Physics 180E Homework #6

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Problem 0.1.

(a)

(b)

Problem 0.2.

(a)

(b)

PHYSICS 180E, WINTER 2025

HOMEWORK 6

(DUE WEDNESDAY MARCH. 12 BY MIDNIGHT ON GRADESCOPE)

1. Assuming a plasma density of $n_e \sim 5 \times 10^{10}$ cm^{-3} and a magnetic field strength of $B = 60$ G (0.006 T):

   (a) Make a plot of the dispersion relation ω (y-axis) versus $k_{||}$ (x-axis) for the frequency range you will use in the lab (say 20 MHz to 140MHz). Recall for pure parallel propagation ($\theta = 0$) the dispersion relation is given by:

   $$\omega = |\omega_{ce}| \frac{k_{||}^2 \delta_e^2}{1 + k_{||}^2 \delta_e^2}$$

   where $\delta_e = c/\omega_{pe}$ is the electron skin depth (inertial length). Note that ω and ω_{ce} are in rad/s not Hz in the expression above.

   (b) What is the parallel wavelength you should measure for 70 MHz waves? How does this compare to the mean free path for electron-neutral collisions in this plasma (assume 1 mTorr neutral gas pressure and recall $\lambda_{mfp} = 1/(n_g \sigma)$)?

2. We can extend the dispersion relation for parallel propagating whistler waves to include the presence of electron neutral collisions as follows:

   $$k_{||}^2 \delta_e^2 = \frac{\omega}{|\omega_{ce}| - (\omega + i\nu)}$$

   where ν is the collision frequency and i indicates an imaginary number.

   (a) Assuming a real frequency, find an expression for the imaginary part of $k_{||}$. (If you like, you can find an approximate answer by assuming $\nu \ll \omega$).

   (b) For the parameters in the first problem and again assuming $f = 70$ Mhz, compute the damping length (inverse of the imaginary part of k). How does this compare to your wavelength? You can assume an electron temperature of 3 eV when doing this calculation; you will need it for the collision frequency, which is $\nu = v_{th,e}/\lambda_{mfp}$.

A useful resource for equations in this lab is the Naval Research Laboratory's "Plasma Formulary", a copy of which you can find on the course website.