PHYC30300- Advanced Laboratory I

# Compton Scattering

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# Abstract

This experiment investigates the phenomenon that is Compton Scattering The purpose of this experiment is to verify the energy dependence of gamma radiation upon scattering angle and to compare the differential cross section obtained from the data with those calculated using the Klein-Nishina formula and classical theory. We also want to investigate the relativistic relation between energy and momentum, measure the rest mass of the electron, and to demonstrate the dependence of p, E and γ on β. Using the gamma rays emitted from Caesium-137, gamma rays were scattered by colliding the gamma rays or photons with electrons in the scattering rod. As a result some of the photon energy was transferred to electrons and this transfer of energy is known as the Compton Effect.

# Introduction

In the early 1900s, physicists used scattering experiments to help build a model for the structure of the atom. Around 1920, Arthur H. Compton performed one of the most important experiments of the era using x-ray photons scattering from electrons in light materials. This experiment was the first to show that photons are not just waves but, in collisions with electrons, should be treated as particles. [1]

In this experiment, the kinematics of Compton scattering are thoroughly investigated using γ-ray photons emitted by a 137Cs source. By considering the photon-electron interaction as an elastic collision between two particles, the outgoing energies of the scattered photon and recoiling electron are compared to the Compton scattering formula. We also delve more deeply into the phenomenon by determining the probabilities of scattering into various angles.[2] The Klein-Nishina scattering formula is a quantum-theoretical prediction of these probabilities. We also want to investigate the relativistic relation between energy and momentum, measure the rest mass of the electron, and to demonstrate the dependence of p, E and γ on β.

# Theory

## Rest Mass of the Electron

Leading on from Compton, we apply the basic results of quantum mechanics and relativity to the scattering process, rather than the classical equations. In the modern picture, light is composed of particle-like photons, containing energy $E = hc/\lambda$ as predicted by quantum mechanics . Light also possesses momentum according to the relativistic equation $p = E/c = h/\lambda$, valid for particles of zero rest mass. [3]

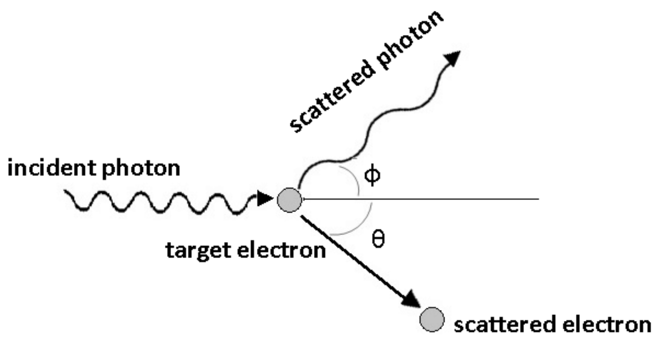



*Fig.1 shows the geometry of Compton scattering [4]*

We shall analyse the kinematics of Compton scattering as an elastic collision between a photon and an electron initially at rest. [2] The colliding photons transfer their energy and momentum to such electrons. Each of these electrons in turn scatters the photon colliding with it, in some definite direction. This definite angle between the directions of the incident photon and scattered photon is called the scattering angle. In Compton scattering, the energy of the scattered photon is always lower than that of the corresponding incident photon. This difference between the energies of the incident photon and the scattered photon is utilised to recoil the struck electron. [1] [4]

$$p_{\gamma} = p - p'_{\gamma} \quad (1)$$

Where $p_{\gamma}$ is the incident gamma ray momentum, $p'_{\gamma}$ is the scattered photon momentum and $p$ is the momentum of the recoiling electron

Conservation of momentum then gives; [5]

$$p_{\gamma}c = p'_{\gamma}c + T \quad (2)$$

This in turn gives the non-relativistic relation;

$$m_{nr}c^2 = \frac{(2E_{\gamma} - T)^2}{2T} \quad (3)$$

*Where $E_{\gamma}$ is the incident gamma ray energy, $T$ is the electron kinetic energy*

In this experiment, we plotted our calculated $m_{nr}c^2$ and it was shown that $m_{nr}c^2$ was not constant. Adding our experimental result of $m_{nr}c^2$ + T/2 we find the central energy-momentum relationship for special relativity;

$$E^2 = (T + m_0c^2)^2 \quad (4)$$ [5]

This equation 4, can then be used to calculate the rest energy of the electron and the relations of p, E and γ on β, which we will conduct later in the paper.

