Problem 6-25

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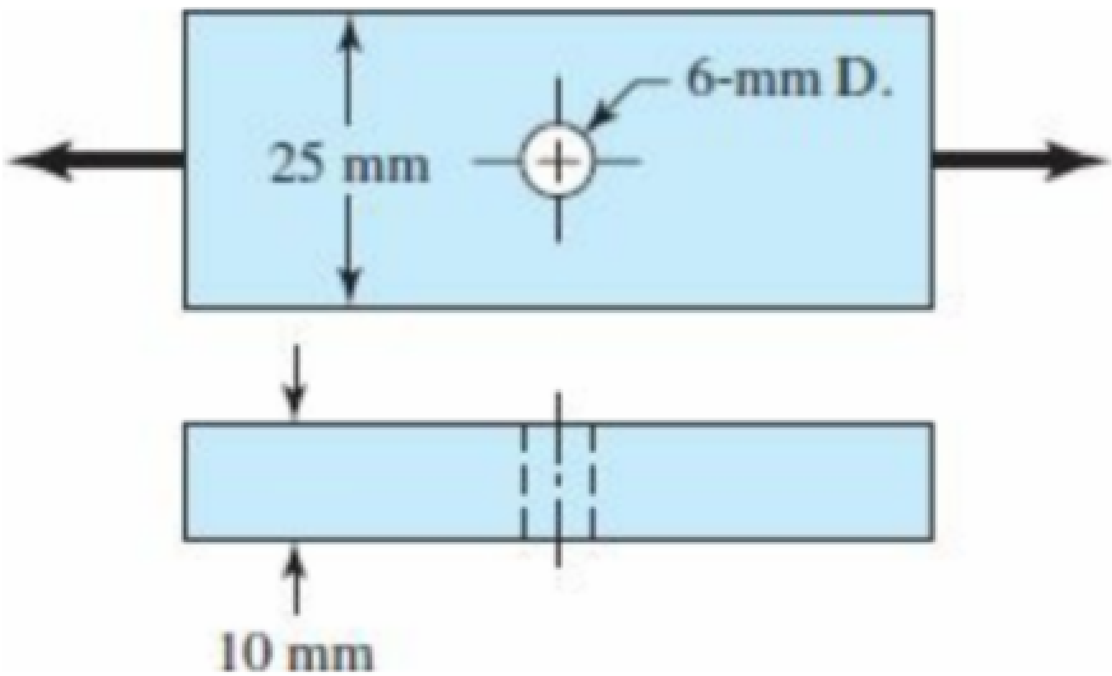
Video

https://youtu.be/Avqz7PnwQXo

Problem Statement

The cold-drawn AISI 1040 steel bar shown in the figure is subjected to a completely reversed axial load fluctuating between 28 kN in compression to 28 kN in tension. Estimate the fatigue factor of safety based on achieving infinite life and the yielding factor of safety. If infinite life is not predicted, estimate the number of cycles to failure.



Solution

For cold-drawn AISI 1040 steel, *Table A-20* tells us that our steel has a yielding strength, $S_y = 490$ MPa, and an ultimate tensile strength, $S_{ut} = 590$ MPa.

First we will look to find our yielding factor of safety. This is related to our yeilding strength, S_y, and our max stress, σ_{max}, which is a result from our highest loading:

$$n_y = \frac{S_y}{\sigma_{max}} \tag{1}$$

$$\sigma_{max} = \frac{F_{max}}{A_{min}} \tag{2}$$

Where $F_{max} = 28$ kN since our load is completly reversible and $A_{min} = 10(25-6)$ mm^2 which is the cross section at the hole. Plugging all these values into equations 2 and 1 yeilds:

[1]:
```
from thermostate import Q_, units
from numpy import sqrt, log10

S_y = Q_(490, "MPa")

F = Q_(28, "kN")
A = Q_(10*(25-6), "mm^2")

sigma_max = F/A

n_y = S_y/sigma_max
print("n_y =", n_y.to("dimensionless").round(4))
```

```
n_y = 3.325 dimensionless
```

$n_y = 3.325$

Similarly, to find our infinite life fatigue factor of safety:

$$n_f = \frac{S_e}{\sigma_a} \tag{3}$$

Where σ_a is the stress amplitude, which for our completly reversible stress is $K_f|\sigma_{max}|$. We have to account for our extra stress concentration factor K_f because of the hole in our element. We will find K_f by:

$$K_f = 1 + q(K_t - 1) \tag{4}$$

With q being the notch sensitivity of the material and K_t being the static consentration factor. To find q we will use the following equation:

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}} \tag{5}$$

And Neuber constant for axial loading between $340 \leq S_{ut} \leq 1700$ MPa:

$$\sqrt{a} = 1.24 - 2.25(10^{-3})S_{ut} + 1.6(10^{-6})S_{ut}^2 - 4.11(10^{-10})S_{ut}^3 \tag{6}$$

