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Supplement 1: Compliance of a Bubble

In this supplement I'll derive the approximation of the compliance of a bubble, $C_{bubble} \approx \mathcal{V}_0/p_0$, which I showed in class without proof. Note that I take this from Bruus. (Thanks Ian for the question!)

Assume the bubble has initial volume $\mathcal{V}_0$ at initial pressure p_0. Then by the ideal gas law we can write

$$p_0\mathcal{V}_0 = nRT$$

Assume an isothermal process $\mathrm{d}T = 0$ and assume also that n does not change. Then after the bubble has been pressurized to final pressure p and achieved final volume $\mathcal{V}$, we can write

$$p\mathcal{V} = nRT$$

And equating these two we have $p\mathcal{V} = p_0\mathcal{V}_0$ which we can rewrite to find the final volume

$$\mathcal{V} = \frac{p_0\mathcal{V}_0}{p}$$

We defined hydraulic compliance in class as

$$C = -\frac{\partial \mathcal{V}}{\partial p}$$

so we can differentiate our expression for $\mathcal{V}$ above to find

$$C = -\frac{\partial}{\partial p}\left(\frac{p_0\mathcal{V}_0}{p}\right) = -\left(\frac{-p_0\mathcal{V}_0}{p^2}\right) = \frac{p_0\mathcal{V}_0}{p^2}$$

Assuming small pressure changes, i.e. that $p \approx p_0$, we have

$$C \approx \frac{p_0\mathcal{V}_0}{p_0^2} = \frac{\mathcal{V}_0}{p_0}$$

which is what we wanted to show.