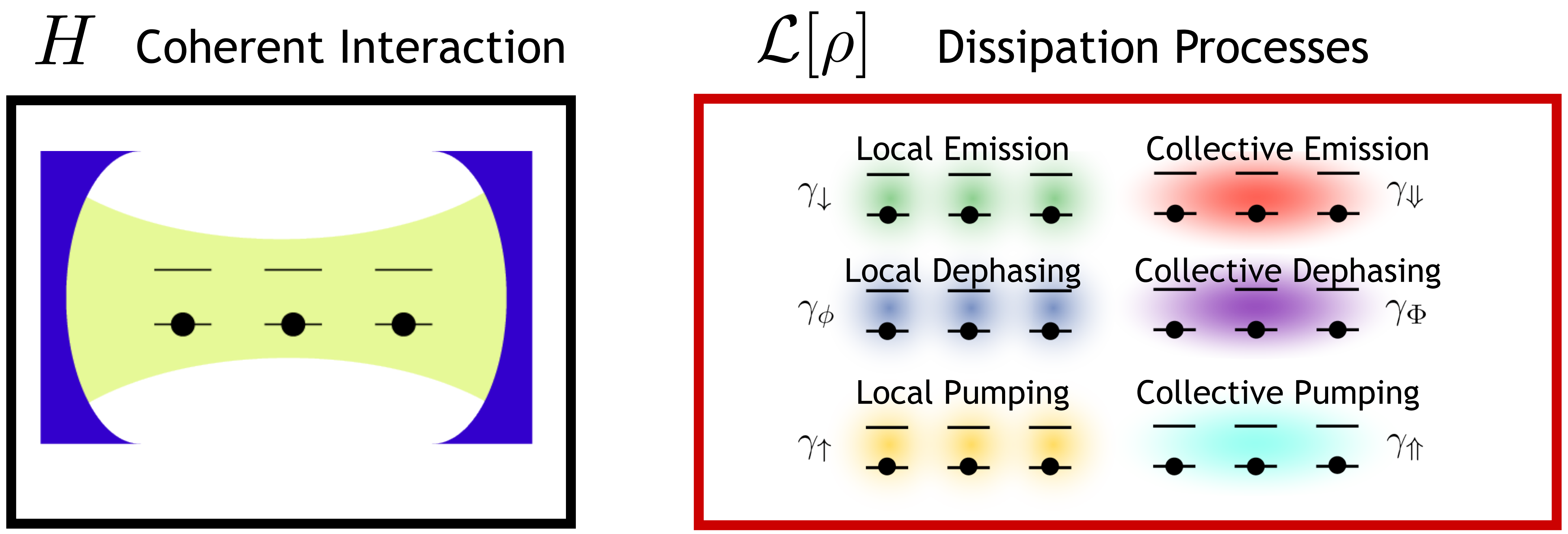
Collective and local effects in the driven-dissipative dynamics of many-body quantum systems



NS *et al.*, Phys. Rev. A **96**, 023863 (2017)
NS *et al*. Phys. Rev. A **98**, 063815 (2018)
M. Cirio, NS, *et al.*, Phys. Rev. Lett. **122**, 190403 (2019)

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Cluster for Pioneering Research
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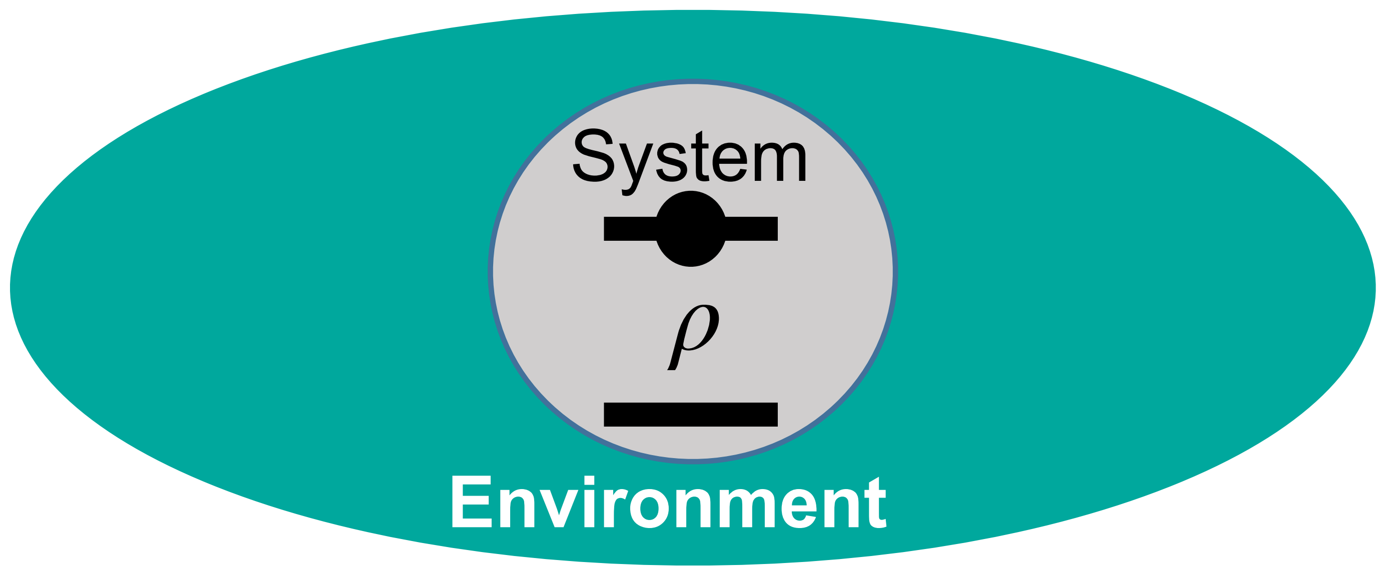
25th June 2019
CM Seminar
ICTP, Trieste, Italy

Open quantum systems: Lindblad master equation

In general, the Liouvillian space grows exponentially as 4^N

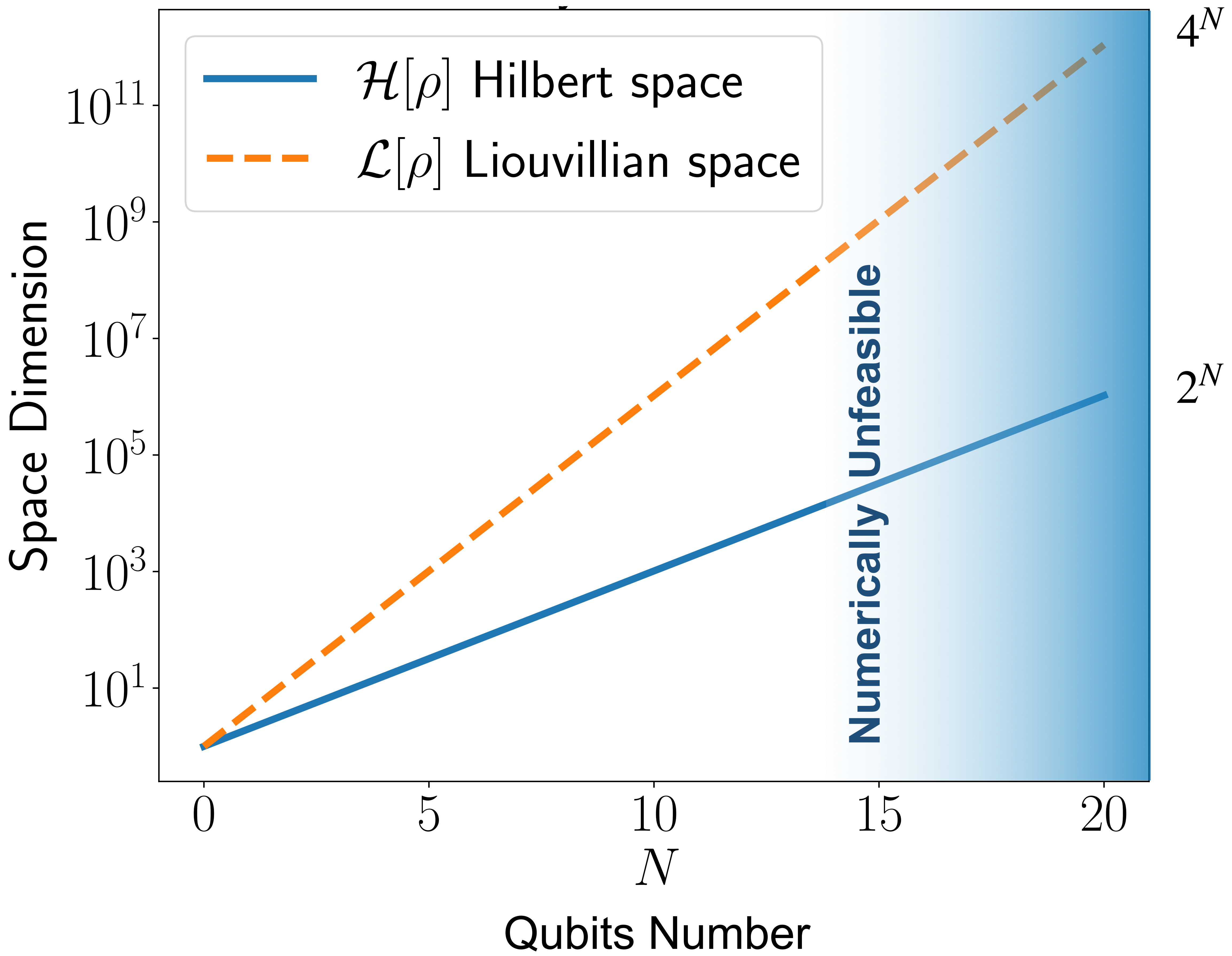


$$\frac{d}{dt}\rho = -\frac{i}{\hbar}[H,\rho] + \sum_{i,j}^{N} \gamma_{ij} \left(L_i \rho L_j^\dagger - \frac{1}{2} L_i^\dagger L_j \rho - \frac{1}{2} \rho L_i^\dagger L_j \right)$$

Approximations:
- **Born**
- **Markov**
- **Rotating wave**

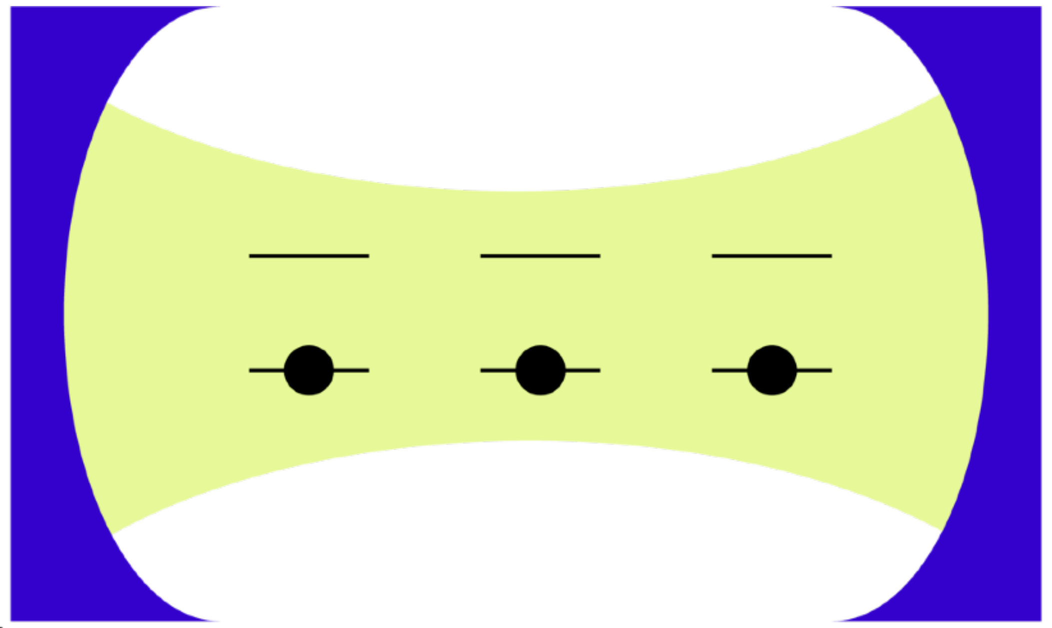
$$\frac{d}{dt}\overrightarrow{\rho} = \mathscr{L}\overrightarrow{\rho}$$

Liouvillian



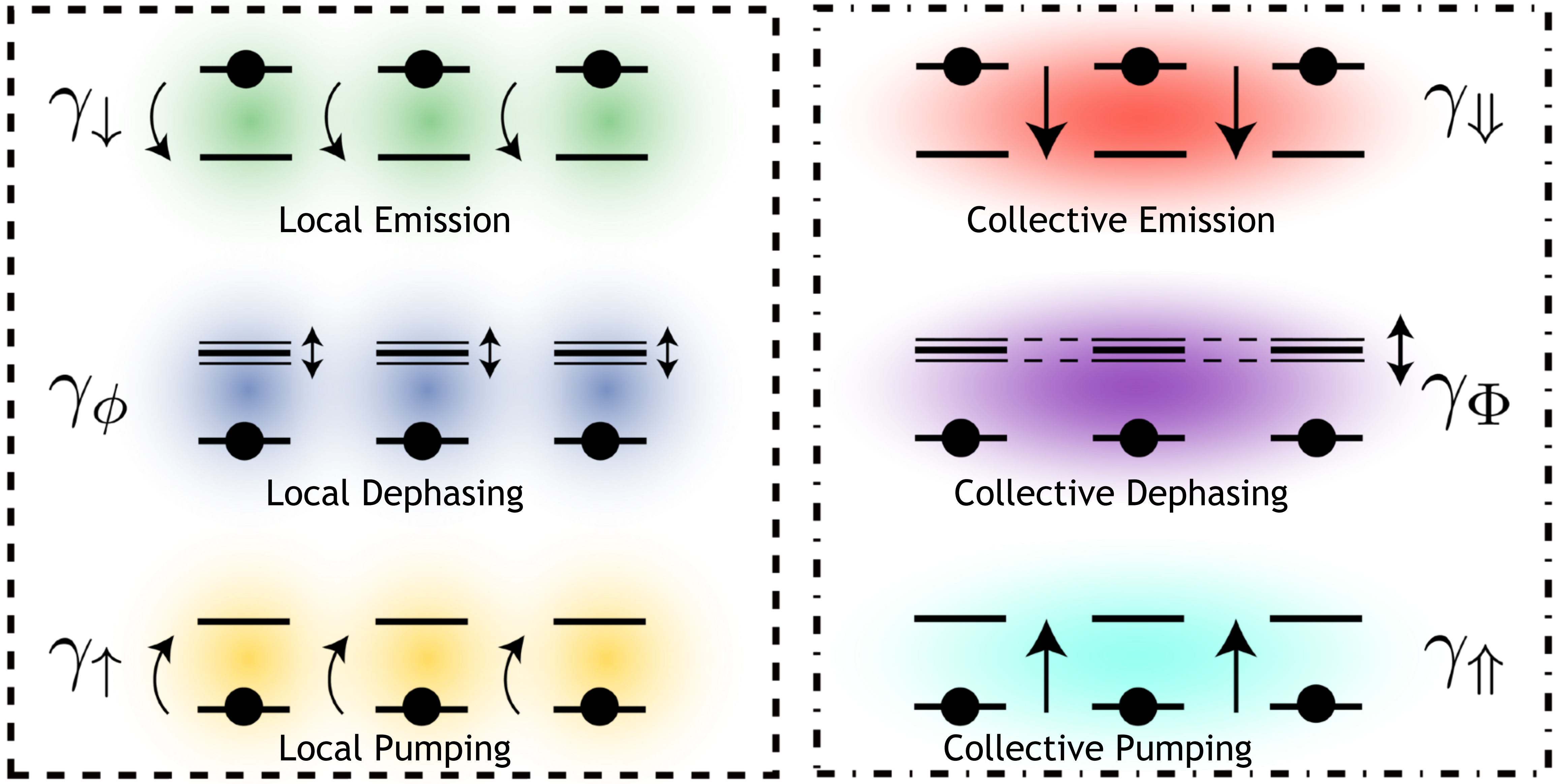
Dissipative open quantum systems simulation

Coherent Dynamics $$i\hbar\frac{d}{dt}\rho = [H,\rho] + \sum_{i,j}^{N}\gamma_{ij}\left(L_i\rho L_j^\dagger - \frac{1}{2}L_i^\dagger L_j\rho - \frac{1}{2}\rho L_i^\dagger L_j\right)$$



$\gamma_{ij} \equiv \gamma\delta_{ij}$ $\qquad$ $\gamma_{ij} \equiv \gamma\ \forall i,j$

Dissipation Processes $\mathcal{L}_{\mathrm{loc}}[\rho]$ Local $\qquad$ $\mathcal{L}_{\mathrm{col}}[\rho]$ Collective



Open quantum systems with local and collective incoherent processes: Efficient numerical simulation using permutational invariance

N. Shammah, S. Ahmed, N. Lambert, S. De Liberato, and F. Nori
Phys Rev A **98**, 063815 (2018) arXiv:1805.05129

Open Quantum Systems

Non-Equilibrium Phase Transitions

Quantum Optics, Spin Squeezing

Ultrastrong Coupling of Light and Matter

Dicke states and collective operators

Defining the algebra to rewrite the Hamiltonian with collective operators

$$H_0 = \boxed{\hbar\omega_0 \sum_{n=1}^{N} J_{z,n}} = \boxed{\hbar\omega_0 J_z} \qquad H_{\text{int}} = \boxed{\mathbf{d}\cdot\mathbf{E}\sum_{n=1}^{N}(J_{+,n} + J_{-,n})} = \boxed{\mathbf{d}\cdot\mathbf{E}(J_{+} + J_{-})}$$

Local Pauli Matrices

$$\boxed{[J_{x,n}, J_{y,n'}] = iJ_{z,n}\delta_{n,n'}} \qquad J_{\pm,n} = J_{x,n} \pm iJ_{y,n}$$

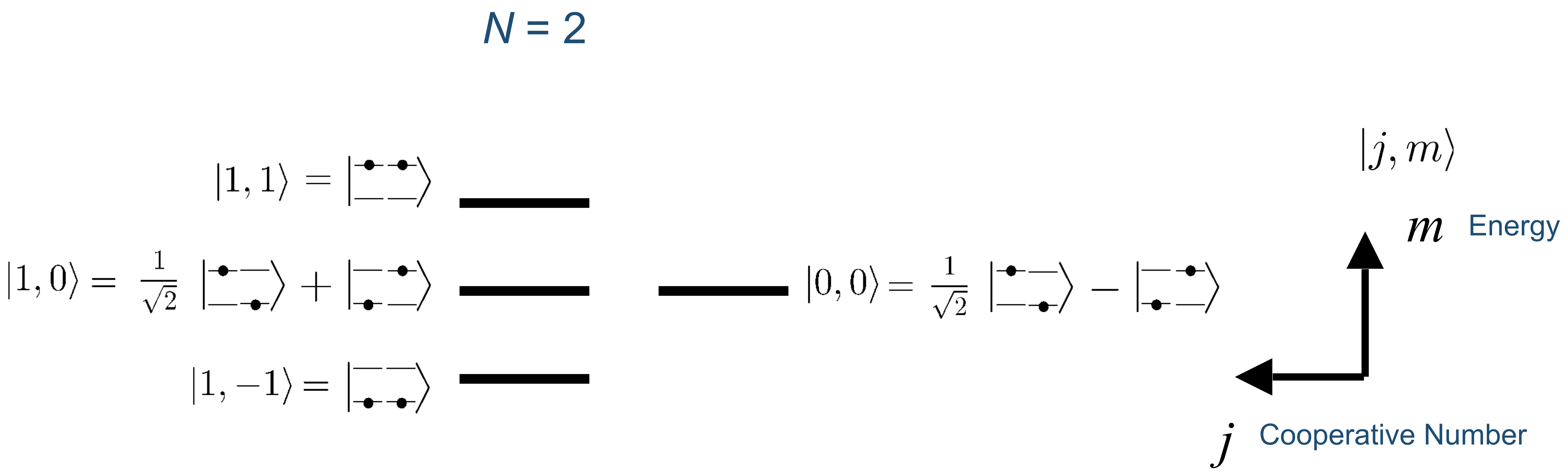
Collective Spin Operators ans Dicke states

$$\boxed{\begin{aligned} J_z|j,m,\alpha\rangle &= m|j,m,\alpha\rangle \\ J^2|j,m,\alpha\rangle &= j(j+1)|j,m,\alpha\rangle \end{aligned}} \qquad \begin{aligned} &|m| \leq j \\ &j = \frac{N}{2}, \frac{N}{2} - 1, ..., j_{\text{min}} + 1, j_{\text{min}} \end{aligned}$$

$$J_{\pm}|j,m,\alpha\rangle = \sqrt{(j \mp m)(j \pm m + 1)}|j,m \pm 1,\alpha\rangle \qquad |j,m,\alpha\rangle \to |j,m\rangle$$

Dicke states for *N*=2

Dicke States are collective spin states



Non-symmetrical Dicke states have a degeneracy attached

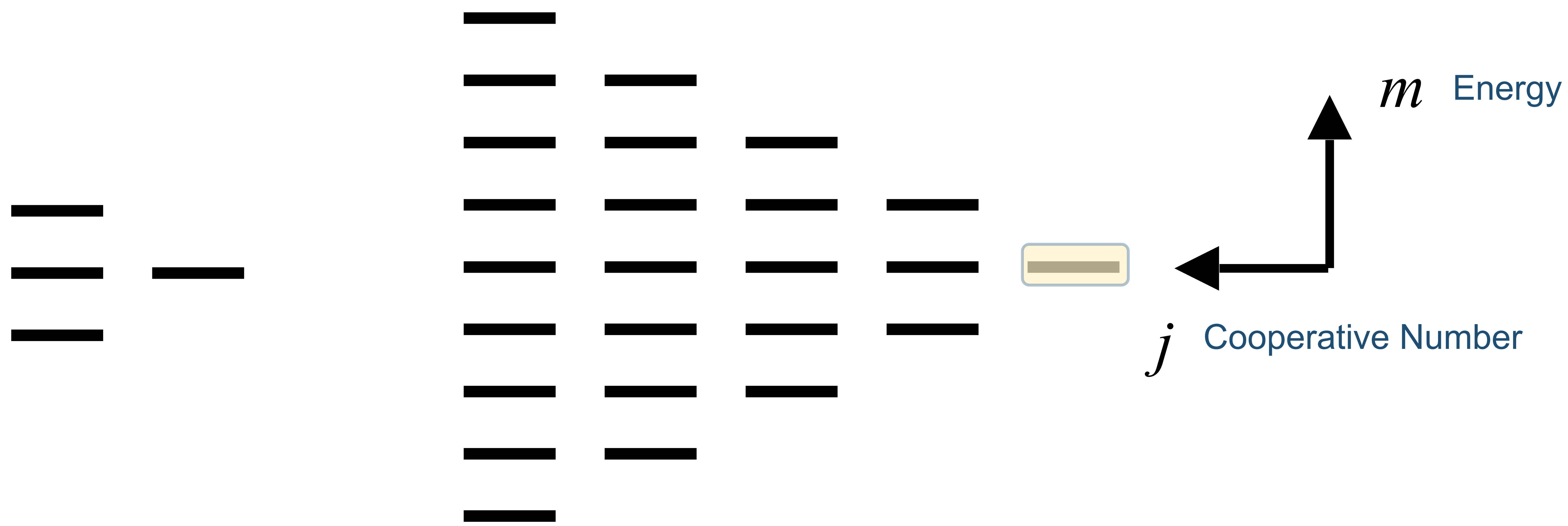
Exploiting permutational invariance

$$|j, m, \alpha\rangle \rightarrow |j, m\rangle$$

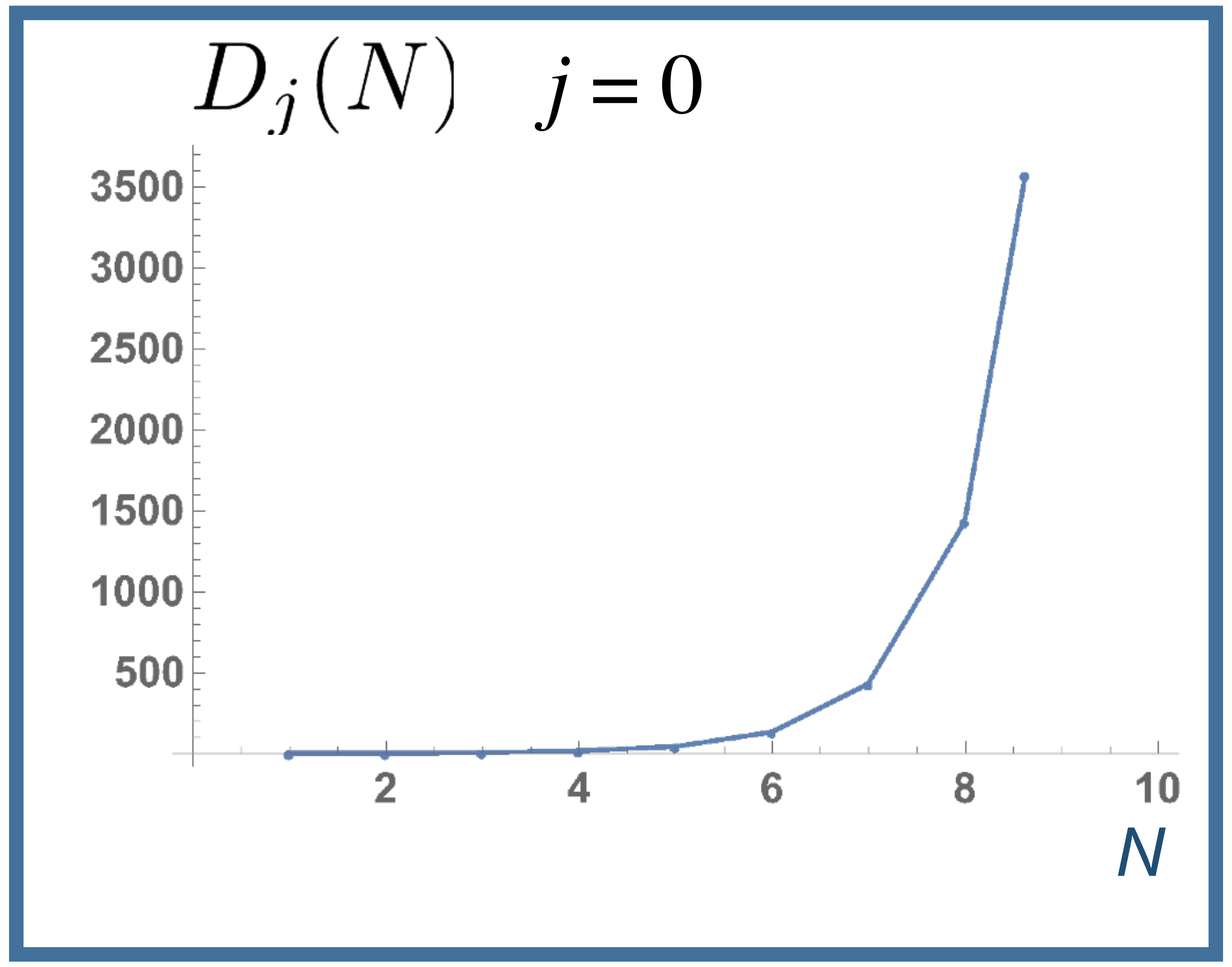
Degeneracy of a Dicke state

$$D_j(N) = \frac{N!(2j+1)}{(\frac{N}{2}+j+1)!(\frac{N}{2}-j)!}$$

N = 2 *N* = 10

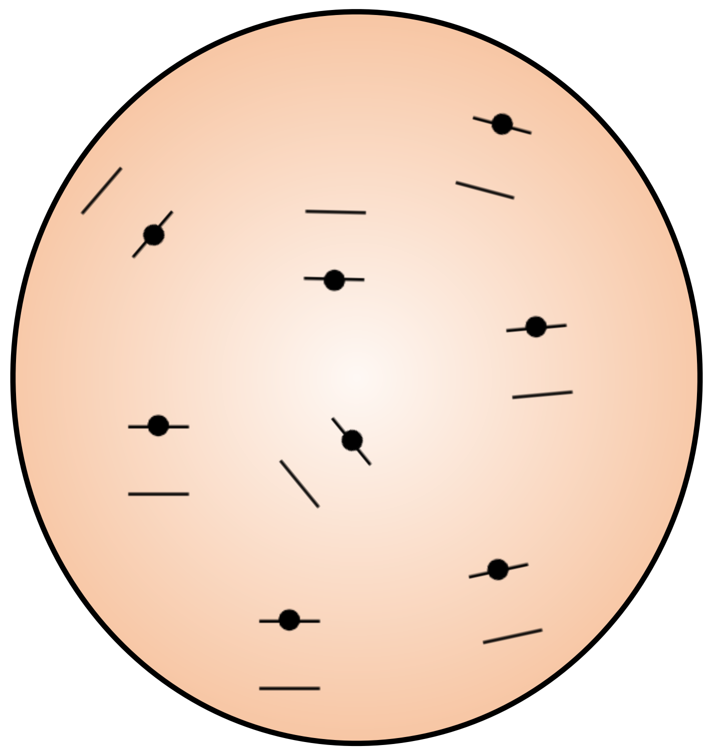


$2^N = 4$ $2^N = 1024$



Dicke superradiance: light emission from an atomic cloud

Probing the dynamics of *many* qubits

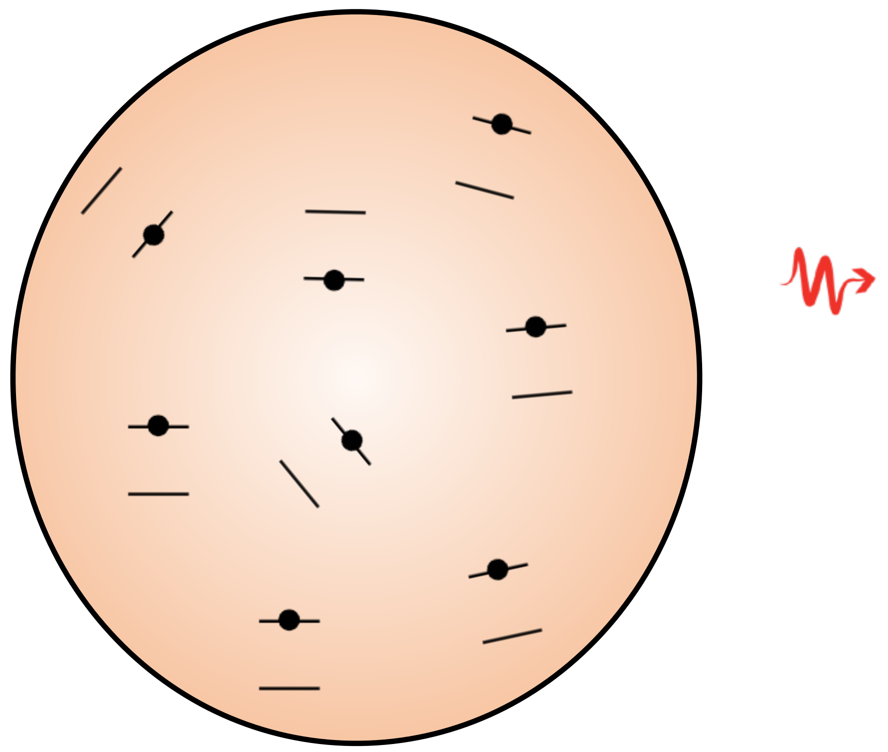


$t = 0$

$|\uparrow\uparrow \dots \uparrow\rangle$

Dicke superradiance: light emission from an atomic cloud

Probing the dynamics of *many* excited qubits / two-level systems



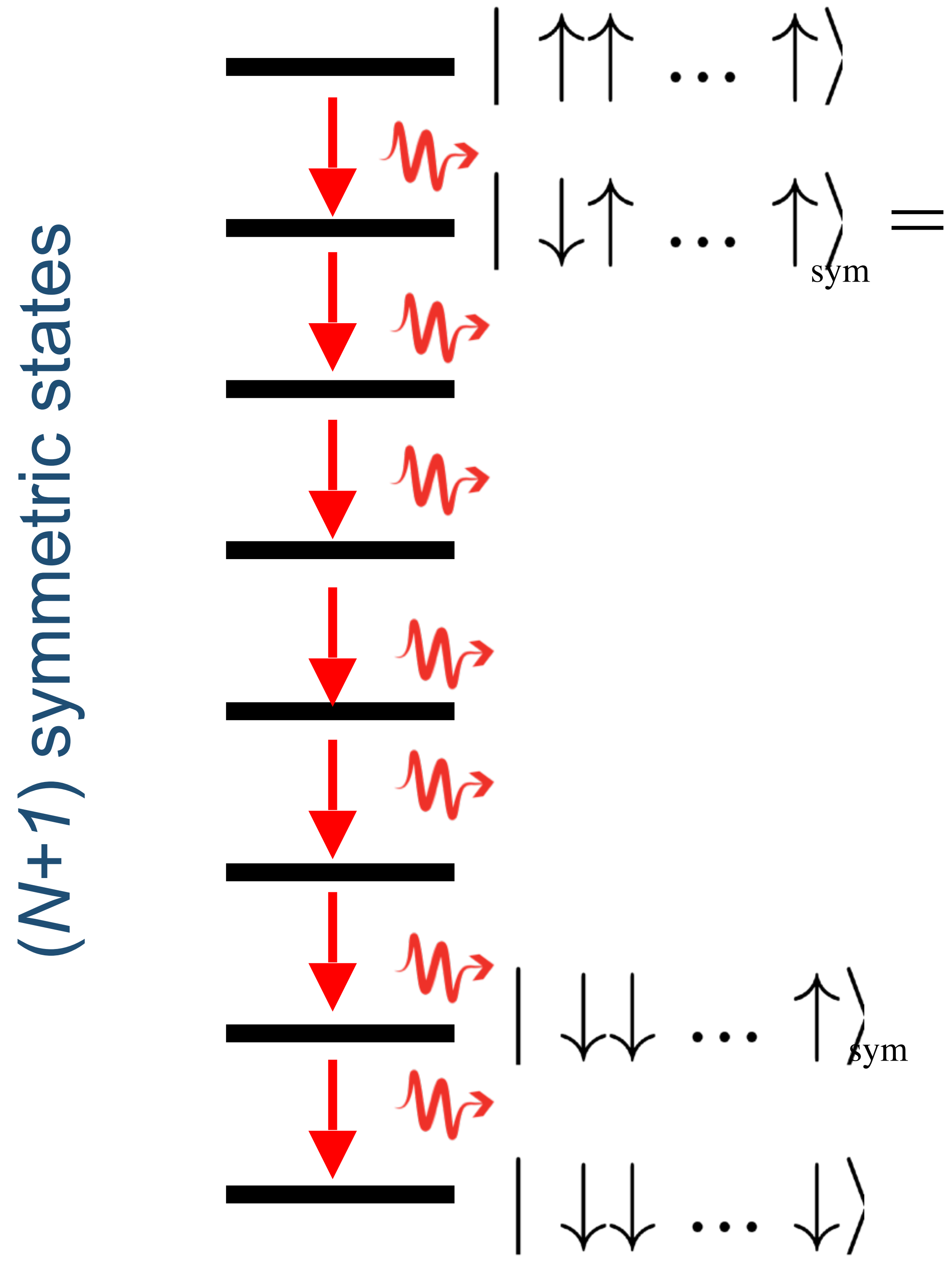
$t = 0$ $\quad |\uparrow\uparrow \dots \uparrow\rangle$

$$\big|\downarrow\uparrow \dots \uparrow\big\rangle_{\text{sym}} = \frac{1}{\sqrt{N}}\left(|\downarrow\uparrow \dots \uparrow\rangle + |\uparrow\downarrow \dots \uparrow\rangle + \dots\right)$$

