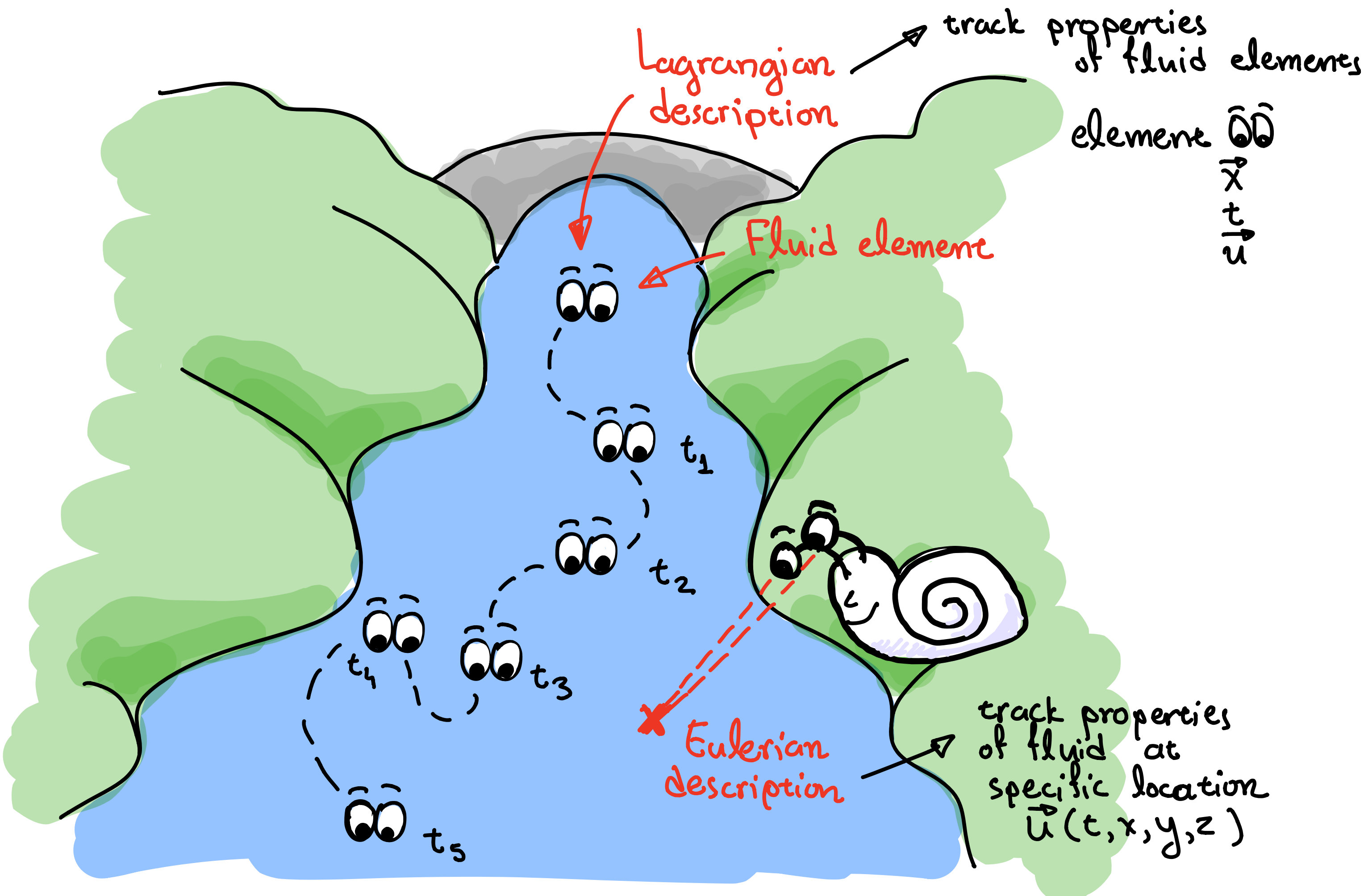
Fluid Kinematics

Chapter 4

