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A new online portfolio selection algorithm based on Kalman Filter and anti-correlation

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HIGHLIGHTS

- Consider both momentum and reversal of price in Anti-correlation algorithm can fully exploit the property of the price fluctuation.
- Anti-Correlation algorithm can fully exploit the property of the price fluctuation.
- Kalman filter with wavelet improve prediction accuracy of online portfolio selection algorithm.
- W-KACM improve portfolio return without any additional computation complexity.

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ABSTRACT

In this paper, we consider both momentum and reversal in the original Anticor algorithm and propose a new online portfolio selection algorithm named the Wavelet de-noise Kalman Momentum anti-correlation algorithm (W-KACM), which can fully exploit the property of the price fluctuation. Our new strategy also employs a improved measure of the cyclically adjusted price relative called the Wavelet de-noise Kalman Filter price relative (WKFPR). WKFPR, unlike the raw price relative that measures only how much the price moves from one period to the next, measures how far the price deviates from the inherent trend value. To demonstrate the effectiveness of our strategy, we extensively simulate on previously untested real market datasets, including Chinese stock market datasets, and make comparison with AC and KACM algorithms. The results of these experiments indicate that our strategy significantly outperforms the Anticor and KACM algorithms without any additional model or computational complexity.

1. Introduction

The portfolio selection problem is a challenging problem for investors, who are required to make sequence decisions about how to select and allocate assets in an uncertain environment. Based on the mean–variance model [1], the modern portfolio theory has studied the problem mainly by using the static model. However, the financial market is an extremely complex system. When investors are making decisions, the environment that they face is constantly changing. A portfolio manager must continually re-adjust the proportion of different assets, rather than hold it over the whole period, to achieve maximize wealth in the long run. Thus, portfolio selection is an online problem. Kelly [2,3]studied the optimal investment proportion to maximize the expected log return of a portfolio in the long-term investment process, and this approach became the basic principle of existing online portfolio selection algorithm.

The existing online portfolio selection strategies are mainly based on the assumption that the stock price follows the mean reversion principle in the long term [4–7]. Those strategies assume that the stock price will revert to its

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inherent value in the future, regardless of whether the present price is higher or lower than this value, which means that the poor security in the current period will be better in the future and that investors can achieve abnormal returns by transferring wealth from a higher-performance stock to poor-performance stock. The strategy includes the Passive Aggressive Mean Reversion (PAMR) [8], Confidence Weighting Mean Reversion (CWMR) [9], Online Moving Average Reversion (OLMAR) [10], Robust Mean Reversion (RMR) [11], Boosting Mean Average Reversion (BMAR) [12] Autoregressive moving average reversion (AMAR) [13] and Anti-correlation (AC) [14], which adopt the mean reversion ideal in different ways and achieve better performance than the benchmark strategy, such as the Universal portfolio (UP) [15].

PAMR and CWMR take advantage of the Passive Aggressive on-line learning algorithm [16] and the confident weighted online learning algorithm [17–20] to follow the mean reversion principle. These approaches make the simple assumption that the next price relative will be inversely proportional to the last price relative. In other words, they assume that the next price will revert to the last price. Although, these two strategies achieve satisfactory performances, the simplicity of this assumption leads to two potential risks. First, all existing mean reversion strategies use the same noisy price data, which often leads to estimation error [21]. Second, the assumption of single-period mean reversion may not be satisfied in the real world [10]. OLMR and RMR, which use multiperiod price information to make predictions, assume that the next price will revert to the moving average (MA, $MA_t(\omega) = \sum_{i=t-\omega+1}^{i=t}(p_i)$) and the $L_1 - median$. However, these two algorithms interpret each period equally, which ignores the temporal heterogeneity of historical information and leads to inaccurate predictions. The BMAR gives different weights to historical periods and reduces the impact of market noise by comprehensively using multiperiod price information. Focus on the issue that the noise data, single period hypothesis and nonstationary prediction are not fully considered in existing mean reversion strategy, AMAR use autoregressive moving average algorithm to predict stock price. Generally, all the algorithms mentioned in the previous article [8–13,22] use the historical price to predict the next price and apply different ways to explore the inherent value of a stock. However, the inherent value of a stock is hidden in the complex market noise, which is challenging to describe accurately using the MA or weighted average of the prices in the multiple past periods. To achieve both sparsity and stability in a portfolio choice problem, some improved sparse and stable portfolio optimization models [23–26] have been introduced to online portfolio selection problem.

Borodin [14] proposed the Anticor algorithm (AC) to exploit the market price fluctuation via correlation. This heuristic algorithm establishes a mechanism that transfers wealth from high-performance stocks to low-performance stocks. Anticor calculates the cross-correlation matrix M_{cor} between two specific market windows and determines the direction of next price change using the statistical relationship between different stocks. Anticor does not predict the price but rather using the statistical relationship to determine the probability of mean reversion. Nevertheless, the historical price relative can only measure how much the price moves from one period to the next, it cannot measure how far the stock price is from its inherent value. Thus, it is not reasonable to use raw price relative to determine whether the mean reversion will occur and which conditions will cause great market risk. Moreover, The Anticor uses simple average to measure the performance of different stocks in window period, which ignores the temporal heterogeneity of price relative and causes inaccuracy of predictions. The Anticor describes the relationship between different stocks only by correlation coefficient, which is easily affected by window's size. When the window size is smaller, the absolute value of correlation coefficient is likely closer to 1; when the window size is bigger, the absolute value of correlation coefficient is likely closer to 0. It cannot accurately measure the real relationship between different stocks.

Price momentum and reversal tend to coexist in the world stock market [27–30]. Considering only one characteristic of the stock price trend (momentum or mean reversion) will lead to missed profit opportunities, a suboptimal strategy. This paper introduces momentum to AC and generalizes AC to account for both mean reversion and momentum. This coexistence of both momentum and mean reversion in our algorithm can fully exploit price fluctuations and achieve greater abnormal returns from stock market peculiarities.

To eliminate the impact of market noise and accurately describe the asset's inherent trend price, many filter methods have been applied into financial field, such as Auto Regressive (AR), Moving Average (MA), Autoregressive Moving Average (ARMA) and Kalman Filter [31,32] and Wavelet transform [33,34]. Xu, Lin and Cai [35–38] have used HAR-type (Heterogeneous Autoregressive) models to find the inherent price and predict future price. Raphael and Alain [39] use the Kalman Filter algorithm (KF) to denoise the stock price. They believe that KF is an optimal filter method that allows us to explore the unobserved inherent component, and propose an alternative measure of the price relative, called cyclically adjusted price relative (CAPR, $CAPR_t = \frac{P_t}{P_t^k}$). Based on the CAPR, they proposed an online portfolio selection algorithm called KACM, which is proved to be better than Anti-correlation strategy. Kalman Filter(KF) is a recursive estimation algorithm, which takes minimum mean square deviation as the best criterion. The KF considers both the previous state estimates and the current observation to calculate the current state estimates. In order to establish an accurate state equation, we must know the transform pattern of system. However, the stock price is a nonstationary and nonlinear time series, it is seemly impossible to find a clear equation to describe the transform pattern of stock price. So, there are some errors between the KF price and the trend price. Focus on this issue, A Wavelet de-noise Kalman Filter (WKF) is employed based on Scale Kalman Filter. The WKF not only has the character of multi-resolution analysis in wavelet domain and remove the properties of self-similarity, but also maintain minimum square deviation estimation of Kalman filtering for unknown time series.