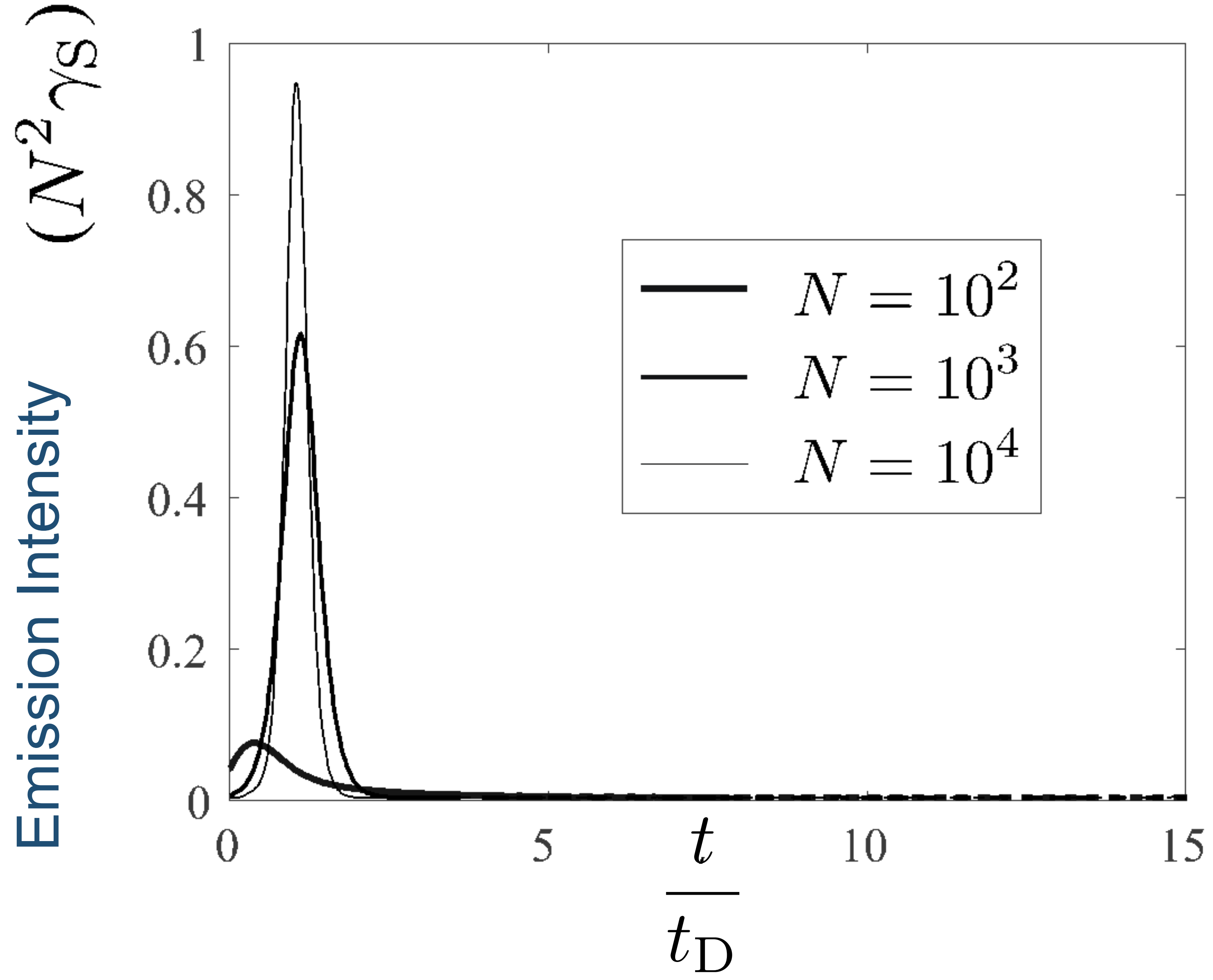
Superradiance emerges from collective coupling

Ideal case

$$\dot{\rho} = -i[\omega_0 J_z , \rho] + \gamma \left(J_- \rho \sigma_+ - J_+ J_- \rho - \rho J_+ J_- \right)$$



$$|\downarrow\uparrow \dots \uparrow\rangle_{\mathrm{sym}} = \frac{1}{\sqrt{N}} \left(|\downarrow\uparrow \dots \uparrow\rangle + |\uparrow\downarrow \dots \uparrow\rangle + \dots \right)$$

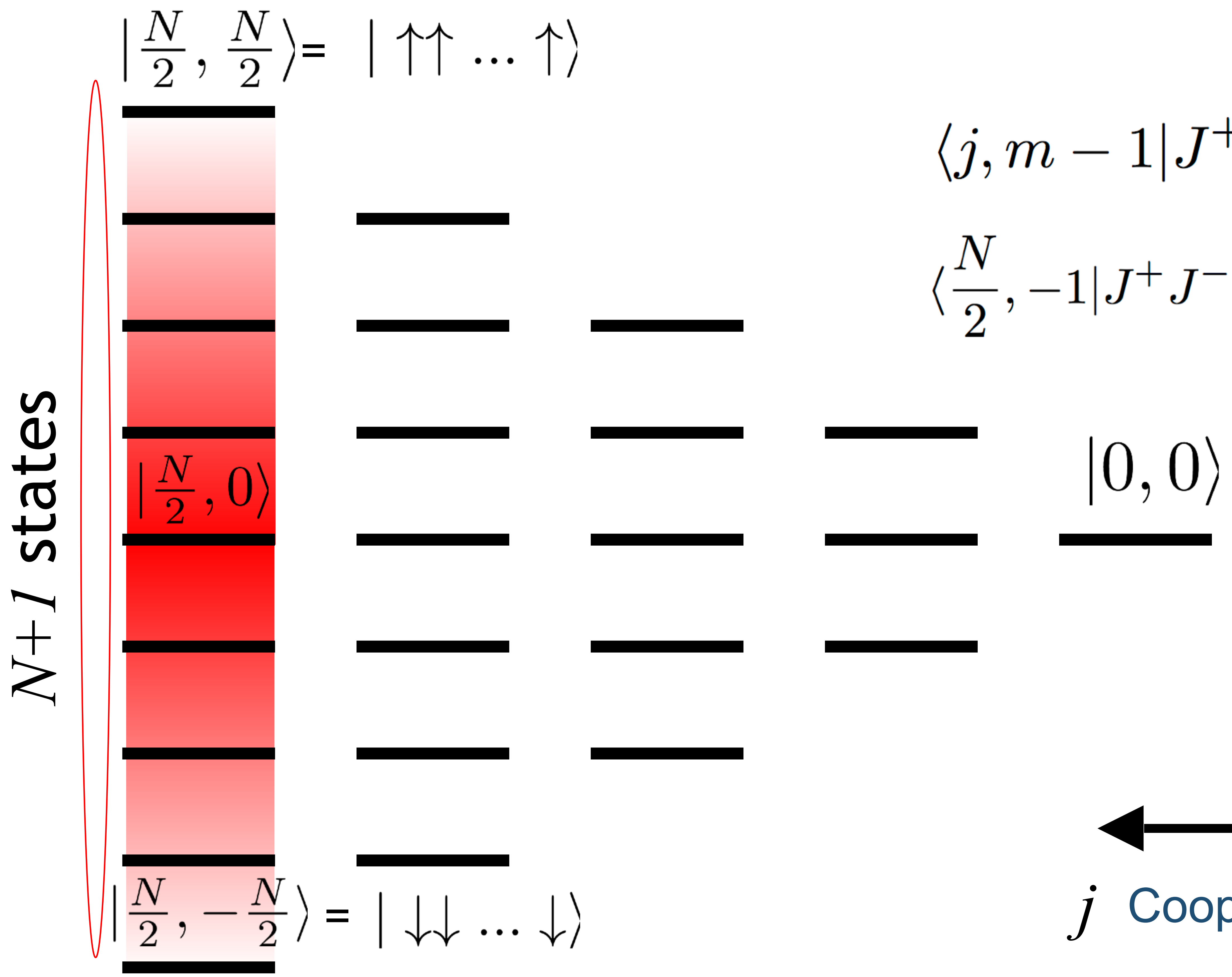


$$t_{\mathrm{D}} = \frac{\log N}{(N \gamma_{\mathrm{S}})}$$

M. Gross & S. Haroche, Physics Reports **93**, 301 (1982)

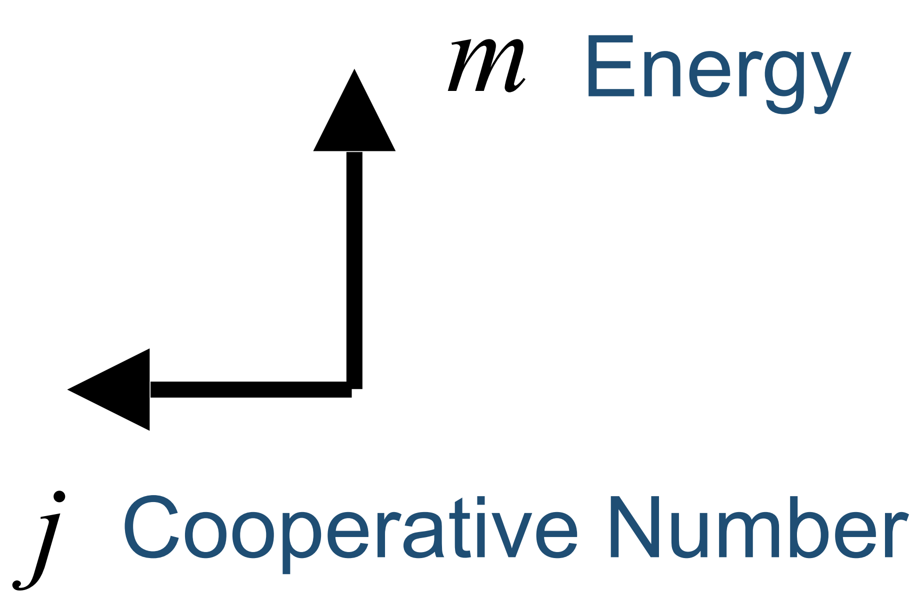
Superradiance in the symmetric limit

Standard study confined in a limited sector of the Hilbert space



$$\langle j,m-1|J^+J^-|j,m\rangle = (j-m)(j+m+1)$$

$$\langle\frac{N}{2},-1|J^+J^-|\frac{N}{2},0\rangle \approx \frac{N^2}{4}$$

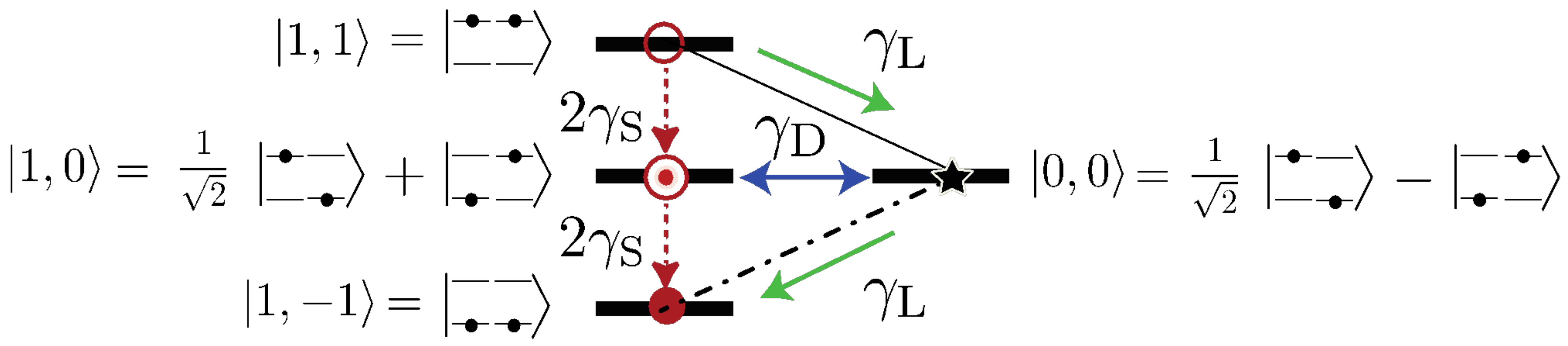


M. Gross & S. Haroche, Physics Reports **93**, 301 (1982)

The Dicke space generalizes the singlet-triplet system

Local processes connect the singlet (dark) state

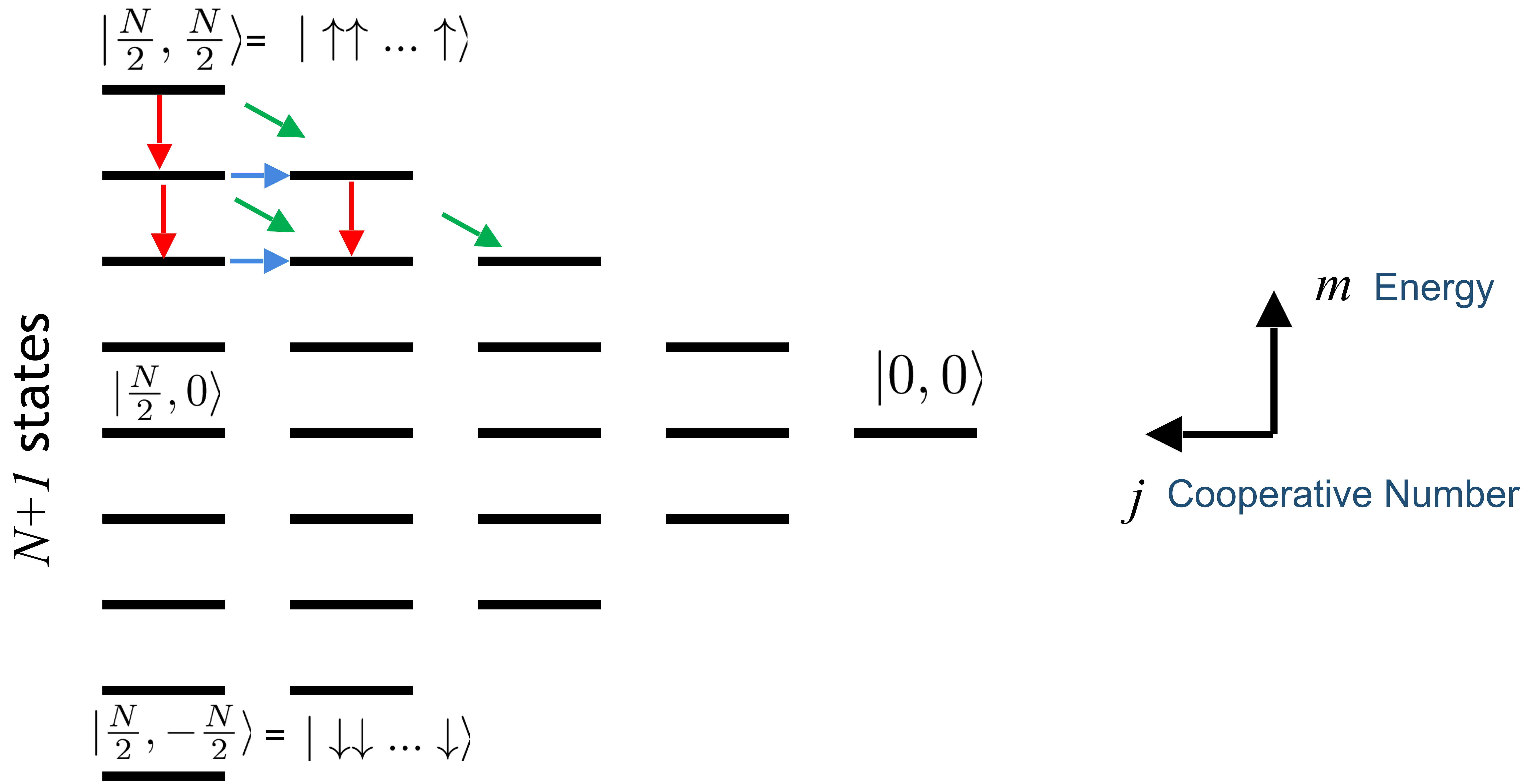
$$\dot{\rho} = i\omega_0[\rho, J_z] + \boxed{\frac{\gamma_{\mathrm{S}}}{2}\mathcal{L}_{J_-}[\rho]} + \boxed{\sum_{n=1}^{N}\frac{\gamma_{\mathrm{L}}}{2}\mathcal{L}_{J_{-,n}}[\rho]} + \boxed{\sum_{n=1}^{N}\frac{\gamma_{\mathrm{D}}}{2}\mathcal{L}_{J_{z,n}}[\rho]}$$



N=2

Superradiance and phase-breaking effects

Exploring inner states by including local incoherent processes



Analytical techniques

Hierarchy Truncation: Higher-moment correlations are neglected

Mean-field approximation $N \gg 2$

Factorization

$$\langle \dot{J}_z^2 \rangle = \gamma_S \left(\langle J^2 \rangle + \langle J_z \rangle - 3\langle J_z^2 \rangle + 2\langle J_z \rangle \langle J_z^2 \rangle - 2\langle J_z \rangle \langle J^2 \rangle \right)$$

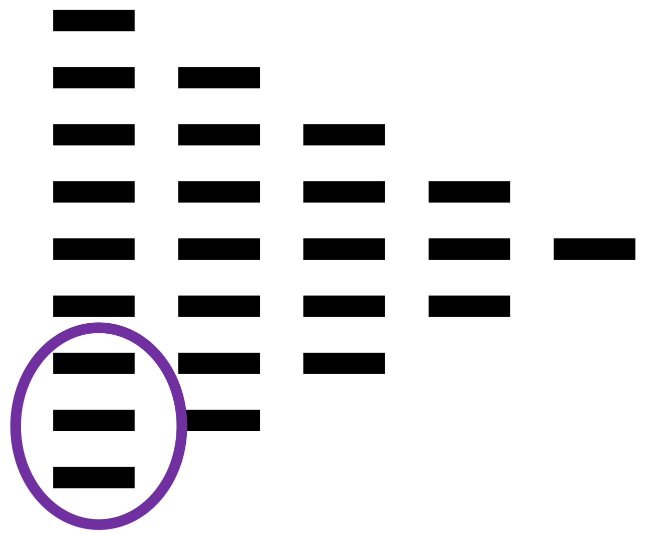
$$\langle J_z^3 \rangle \simeq \langle J_z^2 \rangle \langle J_z \rangle \qquad \langle J_z J^2 \rangle \simeq \langle J_z \rangle \langle J^2 \rangle$$

Bosonic approximation: Holstein-Primakoff (not suitable for local processes)

I. Holstein-Primakoff

$$H_{\text{int}} = g(J_+ + J_-)(a + a^\dagger) \approx \sqrt{2j}\, g(b + b^\dagger)(a + a^\dagger)$$

$$J_z = b^\dagger b - j \qquad J_- = \sqrt{2j}\sqrt{1 - \frac{b^\dagger b}{2j}}\, b \qquad [b, b^\dagger] = 1 \qquad \frac{\langle b^\dagger b \rangle}{2j} \ll 1$$

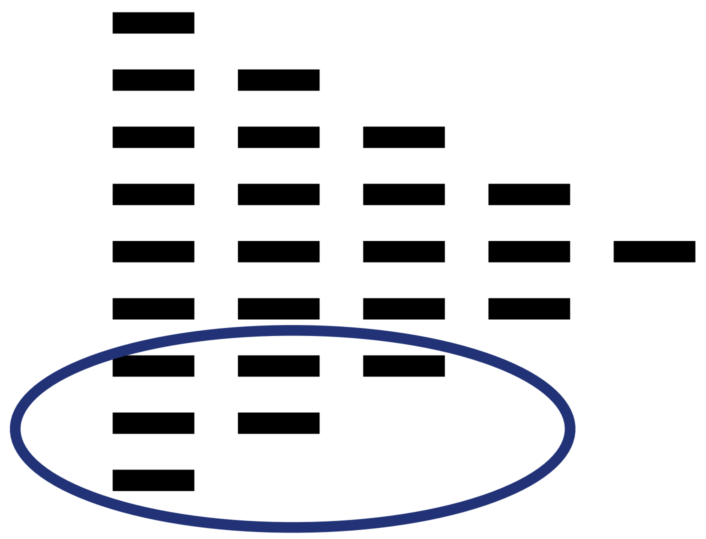


Bosonic approximation: Polariton Modes valid in the dilute regime

II. Dilute regime without cooperative number conservation

$$H_{\text{int}} = \sqrt{N}\mathbf{d} \cdot \mathbf{E}(b_0 + b_0^\dagger)$$

$$b_p^\dagger = \frac{1}{\sqrt{N}} \sum_n f_n^p J_{+,n}, \quad p \in [1, N] \qquad [b_p, b_q^\dagger] = \delta_{p,q}$$



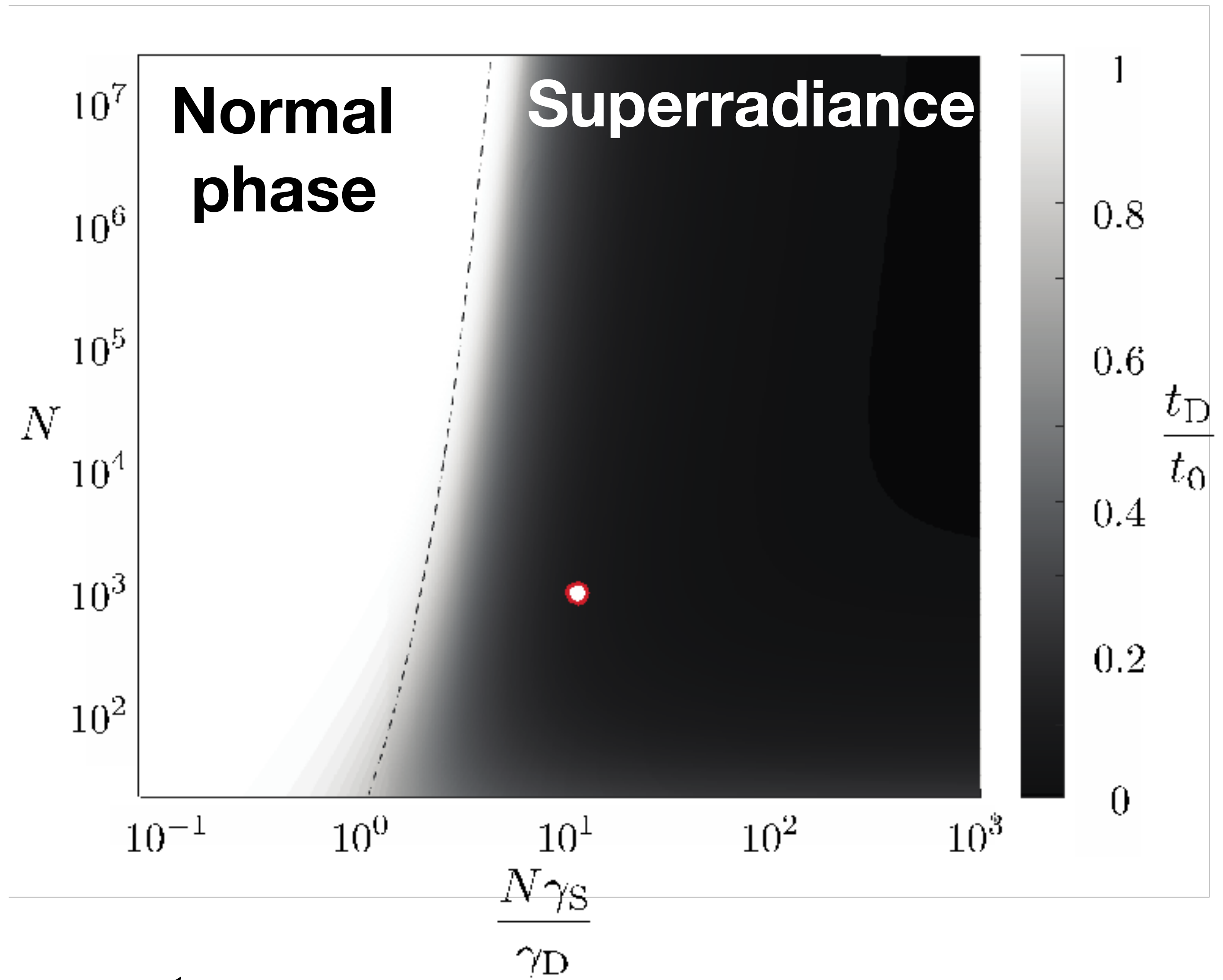
N Shammah *et al.*, Superradiance with local phase-breaking mechanisms, Phys. Rev. A 2017, arXiv 1704.07066

Superradiance and phase-breaking effects

Numerical results in a wide parameter space

Clear onset threshold

$\gamma_L = 10 \quad \gamma_S = 1 \quad \gamma_D \gg \gamma_L, \gamma_S$

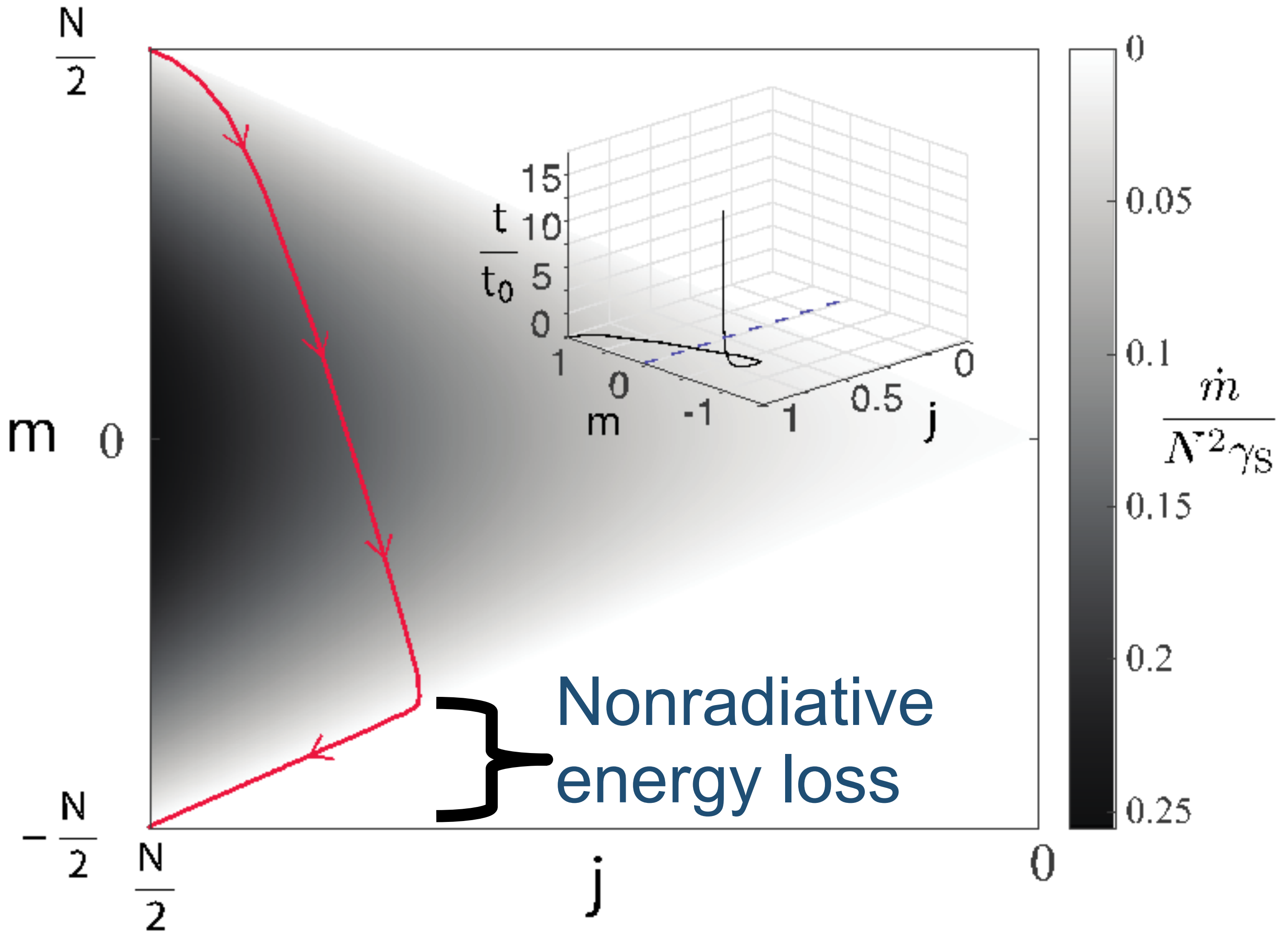


t_0 Non-radiative timescale

t_D Delay time

Sub-optimal superfluorescence

$\gamma_D \gg \gamma_L, \gamma_S \quad N\gamma_S \gg \gamma_D, \gamma_L$

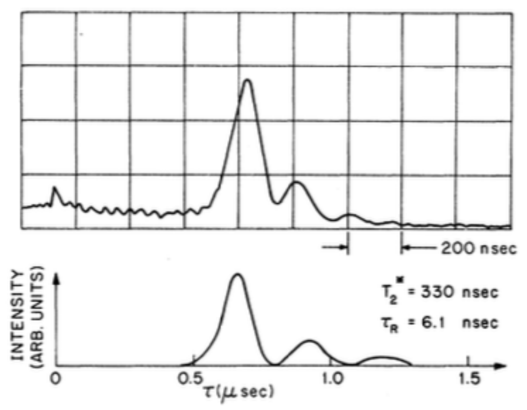


N Shammah *et al.*, Superradiance with local phase-breaking mechanisms
Phys. Rev. A **96,** 023863 (2017) arXiv 1704.07066

Experiments on Superradiant Light Emission

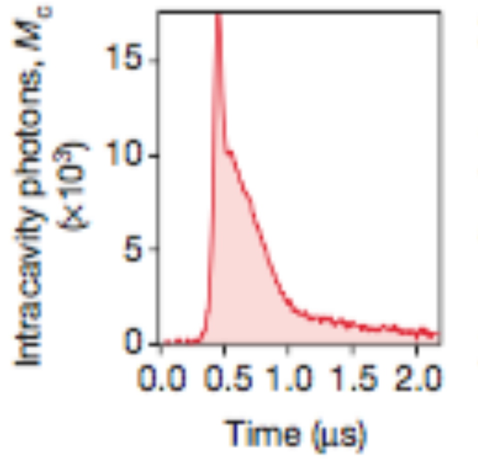
Many platforms for verification

HF Gas



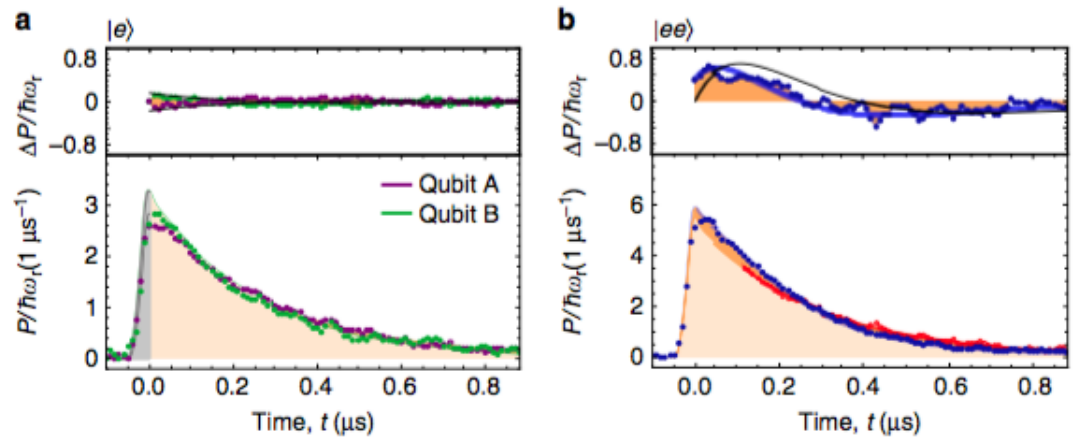
Skribanowitz *et al.*, *PRL* (1973)

Cold Atoms



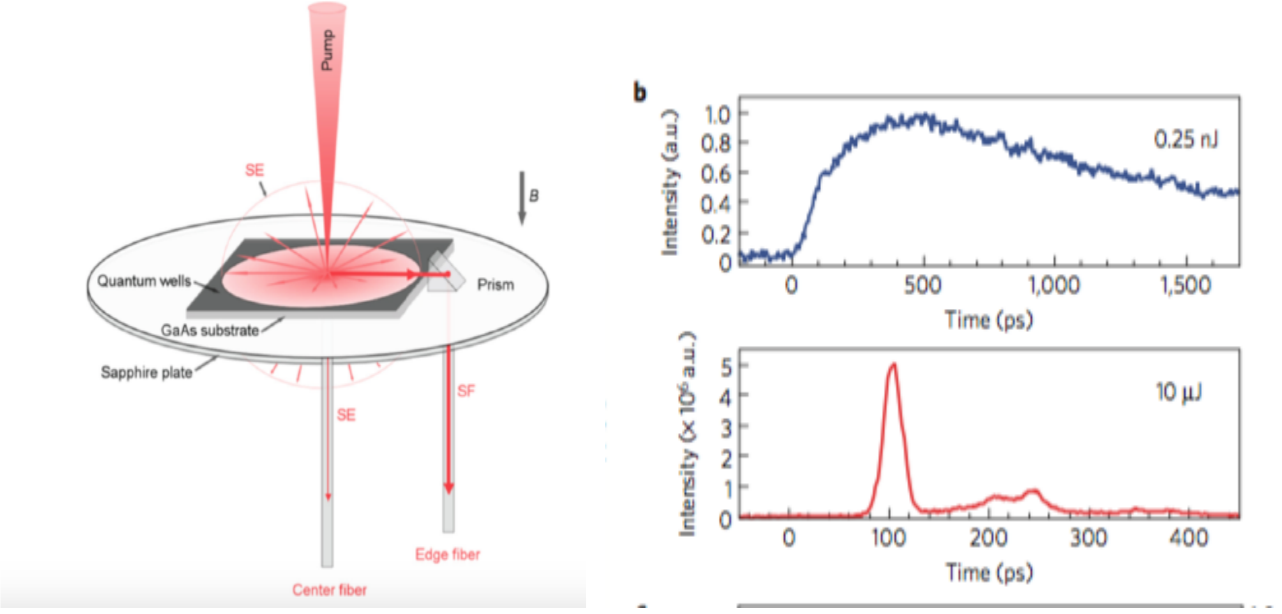
Bohnet *et al.*, *Nature* (2012)

Superconducting Qubits



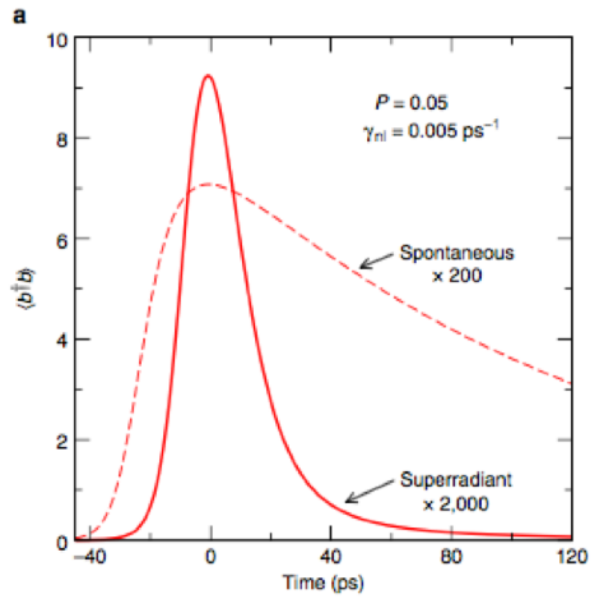
Mlynek *et al.*, *Nat Comm* (2014)