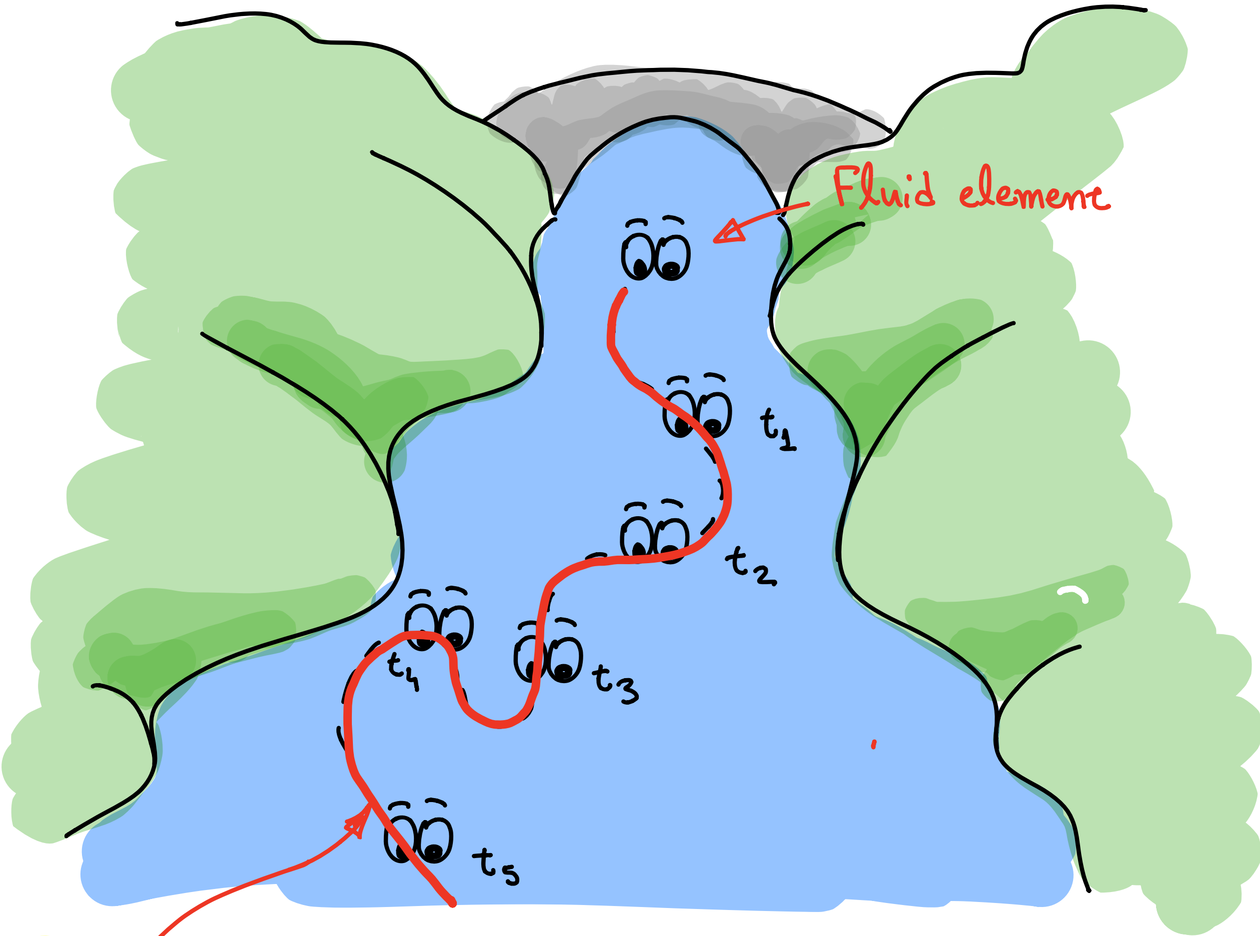
- The velocity field – Ch. 4.1 see also Module 3

