

PHYC40970- Advanced Laboratory II



Surface Tension

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Abstract

This paper describes a simple technique for determining the coefficient of the surface tension of liquids, based on laser light diffraction on capillary waves. We obtain the coefficient of the surface tension of liquids by applying the dispersion relation for capillary waves and analyse the changes in said coefficient when analysed under different conditions such as increasing temperature, adding salt and adding detergent. Also explored in this report is the surface tension of whiskey.

Introduction

Surface tension is a measure of the force per unit length required to extend the surface of a liquid. Surface tension can also be described as a property of water that causes the surface to behave like a film as a result of its tendency to minimise its surface area. The behaviour of a liquid can be predicted based on its surface tension. Factors that affect the surface tension of pure liquids include its polarity, purity, density and temperature[1]. The surface tension of solutions also depends on the solute.[2] The aim of this study was to investigate the property of the surface tension of water at different temperatures in order to better understand the behaviour of water under different conditions. Since some of these conditions are difficult to control, knowledge of surface tension at a variety of conditions is especially useful because it allows for adjustments to be made based on these conditions rather than trying to control the conditions themselves.

As stated above, surface tension can be observed as the surface of liquid behaving like a thin, flexible membrane. This is the result of a liquid's tendency to maintain a minimal surface area.[3] This is caused by an imbalance of cohesive forces at the surface of the liquid. Cohesive forces are attractive forces between molecules of the same kind. In water, hydrogen bonds are responsible for cohesion between the water molecules. Within the bulk of a liquid, each molecule is attracted by all of the molecules around it. This means that the net force on each molecule is zero. However, a molecule at the liquid-gas interface will only experience cohesive forces on one side, assuming that gas is an ideal gas that has no intermolecular forces. Therefore, there will be a net force perpendicular to the surface of the liquid acting upon molecules at the surface.[3] As a result of this, and because a sphere has the smallest possible surface area for a given volume, intermolecular attractive interactions between water molecules cause the droplet to adopt a spherical shape. This maximises the number of attractive interactions and minimises the number of water molecules at the surface. Hence raindrops are almost spherical, and drops of water on a nonpolar surface, which does not interact strongly with water, form round beads. [3]

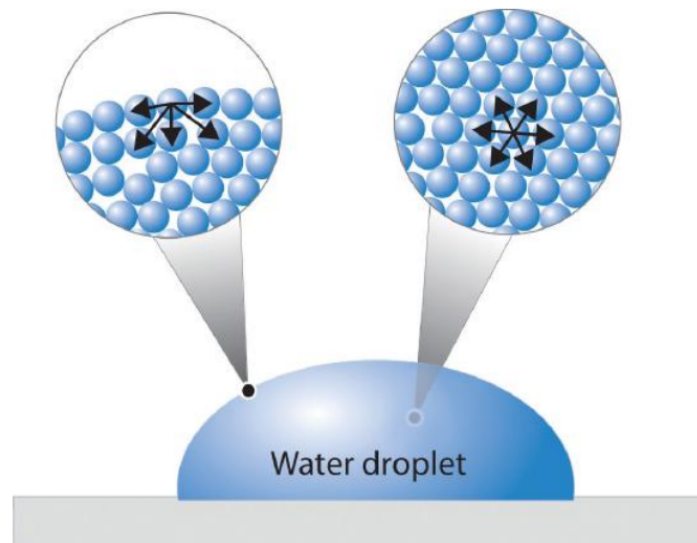


Fig.1 Molecules at the surface of water experience a net attraction to other molecules in the liquid, which holds the surface of the bulk sample together. In contrast, those in the interior experience uniform attractive forces.[3]

The same phenomenon holds molecules together at the surface of a bulk sample of water, almost as if they formed a skin. When filling a glass with water, the glass can be overfilled so that the level of the liquid actually extends above the rim. Similarly, a sewing needle or a paper clip can be placed on the surface of a glass of water where it seems to float, even though steel is much denser than water. Many insects take advantage of this property to walk on the surface of puddles or ponds without sinking.[3] In this report the effect of different conditions, such as temperature change, concentration of salt, concentration of soap and alcohol, on surface tension will be explored.

Theory

Surface waves on fluids with wavelengths of a few millimetres and less are well known capillary waves. Typically the amplitude of these waves is of the order of one micrometre. In this regime the dominant restoring force is the surface tension. In this report we describe a relatively simple method for the generation of capillary waves of known frequency on a liquid surface in the millimetre wavelength range.[1] When the laser beam obliquely impinges on capillary waves at a certain angle, steady and visible diffraction spots are formed. The wavelength data provide a very accurate dispersion relation. In particular, based on the dispersion relation, the relation between the surface tension and the temperature is investigated by laser diffraction.[1]

Since capillary waves serve as a reflective diffraction grating for light, the physical basics of diffraction on a reflection grating. Unlike the transmission diffraction grating in which the light passes through a series of narrow slits located between the opaque parts of a glass or plastic plate, with reflection gratings the light is reflected from a series of parallel, close, smooth surfaces and is absorbed or diffused by the surface areas located between them such as in **Fig.1**.^[4] Consequently, the destructive interference does not occur along every path where the reflected angle is not equal to the incidence angle, but on some of them the constructive interference will occur. As a result, a series of bright spots appears on the screen, rather than just one.

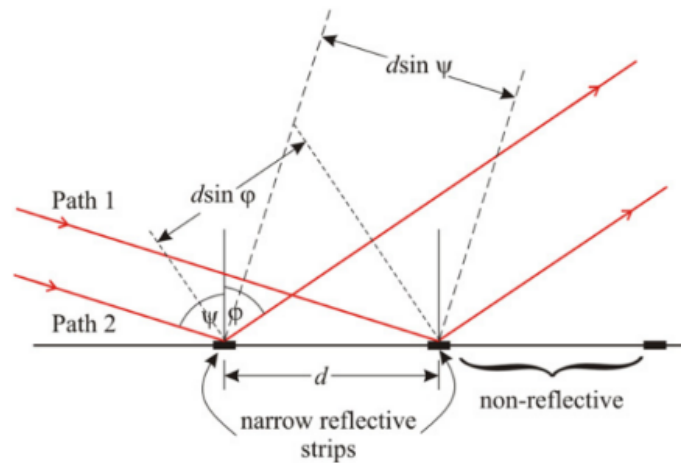


Fig.2 Path difference of rays observed on a reflection diffraction grating.^[4]

Path 1 is longer by $d \sin \psi$ from path 2 before the arrival of rays to the smooth part of the grating, but it is shorter from it by $d \sin \phi$ after reflection (d is the distance between adjacent reflective parts). When this difference is equal to the whole number of wavelengths of light the waves reflected from the adjacent reflective surfaces are in phase and this leads to the constructive interference;^[4]

$$d(\sin(\psi) - \sin(\phi)) = m\lambda$$

where $m = 0, \pm 1, \pm 2, \dots$

The integer m represents the order of the various principal maxima.

For further application it is convenient to introduce the grazing angle θ between the incident ray and the plane of the grating, for which this applies $\theta + \psi = \pi/2$ and the angle δ between the direction of the geometrically reflected ray and the direction of the first constructive interference $\psi - \phi = \delta$ (for $m = 1$).

It is shown that the grating equation for the first-order spot is;^[4]

$$\lambda = 2d \sin\left(\frac{\delta}{2}\right) \sin\left(\theta + \frac{\delta}{2}\right)$$

If we apply the same procedure to the first ray of constructive interference, but on the other side of the geometrically reflected ray for which $\varphi - 1 - \psi = \delta$, by an analogous procedure relation;

$$\lambda = 2 d \sin\left(\frac{\delta}{2}\right) \sin\left(\theta - \frac{\delta}{2}\right)$$

is obtained. Adding these two equations leads to the important formula; [4]

$$\lambda = d \sin\left(\frac{\delta}{2}\right) (\sin\left(\theta + \frac{\delta}{2}\right) + (\sin\left(\theta - \frac{\delta}{2}\right)))$$

The previous relation can be written using the wave number of capillary waves[4]

$$k = \frac{2\pi}{\lambda} \sin\left(\frac{\delta}{2}\right) (\sin\left(\theta + \frac{\delta}{2}\right) + (\sin\left(\theta - \frac{\delta}{2}\right)))$$

Experimental Procedure

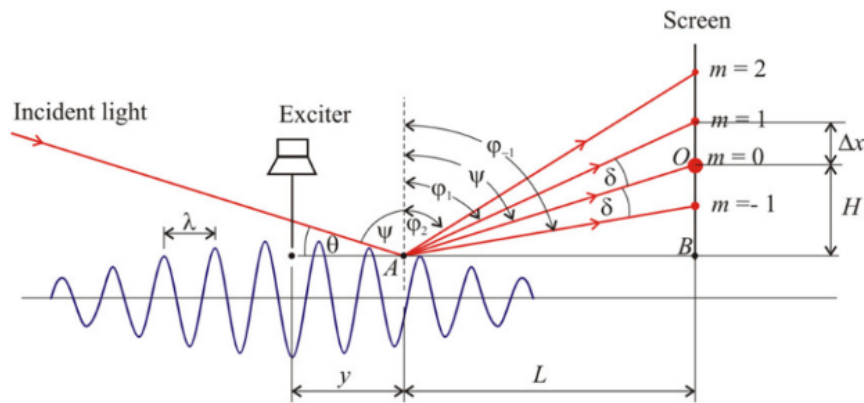


Fig.3 shows a sketch of the experiment.[4]

Using the formula found for k , we can now perform our experiment and experimentally determine the coefficient of surface tension using the following relation;

$$\frac{\rho \cdot \omega^2}{k^3} = \sigma$$

where σ = surface tension of liquid, ω = angular frequency, k = wavenumber

Using this equation for our experiment, it was necessary to know the density of the fluid on which the capillary waves were generated, their wavelength and frequency. A sketch of the experimental set-up for measuring these quantities is shown in **Fig.3**.

The equipment used in this experiment comprised of a digital signal generator, an exciter needle, a dish of about 2 cm in depth for holding the liquid, a screen with a grid of with divided into millimetre squares, a helium–neon (He–Ne) laser of 632.8 nm wavelength, a hot plate and thermometer, salt, detergent, deionized water and a liquid of choice of which I chose whiskey. It was crucial that we used deionized water within this experiment in order to eliminate any impurities having an effect on our data.

The exciter needle was positioned with the tip touching the water surface. It was excited by the digital signal generator by low power sine signals in the frequency range from 25 to 225 Hz, in steps of 25 Hz. We used the He–Ne laser as a light source. Special care should be taken that the laser beam was kept away from the walls of the container and also not hitting the exciter. If this were to happen the diffraction pattern would be skewed and the data could not be used. The position of the capillary wave exciter needle was chosen so that the needle was in the plane with the incidence and the reflected ray. In our experiment, the modulation point, the point where the exciter needle touches the surface of the liquid, was kept roughly 1 cm away from the incidence point of the laser beam. The diffraction patterns for each of the frequencies used were recorded in photographs on graph paper of 1 millimetre squares.

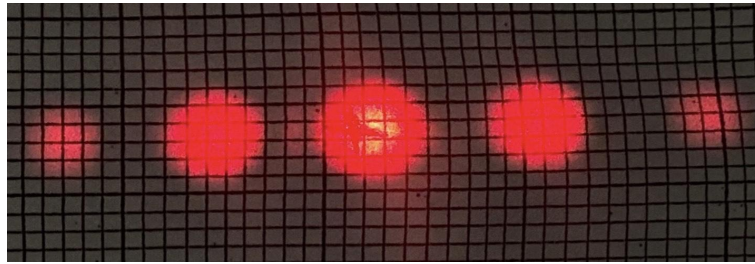


Fig.4 Diffraction pattern for 20 degree water at 175 Hz

The $m = 0$ maximum was marked on the graph paper and used to make sure the value of H stayed consistent throughout the whole experiment. This was important as the smallest change in the height of the water in the tray would change the position of this central maximum. The number of visible diffraction maxima depended on the selected frequency and the output power of the signal generator. Therefore, it was sometimes necessary to lower or raise the amplitude of the signal generator in order to get a clear pattern which data could be extracted from. It is important to note that changing the amplitude of the wave generator does not affect the distance Δx but just allows more maxima to become visible. With increased frequency, we observed an increase in the distance Δx between the first adjacent spots for $m = \pm 1$ and vice versa. As the Δx was such a small distance being measured by hand, it was found that more accurate results were found by measuring the distance between the first order maxima and dividing it in half as opposed to measuring the distance from the first order maximum and the $m = 0$ central maximum. The horizontal distance $L = (5.41 \pm 0.01) \text{ m}$, from the incidence point of the laser beam into the tray with the liquid to the screen, was measured by a tape measure and a spare laser which was used to have a straight accurate line to measure and any deviations from the perpendicular distance would add error to our calculations. This technique was also used to measure the H value for our set up ($H = .40 \pm 0.01 \text{ m}$). The value of this angle is calculated approximately from $\theta = \arctan(\frac{H}{L})$. [4] This value was kept consistent throughout the experiment and constantly monitored. The angle of diffraction δ was determined from the approximate formula for small angles $\delta = \arctan(\frac{\Delta x}{L})$ [4] by measuring the distances Δx and L the value could be found for each step of the experiment. The main source of error in the experiment came from the value of Δx as using the graph paper as a measurement there was a significant error with the smaller of 0.5 millimetres. This had a larger relative error in the smaller frequency recordings than the higher frequencies. Densities of liquids were also measured for each concentration. This was done by using a weighing scale to measure the mass of the liquid and then by knowing the volume of liquid and its mass we could calculate the density and use it in our data analysis.