Magneto-Plasma in a Quantum Well



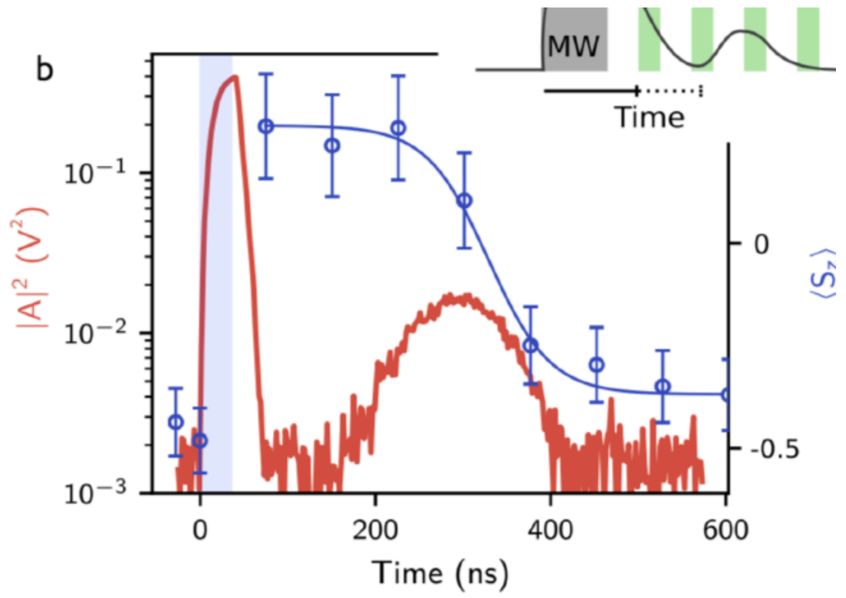
Noe II *et al.*, *Nat Phys* (2012)

Quantum Dot



Jahnke *et al.*, *Nat Comm* (2016)
Rainò et al. *Nature* (2018)

NV Centers and Hybrid



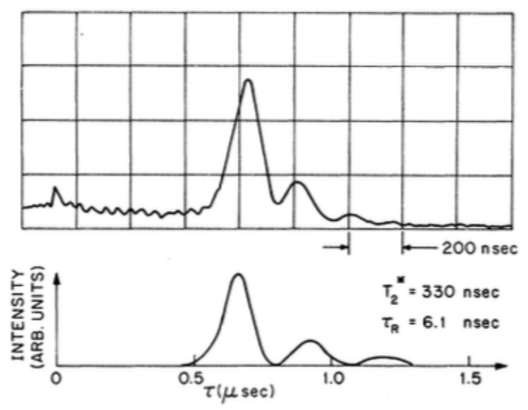
Bradac *et al.*, *Nat Comm* (2017)

Angerer *et al.* Nat Phys (2018)

Experiments on Superradiant Light Emission

Many platforms for verification

HF Gas



Skribanowitz *et al.*, *PRL* (1973)

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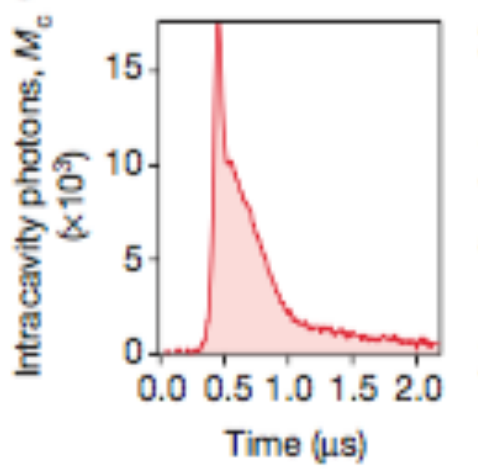

Bohnet *et al.*, *Nature* (2012)

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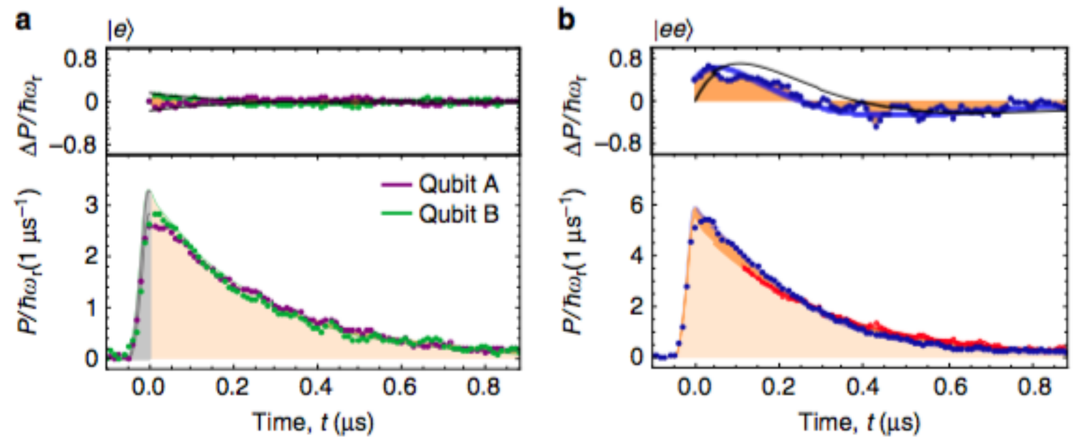

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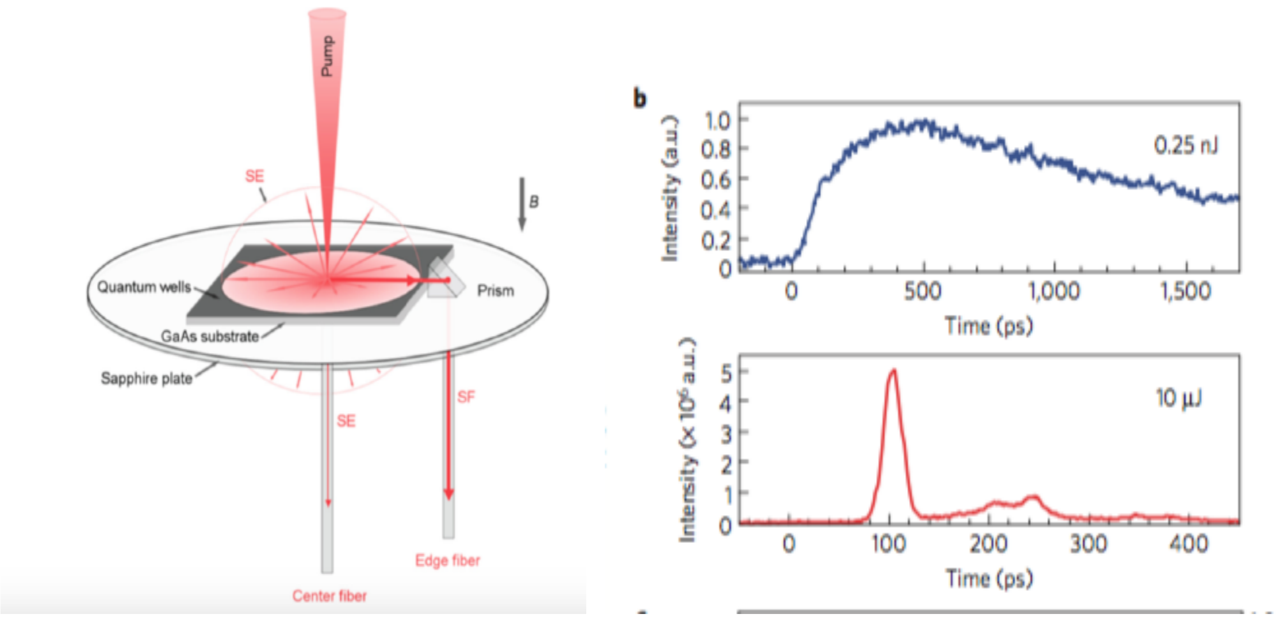

Noe II *et al.*, *Nat Phys* (2012)

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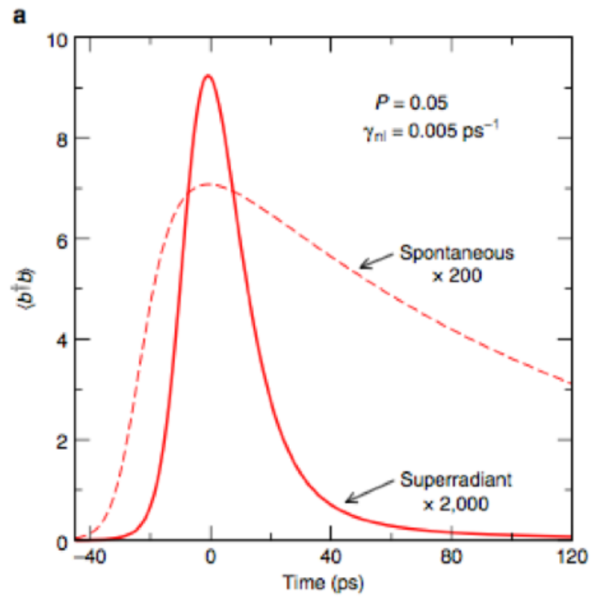

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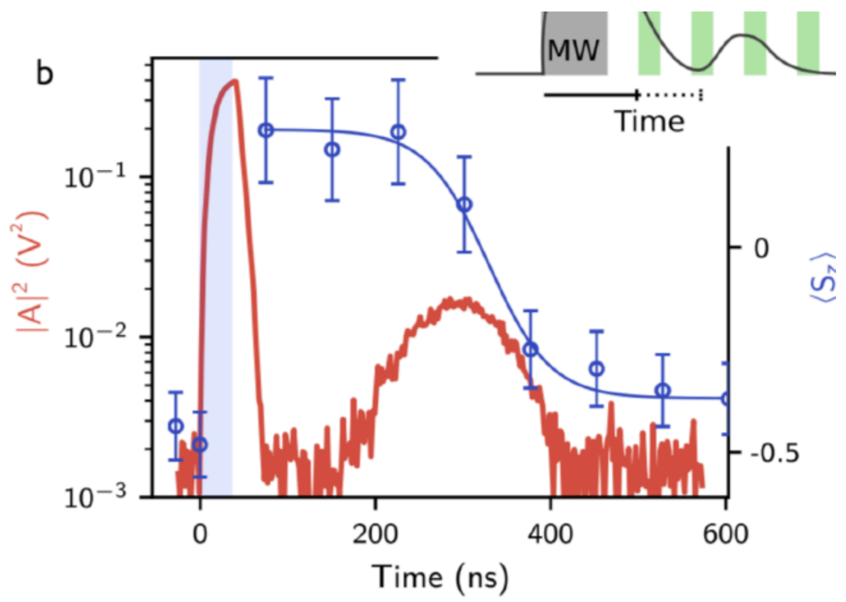
Rainò et al. *Nature* (2018)

NV Centers and Hybrid



Bradac *et al.*, *Nat Comm* (2017)

Angerer *et al.* Nat Phys (2018)

Validate our predictions
N Shammah *et al.*,
Phys. Rev. A **96,** 023863 (2017)

Exploiting permutational symmetry

Validity and Limitations

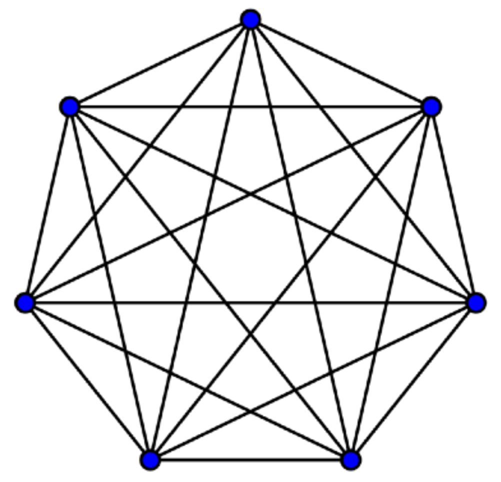
$$\frac{d}{dt}\rho \boxed{= -\frac{i}{\hbar}[H,\rho]} + \boxed{\boxed{\frac{\gamma_\Downarrow}{2}\mathcal{L}_{J_-}[\rho] + \frac{\gamma_\Phi}{2}\mathcal{L}_{J_z}[\rho] + \frac{\gamma_\Uparrow}{2}\mathcal{L}_{J_+}[\rho]} + \boxed{\sum_{n=1}^{N}\left(\frac{\gamma_\downarrow}{2}\mathcal{L}_{J_{-,n}}[\rho] + \frac{\gamma_\phi}{2}\mathcal{L}_{J_{z,n}}[\rho] + \frac{\gamma_\uparrow}{2}\mathcal{L}_{J_{+,n}}[\rho]\right)} + \boxed{\frac{w}{2}\mathcal{L}_{a^\dagger}[\rho] + \frac{\kappa}{2}\mathcal{L}_{a}[\rho]}}$$

TLS Pauli operators $[J_{x,n}, J_{y,n'}] = iJ_{z,n}\delta_{n,n'}$

Lindblad Superoperator $\mathcal{L}_A[\rho] = 2A\rho A^\dagger - A^\dagger A\rho - \rho A^\dagger A$

- Unitary Dynamics
- Non-unitary Dynamics
  - Collective Dissipation
  - Local Dissipation
  - Bosonic Dissipation

Range of applicability



Hamiltonian
Collective spin operators only
Complete graph (fully connected).
Constant edges weight: no lattice distance.

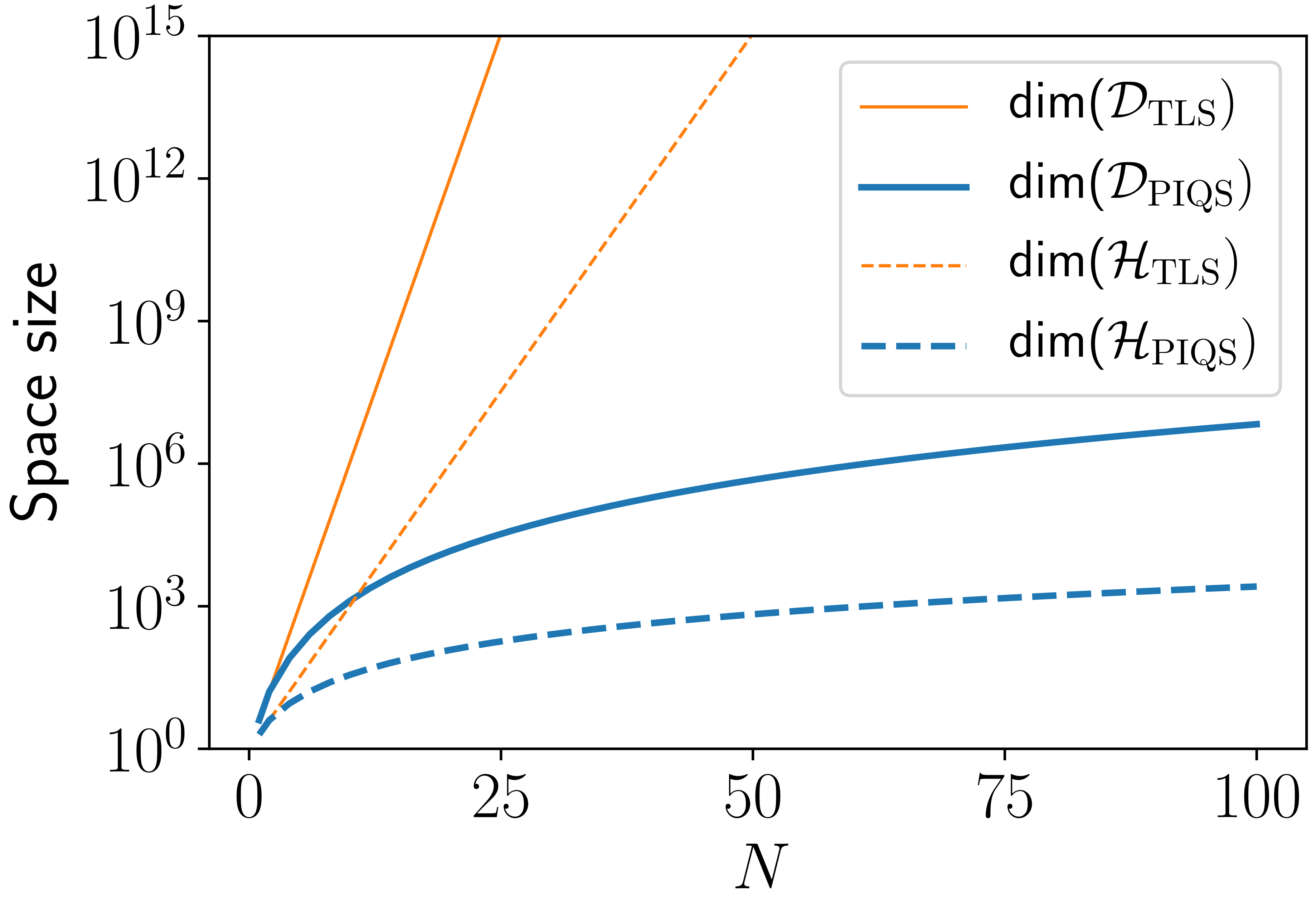
Dissipation
Homogeneous local couplings.

States
Limited to identical qubit states.

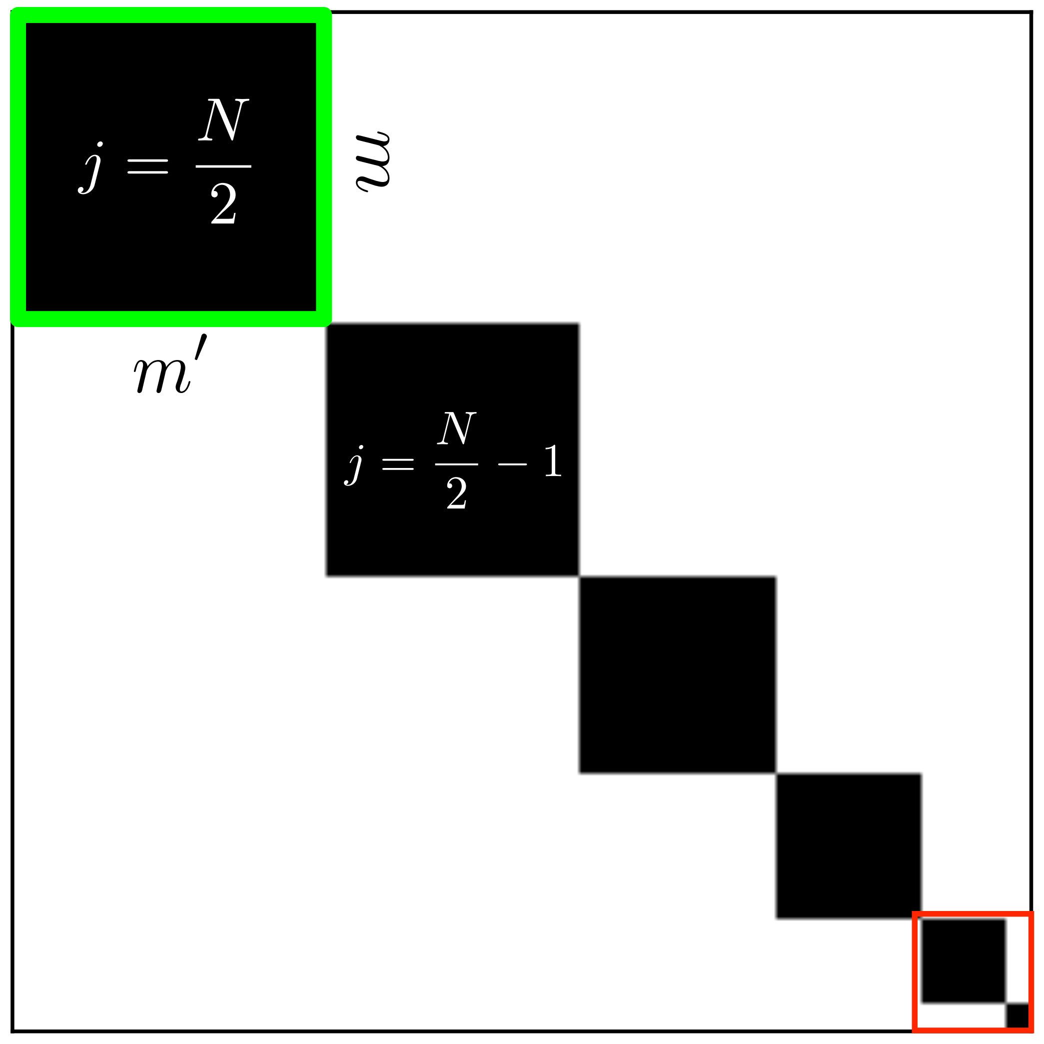
Numerical method in the Dicke space

Homogeneous local dissipation can be included in the dynamical model

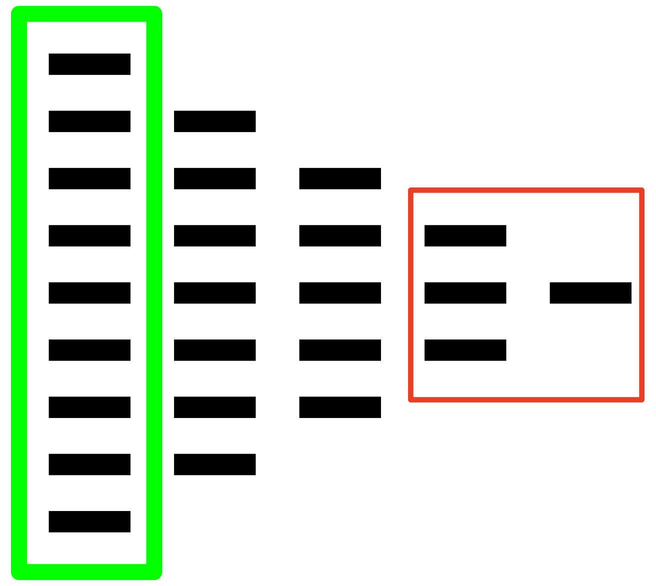
Exponential Space Reduction



Block-Diagonal Density Matrix



Dicke space visualization



Number of Dicke States

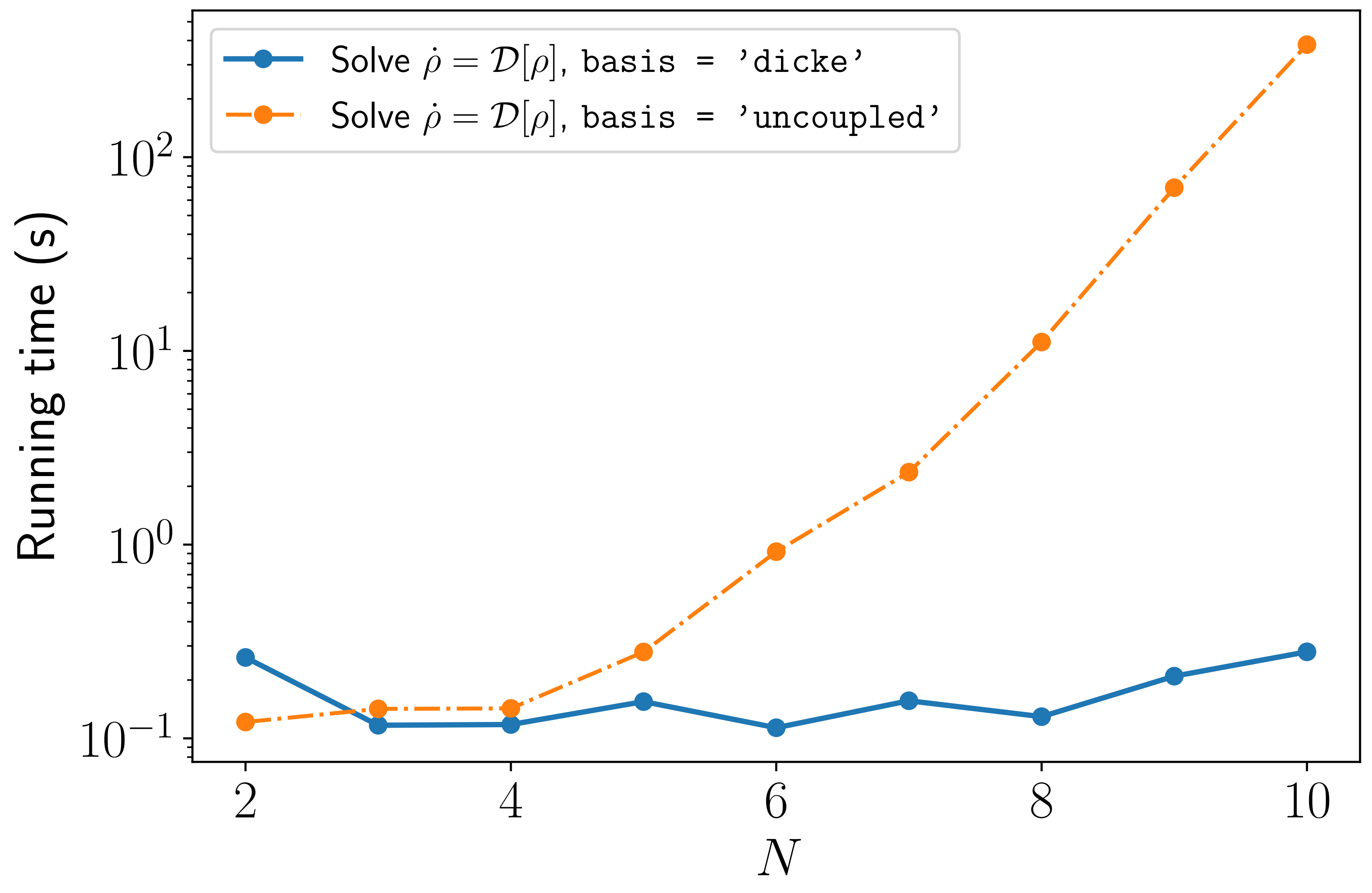
$$n_{\mathrm{DS}}(N) = \sum_{j=0}^{N/2} (2j+1) = \left(\frac{N}{2} + 1\right)^2$$

B. A. Chase and J. M. Geremia, Phys. Rev. A **78**, 052101 (2008)
F. Damanet, D. Braun, and J. Martin, Rev. A **94**, 033838 (2016)
N. Shammah et al. Phys. Rev. A (2017)

Permutational-Invariant Quantum Solver (PIQS)

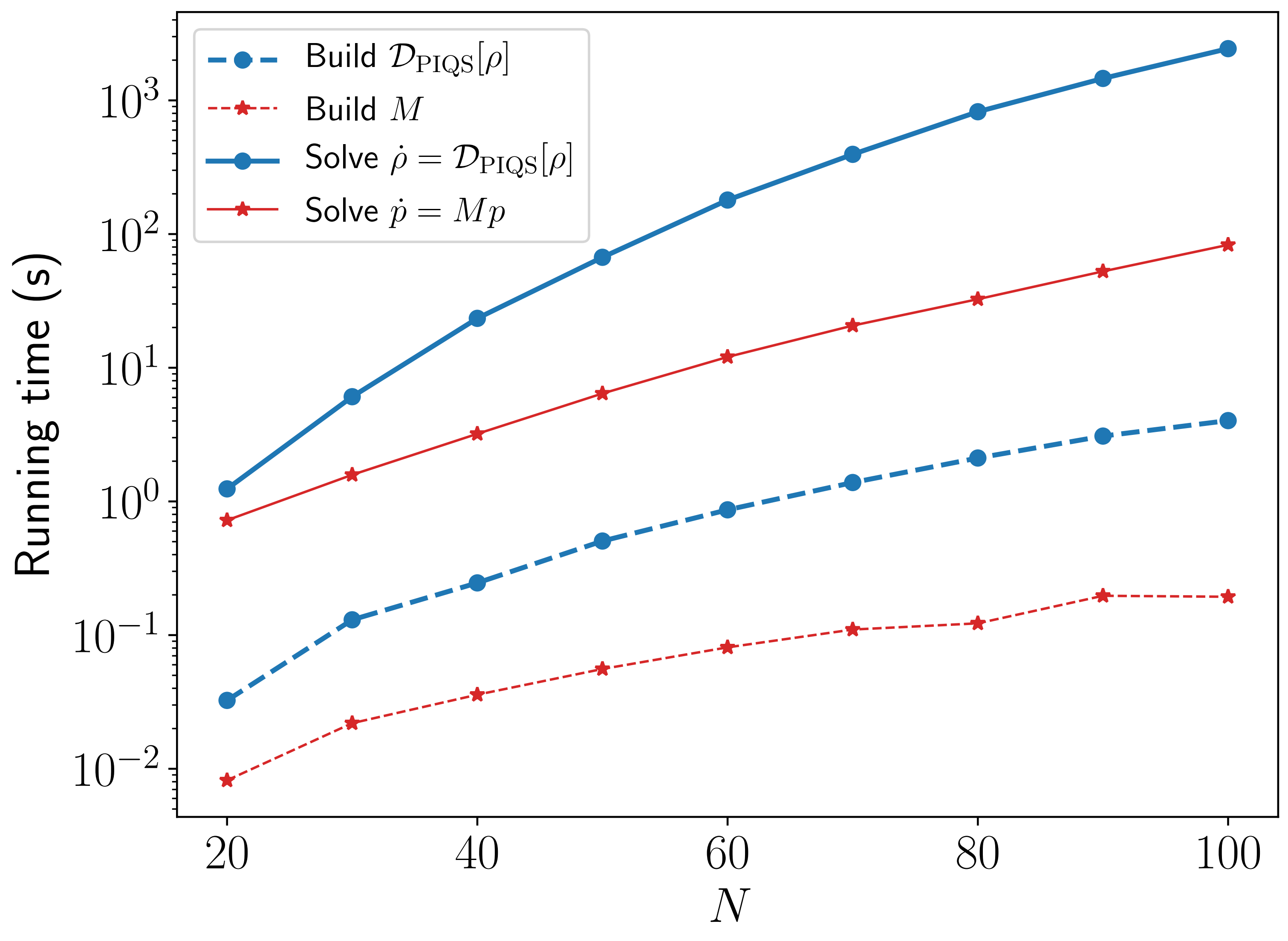
Example: Steady-state superradiance

Solution of the master equation (Small *N*)



Comparison between the Dicke basis used by the permutational symmetric vs. the full uncoupled basis

Definition and Solution of the master equation (Larger *N*)



Generic permutational symmetric vs. a diagonal problem

N. Shammah, S. Ahmed, N. Lambert, S. De Liberato, and F. Nori, PRA 98, 063815 (2018) arXiv:1805.05129

Using the Dicke state formalism

Local dissipation

Light emission | Local Dephasing | Nonradiative losses | Incoherent pump

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] + \boxed{\frac{\gamma_{\mathrm{S}}}{2}\mathcal{L}_{J_-}[\rho]} + \boxed{\frac{\gamma_{\mathrm{D}}}{2}\sum_{n=1}^{N}\mathcal{L}_{J_{z,n}}[\rho]} + \boxed{\frac{\gamma_{\mathrm{L}}}{2}\sum_{n=1}^{N}\mathcal{L}_{\sigma_{-,n}}[\rho]} + \boxed{\frac{\gamma_{\mathrm{P}}}{2}\sum_{n=1}^{N}\mathcal{L}_{\sigma_{+,n}}[\rho]}$$

Using the Dicke state formalism

Local dissipation

Light emission | Local Dephasing | Nonradiative losses | Incoherent pump

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] + \boxed{\frac{\gamma_{\mathrm{S}}}{2}\mathcal{L}_{J_-}[\rho]} + \boxed{\frac{\gamma_{\mathrm{D}}}{2}\sum_{n=1}^{N}\mathcal{L}_{J_{z,n}}[\rho]} + \boxed{\frac{\gamma_{\mathrm{L}}}{2}\sum_{n=1}^{N}\mathcal{L}_{\sigma_{-,n}}[\rho]} + \boxed{\frac{\gamma_{\mathrm{P}}}{2}\sum_{n=1}^{N}\mathcal{L}_{\sigma_{+,n}}[\rho]}$$

$$\boxed{\frac{\gamma_{\mathrm{D}}}{2}\sum_{n=1}^{N}\mathcal{L}_{J_{z,n}}[\rho]} = \frac{\gamma_{\mathrm{D}}}{2}\left(2\left(\sum_{n=1}^{N} J_{z,n}\rho J_{z,n}\right) - \frac{N}{2}\rho\right)$$

$$\boxed{\frac{\gamma_{\mathrm{L}}}{2}\sum_{n=1}^{N}\mathcal{L}_{\sigma_{-,n}}[\rho]} = \frac{\gamma_{\mathrm{L}}}{2}\left(2\left(\sum_{n=1}^{N} \sigma_{-,n}\rho\sigma_{+,n}\right) - J_z\rho - \rho J_z - N\rho\right)$$

$$\boxed{\frac{\gamma_{\mathrm{P}}}{2}\sum_{n=1}^{N}\mathcal{L}_{\sigma_{+,n}}[\rho]} = \frac{\gamma_{\mathrm{P}}}{2}\left(2\left(\sum_{n=1}^{N} \sigma_{+,n}\rho\sigma_{-,n}\right) + J_z\rho + \rho J_z - N\rho\right)$$

Using the Dicke state formalism

Local dissipation

Light emission | Local Dephasing | Nonradiative losses | Incoherent pump

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] + \boxed{\frac{\gamma_\mathrm{S}}{2}\mathcal{L}_{J_-}[\rho]} + \boxed{\frac{\gamma_\mathrm{D}}{2}\sum_{n=1}^{N}\mathcal{L}_{J_{z,n}}[\rho]} + \boxed{\frac{\gamma_\mathrm{L}}{2}\sum_{n=1}^{N}\mathcal{L}_{\sigma_{-,n}}[\rho]} + \boxed{\frac{\gamma_\mathrm{P}}{2}\sum_{n=1}^{N}\mathcal{L}_{\sigma_{+,n}}[\rho]}$$

$$\underline{\sum_{n=1}^{N} J_{r,n}\,|j,m\rangle\langle j,m'|\,J_{q,n}^{\dagger}} = a_{qr}^{N}(j,m)\,|j,m+\tilde{q}\rangle\langle j,m'+\tilde{r}|$$

$$+b_{qr}^{N}(j,m)\,|j-1,m+\tilde{q}\rangle\langle j-1,m'+\tilde{r}|$$

$$+c_{qr}^{N}(j,m)\,|j+1,m+\tilde{q}\rangle\langle j+1,m'+\tilde{r}|$$

$$|j,m\rangle \to |j',m'\rangle$$

S. Sarkar and J.S. Satchell (1987)

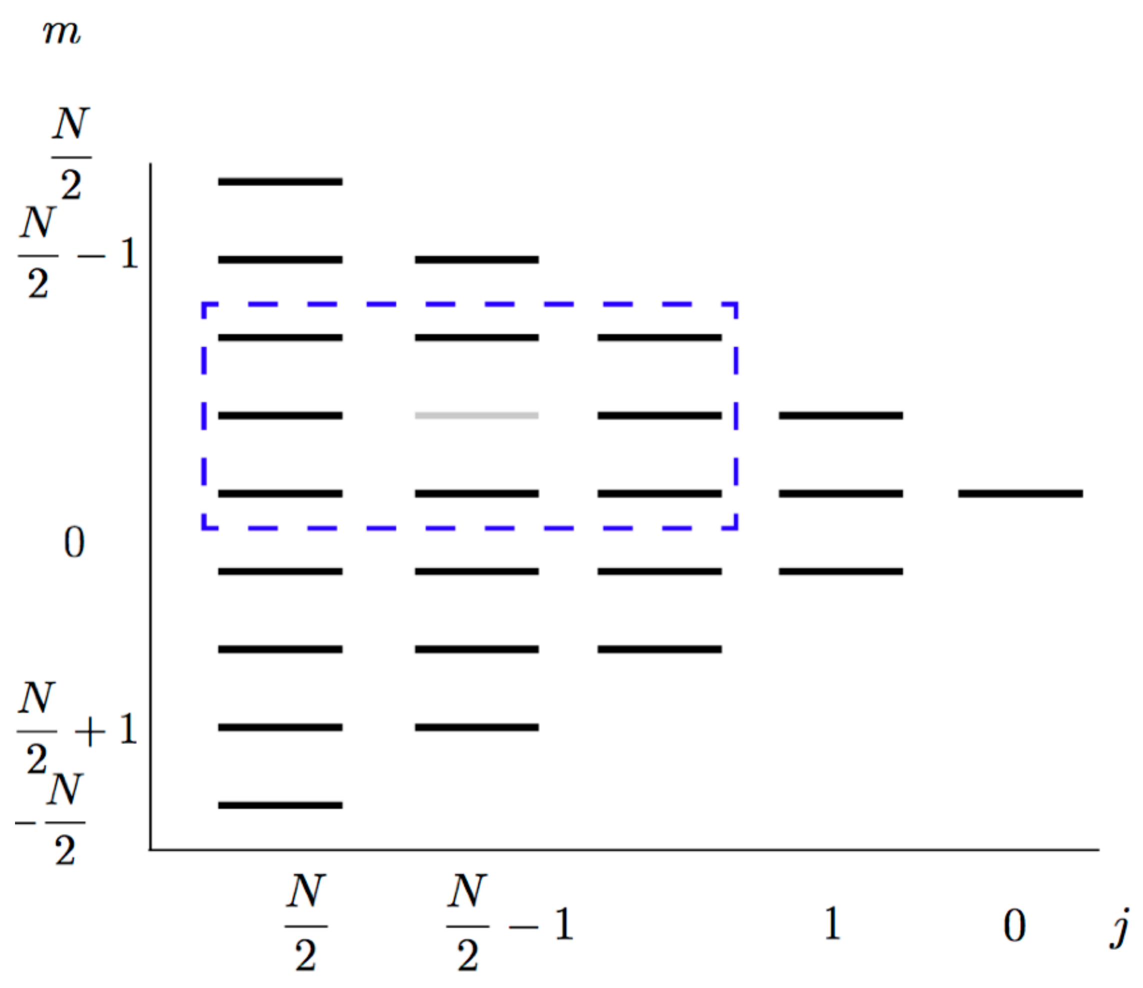
B. A. Chase and J. M. Geremia, Phys. Rev. A (2008)

