Matter Waves

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1 Cool Integral Trick

Before starting the actual notes, here's a fun integral. It's particularly useful for solving all those problems where you average something over a probability distribution. It assumes $\mathrm{Re}(a) > 0$ and $n \in \mathbb{N}$.

$$\begin{aligned}\int_0^\infty x^n e^{-x/a} dx &= \int_0^\infty (a^n y^n) e^{-y} (a\, dy) \\ &= a^{n+1} \int_0^\infty y^n e^{-y} dy \\ &= a^{n+1} \Gamma(n+1) \\ &= n!\, a^{n+1}\end{aligned}$$

2 Another Unrelated Topic

We should memorize this definition of the fine-structure constant:

$$\alpha := \frac{e^2}{4\pi\varepsilon_0 \hbar c} = \frac{e^2}{2\varepsilon_0 hc} \approx \frac{1}{137}$$

It was also recommended that we remember the following approximations:

$$\begin{aligned}\hbar c &\approx 197 \text{ eV nm} \\ \frac{e^2}{4\pi\varepsilon_0} &\approx 1.44 \text{ eV nm}\end{aligned}$$

3 Dispersion Relations

This is pretty much all we need to know:

$$\begin{aligned}v_{\text{phase}} &= \omega/k \\ v_{\text{group}} &= \frac{\partial \omega}{\partial k}\end{aligned}$$

For all matter waves, the geometric mean of the phase velocity and the group velocity is c, the speed of light. For non-dispersive waves, such as light traveling through a vacuum, the phase and group velocities are both c.

4 Uncertainty Principle

5 Fourier Inversion Theorem