Math 132 Homework #2

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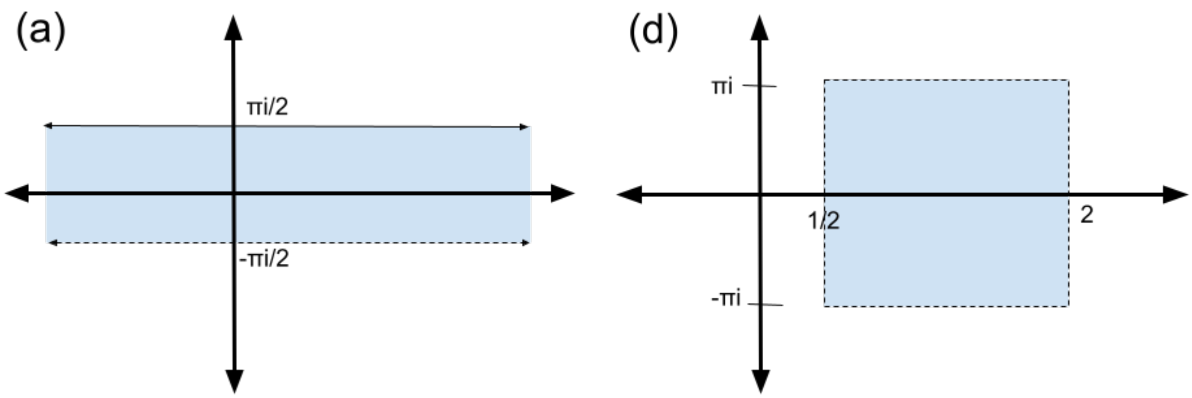
April 23, 2025

Problem 0.1. Chapter I, section 6, exercise 2, parts a & d



Problem 0.2. Chapter I, section 7, exercise 2

$$
\begin{aligned}
\log\left[(1+i)^{2i}\right] &= \log\left[\exp\left(2i\log(1+i)\right)\right] \\
&= \log\left[\exp\left(2i\cdot\left(\log(\sqrt{2})+\frac{\pi i}{4}+2\pi i k_1\right)\right)\right] && k_1\in\mathbb{Z} \\
&= \left(2i\cdot\left(\sqrt{2}+\frac{\pi i}{4}+2\pi i k_1\right)\right)+2\pi i k_2 && k_1,k_2\in\mathbb{Z} \\
&= i\log(2)-\frac{\pi}{2}-4\pi k_1+2\pi i k_2 && k_1,k_2\in\mathbb{Z} \\
&= \left(-\frac{\pi}{2}4\pi k_1\right)+i\left(\log(2)+2\pi k_2\right) && k_1,k_2\in\mathbb{Z}.
\end{aligned}
$$

When you plot all possible values on the complex plane, you get one point (the "principal value") at $z=-\pi/2+2\log(2)i$, and infinitely many copies of that point in a grid with vertical spacing of 2π and horizontal spacing of 4π.

Problem 0.3. Chapter I, section 8, exercise 1

$$
\begin{aligned}
\cos z\cos w-\sin z\sin w &= \left(\frac{e^{iz}+e^{-iz}}{2}\right)\left(\frac{e^{iw}+e^{-iw}}{2}\right)-\left(\frac{e^{iz}-e^{-iz}}{2i}\right)\left(\frac{e^{iw}-e^{-iw}}{2i}\right) \\
&= \frac{1}{4}\left[(e^{iz}+e^{-iz})(e^{iw}+e^{-iw})+(e^{iz}-e^{-iz})(e^{iw}-e^{-iw})\right] \\
&= \frac{1}{4}\left[2e^{iz}e^{iw}+2e^{-iz}e^{-iw}\right] \\
&= \cos(z+w). \\
\sin z\cos w+\cos z\sin w &= \left(\frac{e^{iz}-e^{-iz}}{2i}\right)\left(\frac{e^{iw}+e^{-iw}}{2}\right)-\left(\frac{e^{iz}+e^{-iz}}{2}\right)\left(\frac{e^{iw}-e^{-iw}}{2i}\right) \\
&= \frac{1}{4i}\left[(e^{iz}-e^{-iz})(e^{iw}+e^{-iw})-(e^{iz}+e^{-iz})(e^{iw}-e^{-iw})\right] \\
&= \frac{1}{4i}\left[2e^{iz}e^{iw}-2e^{-iz}e^{-iw}\right] \\
&= \sin(z+w). \\
\cosh(z+w) &= \cos(iz+iw) \\
&= \cos(iz)\cos(iw)-\sin(iz)\sin(iw) \\
&= (\cosh z)(\cosh w)-(i\sinh z)(i\sinh w) \\
&= \cosh z\cosh w+\sinh z\sinh w. \\
\sinh(z+w) &= -i\sin(iz+iw) \\
&= -i\left(\sin(iz)\cos(iw)+\cos(iz)\sin(iw)\right) \\
&= -i\left((i\sinh z)(\cosh w)+(\cosh z)(i\sinh w)\right) \\
&= \sinh z\cosh w+\cosh z\sinh w.
\end{aligned}
$$

