Physics 245 Homework #6

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Problem 0.1.

(a) The given electric field is

$$E = E_0 \exp\left(-\frac{x^2+y^2}{w_0^2}\right) \exp\left(i(kz - \omega t + \phi)\right) \hat{x},$$

the Rabi frequency is

$$\Omega = \frac{d \cdot E}{\hbar},$$

and our formula for the Stark shift is

$$E_{Stark} = -\frac{\hbar \left|\Omega\right|^2}{4\delta}.$$

Therefore the Stark shift on the atom is

$$E_{Stark} = -\frac{d^2 \left|E\right|^2}{4\delta\hbar} = -\frac{d^2 E_0^2}{4\delta\hbar} \exp\left(-2 \cdot \frac{x^2+y^2}{w_0^2}\right).$$

(b) Let $r = \sqrt{x^2+y^2}$, so r is a unitless quantity representing the distance away from the center of the atom. The change in potential is equal to the Stark shift, and to model the new potential with a harmonic oscillator, we need to approximate E_{Stark} to second order in r:

$$E_{Stark} = -\frac{d^2 E_0^2}{4\delta\hbar} e^{-2r^2/w_0^2} \approx -\frac{d^2 E_0^2}{4\delta\hbar}\left(1 - \frac{2r^2}{w_0^2}\right).$$

The coefficient of r^2 in that expression is

$$\frac{V(r)}{r^2} = \frac{m\omega^2}{2} = \frac{d^2 E_0^2}{2\delta\hbar w_0^2}.$$

Now we can solve for the natural frequency:

$$\omega = \frac{dE_0}{w_0\sqrt{m\hbar\delta}}.$$

(c) Using the relations

$$I = \frac{1}{2} c\varepsilon_0 \left|E\right|^2 = c\varepsilon_0 E_0^2 \exp\left(-2 \cdot \frac{r^2}{w_0^2}\right)$$

for the intensity of the laser and

$$\begin{aligned}
P &= \iint I \mathrm{d}A \\
&= c\varepsilon_0 E_0^2 \int_{r=0}^{\infty} \int_{\theta=0}^{2\pi} \exp\left(-2 \cdot \frac{r^2}{w_0^2}\right) (r \mathrm{d}\theta \mathrm{d}r) \\
&= 2\pi c\varepsilon_0 E_0^2 \int_{r=0}^{\infty} r \cdot \exp\left(-2 \cdot \frac{r^2}{w_0^2}\right) \mathrm{d}r \\
&= 2\pi c\varepsilon_0 E_0^2 \cdot \frac{w_0^2}{4} \\
&= \frac{\pi}{2} c\varepsilon_0 E_0^2 w_0^2
\end{aligned}$$

for the power of the laser, the (angular) frequency is

$$\begin{aligned}
\omega &= \frac{d}{w_0\sqrt{m\hbar\delta}} \cdot E_0 \\
&= \frac{d}{w_0\sqrt{m\hbar\delta}} \cdot P\sqrt{\frac{2}{\pi c\varepsilon_0 w_0^2}} \\
&= \frac{d}{w_0^2} \cdot \sqrt{\frac{2P}{\pi c\varepsilon_0 m\hbar\delta}} \\
&= \frac{ea_0}{(10\mu m)^2} \cdot \sqrt{\frac{2P}{\pi c\varepsilon_0 m\hbar(10THz)}} \\
&= \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2} \cdot \frac{e}{(10\mu m)^2} \cdot \sqrt{\frac{2(100mW)}{\pi c\varepsilon_0 m_e \hbar(10THz)}} \\
&= \frac{4\sqrt{2\pi\varepsilon_0\hbar^3}}{e\sqrt{m_e^3 c}} \cdot \frac{\sqrt{100mW}}{(10^{-5}m)^2\sqrt{10^{13}s^{-1}}} \\
&= \frac{4P\sqrt{2\pi\varepsilon_0\hbar^3}}{e\sqrt{m_e^3 c}} \cdot \frac{\sqrt{100mW}}{(10^{-5}m)^2\sqrt{10^{13}s^{-1}}} \\
&= 13.3962824 \text{ megahertz.}
\end{aligned}$$

(d) ω is an angular frequency, so I can multiply it by $\hbar$ to get an energy, then divide that to get a temperature (in Kelvin).

$$\frac{\hbar\omega}{k} = 0.00010232392K \approx 0.1mK.$$

Problem 0.2.

(a) Using the notation from chapter 7 in Griffiths,

$$H = H^0 + \lambda H',$$

where $H' = x^4$. Then the first order correction to ψ_n is

$$\psi_n^1 := \sum_{m \neq n} \frac{\langle \psi_m^0 | \lambda H' | \psi_n^0 \rangle}{E_n^0 - E_m^0} \psi_m^0.$$

See the jupyter notebook for the actual calculations of those corrections.

(b) The second order energy corrections are

$$E_n^2 = \sum_{m \neq n} \frac{\left|\left\langle \psi_m^0 \right| \lambda H' \left| \psi_n^0 \right\rangle\right|^2}{E_n^0 - E_m^0}.$$

Once again, see the jupyter notebook at the end of this doc for the actual calculations.

(c) See jupyter notebook for calculations. I experimented with changing the value of n_{max} and found that increasing n_{max} makes all of my predicted energies more accurate. By "predicted energies", I mean the unperturbed energies plus the first and second order corrections.

Problem 0.3.

For parts (a) through (c), see the jupyter notebook.
For part (d):

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} \otimes \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} \otimes \begin{bmatrix} a_3 \\ b_3 \end{bmatrix} = \begin{bmatrix} a_1 a_2 a_3 \\ a_1 a_2 b_3 \\ a_1 b_2 a_3 \\ a_1 b_2 b_3 \\ b_1 a_2 a_3 \\ b_1 a_2 b_3 \\ b_1 b_2 a_3 \\ b_1 b_2 b_3 \end{bmatrix}.$$

For part (e):
The total state vector is

$$\frac{1}{\sqrt{3}} \begin{bmatrix} a_1 \\ a_1 \\ 0 \\ a_1 \\ b_1 \\ b_1 \\ 0 \\ b_1 \end{bmatrix}.$$

The expectation value of σ_x on the qubit is

$$\begin{aligned}
(|\psi\rangle \otimes |\phi\rangle)^\dagger (\sigma_x \otimes I)(|\psi\rangle \otimes |\phi\rangle) &= (|\psi\rangle \otimes |\phi\rangle)^\dagger (\sigma_x |\psi\rangle \otimes I |\phi\rangle) \\
&= (\langle\psi| \otimes \langle\phi|)(\sigma_x |\psi\rangle \otimes I |\phi\rangle) \\
&= \langle\psi| \sigma_x |\psi\rangle \cdot \langle\phi| I |\phi\rangle \\
&= \langle\psi| \sigma_x |\psi\rangle \\
&= a_1^2 - b_1^2.
\end{aligned}$$