Results and Discussion

Temperature vs Surface Tension

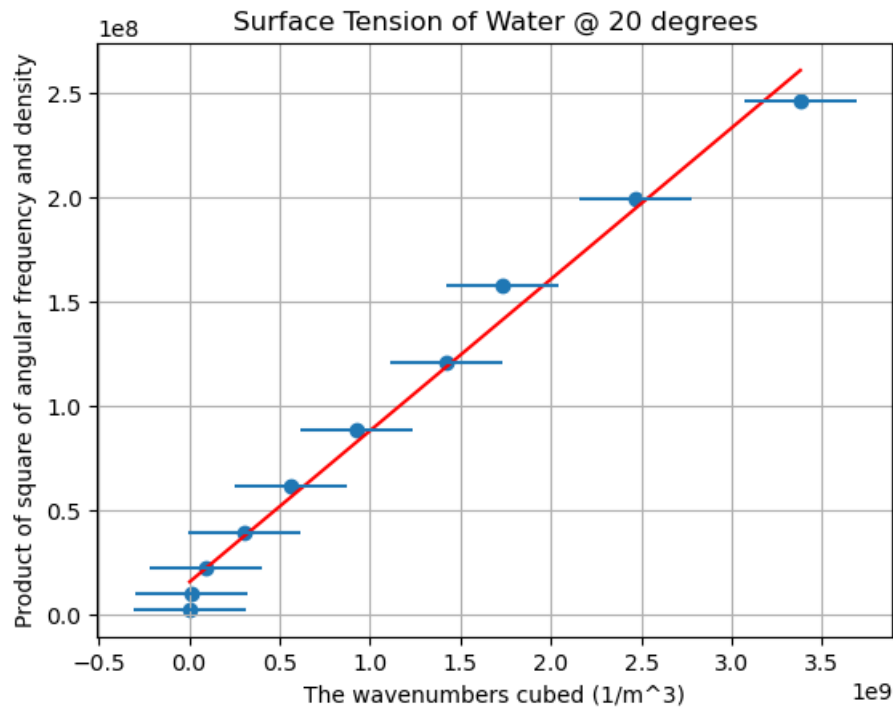


Fig.5 Linearized dependence $\omega^2 \rho$ on the wavenumber k^3 for 20 degree water.
The slope $\sigma = 0.0726 \pm 8.32E-6$ N/m is the value found for surface tension.

Interfacial tension or surface tension is sensitive to temperature. Surface tension has been shown experimentally to decrease nearly linearly with temperature. The increase in temperature causes a corresponding decrease in cohesive intermolecular forces and this surface tension decreases. For example, the surface tension of water and air has an accepted value of 72.8 mN/m at 20°C,[5] while the surface tension at 25°C is 72.0 mN/m. During our experiment, the surface tension of water and air was found to be 72.6 mN/m at 20°C, as seen in **Fig.5**.

If the temperature increases, the surface tension will decrease because the strength of intermolecular forces between water molecules decreases as temperature increases. The cohesive force in water results primarily from hydrogen bonds between water molecules. The strength of hydrogen bonds depends on their length; as the bond length increases, the strength decreases. [6] As temperature increases, water molecules gain kinetic energy and move away from each other. Hence as the temperature of water increases, the hydrogen bond length between water molecules increases and the hydrogen bond strength decreases. Therefore, surface tension should decrease as temperature increases since it is a measure

of the strength of cohesive forces. This hypothesis agrees with the experimentally found data shown in Fig.6. It was found that as temperature increased, the surface tension fell at a rate of $-1.66\text{E-}4 \pm 1.41223330 \text{ E-}10 \text{ N/m} / ^\circ\text{C}$. As we can see, as the temperature increased so did the error bars in x-direction. This was due to the rapid cooling of the water sample over the course of the data acquisition. In hindsight, this error could have been somewhat avoided by the use of a styrofoam tray to hold the water in order to minimise the heat loss.

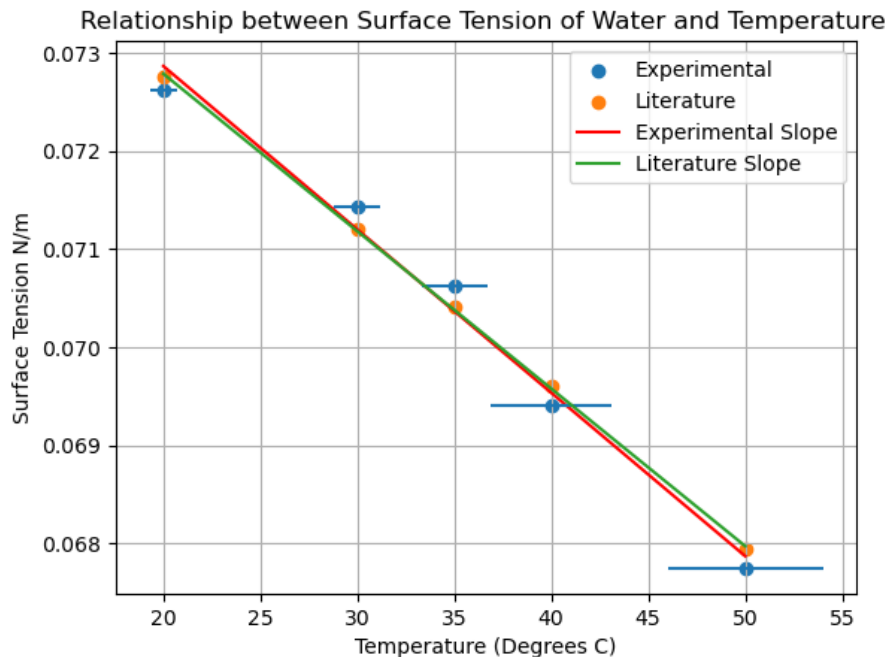


Fig.6 Relationship between Surface Tension of Water and Temperature
The slope = $-1.66\text{E-}4 \pm 1.41 \text{ E-}10 \text{ N/m} / ^\circ\text{C}$

However, comparing our data to the values found in *The International Tables of the Surface Tension of Water* [7], there is an extremely strong agreement with just a 3.6% error between both slopes.

Soap Concentration vs Surface Tension

Surfactants lower surface and interfacial tension by adsorbing to the surface or interface. In general, surfactants are amphiphilic molecules, meaning they have both hydrophobic and hydrophilic components.[5] This allows the soap or detergent to grab onto the grease from a dirty dish and use the other end of the detergent molecule to latch on to water to be washed away. In the presence of a water-air or water-oil interface, for example, the surfactant aligns itself at the interface such that the hydrophilic component is in the water and the hydrophobic component is in the other phase. By attempting to move away from the water molecules, the hydrophobic ends of the detergent molecules push up to the surface. This weakens the hydrogen bonds holding the water molecules together at the surface. The soap in the water will result as a break in the surface tension of water. The presence of the surfactant disrupts the cohesive forces between water molecules at the interface and reduces the surface or interfacial tension. [8]

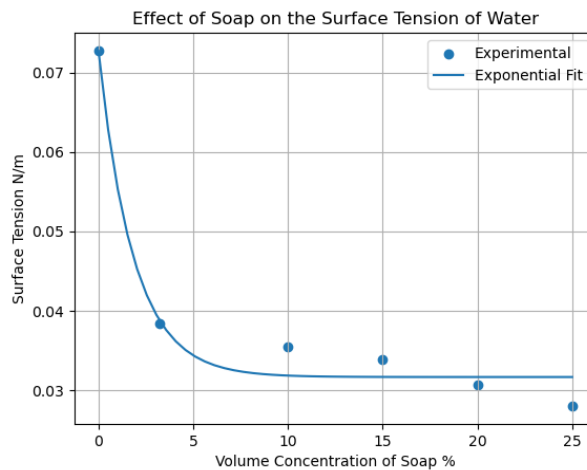
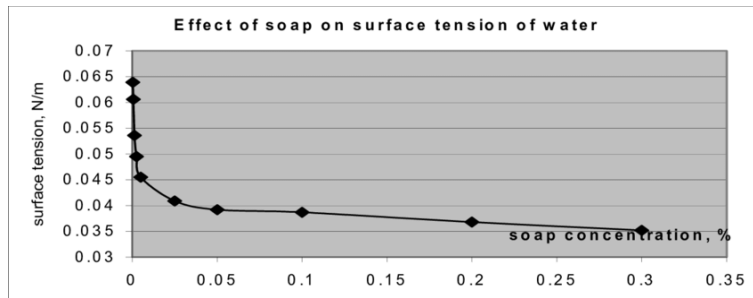


Fig.7 Relationship between Soap Concentration vs Surface Tension

(a) Shows previously found experimental results [9]

(b) Shows the results found in this report

The exponential fit, $y = 0.031637155 + 0.041078873\exp(-0.54213779x)$

In the data acquisition in this segment of the experiment, I underestimated the effect soap would have on the surface tension of water. I made my measurements in steps of 5% concentration of soap whereas ideally I should have taken measurements at considerably smaller concentrations. As we can see from **Fig.7(a)**, the surface tension of water was affected massively by as small of a concentration as 0.01 %. Although the shapes of the fits are similar it is not possible to determine if the fits would agree to a significant degree. If I was to repeat this experiment, I would repeat this segment specifically and take smaller measurements.

However, in the findings we did indeed find the surface tension did decrease at a rate of $-4.78\text{E-}4 \pm 9.9\text{E-}10 \text{ N/m} / \%$ as the concentration was increased, as can be seen in **Fig.8**.

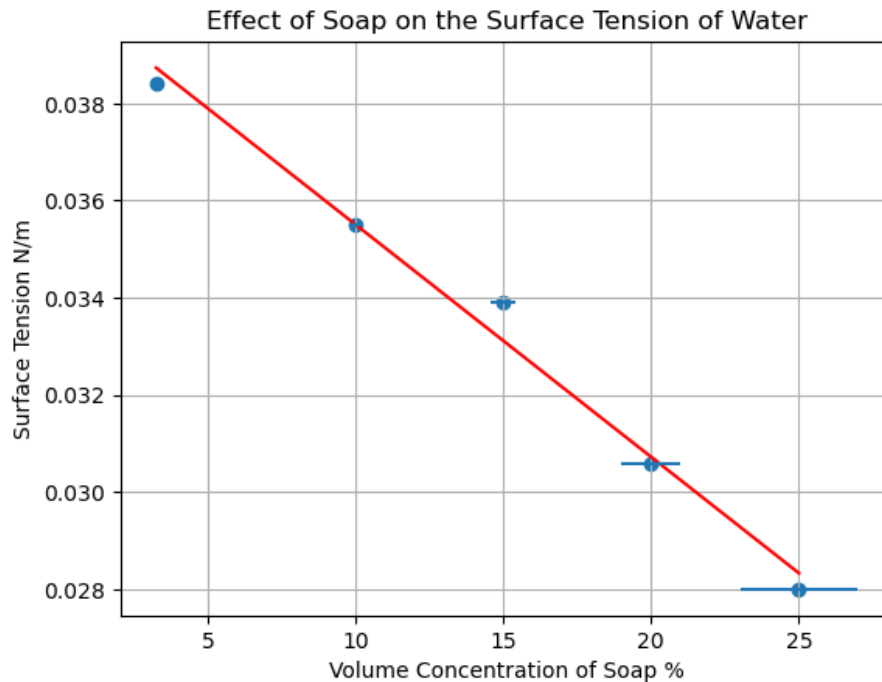


Fig.8 Relationship between Surface Tension of Water and Soap Concentration
The slope = $-4.78E-4 \pm 9.9E-10$ N/m / %

Salt Concentration vs Surface Tension

It is a very common misconception that salt is a compound that either lowers or breaks surface tension otherwise known as a surfactant. However, experiments done with salt water show that surface tension actually increases when salt is added to pure water. [8]

Sodium chloride is a strong electrolyte, which means it completely dissociates into Na cations, and Cl anions, when placed in H₂O. The strong interactions between the Na cations and the partial negative O atoms as well as the Cl anions and the partial positive H atoms, although they disrupt part of the hydrogen bonding that takes place between water molecules, they actually strengthen the water's surface tension because the ion-dipole interactions are stronger than the hydrogen-bonding interactions. [10]

As seen below in **Fig.9**, our experimentally found data shows that with an increasing concentration of salt that the water's surface tension is strengthened. This is one reason we use deionized water instead of just tap water as any presence of impurities or salt in the water may skew our data. When mixing the solution for 30% salt concentration, it was found that the salt would not fully dissolve at room temperature. Therefore, as the solution had reached saturation, a value of 28% was used initially with a 2.5% error in order to compensate as the saturation for salt in water is around this value and it would be impossible to find the exact percentage, however after further review, it was decided that the 30% value would not be used in our plots.

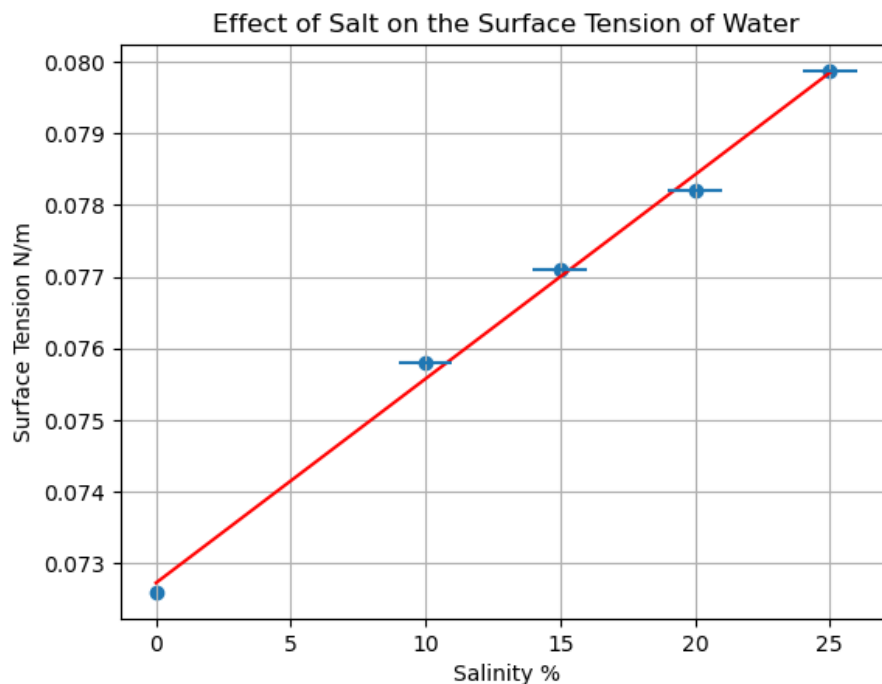


Fig.9 Relationship between Surface Tension of Water and Salinity
The slope = $2.85\text{E-}4 \pm 1.15\text{E-}10$ N/m / %

When comparing our data to literature we see our results are in great agreement with previously found values. As seen in **Fig.10**, our slope lines up strongly with the other two sources. In order to compare our data to the other sources, it was necessary to convert our salinity percentage to a molarity unit instead. This was done by using the molecular weight of salt and the amount of moles added to the water per litre or kilogram of water.

