Calculus 1 with Precalculus
Recitation 12

Nathan Marianovsky

MORE ON LIMITS

Example 1. *Determine the value of:*

$$\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}$$

Solution 1.

$$\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4} = \lim_{x \to 4} \frac{\sqrt{x} - 2}{(\sqrt{x} - 2)(\sqrt{x} + 2)} = \lim_{x \to 4} \frac{1}{\sqrt{x} + 2} = \boxed{\frac{1}{4}}$$

Example 2. *Determine the value of:*

$$\lim_{x \to 2} \frac{\sqrt{x} - 2}{x - 4}$$

Solution 2.

$$\lim_{x \to 2} \frac{\sqrt{x} - 2}{x - 4} = \frac{\sqrt{2} - 2}{2 - 4} = \boxed{\frac{2 - \sqrt{2}}{2}}$$

Example 3. *Determine the value of:*

$$\lim_{x \to 5} \frac{x^2 - 3x - 10}{x - 5}$$

Solution 3.

$$\lim_{x \to 5} \frac{x^2 - 3x - 10}{x - 5} = \lim_{x \to 5} \frac{(x - 5)(x + 2)}{x - 5} = \lim_{x \to 5} (x + 2) = \boxed{7}$$

Example 4. *Determine the value of:*

$$\lim_{x \to \infty} \frac{1 - 3x^3}{2x^3 - 6x + 2}$$

Solution 4.

$$\lim_{x \to \infty} \frac{1 - 3x^3}{2x^3 - 6x + 2} = \lim_{x \to \infty} \frac{\frac{1}{x^3} - 3}{2 - \frac{6}{x^2} + \frac{2}{x^3}} = \boxed{-\frac{3}{2}}$$

Example 5. *Determine the value of:*

$$\lim_{x \to \infty} \frac{x^2 + x - 5}{1 - 2x - x^3}$$

Solution 5.

$$\lim_{x \to \infty} \frac{x^2 + x - 5}{1 - 2x - x^3} = \lim_{x \to \infty} \frac{\frac{1}{x} + \frac{1}{x^2} - \frac{5}{x^3}}{\frac{1}{x^3} - \frac{2}{x^2} - 1} = \boxed{0}$$

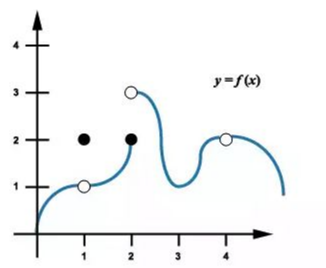
CONTINUITY

Definition 1 (Continuous at a Point)**.** A function is defined to be continuous at a point $x = p$ iff:

$$\lim_{x \to p^-} f(x) = \lim_{x \to p^+} f(x) = f(p)$$

Definition 2 (Continuous on an Interval)**.** A function is said to be continuous on an interval $[a, b]$ iff it is continuous at each point in the interval.

Example 6. *In the given graph:*



is the function continuous at $x = 1$?

Solution 6. *The needed information is:*

$$\lim_{x \to 1^-} f(x) = 1$$
$$\lim_{x \to 1^+} f(x) = 1$$
$$f(1) = 2$$

Even though the limit exists, since it does not equate the function at that point the function is $\boxed{\text{not continuous at } x = 1}$.

Example 7. *For which values of x is the function defined below not continuous?*

$$f(x) = \begin{cases} x^2 & x \leq 2 \\ 9 & x > 2 \end{cases}$$

Solution 7. *The two components of the piecewise function are both continuous, so only the intersection of the two has to be checked. Since the left function evaluates to 4 at $x = 2$ and does not equate to the right function, the function is not continuous at* $\boxed{x = 2}$.

Example 8. *For which values of x is the function defined below not continuous?*

$$f(x) = \frac{x}{x^2 - x}$$

Solution 8. *Any rational function is continuous at all points except the vertical asymptotes which are determined by finding the zeros of the denominator:*

$$0 = x^2 - x$$
$$0 = x(x - 1)$$
$$x = \boxed{0, 1}$$

Example 9. *If a hollow sphere of radius R is charged with one unit of static electricity, then the field intensity $E(x)$ at a point P located x units from the center of the sphere satisfies:*

$$E(x) = \begin{cases} 0 & 0 < x < R \\ \frac{1}{2x^2} & x = R \\ \frac{1}{x^2} & x > R \end{cases}$$

Is $E(x)$ continuous for $x > 0$?

Solution 9. *Since each part is continuous independently, only the point of intersection has to be checked. At $x = R$, the left part gives 0, the middle gives $\frac{1}{2R^2}$, and the right part gives $\frac{1}{R^2}$. Since these are not equal,* $\boxed{E(x) \text{ is not continuous at x = R}}$.

Example 10. *Discuss the continuity of the function:*

$$f(x) = x\left(x + \frac{1}{x}\right)$$

on the open interval $(0, 1)$ and on the closed interval $[0, 1]$.

Solution 10. *Since the function has a removable singularity, at $x = 0$, on the open interval the function is continuous. As for the closed interval, since it contains the singularity it is not continuous on the boundary at $x = 0$.*