Physics 180E Homework #1

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Problem 0.1.

(a) According to the ideal gas law,

$$\frac{N}{V} = \frac{P}{kT} = \frac{P}{(300\,\mathrm{K})\,(1.381 \times 10^{-23}\,\mathrm{J/K})} = \frac{P}{4.142 \times 10^{-21}\,\mathrm{J}}.$$

When the pressure is 1 Torr, that works out to

$$\frac{N}{V} = \frac{1\,\mathrm{Torr}}{4.142 \times 10^{-21}\,\mathrm{J}} \cdot \frac{133.32\,\mathrm{Pa}}{\mathrm{Torr}} = 3.219 \times 10^{22}\,\mathrm{m}^{-3}.$$

So at 1 Torr, there are 3.219×10^{22} atoms per meter cubed. At 100 mTorr, there are 3.219×10^{21} atoms per meter cubed. At 10 mTorr, there are 3.219×10^{20} atoms per meter cubed.

(b) The mean free path is $\lambda = 1/(N\sigma)$. At 1 Torr, that's

$$\lambda = \frac{1}{\left(3.219 \times 10^{22}\,\mathrm{m}^{-3}\right)\left(10^{-15}\,\mathrm{cm}^{-2}\right)} \cdot \frac{10^6\,\mathrm{cm}^3}{\mathrm{m}^3} = \frac{1\,\mathrm{cm}}{32.19} = 0.03106\,\mathrm{cm}.$$

Using that same formula, the mean free path will be 0.3106 cm at 100 mTorr and 3.106 cm at 10 mTorr.

(c) Assuming the argon is an ideal gas, mean speed per atom is

$$v = \sqrt{\frac{8kT}{\pi m_{Ar}}} = \sqrt{\frac{8PV}{\pi N m_{Ar}}}.$$

We calculated in part (a) that $PV/N = 4.142 \times 10^{-21}$ J, and the mass of an argon atom is 6.63×10^{-26} kg, so no matter what the pressure is,

$$v = \sqrt{\frac{8 \cdot (4.142 \times 10^{-21}\,\mathrm{m}^2/\mathrm{s}^2)}{\pi \cdot (6.63 \times 10^{-26})}} = 399\,\mathrm{m/s}.$$

The collision frequency is the mean speed divided by the mean free path, which works out to

$$\frac{399\,\mathrm{m/s}}{0.0311\,\mathrm{cm}} = 1.28 \times 10^6\,\mathrm{s}^{-1}$$

at 1 Torr. Therefore the collision frequency is 1.28×10^5 collisions per second at 100 mTorr and 1.28×10^4 collisions per second at 10 mTorr.

(d) My answers to parts (a) and (b) would not change, because for those, I did not use the fact that the gas was argon. But my answer for part (c) would change because collision frequency is proportional to mean velocity, which is inversely proportional to the square root of the atomic mass.

Of course, that's assuming the cross section doesn't change. In reality though, if pressure and temperature (which were both given) are held constant, helium should have a lower collisional cross section than argon. That means the mean free path would be larger, but the mean speed would also be larger, so I'm not sure whether the collision frequency would increase or decrease.

Problem 0.2.

(a) Since the collisional cross section is the same, and the number density of neutral argon atoms is the same as it was at 100 mTorr in question 1(b), the mean free path of the electron is also 0.3106 cm.

(b) The speed of the electron is

$$v_e = \sqrt{\frac{2 \cdot (100\,\mathrm{eV})}{m_e}} = 5.93 \times 10^6\,\mathrm{m/s},$$

so the collision frequency is

$$\frac{v_e}{\lambda} = \frac{5.93 \times 10^6\,\mathrm{m/s}}{0.003106\mathrm{m}} = 1.91 \times 10^9\,\mathrm{s}^{-1}.$$

Problem 0.3.

(a) The given formula makes no sense, but the answer is

$$\frac{1\,\mathrm{eV}}{k_B} = \frac{1\,\mathrm{eV}}{1.381\,\mathrm{J/K}} \cdot \frac{1\,\mathrm{J}}{6.242 \times 10^{18}\,\mathrm{eV}} = 11600\,\mathrm{K},$$

where k_B is the constant used to convert between temperatures and energies.

(b) Room temperature is 300 K, so the answer is

$$(300\,\mathrm{K})(k_B) = \frac{300\,\mathrm{K}}{11600\,\mathrm{K/eV}} = 0.0259\,\mathrm{eV}.$$

(c) $1\,\mathrm{keV} = 1000 \cdot (11600\,\mathrm{K}) = 1.16 \times 10^7\,\mathrm{K}$.

(d) $50\,\mathrm{keV} = 50000 \cdot (11600\,\mathrm{K}) = 5.80 \times 10^8\,\mathrm{K}$.

(e) "Thermal velocity" can have a few different meanings, so I'll guess it means the RMS speed. The electrons form an ideal non-relativistic gas, so the kinetic energy per electron is $\frac{1}{2} m_e v_{RMS}^2 = \frac{3}{2} k_B T$ (where T is temperature), which means the RMS kinetic energy is

$$v_{RMS} = \sqrt{\frac{3k_BT}{m_e}} = \sqrt{\frac{3\,\mathrm{eV}}{m_e}} = \sqrt{\frac{3\,\mathrm{eV}}{0.511\,\mathrm{MeV\,c^{-2}}}} = 0.00587c = 1.76 \times 10^6\,\mathrm{m/s}.$$

Problem 0.4.

Atom type	First ionization potential
He	24.6 eV
Ne	21.6 eV
Ar	15.8 eV
N	14.6 eV

PHYSICS 180E, WINTER 2025

HOMEWORK 1
(DUE WEDNESDAY JAN. 17 BY MIDNIGHT ON GRADESCOPE)

1. We will be creating plasmas out of low-density gases in a vacuum chamber. Answer the following for room-temperature Argon gas at three pressures: 10 mTorr, 100 mTorr, and 1 Torr.

   (a) What is the number density (atoms per unit volume)?

   (b) What is the mean-free-path for collisions between neutral atoms? You can assume that the cross-section is $\sigma \sim 10^{-15}$ cm^{-2}.

   (c) What is the collision frequency?

   (d) If instead the fill gas was Helium, would any of your answers change? If so, why?

2. Now consider an electron having 100 eV of kinetic energy moving through a 100 mTorr room-temperature Argon gas.

   (a) Assuming the electron-neutral collision cross section is also $\sim 10^{-15}$ cm^{-2}, what is the mean-free-path of electron in the gas?

   (b) What is the collision frequency?

3. Plasma physicists typically quote temperature in electron volts rather than Kelvin. When electron volts (an energy unit) are used in this way, the quantity being quoted is:

$$T_{eV} = \frac{k_B T}{e}$$

   where T_{eV} is the "temperature" in electron volts, k_B is Boltzmann?s constant, and e is the elementary charge.

   (a) What is the temperature in Kelvin associated with 1 eV?

   (b) What is room temperature stated in electron volts?

   (c) What is the temperature in Kelvin at the center of the sun ($T_{eV} \sim 1$ keV)?

   (d) What is the temperature in Kelvin in the hottest tokamak plasmas on record ($T_{eV} \sim 50$ keV)?

   (e) What is the thermal velocity associated with a Maxwellian distribution of electrons with a temperature of 1 eV?

4. Look up and write down the first ionization potentials for the following gases: Helium, Neon, Argon, and Nitrogen. We will be making plasmas out of these gases using electron-impact ionization.