PHYC30300- Advanced Laboratory I

Skin Depth

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Abstract

This experiment highlights the skin depth effect and its dependence on frequency using a high conductivity cylinder placed in a magnetic field with its axis parallel to said magnetic field. By taking measurements of the rms voltage and frequency of three different conductor samples; brass, aluminium and mild steel, a study of rms voltage divided by frequency we can observe a relationship between frequency and the skin depth of a metal.
From our results, we have proved that skin depth does in fact depend on frequency, our results also clearly indicate that conductivity also affects the skin depth.

Introduction

When an AC current is applied to a conductor, the current concentrates near the surface of the conductor and its strength decreases towards the center of the conductor. The skin depth is the depth till which current flows in a conductor.[2]

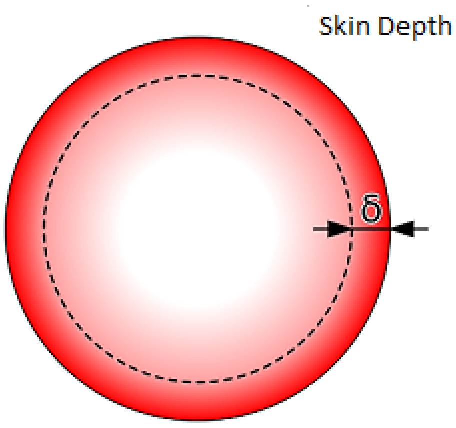


Fig.1 shows the cross section of a cylindrical conductor, the intensity of the red color represents the intensity of the current in a cylindrical conductor such as our metal samples [2]

The Skin Depth can be calculated using the following formulae:

$$Skin\ Depth = \delta = \sqrt{\frac{\rho}{\pi f \mu \mu_r}} = \sqrt{\frac{1}{\pi f \mu \sigma}} \qquad (1)$$

Where, ρ = the resistivity of the conductor , f = frequency, μ = permeability constant, μ_r = relative permeability and, σ = the conductivity of the material.

From these equations we can clearly see that the skin depth is dependent on the frequency of the current and the resistivity of the material. It is inversely proportional to the frequency and directly proportional to the resistivity. As it is directly proportional to the resistivity of the material, this means it is then also inversely proportional to the conductivity of the conductor.

These equations also reveal that the skin depth of the ideal conductor is zero, since its conductivity is infinite.

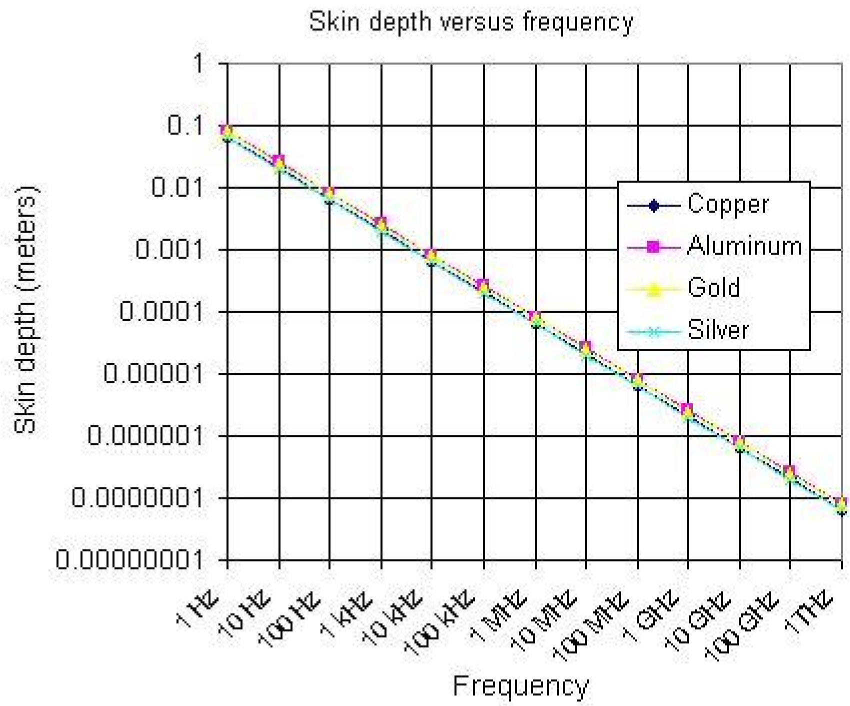


Fig.2 shows the inversely proportional relationship between frequency of current and skin depth for various metal samples [3]

This means that better conductors have a reduced skin depth. The overall resistance of the better conductor remains lower even with the reduced skin depth. However the better conductor will show a higher ratio between its AC and DC resistance, when compared with a conductor of higher resistivity. For example, at 60 Hz, copper conductor has 23% more resistance than it does at DC. The same size conductor in aluminum has only 10% more resistance with 60 Hz AC than it does with DC.[4]
However, as the skin depth shrinks with frequency, conductors can become thinner at higher frequencies with little impact on circuit loss. This allows companies to save money without compromising performance.