The contributions of this paper are as follows. First, we propose a new filter method, Wavelet de-noise Kalman Filter (WKF), to separate the unobserved inherent trend price ("true value") from the noisy price data. We propose the WKFPR method to improve CAPR, this method can measure the price deviation from its inherent trend price more accurate. Second, our algorithm considers not only the mean aversion (momentum) but also the mean reversion, unlike AC which follows only the mean reversion principle. The coincorporation of both momentum and mean reversion is more likely to generate optimal results. Third, Euclidean distance is used as the supplement of correlation coefficient to measure the relationship between different stocks. We also uses time weighted average to replace simple average to describe the performance of different stocks in window period. These improvements will improve the accuracy of predictions. Finally, we conduct extensive experiments on untest real-market datasets and achieve superior results than the AC and KACM algorithms.

2. Methods

2.1. Problem setting

Before we formally introduce the online portfolio selection problem, we first make some assumptions and introduce some notations to be used throughout the article. Consider a financial market with m assets. The market vector $X = (X_1, X_2, \dots, X_n)$ is the sequence of the price relative vector for n trading periods. In the tth period, the assets' price relative are presented by $X_t = (X_t(i), X_t(2), X_t(3), \dots, X_t(m)), X_t \in R_+^m$, where R_+^m is the positive orthant. The ith component of the tth vector $X_t(i) = \frac{p_t(i)}{p_{t-1}(i)}$ denotes the ratio of the closing price to the last closing price of the ith asset on the tth trading period. An investment in the market is specified by a portfolio vector, which is designated $b_t = (b_t(1), b_t(2), b_t(3), \dots, b_t(m))$, where $b_t(i)$ represents the allocation of wealth in the ith stock on the tth period. Generally, we assume that the portfolio is self-financing and that no margin/shorting is allowed, which means that $\sum_{i=1}^{m} b_t(i) = 1$ and $b_t(i) \geq 0$. Let $b = (b^1, b^2, b^3, \dots, b^n)$ be the investment proportions vector for the n trading periods. For the tth trading period, a portfolio b_t produces a *portfolio period return* $s_t = b_t^T \bullet X^t = \sum_{i=1}^{m} b_t(i)x_t(i)$, which represents the increments of wealth. Thus, after n periods, a portfolio strategy b produces a *portfolio cumulative wealth* of $S_n = S_0 \prod_{t=1}^{n} b_t^T \bullet x_t = S_0 \prod_{t=1}^{n} \sum_{i=1}^{m} b_t^T(i) \bullet x_t(i)$, where S_0 denotes the initial wealth invested in the portfolio.

We make the following assumptions:

(1) There is no friction in the market. Transaction costs are an important factor to influence the performance of the strategy [40]. In some online portfolio selection algorithms, transaction costs are directly involved in the PS process [41–43]. However, in this paper, we do not consider the transaction costs in our original algorithmic formulation, although we evaluate the impact of transaction costs in the back test. In addition, we assume that all securities are infinitely separable, which means that we can buy or sell any number of shares of stocks. We also assume that there are adequate liquidity in the market and that all trade orders can be executed at the closing price.

(2) There is no suspension of stock trading during the whole investment period.

(3) In this perfectly Competitive Market, all traders are price takers and their behavior cannot influence the market price.

2.2. Benchmark algorithm

The Buy And Hold portfolio strategy (BAH), the constant Re-balanced portfolio strategy (CBAL) [44] and Exponential Gradient portfolio strategy (EG) [45] are typically used as benchmark strategies to evaluate other online portfolio section strategies.

The most basic strategy is BAH, which buys stock using the initial portfolio b and then never reinvests. After n periods, the cumulative period return can be given as: $S_n = b_0^T \cdot (|_{t=1}^{n} x_t)$, where the $|$ denotes the corresponding multiplicative components in a vector. When $b_t = (\frac{1}{m}, \frac{1}{m}, \frac{1}{m}, \dots, \frac{1}{m})$, the BAH strategy is also called the uniform Buy and Hold strategy (U-BAH), which describes the trend in the market.

The constant rebalanced portfolio strategy is an "active portfolio strategy", that dynamically changes the portfolio during the trading period. Unlike other active portfolio strategies, the constant rebalanced portfolio strategy reinvests wealth each trading day according to a fixed portfolio b. The constant Re-balanced portfolio strategy can take strong advantage of market fluctuations to achieve a more significant return than BAH. The uniform constant re-balancing strategy (U-CBAL) is another natural online benchmark algorithm, which is U-CBAL with $b = (\frac{1}{m}, \frac{1}{m}, \frac{1}{m}, \dots, \frac{1}{m},)$. A natural offline benchmark algorithm is $CBAL^*$, and we let it denote the optimal CBAL where b is given as $b^* = \arg\max_{b(*)} ret_x(CBAL_b)$.

Next, we cover the proposed UP. To asymptotically approach the $CBAL_*$, the UPs use the weighted average of the performance of all CBAL experts. The portfolio can be computed as follows:

$$b_{t+1} = \frac{\int_{\Delta_m} b\dot{S}_t(b)db}{\int_{\Delta_m} S_t(b)db} \tag{1}$$

Where b_1 is uniform over the market. However, the UP strategy is too complex to calculate. To obtain a favorable UP strategy, we need to perform n^m calculations in the n trading periods when the portfolio has m stocks. The regret achieved by Cover's UP strategy is $O(m \log n)$ [46], and the transaction costs of reinvestment are huge. Therefore, the performance of the UP strategy is not as good as we expect in the real market.

Helmbold [45] proposed EG, which use multiplicative updates to maximize the expected logarithmic portfolio daily returns. The strategy *portfolio b* can be generally formulated as follows:

$$b_{t+1}(i) = \frac{\frac{b_t(i)exp(\eta x_t(i))}{b_t^T}}{\frac{\sum_{j=1}{}^m b_t(j)exp(\eta x_t(j))}{b_t^T \dot{x_t}}} \tag{2}$$

where η is the learning rate parameter. Relative to the UP strategy, the regret of the EG strategy is only $O(\sqrt{n} \log m)$, and the running time is $O(mn)$. Therefore, the EG strategy is more suitable to large-scale portfolio calculation.

Cover [15] and Helmbold [45] proved that the UP strategy and EG strategy are both universal portfolio algorithms. In this paper, the new online portfolio selection algorithm is also categorized as a UP algorithm.

2.3. W-KACM algorithm

2.3.1. Wavelet de-noise Kalman Filter

As a special commodity, the stock has an underlying value and follows the law of value. However, the stock price has inherent noise that obscures the true underlying value. Raphael and Alain [39] use KF to eliminate the influence of noise and find the stock's true underlying value. The stock price is a nonstationary and nonlinear time series, it is difficult to find a clear state equation to describe the transform pattern of stock price. There are great errors if we use KF to deal with nonstationary time series directly. To maintain linear unbiased and minimum error variance estimate of Kalman filtering for the unknown signal, we consider both the Wavelet transform and the KF and propose a new filter method called Wavelet de-noise Kalman Filter (WKF). First, the wavelet analysis method is used to do multi-scale decomposition with the data of stock price to the data signal smooth, then the low frequency signal of wavelet decomposition is used as Kalman Filter algorithm's input. The WKF can significantly improve the accuracy of the KF predictions.

