General Theorems for Quantum

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Contents

1 Eigenvalues & eigenvectors of Hermitian matrices

Prove eigenvalues are orthogonal, and discuss when there are repeated eigenvalues. Also, show that determinate states (of an observable) are the same as the eigenvectors (of the corresponding operator).

2 Energy degeneracy in one dimension

Suppose there are two wavefunctions in 1D with the same energy, then prove those wavefunctions are the same.

3 Sturm separation theorem

Use the absence of nodes in the ground state in 1D, along with the Sturm separation theorem, to inductively prove that the nth excited state has $n-1$ nodes.

4 Riesz-Fischer theorem

Prove that the set of L^2 functions has a countable basis.

5 Riesz representation theorem

6 Ehrenfest's theorem

7 Fourier inversion theorem

8 (General) uncertainty principle

9 Observables are conserved iff they commute with Hamiltonian

10 Proof that gaussian minimizes "total uncertainty"

11 Virial theorem (quantum version)

12 Hydrogen fine structure & spin-orbit coupling

13 Aharonov-Bohm effect

14 Conservation of probability current

15 Heisenberg picture

Show that this is equivalent to the Schrödinger picture.

16 The variational principle

17 The WKB approximation

18 Fermi's golden rule

19 The EPR paradox

20 Bell's theorem

21 $SU(2)$ is a double cover of $SO(3)$

22 Spin statistics theorem

Spin is always a multiple of 1/2. Exchange relation is symmetric for bosons and antisymmetric for fermions.