## Cross Section Determination

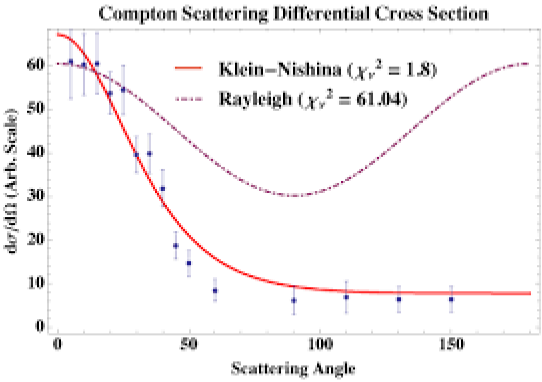



*Fig.2 shows an example of the relationship between Differential Cross Section and Scattering Angle for Compton scattering [3]*

According to the Thompson model, the scattering cross section of the electron can be calculated by the formula: [1]

$$\frac{dI}{dt} = \frac{8\pi e^2}{3me^2} I \quad (5)$$

which predicts the scattering cross section of an electron to be σ = 6.65×10−25 cm2.[1] However, this does not account for the recoil of the electron or the relativistic effects.

Klein and Nishina derived a formula for the differential cross section, equation (6);[1]

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{theory}} = \frac{r_0^2}{2}\left\{\frac{1+\cos^2\theta}{[1+\alpha(1-\cos\theta)]^2}\right\} \times \left\{1+\frac{\alpha^2(1-\cos\theta)^2}{(1+\cos^2\theta)(1+\alpha[1-\cos\theta])}\right\}, \left(\frac{\text{cm}^2}{\text{sr}}\right)$$

where r0 = 2.82×10−13cm (the classical electron radius) , θ is the photon scattering angle and $\alpha$ = 1.29.

The Klein-Nishina formula predicts that the scattering is σ = 2.53 × 10−25 cm2. [1]

# Experimental Procedure

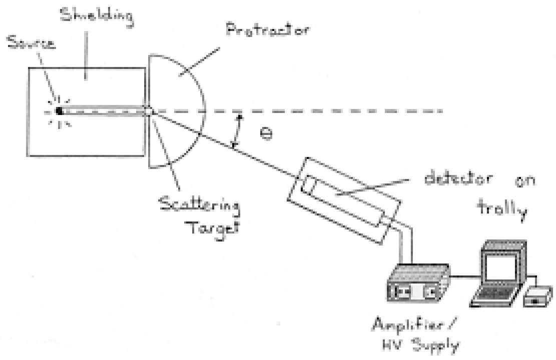




*Fig.3a and Fig.3b  shows the apparatus used in this experiment*

Figure 2 shows the apparatus used in this experiment. As mentioned previously, the source used in this experiment was a high activity 137Cs source, which was encased by a lead shield. The scattering sample was placed directly in front of the source aperture and the NaI(TI) detector was on a movable arm spun about the scattering sample. For scattering angles between 20° and 100°, in steps of 5°, we measured the gamma spectra. The spectra were not counted until at least 10 000 net counts were amassed under the photopeak. We then recorded the energy of the scattered gamma rays corresponding to each angle. We also recorded the net area under the photopeak as well as the live time which the detector was running for each measurement of the angle. We also took the measurements of kinetic energy of the electron and the energy of the gamma ray for many different sources using a constant angle of scattering. We found our kinetic energy of the electron by finding at which energy the Compton edge occurred.

A close relationship between the bin number outputted by the data program and the energy must be known to  measure the different energy values accurately for each of the angles. To determine this relationship, the energy spectrum of two radioactive sources with known energies  Ba-133 (363keV) and Cs-137(661keV) were measured. Each peak intensity value corresponded to a measured bin number and a known energy. This way the detector and programme were calibrated to give accurate readings.

# Results and Discussion

## Verify the energy dependence of gamma radiation upon scattering angle

By solving the conservation of momentum equations in the x and y directions we get the formula; [1]

$$E'_{\gamma} = \frac{E_{\gamma}}{1 + \frac{E_{\gamma}}{m_0 c^2}(1 - cos(\theta)} \quad (7)$$

Using the following values; $m_0 c^2$ is equal to 0.511 MeV and $E_{\gamma}$ is equal to 0.662 MeV for the 137Cs source used in this experiment. Substituting these values into equation 7 gives the following linear relationship;

$$\frac{1}{E'_{\gamma}} = 1.51 + 1.954(1 - cos(\theta) \quad (8)$$

Using our measured values for $E_{\gamma}$ and $\theta$, we plotted $\frac{1}{E'_{\gamma}}$ vs $(1 - cos(\theta)$ and compared our results with equation 8.

