Phys 245 Quantum Computation
Homework 4

1. *[40 + 5] Calibrating a new qubit!* On the class website (on the page with Lecture 7) there is a file which has the experimental result of a Rabi spectroscopy experiment. Specifically, the qubit was initially prepared in $|0\rangle$ and the probability of finding the qubit in $|1\rangle$ measured. The Rabi pulse was applied for 500 ns. Like real data, this experimental result is noisy. By fitting the expected lineshape, extract values for:
   a. [20] The qubit frequency ω_o and the Rabi frequency Ω
   b. [20] What are the uncertainties on these extracted parameters?
   c. [Bonus + 5] What is this qubit?
   (Hint for problem 1: Be careful with the factors of 2π. Remember, in all our work we have been using $H/\hbar$, therefore ω_o and Ω are angular frequencies, but the data, as is typical in the lab, is in Hz.)

2. *[50] Harmonic Oscillators are classic!* In this problem, we'll develop intuition about classical harmonic oscillators, which will prepare us for tackling the quantum problem. Suppose we have a mass on a spring. As the mass is moved from its equilibrium position ($x = 0$) the spring provides a restoring force of $F_x = -kx$, where k is the spring constant.
   a. [5] Show that the motion of the mass is given by a differential equation of the form $\frac{d^2x}{dt^2} = -\omega^2 x$ and determine ω.
   b. [5] Solve the differential equation from part (a) and write it as a single sine or cosine with an amplitude and phase to be found from initial conditions.
   c. [5] Use the solution from (b) to calculate the kinetic energy as a function of time.
   d. [5] Use the solution from (b) to calculate the potential energy as a function of time.
   e. [5] Use the solution from (b) to calculate the total energy as a function of time.
   f. [10] For $\omega = 2\pi \times 1$ Hz and m = 2 kg, make a plot of the position x(t) and momentum p(t) as a function of time for the initial conditions:
      i. x = 1, v = 0
      ii. x = 0, v = 1
      iii. x = 1, v = 1
   g. [10] For the same three cases as part (f), make a phase space plot of the evolution. That is, make a parametric plot of (x(t),p(t)) over one cycle of the oscillation.
   h. [5] Define a scaled position and momentum as:
      $\tilde{x} = \sqrt{\frac{m\omega}{2}}$ x and $\tilde{p} = \frac{p}{\sqrt{2m\omega}}$
      And remake the plots of part(g)
   i. [10] Construct a complex variable a as $a = \tilde{x} + \iota\tilde{p}$ and write it in phasor notation. Now make a plot where the real part of a is on the x axis and the imagine part of a is on the y axis.

3. *[25] Quantum Harmonic Oscillators by hand.* For the states $|n\rangle$:

a. [10] Calculate $\langle\hat{x}\rangle$ and $\langle\hat{p}\rangle$
b. [10] Calculate the uncertainties, $\sigma_{\hat{x}}$ and $\sigma_{\hat{p}}$.
c. [5] Sketch these states on a phase space plot (x vs p) for several values of n.

4. [30] Calculate *Quantum Harmonic Oscillators by QuTip.* For QHO (just set $\hbar = m = 1\ and\ \omega = 2\pi$), use QuTip to do the following:
   a. [10] Show the result of $\hat{a}|0\rangle$ and $\hat{a}^{\dagger}|0\rangle$.
   b. [10] Show the result of $\hat{a}|3\rangle$ and $\hat{a}^{\dagger}|4\rangle$.
   c. [10] Calculate $\langle\hat{x}\rangle$ and $\langle\hat{p}\rangle$ for $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |3\rangle)$.

5. *[40] A driven oscillator*. Suppose a harmonic oscillator is driven by a force on resonance, i.e. $F_o \cos \omega t$. Pick your own parameters for the strength of the force and the harmonic oscillator. Use numerical integration to solve the following.
   a. [20] Solve Newton's equation to find the evolution of the classical harmonic oscillator under this driving force. Plot x vs t, p vs t, and make a phase space plot of (x(t),p(t)). (If you prefer to do this one analytically, that's fine too. But still make the plots.)
   b. [20] Solve Schrodinger's equation (e.g. use sesolve in QuTip) to find the evolution of the quantum harmonic oscillator under this driving force. Plot x vs t, p vs t, and make a phase space plot of (x(t),p(t))