Physics 180E Homework #4

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Problem 0.1.

Assuming we only have first ionization, the ion saturation current is given by

$$I_{sat} = neA \exp\left(-\frac{1}{2}\right)\sqrt{\frac{k_B T_e}{M_{ion}}}.$$

The mass of an argon atom is $M_{ion} = 37.2\,\mathrm{GeV/c^2}$. Solving for the plasma density n, that becomes

$$\begin{aligned}
\frac{1}{n} &= \frac{eA}{I_{sat}} \exp\left(-\frac{1}{2}\right)\sqrt{\frac{k_B T_e}{M_{ion}}} \\
&= \frac{(1.602 \times 10^{-19}\,\mathrm{C})(2\,\mathrm{mm})^2}{10^{-4}\,\mathrm{A}} \exp\left(-\frac{1}{2}\right)\sqrt{\frac{2\,\mathrm{eV}}{3.72 \times 10^{10}\,\mathrm{eV/c^2}}} \\
&= (1.602 \times 10^{-15}\,\mathrm{s})(0.002\,\mathrm{m})^2(0.6065)\sqrt{\frac{2 \cdot (3 \times 10^8\,\mathrm{m/s})^2}{3.72 \times 10^{10}}} \\
&= (1.602 \times 10^{-15}\,\mathrm{s})(4 \times 10^{-6}\,\mathrm{m^2})(0.6065)(3 \times 10^8\mathrm{m/s})\sqrt{\frac{2}{3.72 \times 10^{10}}} \\
&= (1.602 \times 10^{-15})(4 \times 10^{-6})(0.6065)(3 \times 10^8)(7.33 \times 10^{-6})\,\mathrm{m^3} \\
&= 8.55 \times 10^{-18}\,\mathrm{m^3} \\
n &= 1.17 \times 10^{17}\,\mathrm{m^{-3}},
\end{aligned}$$

meaning there are 1.17×10^{17} argon ions per cubic meter.

The plasma potential is given by the following equation:

$$\begin{aligned}
V_p &= V_f + \frac{k_B T_e}{2e}\left(1 - \log\left(\frac{2m_e\pi}{M_{ion}}\right)\right) \\
&= (2\,\mathrm{V}) + \frac{2\,\mathrm{eV}}{2e}\left(1 - \log\left(\frac{2\pi(0.511\,\mathrm{MeV/c^2})}{37.2\,\mathrm{GeV/c^2}}\right)\right) \\
&= (2\,\mathrm{V}) + (1\,\mathrm{V}) \cdot (1 - \log(2\pi(0.511)/37200)) \\
&= (2\,\mathrm{V}) + (1\,\mathrm{V}) \cdot (1 - \log(0.0000863)) \\
&= (2\,\mathrm{V}) + (1\,\mathrm{V}) \cdot (1 - (-9.36)) \\
&= 12.36\,\mathrm{V}.
\end{aligned}$$

Problem 0.2.

I will assume that the electrons don't collide with neutrals enough to heat the neutral temperature much above room temperature (300 Kelvin). If we assume the cross-sectional area of a neutral argon atom (colliding with an electron) is $\sigma = 10^{-15}\,\mathrm{cm}^2 = 10^{-19}\,\mathrm{m}^2$, then the mean free path for electrons is

$$\begin{aligned}
\ell_{mfp} &= \frac{1}{n\sigma} \\
&= \frac{k_B T}{p\sigma} \\
&= \frac{k_B(300\,\mathrm{K})}{(1\,\mathrm{mTorr})(10^{-19}\,\mathrm{m}^2)} \\
&= \frac{4.14\times 10^{-21}\,\mathrm{J}}{(0.00133\,\mathrm{Pa})(10^{-19}\,\mathrm{m}^2)} \\
&= 31.1\,\mathrm{m}.
\end{aligned}$$