Skin depth also varies as the inverse square root of the permeability of the conductor. In the case of a particular mild steel, its conductivity is about 1/6 that of copper. However being ferromagnetic its permeability is about 2000 times greater.[5][6]
This reduces the skin depth for mild steel to about 1/15 that of copper. This makes pure mild steel wire useless for practical use such as AC power lines. However, power cables are usually made not from pure aluminium either but instead have a steel core in the cable. The steel provides a physical solid backing for the cable while the power is kept mainly in the aluminium. This is why power companies often run double or triple cables as they can save money making these cables rather than running singular expensive cables. [4]

Theory

Eddy current results from changing magnetic fields within a conductor. Skin Effect results from these circulating eddy currents. These currents are caused by changing magnetic fields which cancels the current flow in the center of a conductor and reinforces it in the surface of the material.

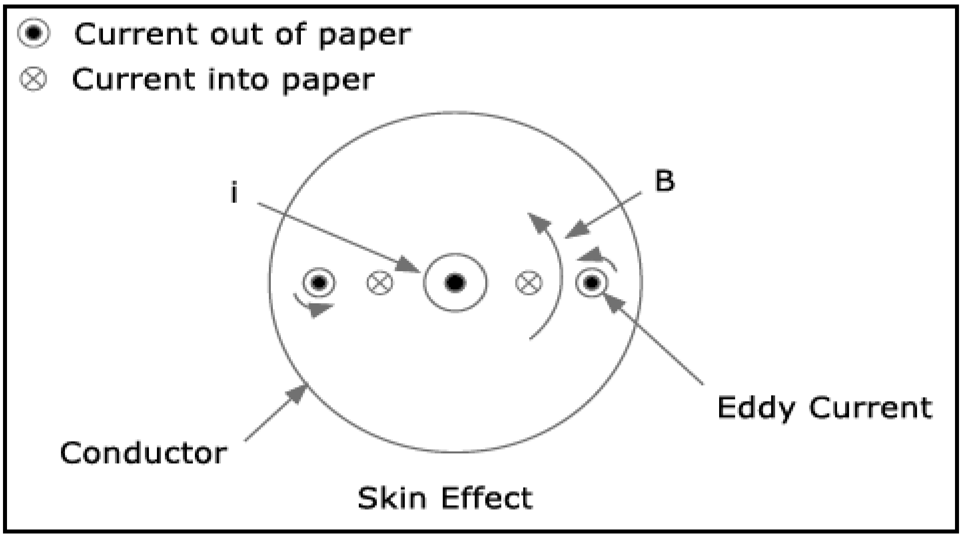


Fig.3 shows the skin effect and eddy currents in a metal cylinder [7]

Fig.3 shows a current, i, coming out from the page. By the Right Hand Rule, this current induces the magnetic field B. The four small circles represent eddy currents circulating within the cylindrical conductor. By Lenz's law, these eddy currents induce opposing fields, indicated by the small curved arrows. The eddy currents flowing into the page oppose the current. Thus the conductor current tends to concentrate toward the outer edge of the conductor's cross section.This then means that current density decreases exponentially moving in from the outer edge of the conductor. [7]

This experiment aims to use the presence of a strong skin depth effect in a high conductivity cylinder placed in a magnetic field of frequency v, and amplitude B, with its axis parallel to said magnetic field. [1] A pick-up coil is wound around the midsection of the cylindrical conductor sample, and the frequency is set high enough so that the skin depth is much smaller than the cylinder radius R but much larger than the thickness of the air gap between the cylinder surface and the mean radius R' of the pick-up coil. [1]

Under these conditions of operation, one expects the rms pick-up voltage V to contain two measurable terms. The first should originate from the field in the air gap and be proportional to the factor; $(2\pi v)(B\pi(R'^2 - R^2)$.

