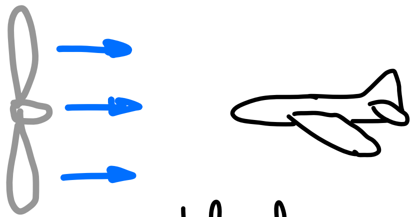
Dimensional Analysis

1. Motivation - Preliminaries

Wind Tunnel



model plane

Atmosphere

Actual Dimensions

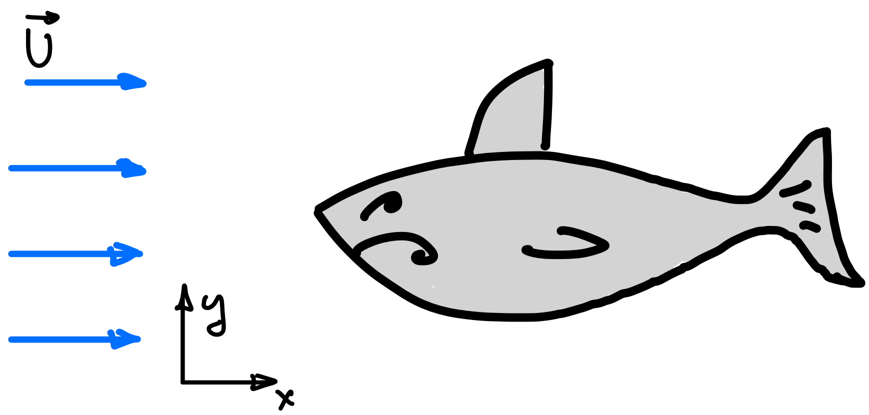
Drag force $F_{D,model}$ = known

Drag force F_D = ?

What is there relation ???

To find this relation we need :
- scaling
- similarity

2. Intoduction to forces on objects in a flow field

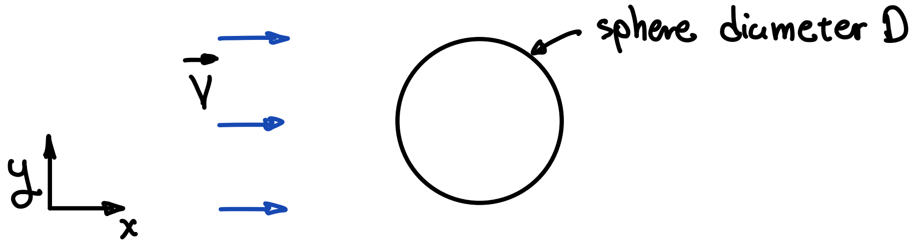


$$\overrightarrow{\text{Force on shark}} = \int_{\substack{\text{surface} \\ \text{of shark}}} T_{ij} \cdot \hat{n}\, dA$$ ← vector

integral of stresses over area

component of $\overrightarrow{\text{Force on shark}}$ in the direction of oncoming flow $\vec{U}$ is called drag

3. Drag on a sphere



- We want to do experiments to measure drag on spheres
- What do we need to measure?

Drag force	F_D
Diameter	D
Velocity	V
Density	ρ
Viscosity	μ

Approach #1: vary each parameter independently

use 10 values for each $D, V, \rho, \mu \rightarrow 10^4 = 10000$ experiments !!!

Is there a better way?

Approach #2: Use Buckingham Pi theorem

the problem can be reduced to $k-r$ "non-dimensional groups"

where k is the number of variables and
r is the number of reference dimensions

Variables		Dimensions
Drag force	F_D	$\frac{ML}{T^2}$
Diameter	D	L
Velocity	V	$\frac{L}{T}$
Density	ρ	$\frac{M}{L^3}$
Viscosity	μ	$\frac{M}{TL}$
	5 variables	3 dimensions

$$\left.\begin{matrix} k=5 \\ r=3 \end{matrix}\right\} k-r=2 \text{ variables}$$

Need to construct 2 <u>new</u> variables

- Variables must be non-dimensional
- Use combinations of variables I have
- Must use all variables

4. Non-Dimensional groups

Reynolds number $\quad Re = \rho\frac{DV}{\mu} = \frac{DV}{\nu}$ ← VERY IMPORTANT

ν ← kinematic viscosity

Drag coefficient $\quad C_D = \frac{F_D}{\frac{1}{2}\rho V^2 D^2}$ usually $C_D = \frac{F_D}{\frac{1}{2}\rho V^2 \text{Area}}$

