-n)$$

(2) (Q) Let $A = \{a, b, c, d\}$. How many relations defined on A are reflexive, symmetric, and transitive while containing the ordered pairs (a, b), (b, c), and (c, d)?

(A) By design we know that there are a total of 16 pairs inside $A \times A$. Choosing (a, b), (b, c), and (c, d) to be included forces the following:

* For the relation to be reflexive we must include the pairs (a, a), (b, b), (c, c), $(d, d) \in \mathrm{R}$.
* For the relation to be symmetric we must include the pairs (b, a), (c, b), $(d, c) \in \mathrm{R}$.
* For the relation to be transitive we must include (a, c), (a, d), (b, d), (d, b), (d, a), $(c, a) \in \mathrm{R}$.

The above tells us that $|\mathrm{R}| \geq 16$. This minimal value corresponds exactly to the unique case in which $\mathrm{R} = A \times A$.