# NYU - Polytechnic School of Engineering
# MA 1124/1424
# Review Problems for Exam 3

(1) Given the region bounded by the curves $y = \sqrt{x}$ and $y = x^3$. Find the area of the region, and also the volume of the solid generated by revolving this region around the line $y = -3$.

(2) Given the region bounded by the curves $y = \sqrt{x}$ and $y = x^3$. Find the volume of the solid generated by revolving this region around the $x$-axis.

(3) Given the region bounded by the curves $x + y = 3$, $2x + y = 6$, and $x = 0$. Find the area of the region, and also the volume of the solid generated by revolving this region about the line $x = \pi$.

(4) Show that the volume of a pyramid whose base is a square with side $a$ and whose height is $h$ is $a^2h/3$.

**For problems 5-8, use the method of cylindrical shells to find the volume of the solid generated by rotating the region bounded by the given curves about the specific axis. A sketch of the region and a typical shell might help.**

(5) $y = 4x^2$, $\quad 2x + y = 6$ $\quad$ about $x$-axis

(6) $x + y = 3$, $\quad x = 4 - (y - 1)^2$, $\quad$ about $x$-axis

(7) $x = y^2$, $\quad y = x^2$, $\quad$ about $y = -1$

(8) $y = x^2$, $\quad y = 0$, $\quad x = -2$, $\quad x = -1$, $\quad$ about the $y$-axis

(9) Find the exact arc length of the following curves between given points.

(a) $y = x^{3/2} + 4$ from $x = 0$ to $x = 5$.

(b) $y = x^2/2$ from $x = 0$ to $x = 2$.

(10) Find the exact arc length of $y = \sqrt{9 - x^2}$ between $x = 0$ to $x = 2$.

**For problems 11-12, find a formula for the general term $a_n$ of the sequence, assuming that the pattern of the first few terms continues.**

(11) $\left\{-\frac{1}{4}, \frac{2}{9}, -\frac{3}{16}, \frac{4}{25}, -\frac{5}{36} \cdots\right\}$

(12) $\{0, \pi, 0, \pi, 0, \pi, ...\}$

**For problems 13-16, determine whether the sequence converges or diverges. If it converges, find the limit.**

(13) $a_n = \dfrac{\sqrt[4]{n} + \sqrt[5]{n}}{\sqrt{n} + \sqrt{6}n}$

(14) $a_n = \dfrac{4^n - 1}{4^n}$

(15) $a_n = \cos(n\pi)$

(16) $a_n = \dfrac{6^n}{(n+1)^5}$

**For problems 17-20, find the sum of each infinite geometric series, if it converges.**

(17) $\displaystyle\sum_{n=0}^{\infty} \frac{7^n + 5}{9^n}$

(18) $\displaystyle\sum_{n=1}^{\infty} 4^{-n} 7^{n+1}$

(19) $\displaystyle\sum_{n=1} 5\left(\frac{e}{4}\right)^n$

(20) $\displaystyle\sum_{k=0} \frac{2^4}{8^k}$

**For problems 21-25, determine if each of the statement is True or False. Explain or give a counter example.**

(21) If $-1 < a < 1$, then $\lim_{n\to\infty} a^n = 0$.

(22) If $0 \le a_n \le b_n$ for all $n$, and $\sum b_n$ diverges, then $\sum a_n$ also diverges.

(23) If $0 \le a_n \le b_n$ for all $n$, and $\sum b_n$ converges, then $\sum a_n$ also converges.

(24) If $\lim\limits_{n\to\infty} a_n = 0$, then $\sum a_n$ is convergent.

(25) If $\sum a_n$ is convergent , then $\lim\limits_{n\to\infty} a_n = 0$.

(26) Find the limit of the sequence

$$\left\{\sqrt{3}, \sqrt{3\sqrt{3}}, \sqrt{3\sqrt{3\sqrt{3}}}, \dots\right\}$$

(27) Write the number $3.\overline{847} = 3.847847847\ldots$ as a ratio of integers.

(28) Given that a ball dropped to the floor rebounds to a height proportional to the height from which it is dropped, find the total distance travelled by a ball dropped from a height of 8 feet if it rebounds initially to a height of 4 feet.

(29) Use the integral test to decide whether the series converge or diverge.

(a) $\sum_{n=1}^{\infty} \frac{n}{e^n}$

(b) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{5n-4}}$

(c) $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

(30) Use the comparison test to determine whether the series converges or diverges.

(a) $\sum_{n=1}^{\infty} \frac{n^3+2}{n^4+n^3}$

(b) $\sum_{n=1}^{\infty} \frac{2n+1}{\sqrt{n^5+1}}$

**For problems 31-37, use an appropriate test to determine whether each of the series converges or diverges.**

(31) $\sum_{n=1}^{\infty} \frac{n}{(n+1)^n}$

(32) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{3n+2}}$

(33) $\sum_{n=1}^{\infty} \frac{(-1)^n \ln(n)}{n}$

(34) $\sum_{n=1}^{\infty} \frac{n^n}{(n+1)!}$

(35) $\sum_{n=1}^{\infty} \frac{(-3)^{2n}}{n^4 4^n}$

(36) $\sum_{n=1}^{\infty} \frac{n^2-n+2}{\sqrt[4]{n^{10}+n^5+3}}$

(37) $\sum_{n=1}^{\infty} n^2 2^{-n^2}$

(38) Find the radius of convergence and the interval of convergence of the series.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-4)^n}{n5^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{(x-1)^n}{\sqrt{n}}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n(x-3)^n}{n+3}$

(39) Find the radius of convergence and the interval of convergence of the series.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(2x-1)^n}{n^3}$

(b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n-1}\right) \frac{(x+2)^n}{2^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\frac{2}{3}\right)^n (x+1)^n$

(40) Suppose that the power series $\displaystyle\sum_{n=1}^{\infty} C_n x^n$ converges when $x = -4$ and diverges when $x = 7$. Which of the following are true, false, or not possible to determine?

(a) The power series converges when $x = 1$.

(b) The power series converges when $x = 4$.

(c) The power series converges when $x = 6$.

(d) The power series converges when $x = 7$.

(e) The power series converges when $x = 10$.

(f) The power series diverges when $x = -1$.

(g) The power series diverges when $x = -4$.

(h) The power series diverges when $x = -6$.

(i) The power series diverges when $x = -7$.

(j) The power series diverges when $x = -10$.