Comparing our slopes to literature it is found they are in strong agreement with our slope; ($m = 1.66\text{E-}4 \pm 4.29\text{E-}9$, and the slopes found in literature $m_1 = 1.57\text{E-}4 \pm 1.31\text{E-}9$, [2] $m_2 = 1.62\text{E-}4 \pm 1.97\text{E-}9$ [11]). The measured slope differs by a percentage difference of 5.4% and 2.4% respectively. In light of this, it is interesting to think about the possible relationship between surface tension and the boiling point of water. It is often said when chefs add salt to water that the boiling point is increased. However as we proved earlier, as the temperature increases, surface tension decreases. At the boiling point, every molecule of a liquid is in motion from bottom to surface, and due to rise in temperature, molecules lose the adhesion and hence, surface tension becomes zero. So with this knowledge also, any effects due to surface tension at the boiling point are negligible. [13]

Water boils when the molecules are able to overcome the vapour pressure of the surrounding air to move from the liquid phase to the gas phase. When you add a solute that increases the amount of energy needed for water to make the transition. The ion-dipole interaction present when salt is added is stronger than the hydrogen bonding between the water molecules, so more energy is needed to move water away from the ions and into the vapour phase and has nothing to do with the surface tension increase due to added salts.

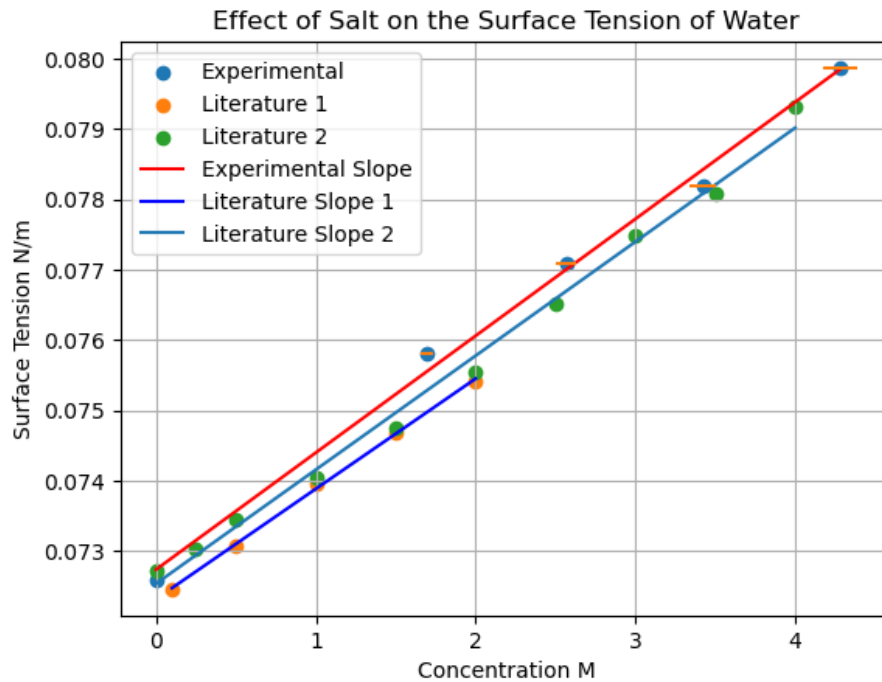


Fig.9 Relationship between Surface Tension of Water and Salt Concentration [2][11]

Surface Tension of Whiskey

The reason I chose to analyse whiskey's surface tension was due to the fact that I have heard many times that adding water to a glass of whiskey enhances the flavour and aroma produced by said whiskey. The flavour typical of whiskey, can be traced to a group of flavour-packed molecules known as phenols, and to one in particular called guaiacol. Laboratory simulations revealed that adding a splash of water makes guaiacol rise to the air-liquid interface and because obviously the whiskey is consumed at the interface first, the aroma and taste is made stronger. [13]

I have also observed when you swirl a glass of whisky, some of the liquid sticks to the side of the glass and then begins to form trails as it flows back into the bottom of the glass. These are known as the "legs".

I wondered why this would be happening. And from my research, I have found it is to do with alcohol having a lower surface tension than water. The swirling or agitation of the whisky results in it coating the inside surface of the glass due to capillary action. The alcohol is more volatile than the water so it starts to evaporate more rapidly and lose strength.

As there is now more of a concentration of water, the surface tension is increased. This then pulls some of the whisky up the sides of the glass. This then coheres and falls back under its own weight into the mass of liquid as "legs". This is known as the Marangoni Effect, which records the flow of liquid caused by surface tension gradients.[14][15]

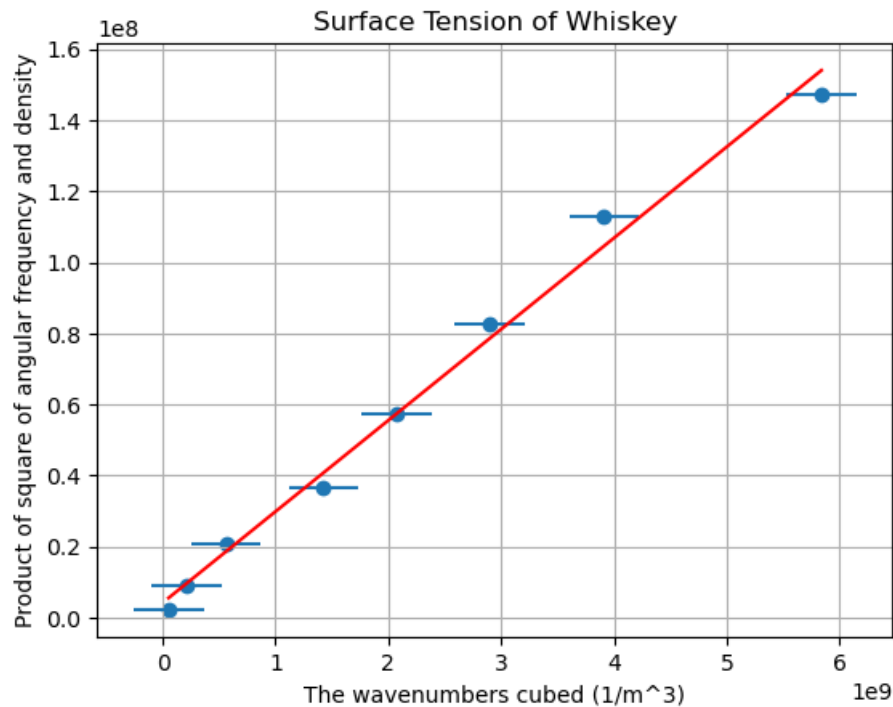


Fig.10 Linearized dependence $\omega^2 \rho$ on the wavenumber k^3 for whiskey. The slope $\sigma = 0.0256 \pm 9.5E-7$ N/m is the value found for surface tension.

For the purpose of simplicity of explanation, I have taken whiskey to just contain ethanol and water, we can then treat it as an ethanol-water solution and ignore any other additives. Alcohol has one O–H bond which is polar but a large portion of the molecule is made up of C–H bonds which are nonpolar. Alcohol molecules do not attract each other as strongly as water molecules and have a weaker surface tension. As stated previously in this report, water molecules are polar and are very attracted to each other. Alcohol molecules are polar in only one area, making them somewhat attracted to each other but they are not as attracted to other alcohol molecules as water is to other water molecules.[8] As seen above in **Fig.10**, whiskey at room temperature has a significantly lower surface tension than water at room temperature as hypothesised previously. Adding water to our whiskey would increase the surface tension due to the concentration of ethanol decreasing as can be seen from previous studies.

Conclusion

From our results, it can be clearly seen that the majority of the data analysis agreed with the expected results.[1][2][11][9] We can clearly see that we were able to calculate the values for surface tension accurately and analyse the relationships of certain conditions on the surface tension effectively. Although, if I were to repeat this experiment again I would have measured smaller increments of soap concentration so I would be able to compare my results accurately to theory and previous carried out experiments. There may have also been errors which arose due to the apparatus for measurement being very sensitive when the measurements are carried out in real life conditions. For example, the flickering of interference pattern images was caused by any external vibrations; people passing in the hall, students speaking too close to the tray of water, even any loud noises down the corridor from other labs. These effects of these occurrences were avoided for the most part could not be completely eliminated and consequently some of the diffraction images obtained were imperfect and possessed vertical vibrations. However, aside from this I believe that this report was effective in investigating the dynamic surface tension of water.

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Appendix - Python Script for Surface Tension

```
In [ ]: import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit
```

```
In [235... def k(theta,delta,A): #wavenumber formula
    return (2*np.pi/A) * np.sin(delta/2)* (np.sin(theta + (delta/2)) + np.sin(theta))

L=5.14 # Value for L - horizontal distance

H=.40 # Value for H - vertical distance

p_water = 997 #density of water kg/m3
p_whiskey = 932.5 #density of whiskey kg/m3

w1 = 2*np.pi*(np.array([25,50,75,100,125,150,175,200,225,250])) # angular frequency
w2 = 2*np.pi*(np.array([25,50,75,100,125,150,175,200])) # different data sets didn't work
w3 = 2*np.pi*(np.array([25,50,75,100,125,150,175,200,225]))

theta = np.arctan(H/L)

k_err = np.array([677]) #error on wavenumber dependant mostly on x but changes were made

def delta(x,L):
    return np.arctan(x/L)

def f(m,x,c): #used with curve_fit to find variances in slopes
    return m*x + c
```

Temperature vs Surface Tension

20 degrees water

```
In [35]: water = np.loadtxt("20_degrees_water.txt")
water = water/2E3
k=k(theta,delta(water,L),632E-9) #wavenumber

plt.scatter((k)**3,(w1**2)*(p_water))

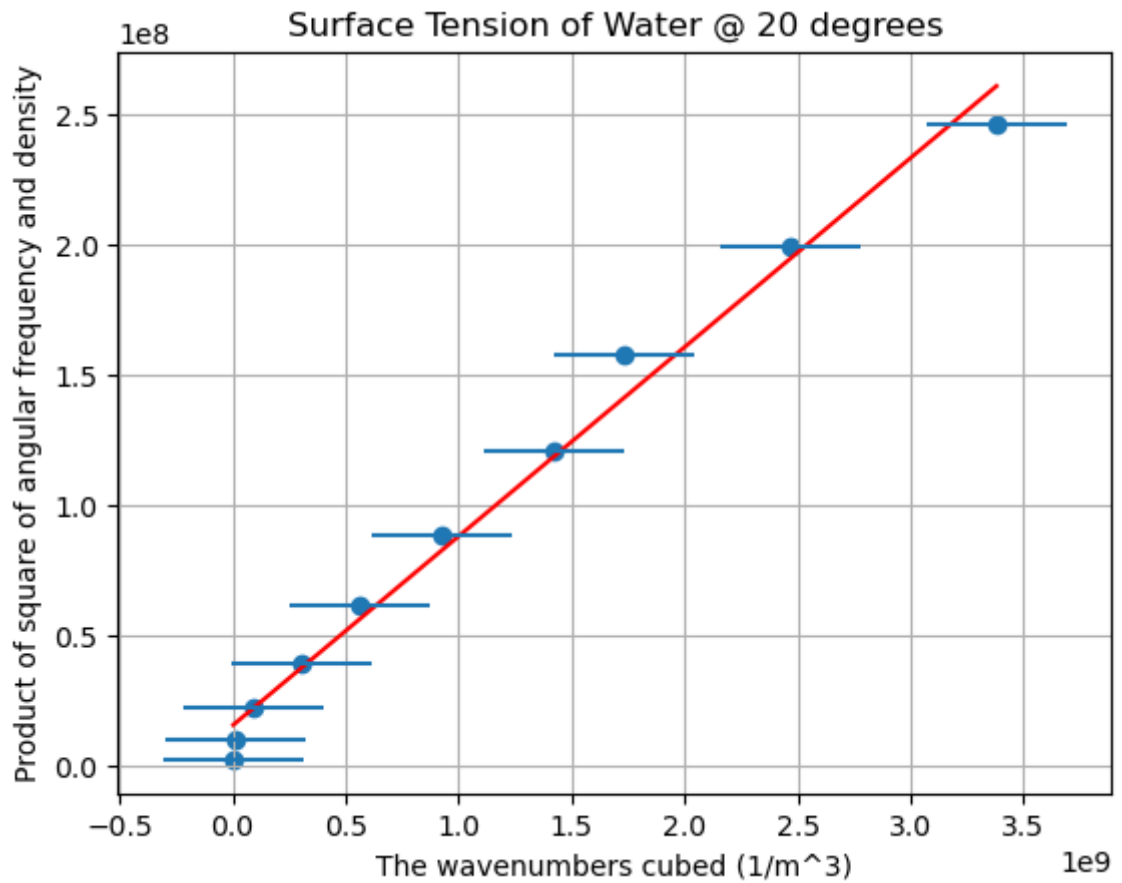
m,c = np.polyfit((k)**3,(w1**2)*(p_water), 1) #slope and intercept
plt.plot((k)**3,((k)**3)*m + c, "r")

plt.errorbar((k)**3,(w1**2)*(p_water),xerr=(k_err)**3, ls="none")

print(m)

plt.grid(True)
plt.ylabel("Product of square of angular frequency and density")
plt.xlabel("The wavenumbers cubed (1/m^3)")
plt.title("Surface Tension of Water @ 20 degrees");

0.07262327996872742
```



