

# Calculus 2

## Review Answer Key for Exam 2

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Problem #	Hint	Answer
1.a	$u = y^2 + 4$	$\frac{61741}{14}$
1.b	Distribute and use power rule	$2\left(\frac{3^{\frac{7}{2}}}{7} + \frac{3^{\frac{5}{2}}}{5} + \sqrt{3}\right)$
1.c	Separate the integral and use geometry to evaluate	$\frac{137}{6}$
2.a	$u = \ln^2(x)$ and $dv = dx$	$2 \ln^2(2) - 4 \ln(2) + 2$
2.b	$u = y$ and $dv = e^{-2y} dy$	$\frac{1}{4} - \frac{3}{4}e^{-2}$
2.c	Use the fact that $\sec^2(x) = 1 + \tan^2(x)$	$\frac{117}{8}$
2.d	Use the fact that $\tan^2(x) = \sec^2(x) - 1$	$\frac{8408}{315}$
2.e	$u = \sin(t)$ followed by $u = \tan(\theta)$	$\ln(\sqrt{2} + 1)$
3.a	$u = \ln(y)$ and $dv = y^2 dy$	$\frac{y^3}{3} \ln(y) - \frac{y^3}{9} + C$
3.b	$u = \ln(3x + 1)$ and $dv = dx$	$x \ln(3x + 1) - \frac{1}{3}(3x - 1)$ $+ \frac{1}{3} \ln  3x + 1  + C$
3.c	$u = \arctan(2y)$ and $dv = dy$	$y \arctan(2y) - \frac{1}{4} \ln  4y^2 + 1  + C$
4	$u = t^2$ followed by $u = \sin(\theta)$	$\frac{1}{4} \left( \arcsin(t^2) + t^2 \sqrt{1 - t^4} \right) + C$
5	$t = 4 \sin(\theta)$	$-\sqrt{16 - t^2} + C$
6	$t = a \sin(\theta)$	$\frac{t}{\sqrt{a^2 - t^2}} - \arcsin\left(\frac{t}{a}\right) + C$
7	$x = 3 \tan(\theta)$	$-\frac{\sqrt{x^2 + 9}}{9x} + C$
8	$x = 5 \sec(\theta)$	$\sqrt{x^2 - 25} - 5 \arccos\left(\frac{5}{x}\right) + C$
9	$u = x^2$ followed by $u = \sin(\theta)$	$\frac{1}{2} \arcsin(x) + C$
10	Use the fact that $\sin^2(x) = 1 - \cos^2(x)$	$-\frac{\cos^9(x)}{9} + \frac{\cos^{11}(x)}{11} + C$
11	Use the fact that $\cos^2(x) = 1 - \sin^2(x)$	$\sin(\theta) - \frac{2}{3} \sin^3(\theta) - \frac{1}{5} \sin^5(\theta) + C$

12	Partial fractions	$\ln \left  \frac{(x+5)^2}{x-2} \right  + C$
13	Simplify the fraction and integrate	$a \ln  x - b  + C$
14	Long division and partial fractions	$\frac{3}{2} - \ln(2) + \ln(3)$
15	Partial fractions	$\ln \left  \frac{x(x-1)}{x+1} \right  + C$
16	Partial fractions	$\frac{1}{49} \ln \left  \frac{x-2}{x+5} \right  + \frac{1}{7x+35} + C$
17	Partial fractions	$\ln  x^2(x+2)^3  + \frac{1}{x} + C$
18	$u = \sin(x)$ followed by partial fractions	$\ln \left  \frac{\sin(X)}{\sin(x)+1} \right  + C$
19	$u = \sqrt{x}$ , long division, and partial fractions	$2 + 2 \ln \left( \frac{5}{3} \right)$
20	Turn the $\infty$ into a limit	Diverges
21	Turn the $\infty$ into a limit	1
22	Turn the 5 on the boundary into a limit	Diverges
23	Turn the 5 on the boundary into a limit	$3(5)^{\frac{1}{3}}$
24	Turn the 2 on the boundary into a limit	$\frac{40}{3}$
25	Turn the 1 on the boundary into a limit	6
26	Turn the infinities into limits	$\frac{\pi}{9}$
27	Turn the $\infty$ into a limit	Diverges
28	Turn the $\infty$ into a limit	$\frac{1}{12}$
29	Turn the $\infty$ into a limit	$\frac{\pi}{3^{\frac{3}{2}}}$
30	Break at $x = 2$ and evaluate with limits on the 2's	Diverges
31	Turn the $-\infty$ into a limit	Diverges
32	Turn the $\infty$ into a limit	$\frac{1}{3}$
33	Turn the 0 on the boundary into a limit	-4
34	Break at $x = 1$ and evaluate with limits on the 1's	$\pi$
35		$\mathbf{L}_2 = 620, \mathbf{R}_2 = 320, \mathbf{T}_2 = 470$ $\mathbf{M}_2 = 450, \mathbf{S}_2 = 460$

36		$\mathbf{L}_4 = 0, \mathbf{R}_4 = 0, \mathbf{T}_4 = 0$ $\mathbf{M}_4 = -\frac{\pi^2}{2}, \mathbf{S}_2 = 0$
37	Function is strictly increasing: $\mathbf{L}_n \leq \mathbf{R}_n$	$\mathbf{L}_4 = 2.95, \mathbf{R}_4 = 4.51$
38	Function is strictly concave up: $\mathbf{M}_n \leq \mathbf{T}_n$	$\mathbf{T}_4 = 20.65, \mathbf{M}_4 = 14.49$
39.a	Right - Left	36
39.b	Top - Bottom, split into two regions	12
39.c	Right - Left	$\frac{22}{3}$
40.a	Use symmetry	$\pi ab$
40.b	Right - Left	$\frac{9}{2}$
40.c	Top - Bottom	$2 - \sqrt{2}$
40.d	Break into three different regions	8