S. Hartmann (arXiv 2012, published 2016)
M. Tieri, M. Xu, M.J., Holland, PRA (2013)
P. Kirton and J. Keeling, PRL (2017)

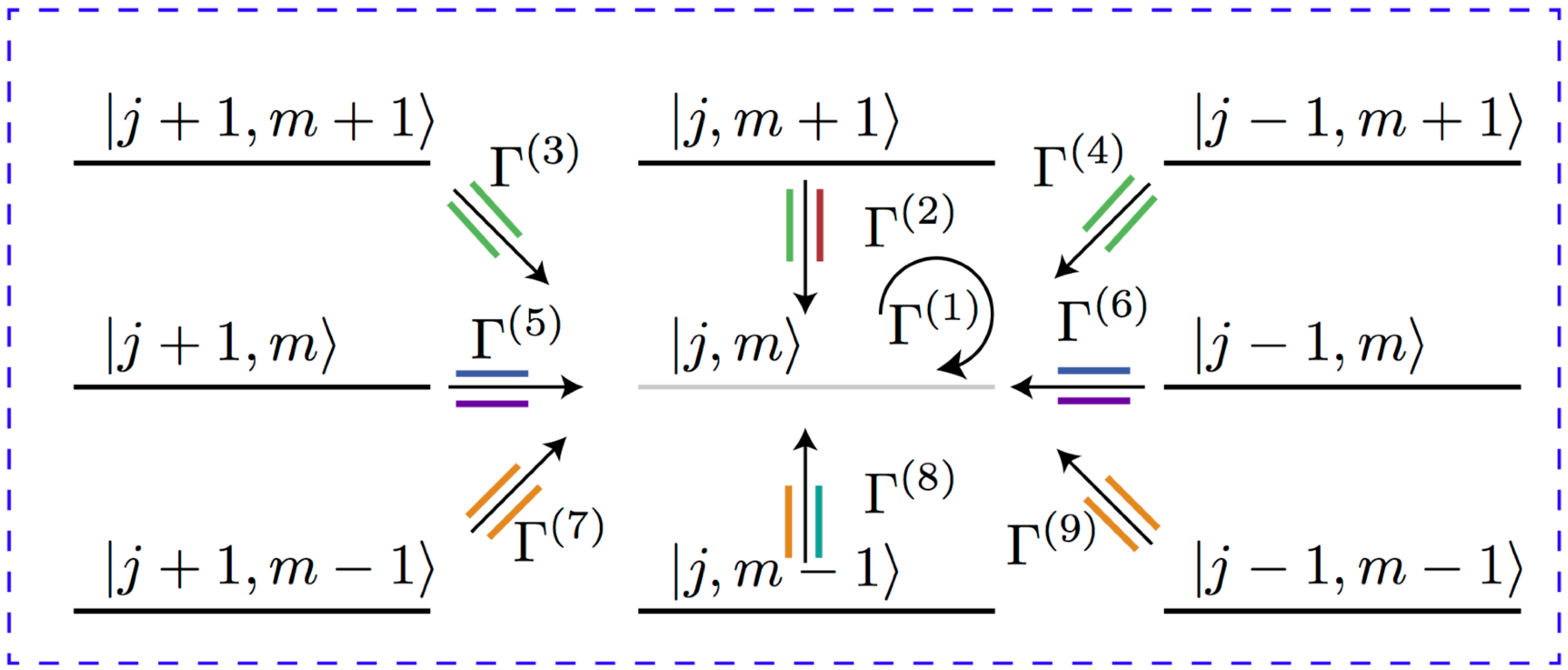
Exploiting permutational invariance and the Dicke space

Exploring the dynamics in the Dicke space

Dicke Space



Dynamics in the Dicke Space



Processes	Emission	Dephasing	Pumping
Local	$\gamma_\downarrow$	γ_ϕ	$\gamma_\uparrow$
Collective	$\gamma_\Downarrow$	γ_Φ	$\gamma_\Uparrow$

Literature overview

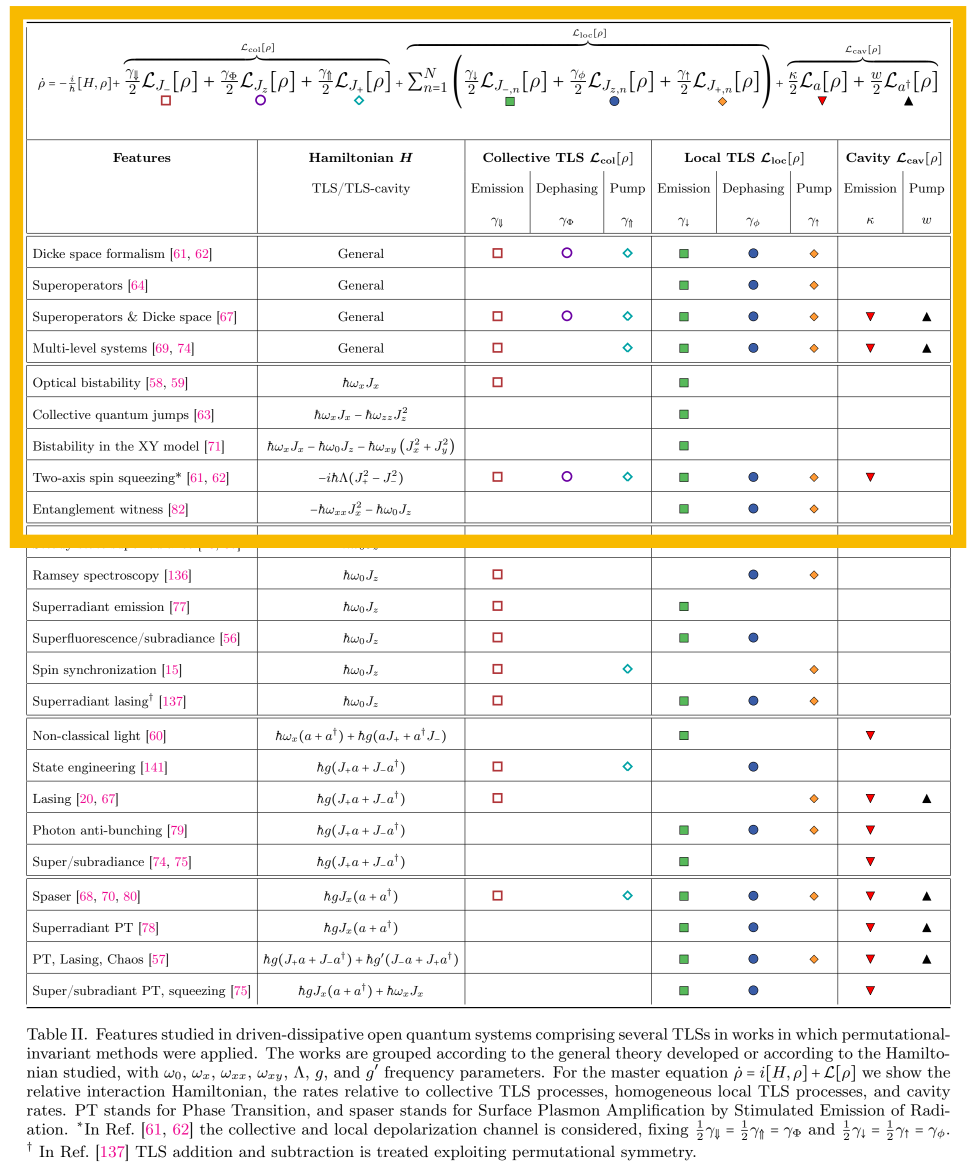
Permutational symmetry in Lindblad dynamics

N. Shammah, S. Ahmed, N. Lambert, S. De Liberato, and F. Nori, PRA **98**, 063815 (2018), arXiv:1805.05129

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] + \overbrace{\frac{\gamma_\Downarrow}{2}\mathcal{L}_{J_-}[\rho] + \frac{\gamma_\Phi}{2}\mathcal{L}_{J_z}[\rho] + \frac{\gamma_\Uparrow}{2}\mathcal{L}_{J_+}[\rho]}^{\mathcal{L}_{\rm col}[\rho]} + \overbrace{\sum_{n=1}^{N}\left(\frac{\gamma_\downarrow}{2}\mathcal{L}_{J_{-,n}}[\rho] + \frac{\gamma_\phi}{2}\mathcal{L}_{J_{z,n}}[\rho] + \frac{\gamma_\uparrow}{2}\mathcal{L}_{J_{+,n}}[\rho]\right)}^{\mathcal{L}_{\rm loc}[\rho]} + \overbrace{\frac{\kappa}{2}\mathcal{L}_{a}[\rho] + \frac{w}{2}\mathcal{L}_{a^\dagger}[\rho]}^{\mathcal{L}_{\rm cav}[\rho]}$$

□ ○ ◇ ■ ● ◆ ▼ ▲

Features	**Hamiltonian H**	**Collective TLS $\mathcal{L}_{\rm col}[\rho]$**			**Local TLS $\mathcal{L}_{\rm loc}[\rho]$**			**Cavity $\mathcal{L}_{\rm cav}[\rho]$**	
	TLS/TLS-cavity	Emission	Dephasing	Pump	Emission	Dephasing	Pump	Emission	Pump
		$\gamma_\Downarrow$	γ_Φ	$\gamma_\Uparrow$	$\gamma_\downarrow$	γ_ϕ	$\gamma_\uparrow$	κ	w
Dicke space formalism [61, 62]	General	□	○	◇	■	●	◆		
Superoperators [64]	General				■	●	◆		
Superoperators & Dicke space [67]	General	□	○	◇	■	●	◆	▼	▲
Multi-level systems [69, 74]	General	□		◇	■	●	◆	▼	▲
Optical bistability [58, 59]	$\hbar\omega_x J_x$	□			■				
Collective quantum jumps [63]	$\hbar\omega_x J_x - \hbar\omega_{zz} J_z^2$				■				
Bistability in the XY model [71]	$\hbar\omega_x J_x - \hbar\omega_0 J_z - \hbar\omega_{xy}\left(J_x^2 + J_y^2\right)$				■				
Two-axis spin squeezing* [61, 62]	$-i\hbar\Lambda(J_+^2 - J_-^2)$	□	○	◇	■	●	◆	▼	
Entanglement witness [82]	$-\hbar\omega_{xx} J_x^2 - \hbar\omega_0 J_z$				■	●	◆		

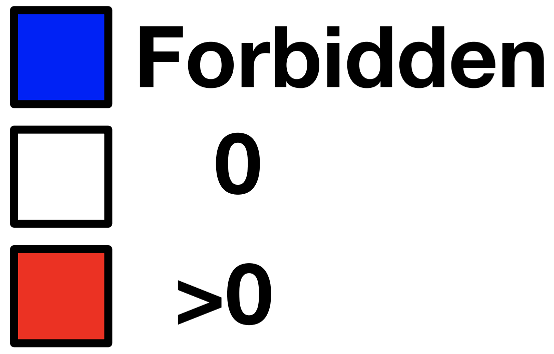


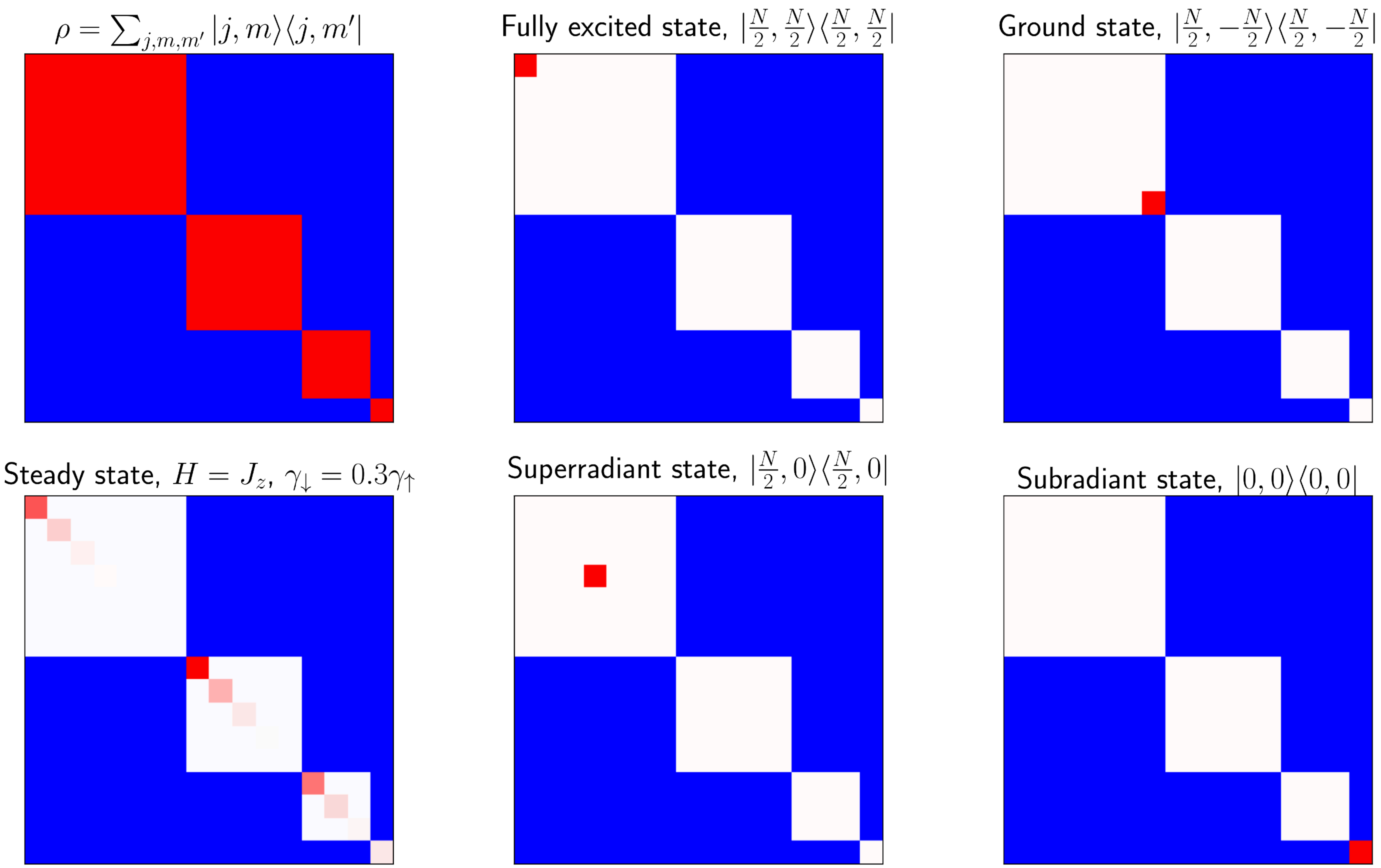
Features	Hamiltonian H	$\gamma_\Downarrow$	γ_Φ	$\gamma_\Uparrow$	$\gamma_\downarrow$	γ_ϕ	$\gamma_\uparrow$	κ	w
Ramsey spectroscopy [136]	$\hbar\omega_0 J_z$	□				●	◆		
Superradiant emission [77]	$\hbar\omega_0 J_z$	□			■				
Superfluorescence/subradiance [56]	$\hbar\omega_0 J_z$	□			■	●			
Spin synchronization [15]	$\hbar\omega_0 J_z$	□		◇			◆		
Superradiant lasing† [137]	$\hbar\omega_0 J_z$	□			■	●	◆		
Non-classical light [60]	$\hbar\omega_x(a + a^\dagger) + \hbar g(aJ_+ + a^\dagger J_-)$				■			▼	
State engineering [141]	$\hbar g(J_+ a + J_- a^\dagger)$	□		◇		●			
Lasing [20, 67]	$\hbar g(J_+ a + J_- a^\dagger)$	□					◆	▼	▲
Photon anti-bunching [79]	$\hbar g(J_+ a + J_- a^\dagger)$				■	●	◆	▼	
Super/subradiance [74, 75]	$\hbar g(J_+ a + J_- a^\dagger)$				■			▼	
Spaser [68, 70, 80]	$\hbar g J_x(a + a^\dagger)$	□		◇	■	●	◆	▼	▲
Superradiant PT [78]	$\hbar g J_x(a + a^\dagger)$				■	●		▼	▲
PT, Lasing, Chaos [57]	$\hbar g(J_+ a + J_- a^\dagger) + \hbar g'(J_- a + J_+ a^\dagger)$				■	●	◆	▼	▲
Super/subradiant PT, squeezing [75]	$\hbar g J_x(a + a^\dagger) + \hbar\omega_x J_x$				■	●		▼	

Table II. Features studied in driven-dissipative open quantum systems comprising several TLSs in works in which permutational-invariant methods were applied. The works are grouped according to the general theory developed or according to the Hamiltonian studied, with ω_0, ω_x, ω_{xx}, ω_{xy}, Λ, g, and g' frequency parameters. For the master equation $\dot{\rho} = i[H,\rho] + \mathcal{L}[\rho]$ we show the relative interaction Hamiltonian, the rates relative to collective TLS processes, homogeneous local TLS processes, and cavity rates. PT stands for Phase Transition, and spaser stands for Surface Plasmon Amplification by Stimulated Emission of Radiation. *In Ref. [61, 62] the collective and local depolarization channel is considered, fixing $\frac{1}{2}\gamma_\Downarrow = \frac{1}{2}\gamma_\Uparrow = \gamma_\Phi$ and $\frac{1}{2}\gamma_\downarrow = \frac{1}{2}\gamma_\uparrow = \gamma_\phi$. † In Ref. [137] TLS addition and subtraction is treated exploiting permutational symmetry.

Dicke state basis as a visualization tool

Density matrices of collective quantum states





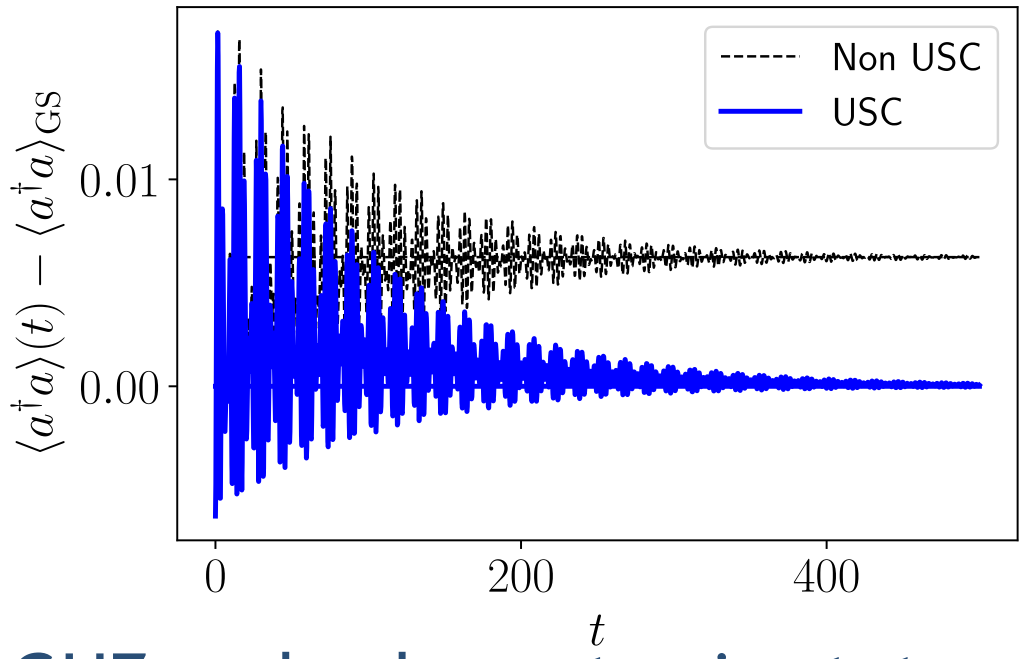
Can also study:
GHZ state
CSS states
Thermal states

PIQS: Driven-dissipative two-level system ensembles

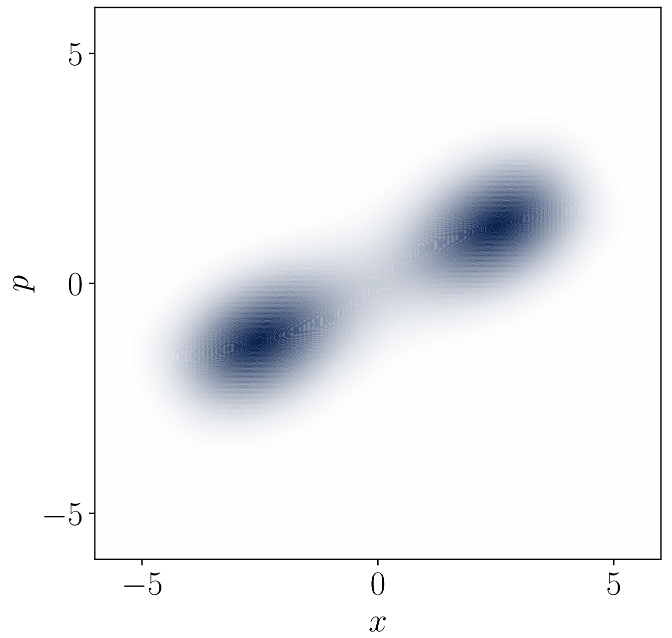
The Permutational Invariant Quantum Solver

$$i\hbar\frac{d}{dt}\rho = [H,\rho] + \gamma_\alpha \mathscr{L}_x[\rho]$$

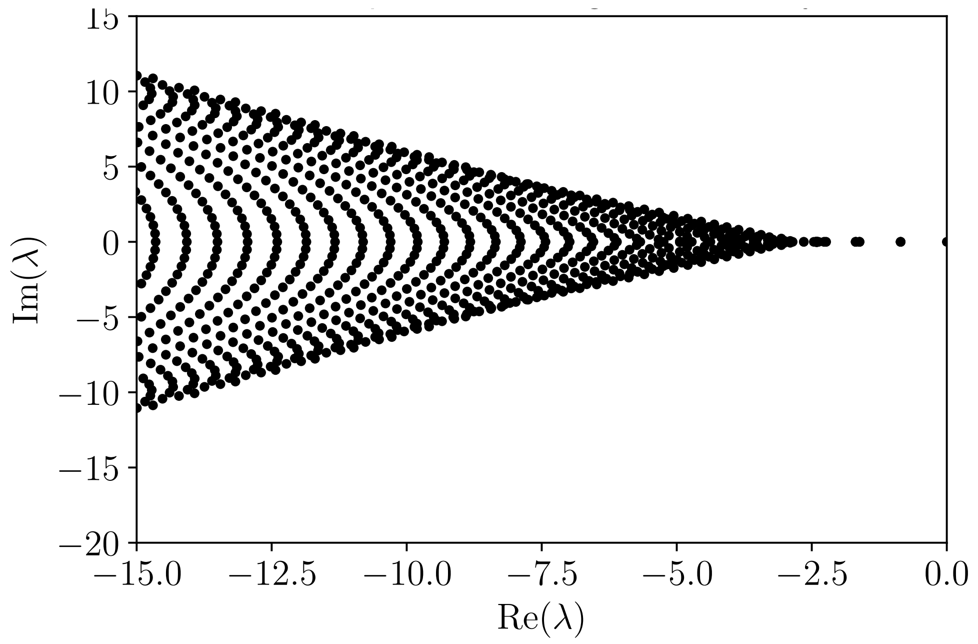
Ultrastrong Coupling (USC)



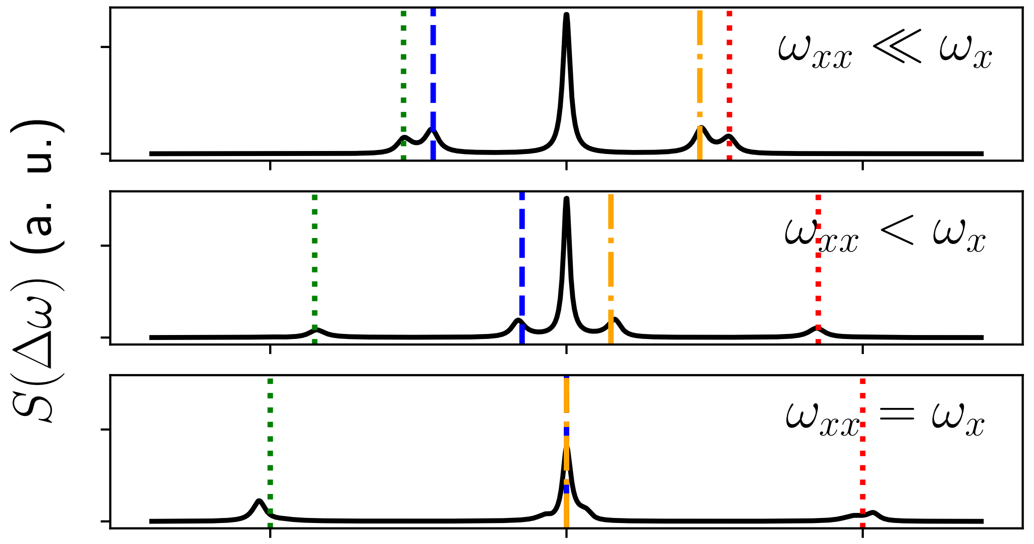
Superradiant PT



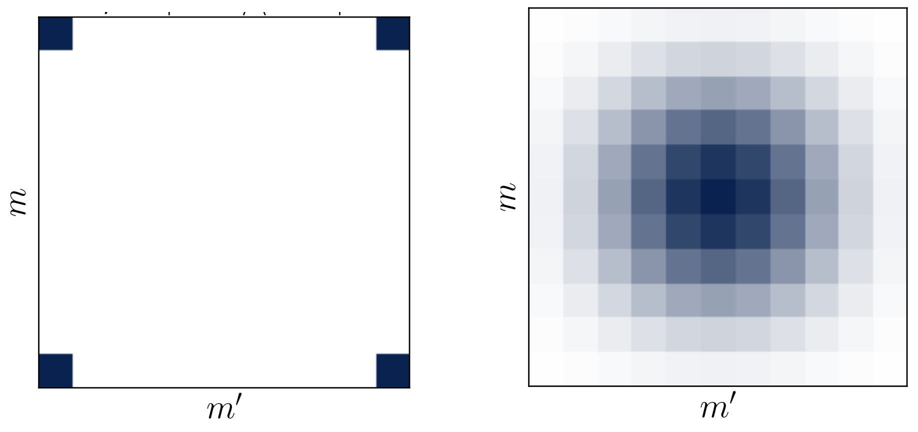
Boundary Time Crystals



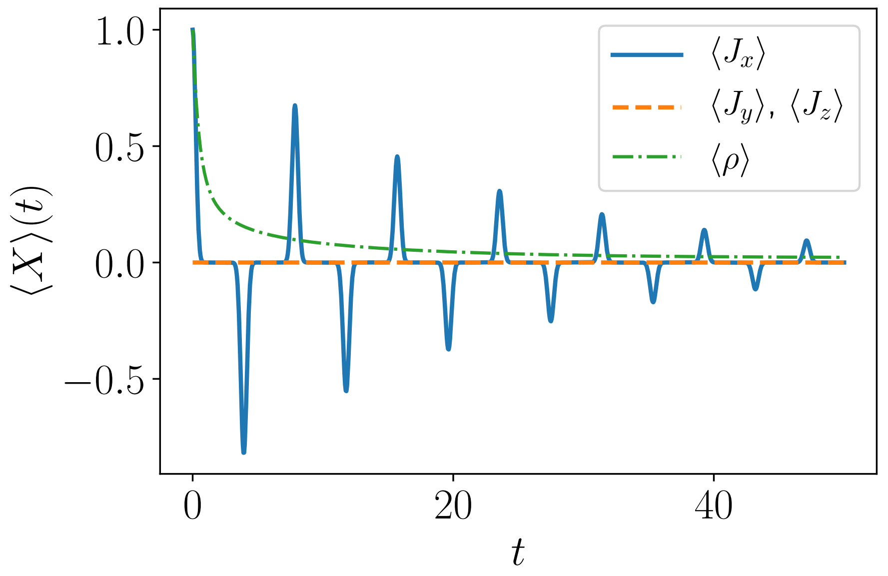
Resonance Fluorescence



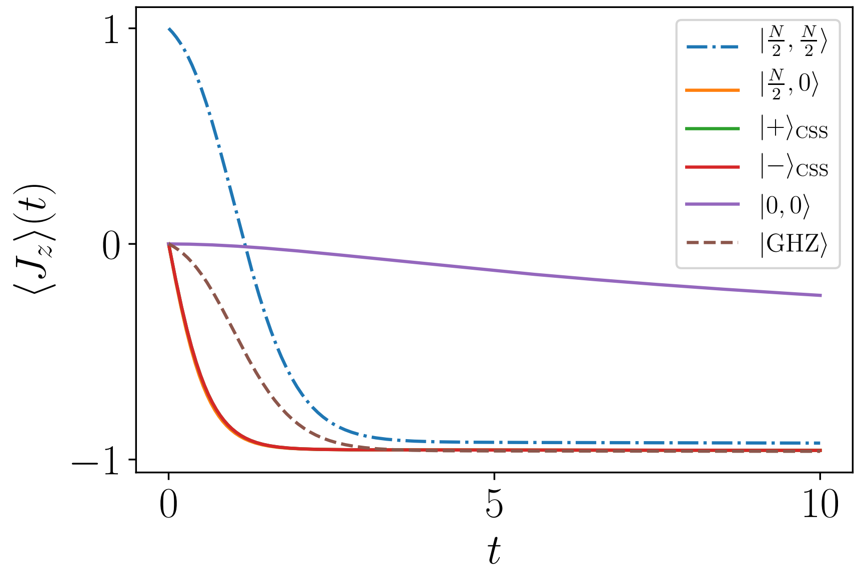
GHZ and coherent spin states



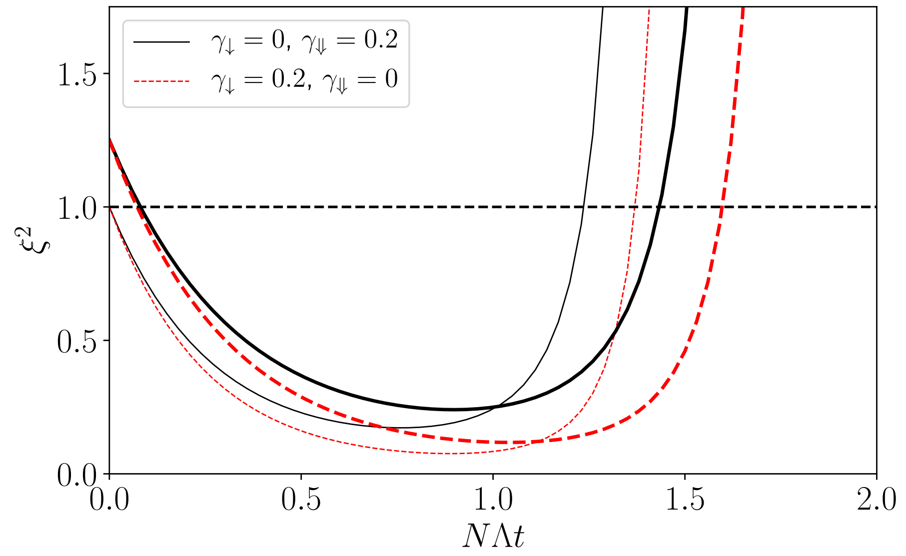
Noisy Heisenberg Model



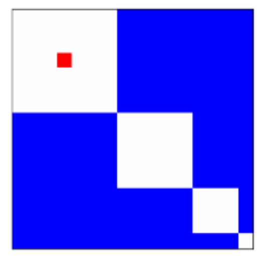
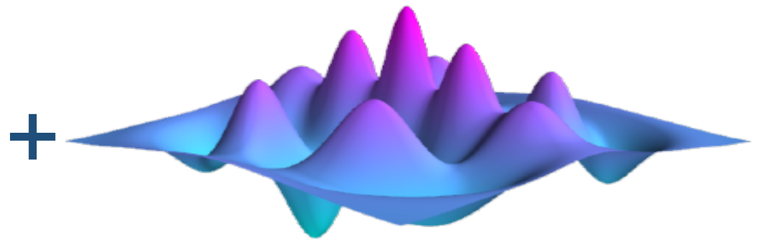
Dicke Superradiance



Spin Squeezing

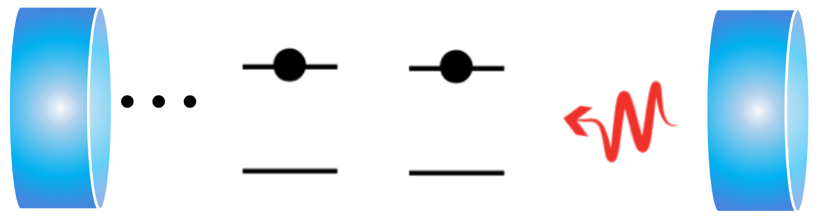


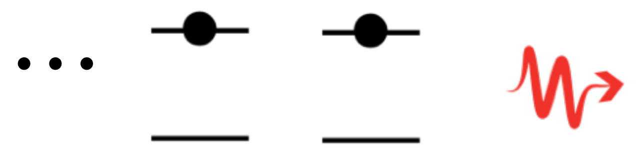
N. Shammah, S. Ahmed, N. Lambert, S. De Liberato, and F. Nori, PRA 98, 063815 (2018) arXiv:1805.05129