30 degrees water

```
In [43]: water = np.loadtxt("30_degrees_water.txt")
water = water/2E3

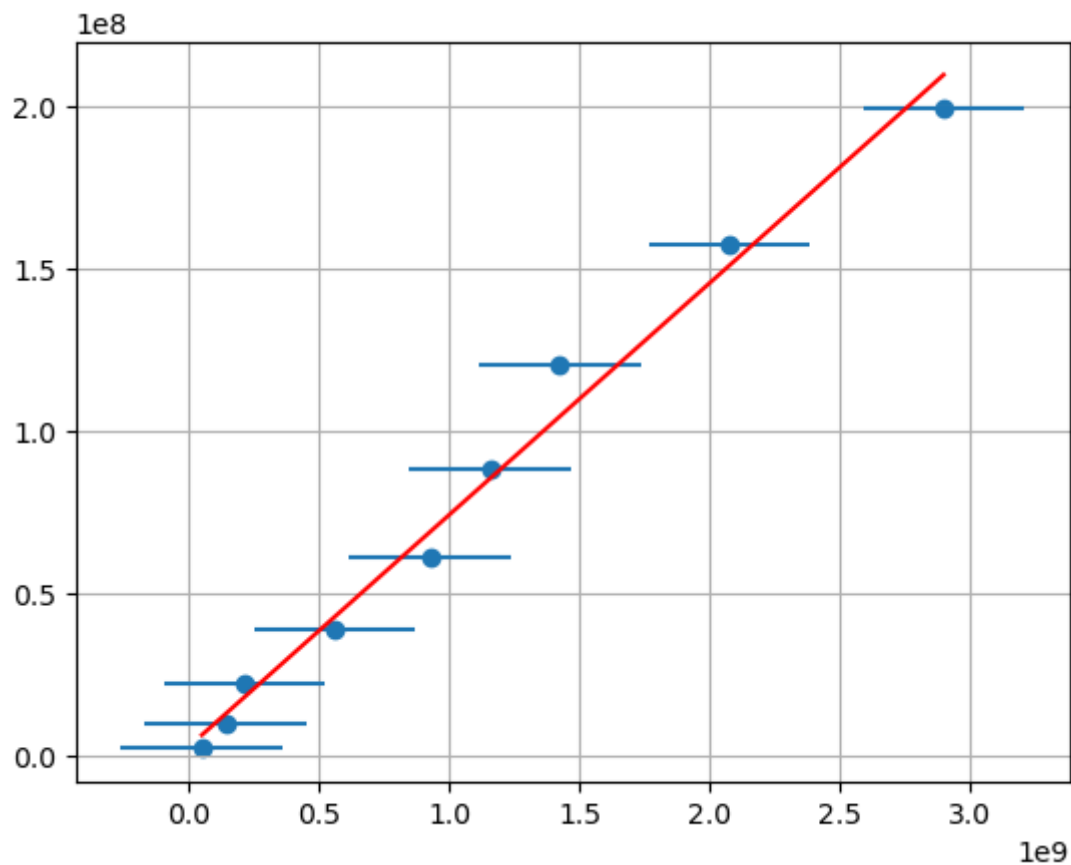
k=k(theta,delta(water,L),632E-9)

plt.scatter((k)**3,(w3**2)*(p_water))

plt.errorbar((k)**3,(w3**2)*(p_water),xerr=(k_err)**3, ls="none")
plt.grid(True)

m,c = np.polyfit((k)**3,(w3**2)*(p_water), 1)
plt.plot((k)**3,((k)**3)*m + c, "r")
m
```

Out[43]: 0.07142600833605048



35 degrees water

```
In [45]: water = np.loadtxt("35_degrees_water.txt")
water = water/2E3

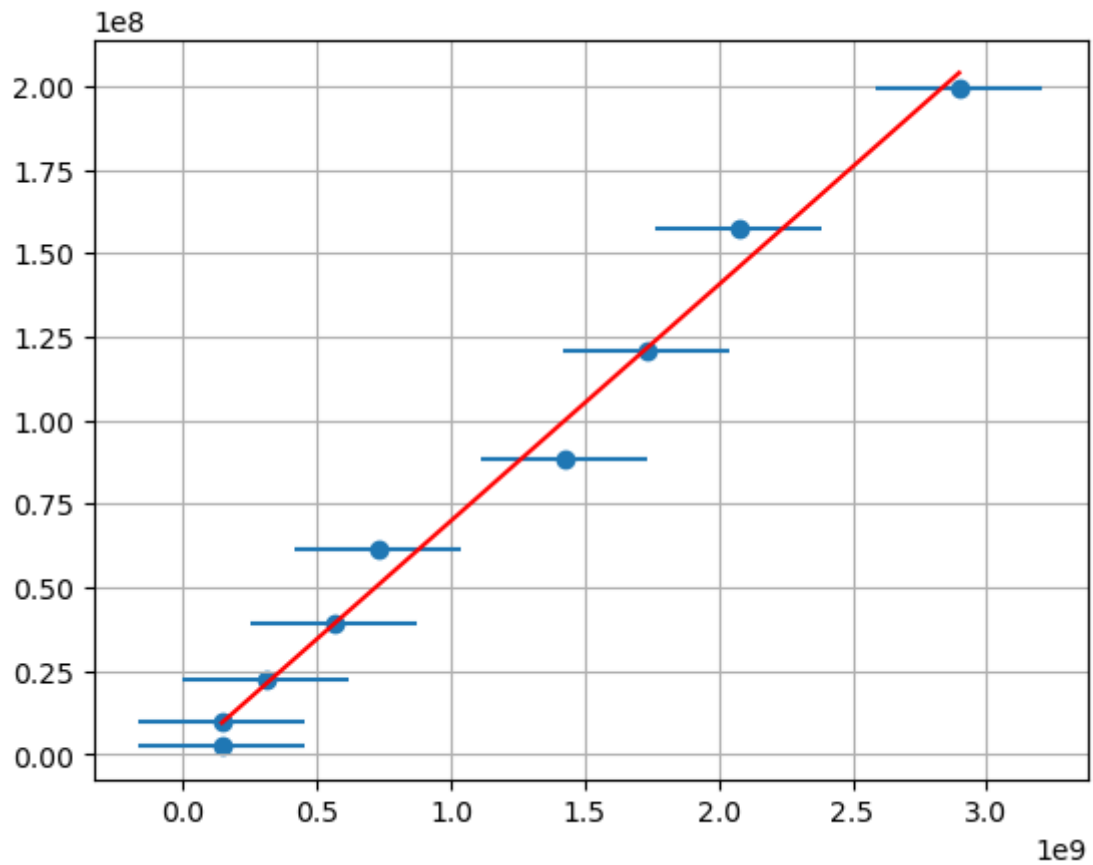
k=k(theta,delta(water,L),632E-9)

plt.scatter((k)**3,(w3**2)*(p_water))

plt.errorbar((k)**3,(w3**2)*(p_water),xerr=(k_err)**3, ls="none")

plt.grid(True)
m,c = np.polyfit((k)**3,(w3**2)*(p_water), 1)
plt.plot((k)**3,((k)**3)*m + c, "r")
m
```

Out[45]: 0.07062695375389566



40 degrees water

```
In [47]: water = np.loadtxt("40_degrees_water.txt")
water = water/2E3

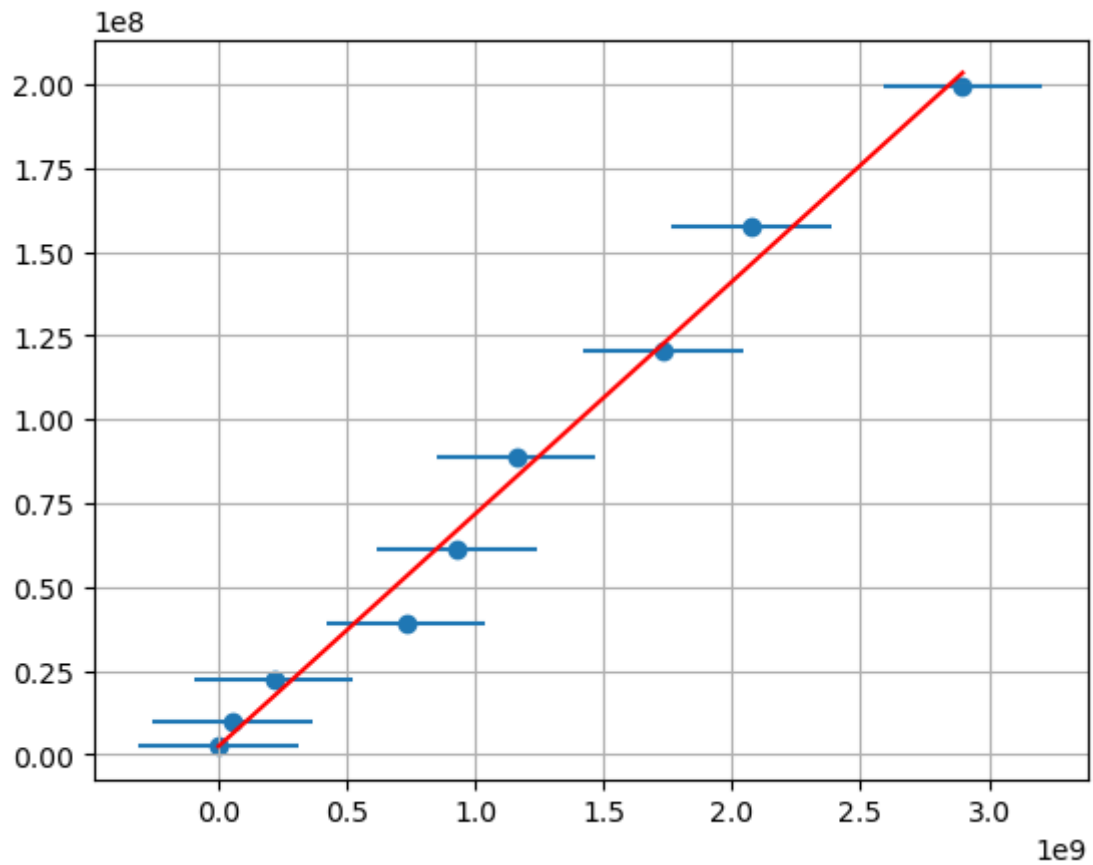
k=k(theta,delta(water,L),632E-9)

plt.scatter((k)**3,(w3**2)*(p_water))

plt.errorbar((k)**3,(w3**2)*(p_water),xerr=(k_err)**3, ls="none")

plt.grid(True)
m,c = np.polyfit((k)**3,(w3**2)*(p_water), 1)
plt.plot((k)**3,((k)**3)*m + c, "r")
m
```

Out[47]: 0.06940824277491699



50 degrees water

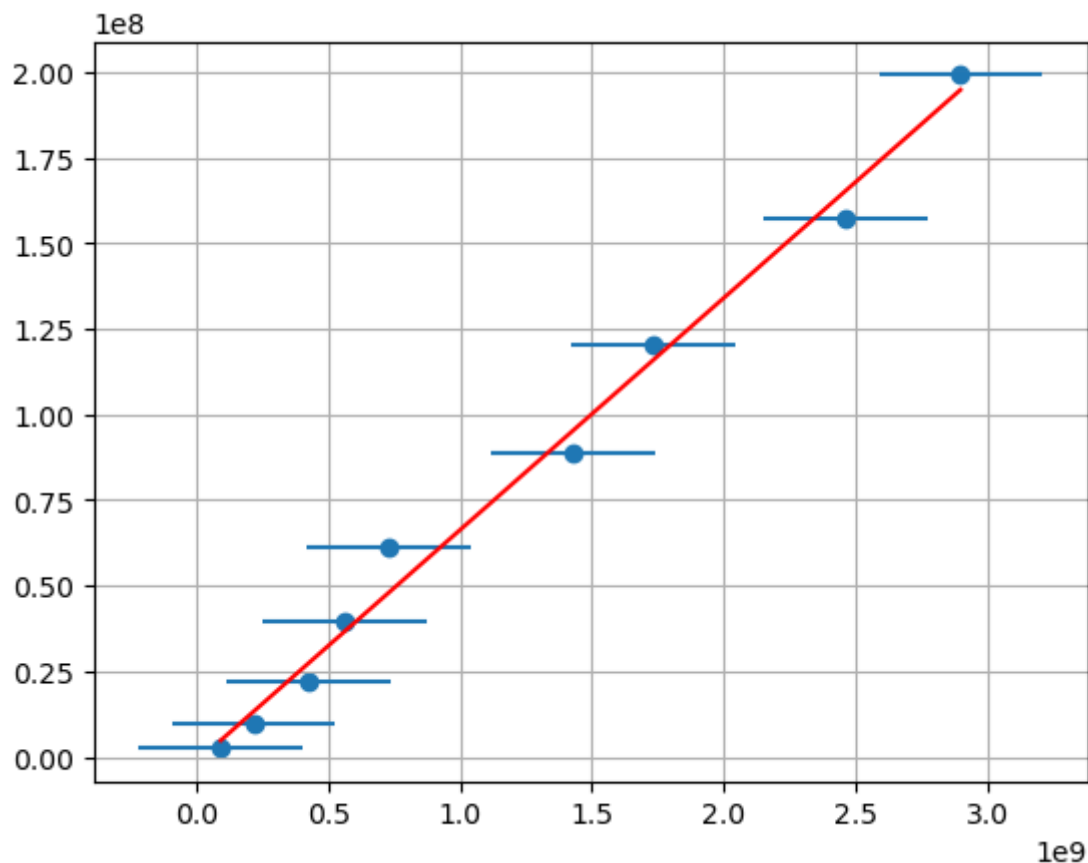
```
In [49]: water = np.loadtxt("50_degrees_water.txt")
water = water/2E3
k=k(theta,delta(water,L),632E-9)

plt.scatter((k)**3,(w3**2)*(p_water))

plt.errorbar((k)**3,(w3**2)*(p_water),xerr=(k_err)**3, ls="none")

plt.grid(True)
m,c = np.polyfit((k)**3,(w3**2)*(p_water), 1)
plt.plot((k)**3,((k)**3)*m + c, "r")
m
```