The second should originate from the field contained in a layer of thickness δ below the surface of the cylinder, and be proportional to the factor $(2\pi v)(B2\pi R\delta)$.[1]

A study of V/v as a function of frequency [1] provides the relationship;

$$\frac{V}{v} = (2\pi v)(B\pi(R'^2 - R^2).\frac{1}{2} + (2\pi v)(B2\pi R\delta) \qquad (2)$$

This in turn relates to our data taken in this experiment as we find that from our study of V/v as a function of frequency the relationship can be simply described as;

$$\frac{V}{v} = \alpha + \beta v^{\frac{-1}{2}} \qquad (3)$$

Where, α = y intercept and β = slope, of our plots of V/v versus square root of the frequency

Experimental Procedure

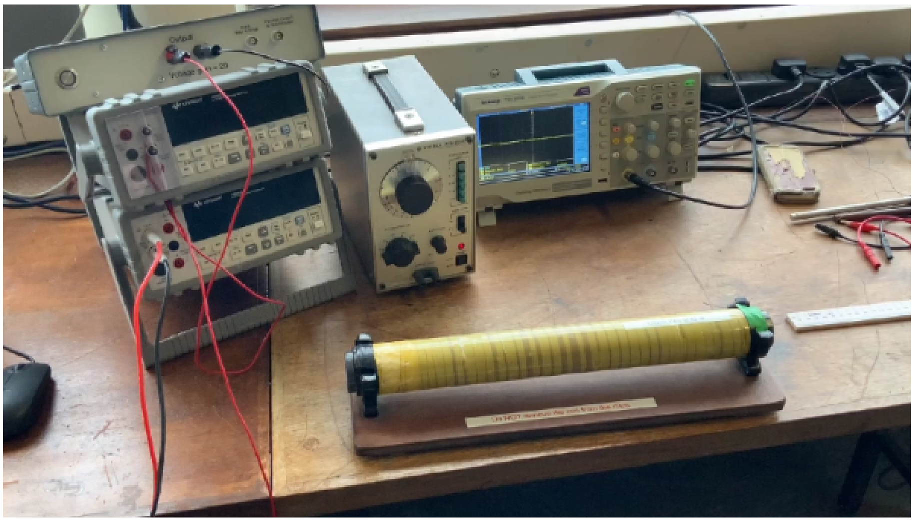


Fig.4 shows the apparatus used in this experiment

In this experiment, an induction field was created using a long solenoid consisting of copper wire wound around a hollow bakelite cylinder. A constant current of 19mA was powered through the solenoid using an AC generator and an amplifier. Two multimeters were then used to measure both current and voltage for each of our three sample metals; brass, aluminium and mild steel. Frequency was measured to an accuracy of 0.5 kHz and voltage to 5x10^-3 mV. Starting at 10 kHz, increasing in 5 kHz stages until 70kHz, secondary voltage measurements were taken off a short solenoid wound on each sample.

Results and Discussion

The graphs (Fig.4.A and Fig.4.B) below show the rms pickup voltage divided by the frequency as a function of the inverse square root of the frequency. We can see the relationship is linear between the two parameters. Fig.4.A shows the rms pickup voltage divided by the frequency as a function of the inverse square root of the frequency for brass and aluminium Fig. 4.B the same relationship but for mild steel. It is clear to see that there is a slight deviation from linearity in our plots in the lower frequencies. This arises as the ratio of δ:R is too large for the simple theory proposed to hold.[1]

Using our data and comparing how it fits the the relation;

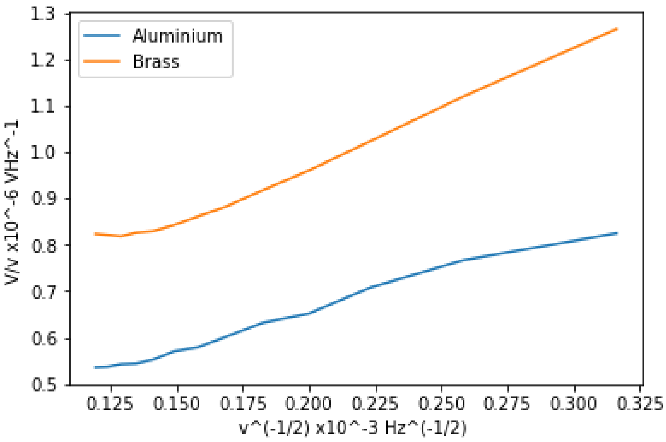
$$\frac{V}{v} = \alpha + \beta v^{\frac{-1}{2}} \quad (3)$$

