Deontic Logic Tutorial

Introduction

Deontic logic is the field of philosophical logic that formalizes concepts such as obligation, permission, and prohibition.

Notation and Operators

Deontic logic extends classical logic with modal operators that express normative concepts:

1. Obligation: OA means "It is obligatory that A."
2. Permission: PA means "It is permitted that A."
3. Prohibition: FA means "It is forbidden that A," or equivalently O¬A or ¬PA

Standard Deontic Logic

SDL is the most basic form of deontic logic, which follows these axioms:

1. **Necessitation Rule (N)**: If A is a tautology, then it ought to be that A:

   $\vDash A \rightarrow \vDash OA$

   (Contradictions are not allowed)

2. **Modal Axiom K**: If it ought to be that A implies B, then if it ought to be that A, it ought to be that B:

   $O(A \rightarrow B) \rightarrow (OA \rightarrow OB)$

3. **Modal Axiom D**: If it ought to be that A, then it is permitted that A:
   $OA \rightarrow PA$

Conditional Obligations and Dyadic Deontic Logic

SDL struggles to represent conditional obligations, such as "If you smoke, then you ought to use an ashtray." Dyadic deontic logic introduces binary operators:

- O(A|B): "It is obligatory that A, given B"
- P(A|B): "It is permissible that A, given B"

Example:

This correctly states that using an ashtray is obligatory if one smokes without implying that one must smoke.

O(ashtray | smoke)