Out[49]: 0.0677551684416101



Relationship between Surface Tension of Water and Temperature

In [229...

```

temps = np.array([20,30,35,40,50]) #temperature

lit_s_t = np.array([.07275,0.0712,.07041,0.06960,0.06794]) #literature surface tension
s_t = np.array([0.07262,0.07143,0.07063,0.06941,0.06775]) # experimental surface tension

plt.scatter(temps,s_t)
plt.scatter(temps,lit_s_t)

m1,c = np.polyfit(temps,s_t, 1)
plt.plot(temps,temps*m1 + c, "r")

temp_err = ([.7,1.2,1.7,3.1,4]) #error on temperature
plt.errorbar(temps,s_t,xerr=(temp_err), ls="none")

y_err = np.array([8.32E-6,1.02E-5,5.88E-6,7.8E-6,6.6E-6]) # error on surface tension
plt.errorbar(temps,s_t,xerr=(y_err), ls="none")

plt.grid(True)

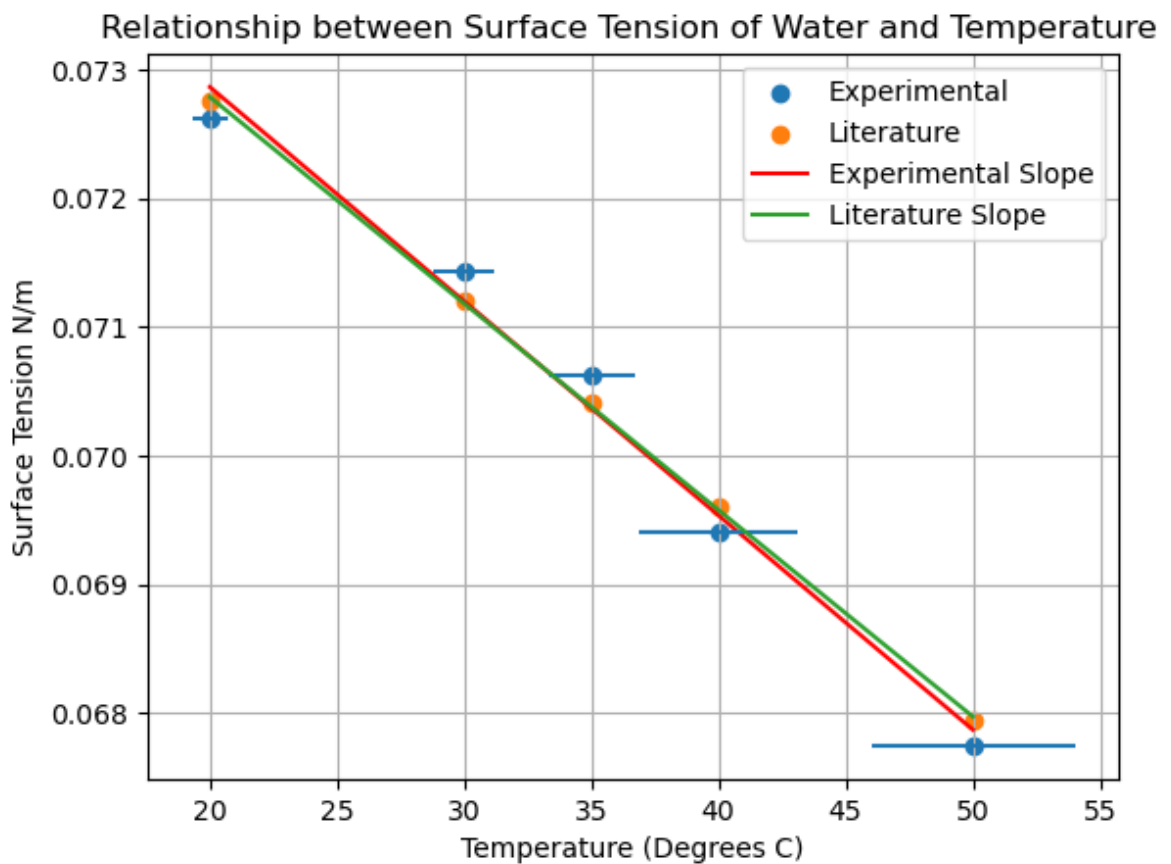
m,c = np.polyfit(temps,lit_s_t, 1)
plt.plot(temps,temps*m + c)

print(m1)
print(m)

plt.grid(True)
plt.xlabel("Temperature (Degrees C)")
plt.ylabel("Surface Tension N/m")
plt.title("Relationship between Surface Tension of Water and Temperature ")
plt.legend(["Experimental","Literature","Experimental Slope", "Literature Slope"],

```

-0.00016630000000000076
-0.00016029999999999997



Soap Concentration vs Surface Tension

10 ml Soap

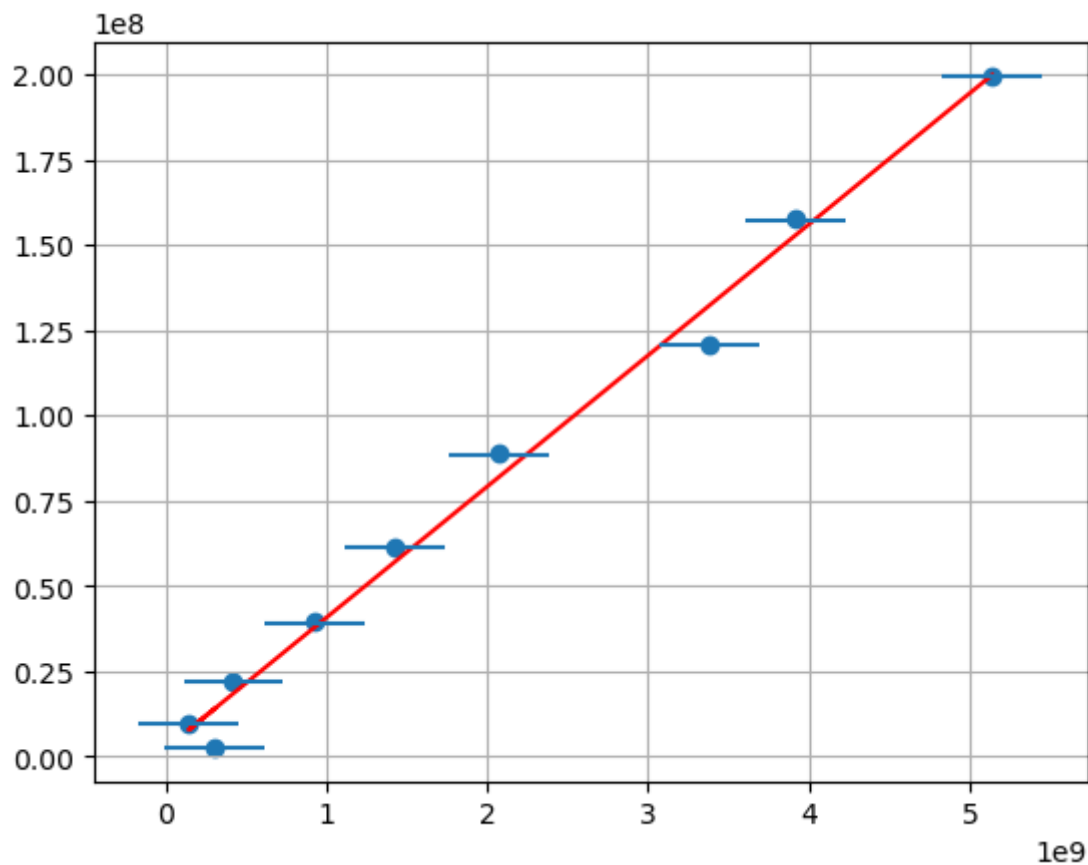
```
In [51]: soap = np.loadtxt("soap-10ml.txt")

soap = soap/2E3
k=k(theta,delta(soap,L),632E-9)

plt.scatter((k)**3,(w3**2)*(999))
m,c = np.polyfit((k)**3,(w3**2)*(999), 1)

plt.plot((k)**3,((k)**3)*m + c, "r")
plt.errorbar((k)**3,(w3**2)*(p_water),xerr=(k_err)**3, ls="none")
plt.grid(True)
m
```

Out[51]: 0.03843644195455484



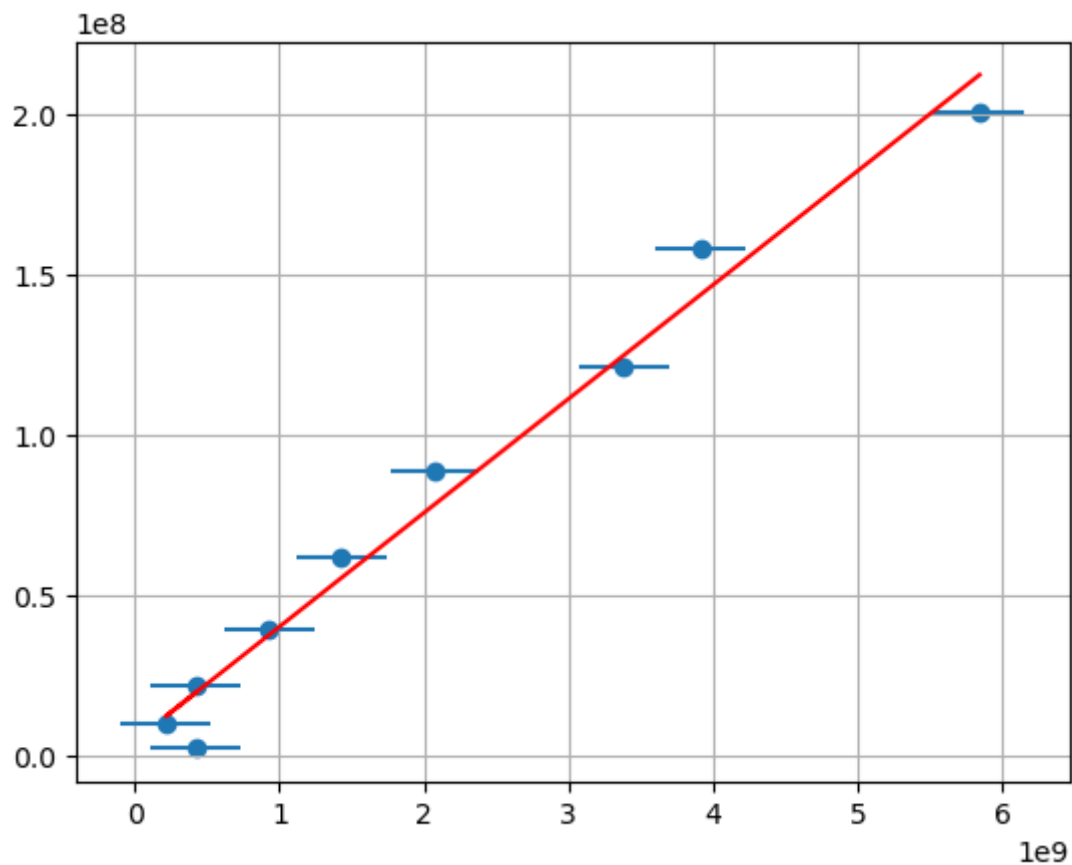
30 ml Soap

```
In [65]: soap = np.loadtxt("soap-30ml.txt")
soap = soap/2E3
k=k(theta,delta(soap,L),632E-9)
plt.scatter((k)**3,(w3**2)*(1002.5))

m,c = np.polyfit((k)**3,(w3**2)*(1002.5), 1)
plt.errorbar((k)**3,(w3**2)*(1002.5),xerr=(k_err)**3, ls="none")

plt.grid(True)
plt.plot((k)**3,((k)**3)*m + c, "r")
m
```

Out[65]: 0.035535518756914346



50 ml Soap

```
In [69]: soap = np.loadtxt("soap-50ml.txt")
         soap = soap/2E3
         k=k(theta,delta(soap,L),632E-9)

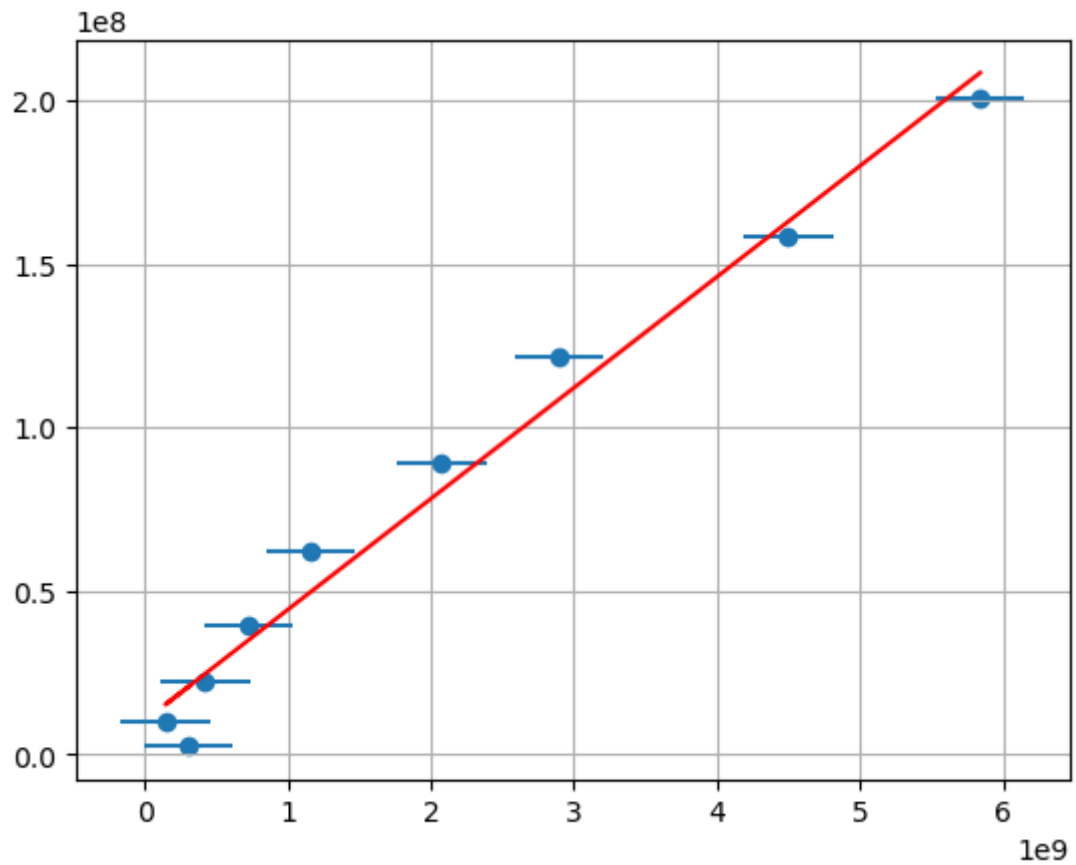
         plt.scatter((k)**3,(w3**2)*(1004.75))

         m,c = np.polyfit((k)**3,(w3**2)*(1004.75), 1)

         plt.errorbar((k)**3,(w3**2)*(1004.75),xerr=(k_err)**3, ls="none")

         plt.grid(True)
         plt.plot((k)**3,((k)**3)*m + c, "r")
         m
```