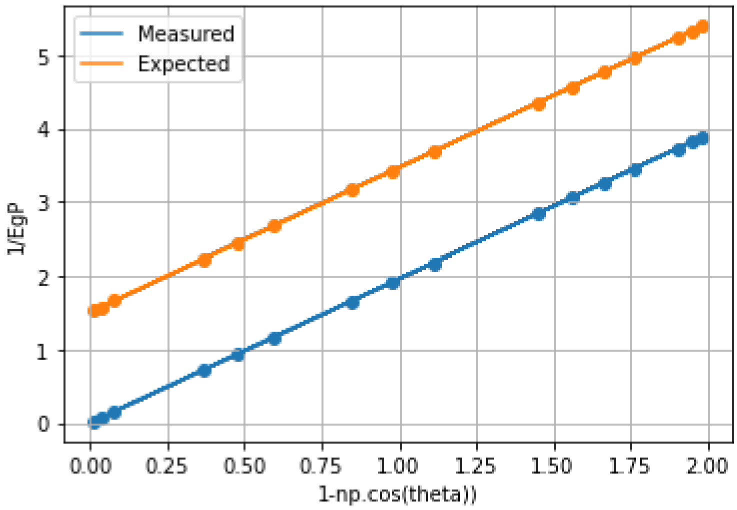



*Fig.5: compares the measured and expected linear relationship of* $\frac{1}{E'_{\gamma}}$ *keV vs* $(1 - cos(\theta)$

It is clear to see from Fig.4 that although the measured and expected relationships have vastly different y-intercepts they do in fact have the same slope. The relative difference between the slopes is 0.00732%. The difference in y-intercept may be due to computational error or possibly an error when taking measurements. This error may be at fault as to why our cross section determination does not turn out as predicted.

## Comparing the differential cross-section

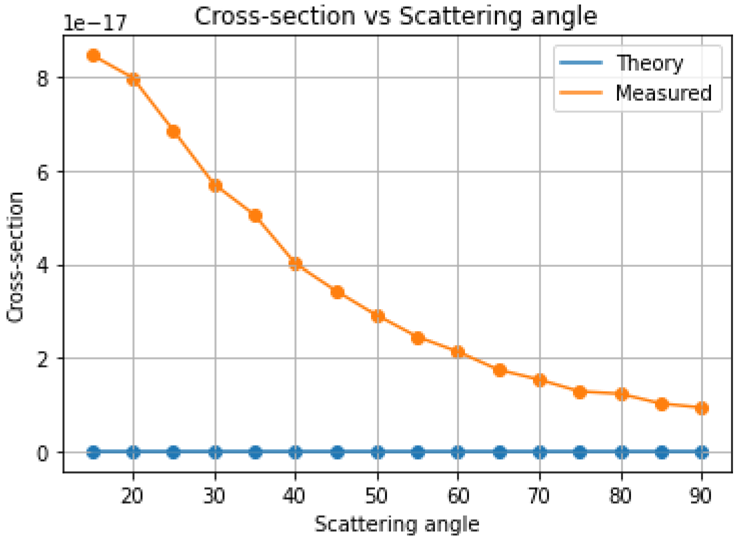



*Fig.6: compares the theoretical and measured differential cross section vs scattering angle relationship. The data obtained is not what was expected and is out by several orders of magnitude.*

The theoretical values for the cross section were obtained using the Klein-Nishina formula, equation (6). The values obtained were close to the expected order of magnitude but they did not fit the relationship with the scattering angle we had hoped to find which was in the region of σ = 2.53 × 10−25 cm^2.[1] The measured cross section did fit the plot shape we were hoping to find; however, the data points were out by an order of 10x-8. These points were found using the formula;

$$\frac{d\sigma}{dt} = \frac{\Sigma'_{\gamma}}{N\Delta\Omega I} \quad (9)$$

*where $\Sigma'_{\gamma}$ = sum under the photopeak divided by the counting time and divided by the intrinsic peak efficiency , $N$ = number of electrons in the scattering sample , $\Delta\Omega$ =solid angle of the detector in steradians, $I$the number of incident gamma rays per cm2 per s at the scattering sample.*

## Rest mass of the electron, and Demonstrate the dependence of p, E and γ on β.

Next, we determined the rest mass of the electron using the energy of the incident gamma ray and the corresponding Compton edge for many different radioactive sources. By finding the corresponding Compton edge to each photo-peak we could determine the kinetic energy of the electron. By rearranging equation (4) we found the following equations; [5]

$$m_0c^2 = \frac{2E_{\gamma}(E_{\gamma} - T)}{T} \quad (10)$$

$$\beta = \frac{T(2E_{\gamma} - T)}{T^2 - 2TE_{\gamma} + 2E_{\gamma}^{\ 2}} \quad (11)$$

$$\gamma = 1 + \frac{T^2}{2E_{\gamma}(E_{\gamma} - T)} \quad (12)$$

$$pc = 2E_{\gamma} - T \quad (13)$$

Using our data and equation 3, we can plot a non-relativistic relationship for the electrons rest energy vs electron kinetic energy;