- **Eulerian and Lagrangian Descriptions**

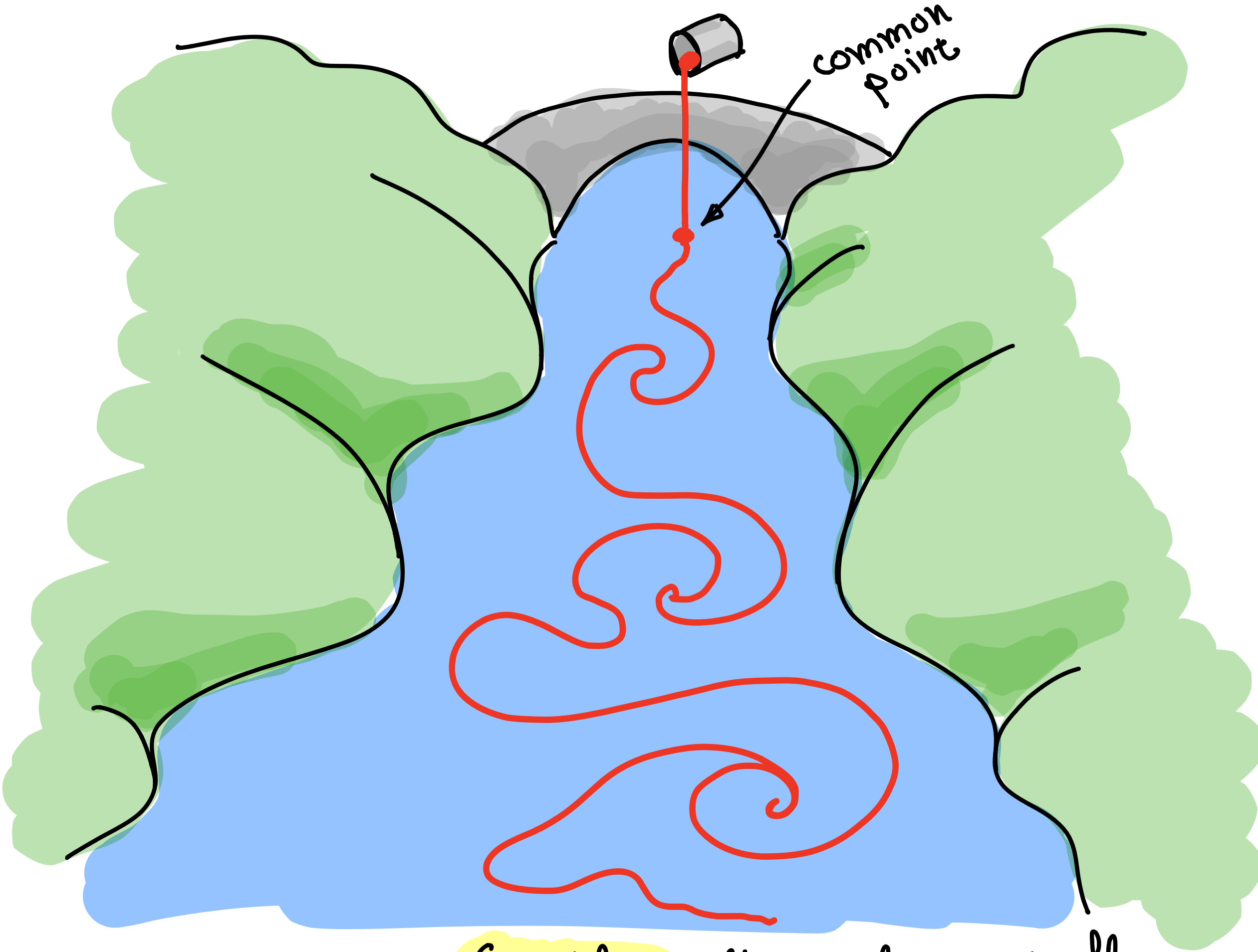


- Eulerian is usually "easier" to use, and most often used
- Lagrangian is useful in measurements

