

## Homework 9-1

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### Imports

```
[1]: from thermostate import Q_, State
```

---

### Definitions

```
[2]: substance = 'ammonia'

x_1 = Q_(1.0, 'dimensionless')
T_1 = Q_(-22.0, 'degC')

p_2 = Q_(16.3, 'bar')
T_2 = Q_(180.0, 'degC')

x_3 = Q_(0.0, 'dimensionless')
p_3 = p_2

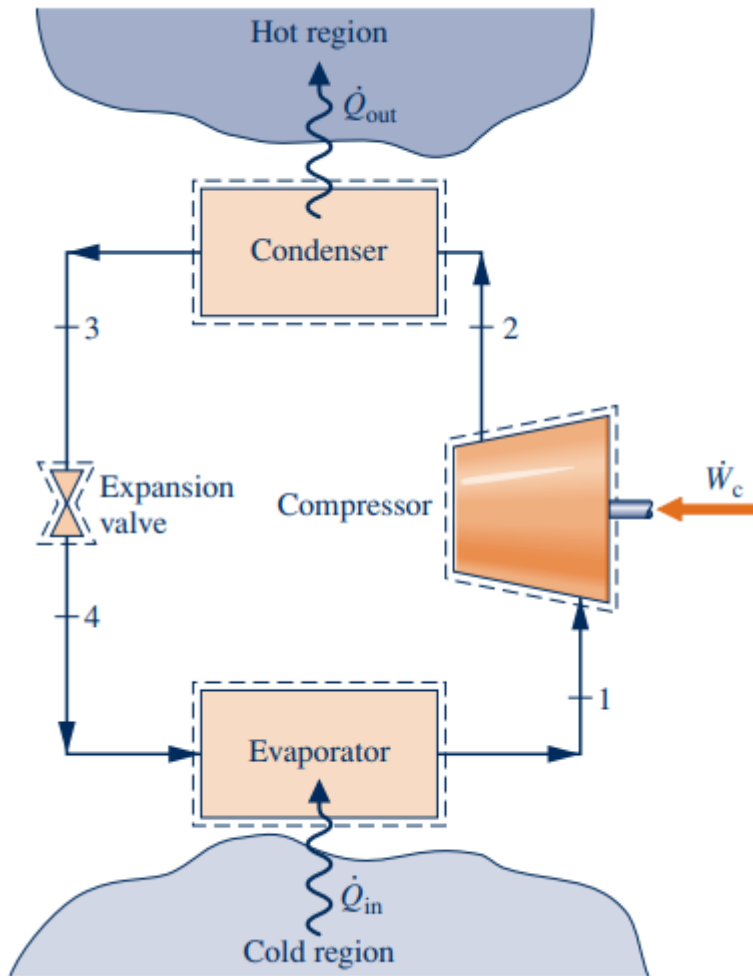
Qdot_in = Q_(127.0, 'kW')
```

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### Problem Statement

In a vapor-compression refrigeration cycle, ammonia exits the evaporator as a saturated vapor at -22 °C. The refrigerant enters the condenser at 16.3 bar and 180 °C, and exits as a saturated liquid. There is no significant heat transfer between the compressor and its surroundings, and the refrigerant passes through the evaporator with negligible change in pressure. If the refrigerating capacity is 127 kW, determine

1. the mass flow rate of the refrigerant, in kg/s,
2. the power input to the compressor, in kW,
3. the coefficient of performance (is it  $\beta$  or  $\gamma$ ?),
4. the isentropic compressor efficiency.



## Solution

Fixing States:

|    | State       | 1 | 2           |
|----|-------------|---|-------------|
| 1  | $T_1$       |   | $x_1$       |
| 2s | $p_2$       |   | $s_2 = s_1$ |
| 2  | $p_2$       |   | $T_2$       |
| 3  | $p_3$       |   | $x_3$       |
| 4  | $p_4 = p_1$ |   | $h_4 = h_3$ |

```
[13]: st_1 = State(substance, T=T_1, x=x_1)
st_2s = State(substance, p=p_2, s=st_1.s)
st_2 = State(substance, p=p_2, T=T_2)
st_3 = State(substance, p=p_3, x=x_3)
st_4 = State(substance, p=st_1.p, h=st_3.h)
```