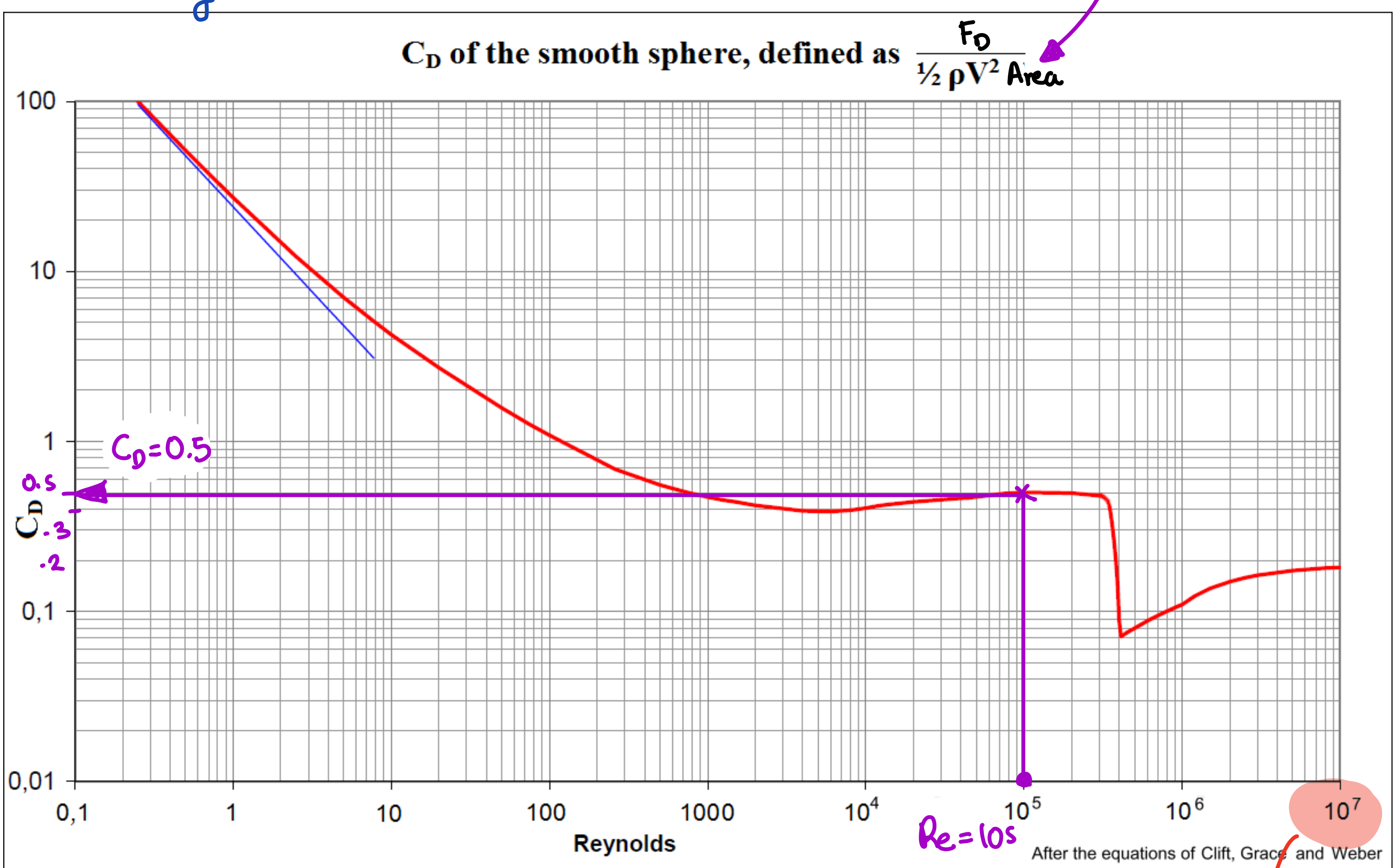
Area ← cross-sectional area

What does this mean?

C_D = function of Re

5. Drag coefficient

Definition



$Re = 10^7$

Example: Find F_D for these two spheres:

Experiment A	Experiment B
$V = 1\ m/s$	$V = 1\ m/s$
$D = 1\ m$	$D = 10\ m$
$\mu = 10^{-5}\ Pas$	$\mu = 0.1\ Pas$
$\rho = 1\ kg/m^3$	$\rho = 1000\ kg/m^3$
$Re = \dfrac{\rho V D}{\mu} = \dfrac{1 \times 1 \times 1}{10^{-5}} = 10^5$	$Re = \dfrac{\rho V D}{\mu} = \dfrac{1 \times 10 \times 1000}{0.1} = 10^5$
$C_D = 0.5$	$C_D = 0.5$
$F_D = \frac{1}{2} C_D \rho V^2 (\pi R^2) = 0.2N$	$F_D = 31.4\ N$

Same Re means same C_D <u>not</u> same F_D!