 +  = **import** qutip.piqs

Superradiance: phase transition vs. light emission

Two kinds of superradiance



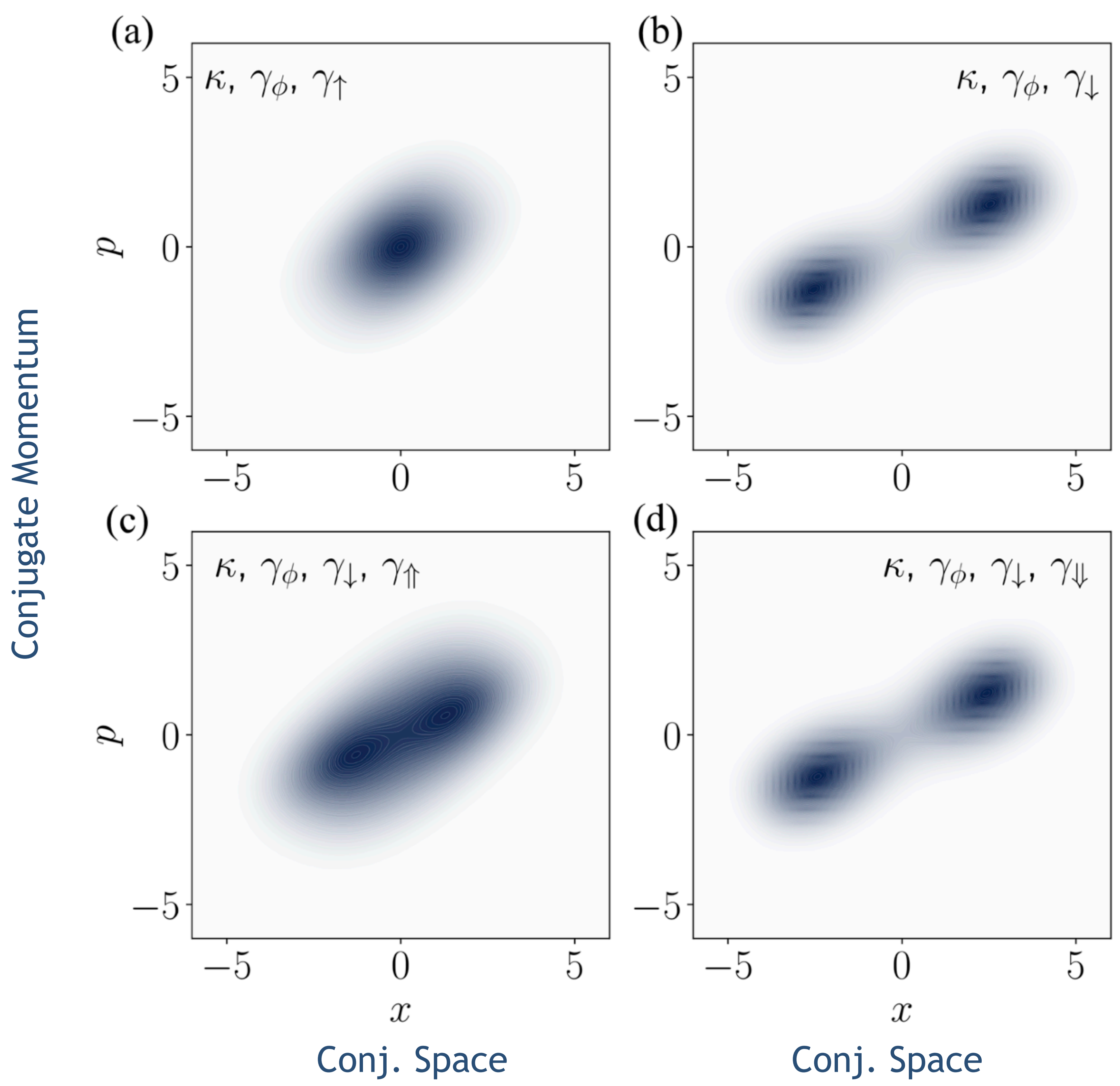


	SUPERRADIANT PHASE TRANSITION (SR PT)	SUPERRADIANT LIGHT EMISSION (SR LE)
Requirements	N two-level systems in a **cavity (cQED)** Typically strong coupling or beyond	N two-level systems, do not require a cavity Fast system initialization
Types	Order parameters: Temperature Light-matter coupling	Initially: Fully-excited → Superfluorescence (SF) Half-excited → Superradiance (SR) One-excitation → Single-photon SR
Quantities	Phase transition: Ground/Thermal state properties Entanglement	Peak delay time, height, width Cooperativity
Features Related Phenomena	Ultrastrong regime, No-go theorems (A^2 term) Light-matter quasiparticles Chaos	Subradiance, Dark states 'Superabsorption' Optical bistability
Experiments	- Bose-Einstein condensate (Dicke model) - Superconducting (SC) flux qubits coupled to a microwave waveguide (N=10^3) - Quantum well (QW) exciton-polariton quasiparticles in a microcavity	• SF: - ^{87}Rb cold atom Raman laser (N=10^6) - SC transmon qubits (small N, e.g., N<3) - QW exciton magnetoplasma (N>10^6) • Single-photon SR: - QW intersubband plasmon - Quantum dot exciton

Superradiant Phase in the Open Dicke Model

Effect of local and collective driven-dissipative processes on the stability of the phase

Wigner Function



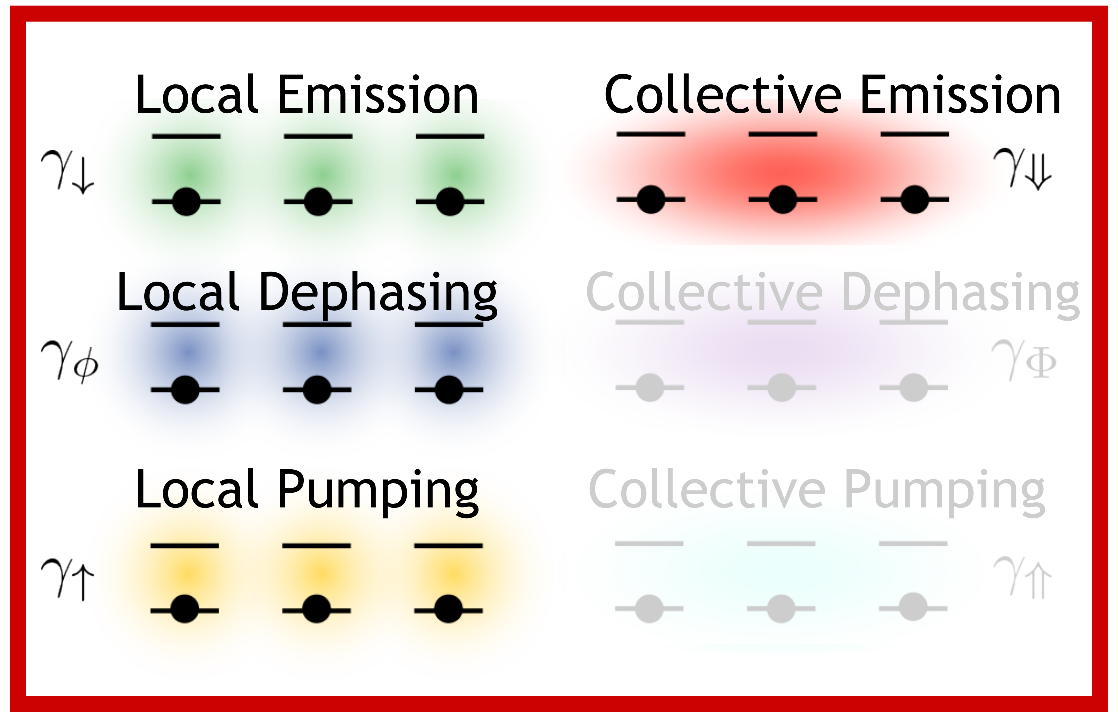
Hamiltonian

$$\boxed{H = \hbar\omega_0 J_z + \hbar\omega_{\mathrm{cav}} a^\dagger a + \hbar g J_x \left(a + a^\dagger\right)}$$

Dissipation $\mathcal{L}[\rho]$

Cavity Emission

$$\frac{\kappa}{2}\mathcal{L}_a[\rho]$$



P.G. Kirton and J. Keeling, Phys Rev Lett (2017)

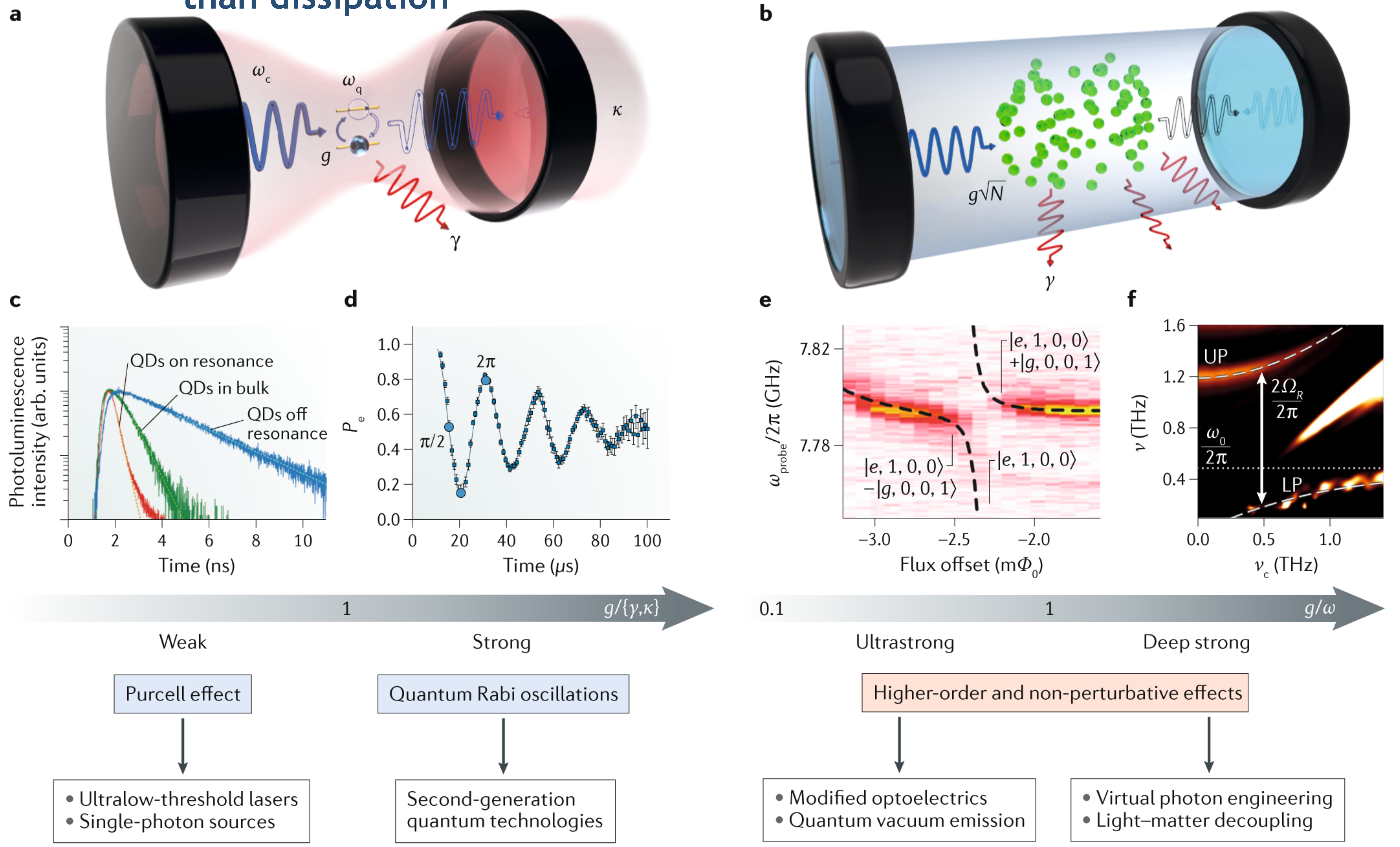
N. Shammah, S. Ahmed, N. Lambert, S. De Liberato, and F. Nori, Phys Rev. A 98, 063815 (2018) arXiv:1805.05129

Ultrastrong Coupling (USC) Regime

A new regime of light-matter coupling

Coupling stronger than dissipation $g/\kappa \gg 1$

Coupling~bare frequencies $g/\omega_0 \geq 0.1$



Hamiltonian

$$\boxed{\begin{aligned} H &= \hbar\omega_0 J_z + \hbar\omega_{\mathrm{cav}} a^\dagger a \\ &+ \hbar g J_x \left(a + a^\dagger\right) \end{aligned}}$$

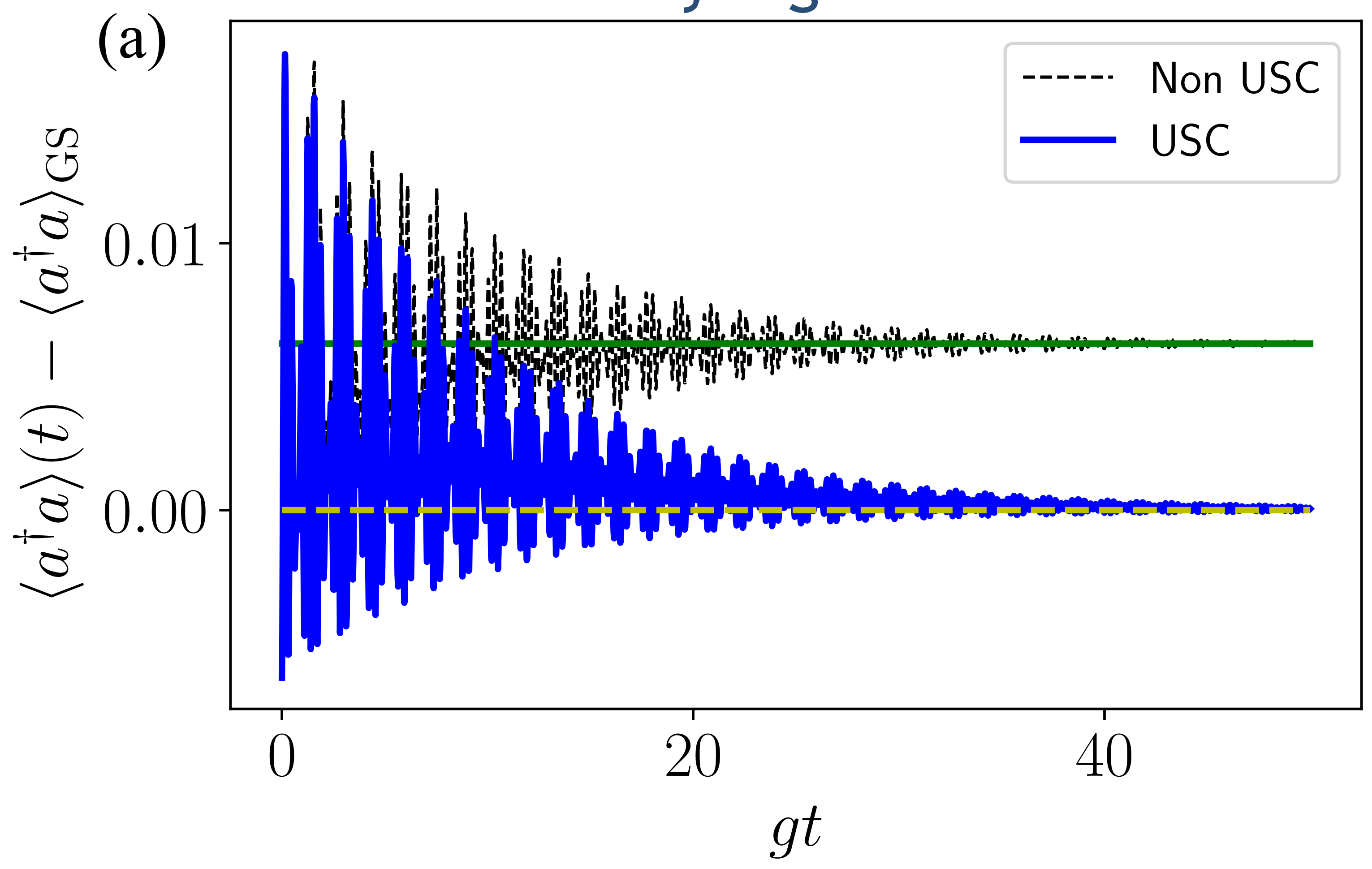
Dissipation $\frac{\kappa}{2}\mathcal{L}_a[\rho]$

A. Frisk Kockum, A. Miranowicz, S. De Liberato, S. Savasta, and F. Nori,
“Ultrastrong Coupling between light and matter”, Nature Reviews Physics **1**, 19 (2019)

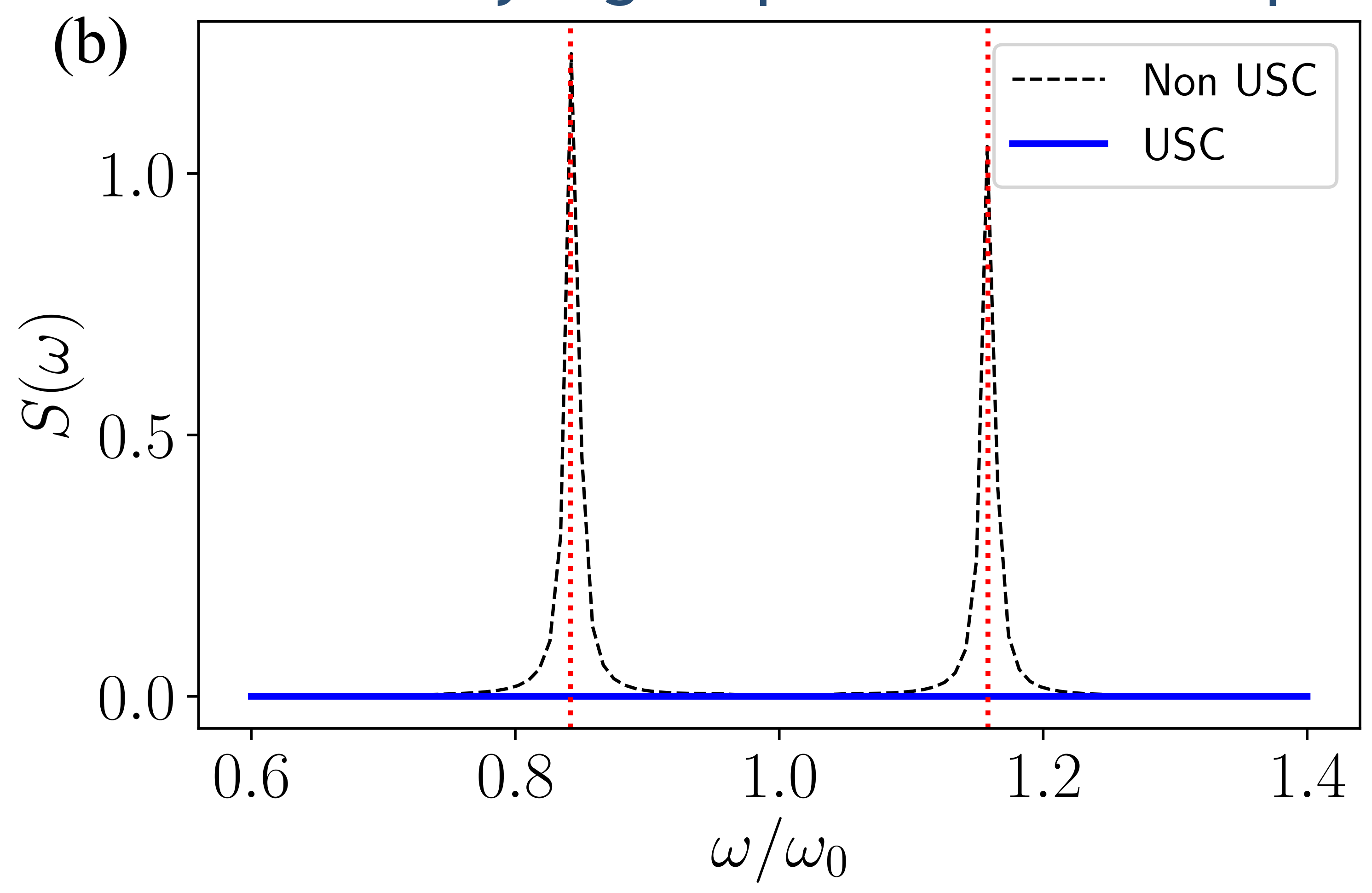
Ultrastrong Coupling (USC) Regime

Relaxation to the true steady state by deriving consistent operators

Extra-cavity Light Field vs. Time



Extra-cavity Light Spectrum vs. Frequency



Hamiltonian

$$\boxed{H = \hbar\omega_0 J_z + \hbar\omega_{\mathrm{cav}} a^\dagger a + \hbar g J_x \left(a + a^\dagger\right)}$$

Dissipation $\mathcal{L}[\rho]$

$$\dot{\rho} = -\frac{i}{\hbar}[H,\rho] + \sum_{r,s>r}\left(\frac{\kappa}{2}|X^{r,s}|^2 + \frac{\gamma_\downarrow}{2}\sum_{n=1}^{N}|J^{r,s}_{x,n}|^2\right)\mathcal{L}[|r\rangle\langle s|](\rho)$$

$$J_{\alpha,n} = \sum_{r,s} J^{r,s}_{\alpha,n}\,|r\rangle\langle s| \qquad \sum_{n=1}^{N}|J^{r,s}_{-,n}|^2_{s>r} = \frac{1}{2}\langle s|\sum_{n=1}^{N}\mathcal{L}_{J_{-,n}}(|r\rangle\langle r|)\,|s\rangle_{s>r}$$

Derive the “dressed” light-matter jump operators for the Lindbladian

Case N=1: F. Beaudoin, J.M. Gambetta, A. Blais, PRA (2011)

Case N>>1: N. Shammah, S. Ahmed, N. Lambert, S. De Liberato, and F. Nori, PRA 98, 2018, arXiv:1805.05129

Light-Matter Dressed Ground State

$$|G\rangle = (1-\frac{\eta^2}{8})|0,g\rangle+\frac{\eta}{2}|1,e\rangle+\frac{\eta^2}{2\sqrt{2}}|2,g\rangle$$

Virtual photons

$$\langle G|a^\dagger a|G\rangle = \frac{\eta^2}{4}$$

$$\eta = \frac{g}{\omega_0}$$

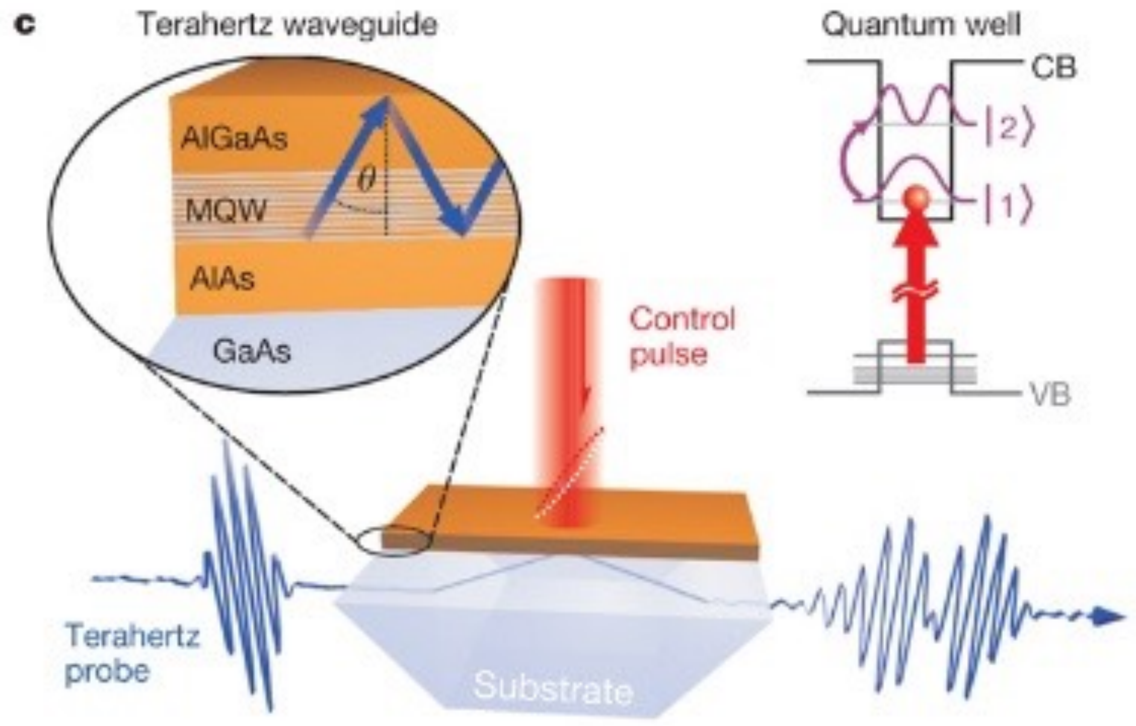
Observing virtual photons: Non-adiabatic modulation coupling

PHYSICAL REVIEW B **72**, 115303 (2005)

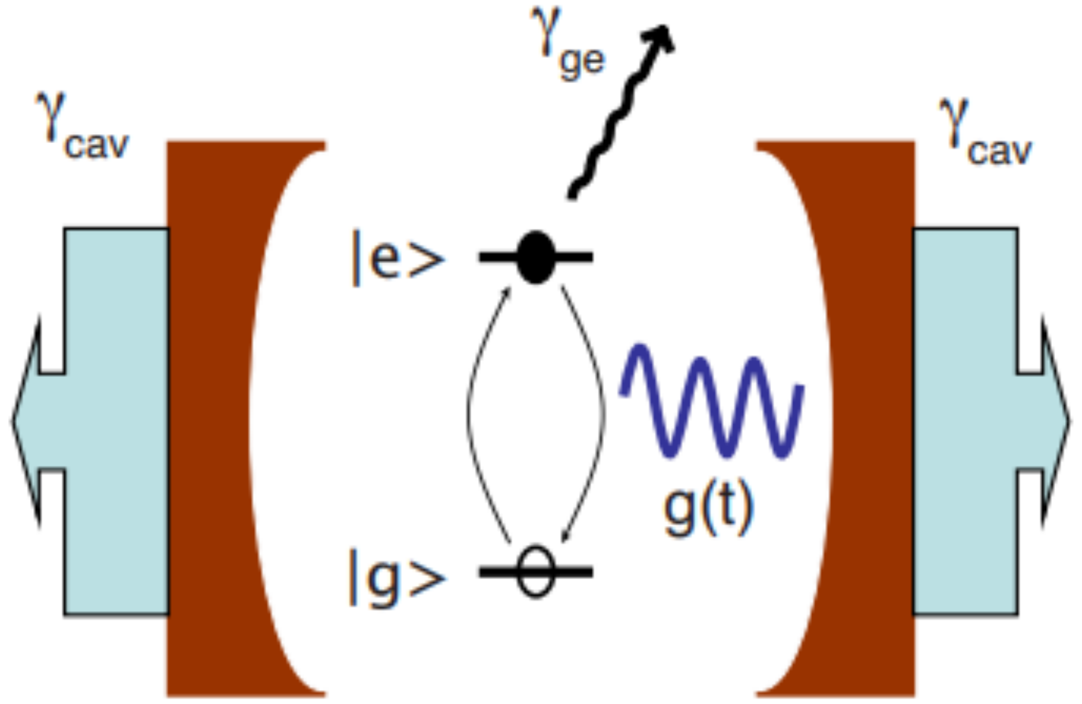
Quantum vacuum properties of the intersubband cavity polariton field

Cristiano Ciuti* and Gérald Bastard
Laboratoire Pierre Aigrain, Ecole Normale Supérieure, 24, rue Lhomond, 75005 Paris, France

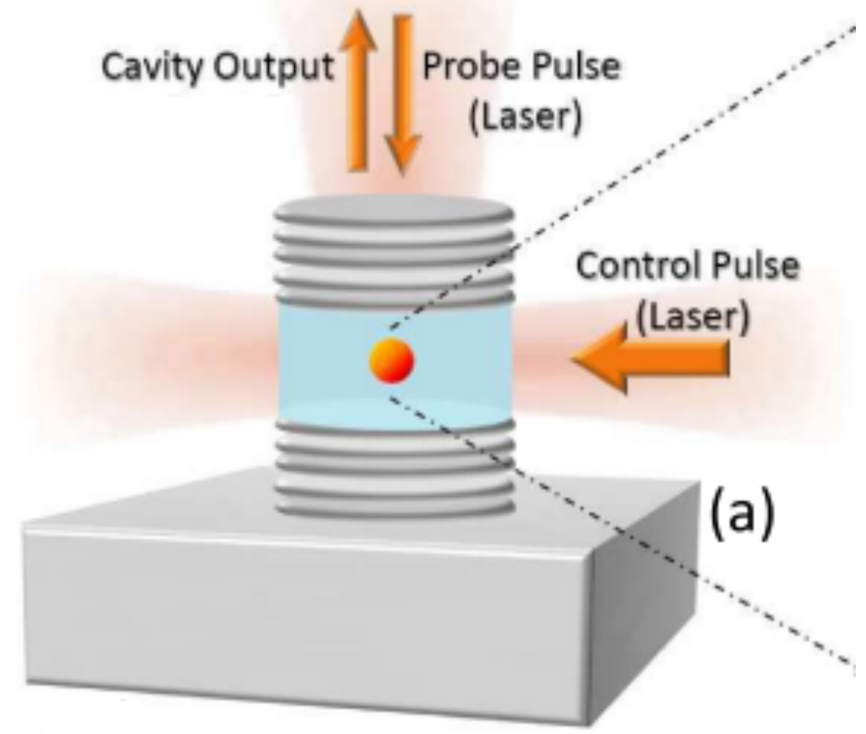
Iacopo Carusotto
CRS BEC-INFM and Dipartimento di Fisica, Università di Trento, I-38050 Povo, Italy



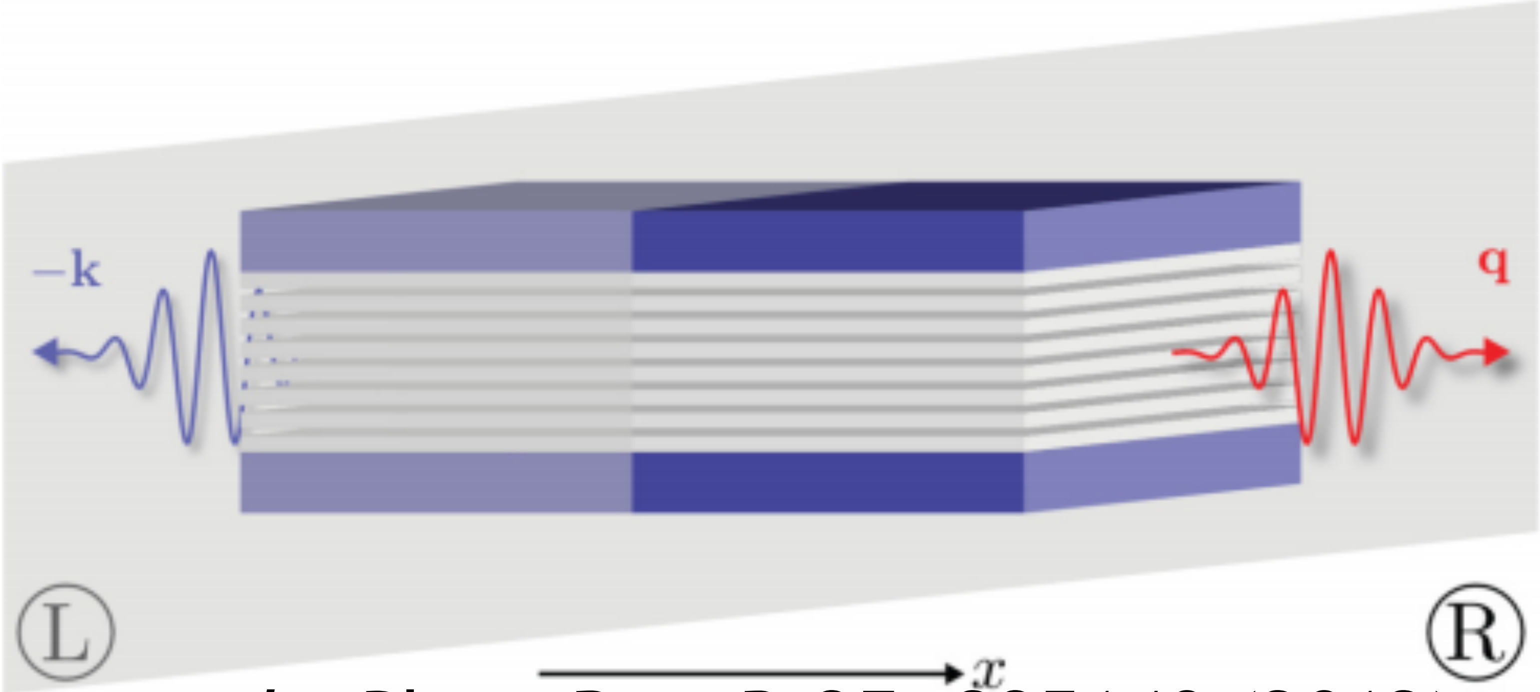
G. Günter, *et al.*, Nature **458**, 178 (2009)



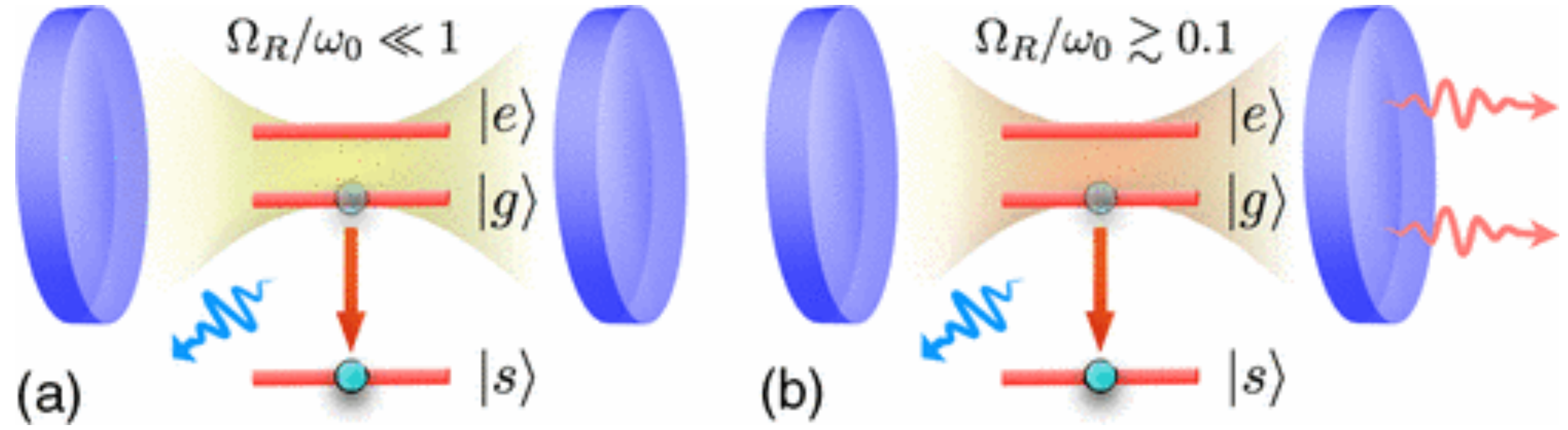
S. De Liberato, *et al.*, Phys. Rev. A **80**, 053810 (2009)



A. Ridolfo, *et al.*, Phys. Rev. Lett. **106**, 013601 (2011)



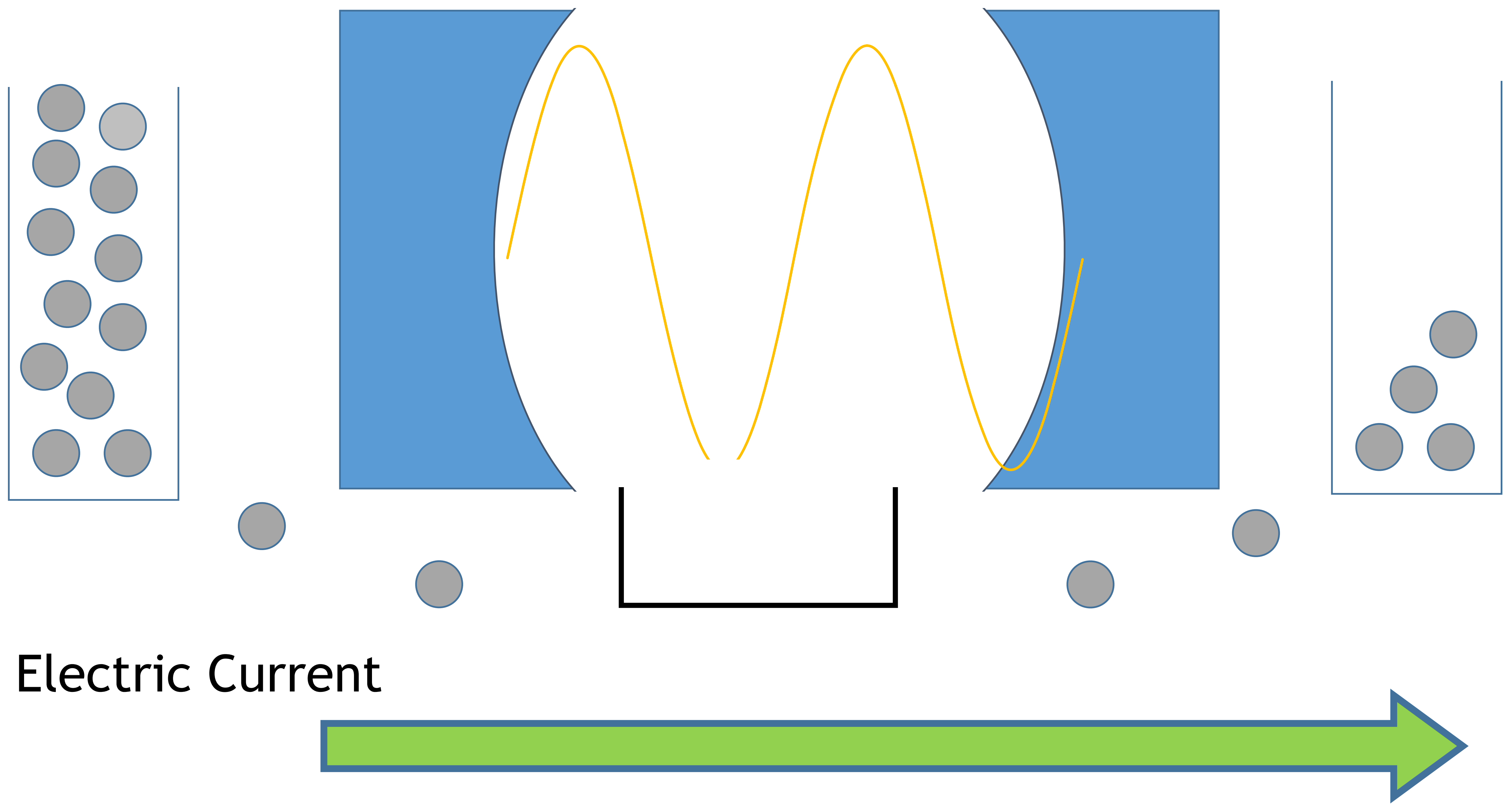
A. Auer, *et al.*, Phys. Rev. B **85**, 235140 (2012)