### 1. Refrigerant Mass Flow (kg/s)

$$\frac{\dot{Q}_{in}}{\dot{m}} = h_1 - h_4 \quad (1)$$

```
[9]: mdot = Qdot_in/(st_1.h-st_4.h)
print(mdot.to("kg/s").round(2))
```

0.12 kilogram / second

**Answer:** 0.12 kg/s

### 2. Compressor Power Input (kW)

$$\dot{W}_{in} = \dot{m}(h_1 - h_2) \quad (2)$$

```
[11]: Wdot_in = mdot * (st_1.h-st_2.h)
print(Wdot_in.to("kW").round(2))
```

-53.17 kilowatt

**Answer:** 53.17 kW (into the compressor)

### 3. Coefficient of Performance (pick the right one!)

$$\beta = \frac{\dot{Q}_{in}}{|\dot{W}_{cycle}|} \quad (3)$$

$$\dot{W}_{cycle} = \dot{W}_{in} \quad (4)$$

```
[12]: beta = Qdot_in/Wdot_in
print(beta.round(2))
```

-2.39 dimensionless

**Answer:** 2.39

### 4. Isentropic Compressor Efficiency

$$\eta = \frac{h_{2s} - h_1}{h_2 - h_1} \quad (5)$$

```
[14]: eta = (st_2s.h-st_1.h)/(st_2.h-st_1.h)
print(eta.round(2))
```

0.79 dimensionless

**Answer:** 79%

## Homework 9-2

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### Imports

```
[1]: from thermostate import Q_, State
```

---

### Definitions

```
[2]: substance = 'r134a'

x_1 = Q_(1.0, 'dimensionless')
x_3 = Q_(0.0, 'dimensionless')

T_C = Q_(-5.0, 'degC')
T_H = Q_(24.0, 'degC')

dT_cond = Q_(11.0, 'delta_degC')
dT_evap = Q_(13.0, 'delta_degC')

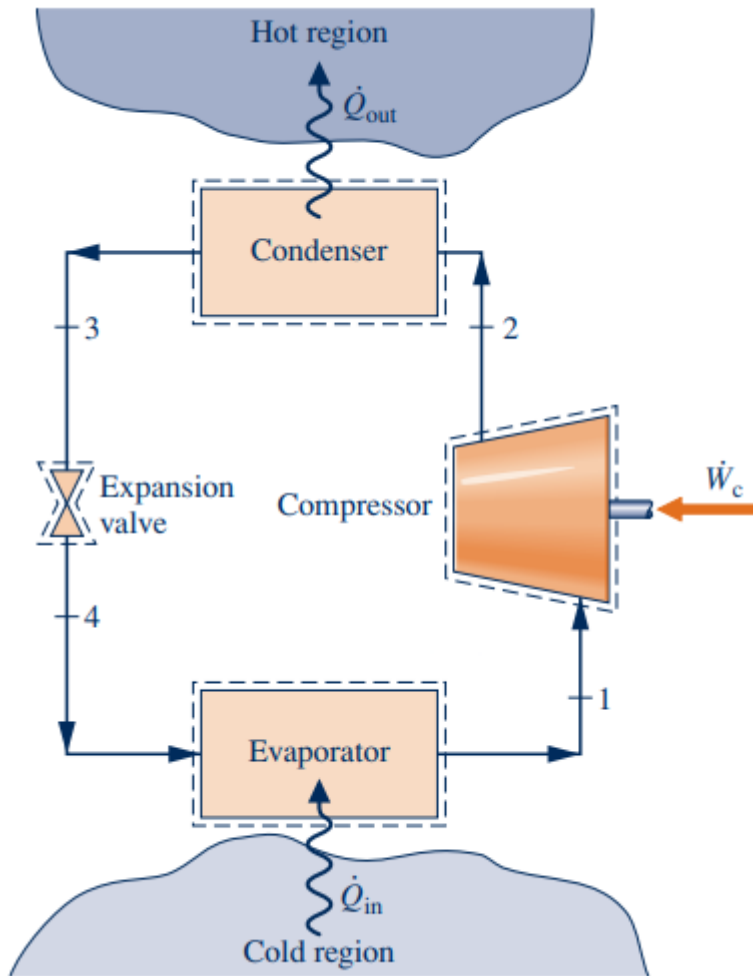
Qdot_out = Q_(-21.5, 'kW')
eta_c = Q_(0.8, 'dimensionless')
```