We can see that our data fits the relationship accurately. Equation 3 is clearly written in the linear form in relation to our data which fits our linear relationship;

$$y = c + mx \quad (4)$$

We can calculate our relative error (due to decimal rounding) and see how well the relationship fits our data by inserting our average values for each parameter. We exclude our data points from the lower-frequency end as they deviate slightly from the linearity.

	y	c	m	x	Relative error
Brass	0.92	0.51	2.33	.18	0.01
Aluminum	0.62	0.33	1.60	.18	0.02
Mild Steel	10.31	1.17	51.5	.18	0.13



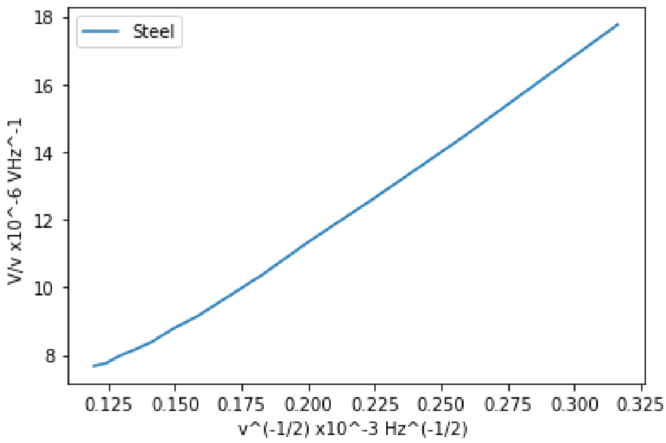


Fig.4.A shows the rms pickup voltage divided by the frequency as a function of the inverse square root of the frequency for brass and aluminium Fig. 4.B the same relationship but for mild steel

As we can see from our table and the graph below, Fig.5, mild steel has significantly higher values than both brass and aluminium. This is due to the fact that mild steel is a ferromagnetic material and also has a significantly lower conductivity. The skin depth for mild steel is much smaller than that of brass and aluminium. Skin depth varies as the inverse square root of the conductivity of a conductor.

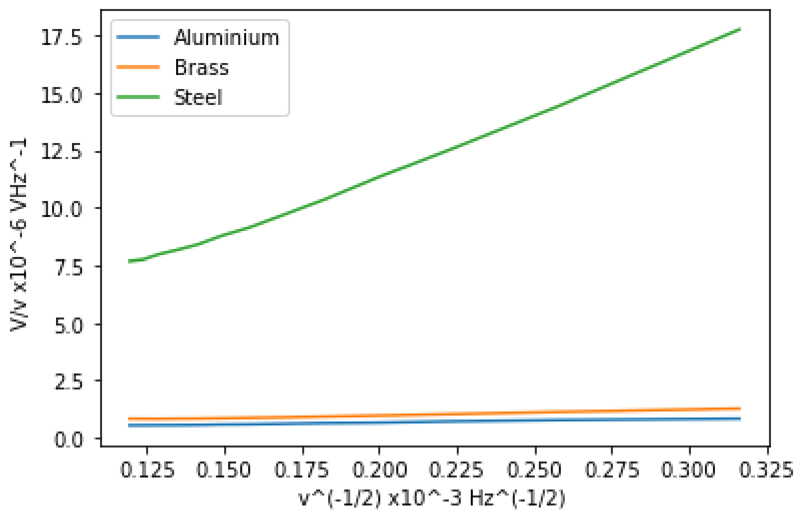


Fig.4.A shows the rms pickup voltage divided by the frequency as a function of the inverse square root of the frequency for brass, aluminium and steel on the same plot

Conclusion

From our results, it can be clearly seen that our data analysis agrees with the expected results.[1 Theoretically, we expected there to be a linear relationship between rms pickup voltage divided by the frequency as a function of the inverse square root of the frequency. We also expected to see that conductivity is inversely proportional to the skin depth of a material at a certain frequency. This was proven to agree with our results as we compared our results for our different samples with vastly different conductivity values. Our main aim was to prove that the skin depth is inversely proportional to frequency. This was consistent with our findings. I believe that this experiment is extremely important and has valuable real life applications i.e in the power industry. I believe that it is important that undergraduates develop a strong understanding of skin depth and how its effect differs in different materials.