A wavelet transform is used to analyze the nonstationary time series in order to generate information on both time and frequency domains. This transform can be regarded as a special type of Fourier transform at multiple scale that decomposes a signal into shifted and scaled versions of a "mother" wavelet[47]. The continuous wavelet transform (DWT) is defined as follows:

$$CWT_x^{\psi}(b, a) = \phi_x^{\psi}(b, a) = \frac{1}{\sqrt{|a|}} \int x(t)\dot{\psi}^*(\frac{t-b}{a})dz \tag{3}$$

Where $x(t)$ is a time series, a is a scale parameter, b is a transform parameter and $*$ is the complex conjugate of $\psi(t)$. The CWT can separate the signal into components at various scales corresponding to successive frequencies. The wavelet de-noise, which is used in this study, can be divided into three steps:

1. The wavelet transform analysis of the time series. According to the feature of the time series, we choice an appropriate wavelet function to do multi-scale wavelet decomposition.
2. Threshold quantitative process. The detail coefficients and the approximation coefficient are modified via threshold.
3. Wavelet reconstruction.

KF is considered as the optimal filter, which uses the current observation to predict the next period's unobservable value and then uses the realization of the next period to update that prediction. In KF, the state-space model [48] can suitably address this optimal filter problem. This model introduces the state equation and observation equation to produce estimates of the time series of unobservable variables. The linear state-space model as follows:

$$X(t+1) = A \cdot X(t) + B \cdot U(t+1) + W(t+1) \tag{4}$$

$$Z(t+1) = H \cdot X(t+1) + V(t+1) \tag{5}$$

where $X(t+1)$ is the system state at time $t+1$, $U(t+1)$ is the system control variable at time $t+1$, $Z(t+1)$ is the system observation at time $t+1$, coefficients A and B are both system parameters, $W(t+1)$ is the system noise and $V(t+1)$ is the observation noise. Both noises are white Gaussian noise.

The KF recursive estimation algorithm works as follows. At time t the process starts with the initial estimation $X(t)$ for the realization $Z(t)$.

1. Compute the system state estimation $X(t+1|t)$.

$$X(t+1|t) = A \cdot X(t|t) + B \cdot U(t+1) \tag{6}$$

2. Estimate the covariance. Here $P(t+1|t)$ is the covariance that corresponds to $X(t+1|t)$ and Q denotes the system noise covariance.

$$P(t+1|t) = A \cdot P(t|t) \cdot A' + Q \tag{7}$$

3. Compute the Kalman gain coefficient. Here K is the gain coefficient, and R denotes the observation noise covariance.

$$K(t+1) = \frac{P(t+1|t) \cdot H'}{H \cdot P(t+1|t) \cdot H' + R} \tag{8}$$

4. Update the covariance. Here I is the identity matrix.

$$P(t+1|t+1) = (I - K(t) \cdot H) \cdot P(t+1|t) \tag{9}$$

5. Update the system estimation.

$$X(t+1|t+1) = X(t+1|t) + K(t) \cdot (Z(t)H \cdot X(t+1|t)) \tag{10}$$

The market noise causes volatility in the stock market. However, many empirical studies have demonstrated that the stock price follows the mean reversion phenomenon, which means that no matter how much the stock price rises or falls, it must revert to the underlying value. By measuring how much the current price deviates from the underlying value, we can accurately predict the stock price in the next period. In this paper, we use the filtered price as the stock's unobservable underlying value. We proposes an improved measure of the CAPR, used in KACM algorithm, called the *WKFPR* based on the Wavelet de-noise Kalman Filter. If p_t represents the stock price and p_t^{wk} is the filtered or underlying price at time t, then *WKFPR* is defined as follows:

$$WKFPR = \frac{P_t}{p_t^{wk}} \tag{11}$$

WKFPR is an powerful price indicator, that can more accurately measure whether the market price is overestimated or underestimated than the CAPR. When $WKFPR > 1$, the price is overestimated, and the price will fall in the future. Conversely, when $WKFPR < 1$, the price is underestimated, and the price will rise in the future. The further the price is from its own underlying value, the more attractive the stock is for purchase or for sale in our algorithm. The heuristic algorithms Anti-correlation exploit the statistical relationship between different stocks and different periods to predict the price trend in the future. WKFPR can significantly improve the accuracy of AC and realize greater returns.

2.3.2. AC Revisited

Borodin [14] proposed the heuristic algorithm Anticor in 2006. AC, which is derived from mean reversion theory and establishes a mechanism to transfer the wealth from higher-performance stocks to lower-performance stocks. The algorithm evaluates the performance of the stock by dividing the historical price relative series into equal-sized periods called windows, each with a length of ω days where ω is a adjustable parameter. This online reversal algorithm mainly makes three assumptions as follows:

1. The growth rate of stock i exceeds that of stock j in the current window.
2. In the next window, stock i reproduces the same performance of stock j in the past window.
3. The growth rate of stock i over the first window is positively related to the growth rate of stock j over the second window.

In AC, Borodin defined LX_1 and LX_2 as two $\omega \times n$ matrices over the consecutive time windows.

$$LX_1 = (\log(X_{t-2\omega+1}))^T, \ldots, (\log(X_{t-\omega}))^T andLX_2 = (\log(X_{t-\omega+1}))^T, \ldots, (\log(X_t))^T \tag{12}$$

where $\log(X_k)$ denotes$(\log(x_k(1)), \ldots, \log(x_k(m)))$. LX_1 and LX_2 are constructed by taking the logarithm over two consecutive time windows $[t-2\omega+1, t-\omega]$ and $[t-\omega+1, t]$. $LX_k(j)$ denotes the jth column of LX_k. Let $\mu_k(j)$ be the mean of $LX_k(j)$ and let $\sigma_k(j)$ be the standard deviation of $LX_k(j)$. The cross-correlation matrix between the column vectors in LX_1 and LX_2 are defined as follows:

$$M_{cov}(i,j) = \frac{1}{\omega - 1}(LX_1(i) - \mu_1(i))(LX_2(j) - \mu_2(j))^T \tag{13}$$

$$M_{cor}(i,j) = \begin{cases} \frac{M_{cov}(i,j)}{\sigma_1(i)\sigma_2(j)} & \sigma_1(i), \sigma_2(j) \neq 0 \\ 0 & \text{Otherwise} \end{cases} \tag{14}$$

$M_{cor}(i,j) \in [-1, 1]$ measures the correlation between the log-relative prices of stock i over the first window and stock j over the second window. Then, when $\mu_2(i) \geq \mu_2(j)$ and $M_{cor} > 0$, the Borodin metric is defined as follows:

$$claim_{i \to j} = M_{cor}(i,j) + max(-M_{cor}(i,i), 0) + max(-M_{cor}(j,j), 0) \tag{15}$$

Table 1
Differences between the four algorithms.