---

### Problem Statement

An office building requires a heat transfer rate of 21.5 kW to maintain the inside temperature at 24 °C when the outside temperature is -5 °C. A vapor-compression heat pump with r134a as the working fluid is to be used to provide the necessary heating. The compressor operates adiabatically with an isentropic efficiency of 80 %. The minimum temperature difference between the condenser and its surroundings is 11 °C, and between the evaporator and its surroundings it is 13 °C. The refrigerant exits the evaporator as a saturated vapor and exits the condenser as a saturated liquid. Determine

1. the necessary pressures in the evaporator and condenser, in bar,
2. the mass flow rate of the refrigerant, in kg/s,
3. the compressor power, in kW,
4. the coefficient of performance of the cycle (is it  $\beta$  or  $\gamma$ ?),
5. the coefficient of performance of the *Carnot cycle* operating between the same values of  $T_C$  and  $T_H$ . Briefly discuss some comparisons between the two cycles.
6. The system diagram for this cycle is *identical* to the refrigeration cycle in the previous problem (albeit with different property values at each state). Briefly state how it is possible that the same system can be either a refrigeration or a heat pump cycle.



## Solution

Fixing States:

|    | State       | 1 | 2              |
|----|-------------|---|----------------|
| 1  | $T_1$       |   | $x_1$          |
| 2s | $p_2 = p_3$ |   | $s_{2s} = s_1$ |
| 2  | $p_2 = p_3$ |   | $h_2$          |
| 3  | $T_3$       |   | $x_3$          |
| 4  | $p_4 = p_1$ |   | $h_4 = h_3$    |

```
[3]: T_3 = T_H + dT_cond
T_1 = T_C - dT_evap
st_3 = State(substance, T=T_3, x=x_3)
st_1 = State(substance, T=T_1, x=x_1)
st_2s = State(substance, p=st_3.p, s=st_1.s)
h_2 = (st_2s.h-st_1.h)/eta_c +st_1.h
```

```
st_2 = State(substance, p=st_3.p, h=h_2)
st_4 = State(substance, p=st_1.p, h=st_3.h)
```

```
[10]: print(st_1.T.to("K"))
      print(st_1.s.to("kJ/K/kg"))
      print(st_4.T.to("K"))
      print(st_4.s.to("kJ/K/kg"))
```

```
268.15 kelvin
1.7300319301342555 kilojoule / kelvin / kilogram
255.15000000023684 kelvin
1.1151941097716505 kilojoule / kelvin / kilogram
```

### 1. Necessary Pressures in the Evaporator and Condenser (bar)

```
[5]: p_cond = st_3.p.to("bar").round(2)
      p_evap = st_1.p.to("bar").round(2)
      print(p_cond)
      print(p_evap)
```

```
8.87 bar
1.45 bar
```

**Answer:** The condenser pressure will operate at 8.87 bar and the evaporator will operate at 1.45 bar

### 2. Refrigerant Mass Flow Rate (kg/s)

$$\dot{Q}_{out} = \dot{m}(h_3 - h_2) \quad (1)$$

```
[6]: mdot = Qdot_out/(st_3.h-st_2.h)
      print(mdot.to("kg/s").round(2))
```

```
0.12 kilogram / second
```

**Answer:** 0.12 kg/s

### 3. Compressor Power (kW)

$$\dot{W}_{in} = \dot{m}(h_1 - h_2) \quad (2)$$

```
[7]: Wdot_in = mdot*(st_1.h-st_2.h)
      print(Wdot_in.to("kW").round(2))
```

```
-5.46 kilowatt
```

**Answer:** 5.46 kW into the compressor

#### 4. Cycle Coefficient of Performance (pick the right one!)

$$\gamma = \frac{\dot{Q}_{out}}{\dot{W}_{cycle}} \quad (3)$$

```
[8]: gamma = Qdot_out/Wdot_in  
print(gamma.round(2))
```