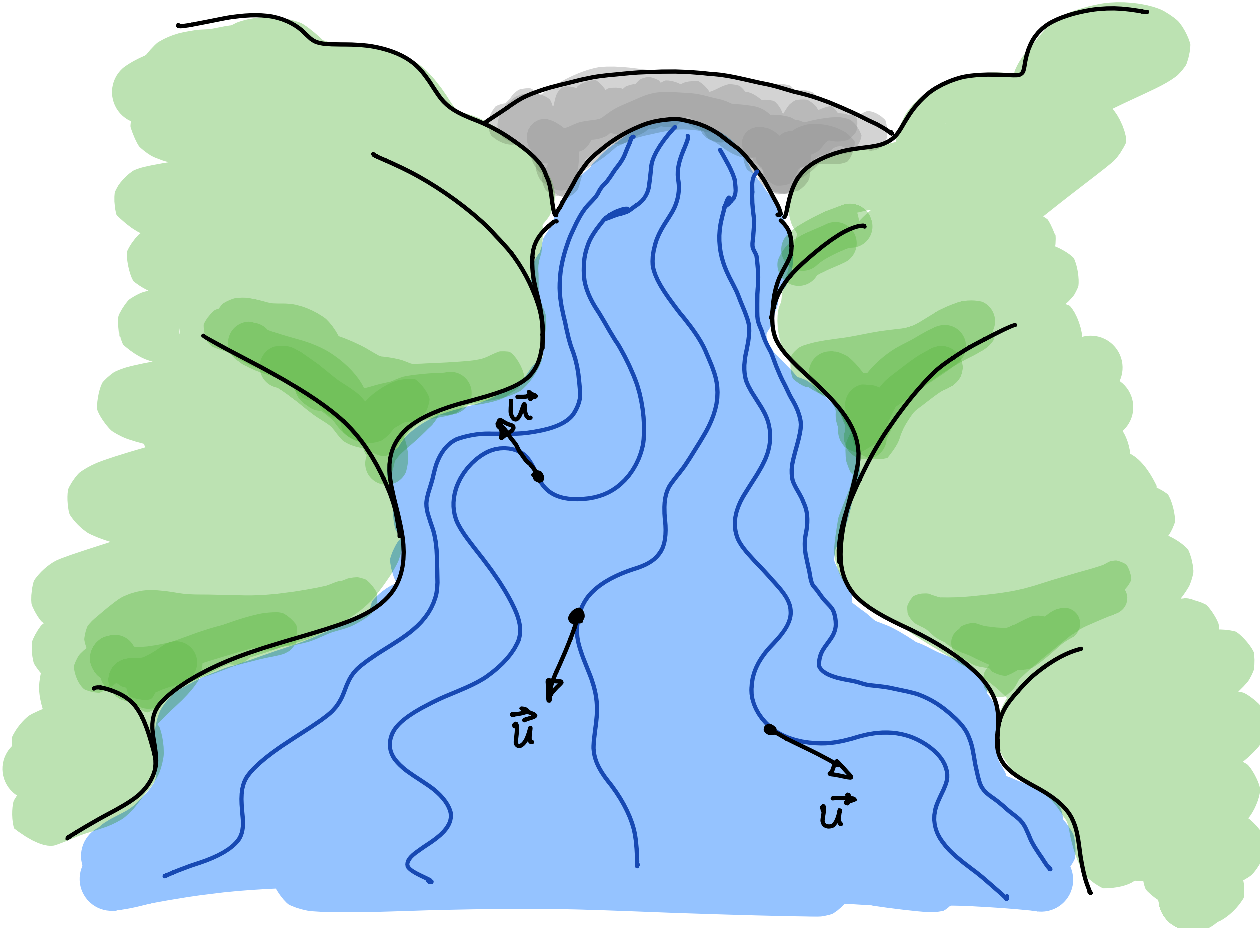
- Streamlines, Streaklines, and Pathlines (Ch. 4.1.4)



Pathline: trajectories of fluid elemenst



Streakline: all particles in the flow that have previously passed through a common point



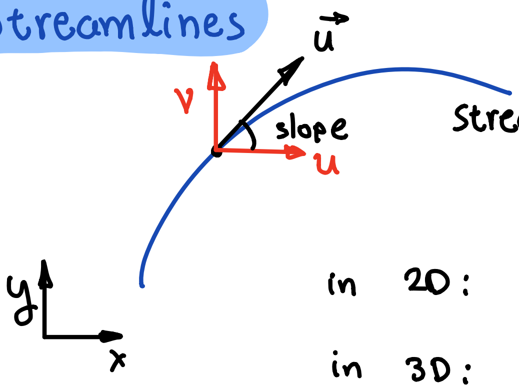
Streamlines: lines where velocity vector is tangent

What is the relation?