Algorithm	Input variable	Momentum	Mean reversion
AC	Raw price relative	No	Yes
KACM	CAPR	Yes	Yes
W-KACM	WKFPR	Yes	Yes

$$transfer_{i\to j} = b_{t-1}(i) \cdot \frac{claim_{i\to j}}{\sum_j claim_{i\to j}} \tag{16}$$

Anticor shifts the investment from stock i to stock j using equations (15) and (16), and ultimately updates the portfolio as follows:

$$\hat{b}_t = \frac{1}{b_t \cdot X_t}(b_t(1)X_t(1), \ldots, b_t(m)X_t(m)) \tag{17}$$

$$b_t = \hat{b}_t(i) + \sum_{j\neq i}(transfer_{j\to i} - transfer_{i\to j}) \tag{18}$$

AC exploits the statistical information to reveal the relationship between different stocks and adopts the mean reversion idea to update the portfolio.

2.3.3. Combine with momentum in anticor

Over the past ten years, an extensive study of recent behavioral finance documented two different price movement paths. One is a reversal, where a price rise is more likely to be followed by a price fall, and the other is momentum, where price fall is likely to be followed by a price rise. AC only relies on one assumption: that of a price reversal. Although, the performance of Anticor surpasses the benchmark algorithm in most datasets, this algorithm cannot fully exploit the price fluctuations in the market. Combined with the momentum in Anticor, this approach can effectively capture the properties of the stock price and significantly improve the performance of the algorithm.

To account for the price aversion, we expand the benchmark Anticor as follows. Here $\mu_2(i) \geq \mu_2(j)$ and $M_{cor} \leq 0$.

$$claim_{i\to j} = -M_{cor}(i,j) + max(M_{cor}(i,i), 0) + max(M_{cor}(j,j), 0) \tag{19}$$

$$transfer_{i\to j} = b_{t-1}(i) \cdot \frac{claim_{i\to j}}{\sum_j claim_{i\to j}} \tag{20}$$

Although, the Anticor can achieve great return in many datasets, there are two deficiencies in the AC. First, the Anticor uses simple average to measure the performance of different stocks in window period, which ignores the temporal heterogeneity of price relative and causes inaccuracy of predictions. Usually, we believe that the price at current period is closer to the recent period due to the continuity of price changes [12]. A time weighted average is used in this algorithm. Second, The Anticor describes the relationship between different stocks only by correlation coefficient, which is easily affected by window's size. We use the Euclidean distance as the supplement of correlation coefficient to measure the relationship between different stocks and update the portfolio.

In this paper, we propose another new online portfolio selection algorithm called W-KACM where the input variable is WKFPR as derived from the Wavelet de-noise Kalman Filter. To prove the effectiveness of the improvements, we compare our new algorithm W-KACM with the benchmark AC and KACM. The differences of these algorithms are shown in Table 1.

In Table 2, we show the pseudo code of the W-KACM algorithm.

3. Datasets

In our empirical study, we exploit the untested historical market data to compare the performance of our proposed algorithm with selected benchmark strategies. The historical market data used in this paper all come from the Wind database, and the stock prices adjusted for stock split splits and dividends have been included in this simulations. We adopt four datasets from several types of financial markets, the stocks we select for the portfolio are all the sample stocks of the market index, such as Nasdaq100 index, Hang Seng index, Shangzhen180 index and ShenZhen300 index. There are two principles of stock selection in this study. First, the stock has less suspension. Second, the stock trades actively. To avoid the interference of the overall market decline on the algorithms' evaluations, the simulation period selected in this paper exclude several market crash. The four datasets from several types of financial markets are summarized in Table 3.

Table 2
W-KACM algorithm pseudo code.

Algorithm
Input:
1. ω : window size
2. t: index of trading day
3. $X_t = x_1, \dots, x_t$: historical market sequence
4. $WKFPR = x_t = \frac{p_t}{p_t^{wk}}$: Wavelet de-noise Kalman Filter price relative
5. b_0: initial portfolio weighted $b_0 = (\frac{1}{m}, \frac{1}{m}, \dots, \frac{1}{m})$
6. $\hat{b_t}$: current portfolio(by the end of the trading day t)
7. ed: the threshold of Euclidean distance
Output:
1. Return the current portfolio b_t if $t < 2\omega$
2. compute LX_1 and LX_2, μ_1 and μ_2
$LX_1 = (log(X_{t-2\omega+1}), \dots, log(X_{t-\omega}))^T$
$LX_2 = (log(X_{t-\omega+1}), \dots, log(X_t))^T$
$\mu_1(i) = \frac{1}{1+2+\cdots+\omega}(1 \cdot \log X_{t-\omega+1}(i) + 2 \cdot \log X_{t-\omega+2}(i) + \cdots + \omega \cdot \log X_t(i))$
$\mu_2(i) = \frac{1}{1+2+\cdots+\omega}(1 \cdot \log X_{t-2\cdot\omega+1}(i) + 2 \cdot \log X_{t-2\cdot\omega+2}(i) + \cdots + \omega \cdot \log X_{t-\omega}(i))$
3. compute $M_{cor}(i,j)$
$M_{cov}(i,j) = \frac{1}{\omega-1}(LX_1(i) - \mu_1(i))^T(LX_2(j) - \mu_2(j))$
$M_{cor}(i,j) = \begin{cases} \frac{M_{cov}(i,j)}{\sigma_1(i)\sigma_2(j)} & \sigma 1(i), \sigma_2(j) \neq 0 \\ 0 & \text{otherwise} \end{cases}$
4. calculate the Euclidean distance: $ED(i,j) = \sqrt{\sum_{k=1}^{\omega}(LX_1(i)(k) - LX_2(j)(k))^2}$
5. calculate $claim$:for $1 \le i, j \le m$, initial $claim_{i\to j} = 0$
Mean reversion:
if $\mu_2(i) \ge \mu_2(j)$ and ED<ed and $M_{cor}(i,j) > 0$ then
$claim_{i\to j} = claim_{i\to j} + M_{cor}(i,j) + max(-M_{cor}(i,i), 0) + max(-M_{cor}(j,j), 0)$
Momentum:
if $\mu_2(i) \ge \mu_2(j)$ and ED(i,j)<ed and $M_{cor}(i,j) \le 0$ then
$claim_{i\to j} = claim_{i\to j} - M_{cor}(i,j) + max(M_{cor}(i,i), 0) + max(M_{cor}(j,j), 0)$
6. calculate new portfolio: initial $b_{t+1} = \hat{b_t}$, for $1 \le i, j \le m$
$transfer_{i\to j} = b_{t+1}(i) \cdot \frac{claim_{i\to j}}{\sum_j claim_{i\to j}}$
$b_{t+1}(i) = b_{t+1}(i) - transfer_{i\to j} + transfer_{j\to i}$

Table 3
Summary of four real stock market datasets.

Datasets	Time frame	Trade days	Stocks	Market
Nasdaq	Jan.4th 2002–Apr.11th 2018	3944	79	US
HSI	Jan.4th 2006–Apr.30th 2015	2299	40	HONG KONG
SH	Jan.4th 2006–Apr.30th 2015	2263	100	CN
SZ	Jan.4th 2006–Apr.30th 2015	2263	100	CN

4. Results

This section presents simulation results obtained by applying the above three algorithms (AC, KACM and W-KACM) to four untested financial market datasets (see the Methods section). Fig. 1 shows the total wealth achieved by various algorithms on the four market datasets as a function of the window size $\omega = 2, \dots, 50$. ω is a critical parameter in our algorithms and the performance of these algorithms depends significantly on the window size. However, the improved algorithms proposed in this paper are superior to the benchmark AC and KACM in most window sizes.