Out[69]: 0.0339427175903509



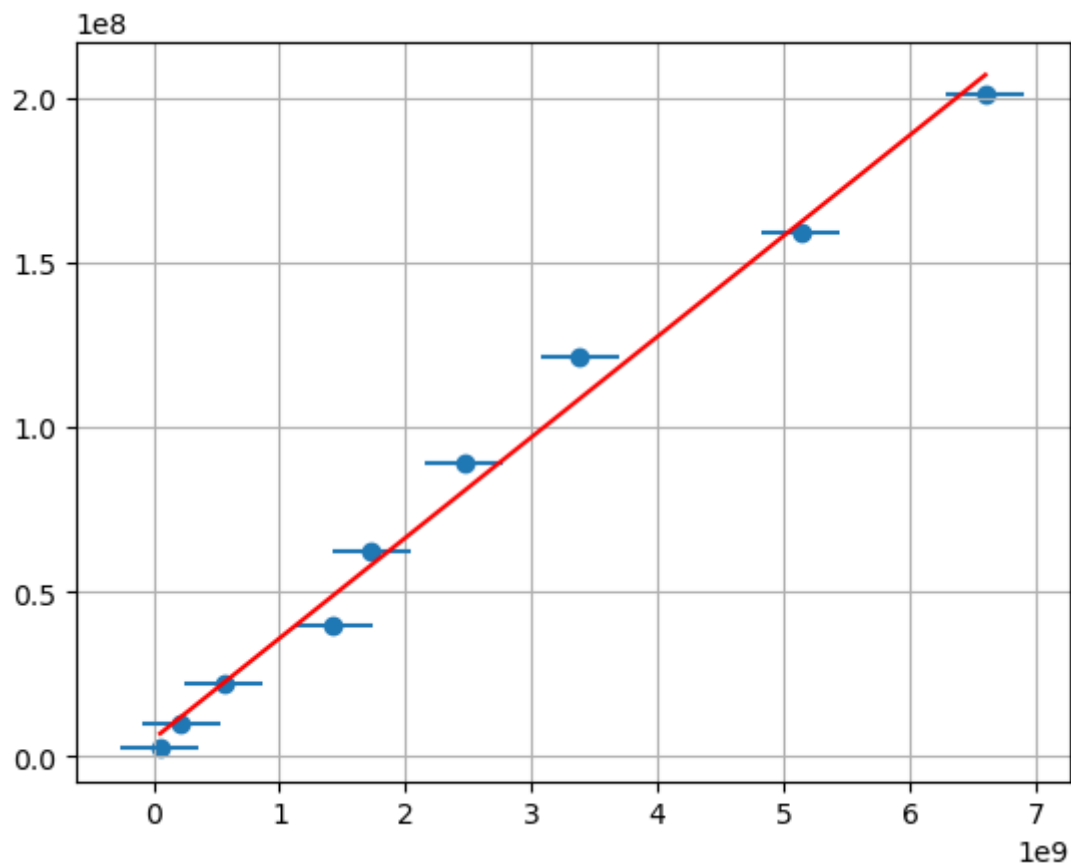
75 ml Soap

```
In [67]: soap = np.loadtxt("soap-75ml.txt")
soap = soap/2E3
k=k(theta,delta(soap,L),632E-9)

plt.scatter((k)**3,(w3**2)*(1007))
m,c = np.polyfit((k)**3,(w3**2)*(1007), 1)

plt.errorbar((k)**3,(w3**2)*(1007),xerr=(k_err)**3, ls="none")
plt.grid(True)
plt.plot((k)**3,((k)**3)*m + c, "r")
m
```

Out[67]: 0.030590884577583875



100 ml Soap

```
In [71]: soap = np.loadtxt("soap-100ml.txt")
soap = soap/2E3
k=k(theta,delta(soap,L),632E-9)

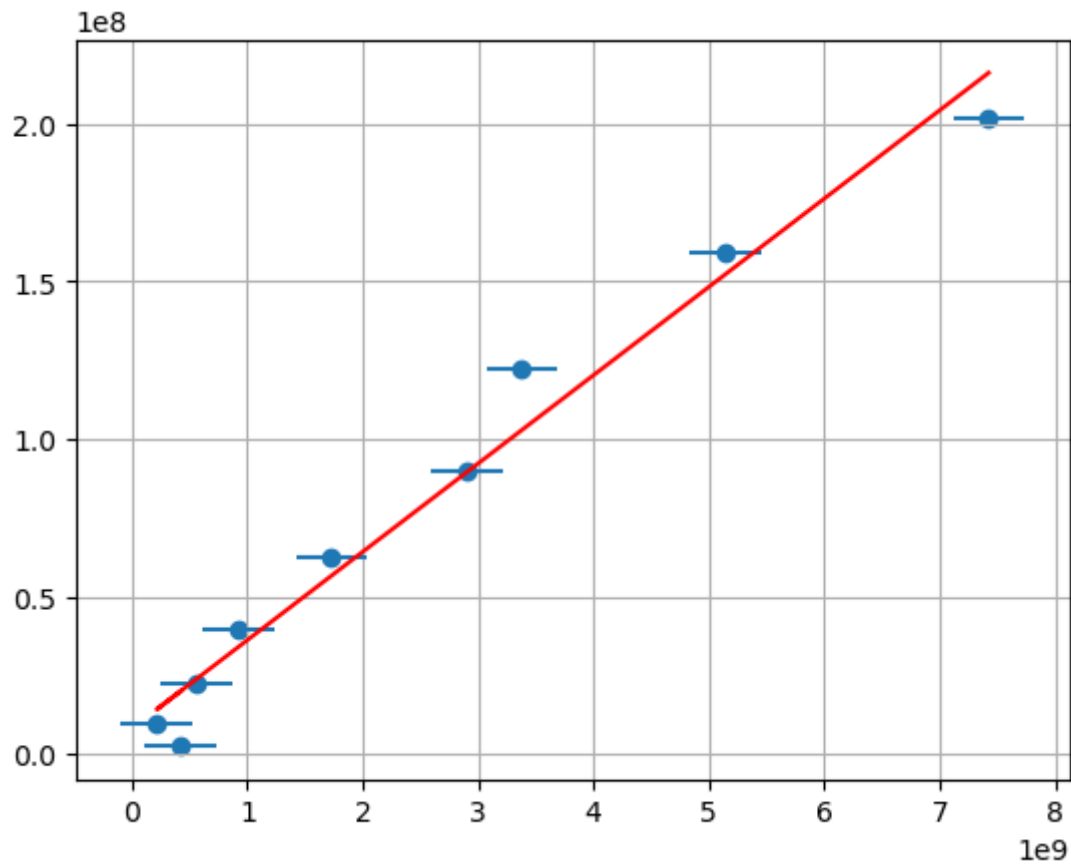
plt.scatter((k)**3,(w3**2)*(1009.25))

m,c = np.polyfit((k)**3,(w3**2)*(1009.25), 1)

plt.errorbar((k)**3,(w3**2)*(1009.25),xerr=(k_err)**3, ls="none")

plt.grid(True)
plt.plot((k)**3,((k)**3)*m + c, "r")
m
```

Out[71]: 0.02802741608340994



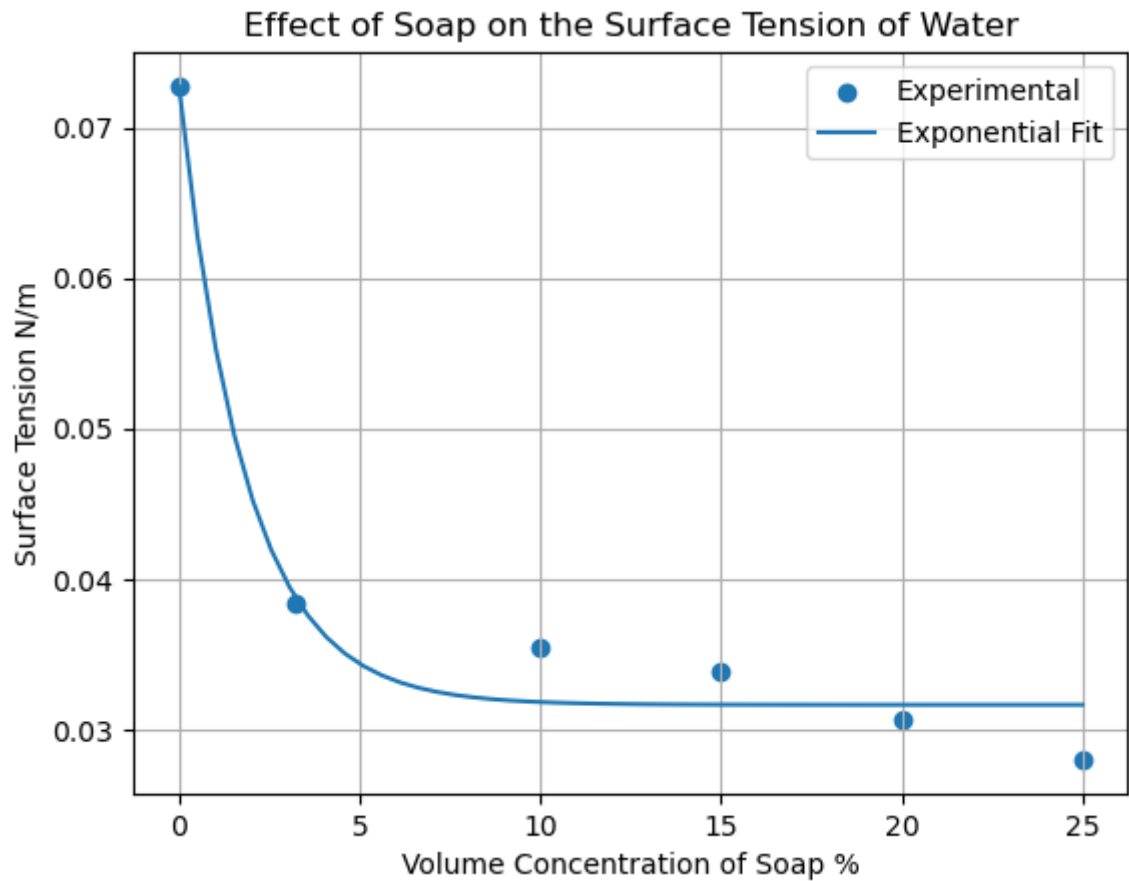
Effect of Soap on the Surface Tension of Water

```
In [112... conc = np.array([0,3.25,10,15,20,25]) #concentration percentage mL/mL
s_t = np.array([0.07275,0.0384,0.0355,0.0339,0.0306,0.028]) #surface tension

plt.scatter(conc,s_t)

x = np.linspace(0,25)
plt.plot(x,0.031637155 + 0.041078873*np.exp(-0.54213779*x))

plt.grid(True)
plt.ylabel("Surface Tension N/m")
plt.xlabel("Volume Concentration of Soap %")
plt.title("Effect of Soap on the Surface Tension of Water")
plt.legend(["Experimental","Exponential Fit"], loc="upper right");
```



In [224...

```

conc = np.array([3.25,10,15,20,25])

s_t = np.array([0.0384,0.0355,0.0339,0.0306,0.028])

plt.scatter(conc,s_t)

m1,c = np.polyfit(conc,s_t, 1)
plt.plot(conc,conc*m1 + c, "r")

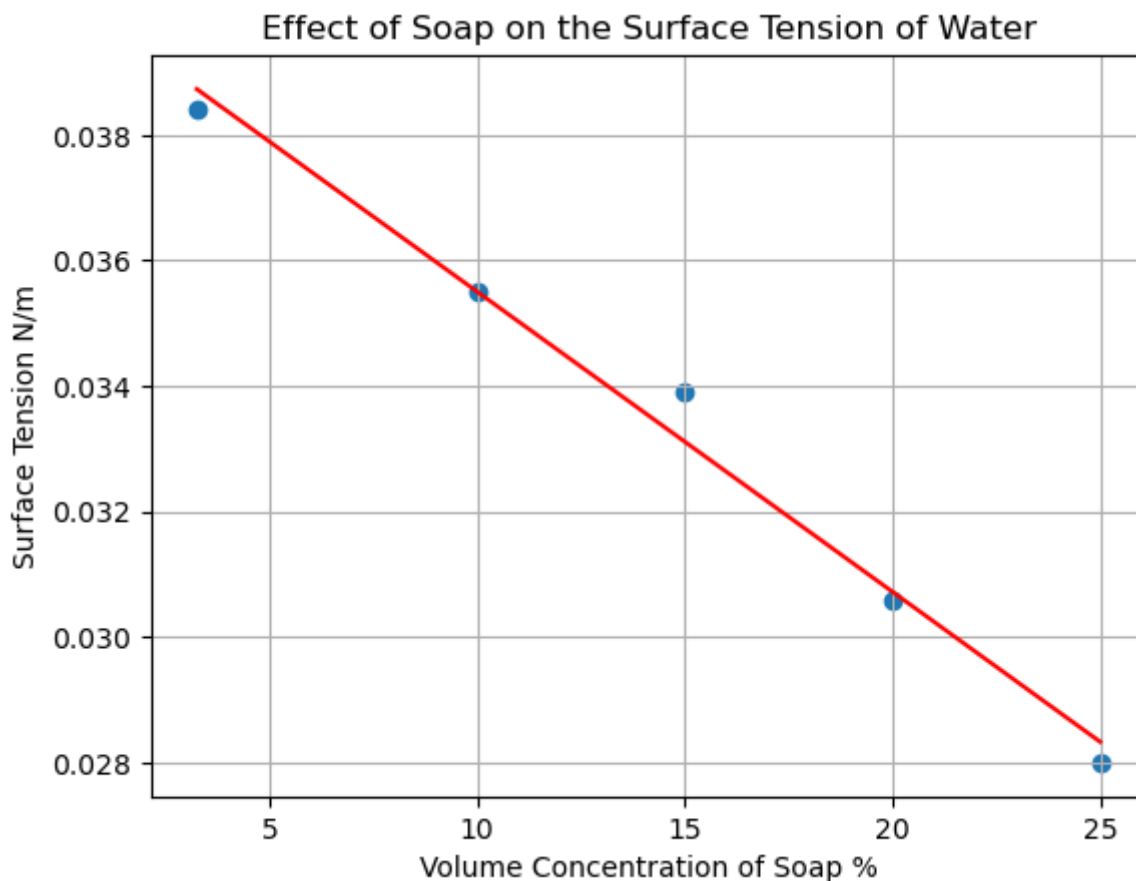
print(m1)
plt.grid(True)
plt.ylabel("Surface Tension N/m")
plt.xlabel("Volume Concentration of Soap %")
plt.title("Effect of Soap on the Surface Tension of Water");