Only when flow is steady $\frac{\partial \vec{u}}{\partial t} = 0$ then

streamlines, pathlines and streaklines are the same

Streamlines



Streamline $y(x)$

in 2D: slope $\dfrac{dy}{dx} = \dfrac{v}{u}$

in 3D: $\dfrac{dx}{u} = \dfrac{dy}{v} = \dfrac{dz}{w}$

in 2D: start with $\dfrac{dy}{dx} = \dfrac{v}{u} \xrightarrow{\text{integrate}} y(x)$

Example 4.2:

Find equation for streamlines given velocity field

$\vec{u}$ components: $u(x,y) = -\dfrac{V_0}{\ell}x$

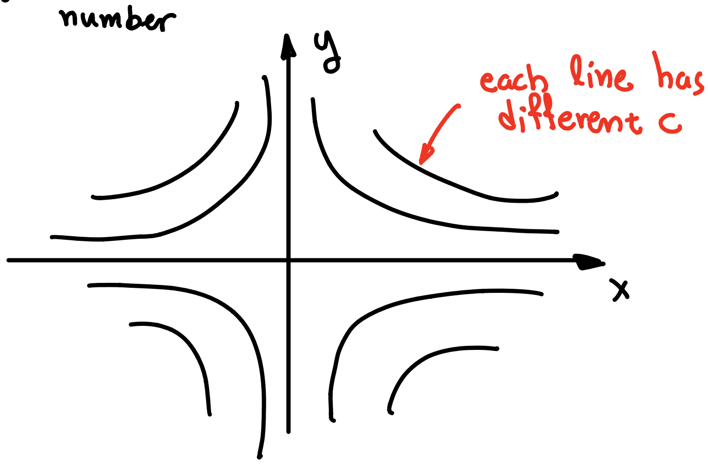
$$v(x,y) = \frac{V_0}{\ell}y$$

streamlines: $\dfrac{dy}{dx} = \dfrac{\frac{V_0}{\ell}y}{-\frac{V_0}{\ell}x} = -\dfrac{y}{x}$

$$\frac{dy}{y} = -\frac{dx}{x}$$

$$\int \frac{dy}{y} = -\int \frac{dx}{x} \rightarrow \ln y + \ln x = c_0 \rightarrow \ln xy = c_0$$

$$xy = \underbrace{e^{c_0}}_{\text{number}} \rightarrow xy = c$$



• Acceleration Field (Ch. 4.2)

$$\vec{a} = \frac{D\vec{u}}{Dt}$$

Material Derivative

guy we are operating on

$$\frac{D\,\text{ôô}}{Dt} = \frac{\partial\,\text{ôô}}{\partial t} + (\vec{u}\cdot\nabla)\,\text{ôô}$$

operator

$\frac{D}{Dt}$ is a differential operator and depents on the coordinate system.

Components

Cartesian 3D: $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

component

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

Convective part

transport of $\vec{u}$ by itself

unsteady

Reynolds Transport Theorem (Chapter 4.4)

Four important ingredients:

1. Extensive property = Mass * Intensive property

	B	$=$	$m * b$
mass	m	$=$	$m * 1$
momentum	$m\vec{u}$	$=$	$m * \vec{u}$
Kinetic energy	$\frac{1}{2}mV^2$	$=$	$m * \frac{1}{2}V^2$

2. Amount of B in a volume $\mathcal{V}$:

$$B = \int_{\mathcal{V}} \rho\, b\, d\mathcal{V}$$

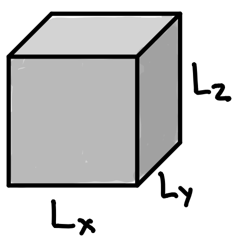
differential volume element

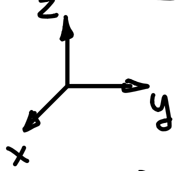
intensive property

density

volume integral

$$\text{e.g. } \overrightarrow{\text{momentum}} = \int_{\mathcal{V}} \rho\, \vec{u}\, d\mathcal{V}$$

