KCL at node X:

$$\frac{V_x - V_{in}}{Z_1} + \frac{V_x - V_{out}}{Z_3} + \frac{V_x - V_+}{Z_2} = 0$$

$$\frac{V_{out} - V_x}{Z_2} + \frac{V_{out}}{Z_4} = 0$$

$$V_x = V_{out}(\frac{Z_2}{Z_4} + 1)$$

$$\frac{V_{out}}{V_{in}} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_3(Z_1 + Z_2) + Z_3 Z_4}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + sC_1(R_1 + R_2) + s^2 C_1 C_2 R_1 R_2}$$
$$= \frac{1}{1 + \frac{2\zeta s}{\omega_o} + \frac{s^2}{{\omega_o}^2}}$$

Butterworth Derivation:

$$|H(\omega)|^2 = \frac{1}{(1 - (\frac{\omega}{\omega_o})^2)^2 + (\frac{2\zeta\omega}{\omega_o})^2}$$

$$\frac{\delta \frac{1}{|H(\omega)^2}}{\delta\omega} = \frac{\delta}{\delta\omega}[(1 - (\frac{\omega}{\omega_o})^2)^2 + (\frac{2\zeta\omega}{\omega_o})^2] = 0$$

$$\frac{\delta}{\delta\omega}[(1 - \omega^2)^2 + (2\zeta\omega)^2] = 0$$

$$2(1 - \omega^2)(-2\omega) + 8\omega^2\omega = 0$$

$$\omega = 0, = \sqrt{1 - 2\zeta^2}$$

No ripple occurs in the passband if $\omega = \sqrt{1 - 2\zeta^2} = 0$ so $\zeta = \frac{1}{\sqrt{2}}$.