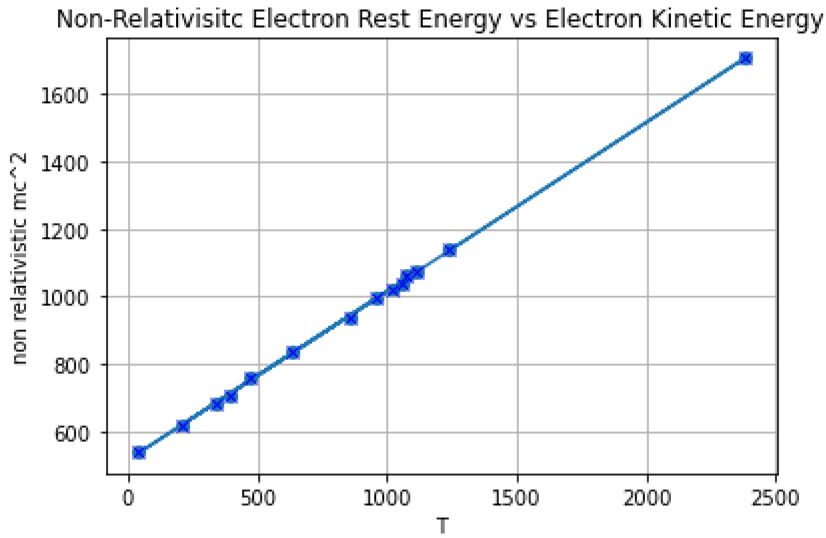



*Fig.7: shows the non-relativistic relationship for the electrons rest energy keV vs electron kinetic energy keV*

As previously discussed, this plot implies that the electrons rest mass is not constant. The y-intercept gives us a value for the electron's rest energy which was found to be 514.2 ∓ 6 keV which is significantly close and agrees with the theoretical value of 511 keV. [5]

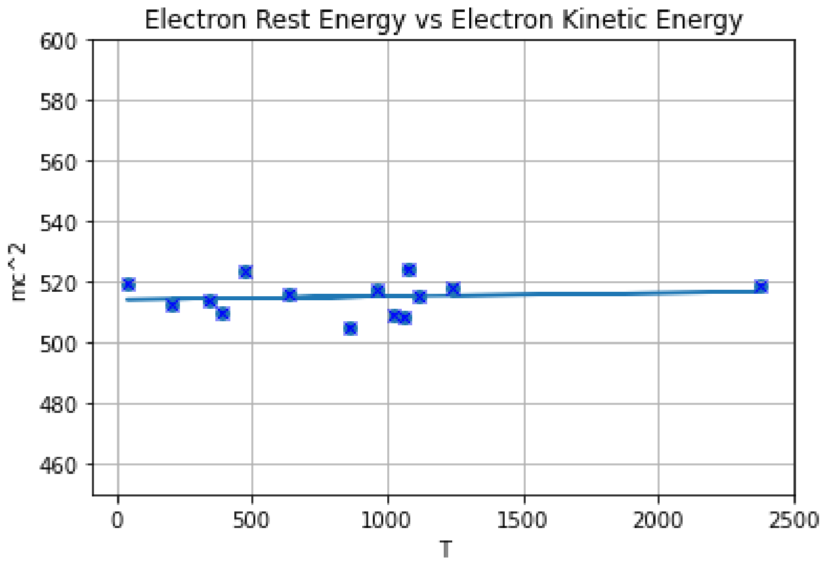



*Fig.8: shows the relativistic relationship for the electrons rest energy keV vs electron kinetic energy keV*

Using equation 10, we can find our relativistic rest energy for an electron. In Fig.6, we see that the electrons rest mass is a constant with a slope of zero and also with a y-intercept 514.2 ∓ 6 keV. Our values for the electrons rest mass agree to the 13th decimal place.

Using equation 11, 12 and 13 we investigated the relationships of p and γ with β.