```

```

-0.0004782049052009047

```



Exponential Fit of Function

```
In [116... def f(x,a,b,c):
            return a + b*np.exp(-c*x)
popt,pcov=curve_fit(f,conc,s_t)
popt
```

```
Out[116]: array([0.03189767, 0.04082229, 0.55289657])
```

Salt

10% Salinity

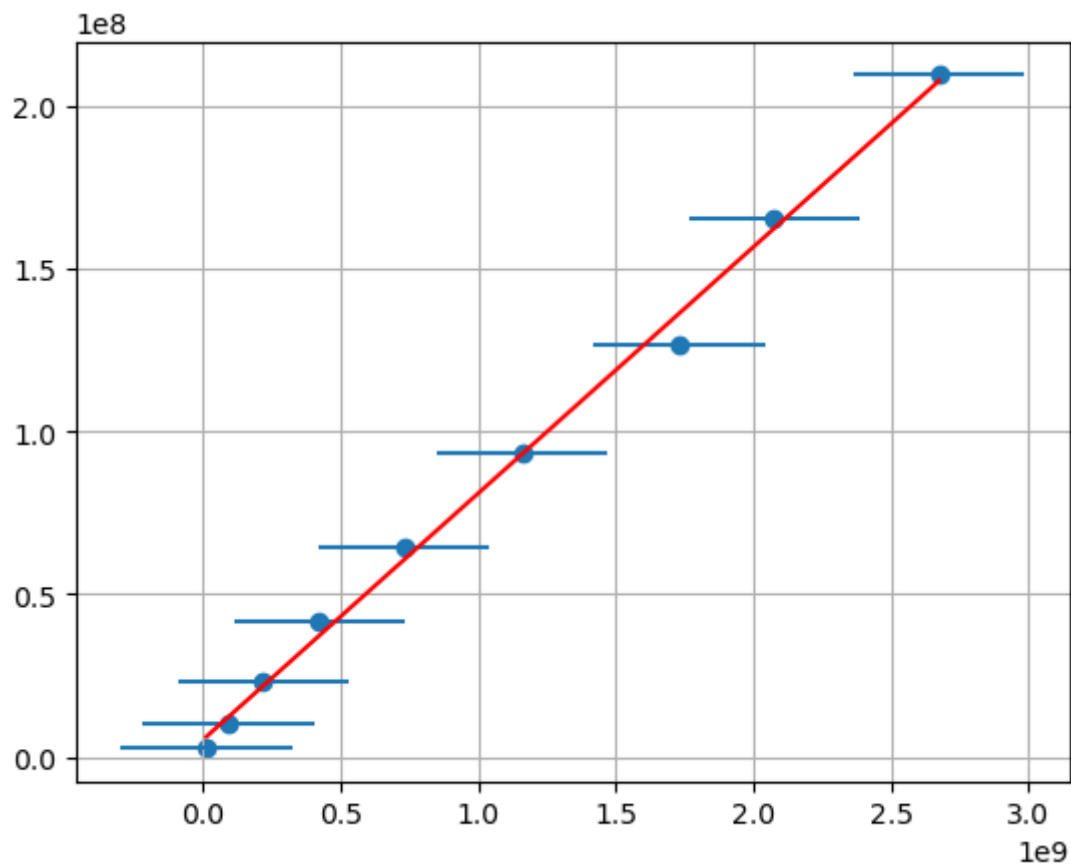
```
In [73]: salt = np.loadtxt("salt_10.txt")
salt = salt/2E3

k=k(theta,delta(salt,L),632E-9)

plt.scatter((k)**3,(w3**2)*(1050))
plt.errorbar((k)**3,(w3**2)*(1050),xerr=(k_err)**3, ls="none")

plt.grid(True)
m,c = np.polyfit((k)**3,(w3**2)*(1050), 1)
plt.plot((k)**3,((k)**3)*m + c, "r")
m
```

```
Out[73]: 0.07579628786749355
```



15% Salinity

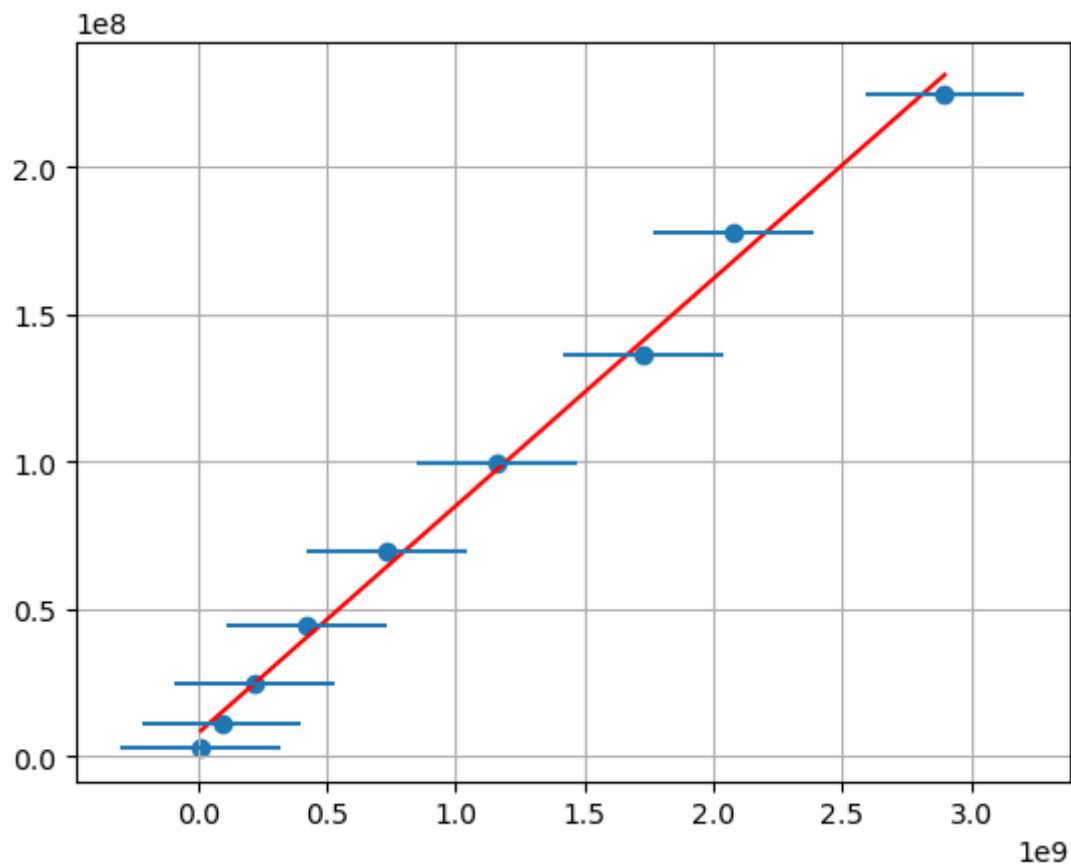
```
In [77]: salt = np.loadtxt("salt_15.txt")
salt = salt/2E3
k=k(theta,delta(salt,L),632E-9)

plt.scatter((k)**3,(w3**2)*(1125))
m,c = np.polyfit((k)**3,(w3**2)*(1125), 1)

plt.plot((k)**3,((k)**3)*m + c, "r")

plt.errorbar((k)**3,(w3**2)*(1125),xerr=(k_err)**3, ls="none")
plt.grid(True)
m
```

Out[77]: 0.07712190188569076



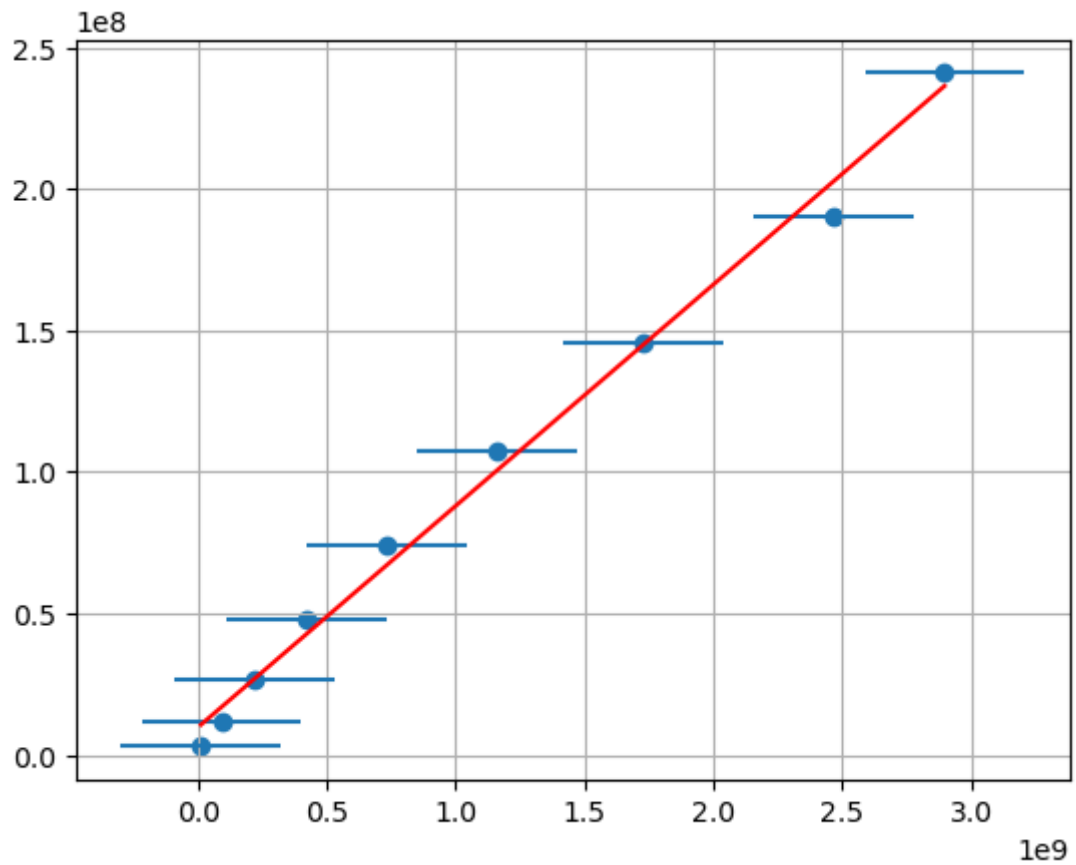
20% Salinity

```
In [160... salt = np.loadtxt("salt_20_.txt")
salt = salt/2E3
k=k(theta,delta(salt,L),632E-9)#did not fully dissolve
plt.scatter((k)**3,(w3**2)*(1205.8))

plt.errorbar((k)**3,(w3**2)*(1205.8),xerr=(k_err)**3, ls="none")
plt.grid(True)

m,c = np.polyfit((k)**3,(w3**2)*(1205.8), 1)
plt.plot((k)**3,((k)**3)*m + c, "r")
m
```

Out[160]: 0.07820866209374372



25% Salinity

```

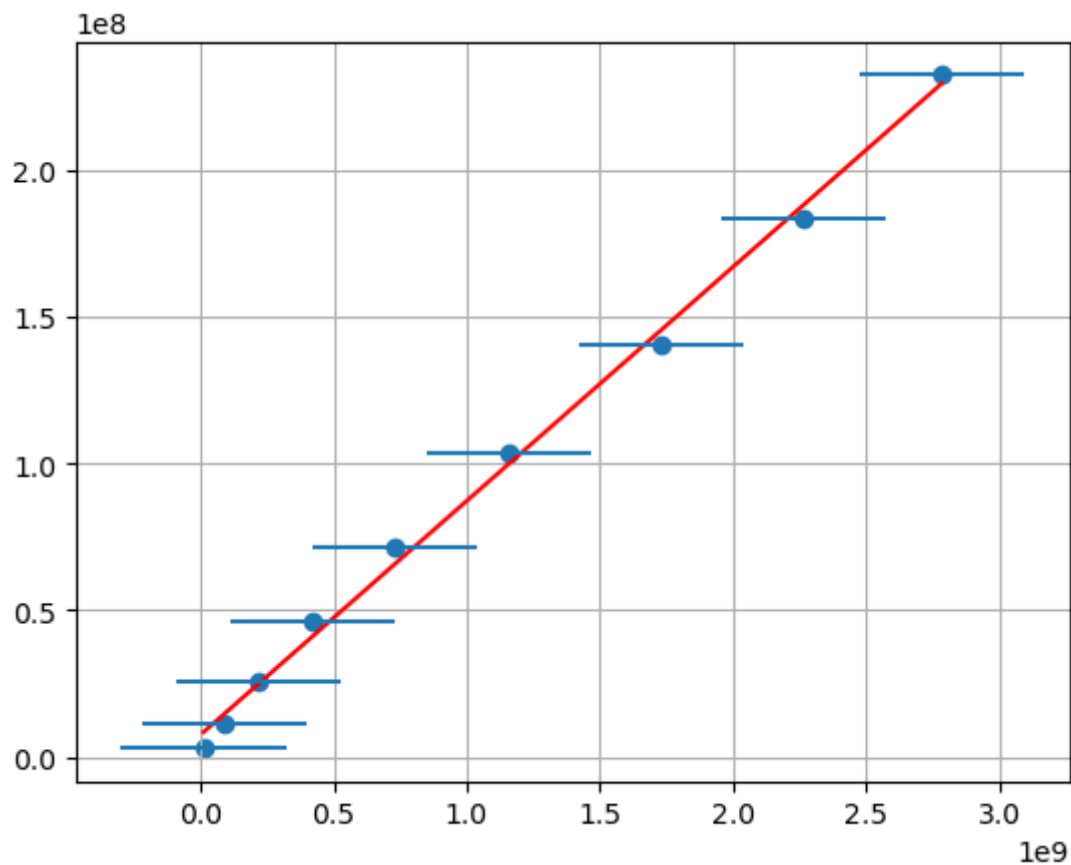
In [179... salt = np.loadtxt("salt_25_.txt")
salt = salt/2E3
k=k(theta,delta(salt,L),632E-9)

plt.scatter((k)**3,(w3**2)*(1165))
m,c = np.polyfit((k)**3,(w3**2)*(1165), 1)

plt.plot((k)**3,((k)**3)*m + c, "r")
plt.errorbar((k)**3,(w3**2)*(1165),xerr=(k_err)**3, ls="none")
plt.grid(True)
m

