Math 151A
HW #7, due on Friday, November 29, 2024 at 11:59pm PST.

[1] Approximate the following integrals using Trapezoidal rule.

(a) $\int_0^1 x^2 e^{-x}\, dx$

(b) $\int_1^{1.6} \frac{2x}{x^2-4}\, dx$

[2] The Trapezoidal Rule applied to $\int_0^2 f(x)dx$ gives the value 4, and Simpson's rule gives the value 2. What is $f(1)$?

[3] [*Composite quadrature rules*]

Use the Composite Trapezoidal and Composite Simpson's rules to approximate the integral

$$\int_1^2 x\ln(x)dx$$

with $n = 4$ subintervals. What are the relative errors? (*Hint*: to compute the true value of the integral, integrate by parts.)

[4] Using Intermediate Value Theorem show that the error for Composite Simpson's Rule can be estimated by:

$$|\frac{h^5}{90}\sum_{j=1}^{n/2} f^{(4)}(\xi_j)| \leq \frac{h^4}{180}(b-a)|f^{(4)}(\xi)|$$

Hint: Use similar steps as for the error in composite Trapezoidal rule.

[5] [*Computational cost as a function of error tolerance*]

Recall from lecture that the error in the Composite Trapezoidal Rule (CTR) using n subintervals of width h is given by

$$\frac{-h^2}{12}(b-a)\, f''(\mu) \tag{1}$$

for some $\mu \in (a, b)$.

(a) Determine the values of n and h that are sufficient to approximate

$$\int_1^2 x \ln(x) dx \tag{2}$$

to within an error tolerance of $\tau = 10^{-5}$; that is, determine n and h so that the error when applying the CTR to (2) is smaller (in absolute value) that τ.

(b) Repeat part (a) for the case of Composite Simpson's Rule.

[6] Find constants a, b, c, d such that the quadrature rule below has degree of precision 3.

$$\int_{-1}^{1} f(x)\, dx = a\, f(-1) + b\, f(1) + c\, f'(-1) + d\, f'(1)$$

[7] Computational exercise Consider the nonlinear equation for x:

$$\int_0^x \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt = 0.45.$$

Note that t is just a 'dummy' variable of integration.

(a) Define

$$f(x) := \int_0^x \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt - 0.45.$$

Using the Fundamental Theorem of Calculus, write down Newton's method applied to f.

(b) Each step of Newton's method derived in (a) requires of an evaluation of $f(x)$. Rewrite the method you derived in (a) using Composite Trapezoidal Rule to estimate $f(x)$. Indicate with N the number of subintervals.

(c) Implement in MATLAB the method derived in part (b) to find the solution x to the equation $f(x) = 0$; terminate the iteration when the *residual* is smaller than $\tau = 10^{-5}$. Use $x_0 = 0.5$ as an initial guess and $N = 50$ for composite trapezoidal rule.