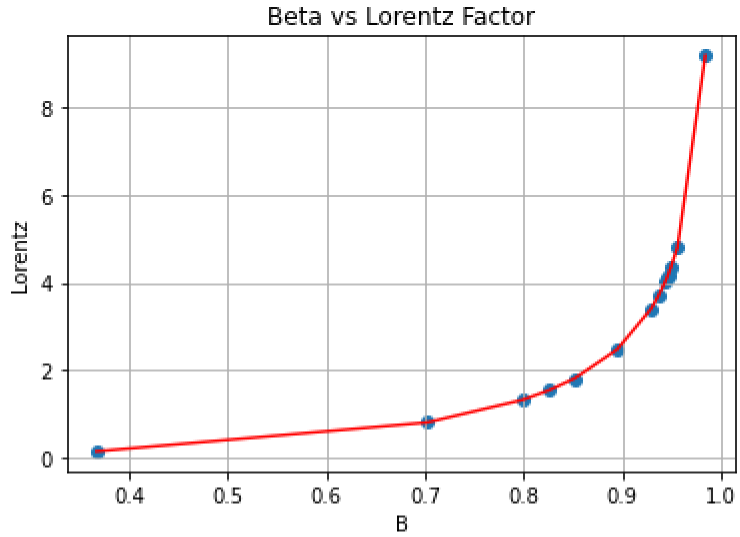



*Fig.9: shows the relationship for the Lorentz Factor γ vs β*

Here, in Fig.8, we can see that our line of best fit follows the theoretical formula for the Lorentz factor;

$$\gamma = \frac{1}{(1-\beta^2)^{\frac{1}{2}}} \quad (14)$$ [5]

This plot could also be fitted using the relation of $\gamma = \frac{E}{m_0 c^2}$.

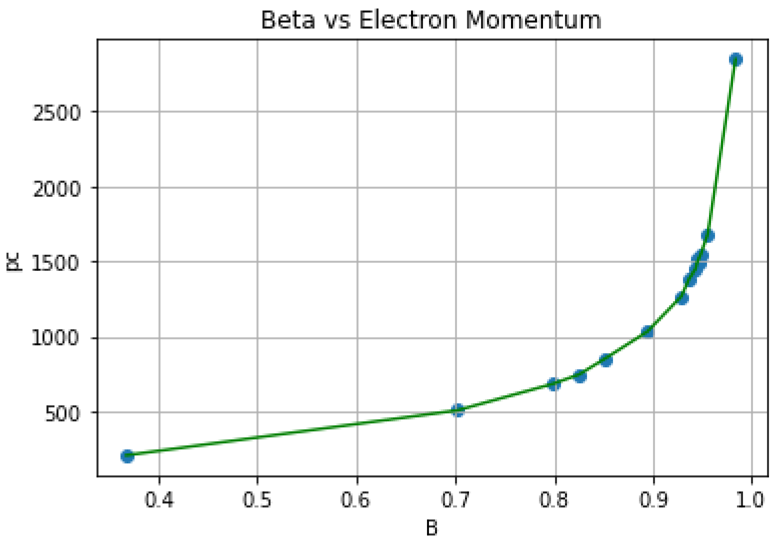



*Fig.10: shows the relationship for the Electron Momentum keV vs β*

Fig.8: shows the relationship for pc vs β. As we can see from the 208TI gamma ray, the plot shows the expected behaviour as β approaches 1.[5] The line of best fit follows the expected relationships of $\frac{1}{E}$ and $\frac{\gamma}{m_0 c^2}$.

# Conclusion

From our results, it can be clearly seen that not all of our data analysis agrees with the expected results.[5][1] We can clearly see that we were unable to verify the equations for both the energy dependence of gamma radiation upon scattering angle and the cross section for Compton Scattering We expected the theoretical values to match our measured data, however this was not the case. This may be due to some computational errors or down to errors made when measuring our data itself. However, we were able to successfully obtain values for the rest energy of the electron both relativistically and classically. Theoretically, we expected our value to be 511 keV.[5] This was proven to agree with our results as we calculated a value of 514.2 $\mp$ 6 keV. We also successfully demonstrated the dependence of p, E and $\gamma$ on $\beta$. All our measured relationships between the variables agreed with their respective theoretical dependencies. Although we did not succeed in all of our findings in this experiment, I believe that it is important that undergraduates develop a strong understanding of Compton Scattering and how it can be used to determine electron rest mass and other important variables.

# References


[1]Compton Scattering and Attenuation Rachel Bowens-Rubin∗ MIT Department of Physics (Dated: January 31, 2010)

[2]19/5/2019 Experiments 32a and 32b COMPTON SCATTERING, California Institute of Technology, Date Last Accessed; 25/2/2022

[3]Kim, T.H., 2008. Compton Scattering., MIT Department of Physics

[4] ResearchGate. (n.d.). FIG. 1: The geometry of Compton scattering showing the directions of... [online] Available at: https://www.researchgate.net/figure/The-geometry-of-Compton-scattering-showing-the-directions-of-the-scattered-photon-and_fig1_236737231 [Accessed 25 Feb. 2022].