The cycle of mean reversion (or mean aversion) is uncertain, as one cannot accurately determine when the reversal happens and it is challenging to select the optimal window size. This condition causes the performances of these algorithms (AC, KACM and W-KACM) to be instable, and the portfolio manager faces great risk when using these

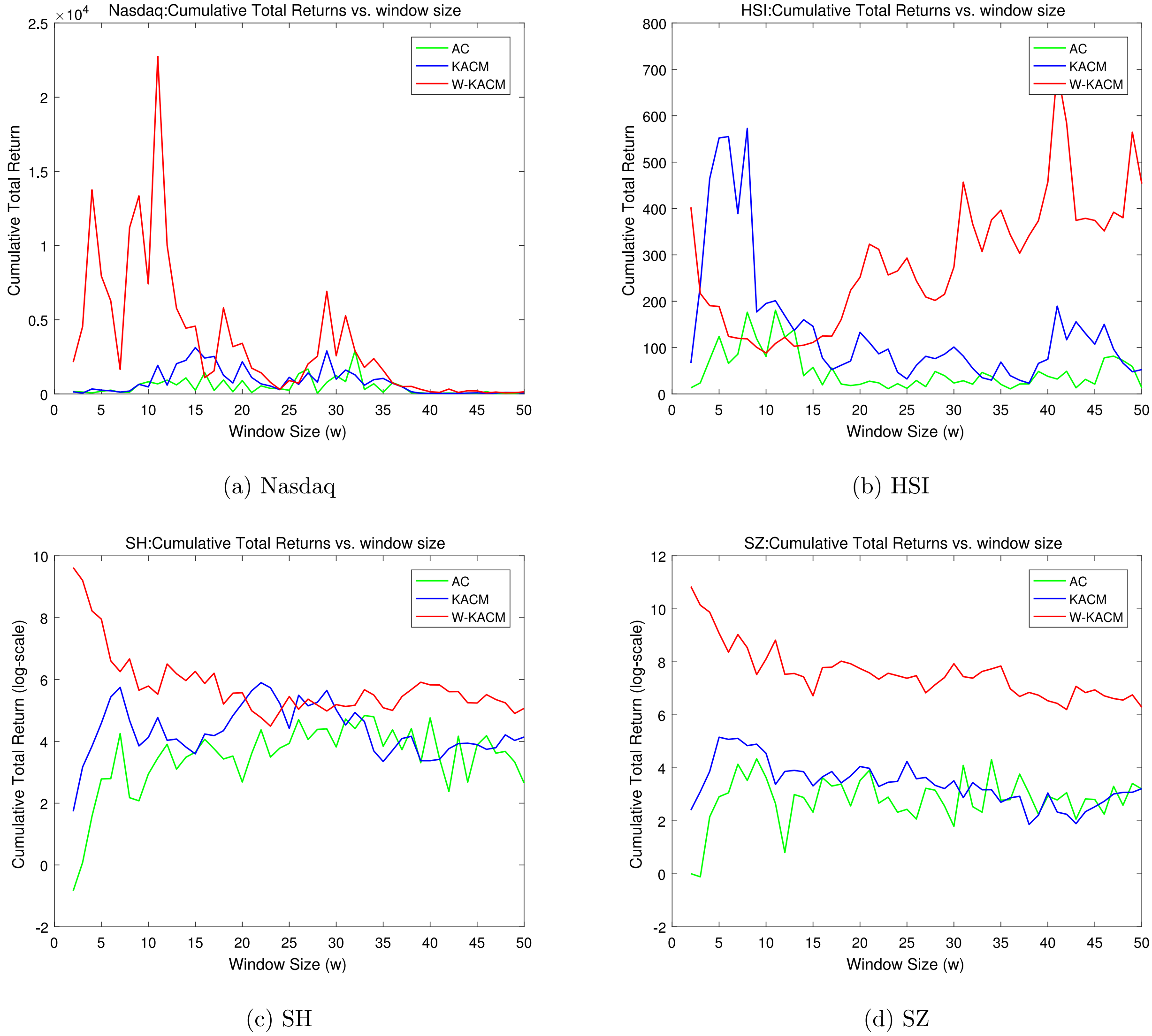
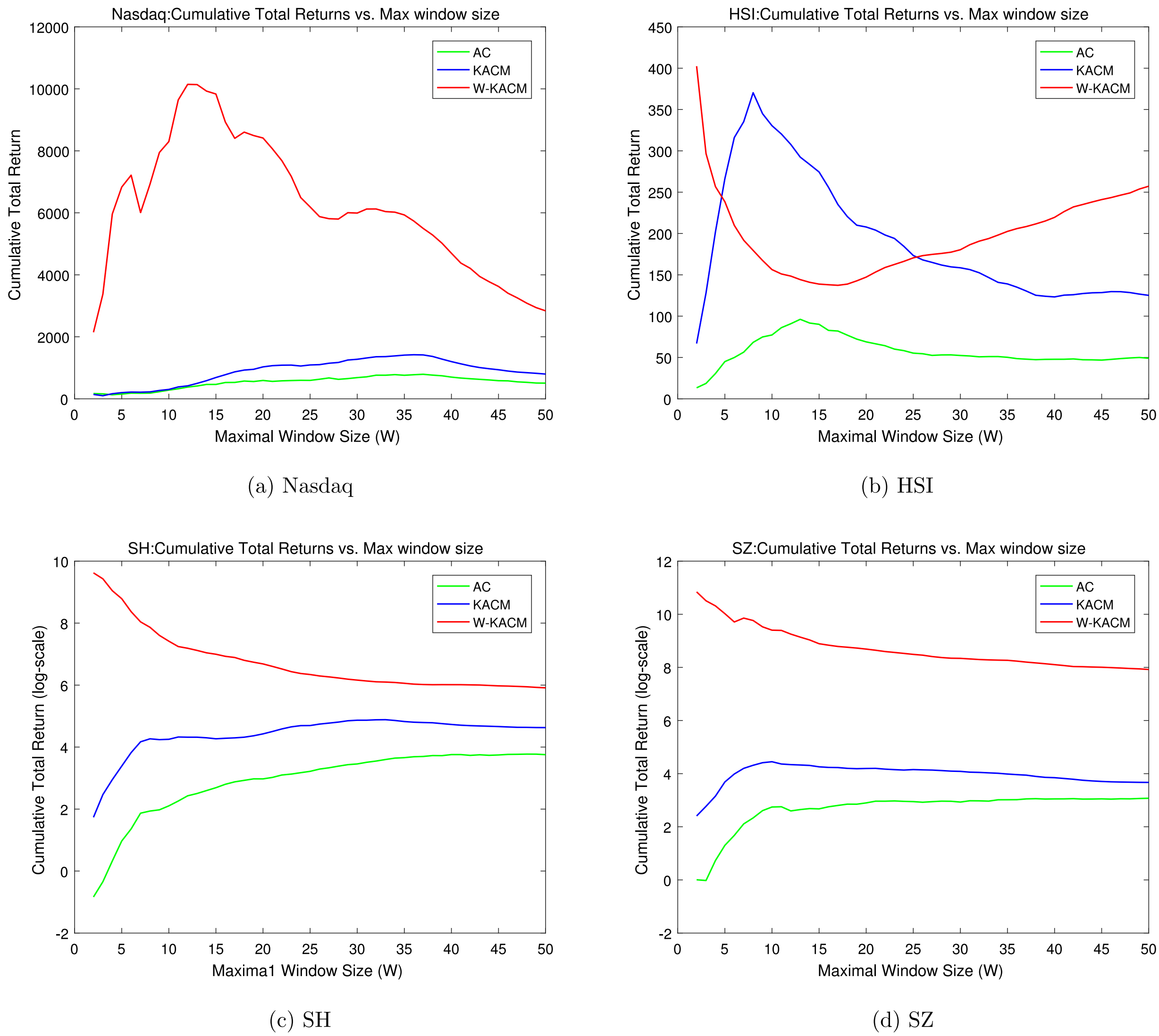