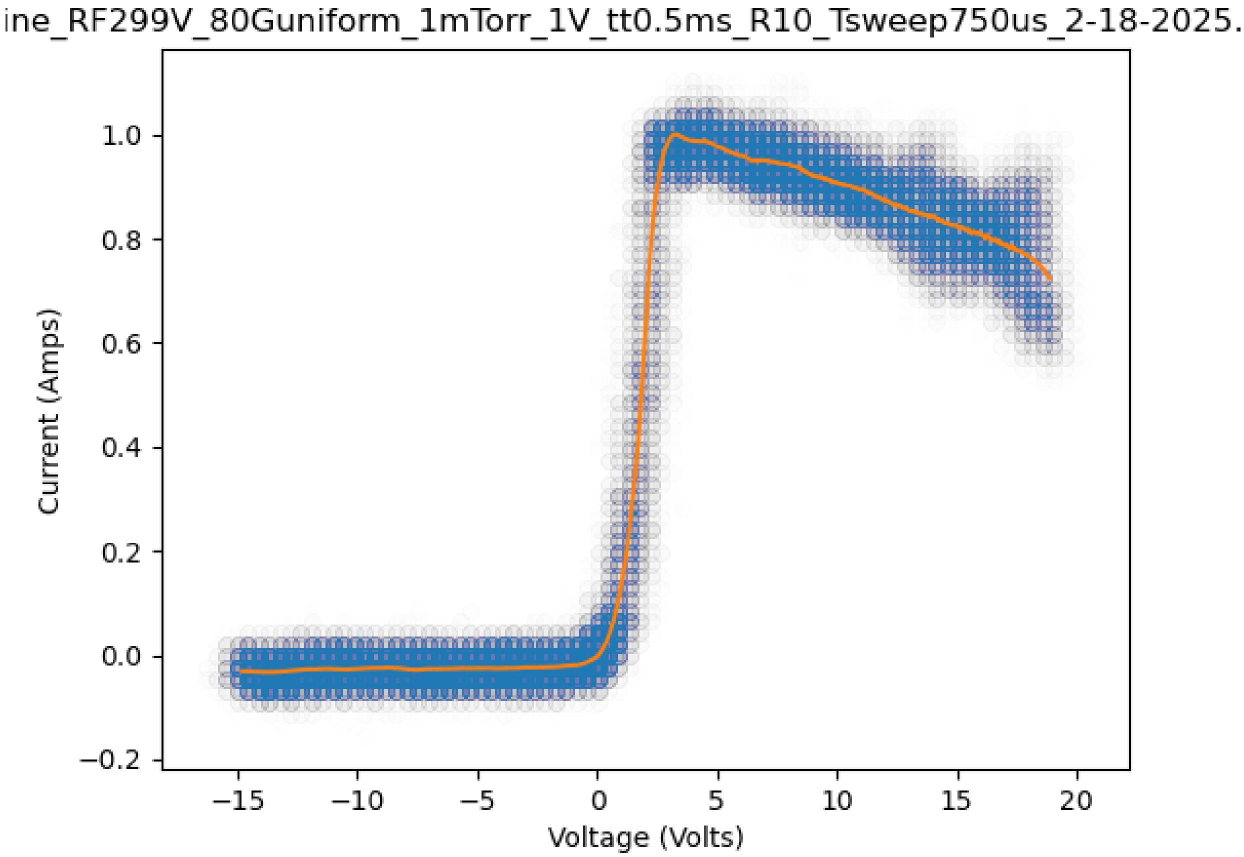
The Debye length of the plasma is

$$\begin{aligned}
\lambda_D &= \sqrt{\frac{\varepsilon_0 k_B T_e}{e^2 n}} \\
&= \sqrt{\frac{\varepsilon_0(5\,\mathrm{eV})}{e^2(10^{19}\,\mathrm{m}^{-3})}} \\
&= \sqrt{\varepsilon_0(5\,\mathrm{V})(10^{-19}\,\mathrm{m}^3)/e} \\
&= \sqrt{(5.526\times 10^7\,\mathrm{e/V/m})(5\,\mathrm{V})(10^{-19}\,\mathrm{m}^3)/e} \\
&= \sqrt{(5.526\times 10^7)(5\times 10^{-19}\,\mathrm{m}^2)} \\
&= 5.26\,\mu\mathrm{m}.
\end{aligned}$$

The mean free path of electrons is much, much larger than the Debye length, meaning collisions don't occur much within the sheath around the Langmuir probe. In fact, since the mean free path is very large, we can think of collisions as being fairly rare, which is why the neutrals and the electrons are allowed to have different temperatures – if they collided very often, they would exchange heat and approach thermal equilibrium.

Problem 0.3.