C_D depends mostly on shape of objects

$$F_D = \frac{1}{2} \rho V^2 \, Area \, C_D$$

fluid

flow

how big ← twice large → twice the drag

shape

double speed => four times the drag!

5. More non-dimensional "numbers"
... or... how fast is fast!

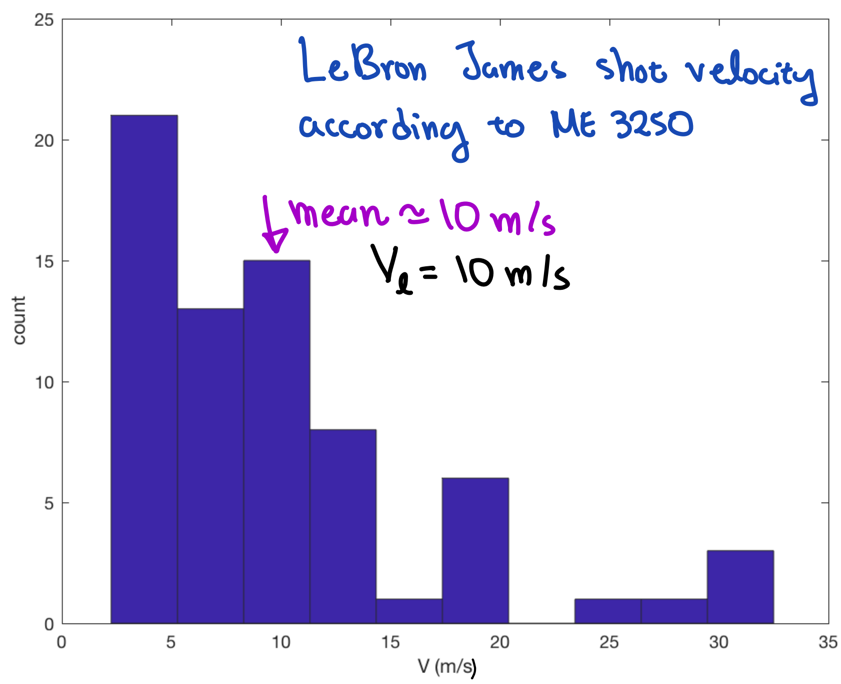
$$Re = 10^7 = \frac{\text{Velocity} * \text{Length}}{\text{kinematic viscosity}}$$

- Sphere is a basketball : Diameter = 0.25 m
- Fluid is air : kinematic viscosity = 10^{-5} m^2/s

Find V for $Re = 10^7$

$$Re = \frac{VD}{\nu} \Rightarrow V = \frac{\nu Re}{D} = \frac{10^{-5}\, 10^{7}}{0.25} = 400 \text{ m/s}$$

is this fast or slow??? we need a reference...



$$\text{LeBron James Number} = \frac{V}{V_\ell} = \frac{400}{10} = 40$$ (no units!)

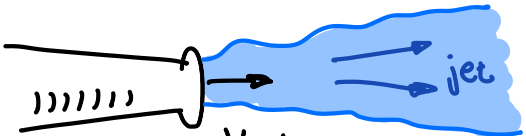
Another comparison:

$$\text{Mach Number} = \frac{V}{c} = \frac{400}{\sqrt{\gamma R T}} = 1.17 > 1$$ supersonic basketball!

speed of sound

6. Examples

6.1



$V = 1\, m/s$

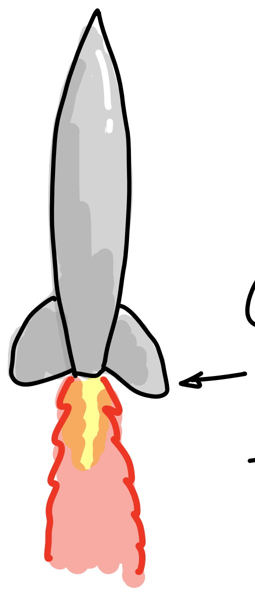
$D = 0.01\, m$

$\nu = 10^{-6}\, m^2/s$

What is the Reynolds number of the water jet?