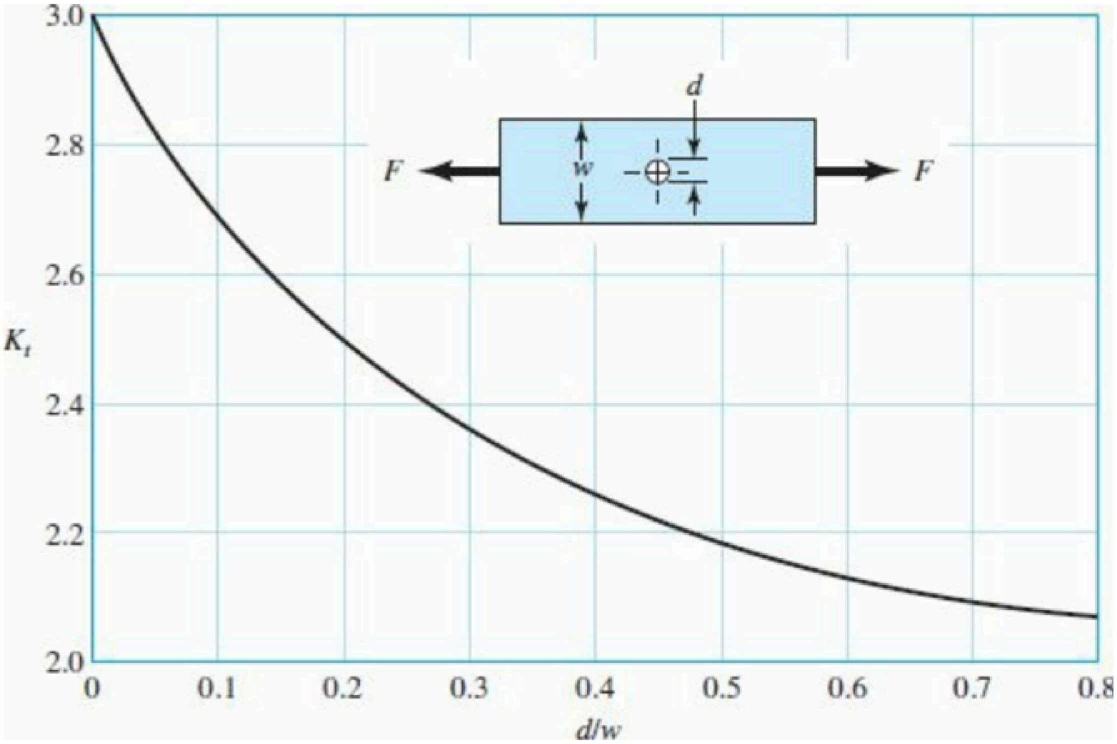
[2]:
```
S_ut = Q_(590, "MPa")
S_utm = S_ut.magnitude

rad_a = 1.24 - 2.25e-3*S_utm + 1.6e-6*S_utm**2 - 4.11e-10*S_utm**3
q = 1/(1+rad_a/sqrt(3))
```

```
print("q =", q)
```

```
q = 0.8181242152070024
```

We will then find the static stress consentration factor K_t using *Figure A-15-1* seen below:



With $d/w = 0.24$, we can say $K_t \approx 2.43$. Plugging in K_t and q into equation 4 yields:

[3]:
```
K_t = 2.43
K_f = 1 + q*(K_t-1)
print("K_f =", K_f.round(3))
```

```
K_f = 2.17
```

And similarly we can now find that σ_a is:

[4]:
```
sigma_a = K_f*sigma_max
print("sigma_a =", sigma_a.to("MPa").round(2))
```

```
sigma_a = 319.78 megapascal
```

Our last step in finding the infinite life fatigue factor of safety is find S_e. Using Marin factors, we will say:

$$S_e = k_a k_b k_c S_e' \tag{7}$$

Where our surface factor for cold drawn steel is:

$$k_a = 3.04S_{ut}^{-0.217} \tag{8}$$

Size factor for ossilating rectanguler cross-section loads with effective diameter betweem $7.62 \leq d_e \leq 51$ mm is:

$$k_b = 1.24(0.808\sqrt{10 \cdot 25})^{-0.107} \tag{9}$$

Load factor for axial loading is:

$$k_c = 0.85 \tag{10}$$

and estimated endurance limit for ultimate strengths less than 1400 MPa is:

$$S'_e = 0.5S_{ut} \tag{11}$$

Plugging all of these in gives us our infinite life strength of:

[5]:
```
S_ep = 0.5*S_ut
k_a = 3.04*S_utm**(-0.217)
print(k_a)
k_b = 1.24*(0.808*sqrt(10*25))**(-0.107)
k_c = 0.85

S_e = k_a*k_b*k_c*S_ep
print("S_e =", S_e)
```

```
0.7613751482674179
S_e = 180.25134659663473 megapascal
```

Using our stress amplitude and infinite life strength, we can say our fatigue factor of safety based on achieving infinite life is:

[6]:
```
n_f = S_e/sigma_a
print("n_f =", n_f.to("dimensionless").round(3))
```

```
n_f = 0.564 dimensionless
```

$$\boxed{n_f = 0.564}$$

This means that we will no see our machine element reach infinte life. To calculate the number of cycles to failure, we will use Basquin's equation for completly reversible stress which is:

$$N = \left(\frac{\sigma_a}{a}\right)^{1/b} \tag{12}$$

With constants:

$$a = \frac{(fS_{ut})^2}{S_e} \tag{13}$$

$$b = \frac{-1}{3} log\left(\frac{fS_{ut}}{S_e}\right) \tag{14}$$

$$f = 1.06 - 4.1(10^{-4})S_{ut} + 1.5(10^{-7})S_{ut}^2 \tag{15}$$

Plugging these in gives a value of N of:

[7]:
```
f = 1.06 - 4.1e-4*S_utm + 1.5e-7*S_utm**2
a = (f*S_ut)**2/S_e
b = -1/3 * log10(f*S_ut/S_e)

N = (sigma_a/a)**(1/b)
print(N.to("dimensionless"))
```

```
22760.471466809722 dimensionless
```

$N = 23,000$ cycles