```

Out[179]: 0.07986161453559722



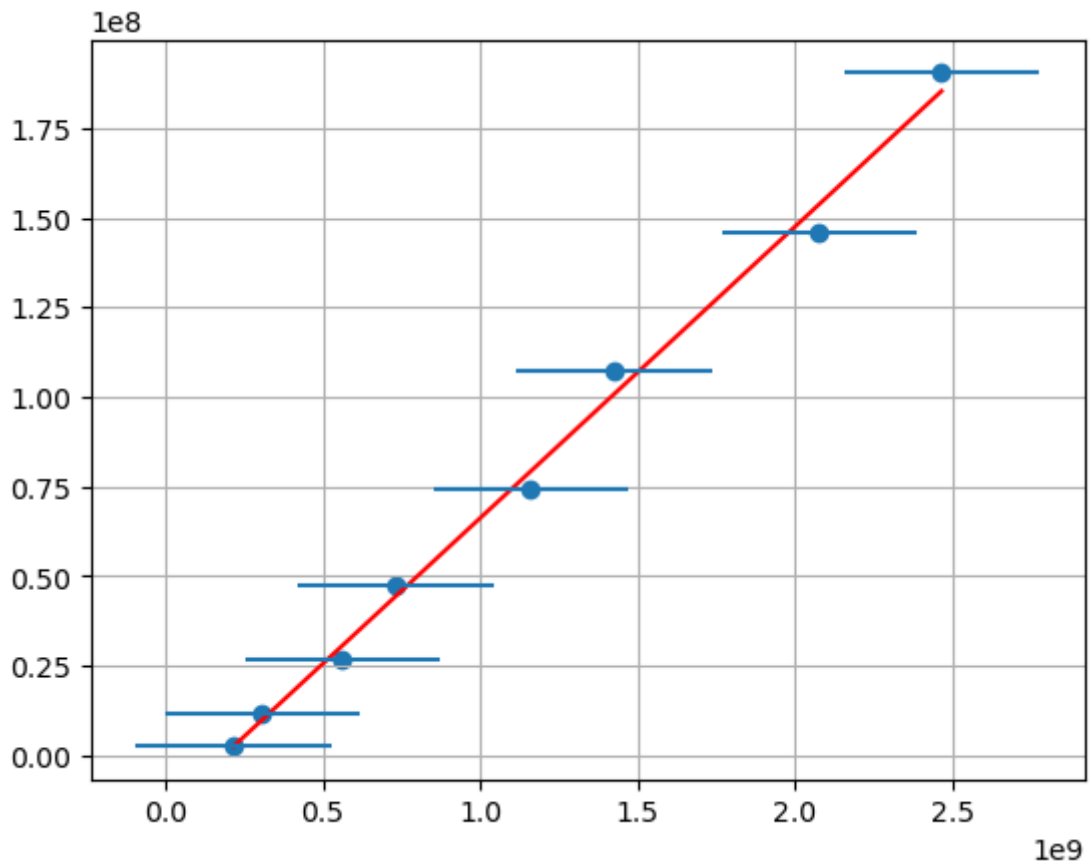
Around 30% Salinity (did not fully dissolve)

```
In [83]: salt = np.loadtxt("salt_30_1.txt")
salt = salt/2E3
k=k(theta,delta(salt,L),632E-9)
plt.scatter((k)**3,(w2**2)*(1205.8))

m,c = np.polyfit((k)**3,(w2**2)*(1205.8), 1)
plt.plot((k)**3,((k)**3)*m + c, "r")

plt.errorbar((k)**3,(w2**2)*(1205.8),xerr=(k_err)**3, ls="none")
plt.grid(True)
m
```

Out[83]: 0.08130876681789304



Effect of Salinity on Surface Tension of Water

In [225...

```

conc = np.array([0,10,15,20,25]) #concentration kg/kg
s_t = np.array([0.0726,0.0758,0.0771,0.0782,0.07987]) #surface tension

plt.scatter(conc,s_t)
m1,c = np.polyfit(conc,s_t, 1)

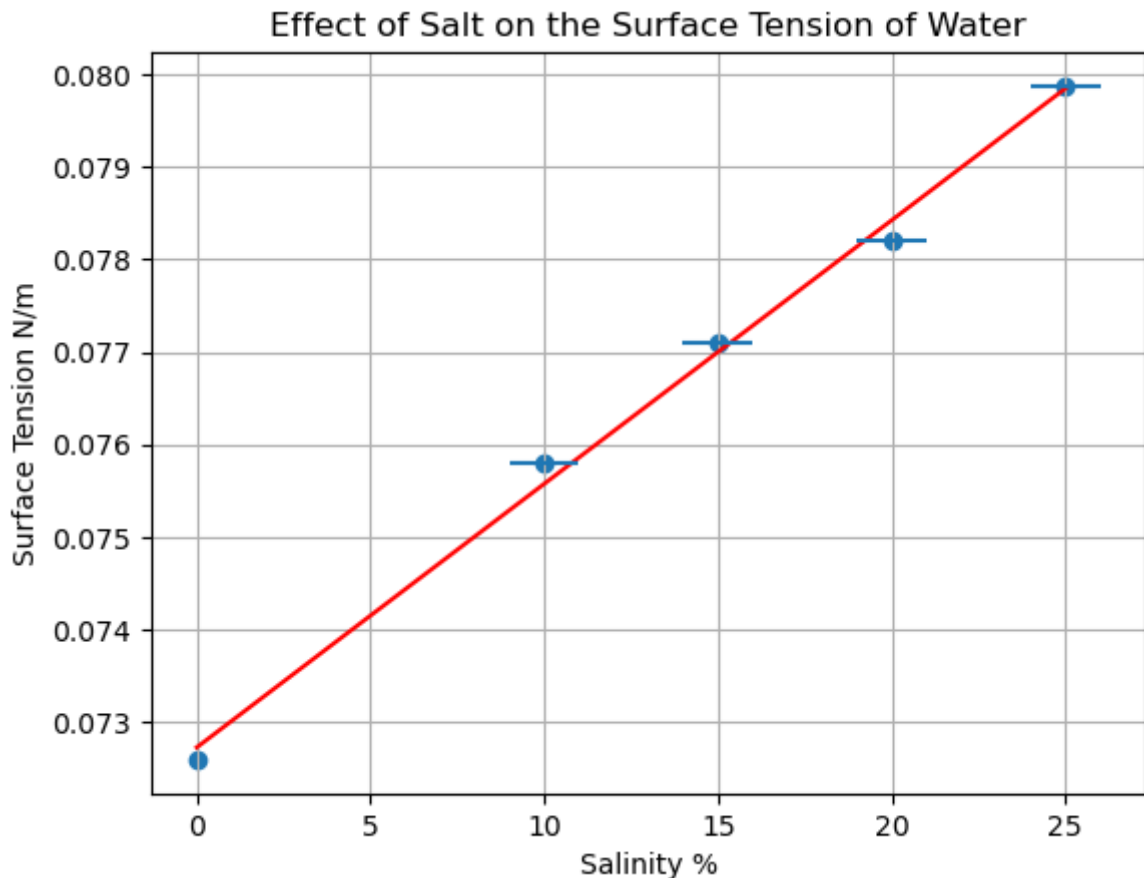
plt.plot(conc,conc*m1 + c, "r")
print(m1)
plt.grid(True)

s_err = np.array([0,1,1,1,1]) #error on conc

plt.errorbar(conc,s_t,xerr=(s_err), ls="none")
plt.ylabel("Surface Tension N/m")
plt.xlabel("Salinity %")
plt.title("Effect of Salt on the Surface Tension of Water");

```

0.0002845135135135136



In [231...

```

conc = (1000/58.44)*(np.array([0,33/333,53/353,75/375,100/400])) #conc in molarity

conc_lit = (np.array([0.1,0.5,1,1.5,2])) #conc from literature 1
conc_lit_2 = (np.array([0,0.25,.5,1,1.5,2,2.5,3,3.5,4])) #conc from literature 2

s_t_lit_2 = np.array([0.07272,0.07303,0.07346,0.07405,0.07475,0.07554,0.07652,0.0775,0.0785,0.0795])
s_t_lit = np.array([0.07246,0.07307,0.07396,0.07469,0.07541]) #sft from literature
s_t = np.array([0.0726,0.0758,0.0771,0.0782,0.07987]) # measured sft

plt.scatter(conc,s_t)
plt.scatter(conc_lit,s_t_lit)
plt.scatter(conc_lit_2,s_t_lit_2)

m1,c = np.polyfit(conc,s_t, 1)
plt.plot(conc,conc*m1 + c, "r")
print(m1)
m,c = np.polyfit(conc_lit,s_t_lit, 1)
plt.plot(conc_lit,conc_lit*m + c, "b")
print(m)
m2,c = np.polyfit(conc_lit_2,s_t_lit_2, 1)
plt.plot(conc_lit_2,conc_lit_2*m2 + c)
print(m2)

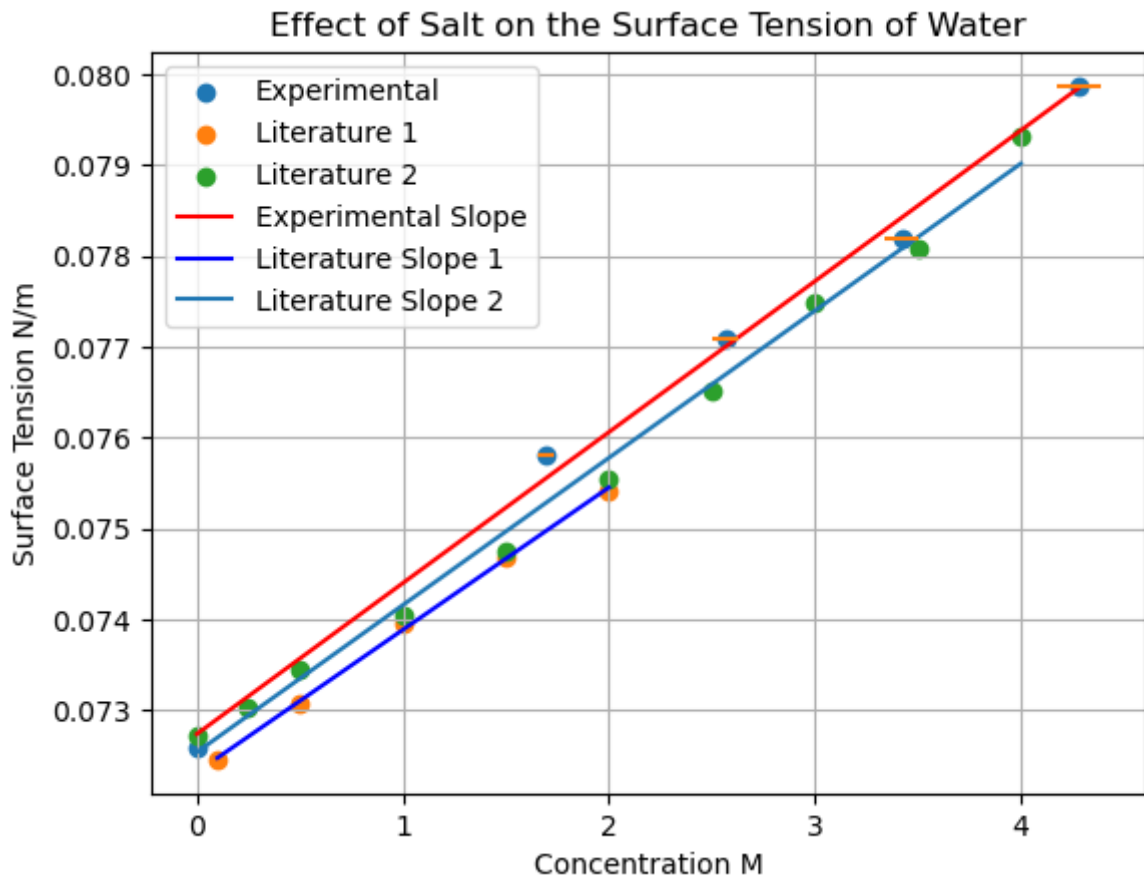
plt.grid(True)
s_err = conc*0.025 #standard 2.5% error
plt.errorbar(conc,s_t,xerr=(s_err), ls="none")
plt.ylabel("Surface Tension N/m")
plt.xlabel("Concentration M")
plt.title("Effect of Salt on the Surface Tension of Water")
plt.legend(["Experimental","Literature 1","Literature 2","Experimental Slope", "Literature 1 Slope"],
           loc = "upper left");

```

0.001660689742414433

0.0015659445407279007

0.0016193593804998208



```
In [237...] popt, pcov = curve_fit(f,(k)**3,(w2**2)*(p_whiskey))
pcov
```

```
Out[237]: array([[ 9.48468078e-07, -2.01336512e+03],
 [-2.01336512e+03,  7.64819132e+12]])
```

Surface Tension of Whiskey

```
In [236...] wh = np.loadtxt("whiskey.txt")
wh = wh/2E3

k=k(theta,delta(wh,L),632E-9)

plt.scatter((k)**3,(w2**2)*(p_whiskey))
m,c = np.polyfit((k)**3,(w2**2)*(p_whiskey), 1)

plt.errorbar((k)**3,(w2**2)*(p_whiskey),xerr=(k_err)**3, ls="none")
plt.plot((k)**3,((k)**3)*m + c, "r")
print(m)

plt.grid(True)
plt.ylabel("Product of square of angular frequency and density")
plt.xlabel("The wavenumbers cubed (1/m^3)")
plt.title("Surface Tension of Whiskey");

0.02565827950412056
```