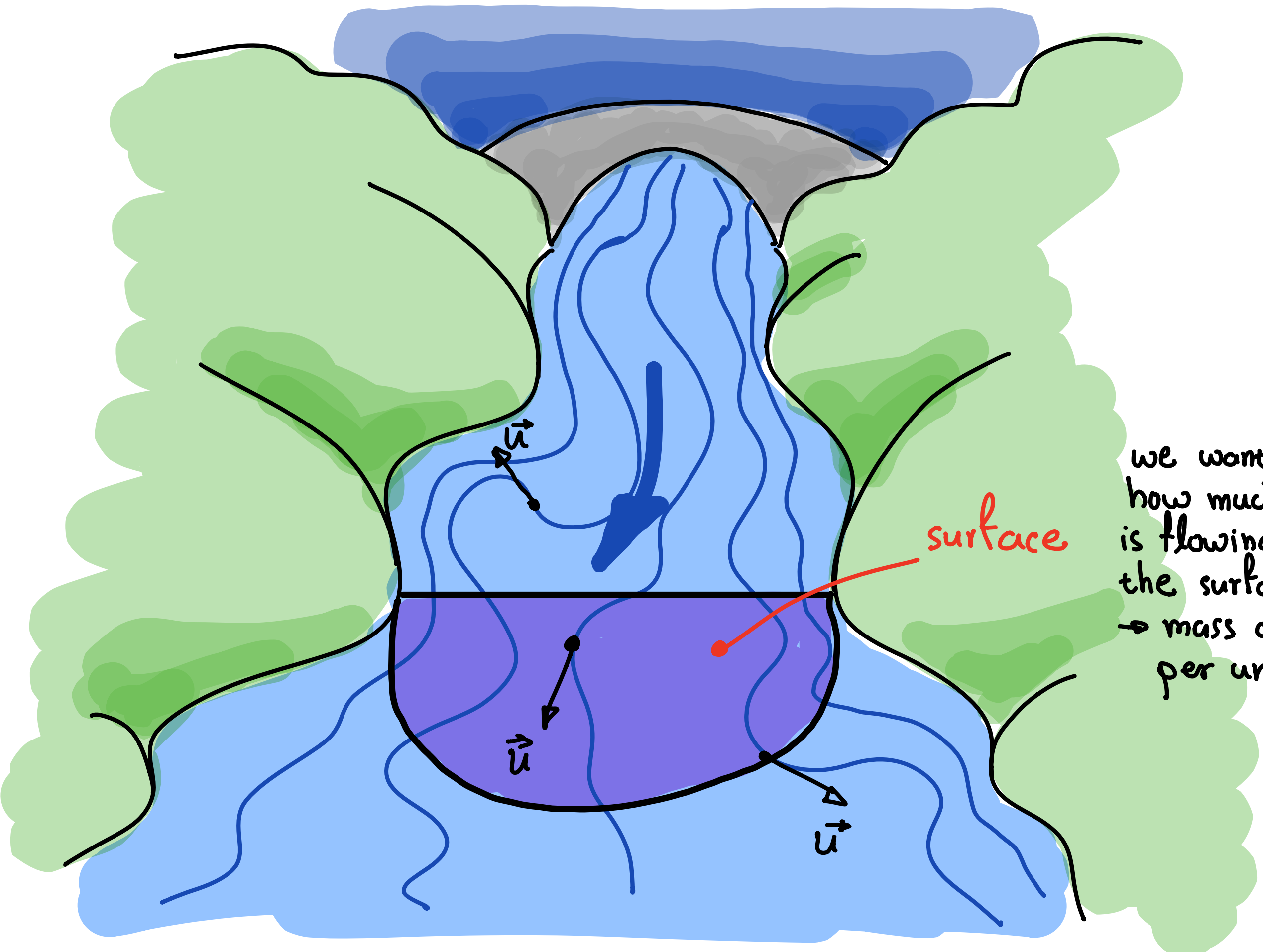
$\mathcal{V} =$ 



$$\overrightarrow{\text{momentum}}(t) = \int_0^{L_z}\int_0^{L_y}\int_0^{L_x} \rho(t,x,y,z)\, \vec{u}(t,x,y,z)\, dx\, dy\, dz$$