Problem 0.4. Chapter I, section 8, exercise 2

If $z = x + iy$, where $x, y \in \mathbb{R}$, then

$$\begin{aligned}
|\cos z|^2 &= |\cos(x+iy)|^2 \\
&= |\cos(x)\cos(iy) - \sin(x)\sin(iy)|^2 \\
&= |(\cos x)(\cosh y) - (\sin x)(i \sinh y)|^2 \\
&= (\cos x \cosh y)^2 + (\sin x \sinh y)^2 \\
&= \cos^2 x \cosh^2 y + \sin^2 x \sinh^2 y \\
&= \cos^2 x (\sinh^2 y + 1) + \sin^2 x \sinh^2 y \\
&= \cos^2 x + (\cos^2 x + \sin^2 x) \sinh^2 y \\
&= \cos^2 x + \sinh^2 y.
\end{aligned}$$

Problem 0.5. Chapter II, section 1, exercise 2

The sequence z^n is bounded iff $|z| \leq 1$, because then all powers of z also lie in the open unit disk.

The sequence z^n converges to zero iff $|z| < 1$, because then $|z^n| = |z|^n$, and we know the sequence r^n converges to zero whenever $r \in (-1, 1)$.

Problem 0.6. Chapter II, section 1, exercise 15

(a) Open

(b) Closed

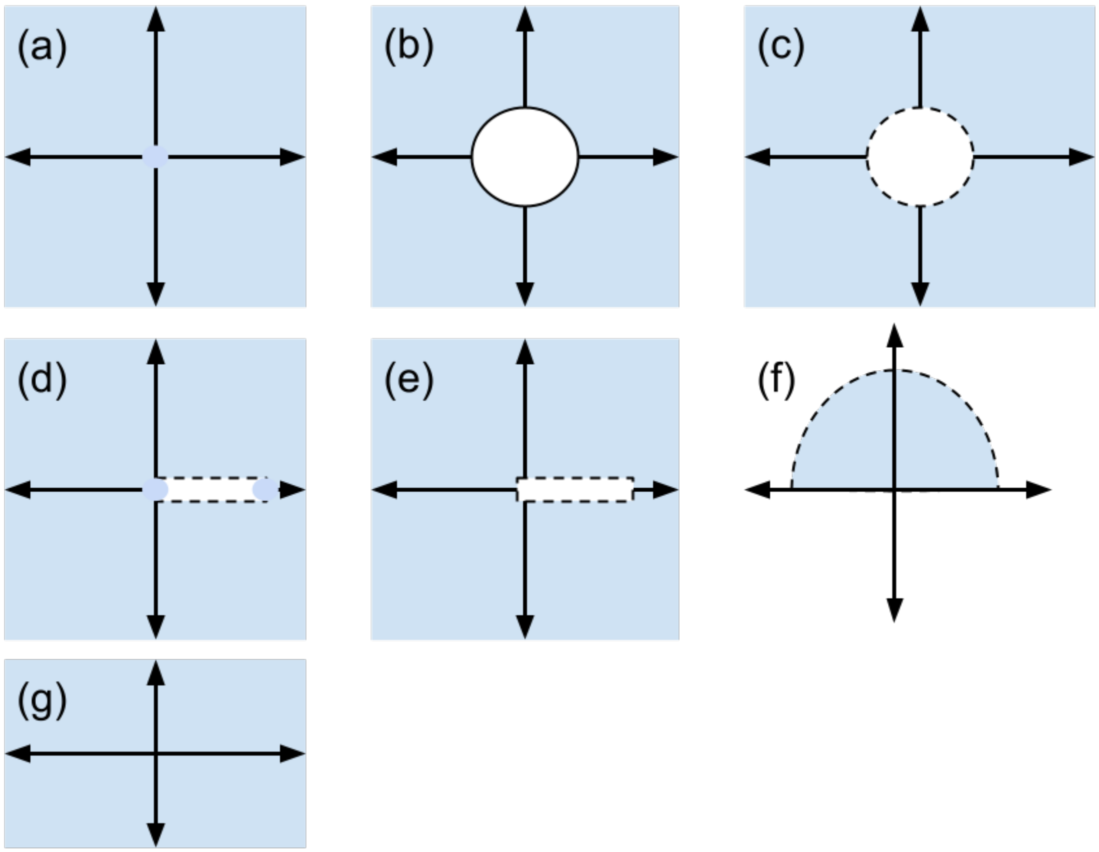
(c) Open

(d) Neither open nor closed

(e) Open

(f) Neither open nor closed

(g) Open and closed


(a)
(b)
(c)
(d)
(e)
(f)
(g)

Homework Assignment 2

MATH 132 LEC 1&2

Due April 13th, Sunday 11:59 PM

Please submit your work to Gradescope!

- I.6 Exercises: #2(a), #2(d),
- I.7 Exercises: #2,
- I.8 Exercises: #1, #2,
- II.1 Exercises: #2, #15.