Using that same trick with the tensor products, the expectation of N on the harmonic oscillator, which is the expectation value $\langle I \otimes N\rangle = (|\psi\rangle \otimes |\phi\rangle)^\dagger (I \otimes N)(|\psi\rangle \otimes |\phi\rangle)$, can be rewritten as $\langle\phi| N |\phi\rangle$:

$$\begin{aligned}
\langle I \otimes N\rangle &= \langle\phi| N |\phi\rangle \\
&= \left(\frac{1}{\sqrt{3}}\right)^2 (0 + 1 + 3) \\
&= \frac{4}{3}.
\end{aligned}$$

In the composite statevector, the coefficient of $|1\rangle \otimes |3\rangle$ is $b_1/\sqrt{3}$, so the probabilitiy of finding the qubit in state $|1\rangle$ and the harmonic oscillator in state $|3\rangle$ is $|b_1|^2/3$.

Problem 0.4.

See the jupyter notebook.

notebook

November 19, 2024

[1]:
```
import qutip as qt
import matplotlib.pyplot as plt
import numpy as np
```

[2]:
```
# Problem 2
N = 10
H = qt.num(N) + qt.qeye(N) / 2
lambd = .01

def state_correction(n):
    total = 0 * qt.basis(N)
    psi_n = qt.basis(N, n)
    for m in range(N):
        if m == n:
            continue
        psi_m = qt.basis(N, m)
        total += psi_m * (psi_m.dag() * lambd * qt.position(N)**4 * psi_n / (n␣
 ↪- m))
    return total

print(f"First order correction to state 0:\n{state_correction(0)}")
print(f"\nFirst order correction to state 1:\n{state_correction(1)}\n")

def first_energy_correction(n):
    return qt.expect(lambd * qt.position(N)**4, qt.basis(N, n))

def second_energy_correction(n):
    total = 0
    psi_n = qt.basis(N, n)
    for m in range(N):
        if m == n:
            continue
        psi_m = qt.basis(N, m)
        total += (psi_m.dag() * lambd * qt.position(N)**4 * psi_n)**2 / (n - m)
    # imaginary component will only exist because of rounding errors
    return np.real(total)
```

```
for i in range(N):
    print(f"First order correction to energy of state {i}:␣
 ↪{first_energy_correction(i):.5f}")
print('\n')
for i in range(N):
    print(f"Second order correction to energy of state {i}:␣
 ↪{second_energy_correction(i):.5f}")
print("\nPredicted energies, based on unperturbed energy" \
      " plus first and second order corrections:")
for i in range(N):
    predicted_energy = i + 0.5 + \
        first_energy_correction(i) + \
        second_energy_correction(i)
    print(f"Predicted energy of state {i}: {predicted_energy:.5f}")

# Make new Hamiltonian
H += lambd * qt.position(N)**4
actual_energies = H.eigenenergies()
print('\n')
for i in range(N):
    print(f"Actual energy of state {i}: {actual_energies[i]:.5f}" +
          f" = {2*i+1}/2 + {actual_energies[i] - (i+1/2):.5f}")
```

```
First order correction to state 0:
Quantum object: dims=[[10], [1]], shape=(10, 1), type='ket', dtype=Dense
Qobj data =
[[ 0.        ]
 [ 0.        ]
 [-0.0106066 ]
 [ 0.        ]
 [-0.00306186]
 [ 0.        ]
 [ 0.        ]
 [ 0.        ]
 [ 0.        ]
 [ 0.        ]]

First order correction to state 1:
Quantum object: dims=[[10], [1]], shape=(10, 1), type='ket', dtype=Dense
Qobj data =
[[ 0.        ]
 [ 0.        ]
 [ 0.        ]
 [-0.03061862]
 [ 0.        ]
 [-0.00684653]
 [ 0.        ]
```

```
 [ 0.        ]
 [ 0.        ]
 [ 0.        ]]

First order correction to energy of state 0: 0.00750
First order correction to energy of state 1: 0.03750
First order correction to energy of state 2: 0.09750
First order correction to energy of state 3: 0.18750
First order correction to energy of state 4: 0.30750
First order correction to energy of state 5: 0.45750
First order correction to energy of state 6: 0.63750
First order correction to energy of state 7: 0.84750
First order correction to energy of state 8: 0.86250
First order correction to energy of state 9: 0.38250


Second order correction to energy of state 0: -0.00026
Second order correction to energy of state 1: -0.00206
Second order correction to energy of state 2: -0.00769
Second order correction to energy of state 3: -0.01969
Second order correction to energy of state 4: -0.04061
Second order correction to energy of state 5: -0.07301
Second order correction to energy of state 6: -0.11156
Second order correction to energy of state 7: -0.03956
Second order correction to energy of state 8: 0.16013
Second order correction to energy of state 9: 0.13433

Predicted energies, based on unperturbed energy plus first and second order
corrections:
Predicted energy of state 0: 0.50724
Predicted energy of state 1: 1.53544
Predicted energy of state 2: 2.58981
Predicted energy of state 3: 3.66781
Predicted energy of state 4: 4.76689
Predicted energy of state 5: 5.88449
Predicted energy of state 6: 7.02594
Predicted energy of state 7: 8.30794
Predicted energy of state 8: 9.52263
Predicted energy of state 9: 10.01683


Actual energy of state 0: 0.50726 = 1/2 + 0.00726
Actual energy of state 1: 1.53565 = 3/2 + 0.03565
Actual energy of state 2: 2.59085 = 5/2 + 0.09085
Actual energy of state 3: 3.67110 = 7/2 + 0.17110
Actual energy of state 4: 4.77501 = 9/2 + 0.27501
Actual energy of state 5: 5.90070 = 11/2 + 0.40070
Actual energy of state 6: 7.03714 = 13/2 + 0.53714
```

```
Actual energy of state 7: 8.25292 = 15/2 + 0.75292
Actual energy of state 8: 9.50224 = 17/2 + 1.00224
Actual energy of state 9: 10.05213 = 19/2 + 0.55213
```