$$Re = \frac{VD}{\nu} = \frac{1 \times 0.01}{10^{-6}} = 10^4$$

6.2



Given:

$D = 2m$

$\nu = 10^{-5}\, m^2/s$

$T = 3000\, K$

Mach number $= 2$

Find the Reynolds number of Jet at nozzle

Solution:

$$Ma = \frac{V}{c} \Rightarrow$$

$$V = Ma\, c$$

$$= 2\sqrt{\gamma R T}$$

this is Kelvin

$$= 2\sqrt{1.4 \times 300 \times 3000}$$

$$V = 2.2 \times 10^3\, m/s$$

$$Re = \frac{VD}{\nu} = \frac{2.2 \times 10^3 \times 2}{10^{-5}} = 4.4 \times 10^8$$

this is the Reynolds number of rocket exhaust jet

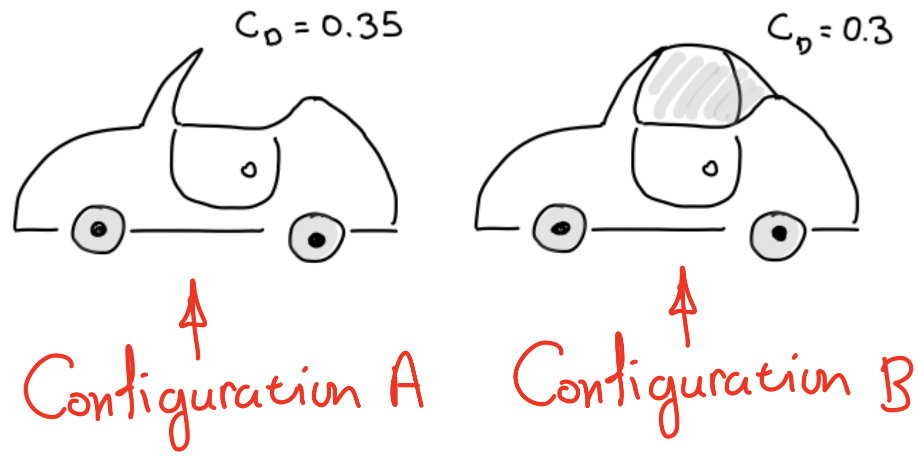
we can define the Reynolds number of the rocket

$$\text{Reynolds rocket} = \frac{V_{rocket} * \text{Length Rocket}}{\nu_{air}}$$

NOTE: Flows can have many Reynolds numbers!!!

6.3

For a convertible car, the drag of the car changes if the top is up or down. When the top is up, the car has a more streamlined shape and the drag coefficient is $C_D = 0.3$. When the top is down the drag coefficient increases to $C_D = 0.35$. With the top down, at what speed is the amount of power needed to overcome the drag the same as it is when the car is moving at 30 m/s with the top up? Assume that the frontal area remains the same when the top is either up or down. [Hint: power = force times velocity]



Solution:

Drag: $F_D = \frac{1}{2}\rho V^2 A C_D$

unknown

Configuration A: $F_{DA} = \frac{1}{2}\rho V_A^2 A C_{DA}$

Configuration B: $F_{DB} = \frac{1}{2}\rho V_B^2 A C_{DB}$

We know $Power_A = Power_B$

$$F_{DA} V_A = F_{DB} V_B$$

$$\frac{1}{2}\rho V_A^2 A C_{DA} V_A = \frac{1}{2}\rho V_B^2 A C_{DB} V_B$$

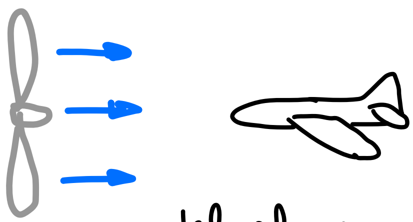
$$V_A = \left(\frac{C_{DB}}{C_{DA}}\right)^{1/3} V_B = \left(\frac{0.3}{0.35}\right)^{1/3} 30 = 28.5\ \frac{m}{s}$$

6. Drag of real planes

or any other force

What have we learned?

Wind Tunnel



model plane

Drag force $F_{D,model}$ = known

Atmosphere

Actual Dimensions

Drag force F_D = ?

1. Measure V, Area, ρ, μ and $F_{D,model}$
2. Calculate C_D Drag coefficient

   for sufficiently large Reynolds number, C_D does not strongly depend on Reynolds number

3. Use C_D to calculate F_D of real plane

$$F_D = \frac{1}{2} \rho_{air} V_{plane}^2 C_D (\text{Area real plane})$$