R. Stassi, *et al.*, Phys. Rev. Lett. **110**, 243601 (2013)

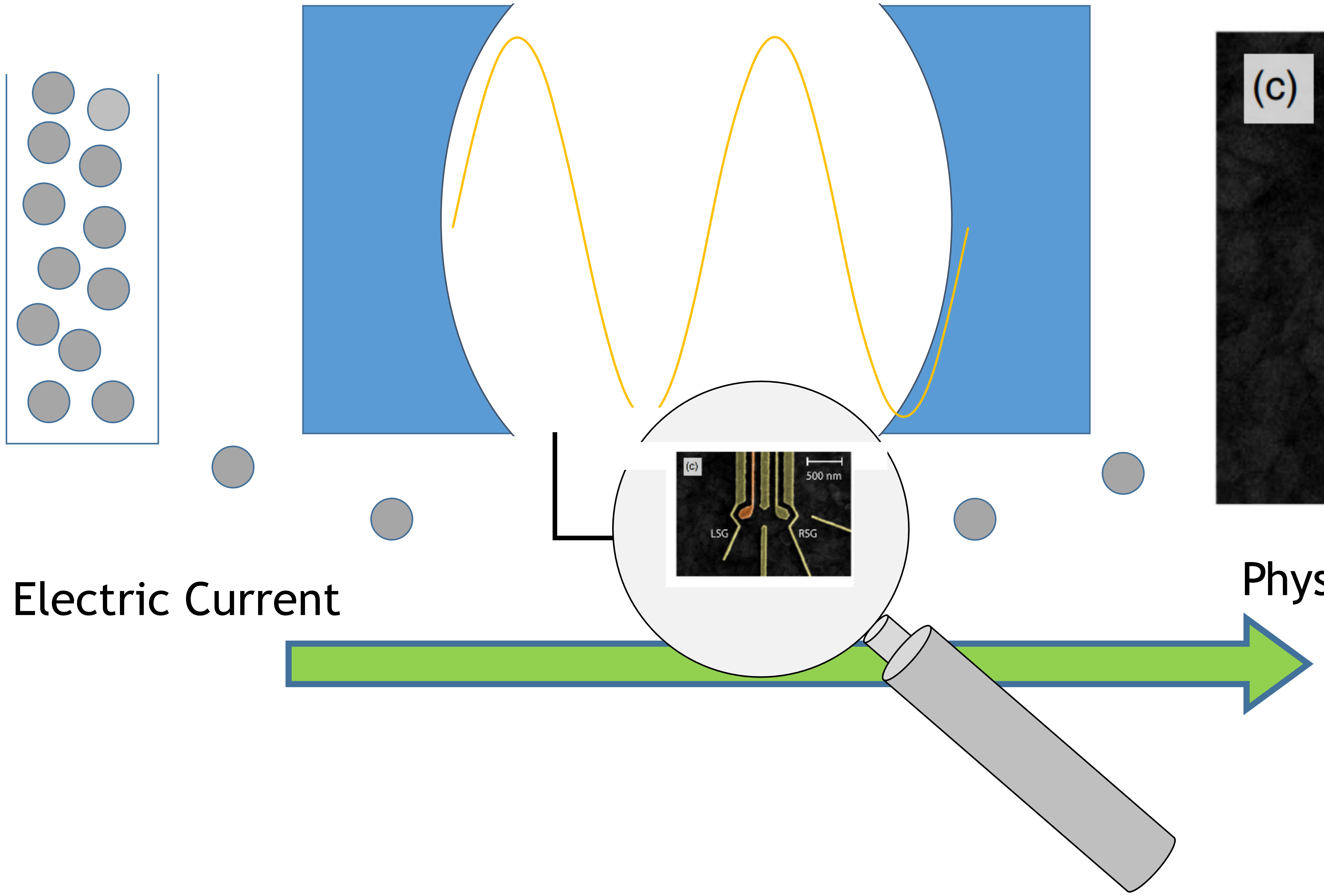
Ground State Electroluminescence

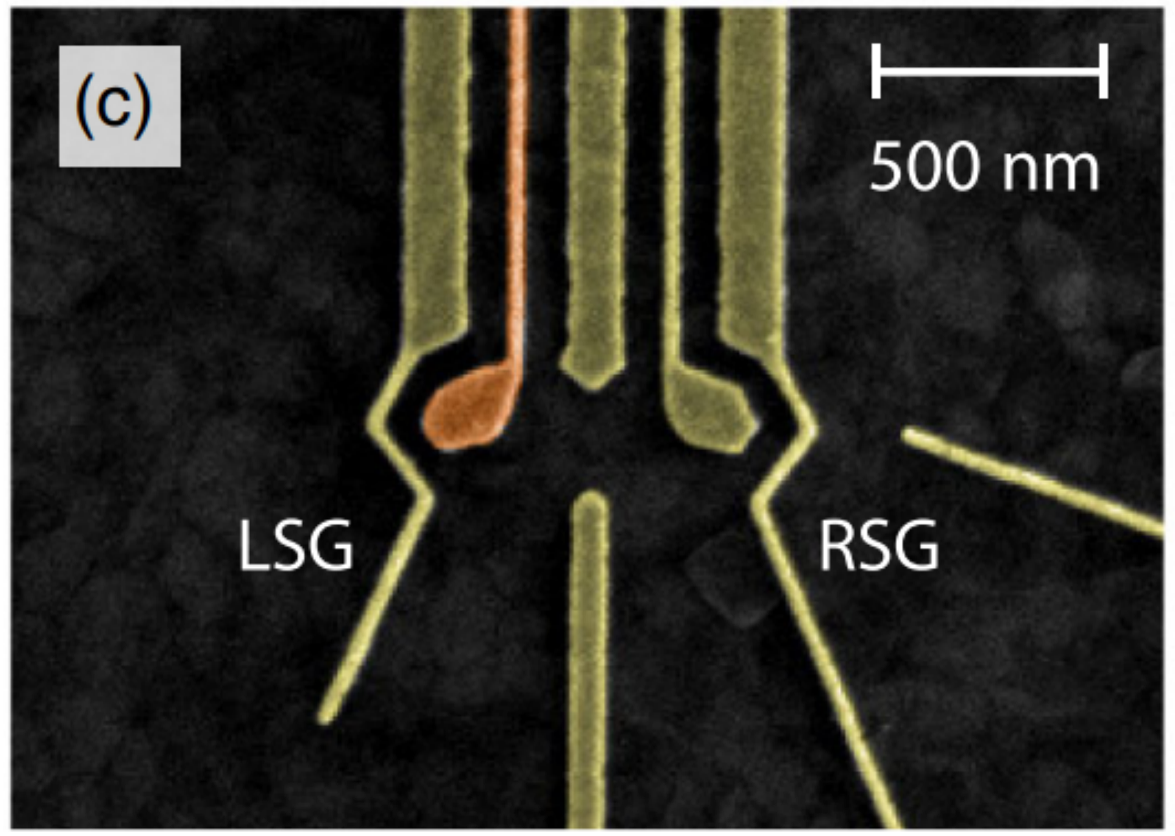
Electric Current prompts Light Emission via Light-matter Coupling Modulation



Ground State Electroluminescence

Electric Current prompts Light Emission





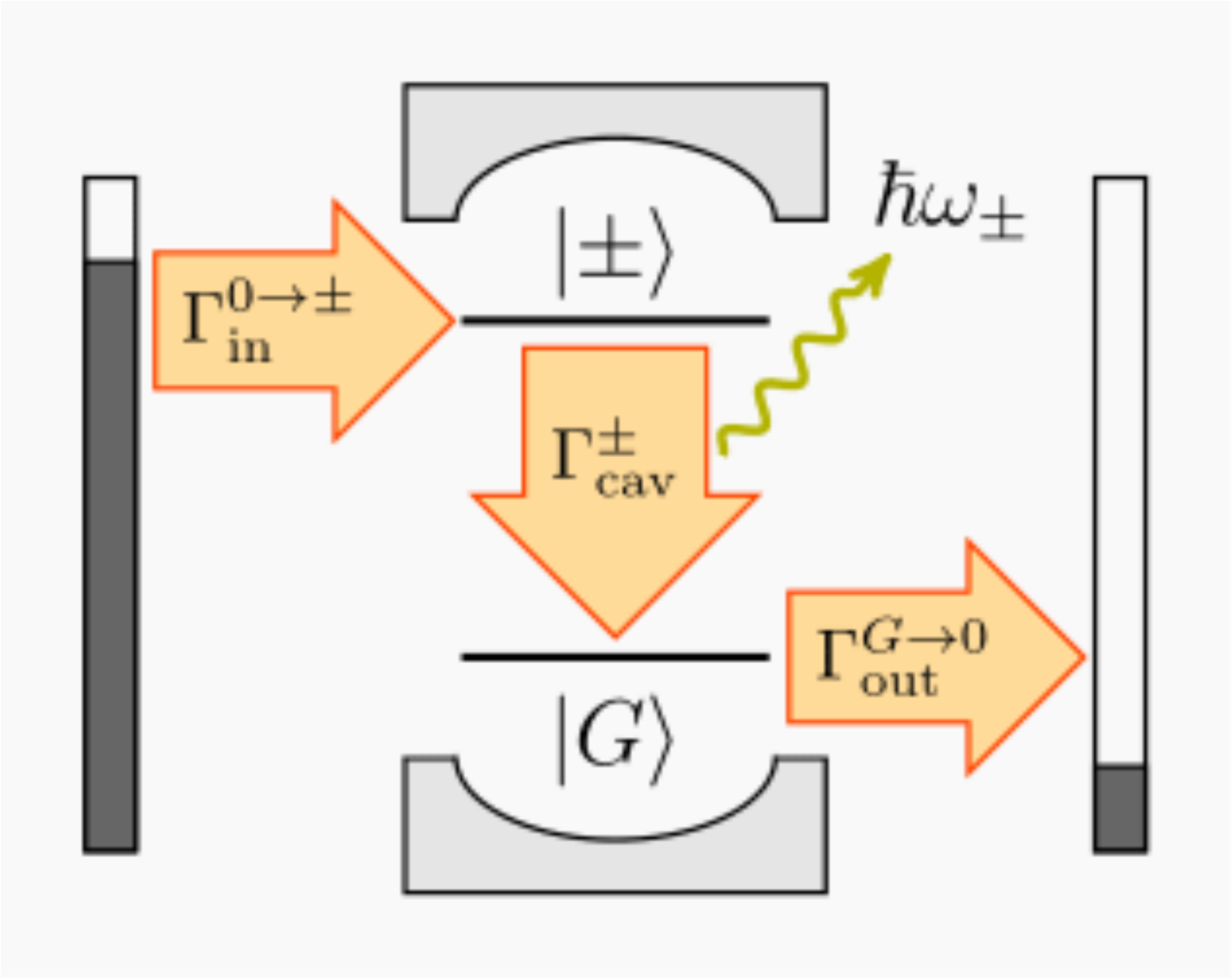
Phys. Rev. X **7**, 011030 (2017)

Ground State Electroluminescence

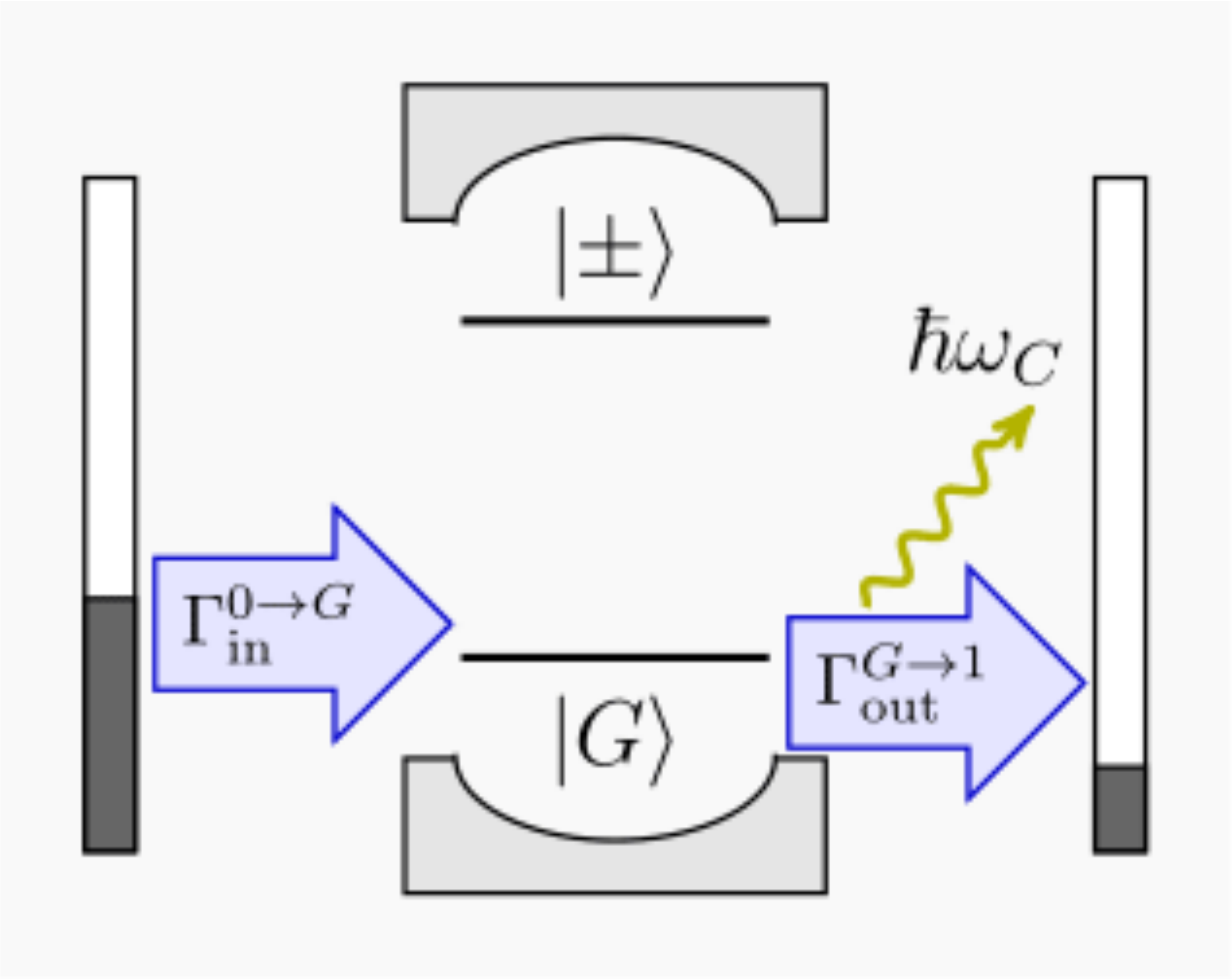
Electric Current prompts Light Emission

Electroluminescence

Regular



Ground State



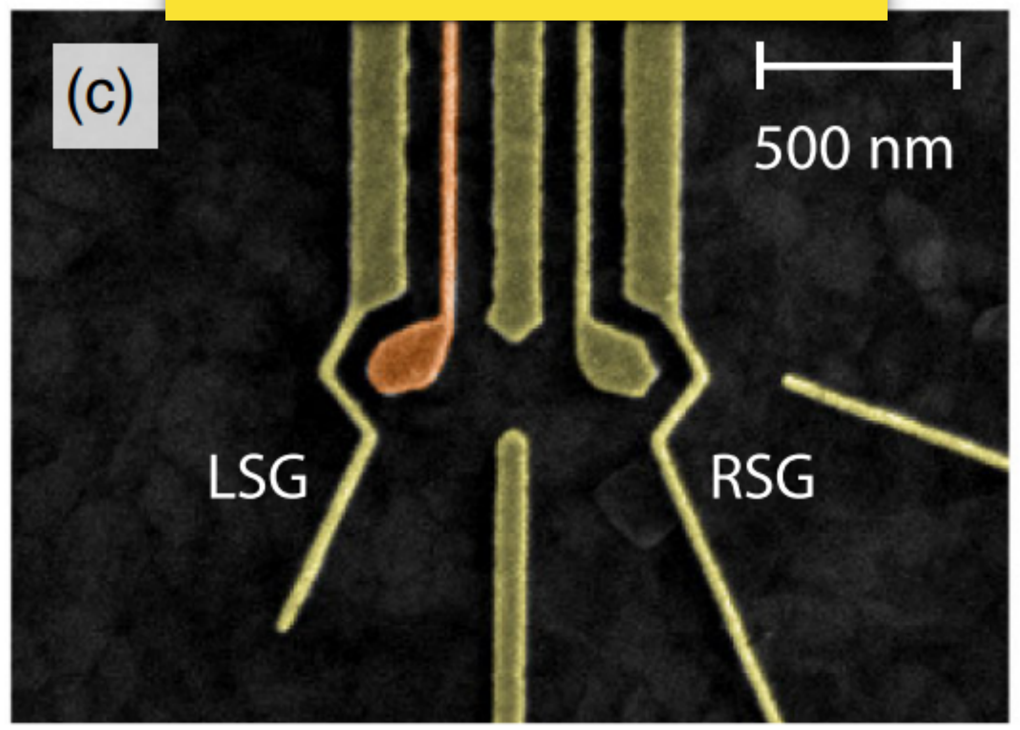
Ground State Electroluminescence
M. Cirio, N. Lambert, F. Nori, and S. De Liberato
Phys. Rev. Lett. **116**, 113601 (2016)
1508.05849

Strong and Ultrastrong Coupling with Transport

Many Platforms could probe ground state electroluminescence

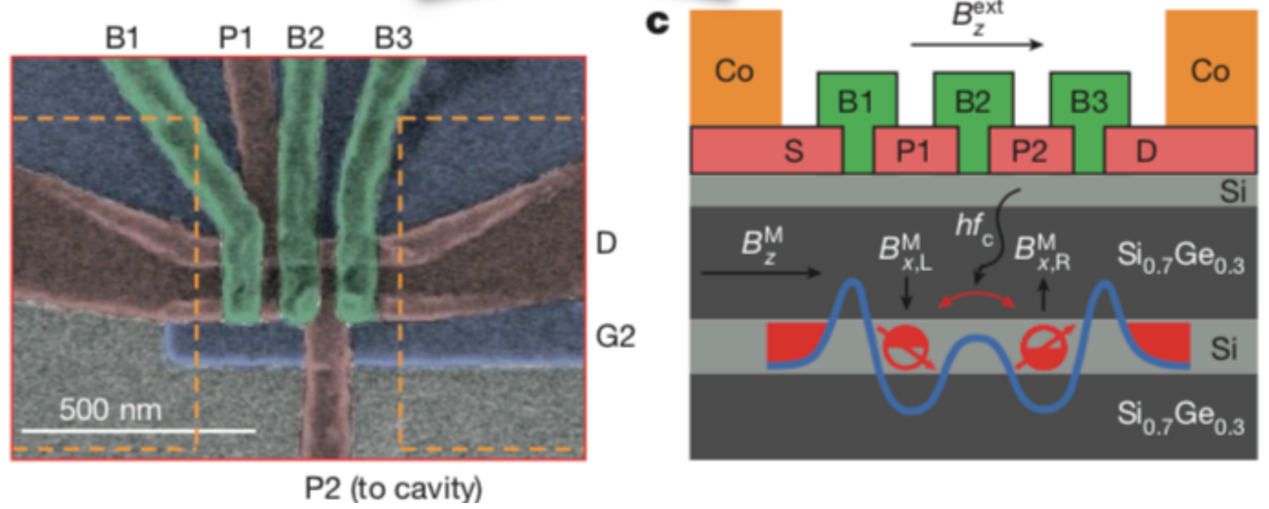
Single Spins

GaAs Double QD



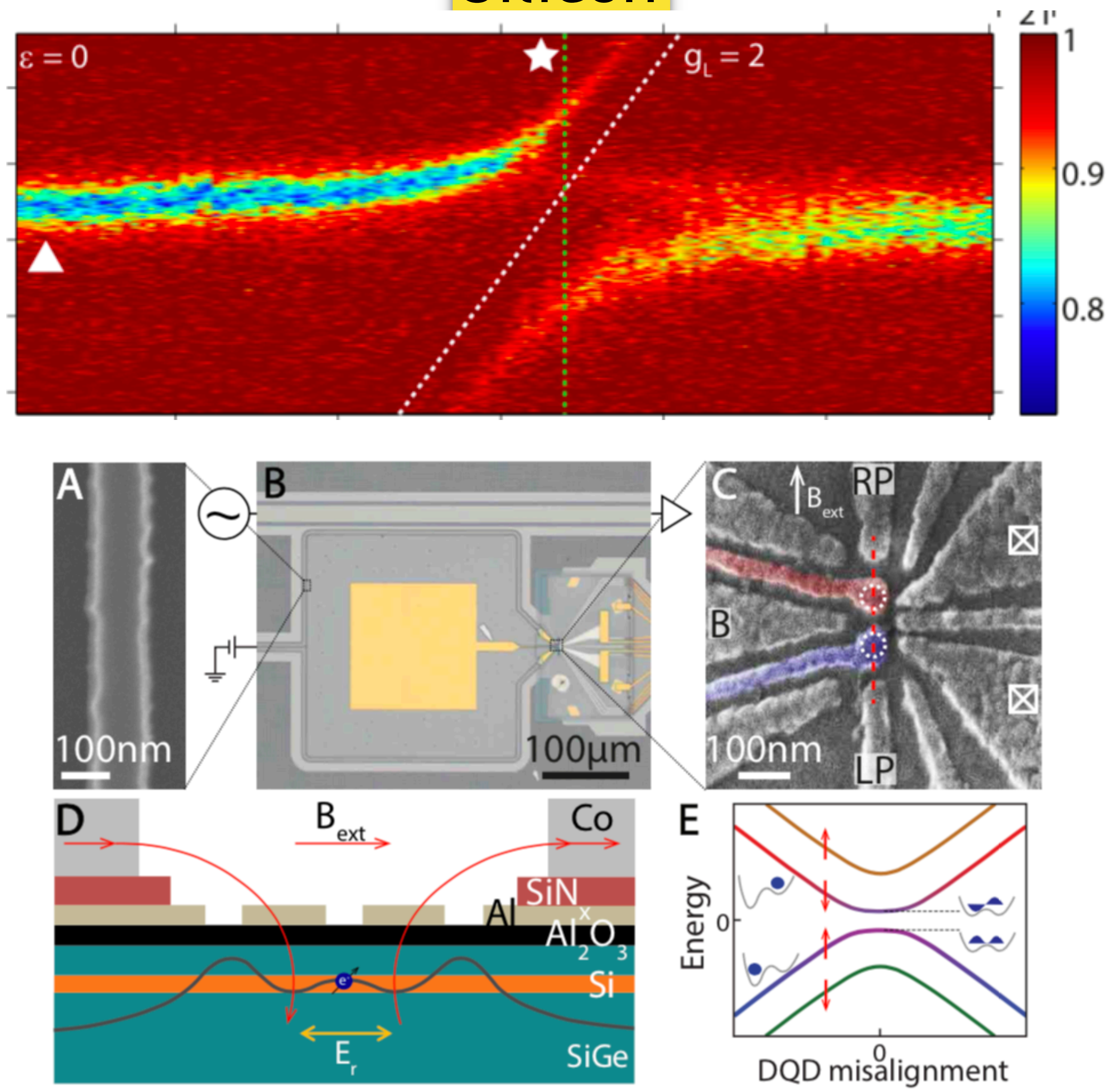
A. Stockklauser, *et al.*, Phys. Rev. X **7**, 011030 (2017)

Silicon



X. Mi, *et al.*, **555**, 599 Nature (2018)
A. Landig, *et al.*, **560**, 179 Nature (2018)

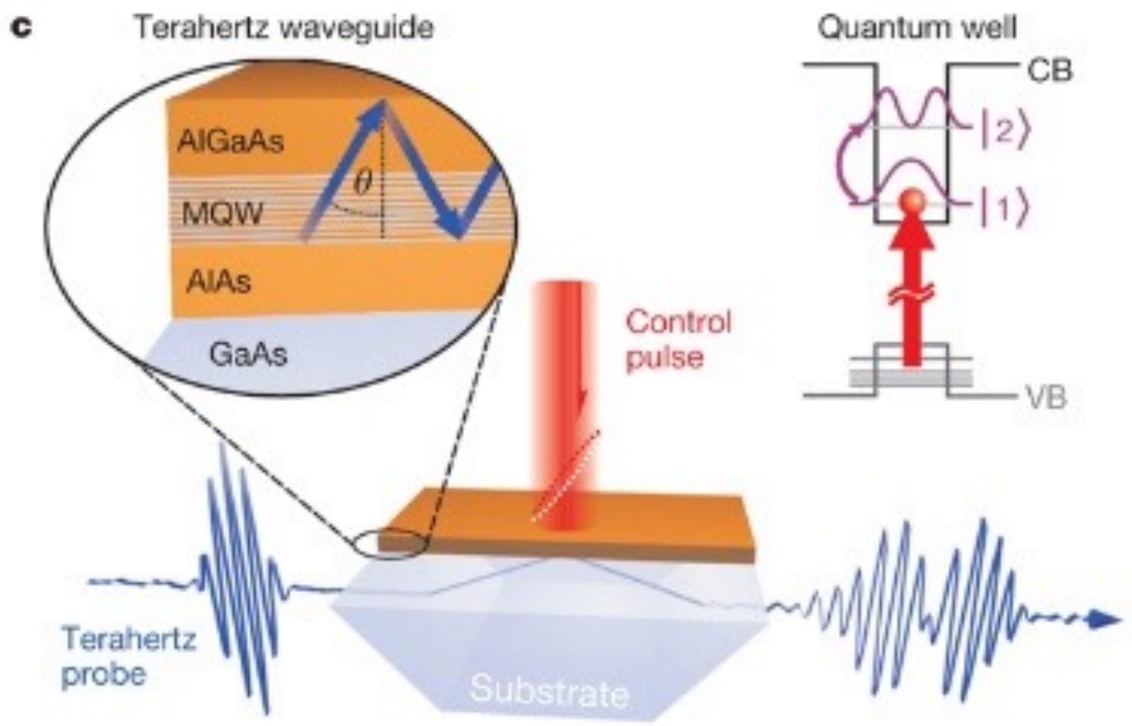
Silicon



N. Samkharadze, *et al.*, Science **359**, 1123 (2018)

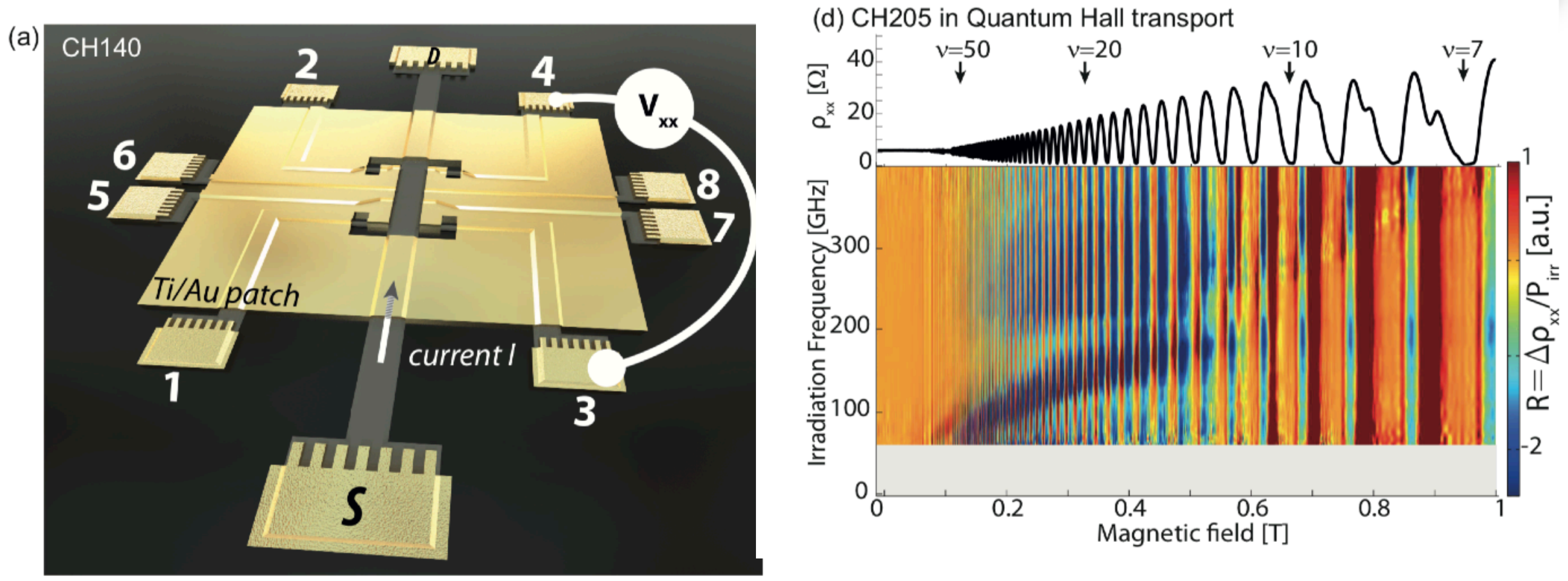
2D Electron Gas

Dynamical USC Switch



G. Günter, *et al.*, Nature **458**, 178 (2009)

Landau Polaritons & Magneto transport



G. Paravicini-Bagliani, *et al.*, Nature Phys **15**, 186 (2019)

Steady-state light emission and transport

Dissipation and ultrastrong coupling regime

Electron transport + Light-matter coupling

Ground State Electroluminescence
M. Cirio*, N. Lambert, F. Nori, and S. De Liberato
Phys. Rev. Lett. **116**, 113601 (2016)

Multielectron Ground State Electroluminescence
M. Cirio*, N. Shammah*, N. Lambert, S. De Liberato, and F. Nori
Phys. Rev. Lett. **122**, 190403 (2019) arXiv: 1811.08682

Features

- **Cooperative effect**
- **Robust with *N***

Possible Experiments

- Quantum wells (Faist)
- Quantum dots (Vandersypen, Petta)
- Superconducting circuits
- Hybrid devices (Wallraff)

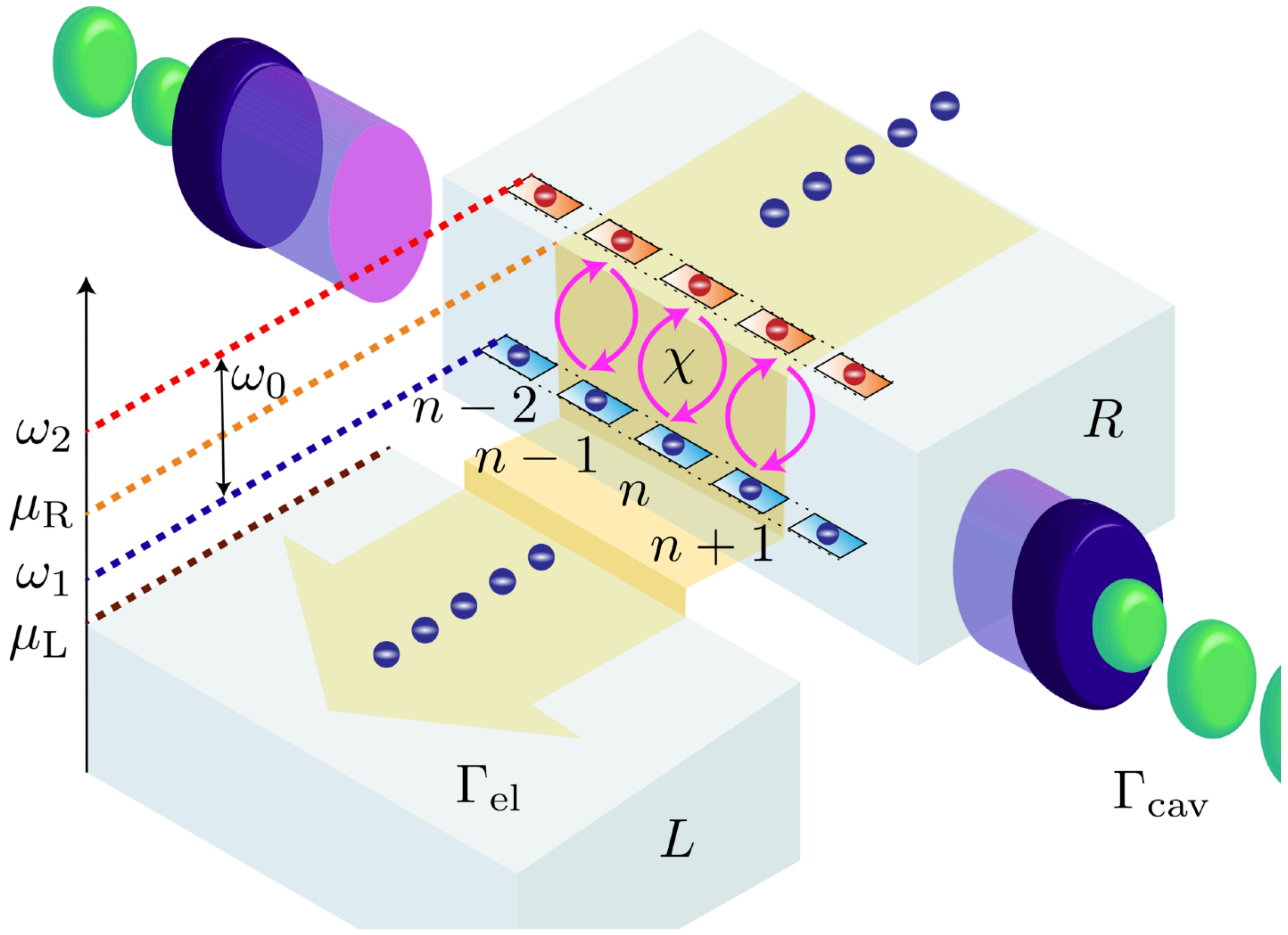


Figure: Solid-state device.