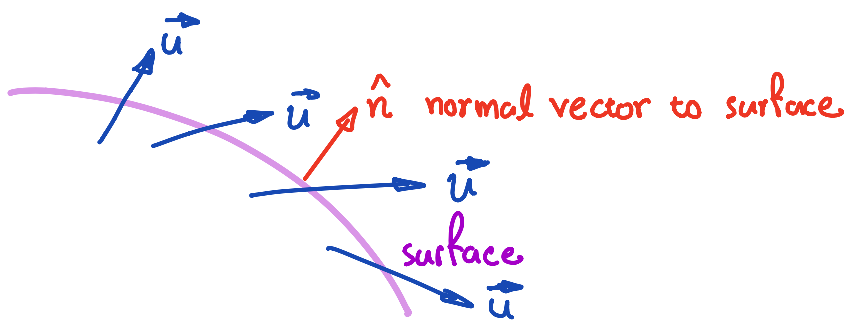
3. Amount of stuff flowing through a surface

→ mass per unit time

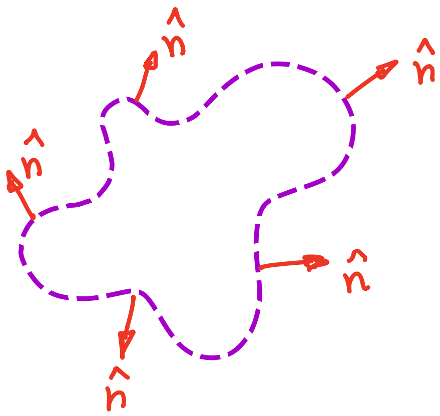
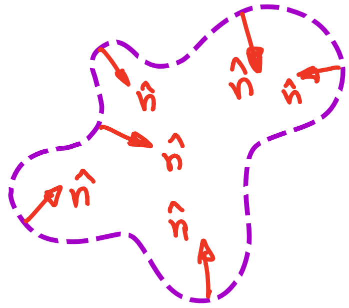
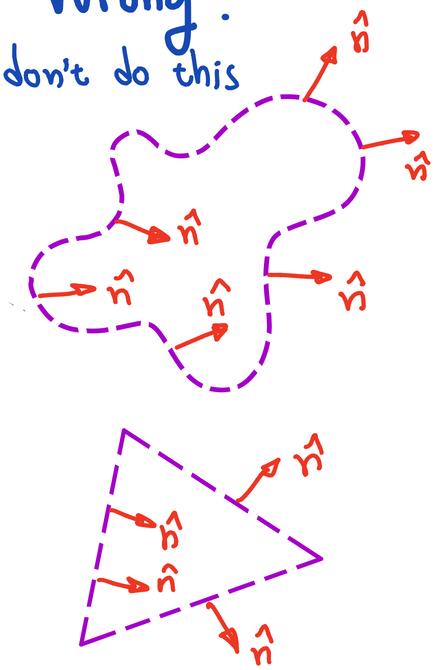


we want to find how much water is flowing through the surface
→ mass of water per unit time

in 2D:



- Choise of $\hat{n}$ direction is important because it defines positive flow direction

Best choice	OK choice (not recommended)	Wrong! don't do this
		
$\hat{n}$ points outwards	$\hat{n}$ points inwards	

surface integral

$$\dot{B} = \int_{surface} \rho\, b\, \underbrace{\vec{V}\cdot\hat{n}}_{\text{scalar}}\, ds$$

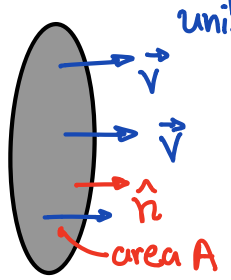
scalar : dot product of two vectors

intensive property

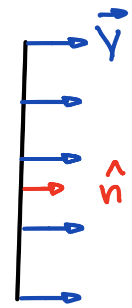
amount of stuff flowing

Example 1: Mass flux through area

uniform velocity $\vec{V}$



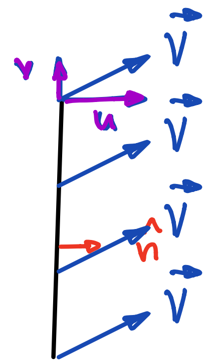
$\vec{V} = V\,\hat{i}$



$$\dot{m} = \int_A \rho \underbrace{b}_{1} \underbrace{\vec{V}\cdot\hat{n}}_{V\hat{i}\cdot\hat{i} \,=\, V} ds = \rho V A$$

