

K is the gain of the amplifier and AB is the loop gain. Let  $\tau = CR$ .

$$\begin{aligned}
A &= K = \frac{R_1 + R_2}{R_2} \\
B &= \frac{-V_+}{V_{out}} = \frac{Z_2}{Z_1 + Z_2} \\
Z_1 &= \frac{1}{sC} + R = \frac{1 + sCR}{sC} \\
Z_2 &= \frac{1}{sC} || R = \frac{R}{1 + sCR} \\
B &= -\frac{R}{R + \frac{(1+sCR)^2}{sC}} = -\frac{sCR}{1 + 3sCR + s^2C^2R^2} \\
AB &= -\frac{-sK\tau}{1 + 3s\tau + s^2\tau^2}
\end{aligned}$$

This transfer function has a zero at  $\omega = 0$  and two poles at  $\omega_o = \frac{1}{\tau}$ .  $-\angle AB(\omega_o) = \frac{K}{3}$ . A Nyquist plot is a plot of the frequency response of a system. The stability of an LTI closed-loop system is done by examining the Nyquist plot of the open loop system. If the point  $-1 + j0$  is encircled by the Nyquist plot, then the closed-loop system is unstable. If the plot does not encircle the  $-1 + j0$  point, then it is stable. For an oscillator, the poles are ideally on the imaginary axis. This is expressed in the Nyquist plot by the loop passing through the  $-1 + j0$  point. For the Wien-Bridge oscillator, the Nyquist plot passes through the  $-1 + j0$  point when  $K = 3$ . This results in a closed loop gain of:

$$\begin{aligned}
H(s) &= \frac{V_{out}}{V_{in}} = \frac{A}{1 + AB} \\
&= \frac{3}{1 - \frac{3s\tau}{1 + 3s\tau + s^2\tau^2}} = \frac{3(1 + 3s\tau + s^2\tau^2)}{1 + s^2\tau^2}
\end{aligned}$$

with poles at  $\pm \frac{j}{\tau}$ . This will produce an output sine wave that oscillates from rail to rail at a frequency of  $f = \frac{1}{2\pi RC}$ . There are a variety of methods that can be used to limit the output of the oscillator, however many of them will produce additional harmonics. The Wien-Bridge oscillator was the first product made by Hewlett-Packard. Their original design used a lightbulb in place of  $R_2$  which provided a soft rise to a limited output, reducing the amount of harmonics.