Physics 105B Lecture Notes, Fall 2024

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The class textbook is *Classical Mechanics of Particles and Systems, 5th edition* by Marion & Thornton. These notes are a supplement for the textbook, not a replacement, so I won't cover everything from the course.

Contents

1 Review of Lagrangian mechanics (physics 105A)

1.1 Calculus of variations

Use catenary and brachiostrome as examples

1.2 Least action principle

1.3 Generalized momenta

1.4 Lagrange multipliers, forces from constraints

2 Phase diagrams

For lots of systems, the state can be described by generalized positions $(q_1, q_2, \dots)$ and generalized velocities $(\dot{q}_1, \dot{q}_2, \dots)$. For example, the state of a pendulum at any time can be described by the vector

$$\begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix},$$

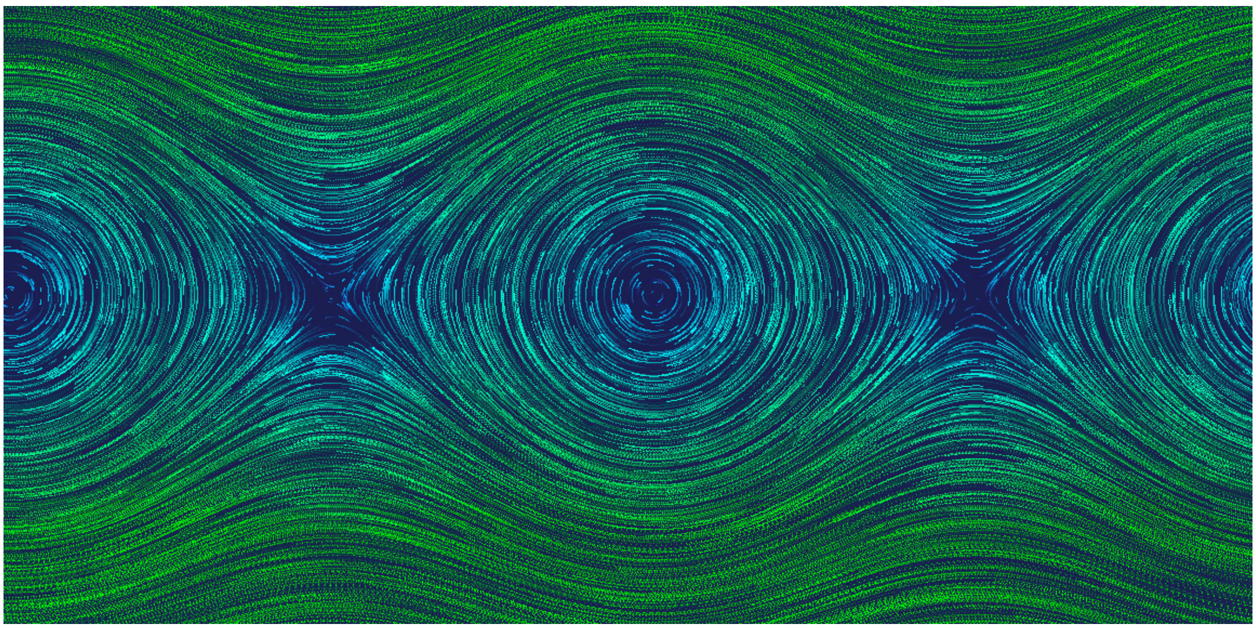
and if we know the state, we can calculate the rate of change of the state. That is,

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix}$$

is a function of the state. We can visualize this as a vector field, where $x = \theta$ and $y = \dot{\theta}$. One amazing tool for animating vector fields is `https://anvaka.github.io/fieldplay/`

For the pendulum example, copy this code:

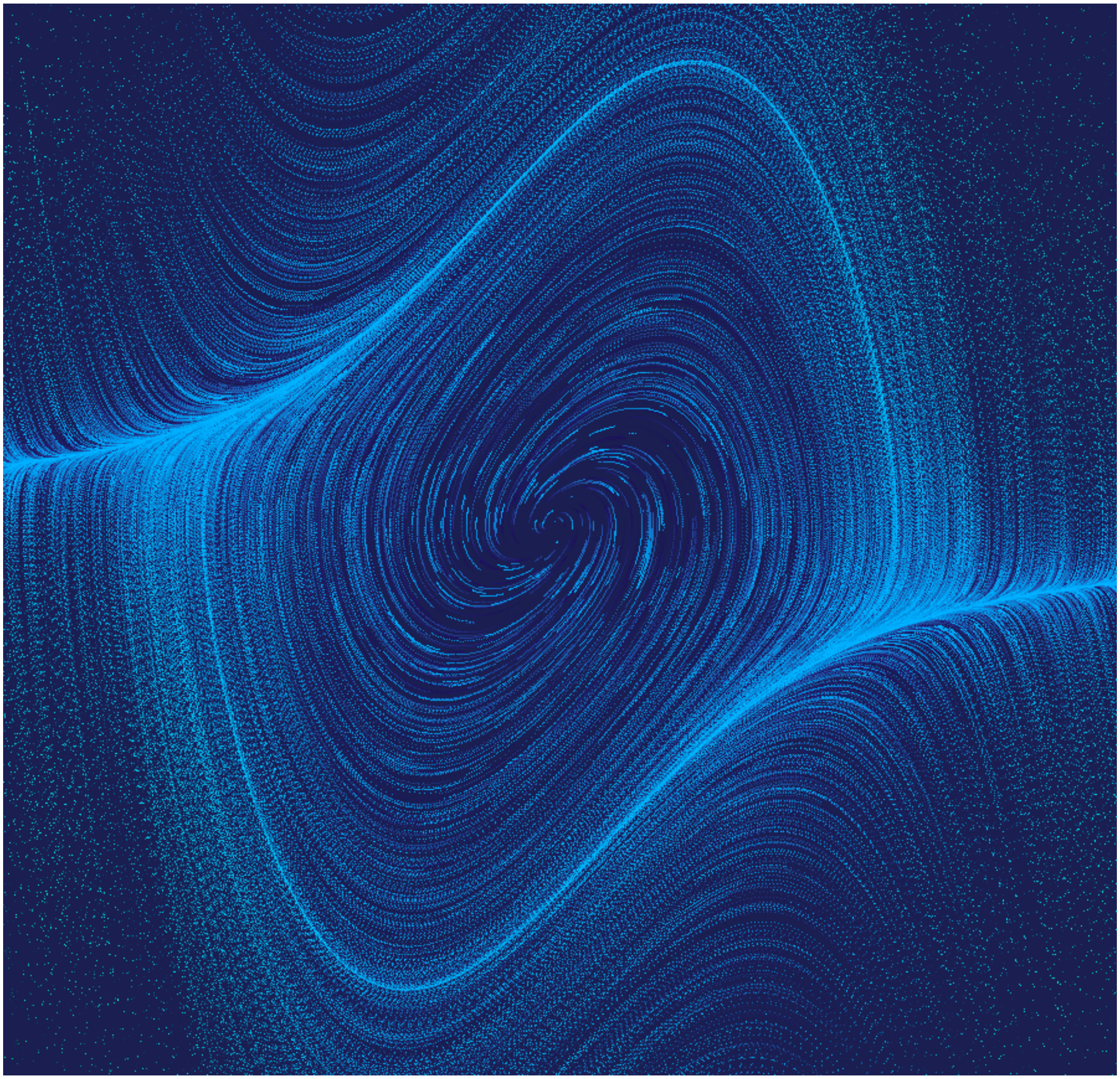
```
// p.x and p.y are current coordinates
// v.x and v.y is a velocity at point p
vec2 get_velocity(vec2 p) {
  vec2 v = vec2(0., 0.);
  v.x = p.y;
  v.y = cos(p.x);
  return v;
}
```



For a Van der Pol oscillator, copy in this code, and experiment with changing μ (mu):

```
float mu = 1.;
vec2 get_velocity(vec2 p) {
  return vec2(p.y, mu*(1.-p.x*p.x)*p.y-p.x);
}
```

Here is a Van der Pol oscillator with $\mu = 1$:



3 Rotating reference frames

Chapter 10 in the textbook. Copy example 10.5 (Foucault pendulum) to these notes

4 Synchronized oscillators

Include links to and screenshots of synchronizing oscillators from the “Explorable explanations” website (although that’s different from the oscillators we are talking about here).

5 Moment of inertia tensor

Compare the off-diagonal elements (“products of inertia”) to the formula for Pearson’s correlation coefficient, which measures how much stuff is on a diagonal line between the x and y axes. Use this to give an intuitive explanation of what diagonalizing I represents.

5.1 Stress tensor

A lot of your intuition for the moment of inertial tensor can be applied to the stress tensor σ, which is another contravariant, symmetric, second-order tensor. It represents the forces on each point in a material. Thinking of it as a 3 by 3 matrix, the diagonal elements represent the tensile stress along each axis. So for example, $\sigma_{1,1}$ would be positive if the material it being stretched along the x axis at that point, and negative if it is being squished along the x axis at that point.

The off-diagonal elements of σ represent shear forces. For example, if $\sigma_{1,2} = \sigma_{2,1} > 0$, then there is some shearing force in the x, y plane. Such a force could also be treated as a pulling force in the direction of the unit vector $\pm\frac{1}{\sqrt{2}}(\hat{x} + \hat{y})$. By treating all of the shearing forces as pulling or squishing along some axis other than $\hat{x}$, $\hat{y}$, or $\hat{z}$, we can describe the stress tensor with just the tensile stress along those 3 axes, which we can ensure are orthogonal, because σ is symmetric. This is equivalent to diagonalizing the stress tensor.

To intuitively understand these off-diagonal elements, imagine a square pice of sheet metal lying in the x, y plane, with one edge flushed to the x axis and another flushed to the y axis. Then apply a shear force $F \propto y\hat{x}$ to it, which points along the x axis and is proportional to the y coordinate. This force acts to stretch the square into a rhombus, effectively pulling (that is, applying tensile stress) in the $\frac{1}{\sqrt{2}}(\hat{x} + \hat{y})$ direction.

Stress has units of force per area. One Pascal is a Newton per meter squared, and one atmosphere is about 101.3 kilopascals. If the stress tensor is some scalar $\sigma_{1,1}$ times the identity tensor, then we can say that the pressure is $-\sigma_{1,1}$. Air pressure at sea level is very close to 1 atm, and air pressure pretty much decays exponentially with altitude.