

# General Theorems for Quantum

Nathan Solomon

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## 1 Eigenvalues & eigenvectors of Hermitian matrices

Prove eigenvalues are orthogonal, and discuss when there are repeated eigenvalues. Also, show that determinate states (of an observable) are the same as the eigenvectors (of the corresponding operator).

## **2 Energy degeneracy in one dimension**

Suppose there are two wavefunctions in 1D with the same energy, then prove those wavefunctions are the same.

## **3 Riesz-Fischer theorem**

Prove that the set of  $L^2$  functions has a countable basis.

## **4 Ehrenfest's theorem**

## **5 Fourier inversion theorem**

## **6 (General) uncertainty principle**

## **7 Observables are conserved iff they commute with Hamiltonian**

## **8 Proof that gaussian minimizes “total uncertainty”**

## **9 Virial theorem (quantum version)**

## **10 Conservation of probability current**

## **11 Heisenberg picture**

Show that this is equivalent to the Schrödinger picture.

## **12 Spin statistics theorem**

Spin is always a multiple of  $1/2$ . Exchange relation is symmetric for bosons and antisymmetric for fermions.