References

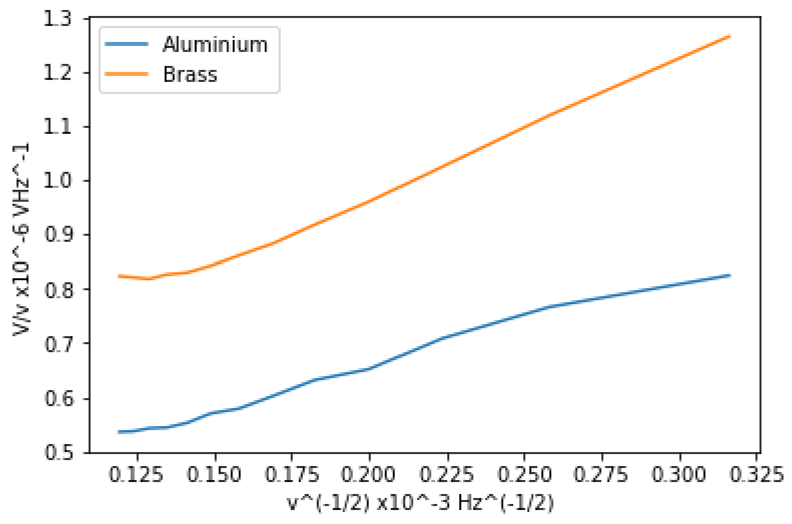
[1]Wiederick, H.D. and Gauthier, N., 1983. Frequency dependence of the skin depth in a metal cylinder. *American Journal of Physics*, *51*(2), pp.175-176.
[2]everything RF, *What is Skin Depth? - everything RF*, Everythingrf.com, Date Last Accessed; 15/1/2022,https://www.everythingrf.com/community/what-is-skin-depth
[3]Microwaves101 | Skin Depth, Microwaves101.com, Date Last Accessed; 14/1/2022 https://www.microwaves101.com/encyclopedias/skin-depth
[4] Fink, Donald G.; Beatty, H. Wayne, eds. (1978), *Standard Handbook for Electrical Engineers* (11th ed.), McGraw Hill, p. Table 18–21
[5] Keith Gibbs, 2020, Relative Permeability, School Physics, Data Last Accessed; 17/01/2022

https://www.schoolphysics.co.uk/age16-19/Electricity%20and%20magnetism/Electromagnetism/text/Relative_permeablity/index.html
[6]Engineering ToolBox, (2003). *Resistivity and Conductivity - Temperature Coefficients Common Materials*. [online], Data Last Accessed; 17/01/2022, Available at: https://www.engineeringtoolbox.com/resistivity-conductivity-d_418.html
[7] Dataforth, Dataforth Tech Note TN905,Eddy Current - Skin, and Proximity Effects, Data Last Accessed; 17/01/2022 https://www.dataforth.com/eddy-current-skin-proximity-effects.aspx

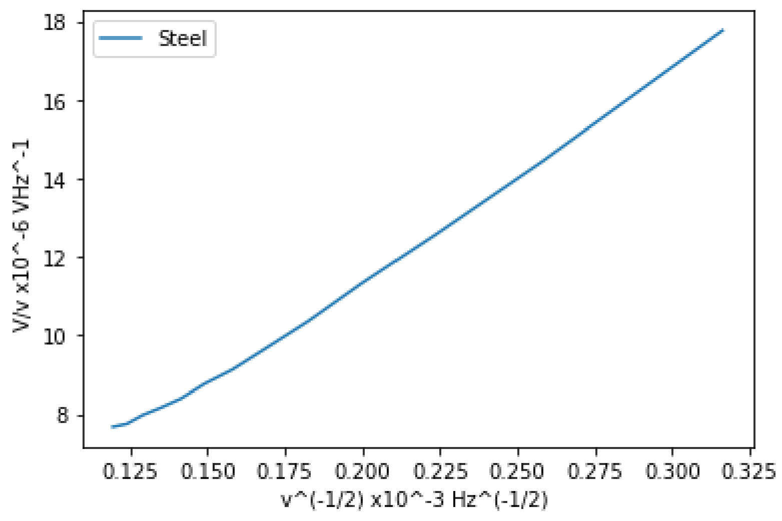
In [33]:
```python
import numpy as np
import matplotlib.pyplot as plt
f,v_brass = np.loadtxt("Brass.txt", unpack=True)
f,v_al = np.loadtxt("Aluminum.txt", unpack=True)
f,v_steel = np.loadtxt("Steel.txt", unpack=True)
#import data
```

