MATH 131B Homework #9

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Problem 0.1. Exercise 5.3.3: Prove corollary 5.3.6.

If $-N \leq n \leq N$, then by the linearity (in the first argument) of the inner product,

$$\langle f, e_n \rangle = \sum_{m=-N}^{N} c_m \langle e_m, e_n \rangle .$$

Lemma 5.3.5 tells us that

$$\langle e_m, e_n \rangle = \delta_{m,n} := \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases},$$

so all of the terms except the one where $m = n$ are zero, and we are left with

$$\langle f, e_n \rangle = c_n.$$

If $n < -N$ or $n > N$, then

$$\langle f, e_n \rangle = \sum_{m=-N}^{N} c_m \langle e_m, e_n \rangle = 0,$$

because m will never be equal to n.

Lastly, we have the identity

$$\begin{aligned} \|f\|^2 &= \langle f, f \rangle \\ &= \left\langle \sum_{n=-N}^{N} c_n e_n, \sum_{m=-N}^{N} c_m e_m \right\rangle \\ &= \sum_{n=-N}^{N} c_n \sum_{m=-N}^{N} \overline{c_m} \langle e_n, e_m \rangle \\ &= \sum_{n=-N}^{N} c_n \sum_{m=-N}^{N} \overline{c_m} \delta_{n,m} \\ &= \sum_{n=-N}^{N} \|c_n\|^2 . \end{aligned}$$

Problem 0.2. Exercise 5.4.2

Problem 0.3. Exercise 5.5.2

Problem 0.4. Exercise 5.5.4

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(1) Exercise: 5.3.3, 5.4.2, 5.5.2, 5.5.4.