Steady-state light emission and transport

Dissipation and ultrastrong coupling regime

Electron transport + Light-matter coupling

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Photon emission rate

$$\Gamma^{\pm}_{em} = O(\eta^2) = O(N\chi^2/\omega_0^2)$$

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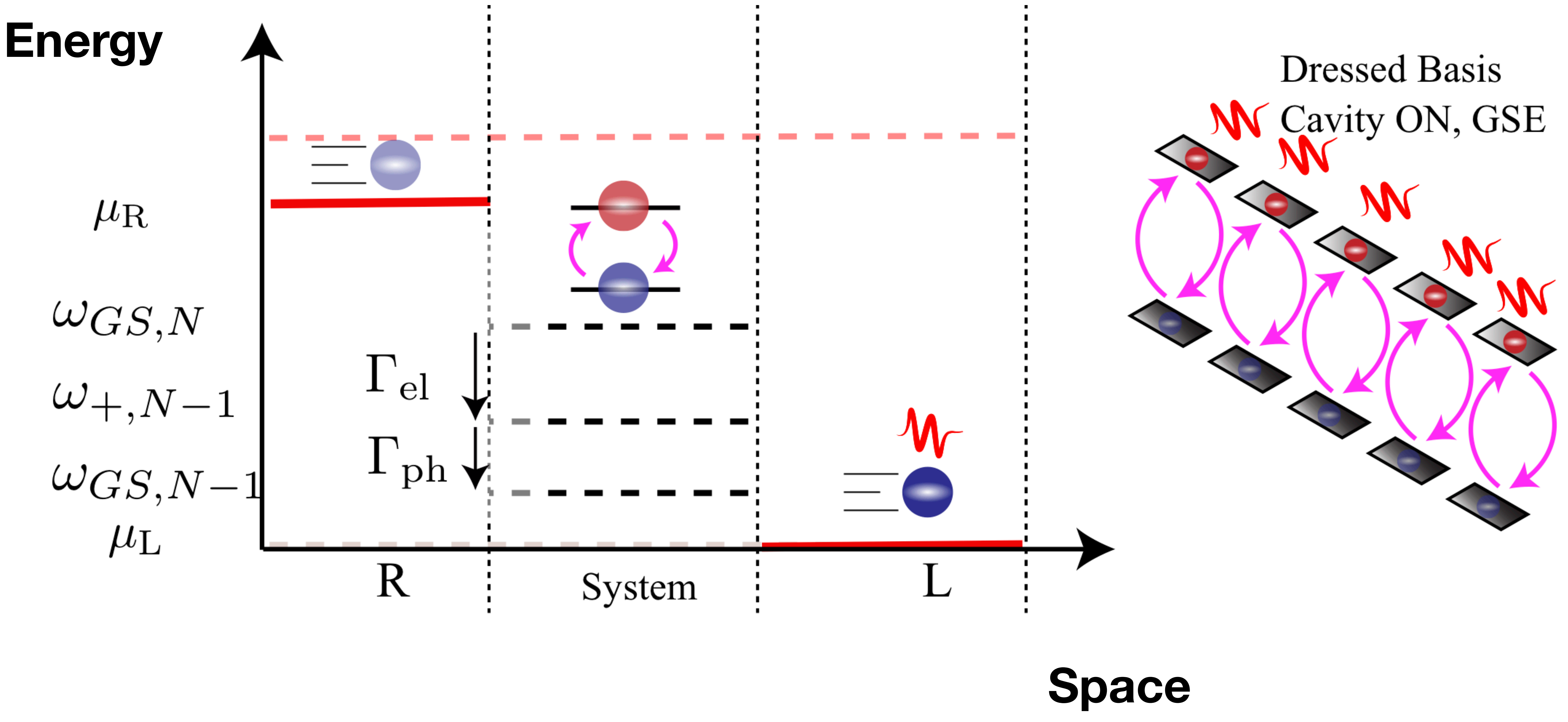

QuTiP: The Quantum *Physics* Simulator

The **Qu**antum **T**oolbox **i**n **P**ython: A toolbox to study the **open** quantum dynamics of realistic systems.

QuTiP

Interactive Lectures @ ICTP, Leonardo Building

Take a snapshot

Tue 25th June - 11:45am, Seminar Room –	Driven-dissipative models in quantum physics
Wed 26th June - 11am, Seminar Room –	Quantum Open Source & Introduction to QuTiP
Thur 27th June - 9am, Computer Room –	Hands-on session on QuTiP's main features
Mon 1st July - 9am, Computer Room –	QuTiP stochastic solvers
Tue 2nd July - 9am, Computer Room –	PIQS and How to Build your Own Software
(Wed 3rd July - 9am, Computer Room –	Extra meeting: SISSA/ICTP projects)

https://github.com/nathanshammah/interactive-notebooks

Acknowledgements and funding

Shahnawaz Ahmed
RIKEN
Chalmers (Sweden)

Dr. Neill Lambert
RIKEN

Dr. Mauro Cirio
RIKEN
GSCAEP (China)

Prof. Simone De Liberato
U. of Southampton (UK)

Prof. Franco Nori
RIKEN
U. of Michigan (USA)

RIKEN

日本学術振興会 Japan Society for the Promotion of Science

JST Japan Science and Technology Agency

THE ROYAL SOCIETY

JOHN TEMPLETON FOUNDATION

Thank you for your attention

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GitHub: nathanshammah
LinkedIn: Nathan Shammah

medium.com/quantum-tech

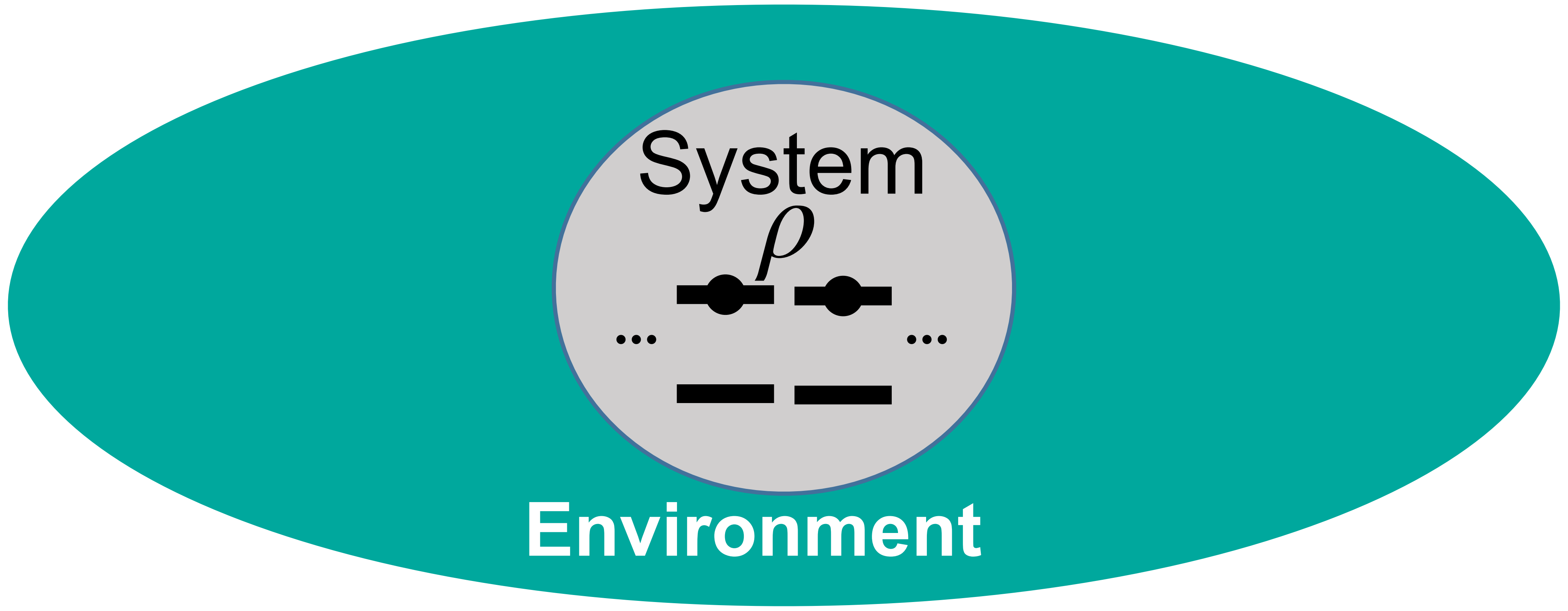
quantika.co

`qutip.piqs`: Exploiting permutational symmetry

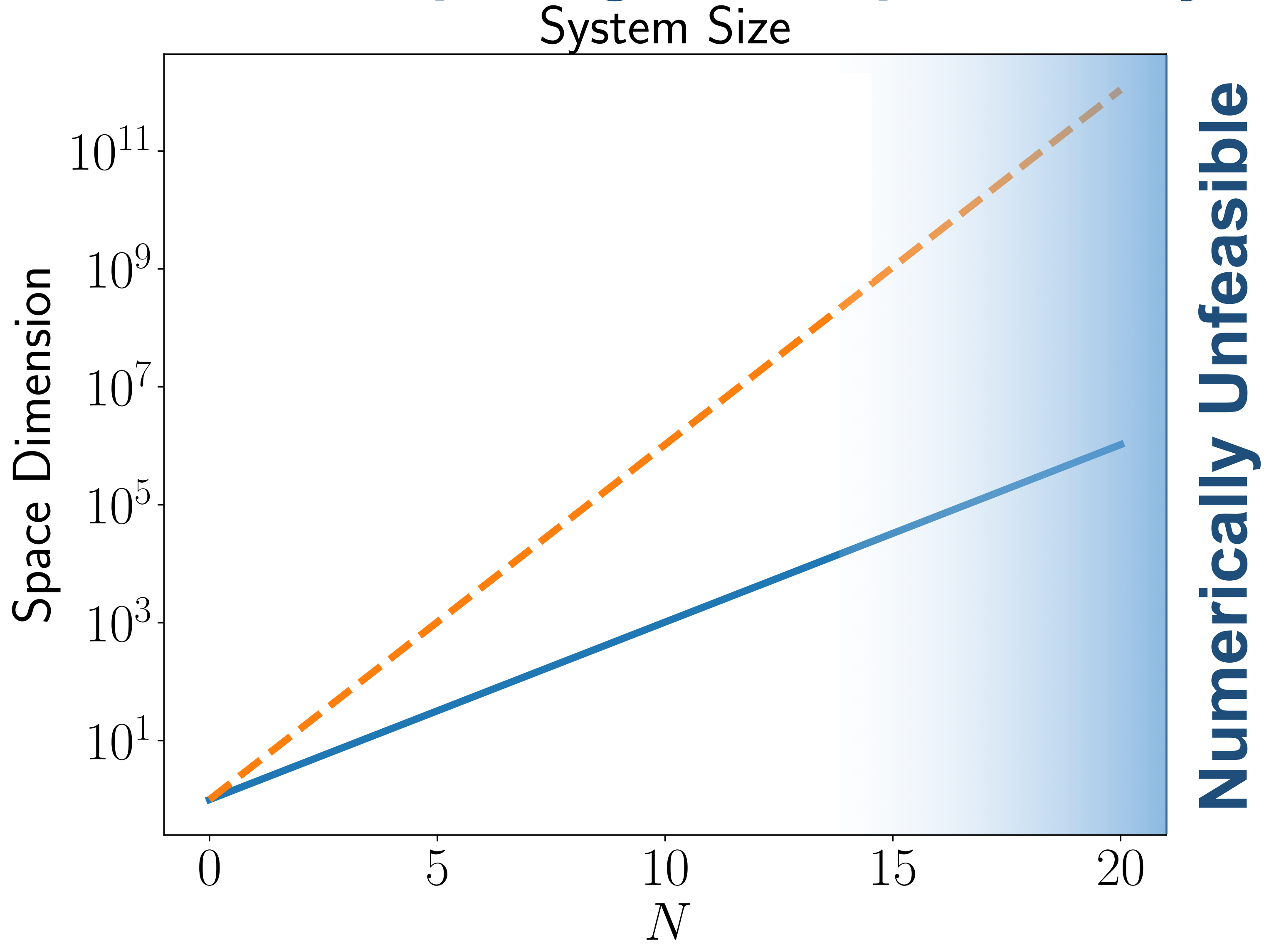
About a recent QuTiP module simulating the dynamics of *many* qubits

$$i\hbar\frac{d}{dt}\rho = [H,\rho] + \gamma\sum_i^N\left(L_i\rho L_i^\dagger - \frac{1}{2}L_i^\dagger L_i\rho - \frac{1}{2}\rho L_i^\dagger L_i\right)$$

Problem



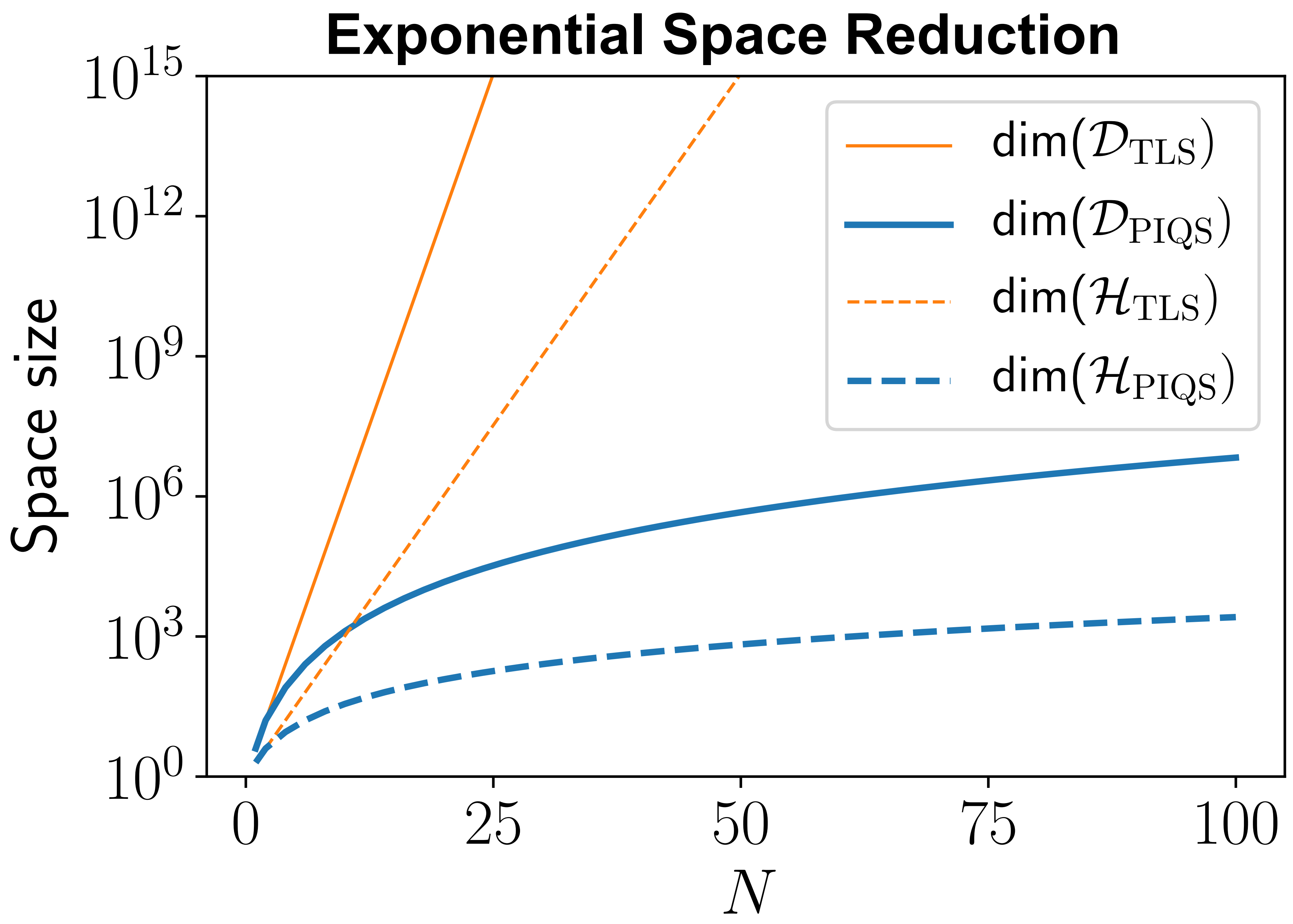
The Liouvillian space grows exponentially as 4^N



Solution

Permutational symmetry

The Su(2) Pauli operators can be written using Su(4) generators in Liouvillian space **obtaining an exponential size reduction.**



Superradiant Light Emission i.e. Dicke Superradiance

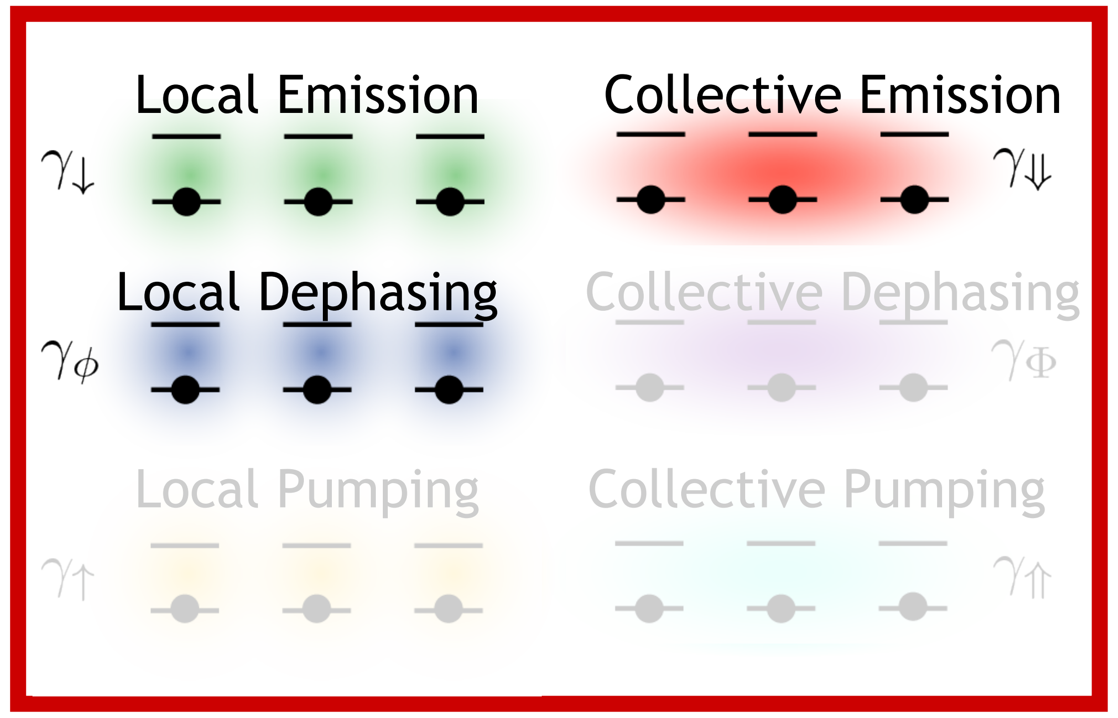
Effect of dephasing on different state preparations

Hamiltonian

$$\boxed{H = \hbar\omega_0 J_z}$$

PIQS: Permutational-Invariant Quantum Solver

Dissipation $\mathcal{L}[\rho]$



N Shammah *et al.*, Phys. Rev. A **96,** 023863 (2017)

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Steady-state Superradiant Light Emission

Effect of thermal equilibrium on the critical coupling

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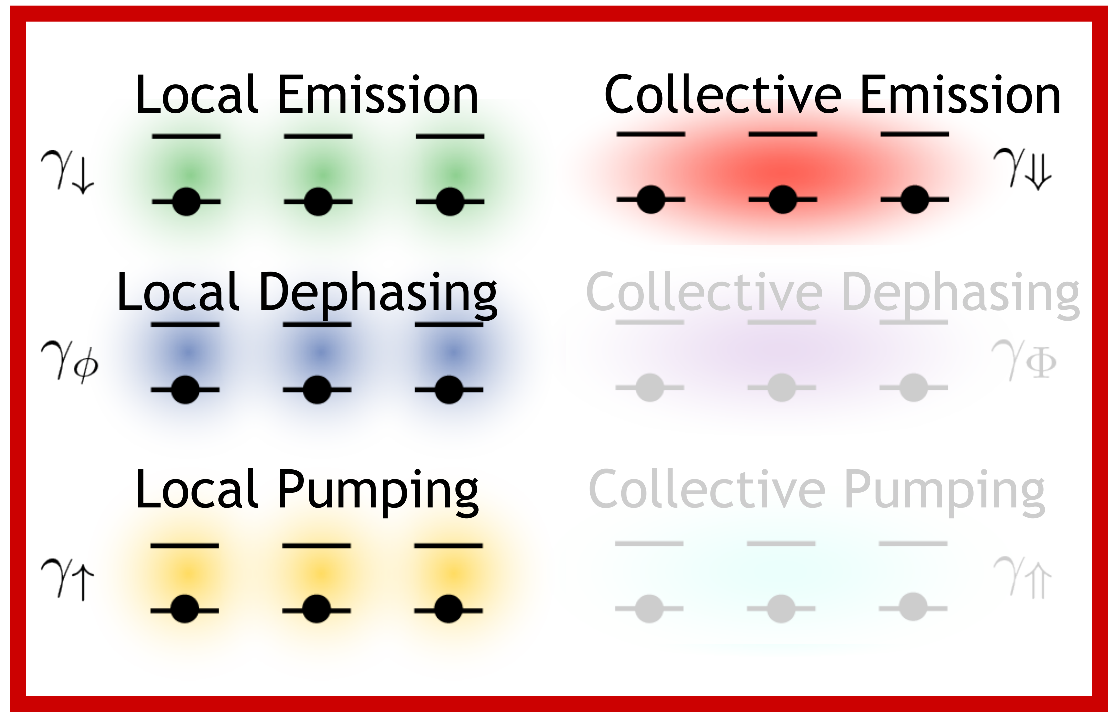

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Spin Squeezing with noise

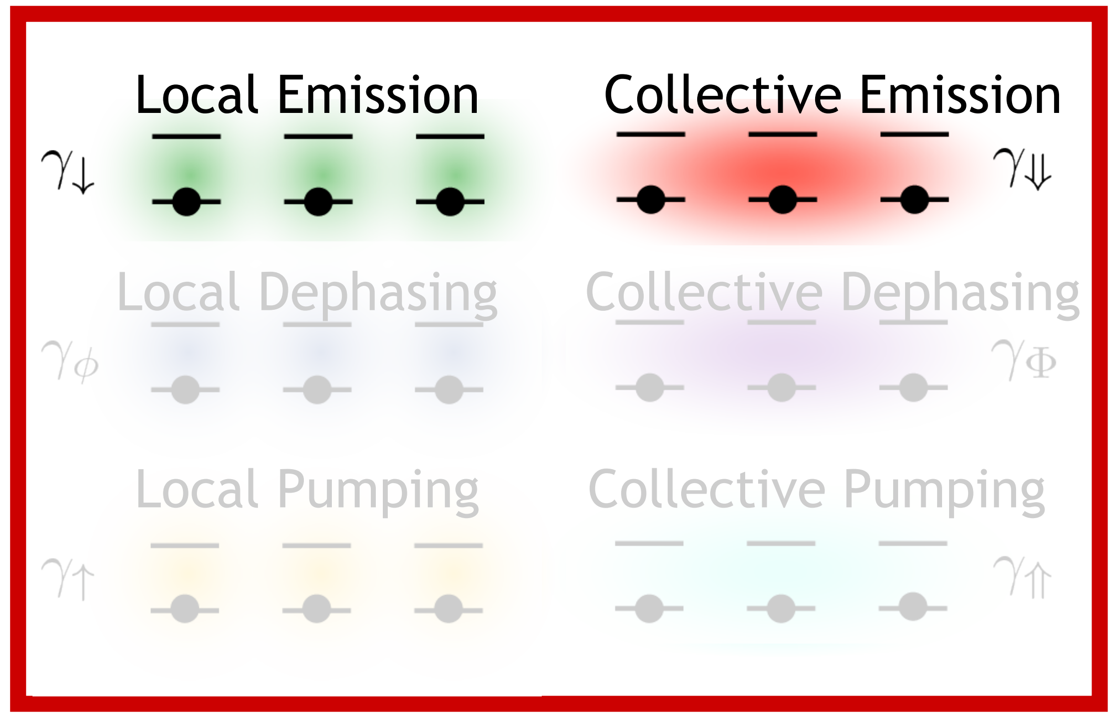
Effect of local vs. collective dissipation on the spin-squeezing parameter

PIQS: Permutational-Invariant Quantum Solver

Hamiltonian

$$\boxed{H = -i\hbar\Lambda\left(J_+^2 - J_-^2\right)}$$

Dissipation $\mathcal{L}[\rho]$



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Boundary Time Crystals

Incommensurate oscillations arising from the competition of superradiant relaxation and coherent drive

Hamiltonian

$$\boxed{H = \hbar\omega_x J_x}$$

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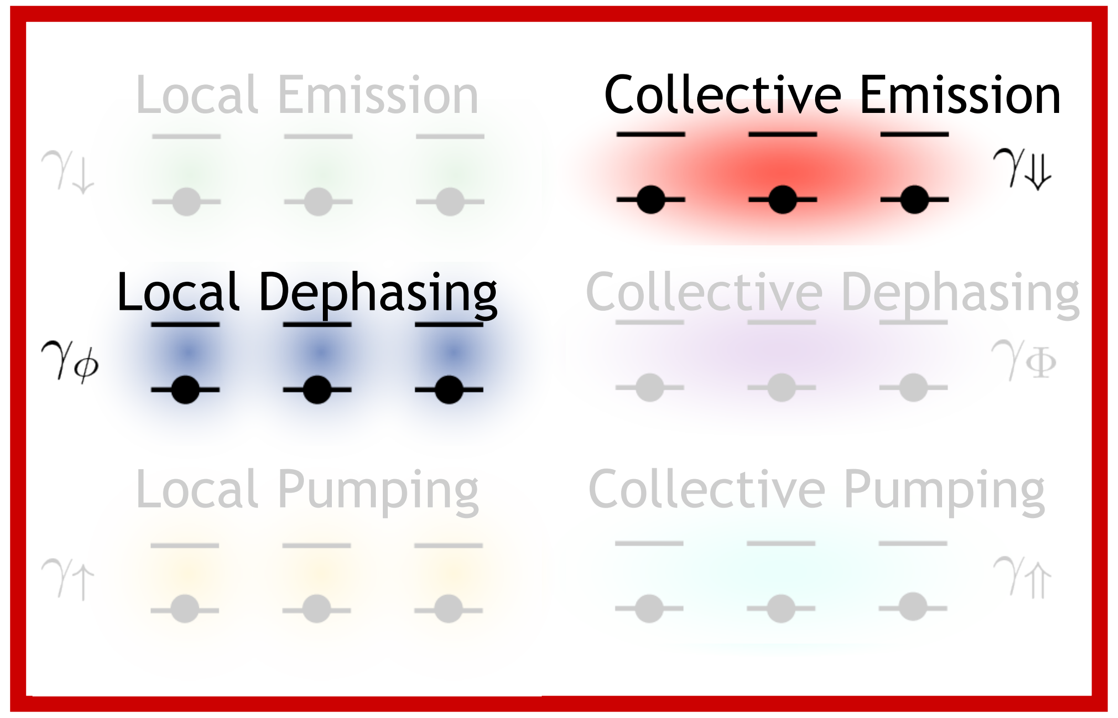

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Multiple Qubit Ensembles

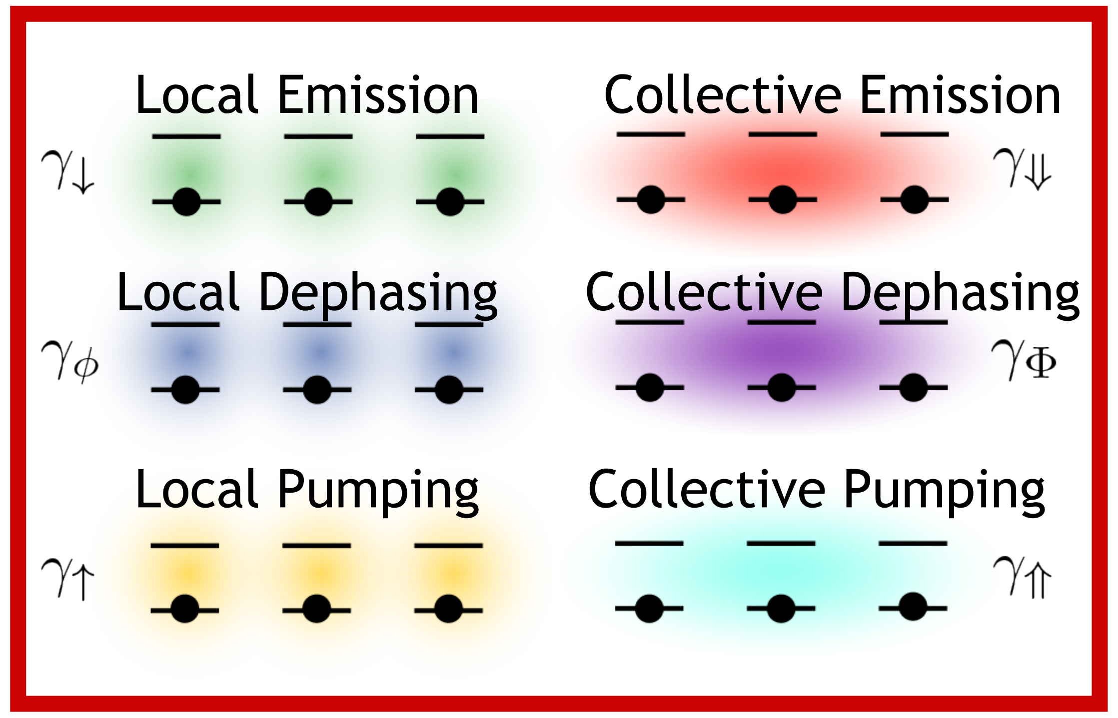
Local and collective processes are detrimental to negative-temperature effects

Hamiltonian
Of Each Ensemble

$$\boxed{H = \hbar\omega_0 J_z}$$

PIQS: Permutational-Invariant Quantum Solver

Dissipation
Of Each Ensemble



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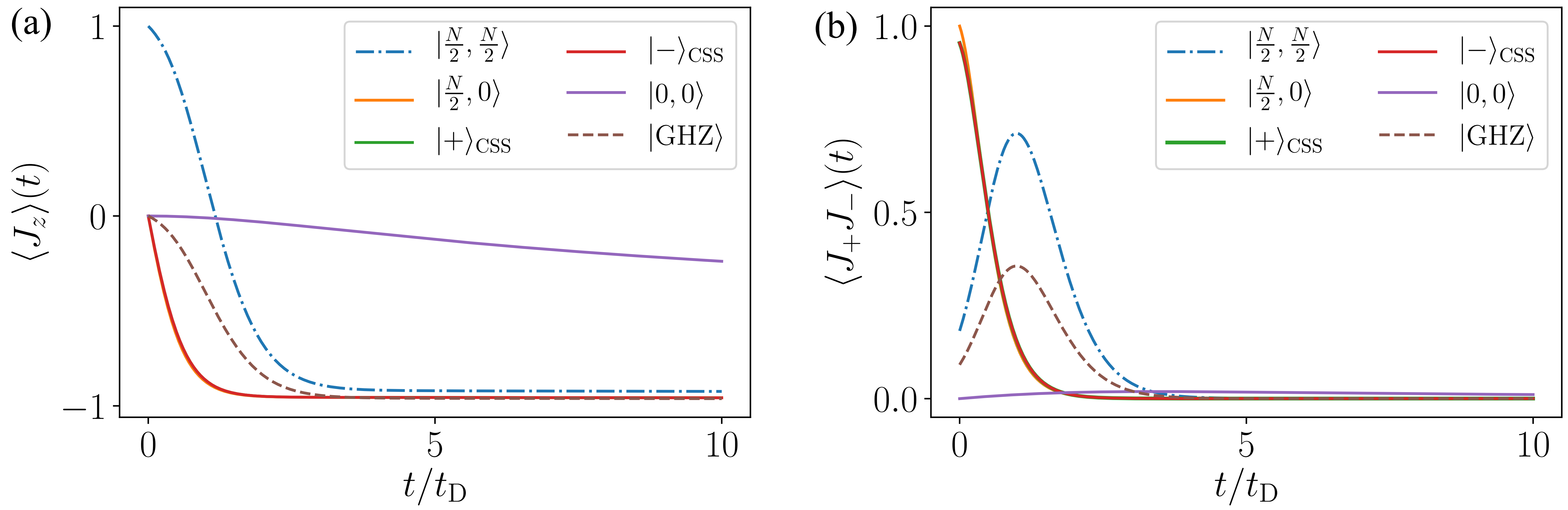
Population Inversion

Light Emission



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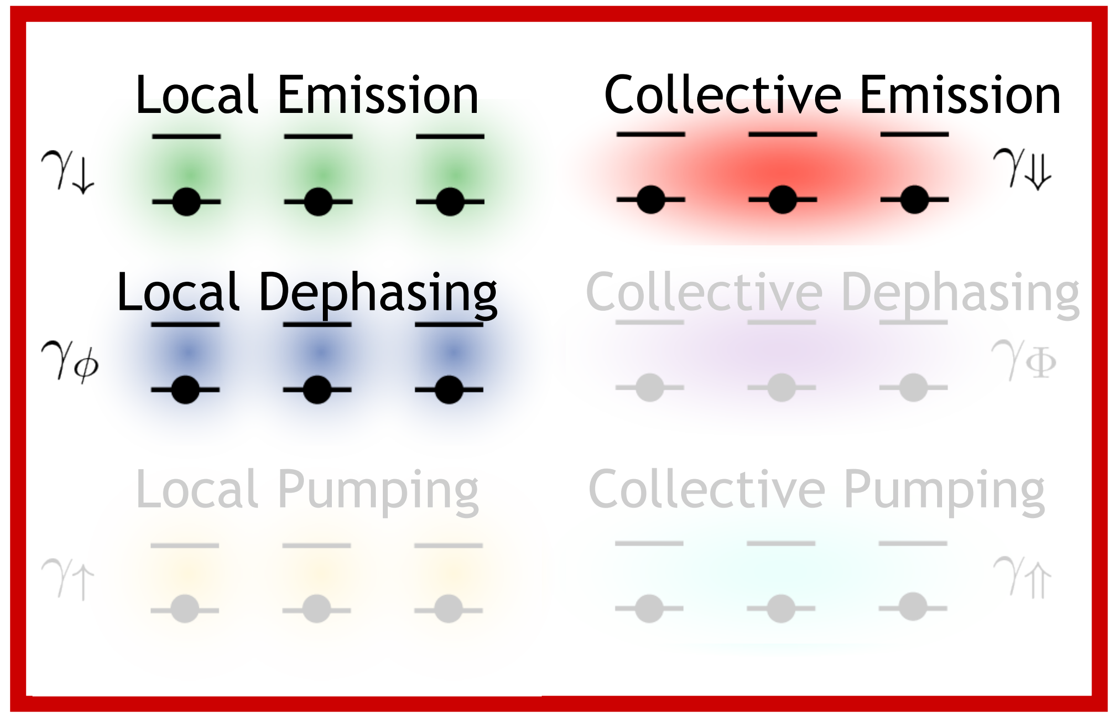

