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CHEM 420: Selected Topics in Analytical Chemistry — Analytical Applications of Microfluidics Module
Due March 15, 2024

Assignment

Note: Where necessary, please use your favourite graphing software (e.g. MS Excel, Origin, others) to complete the following questions. Responses can be typed or handwritten as long as they are legible. *Total marks: 24*

Question 1. *3 marks* Beginning with the Navier-Stokes equation (ignoring body forces, shown below), non-dimensionalize the equation and show that a single dimensionless number, the Reynolds number, $\mathrm{Re} = \rho V_0 L_0/\eta$ can be obtained.

$$\rho(\partial_t \boldsymbol{v} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v}) = -\nabla p + \eta \nabla^2 \boldsymbol{v}$$

Question 2. *2 marks* What is the physical significance of the Reynolds number? Comment on possible values of Re and the implications for fluid flow.

Question 3. *3 marks* Recall that for a circular channel of radius a, we obtained a solution for the z-component of the velocity $v_z(x, y) = -\frac{\Delta p}{4\eta L}(a^2 - x^2 - y^2)$. Verify that this is a solution of the Stokes equation, $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) v_z(x, y) = -\frac{\Delta p}{\eta L}$. Prove that this solution complies with the non-slip boundary condition, i.e. that $v_z = 0$ for (x, y) on $\partial\mathcal{C}$. *Hint: The Pythagorean theorem is helpful.*

Question 4. *10 marks total* In the following dataset, a pressure Δp was applied across a microfluidic device and water was collected at the outlet for 300 s. The mass of water, m, was measured and is reported in the table for each pressure.

pressure Δp / mbar	**mass water, m / g**
50	0.0348
250	0.2327
500	0.484
750	0.7175
950	0.9061

1) *5 marks* Use the Hagen-Poiseuille Law, $\Delta p = QR$, and the following data set to determine the hydraulic resistance, R_{device}, of the microfluidic device. Include a graph and report your answer with units of $\mathrm{Pa\,s/m^3}$.
2) *2 marks* The hydraulic resistance of the tubing, R_{tubes}, was subsequently calculated to be about $1 \times 10^8\ \mathrm{Pa\,s/m^3}$. How does this affect the result?
3) *2 marks* This device has the following dimensions: channel width $w = 650\,\mu\mathrm{m}$ and height $h = 30\,\mu\mathrm{m}$. If we apply a pressure $\Delta p = 150\,\mathrm{mbar}$ to the channel, what is the volumetric flow rate Q and the average linear flow velocity V_0? Report your answer with units of $\mathrm{m^3\,s^{-1}}$ and $\mathrm{m\,s^{-1}}$, respectively.
4) *1 mark* Assuming the use of water at 20 °C, calculate the Reynolds number, Re, using the average linear flow velocity V_0 from Question 3.3. Is this flow turbulent or laminar? Note that the dynamic viscosity of water at 20 °C is $\eta = 1.0\,\mathrm{mPa\,s}$.

Question 5. *6 marks total* You are working for a start-up company developing a microfluidic filtering method for whole blood. As a good analytical chemist, you know that whole blood basically has the following three components:

- dissolved ions and solutes - $D = 2 \times 10^{-9}\ \mathrm{m^2\,s^{-1}}$
- proteins - $D = 6 \times 10^{-11}\ \mathrm{m^2\,s^{-1}}$
- red and white blood cells and platelets - $D = 2 \times 10^{-14}\ \mathrm{m^2\,s^{-1}}$

Your boss wants you to design an H-filter to capture the first two components, discarding the blood cells and platelets.

1) *3 marks* The engineering team says the device needs to run with average velocity $V_0 = 1$ mm/s, channel width $w = 100$ µm, and height $h = 20$ µm. What should the channel length L be in order to achieve complete mixing of only the dissolved ions, solutes, and proteins? What about for only the dissolved ions and solutes?
2) *2 mark* The engineering team says they can't fit anything longer than a 50 mm channel on the chip. Given that constraint, which component(s) of whole blood can be captured?
3) *2 marks* Your boss thinks you should be able to recover 100% of the desired analyte(s) in a single H-filter. Is this true? Why or why not? *Hint: What does the concentration profile look like when diffusion is complete?*