[5] Jolivette, P.L. and Rouze, N., 1994. Compton scattering, the electron mass, and relativity: A laboratory experiment. *American Journal of Physics*, *62*(3), pp.266-271.

[6]March 13, 2009, Compton Scattering, Brown University

[7] Ip, S., 2012. Compton Scattering of Gamma Rays, Department of Physics and Astronomy, University College London

In [204…

```python
# uploading data sets
import numpy as np
import matplotlib.pyplot as plt
import scipy.optimize
angle,peak,area,err_area,time = np.loadtxt("Compton Scattering.txt", unpack=True)
gamma, T , err = np.loadtxt("Compton 2.txt",usecols=(1,2,3),  unpack=True)
```

In [205…

```python
EgP= peak/(1+(1-np.cos(angle))*(peak/.511)) # our measured value of Eg'
```

In [206…

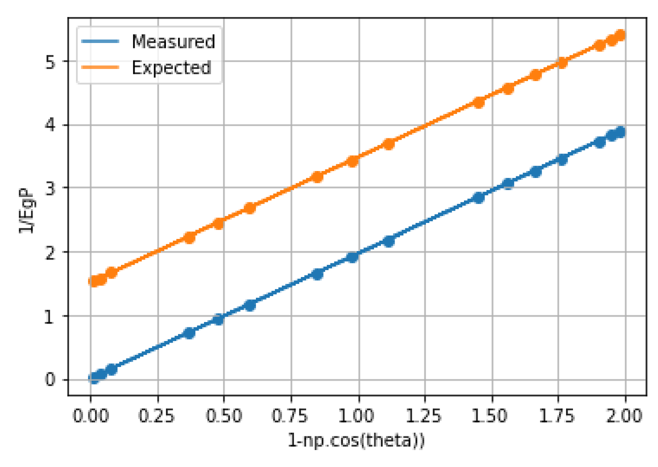
```python
EgP2= 0.662/(1+(1-np.cos(angle))*(0.662/.511)) # theoretical value of Eg' using mc^2 as 511 MeV and gamma energy as 662 MeV
```

## Verify the energy dependence of gamma radiation upon scattering angle

In [207…

```python
plt.scatter((1-np.cos(angle)),1/EgP,) #plotting our measured values
plt.ylabel("1/EgP")
plt.xlabel("1-np.cos(theta))")
plt.grid(True)
a, b = np.polyfit((1-np.cos(angle)), 1/EgP, 1)
plt.plot((1-np.cos(angle)), a*(1-np.cos(angle))+b)

plt.scatter((1-np.cos(angle)),1/EgP2,) #plotting our expected values using mc^2 as 511 MeV and gamma energy as 662 MeV
m, c = np.polyfit((1-np.cos(angle)), 1/EgP2, 1)
plt.plot((1-np.cos(angle)), m*(1-np.cos(angle))+c)
plt.legend(["Measured","Expected"]); # our slopes are matching however our intercepts vary
```




In [208…

```python
(a-m)/a   # the relative difference in the slopes
```

Out[208… 7.3233675276385e-05

In [209…

```python
(c-b)/c # relative difference in y intercepts
```

Out[209… 0.9984752558539041

In [210…

```python
np.mean(EgP/(1 - np.cos(angle)) - a) # prediction for the rest energy of an electron
```

Out[210… 405.0421795978971

## Determination of Cross Section

In [211…

```python
r0 = 2.82E-13
alpha = 1.29
m1 = (((r0**2)/2) * ((1 + (np.cos(angle))**2)/(1 + alpha*(1 - np.cos(angle)))**2))
m2 = (1) + (((alpha**2)*((1-np.cos(angle))**2))/((1+(np.cos(angle))**2)*(1+(alpha*(1-np.cos(angle))))))
theory = m1*m2
theory # could not get a good fit for differential cross section
```

Out[211…

```
array([1.17243359e-26, 1.91399969e-26, 7.70733136e-26, 1.44871141e-26,
       1.18590298e-26, 1.16688665e-26, 2.30732043e-26, 7.03609879e-26,
       1.32494167e-26, 1.19148396e-26, 1.16463197e-26, 2.84590596e-26,
       6.09805948e-26, 1.24711507e-26, 1.19537123e-26, 1.16859590e-26])
```