Fig. 1. AC, KACM and W-KACM's total return (per \$1 investment) vs. window size for the NASDAQ, HSI, SH and SZ.

algorithms to invest. To eliminate the influence of the window size, Borodin [14] views the different windows in the algorithm as experts. We can adaptively learn from and invest in some weighted averages of all window sizes in the algorithms with ω less than some maximum W. In this paper, we adopt one simplest case, which is the uniform investment on all the windows of the algorithms. The uniform buy-and-hold (BAH) investment on the windows ($\omega \in [2, M]$) can fully explore the property of the algorithm windows without any additional computational demands and time complexity. Fig. 2 graphs the cumulative returns of the AC, KACM and W-KACM algorithms as functions of W for all values of $2 \leq W \leq 50$ for the four databases.

As show in Fig. 2, with the increase in the maximum window, the cumulative returns of algorithms tend to stabilize. In these graphs, we let $W = 30$. The uniform maximum window on AC, KACM and W-KACM necessitates a unit standard to compare the performance of these algorithms, although this choice is clearly not optimal. Of course, we aim to optimize the parameter W for each datasets in the practical application.

We establish selected settings about the benchmark algorithm. In the Exponential Gradient algorithm (EG), we set the learning parameter as $\eta = 0.05$. In addition, we use the market index as the market performance. In Table 4 we provide the cumulative returns, annualized returns, annualized standard deviations, Sharpe ratios and maximum drawdowns for all market datasets and algorithms with $W = 30$ considered here.



(a) Nasdaq (b) HSI

(c) SH (d) SZ

Fig. 2. AC, KACM and W-KACM's total return (per $1 investment) as a function of the maximal window: NASDAQ, HSI, SH, SZ.

In the online portfolio selection problem, we need to use multiple indicators to evaluate the algorithm of online portfolio selection. In Table 4, the standard risk measure is the annualized standard deviation and maximum drawdown, the returns are measured by the cumulative returns and annualized returns and the Sharpe ratio can be used to comprehensively evaluate the performance of an investment strategy. In all datasets, the AC, KACM and W-KACM algorithms provide better returns than the benchmark algorithms. In addition, these algorithms proposed in this paper have more risks than benchmark algorithm. Overall, the performance of the AC, KACM and W-KACM algorithms significantly surpasses that of the benchmark algorithms and market and the W-KACM algorithm features the best performance of the group.

When the investor needs to hold one portfolio that consists of various different stocks for a long time, they can adopt the W-KACM algorithm to manage their portfolio. The W-KACM algorithm can significantly increase the portfolio wealth without an additional investment. However, the W-KACM algorithm is not always effective, particularly in some special market periods such as a financial crisis. As shown in Fig. 3, the total wealth of the portfolio significantly increase when the market is steady, and the portfolio wealth collapses when all stock prices decrease in the market depression. The benchmark AC, KACM and W-KACM algorithms exploit the statistical information from historical markets to transfer weights from the winner stock (overestimated stock) to the loser stock (underestimated stock). When all stocks decrease, there is no winner stock (or overestimated stock) in the portfolio and all the stocks in the portfolio are loser stocks (or underestimated stocks), which means that the AC KACM and W-KACM algorithms are no longer effective and that

Table 4
Monetary return in dollars (per $1 investment), cumulative returns, annualized returns, annualized standard deviations (risk), sharpe ratios and maximum drawdowns of various algorithms for four different datasets the winner and runner-up for each market appear in boldface.[1]

Datasets	Algorithm	Cumulative return	Annualized return	Risk	Sharpe ratio	Maximum drawdown
NASDAQ	Market	3.5009	0.0844	**0.0137**	0.2491	0.5563
	U-BAH	37.5997	0.2643	0.0170	0.4621	0.4888
	U-CBAL	16.8686	0.2004	0.0139	0.4876	0.4902
	EG	17.2736	0.2023	0.0139	**0.4902**	**0.4485**
	AC	683.6176	0.5250	0.0229	0.4601	0.6755
	KACM	1275.0579	0.5878	0.0303	0.3038	0.7512
	W-KACM	**5992.3012**	**0.7548**	0.0303	0.3947	0.7837
HSI	Market	1.9633	0.0777	0.0163	0.1897	0.6742
	U-BAH	6.8250	0.2374	0.0169	0.4401	0.6495
	U-CBAL	6.0483	0.2209	**0.0158**	0.4529	0.6303
	EG	6.0693	0.2214	0.0159	0.4507	0.6311
	AC	52.3877	0.5513	0.0214	**0.5013**	0.5988
	KACM	158.5168	0.7540	0.0251	0.4763	**0.5142**
	W-KACM	**202.5492**	**0.8023**	0.0319	0.3824	0.7509
SZ	Market	7.8237	0.2609	**0.0188**	0.4152	0.7101
	U-BAH	12.9888	0.3350	0.0189	0.4649	0.7105
	U-CBAL	20.2786	0.4037	0.0192	0.**4953**	0.6998
	EG	19.8463	0.4003	0.0192	0.4936	0.7007
	AC	18.7370	0.3913	0.0229	0.4099	0.7599
	KACM	59.5077	0.5848	0.0246	0.4455	**0.6357**
	W-KACM	**4189.5417**	**1.5595**	0.0356	0.4158	0.7027
SH	Market	3.6400	0.1567	**0.0170**	0.3423	0.7198
	U-BAH	16.1775	0.3684	0.0190	0.4817	0.7073
	U-CBAL	21.8527	0.4156	0.0183	0.5259	**0.6738**
	EG	21.5678	0.4135	0.0184	0.5219	0.6756
	AC	31.7039	0.4762	0.0222	0.4575	0.8022
	KACM	129.9374	0.7305	0.03817	0.3099	0.7896
	W-KACM	**473.9873**	**1.002**	0.0156	**0.8377**	0.798507463

any adjustment to the portfolio is redundant except to increase transaction costs. When the crisis is coming, AC, KACM, W-KACM and other online portfolio selection algorithms will experience losses.