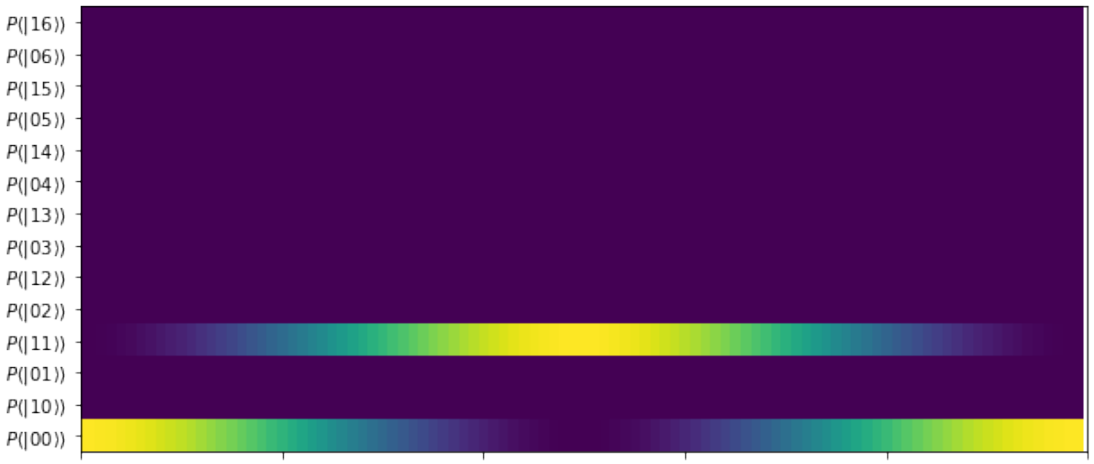
[4]:
```python
# Problem 3

# Part (a)
Psi_1 = (qt.ket("0") + qt.ket("1")).unit()
Psi_2 = qt.ket("0")
Psi = qt.tensor(Psi_1, Psi_2)
print("Part (a)")
print(f"Total state vector:\n{Psi}")
expectation_value = qt.expect(qt.tensor(qt.sigmaz(), qt.qeye(2)), Psi)
print(f"Expectation value of sigma_z for qubit 1: {expectation_value}")
expectation_value = qt.expect(qt.tensor(qt.qeye(2), qt.sigmaz()), Psi)
print(f"Expectation value of sigma_z for qubit 2: {expectation_value}")
expectation_value = qt.expect(qt.tensor(qt.sigmaz(), qt.sigmaz()), Psi)
print(f"Expectation value of sigma_z tensor sigma_z: {expectation_value}")

# Part (b)
Psi_1 = (qt.ket("0") + qt.ket("1")).unit()
Psi_2 = Psi_1
Psi = qt.tensor(Psi_1, Psi_2)
print("\n\nPart (b)")
print(f"Total state vector:\n{Psi}")
expectation_value = qt.expect(qt.tensor(qt.sigmaz(), qt.sigmaz()), Psi)
print(f"Expectation value of sigma_z tensor sigma_z: {expectation_value}")
expectation_value = qt.expect(qt.tensor(qt.sigmax(), qt.sigmax()), Psi)
print(f"Expectation value of sigma_x tensor sigma_x: {expectation_value}")

# Part (c)
Psi = (qt.ket("00") + qt.ket("11")).unit()
print("\n\nPart (c)")
print(f"Total state vector:\n{Psi}")
expectation_value = qt.expect(qt.tensor(qt.sigmaz(), qt.sigmaz()), Psi)
print(f"Expectation value of sigma_z tensor sigma_z: {expectation_value}")
expectation_value = qt.expect(qt.tensor(qt.sigmax(), qt.sigmax()), Psi)
print(f"Expectation value of sigma_x tensor sigma_x: {expectation_value}")
```

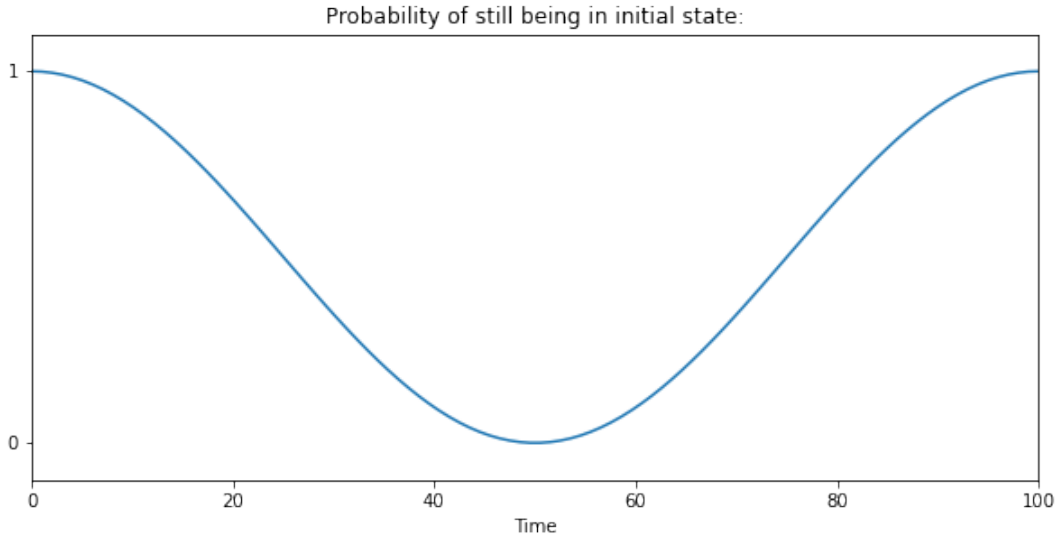
```
Part (a)
Total state vector:
Quantum object: dims=[[2, 2], [1, 1]], shape=(4, 1), type='ket', dtype=Dense
Qobj data =
[[0.70710678]
 [0.        ]
 [0.70710678]
 [0.        ]]
Expectation value of sigma_z for qubit 1: 0.0
```

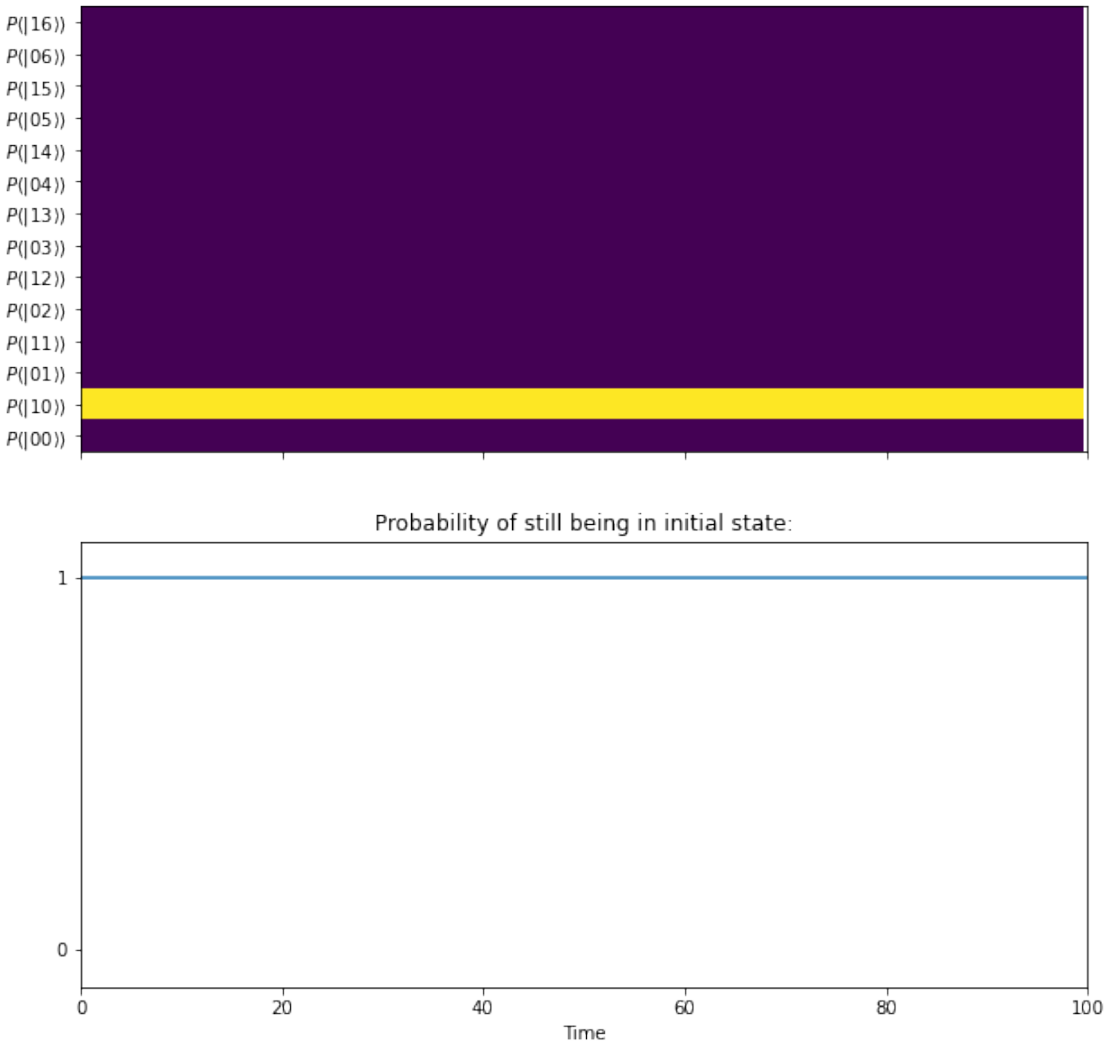
```
Expectation value of sigma_z for qubit 2: 0.9999999999999998
Expectation value of sigma_z tensor sigma_z: 0.0


Part (b)
Total state vector:
Quantum object: dims=[[2, 2], [1, 1]], shape=(4, 1), type='ket', dtype=Dense
Qobj data =
[[0.5]
 [0.5]
 [0.5]
 [0.5]]
Expectation value of sigma_z tensor sigma_z: 0.0
Expectation value of sigma_x tensor sigma_x: 0.9999999999999996


Part (c)
Total state vector:
Quantum object: dims=[[2, 2], [1, 1]], shape=(4, 1), type='ket', dtype=Dense
Qobj data =
[[0.70710678]
 [0.        ]
 [0.        ]
 [0.70710678]]
Expectation value of sigma_z tensor sigma_z: 0.9999999999999998
Expectation value of sigma_x tensor sigma_x: 0.9999999999999998
```

[5]:
```python
# Problem 4, part (a)
N = 7
omega_0 = 2 * np.pi
g = omega_0 / 100
a = qt.destroy(N)
a_dag = qt.create(N)
def H(omega):
    return (omega_0 / 2) * qt.tensor(qt.sigmaz(), qt.qeye(N)) + \
        omega * (qt.tensor(qt.qeye(2), qt.num(N)) + qt.tensor(qt.qeye(2), qt.
 ↪qeye(N))) + \
        (g / 2) * (qt.tensor(qt.sigmap(), qt.destroy(N)) + qt.tensor(qt.
 ↪sigmam(), qt.create(N)))

H_a = H(omega_0)

def plot_time_evo(initial_state, hamiltonian):
    times = np.linspace(0, 100, 100)
    evolved_states = []
    for t in times:
        U = (-1j * hamiltonian * t).expm()
```

```
        evolved_states.append(U * initial_state)

    _, (ax1, ax2) = plt.subplots(2, 1, sharex=True, figsize=(10, 10))
    # Make heatmap
    basis_states = [qt.ket([i % 2, i // 2], [2, N]) for i in reversed(range(2 *␣
 ↪N))]
    heatmap = np.matrix([[qt.expect(b.proj(), s) for s in evolved_states]
                         for b in basis_states])
    ax1.imshow(heatmap, aspect="auto")
    ax1.set_yticks(range(2 * N))
    ax1.set_yticklabels([f"$P(|{i%2}{i//2}\\rangle)$" for i in reversed(range(2␣
 ↪* N))])
    # Make line graph
    y_values = [qt.expect(initial_state.proj(), s) for s in evolved_states]
    plt.xlim(0, 100)
    plt.ylim(-.1, 1.1)
    ax2.set_yticks([0, 1])
    ax2.set_title("Probability of still being in initial state:")
    ax2.plot(times, y_values)
    plt.xlabel("Time")
    plt.show()


for Psi in [
    qt.ket("00", [2, N]),
    qt.ket("10", [2, N]),
    qt.ket("01", [2, N]),
    qt.ket("05", [2, N]),
    qt.ket("16", [2, N])
]:
    plot_time_evo(Psi, H_a)
```


P(|16⟩)
P(|06⟩)
P(|15⟩)
P(|05⟩)
P(|14⟩)
P(|04⟩)
P(|13⟩)
P(|03⟩)
P(|12⟩)
P(|02⟩)
P(|11⟩)
P(|01⟩)
P(|10⟩)
P(|00⟩)


Probability of still being in initial state:
1
0
0
20
40
60
80
100
Time


P(|16⟩)
P(|06⟩)
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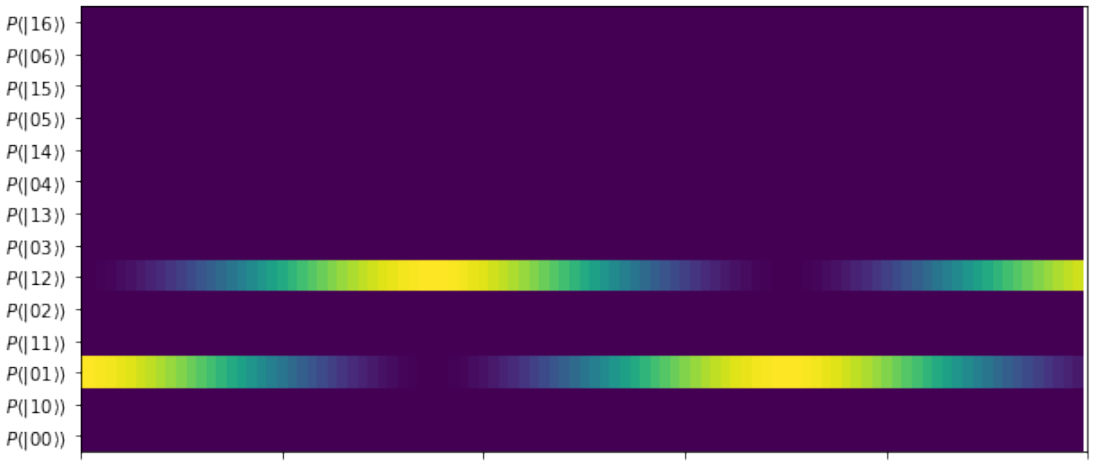
P(|12⟩)
P(|02⟩)
P(|11⟩)
P(|01⟩)
P(|10⟩)
P(|00⟩)
Probability of still being in initial state:
1
0
0
20
40
60
80
100
Time


P(|16⟩)
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P(|03⟩)
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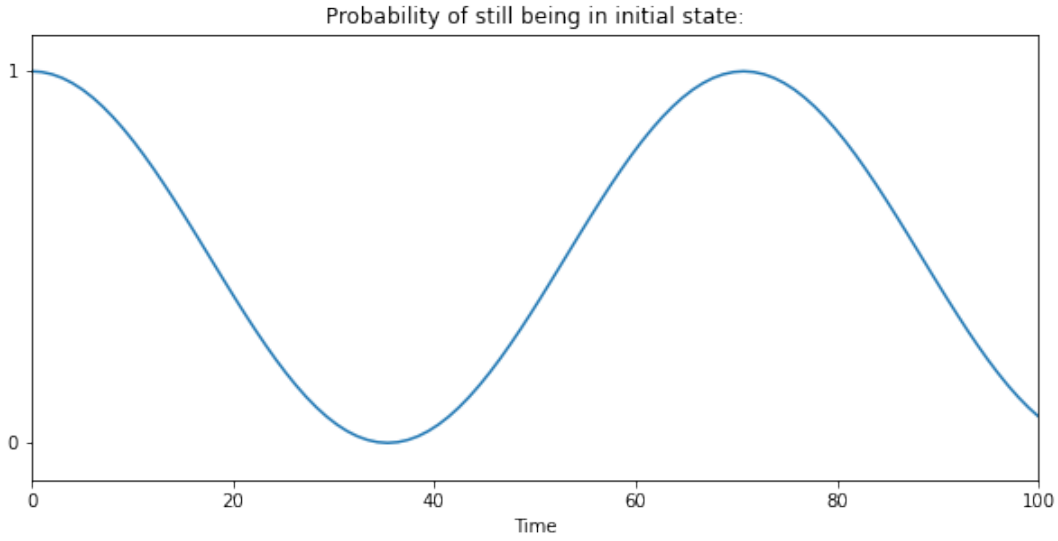
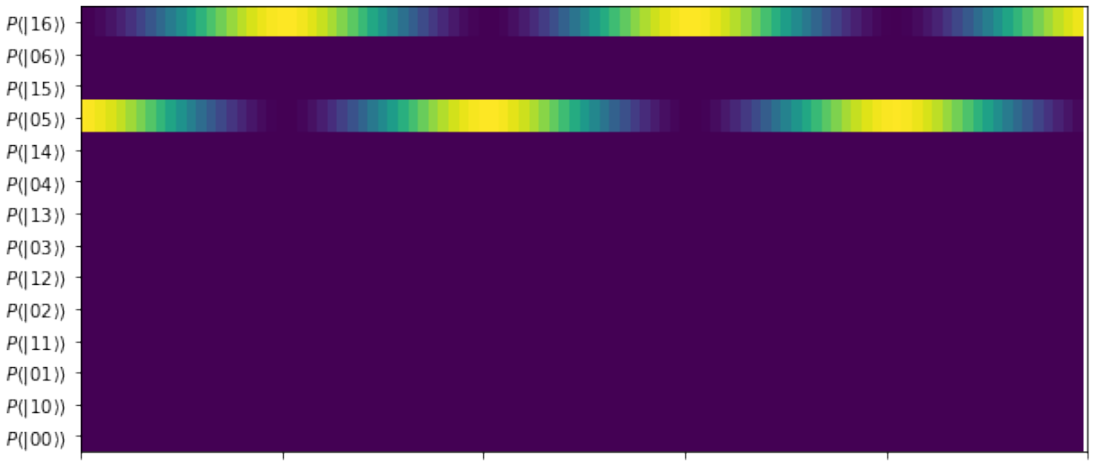
P(|01⟩)
P(|10⟩)
P(|00⟩)


Probability of still being in initial state:
1
0
0
20
40
60
80
100
Time


P(|16⟩)
P(|06⟩)
P(|15⟩)
P(|05⟩)
P(|14⟩)
P(|04⟩)
P(|13⟩)
P(|03⟩)
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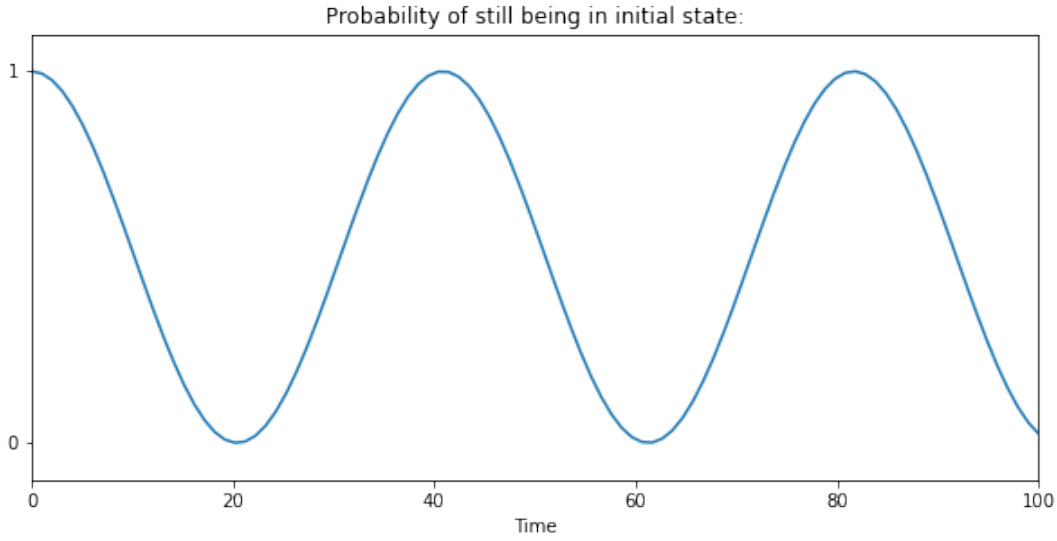
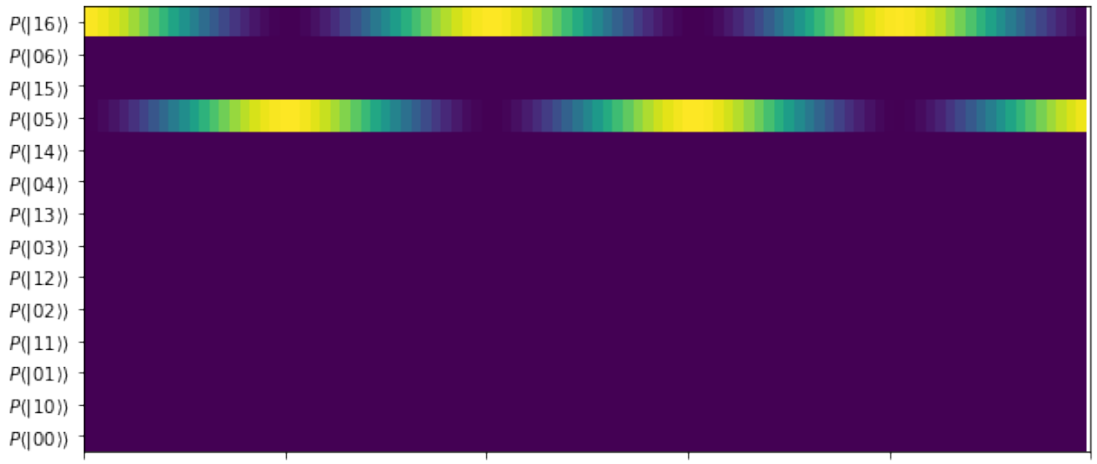
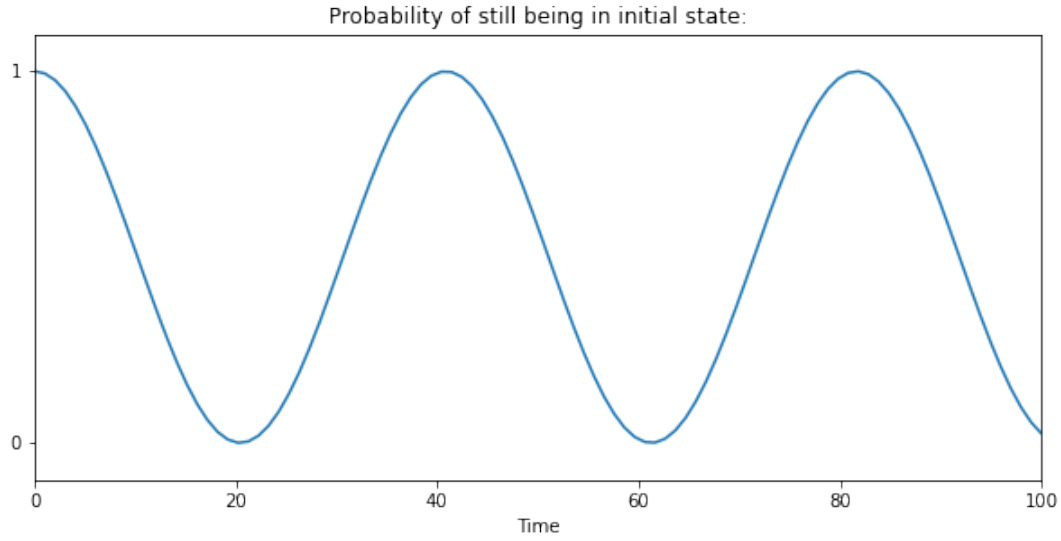
P(|10⟩)
P(|00⟩)

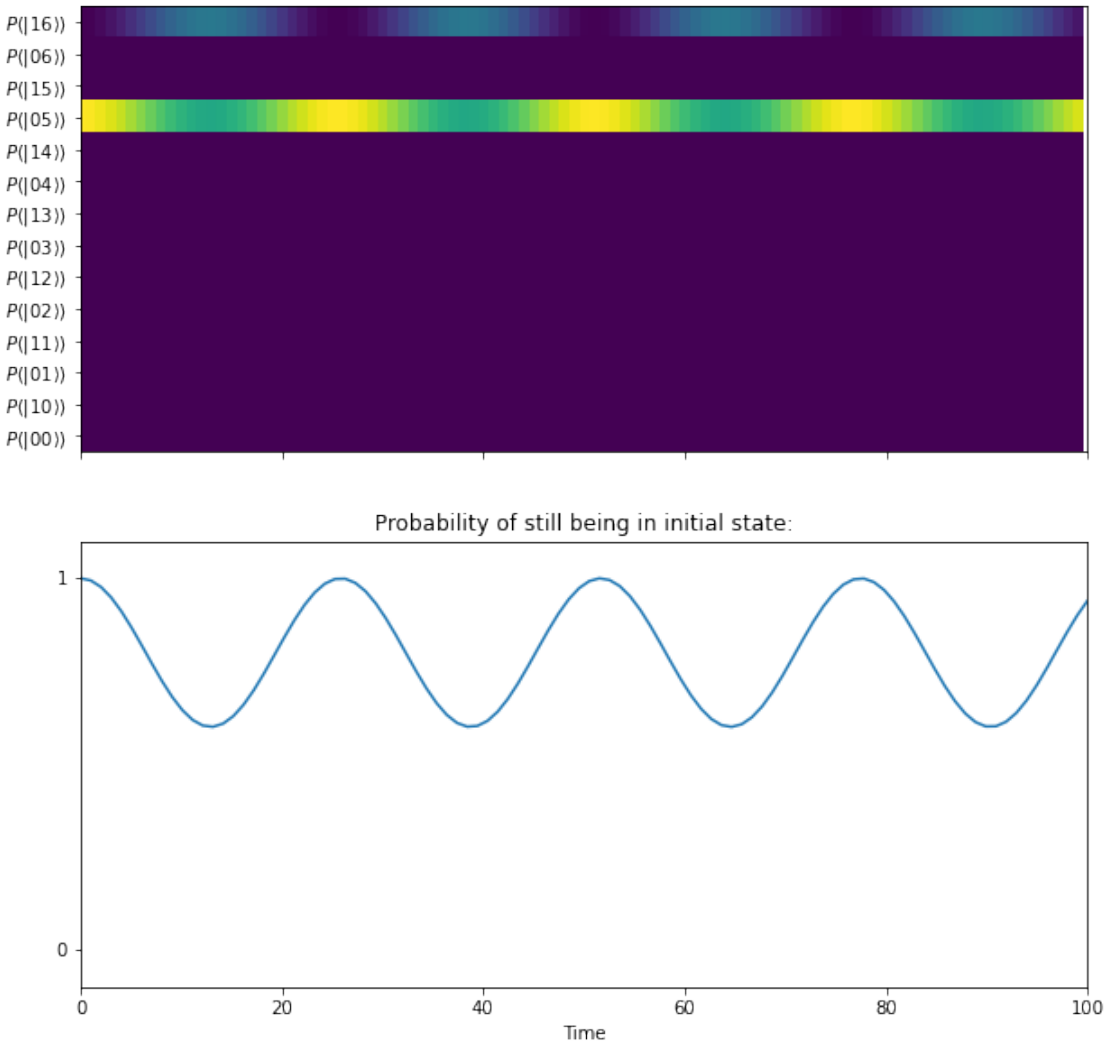

Probability of still being in initial state:
1
0
0
20
40
60
80
100
Time

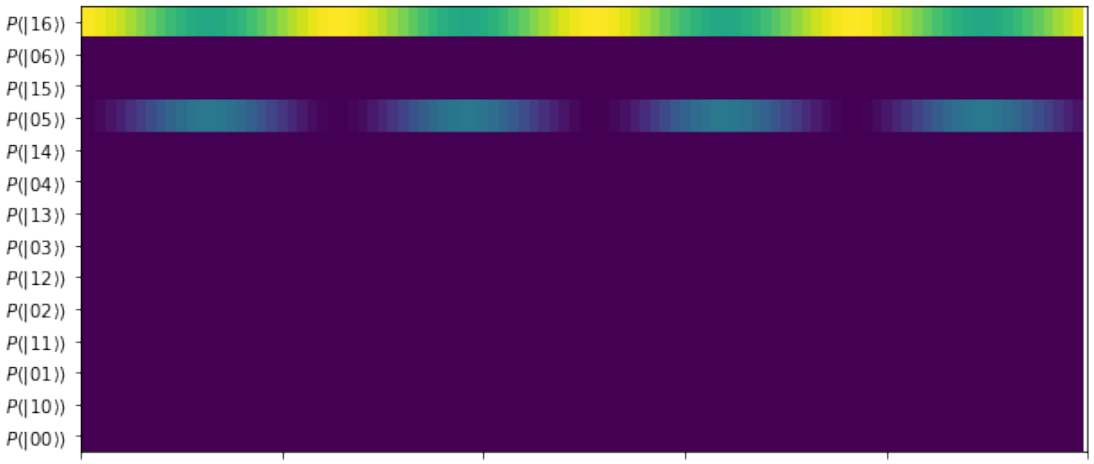


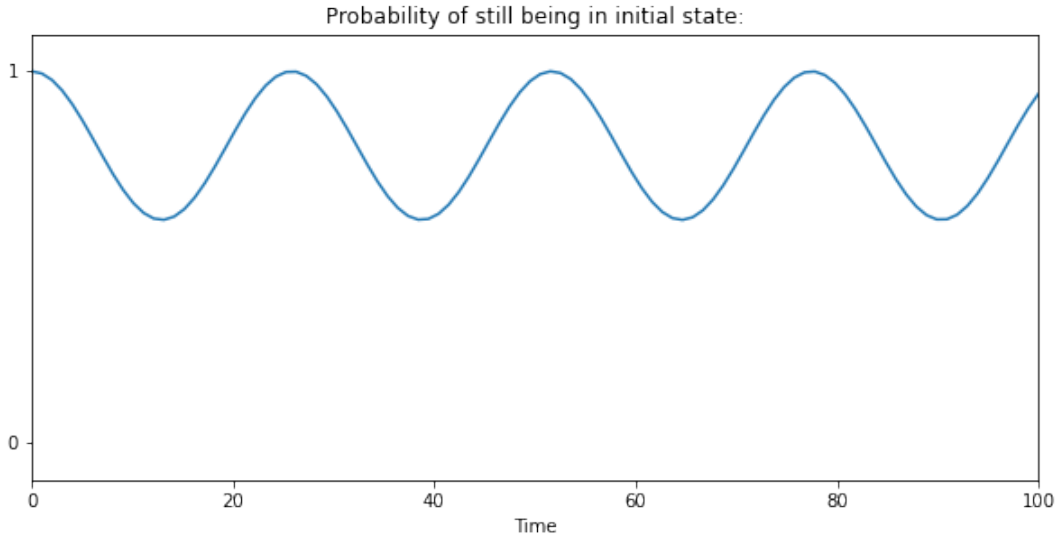


[6]:
```
# Problem 4, part (b)
H_b = H(omega_0 * 1.03)

for Psi in [
    qt.ket("05", [2, N]),
    qt.ket("16", [2, N])
]:
    plot_time_evo(Psi, H_b)
```


P(|16⟩)
P(|06⟩)
P(|15⟩)
P(|05⟩)
P(|14⟩)
P(|04⟩)
P(|13⟩)
P(|03⟩)
P(|12⟩)
P(|02⟩)
P(|11⟩)
P(|01⟩)
P(|10⟩)
P(|00⟩)
Probability of still being in initial state:
1
0
0
20
40
60
80
100
Time


P(|16⟩)
P(|06⟩)
P(|15⟩)
P(|05⟩)
P(|14⟩)
P(|04⟩)
P(|13⟩)
P(|03⟩)
P(|12⟩)
P(|02⟩)
P(|11⟩)
P(|01⟩)
P(|10⟩)
P(|00⟩)


Probability of still being in initial state:
1
0
0
20
40
60
80
100
Time

This one ended up being really hard for them and way too long. I guess they hadn't all heard of perturbation theory etc. Shorten this one up.

Phys 245 Quantum Computation
Homework 6

1. *[31] Neutral atom tweezers as a QHO*. Neutral atom tweezers function by shining a very red detuned laser onto an atom. In class, we derived the Stark shift due to such a laser as $E = -\frac{\hbar|\Omega|^2}{4\delta}$, where δ is the detuning from resonance. For a laser focused on an atom the Rabi frequency can be written as $\Omega = dE/\hbar$, where d is the atomic dipole moment (just a constant and a property of the atom) and E is the electric field of the laser. The intensity of the laser can be found from the electric field as $I = \frac{1}{2}c\epsilon_o|E|^2$. Assuming at the focus of the laser beam the electric field of the laser is described by a Gaussian beam of the form $E = E_o e^{-\frac{x^2+y^2}{w_o}} e^{\iota(kz-\omega t+\phi)}\hat{x}$. Do the following:
    a. [10] Find the Stark shift on the atom
    b. [10] Approximate the result from part (a) to describe the confining potential as a harmonic oscillator and identify the trap frequency ω.
    c. [10] Assume that $d = e\, a_o$ (e is the electron charge and a_o the Bohr radius), the laser power is 100 mW, the spot size is $w_o = 10\ \mu$m, and the detuning is $\delta = 10$ THz, what is ω?
    d. [1] What is $\hbar\omega$ in temperature units?

2. *[40] An anharmonic oscillator.* Suppose a harmonic oscillator with frequency ω is modified such that its potential goes from $V = \frac{1}{2}m\omega^2x^2 \rightarrow$ V = $\frac{1}{2}m\omega^2x^2 + \lambda x^4$. For this problem let $\omega = 1$ and $\lambda = 0.01$.
    a. [10] Use perturbation theory to calculate the first-order correction to the states $|0\rangle$ and $|1\rangle$.
    b. [15] Use perturbation theory to calculate the second-order correction to the energy. For this problem you'll have to deal with the problem of having an infinite basis. You'll need to choose a maximum n at which to truncate the basis. I recommend doing this step numerically and then exploring the effect of changing n_{max} on your final energy.
    c. [15] Instead of using perturbation theory, write the Hamiltonian as an $n_{max} \times n_{max}$ matrix and diagonalize it to find the eigenenergies – tip: QuTip has a built-in function for finding eigenvalues. For the same n_{max} do the methods of part (b) and part (c) agree or disagree?

3. [80] *Composite quantum systems.* In this exercise, we'll try out our new tools for dealing with composite quantum systems.

a. [20] For a system composed of two qubits with qubit one in the state $|\psi_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and qubit two in the state $|\psi_2\rangle = |0\rangle$, do the following:
   i. [5] Write down the total state vector.
   ii. [5] From the composite state vector, find the expectation value of the $\hat{\sigma}_z^{(1)}$, i.e. the expectation value of $\hat{\sigma}_z$ for qubit 1.
   iii. [5] From the composite state vector, find the expectation value of the $\hat{\sigma}_z^{(2)}$, i.e. the expectation value of $\hat{\sigma}_z$ for qubit 2.
   iv. [5] From the composite state vector, find the expectation value of $\hat{\sigma}_z^{(1)} \otimes \hat{\sigma}_z^{(2)}$.

b. [15] For a system composed of two qubits with qubit one in the state $|\psi_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and qubit two in the state $|\psi_2\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, do the following:
   i. [5] Write down the total state vector.
   ii. [5] From the composite state vector, find the expectation value of $\hat{\sigma}_z^{(1)} \otimes \hat{\sigma}_z^{(2)}$.
   iii. [5] From the composite state vector, find the expectation value of $\hat{\sigma}_x^{(1)} \otimes \hat{\sigma}_x^{(2)}$.

c. [10] Assume that the total state vector is $|\psi_{Tot}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ and do the following:
   i. [5] From the composite state vector, find the expectation value of $\hat{\sigma}_z^{(1)} \otimes \hat{\sigma}_z^{(2)}$.
   ii. [5] From the composite state vector, find the expectation value of $\hat{\sigma}_x^{(1)} \otimes \hat{\sigma}_x^{(2)}$.

d. [15] Write down the total statevector for a general three qubit system. That is, starting with $|\psi_1\rangle = (a_1|0\rangle + b_1|1\rangle)$, $|\psi_2\rangle = (a_2|0\rangle + b_2|1\rangle)$, and $|\psi_3\rangle = (a_3|0\rangle + b_3|1\rangle)$ find the total statevector in the composite Hilbert space.

e. [20] For a system composed of a qubit in the state $|\psi\rangle = (a_1|0\rangle + b_1|1\rangle)$ and a quantum harmonic oscillator in a superposition of Fock states $|\phi\rangle = \frac{1}{\sqrt{3}}(|0\rangle + |1\rangle + |3\rangle)$, do the following:
   i. [5] Write down the total state vector.
   ii. [5] From the composite statevector, find the expectation value of $\hat{\sigma}_z$ for the qubit.
   iii. [5] From the composite statevector, find the expectation value of $\widehat{N}$ for the harmonic oscillator.
   iv. [5] From the composite statevector, what is the probability of finding the system with qubit in state $|1\rangle$ and the harmonic oscillator in state $|3\rangle$?

4. [70] *Jayne says!* The Jaynes-Cummings Hamiltonian is:

$$H/\hbar = \frac{\omega_o}{2}\hat{\sigma}_z + \omega\left(\hat{a}^\dagger\,\hat{a} + \frac{1}{2}\right) + \frac{g}{2}(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a}^\dagger)$$

Implement the Jaynes-Cummings Hamiltonian in QuTip and use it to find the time dynamics of the following situations. For simplicity assume $\omega_o = 2\pi$, and $g = \omega_o/100$. Since you cannot use an infinite dimensioned Hilbert space in QuTip you'll need to truncate the basis at some maximum $|n\rangle$. Make sure and choose that maximum n large enough to not affect your answer.

a. [50] For $\omega = \omega_o$ find the time evolution of the population in the initial state for the following initial states over one period of oscillation:
   i. [10] $|\psi(t=0)\rangle = |00\rangle$ -- that is the qubit in the excited state and the harmonic oscillator in its ground state
   ii. [10] $|\psi(t=0)\rangle = |10\rangle$ -- that is the qubit in the ground state and the harmonic oscillator in its ground state
   iii. [10] $|\psi(t=0)\rangle = |01\rangle$ -- that is the qubit in the excited state and the harmonic oscillator in its first excited state
   iv. [10] $|\psi(t=0)\rangle = |05\rangle$ -- that is the qubit in the excited state and the harmonic oscillator in its fifth excited state
   v. [10] $|\psi(t=0)\rangle = |16\rangle$ -- that is the qubit in the excited state and the harmonic oscillator in its fifth excited state

b. [20] For $\omega = 1.03\,\omega_o$ find the time evolution of the population in the initial state for the following initial states over one period of oscillation:
   i. [10] $|\psi(t=0)\rangle = |05\rangle$ -- that is the qubit in the excited state and the harmonic oscillator in its fifth excited state
   ii. [10] $|\psi(t=0)\rangle = |16\rangle$ -- that is the qubit in the excited state and the harmonic oscillator in its fifth excited state