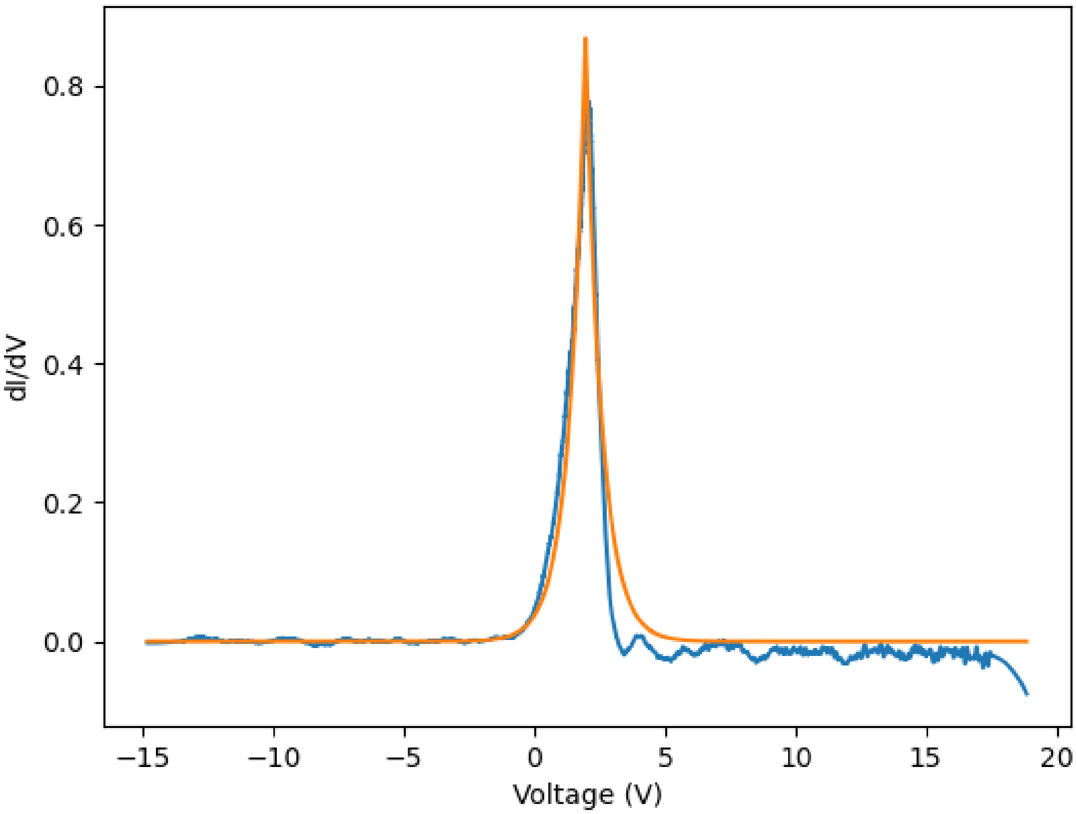
Here is the IV curve for a single Langmuir sweep (before and after filtering):



I then took the derivative of that data and fitted the result to an equation of the form

$$\frac{\partial I_{probe}}{\partial \phi} = ne^2 A \sqrt{\frac{1}{2\pi k_B m_e T_e}} \exp\left(-\frac{e(\phi - V_0)}{2k_B T_e}\right)$$

in order to extract the electron temperature (T_e), the plasma potential (V_0), and the electron density (n).



- Electron temperature: 0.31045 eV
- Plasma potential: 1.9565 V
- Electron density: $4.5136 \times 10^{18}\,\mathrm{m}^{-3}$

Problem 0.4.

(a) Yes, they look like roughly the right order of magnitude.

(b) The first ionization energy of argon is 15.76 eV, which is much larger than the electron temperature I found. That means that the temperature is way too small for “bulk ionization”. For thermal collisions between electrons and neutrals to ionize a non-negligible proportion of the argon atoms, the electron temperature would have to bean close to the ionization energy of argon.

PHYSICS 180E, WINTER 2025

HOMEWORK 4

(DUE WEDNESDAY FEB. 19 BY MIDNIGHT ON GRADESCOPE)

1. A planar Langmuir probe is made of a tantalum foil and is square with 2 mm sides – assume one face of the foil is insulating. At ion saturation, the probe draws 100 μA from an Argon plasma. If the electron temperature is 2 eV, what is the plasma density? In the same plasma, the floating potential is measured to be +2 V relative to chamber ground. What is the plasma potential?

2. Estimate the mean free path for electrons colliding with neutrals in a plasma with similar parameters to what you will be doing experiments with: neutral gas pressure 1 mTorr, electron density 10^{11} cm^{-3}, electron temperature 5 eV. How does the mean free path compare to the Debye length in this plasma (in other words, do the electrons undergo collisions in the sheath around the Langmuir probe? If so, our theory is not valid!). Recall that the Debye length is:

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T_e}{e^2 n}}$$

Hint: you can estimate the cross-section of a neutral atom as the approximate cross-sectional area of the atom. Google it, but roughly 10^{15} cm^2 works.

3. Using data from your first Langmuir lab session, analyze the I-V characteristics of a single Langmuir sweep to calculate the following plasma parameters:

   (a) Electron temperature
   (b) Plasma potential
   (c) Electron density

4. Answer the following questions using the data obtained from your Langmuir sweep analysis above.

   (a) Are your derived measurements for density and temperature reasonable? Does your measured plasma potential make sense? Why or why not? Recall the discussion in lecture over how the plasma potential gets established (by flow of charge from the plasma to the wall and vice versa)

   (b) How does your electron temperature compare to the ionization potential of Argon? Recall we discussed different types of ionization in lecture, including "bulk" ionization (plasma electrons with the temperature you measure colliding with Argon neutrals and ionizing them) and ionization by primary electrons (the electrons accelerated by the discharge voltage). Is your electron temperature large enough that the average electron can ionize Argon? Under what conditions would bulk ionization dominate over primary ionization?

A useful resource for equations in this lab is the Naval Research Laboratory's "Plasma Formulary", a copy of which you can find on the course website.