In [212…

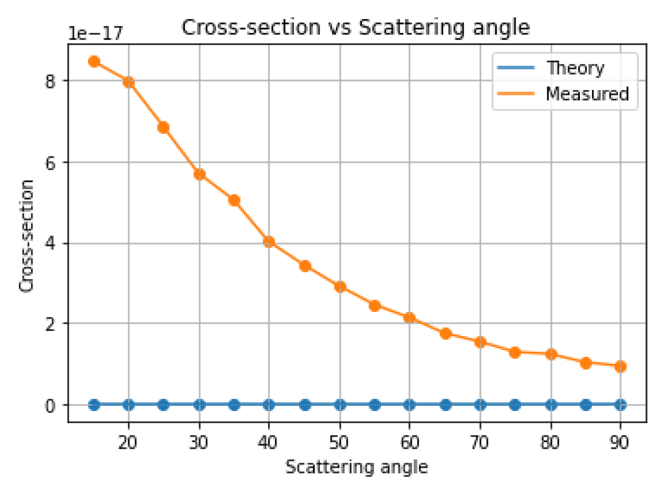
```python
eff = (0.1522)*(peak)**(-1.1325)
N = (73.9)*(55)*(6.0221409E23)/(139)
t = 2022.25-1977.66
I = (1.013E6)*(np.exp(-t/43.48))
omega = (np.pi)*(.9)**(2) / (26**2)
E = (area/time)/eff
measured = E/(N)*(omega)*(I)
measured # suspect cross section to be off considerably by around 10E-8
```

Out[212…

```
array([8.46484993e-17, 7.98094737e-17, 6.85567154e-17, 5.70669821e-17,
       5.04864184e-17, 4.01534070e-17, 3.43809943e-17, 2.91362158e-17,
       2.45057838e-17, 2.13135546e-17, 1.74437847e-17, 1.53455310e-17,
       1.28073548e-17, 1.23052566e-17, 1.02244330e-17, 9.35030501e-18])
```

In [213…

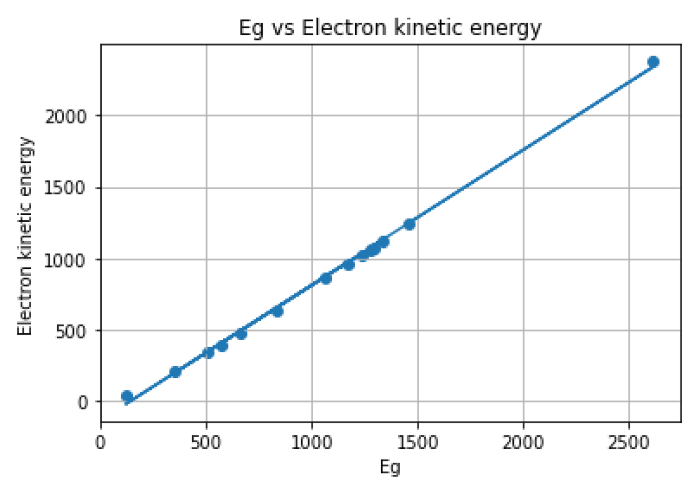
```python
plt.scatter(angle,theory)
plt.plot(angle,theory)
plt.ylabel("Cross-section")
plt.xlabel("Scattering angle")
plt.title("Cross-section vs Scattering angle")
plt.grid(True)
plt.scatter(angle,measured)
plt.plot(angle,measured)
plt.legend(["Theory","Measured"]);
```




## Rest mass of the electron, and Demonstrate the dependence of p, E and γ on β.

In [214…

```python
plt.scatter(gamma,T) # plot energy of the gamma peaks vs Electron kinetic energy
m2, c2 = np.polyfit(gamma, T, 1)
plt.plot(gamma, m2*(gamma)+c2)
plt.ylabel("Electron kinetic energy")
plt.xlabel("Eg")
plt.title("Eg vs Electron kinetic energy")
plt.grid(True);
```



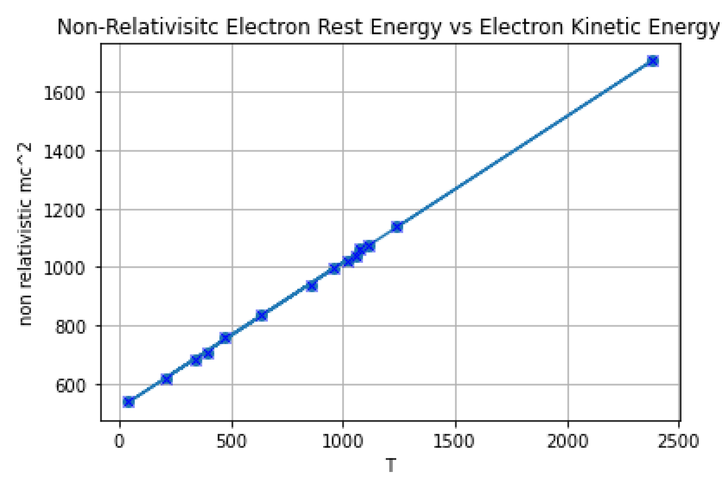

In [215…

```python
mc_2 = ((2*gamma)*(gamma-T))/T #relatvisitic electron rest energy
mc_nr = ((2*gamma)-T)**2/(2*T) # nonrelatvisitic electron rest energy
```