4.1. Trading cost, trading friction and other caveats

Previous empirical studies have focused on the no-friction market. However, transaction costs are an important factor effecting the performance of the algorithm. Each transaction of the portfolio adjustment is charged transaction costs, and the portfolio manager cannot change the properties of these costs. In this section, we assume that there are costs on all transactions that are equal to a fixed percentage of the amount transacted. We consider the fixed transaction cost rate of $\gamma \in (0, 1)$ such that the investor pays at rate of $\frac{\gamma}{2}$ for each buy and sell event. At the end of period t, the portfolio manager needs to rebalance the portfolio's wealth into a new portfolio b_t, from the current closing price adjusted portfolio $\hat{b}_{t-1}$, the transaction costs are calculated as follows:

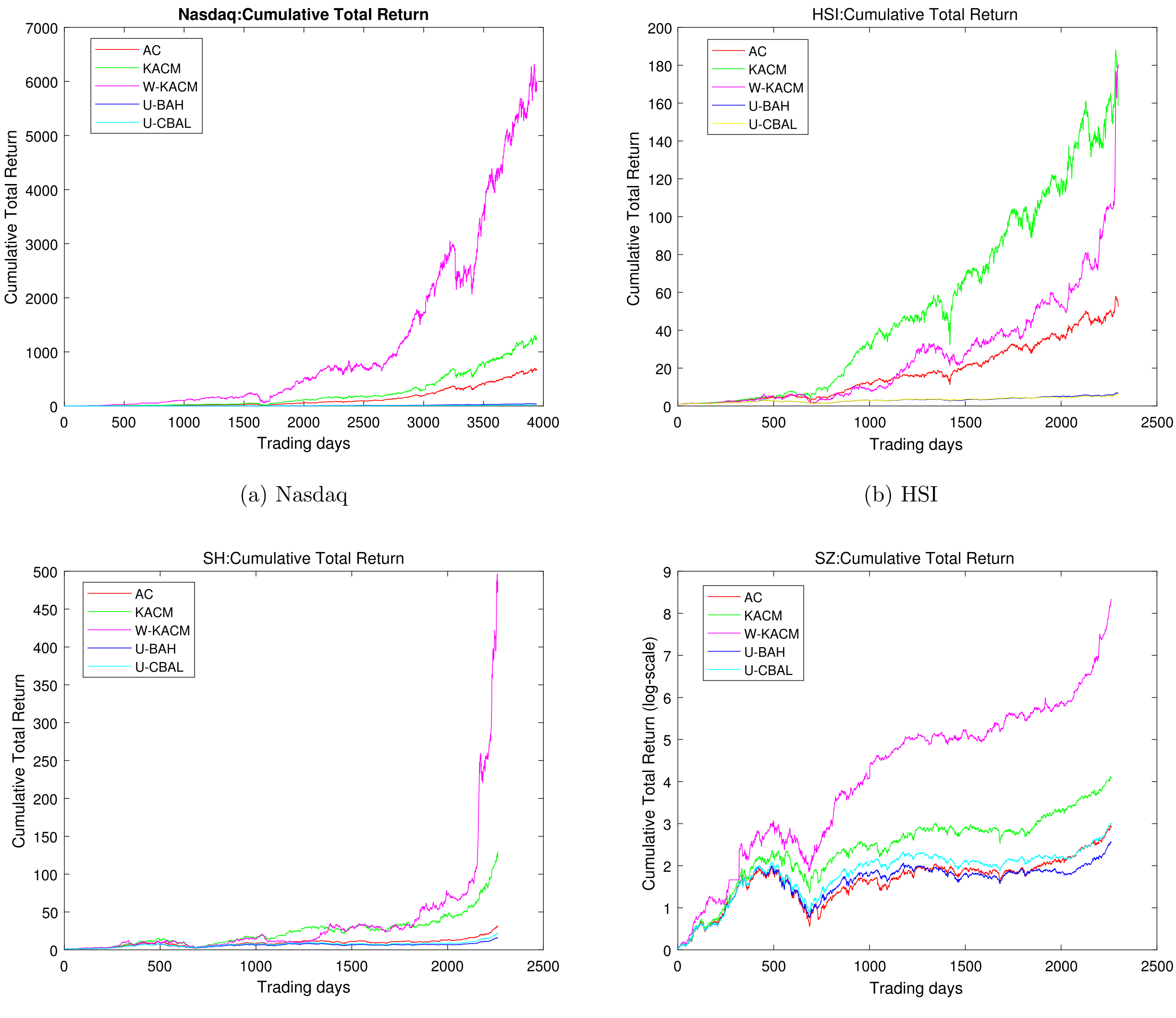
$$cost = \frac{\lambda}{2} \times \sum_{i}^{m}(|b_t(i) - \hat{b}_t(i)|) \tag{21}$$

In addition the cumulative returns after period n can be expressed as follows:

$$S_n = S_0 \prod_{t=1}^{n}[b_t^T \cdot X_t(1 - \frac{\lambda}{2} \times \sum_{i} |b_t(i) - \hat{b}_t(i)|)] \tag{22}$$

Fig. 4 depicts the total returns of the AC, KACM and W-KACM algorithms with proportional transaction cost rates of $\gamma = 0.1\%, 0.2\%, \ldots, 2\%$. These algorithms can accommodate the proportional transaction cost rate. When $\gamma^*_{Nasdaq} = 0.905\%$, $\gamma^*_{HSI} = 29.62\%$, $\gamma^*_{SH} = 10.23\%$, $\gamma^*_{SZ} = 11.03\%$, the total returns of the W-KACM algorithm equal to the BAH strategy in different datasets. The transaction cost rate at the equilibrium point is higher than the real market commission rate,

[1] The cumulative return is the ratio of total wealth on the last portfolio day divided by the initial wealth. The annualized return is the geometric mean of the individual daily return. The risk is the annualized standard deviation of these daily return multiplied by $\sqrt{252}$ where 252 is assumed standard number of trading days per year. The Sharpe ratio is the ratio of the annualized return minus risk-free return (taken by 3.5%) divided by the annualized standard deviation. The maximum drawdown is the maximum loss from a peak to a trough of a portfolio, before a new peak is attained.



(a) Nasdaq (b) HSI

(c) SH (d) SZ

Fig. 3. Cumulative return of AC, KACM, W-KACM, U-BAH and U-CBAL in the Nasdaq, HSI, SH and SZ.

which means that our W-KACM strategy can achieve abnormal returns in a real trading environment. Moreover, the W-KACM keep both high return and high equilibrium cost rate, which means W-KACM increases the return without any additional transaction. It demonstrates that the W-KACM is more efficient than the KACM.

In this paper, we assume that all trades are done at the closing price. We use closing price data in an empirical test mainly because it is easy to obtain. Our algorithm can be implemented at any moment of the trading day. An additional assumption is that all portfolio transactions can be implemented immediately with the quote price. All the transaction orders can be submitted to the exchange trading system instantly. However, there is no guarantee that all orders can be implemented with the quote price instantly. When market liquidity disappears, there is a risk that the order cannot be executed immediately. We can take some measures to eliminate the influence of trading friction. For example, we can choose some active stocks that trade frequently to construct the portfolio. An appropriate order submission strategy can also avoid the nonexecution risk.