In [34]:
```python
def func1(x):
    return (1/(x**(0.5)))
```

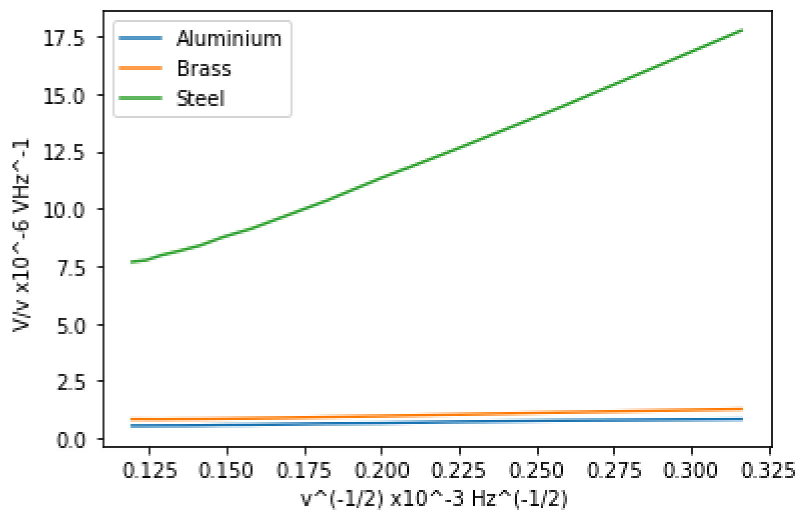
In [66]:
```python
plt.plot(func1(f),v_al*(1/f))
plt.plot(func1(f),v_brass*(1/f))
plt.xlabel("v^(-1/2) x10^-3 Hz^(-1/2) ")
plt.ylabel("V/v x10^-6 VHz^-1 ")
plt.legend(["Aluminium","Brass"]);
```



In [53]:
```python
plt.plot(func1(f),v_steel*(1/f))
plt.xlabel("v^(-1/2) x10^-3 Hz^(-1/2) ")
plt.ylabel("V/v x10^-6 VHz^-1 ")
plt.legend(["Steel"]);
```



In [54]:
```python
plt.plot(func1(f),v_al*(1/f))
plt.plot(func1(f),v_brass*(1/f))
plt.plot(func1(f),v_steel*(1/f))
plt.xlabel("v^(-1/2) x10^-3 Hz^(-1/2) ")
plt.ylabel("V/v x10^-6 VHz^-1 ")
plt.legend(["Aluminium","Brass","Steel"]);
```



In [51]:
```python
from scipy.stats import linregress
linregress(func1(f), v_al*(1/f))
```

Out[51]:
```
LinregressResult(slope=1.6021812999007368, intercept=0.33473248135654454, rvalue=0.994332542887967, pvalue=4.6344264260796795e-12, stderr=0.05165074281166071, intercept_stderr=0.009613065026183726)
```

In [49]:
```python
linregress(func1(f), v_brass*(1/f))
```

Out[49]:
```
LinregressResult(slope=2.3349016683394592, intercept=0.5086507542195661, rvalue=0.9916073169506513, pvalue=3.995358325454017e-11, stderr=0.09178776559613433, intercept_stderr=0.017083234649716325)
```

In [50]:
```python
linregress(func1(f), v_steel*(1/f))
```

Out[50]:
```
LinregressResult(slope=51.49736954723, intercept=1.1730159095897434, rvalue=0.9979852232316443, pvalue=1.5799139801113187e-14, stderr=0.9871296478107101, inte
rcept_stderr=0.18372129764483996)
```

In [68]:
```python
np.mean(func1(f))
```

Out[68]:
```
0.1773647646868714
```

In [69]:
```python
np.mean(v_steel*(1/f))
```

Out[69]:
```
10.30683474132705
```

In [77]:
```python
10.31-(1.17)-(51.5)*(.18)
```

Out[77]:
```
-0.129999999999999
```

In [74]:
```python
np.mean(v_brass*(1/f))
```

Out[74]:
```
0.9227800391915777
```

In [79]:
```python
0.92-(0.51)-(2.33)*(.18)
```

Out[79]:
```
-0.009399999999999964
```

In [80]:
```python
np.mean(v_al*(1/f))
```

Out[80]:
```
0.6189029905991444
```

In [81]:
```python
0.62-(0.33)-(1.60)*(.18)
```

Out[81]:
```
0.0020000000000000018
```

In [ ]: