Math 151A Homework #8

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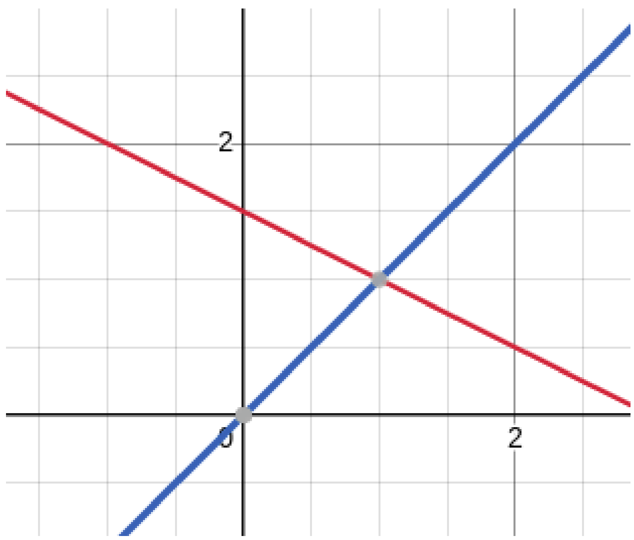
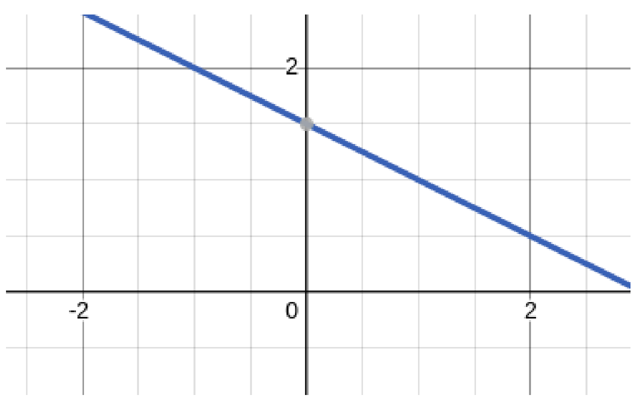
Problem 0.1.

(a) $x_1 = x_2 = 1$



(b) The red and blue lines completely overlap, so there are infinitely many solutions.



Problem 0.2.

(a)

(b)

Problem 0.3.

Let

$$A := \begin{bmatrix} 2 & -6\alpha \\ 3\alpha & -1 \end{bmatrix}.$$

The determinant of this matrix is

$$\det(A) = (2)(-1) - (-6\alpha)(3\alpha) = 18\alpha^2 - 2.$$

Therefore A is singular iff $\alpha = \pm 1/3$. The equation

$$A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3/2 \end{bmatrix}$$

has exactly one solution if A is nonsingular, so we just need to test two possible values of α. If $\alpha = 1/3$, the system of equations is equivalent to

$$\begin{aligned} 2x_1 - 2x_2 &= 3 \\ x_1 - x_2 &= 3/2, \end{aligned}$$

which has infinitely many solutions because for any $x_1 \in \mathbb{R}$, $x_2 = x_1 - 3/2$ is a solution. If $\alpha = -1/3$, the system of equations is

$$\begin{aligned} 2x_1 + 2x_2 &= 3 \\ -x_1 - x_2 &= 3/2, \end{aligned}$$

which has no solutions, because doubling the second equation and adding it to the first gives $0 = 6$, which is never true.

If $\alpha \notin \{-1/3, 1/3\}$, then

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = A^{-1} \begin{bmatrix} 3 \\ 3/2 \end{bmatrix} = \frac{1}{\det(A)} \begin{bmatrix} -1 & 3\alpha \\ -6\alpha & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 3/2 \end{bmatrix} = \frac{1}{18\alpha^2 - 2} \begin{bmatrix} 4.5\alpha - 3 \\ 3 - 18\alpha \end{bmatrix}.$$

Problem 0.4.

Problem 0.5.

By Sarrus' rule,

$$\begin{aligned} \det(A) &= (1)(2)(-1.5) + (-1)(1)(0) + (\alpha)(2)(\alpha) - (\alpha)(2)(0) - (-1)(2)(-1.5) - (1)(1)(\alpha) \\ &= -3 + 2\alpha^2 - 3 + \alpha \\ &= \alpha(2\alpha + 1). \end{aligned}$$

So A is singular iff $\alpha \in \{0, 1/2\}$.

Math 151A

Extra Credit HW #8, due on Friday, December 6, 2024 at 11:59pm PST.

[1] For the following systems obtain a solution (if it exists) via a graphical method.

a)

$$\begin{aligned} x_1 + 2x_2 &= 3 \\ x_1 - x_2 &= 0 \end{aligned}$$

b)

$$\begin{aligned} x_1 + 2x_2 &= 3 \\ 2x_1 + 4x_2 &= 6 \end{aligned}$$

[2] Use Gaussian Elimination to solve the following linear systems, if possible, and determine if row interchanges are necessary.

a)

$$\begin{aligned} 2x_1 - 1.5x_2 + 3x_3 &= 1 \\ -x_1 + 2x_3 &= 3 \\ 4x_1 - 4.5x_2 + 5x_3 &= 1 \end{aligned}$$

b)

$$\begin{aligned} x_2 - 2x_3 &= 4 \\ x_1 - x_2 + x_3 &= 6 \\ x_1 - x_3 &= 2 \end{aligned}$$

[3] Given the linear system

$$\begin{aligned} 2x_1 - 6\alpha x_2 &= 3 \\ 3\alpha x_1 - x_2 &= 3/2 \end{aligned}$$

a) Find the value(s) of α for which the system has no solutions.

b) Find the value(s) of α for which the system has infinite solutions.

c) Assuming a unique solution exists for a given α, find a formula for the solution.

[4] Use Gaussian Elimination with and without partial pivoting and three-digit chopping arithmetic to solve the following system. Compare the result you obtain with the two methods with to the actual solution and comment on your results.

$$0.03x_1 + 58.9x_2 = 59.2$$
$$5.31x_1 - 6.10x_2 = 47.0$$

Actual solution $x_1 = 10, x_2 = 1$

[5] Find the values of α that make the following matrix singular

$$A = \begin{bmatrix} 1 & -1 & \alpha \\ 2 & 2 & 1 \\ 0 & \alpha & -\frac{3}{2} \end{bmatrix}$$