Example 2: Mass flux through area

$$\vec{V} = u\,\hat{i} + v\,\hat{j}$$



$$\dot{m} = \int_A \rho \underbrace{b}_{1} \vec{V}\cdot\hat{n}\, ds = \rho u A$$

$$(u\hat{i} + v\hat{j})\cdot\hat{i} = u$$

Example 3: x-component of momentum through area A, flow as example 2

$$\dot{x}\text{-mom} = \int_A \rho b\, \vec{V}\cdot\hat{n}\, ds = \int_A \rho\, u\, \vec{V}\cdot\hat{n}\, ds = \rho u u A = \rho u^2 A$$

same as example 2

x-momentum component (intensive)

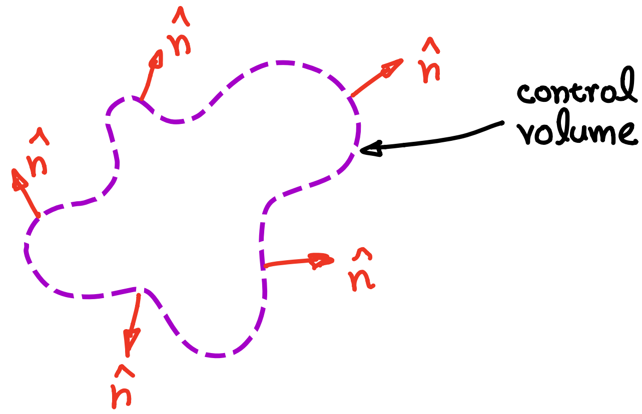
Example 4: y-component of momentum through area A, flow as example 2

$$\dot{y}\text{-mom} = \int_A \rho b\, \vec{V}\cdot\hat{n}\, ds = \int_A \rho\, v\, \vec{V}\cdot\hat{n}\, ds = \rho v u A = \rho u v A$$

same as example 2

y-momentum component (intensive)

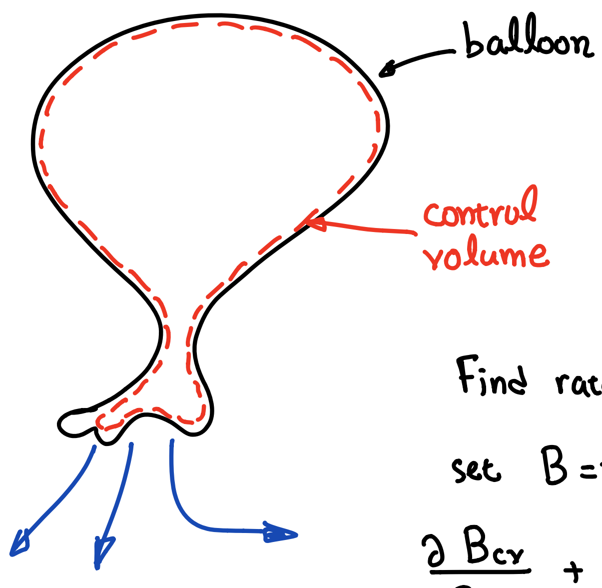
4. "Accounting" for fluid dynamics



Reynolds Transport Theorem (simple version)

$$\underbrace{\frac{\partial B_{cv}}{\partial t}}_{\text{unsteady term}} + \underbrace{\oint_{cs} \rho b \vec{V} \cdot \hat{n}\, ds}_{\text{flux of b through the CV boundary}} = 0$$

Example:



Find rate of change of mass in balloon.

set $B = m$ and $b = 1$

$$\frac{\partial B_{cv}}{\partial t} + \oint_{cs} \rho b \vec{V} \cdot \hat{n}\, ds = 0$$

$$\frac{\partial M}{\partial t} + \oint_{cs} \rho \vec{V} \cdot \hat{n}\, ds = 0$$

$$\underbrace{\frac{\partial M}{\partial t}}_{\text{rate of change of air mass in balloon}} = - \underbrace{\oint_{cs} \rho \vec{V} \cdot \hat{n}\, ds}_{\text{flux of air out of balloon}}$$