Physics 231B Homework #3

Nathan Solomon

April 23, 2024

Problem 0.1. Prove that $A_4 \cong (\mathbb{Z}_2 \times \mathbb{Z}_2) \rtimes \mathbb{Z}_3$.

Problem 0.2. Prove that A_4 has no subgroup of size 6.

Every group of order 6 is either C_6 or $C_2 \times C_3$.

Problem 0.3. **Artin Chapter 6 Problem 12.2, page 193.**

Problem 0.4. **Artin Chapter 6 Problem M.1, page 193.**

Problem 0.5. Show that $\mathbb{R}^d \rtimes O(d)$ is isomorphic to the subgroup of $GL(d+1, \mathbb{R})$ consisting of matrices of the form

$$\begin{pmatrix} M & v \\ 0 \cdots 0 & 1 \end{pmatrix},$$

where $v \in \mathbb{R}^d$ is a column vector and $M \in O(d)$.

Problem 0.6. Consider the wallpaper group $\mathbb{Z}^2 \rtimes C_4$ we discussed in class acting on $\mathbb{R}^2$. What are the points in $\mathbb{R}^2$ which have non-trivial stabilizers? What are their stabilizer groups?