Midterm Notes

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1 Notecard for midterms & final

This document has all of the most important equations to include on your notecard EXCEPT for the stuff that was in notes from previous weeks (e.g. the Maxwell-Boltzmann speed distribution, the blackbody radiation equation, et cetera).

2 Partition function Z

$$Z := \sum_i g_i e^{-\beta E_i} = \int g e^{-\beta E} dE$$

$$E_{avg} = \frac{\sum_i E_i e^{-\beta E_i}}{\sum_i e^{-\beta E_i}} = \frac{\int E e^{-\beta E} dE}{\int e^{-\beta E} dE} = -\frac{\partial}{\partial \beta} \ln Z$$

3 Spherical coords

$$\begin{aligned} x &= r \sin\theta \cos\phi \\ y &= r \sin\theta \sin\phi \\ z &= r \cos\theta \\ dx\,dy\,dz &= r^2 \sin\theta\,dr\,d\theta\,d\phi \end{aligned}$$

4 Summation identities

The following Taylor series converges on $x \in (-1, 1)$.

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

Differentiate that and multiply by x to get this formula:

$$\sum_{n=0}^{\infty} n x^n = \frac{x}{(1-x)^2}$$

5 Binomial coefficients

$$\binom{n}{k} := \frac{n!}{k!(n-k)!}$$

Pascal's identity:

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$

Binomial theorem:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$

6 Energy-momentum equation

In general, I'll use m to denote the rest mass, and γm to mean the relativistic mass.

$$E^2 = m^2c^4 + p^2c^2$$

In that equation, E is the kinetic energy plus the rest energy. The momentum is $p = \gamma mv$, and the rest energy is mc^2, so the kinetic energy must be

$$\begin{aligned} K &= E - mc^2 \\ &= \sqrt{m^2c^4 + p^2c^2} - mc^2 \\ &= \sqrt{m^2c^4 + \frac{m^2v^2c^2}{1-\frac{v^2}{c^2}}} - mc^2 \\ &= mc\sqrt{\frac{(c^2-v^2)+v^2}{1-\frac{v^2}{c^2}}} - mc^2 \\ &= mc^2\left(\sqrt{\frac{1}{1-\frac{v^2}{c^2}}} - 1\right) \\ &= (\gamma - 1)mc^2 \end{aligned}$$

7 Maxwell's equations

Let $D := \varepsilon_0 E + P$ where P is the polarization field, and let $H := \frac{B}{\mu_0} - M$ where M is the magnetization field. Then the following version of Maxwell's equations work even if you aren't in a vacuum:

$$\begin{aligned} \nabla \cdot D &= \rho \\ \nabla \cdot B &= 0 \\ \nabla \times E &= -\frac{\partial B}{\partial t} \\ \nabla \times H &= J + \frac{\partial D}{\partial t} \end{aligned}$$

8 Constants

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\begin{aligned}
k_B &\approx 1.381 \times 10^{-23}\,\frac{\mathrm{J}}{\mathrm{K}} \approx 8.617 \times 10^{-5}\,\frac{\mathrm{eV}}{\mathrm{K}} \\
h &\approx 6.626 \times 10^{-34}\ \mathrm{J\ s} \approx 4.136 \times 10^{-15}\ \mathrm{eV\ s} \\
\hbar := \frac{h}{2\pi} &\approx 1.055 \times 10^{-34}\ \mathrm{J\ s} \approx 6.582 \times 10^{-16}\ \mathrm{eV\ s} \\
c &\approx 2.998 \times 10^{8}\,\frac{\mathrm{m}}{\mathrm{s}^2} \\
k := \frac{1}{4\pi\varepsilon_0} &\approx 8.988 \times 10^{9}\,\frac{\mathrm{N\ m}^2}{\mathrm{C}^2} \\
\varepsilon_0 &\approx 8.854 \times 10^{-12}\,\frac{\mathrm{F}}{\mathrm{m}} \\
\mu_0 &\approx 1.257 \times 10^{-6}\,\frac{\mathrm{N}}{\mathrm{A}^2} \\
1\ \mathrm{mol} &\approx 6.022 \times 10^{23}\ \text{molecules} \\
\alpha := \frac{e^2}{2\varepsilon_0 hc} &\approx 0.007297 \approx \frac{1}{137} \\
a_0 = \frac{\varepsilon_0 h^2}{\pi e^2 m_e} &\approx 5.292 \times 10^{-11}\ \mathrm{m} \\
G &\approx 6.674 \times 10^{-11}\,\frac{\mathrm{N\ m}^2}{\mathrm{kg}^2} \\
e &\approx 1.602 \times 10^{-19}\ \mathrm{C} \\
m_e &\approx 9.109 \times 10^{-31}\ \mathrm{kg} \\
m_p &\approx 1.673 \times 10^{-27}\ \mathrm{kg} \approx 1836\, m_e
\end{aligned}
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