In [216…

```python
plt.scatter(T,mc_nr) #  Plotting Non-Relativisitc Electron Rest Energy vs Electron Kinetic Energy
m3, c3 = np.polyfit(T, mc_nr, 1)
plt.plot(T, m3*T+c3)
print(c3)
plt.ylabel("non relativistic mc^2")
plt.xlabel("T")
plt.title("Non-Relativisitc Electron Rest Energy vs Electron Kinetic Energy")
plt.grid(True)
plt.errorbar(T,mc_nr, xerr=err, fmt="x", color = "b");
```

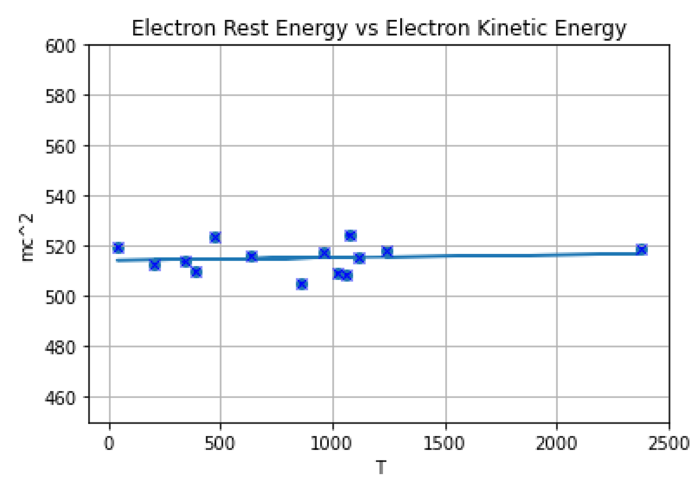
514.2247076960138




In [217…

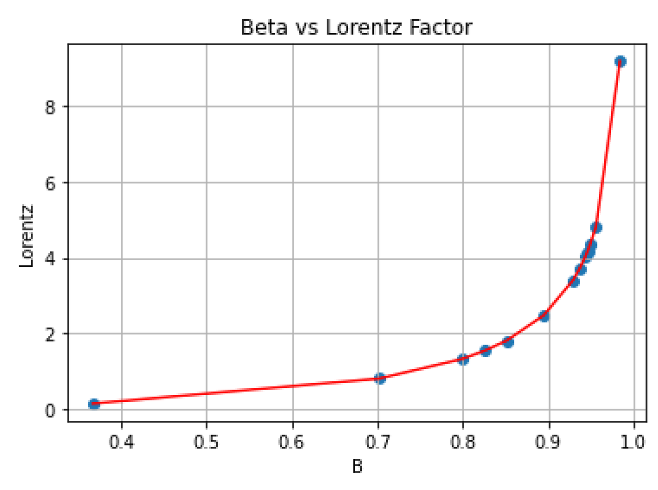
```python
plt.scatter(T,mc_2) # Plotting Electron Rest Energy vs Electron Kinetic Energy
m1, c1 = np.polyfit(T, mc_2, 1)
plt.plot(T, m1*T+c1)
plt.ylim(450,600)
plt.ylabel("mc^2")
plt.xlabel("T")
plt.title("Electron Rest Energy vs Electron Kinetic Energy")
plt.grid(True)
plt.errorbar(T,mc_2, xerr=err, fmt="x", color = "b")
np.mean(err)
c1 # y intercept = rest energy
```

Out[217… 514.2247076960132




In [218…

```python
lorentz = T**2 / (gamma*(gamma-T)) # Plotting Beta vs Lorentz Factor
beta = T*(2*gamma - T)/ (T**2- 2*gamma*T + 2*gamma**2)
plt.scatter(beta,lorentz)
plt.ylabel("Lorentz")
plt.xlabel("B")
plt.title("Beta vs Lorentz Factor")
plt.grid(True)
plt.plot(np.sort(beta),np.sort(lorentz), color = "r");
```




In [219…

```python
plt.scatter(beta,2*gamma - T) #Plotting Beta vs Electron Momentum
plt.ylabel("pc")
plt.xlabel("B")
plt.title("Beta vs Electron Momentum")
plt.grid(True)
np.polyfit(np.log(beta), 2*gamma - T, 1)
plt.plot(np.sort(beta), np.sort(2*gamma - T), color="g");
```