An important problem associate with trading is bankruptcies and suspensions. All of the historical datasets are conditioned on the fact that all stocks are traded everyday and that there are no bankruptcies. It is necessary for the KACM strategy to take any bankruptcies and suspensions into account when trading. One realistic measure to this problem is to use a similar stock to replace the bankrupt or suspended stock in the portfolio.

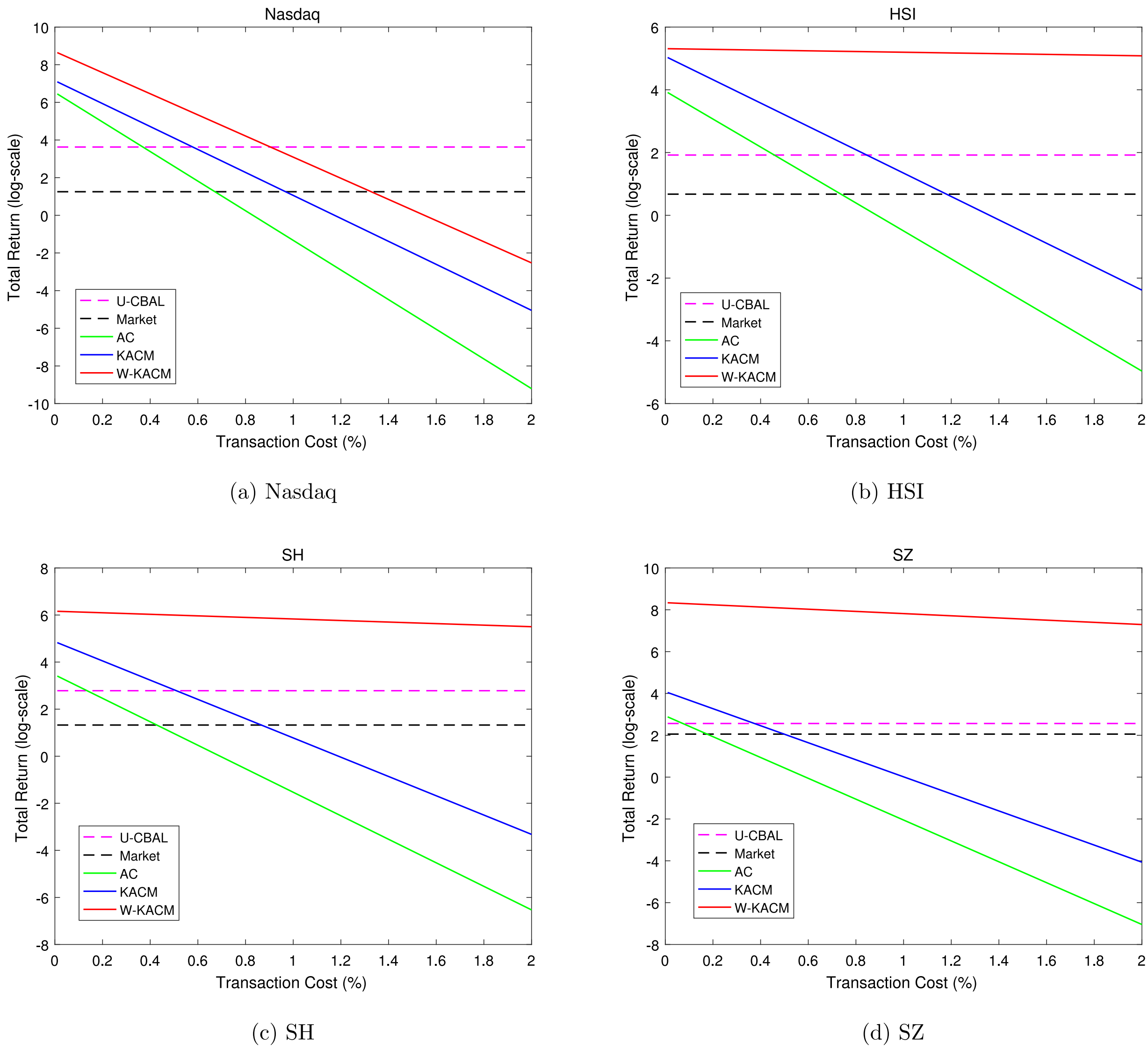


Fig. 4. Total return of AC, KACM and W-KACM, with proportional commission $\omega = 0.1\%, 0.2\%, \ldots, 2\%$.

5. Conclusion

We investigate the online portfolio selection problem and propose a new portfolio algorithm called the W-KACM. The W-KACM is based on the AC, which was proposed by Borodin in 2006. AC assumes that the stock price follows the mean reversion principle and makes bets on the consistency of the positive lagged cross-correlation and the negative autocorrelation. Based on the statistical predictions, AC transfers wealth from great stocks to losing stocks. AC has been proved to be a strongly performing algorithm in most historical datasets.

Our goal is to improve the performance of KACM and fully exploit the fluctuation of stock prices. Thus, we propose a new approach for online portfolio selection, namely W-KACM, to capture the properties of mean reversion and momentum in stock prices. Furthermore, to ensure the accuracy of statistical bets, the W-KACM combines the online portfolio selection algorithm with the Wavelet de-noise Kalman Filter. We use an improved measure of the CAPR, namely, WKFPR as the input of the algorithm. The WKFPR is a well-matched input for the online portfolio algorithm, that can more accurately measure how much the market price deviates from its own underlying trend price than the CAPR. The WKFPR makes it possible for the algorithm to accurately predict when the mean reversion occurs.

We use four untested historical datasets (including Chinese stock market datasets) to evaluate the performance of our new algorithm. The empirical results indicate that the W-KACM algorithm can beat the KACM and achieve strong abnormal returns without transaction costs in all datasets. Furthermore, we test the impacts of the transaction costs on the algorithms' performance and find that W-KACM can also realize profits after the real market transaction costs. Meanwhile, the W-KACM can tolerate higher market transaction cost rate than KACM, which means it obtain higher abnormal return without any additional transaction.

Meanwhile, the W-KACM can be directly applied to the real market without any modification. In our empirical, in order to simplify the testing process, we choose sample stocks to build a portfolio according to some selection principles. Actually, our new online portfolio selection algorithm can be used to any portfolio management in real market transaction. The W-KACM provide a optimal way for any type investors to manage their stocks and help they achieve more return than simple hold by regularly adjusting each stock's investment weight.

In conclusion, this paper propose a new online portfolio selection algorithm, W-KACM, that successfully combines Wavelet de-noise Kalman Filter with the benchmark AC to exploit the properties of stock price fluctuations. Empirically, W-KACM surpasses market, EG, benchmark Anticor and KACM in both simulation testing and in real markets.

CRediT authorship contribution statement

Gang Chu: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Software, Visualization, Writing - original draft, Writing - review & editing. **Wei Zhang:** Conceptualization, Funding acquisition, Project administration, Resources, Supervision, Validation. **Guofeng Sun:** Data curation, Formal analysis. **Xiaotao Zhang:** Conceptualization, Data curation, Methodology, Funding acquisition,Project administration, Resources,Supervision, Writing - review & editing.

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Additional information

Competing financial interests: The authors declare no competing financial interests.

Data Availability

The datasets generated and analyzed during the current study are available from the corresponding author on reasonable request.

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