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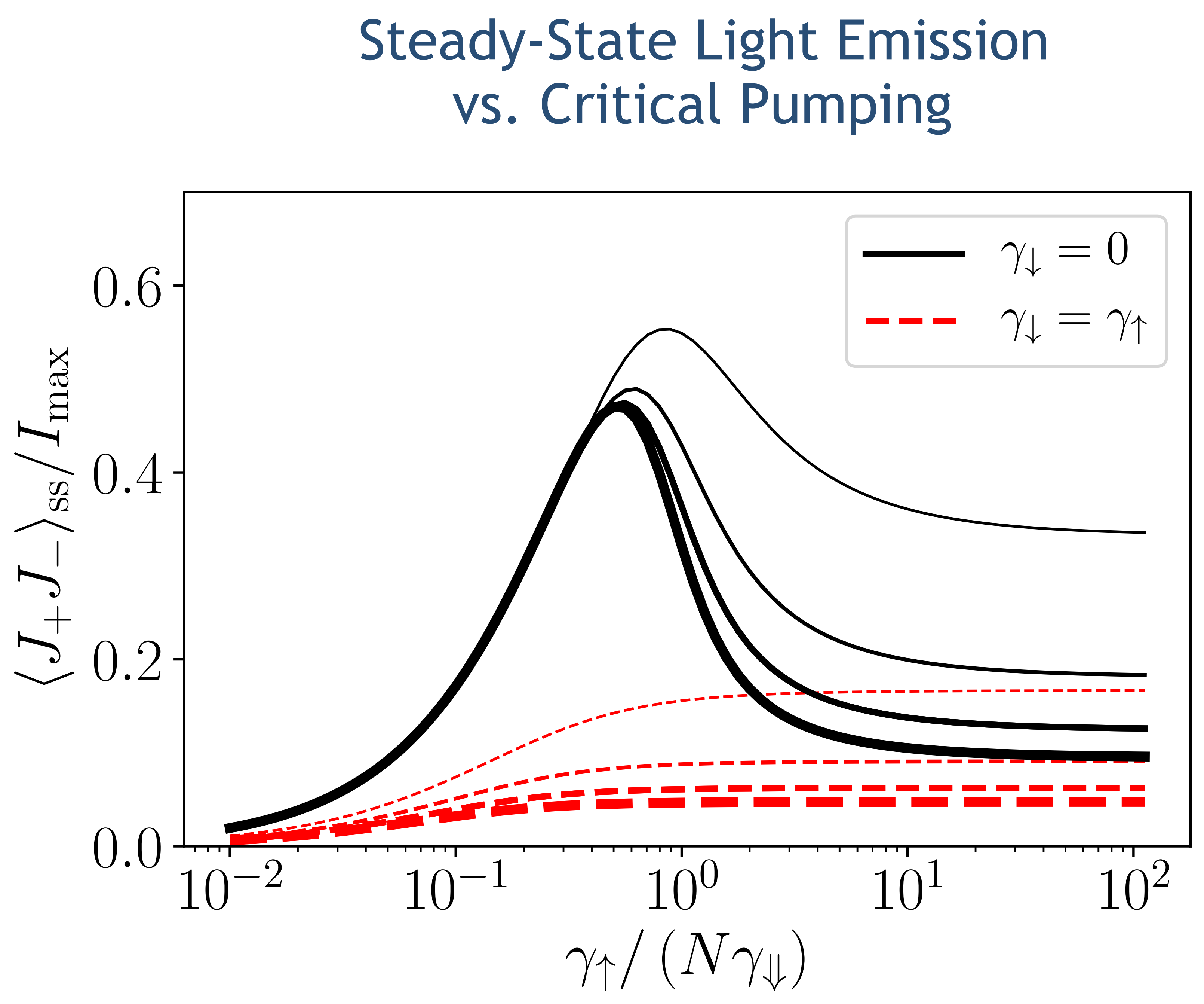

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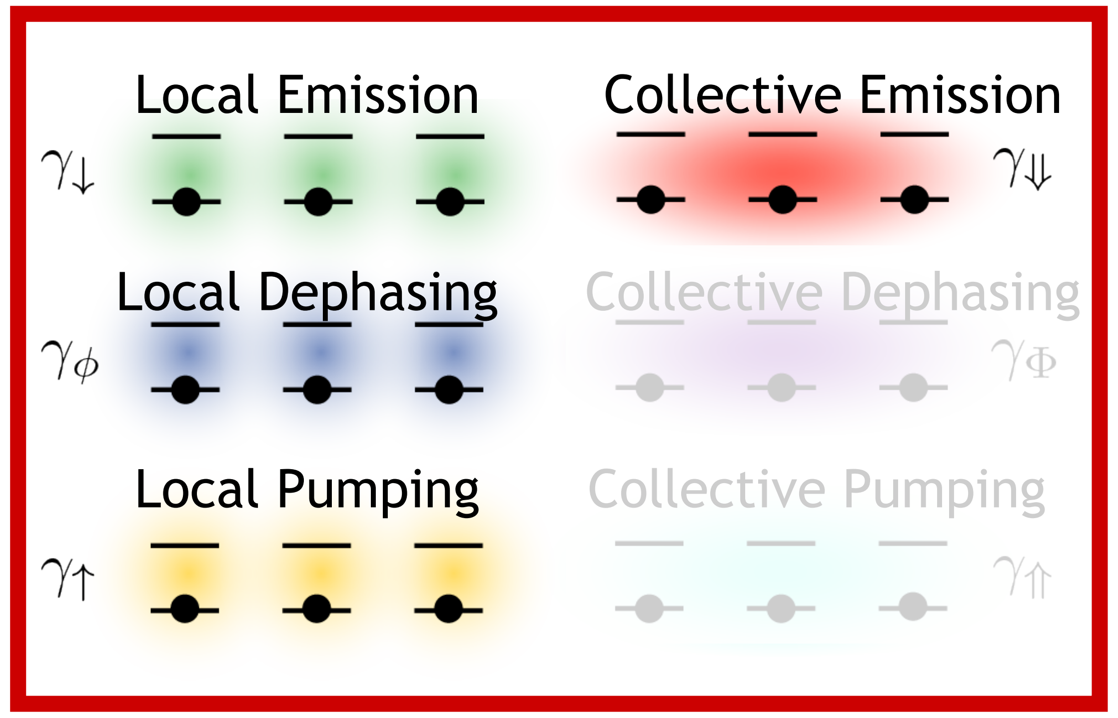

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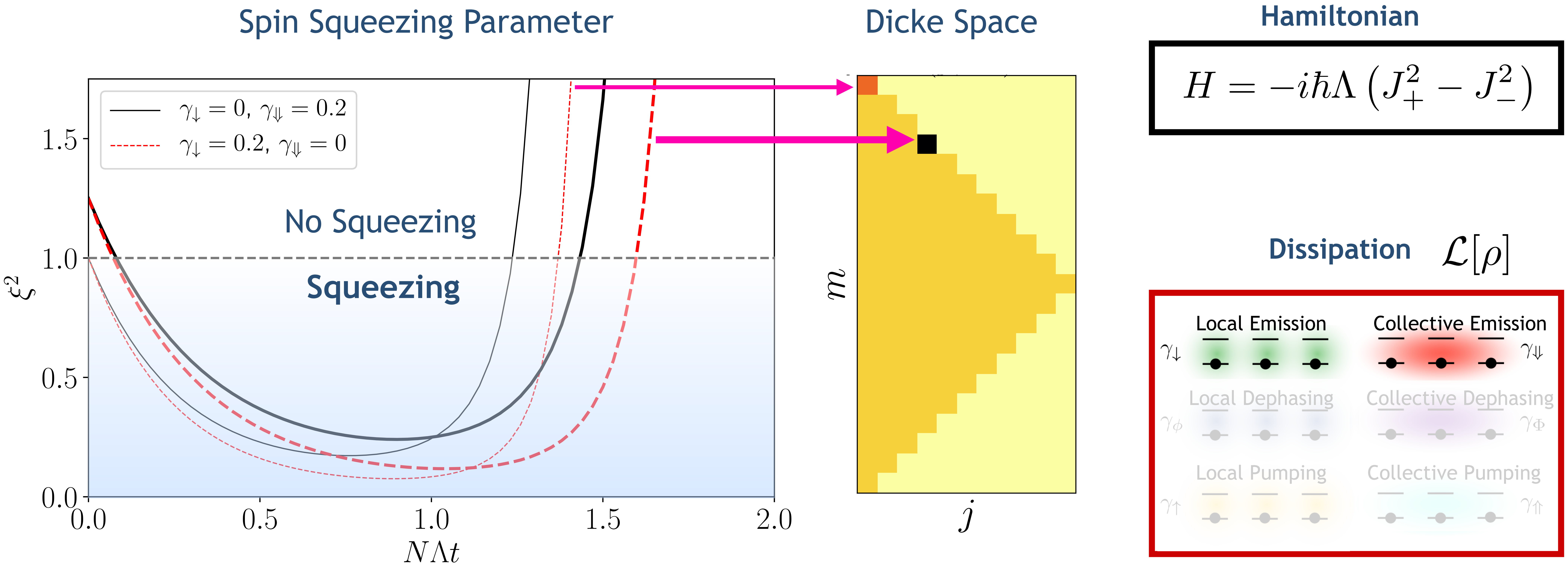

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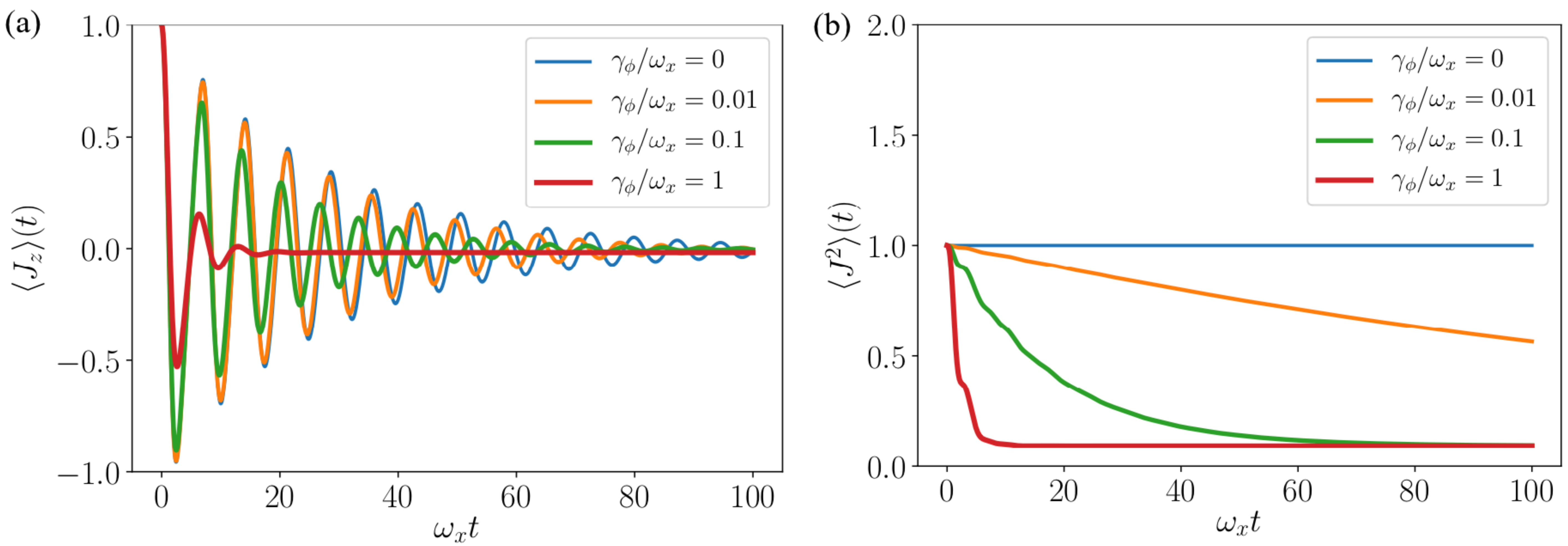
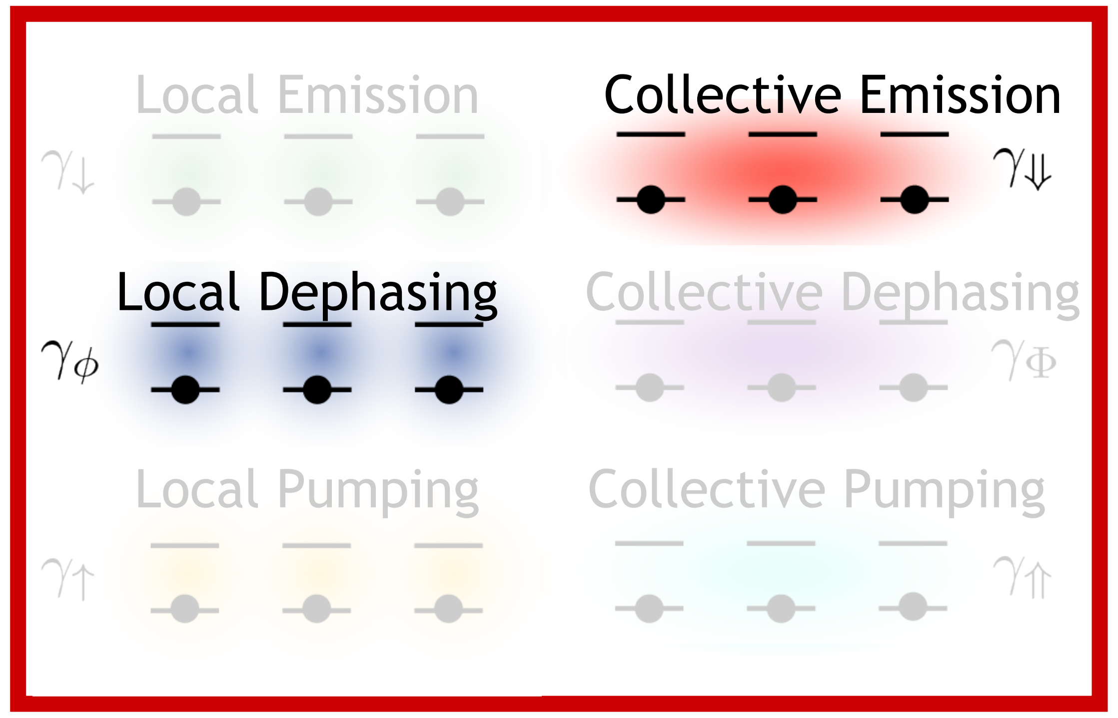


Hamiltonian

$$H = \hbar \omega_x J_x$$

Dissipation $\mathcal{L}[\rho]$



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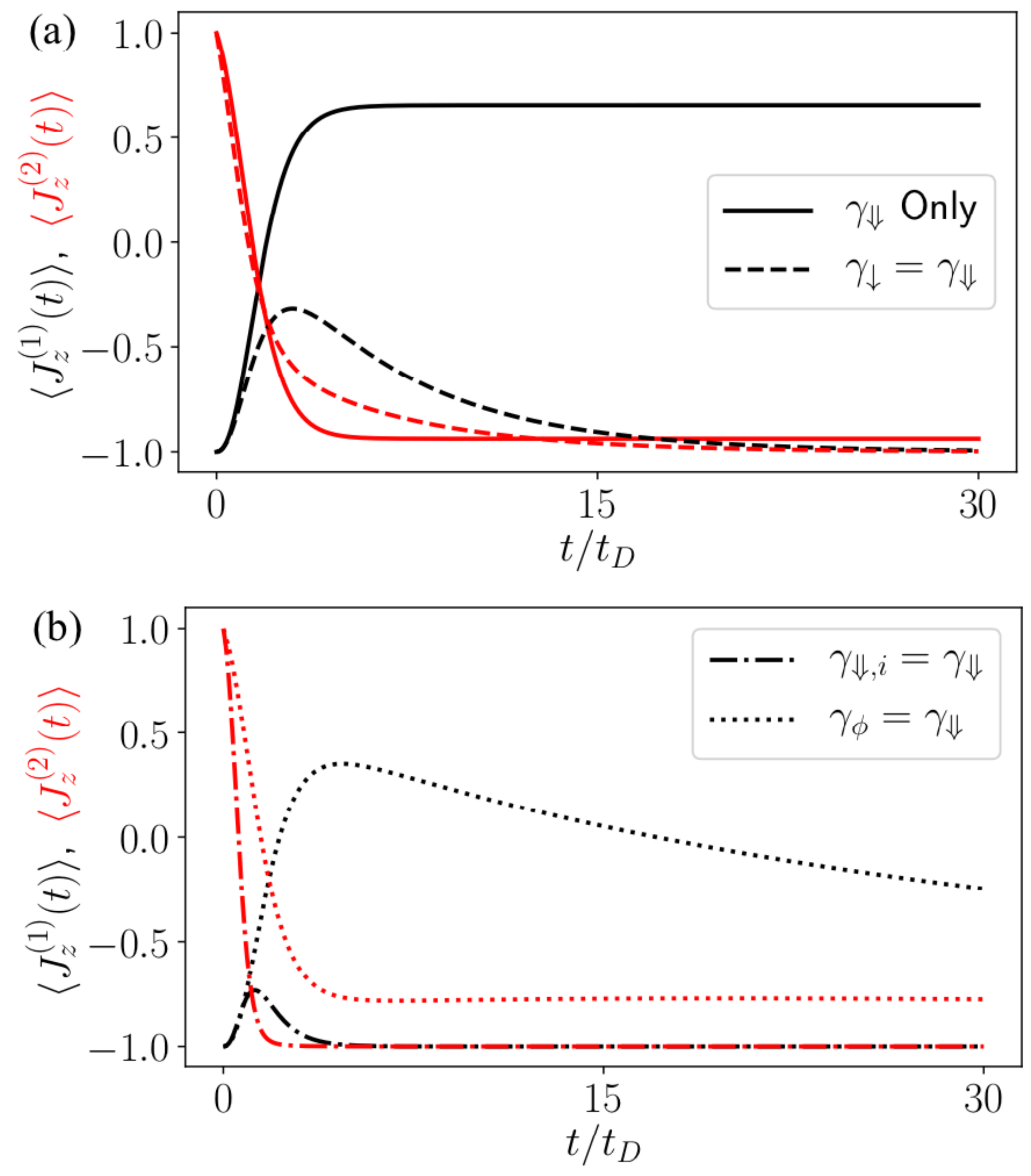
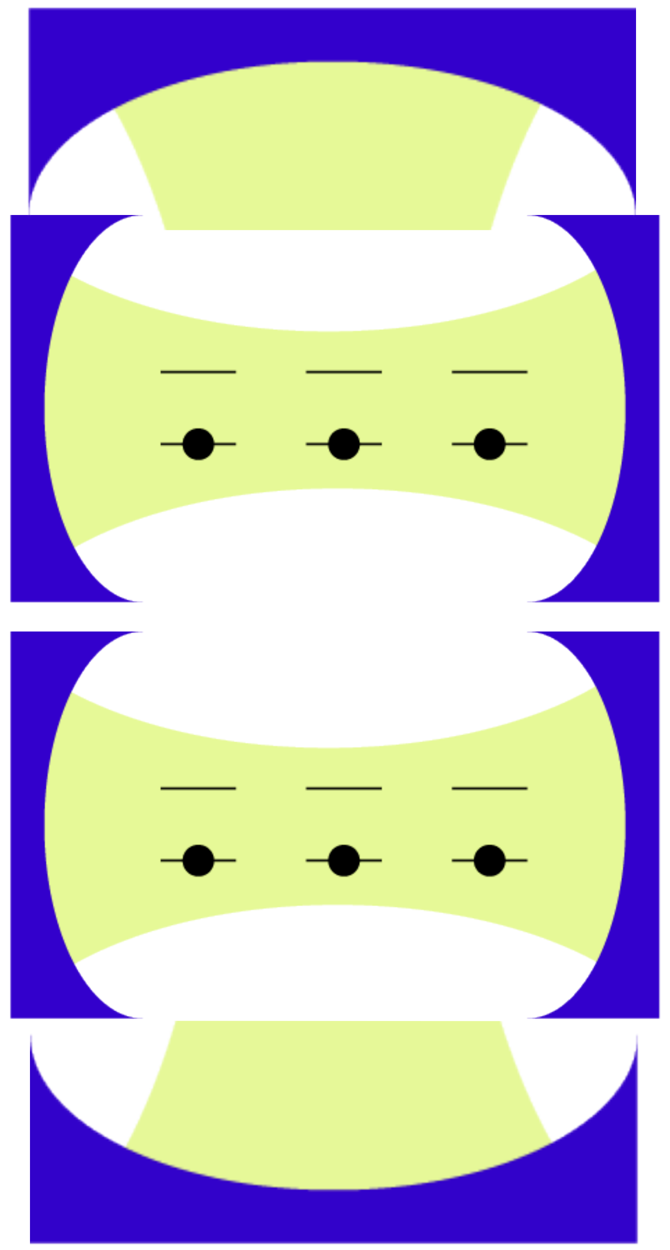
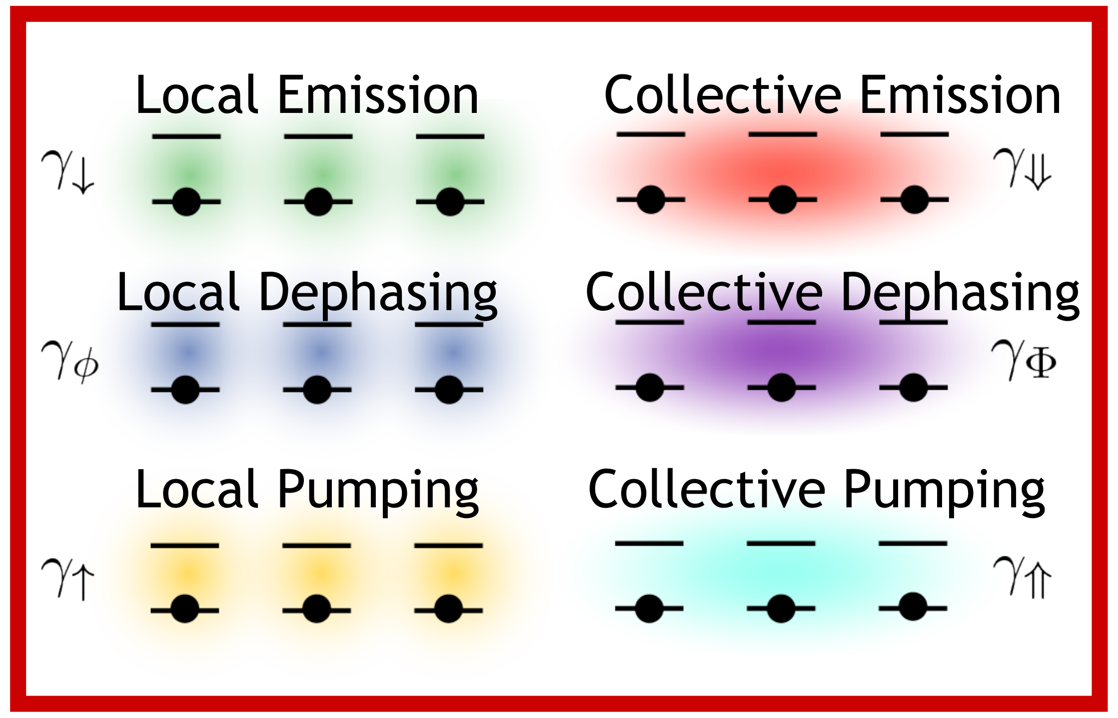




Hamiltonian
Of Each Ensemble

$$\boxed{H = \hbar\omega_0 J_z}$$

Dissipation
Of Each Ensemble



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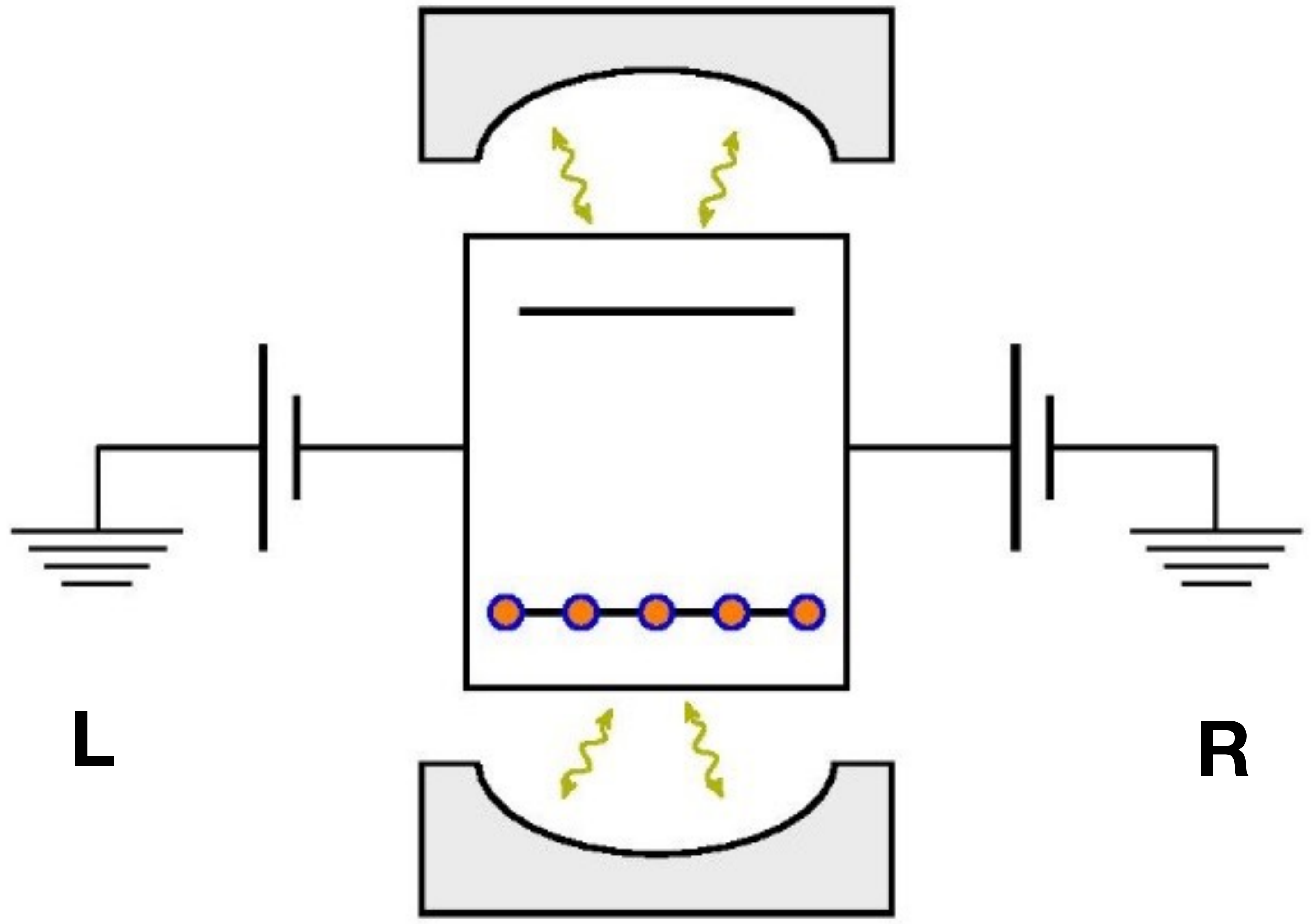
The Model: Many Body Fermionic Bands (I)

From a 2DEG to the Dicke Model



USC System

$$H = \omega_c a^\dagger a + \sum_n \left[\sum_i \omega_i c^\dagger_{i,n} c_{i,n} + \chi (a + a^\dagger)(c^\dagger_{2,n} c_{1,n} + c^\dagger_{1,n} c_{2,n}) \right]$$

Left (L) Reservoir

$$H^I_L = \lambda \sum_{n,\zeta} [(c_{1,n} + c_{2,n}) c^\dagger_{L;n,\zeta} + h\,.\,c.]$$

Photon out

Left (L) Reservoir

$$|G\rangle = | \quad \cdots \rangle + \frac{\eta}{\sqrt{N}} \sum_i | \quad \quad {}_i \cdots , \quad \rangle$$

The Model: Many Body Fermionic Bands (I)

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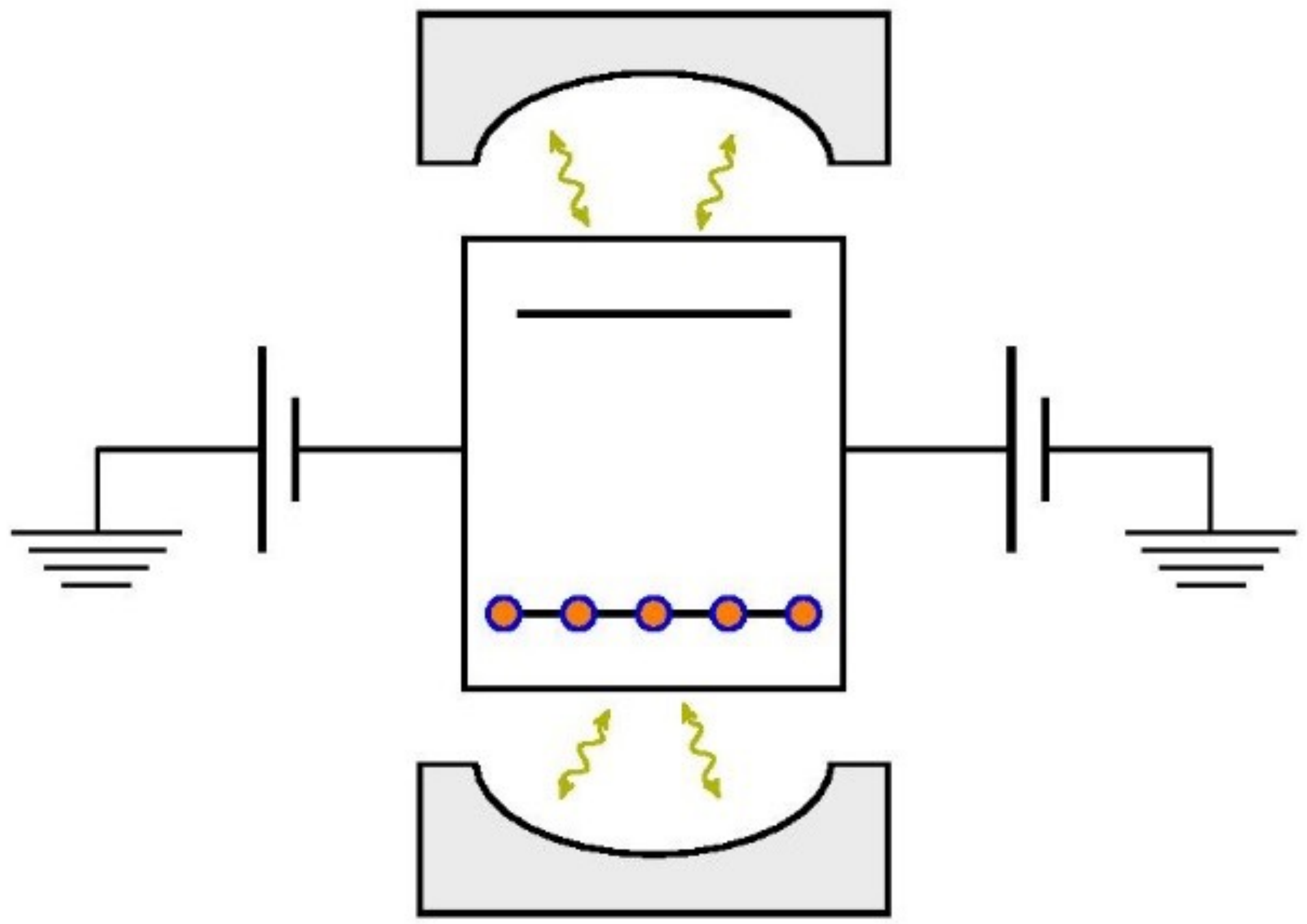


$$H = \omega_c a^\dagger a + \sum_n \left[\sum_i \omega_i c^\dagger_{i,n} c_{i,n} + \chi(a + a^\dagger)(c^\dagger_{2,n} c_{1,n} + c^\dagger_{1,n} c_{2,n}) \right]$$

$$S^\pm = \sum_n \frac{c^\dagger_{1,n} c_{2,n}}{2}$$

$$H = \omega_c a^\dagger a + \omega S^Z + \chi(a^\dagger S^- + a S^+) + \chi(a^\dagger S^+ + a S^-)$$

The Model: Collective Spins

From a 2DEG to the Dicke Model

$$H = \omega_c a^\dagger a + \omega S^Z + \chi(a^\dagger S^- + aS^+) + \chi(a^\dagger S^+ + aS^-)$$

Matter states:

$$|j, m\rangle$$

$$j = \frac{N}{2}, \frac{N}{2} - 1, \frac{N}{2} - 2, \cdots$$

$$m = -j, \cdots, +j$$

$$S^{\pm}|j, m\rangle = \sqrt{\frac{(j \mp m)(j \pm m + 1)}{2}}|j, m \pm 1\rangle$$

Effective collective enhancement of the light-matter coupling

$$\chi \mapsto \chi\sqrt{N}$$

The Model: Bosonic Matter

From a 2DEG to the Dicke Model to a bosonic model

$$H = \omega_c a^\dagger a + \omega S^Z + \chi(a^\dagger S^- + aS^+) + \chi(a^\dagger S^+ + aS^-)$$

Dicke Model

Holstein-Primakoff transformation $S^- \simeq \sqrt{N} b, \quad S^z = b^\dagger b - j_N$

$$H_{bos} = \omega_c a^\dagger a + \omega b^\dagger b + g_N (a^\dagger + a)(b + b^\dagger) \qquad g_N = \sqrt{N}\chi$$

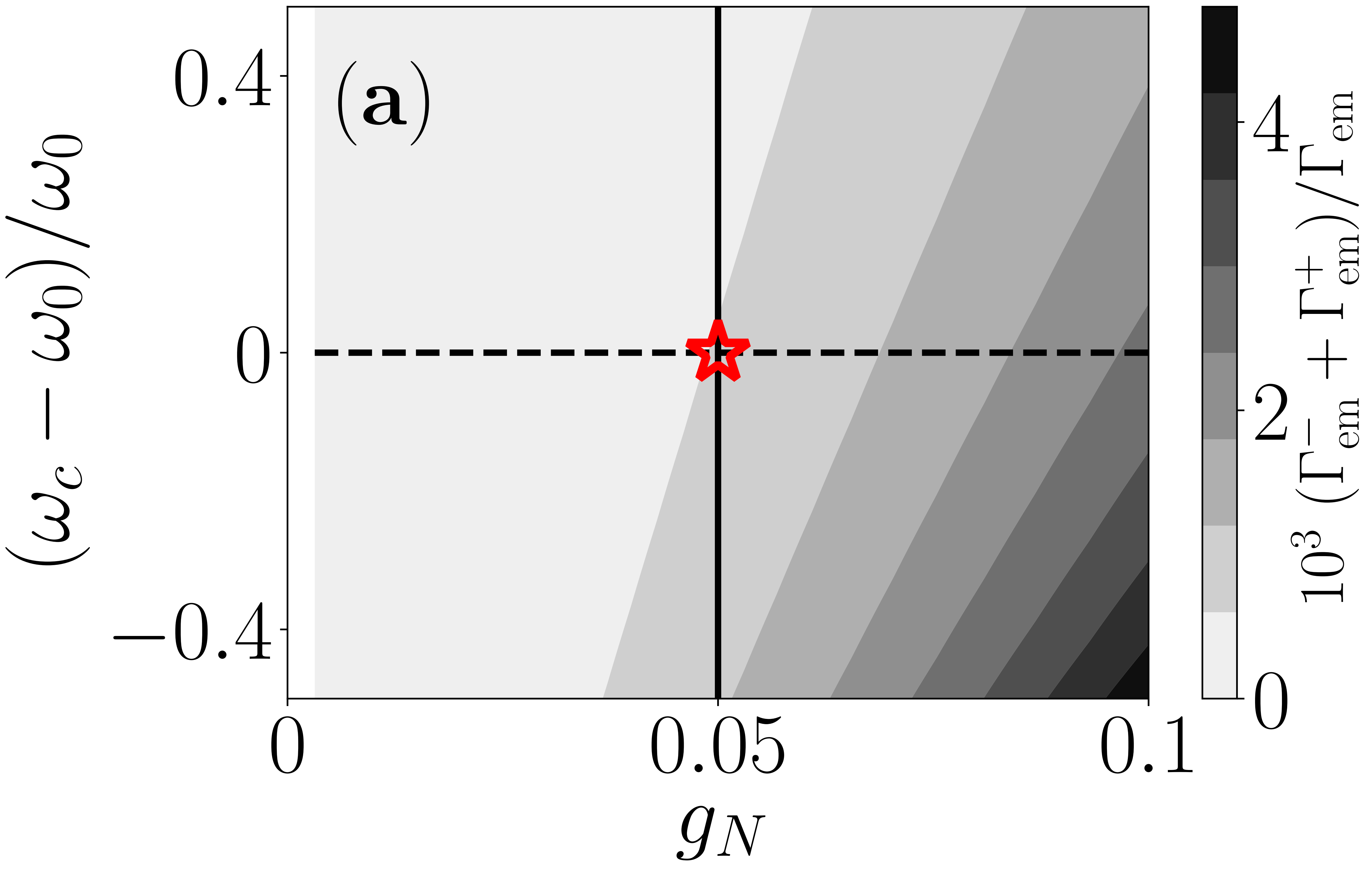
Bosonic Fields

Hopfield-Bugoliubov Transformation

Polariton Model $$H_{pol} = \omega_+ p_+^\dagger p_+ + \omega_- p_-^\dagger p_-$$

The Results: Ground State Electroluminescence

From a 2DEG to the Dicke Model to a bosonic model



$$g_N = \sqrt{N}\chi$$