3.94 dimensionless

**Answer:** 3.94

#### 5. Coefficient of Performance of the Carnot Cycle

Fixing States:

|   | <hr/>       |             |
|---|-------------|-------------|
|   | State       | 1 2         |
|   | <hr/>       |             |
| 1 | $T_C$       | $x_1$       |
| 2 | $T_2 = T_3$ | $s_2 = s_1$ |
| 3 | $T_H$       | $x_3$       |
| 4 | $T_4 = T_1$ | $s_4 = s_3$ |
|   | <hr/>       |             |

```
[9]: st_3 = State(substance, T=T_H, x=x_3)  
st_1 = State(substance, T=T_C, x=x_1)  
st_2 = State(substance, T=st_3.T, s=st_1.s)  
st_4 = State(substance, T=st_4.T, s=st_3.s)  
  
Qdot_out = mdot * (st_3.h-st_2.h)  
Wdot_in = mdot * (st_1.h-st_2.h)  
gamma = Qdot_out/Wdot_in  
print(gamma.round(2))
```

10.09 dimensionless

**Answer:** 10.09



6. How can the same physical system be both a refrigeration *and* a heat pump cycle?

**Answer:** The system can be both a refrigeration and a heat pump cycle depending on if the system you are acting on is the cold region or the warm region. The system makes the cold region colder and the hot region hotter. That's why it can make something cold or something hot, depending on where that "thing" is.

## Homework 9-3

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### Imports

```
[ ]: from thermostate import Q_, State
     from numpy import sqrt
```

---

### Definitions

```
[ ]: substance = 'water'

T = Q_(375.0, 'degC').to('K')
V = Q_(17.8, 'm**3')
m = Q_(513.0, 'kg')
p_limit = Q_(80.0, 'bar')

MW = Q_(18.0, 'g/mol') # molecular weight of water
Rbar = Q_(8.314462, 'J/mol/K')

# van der Waals constants (from Table A-24)
a_vdW = Q_(5.531, 'bar*m**6/kmol**2')
b_vdW = Q_(0.0305, 'm**3/kmol')

# Redlich-Kwong constants (from Table A-24)
a_RK = Q_(142.59, 'bar*m**6*K**(1/2)/kmol**2')
b_RK = Q_(0.02111, 'm**3/kmol')
```

---

### Problem Statement

Rocket Crab, a New Zealand-based company founded by rocket enthusiast and *actual* crab Peter Brine, is developing an interplanetary spacecraft to carry crabs to Europa, a watery moon of Jupiter. The spacecraft contains a 17.8 cubic meter water tank, in which the crabs live. The pressure within the tank cannot exceed 80 bar. The spacecraft requires 513 kg of water for the mission, and in the current design it is stored at 375 °C (yes, that's above the cooking temperature of crabs, but that's what you get when you don't write good design requirements, which is understandable, since—and this is true—crabs cannot read).

Evaluate the actual pressure and determine whether the safe pressure is exceeded using

1. the ideal gas equation of state (yes, even though water vapor is *not* an ideal gas):

$$pV = mRT$$

2. the van der Waals equation of state:

$$p = \frac{\bar{R}T}{\bar{v} - b} - \frac{a}{\bar{v}^2}$$

3. the Redlich-Kwong equation of state:

$$p = \frac{\bar{R}T}{\bar{v} - b} - \frac{a}{\bar{v}(\bar{v} + b)\sqrt{T}}$$

4. Thermostate.

5. Using your engineering judgement, do you think the design will withstand the pressure?

## Solution

### 1. Ideal Gas Equation of State

```
[ ]: R = Rbar/MW
p_IG = m*R*T/V
print(p_IG.to("bar").round(2))
```

**Answer:** 86.28 bar, this will cause the vessel to explode

### 2. van der Waals Equation of State

```
[ ]: n = m/MW
vbar = V/n
p_vdW = Rbar*T/(vbar - b_vdW) - a_vdW/vbar**2
print(p_vdW.to("bar").round(2))
```

**Answer:** 76.54 bar, this means our crabs are safe

### 3. Redlich-Kwong Equation of State

```
[ ]: p_RK = Rbar * T/(vbar-b_RK) - a_RK/(vbar*(vbar-b_RK)*sqrt(T))
print(p_RK.to("bar").round(2))
```

**Answer:** 74.44 bar, this means the vessel will not explode

### 4. Thermostate

```
[ ]: st = State(substance, T=T, v=V/m)
print(st.p.to("bar").round(2))
```

**Answer:** 75.18 bar, this means our spacecraft is fine

**5. Will the design withstand the pressure?**

**Answer:** Seeing these, I would say the crabs were correct and these temperatures and pressures will withstand the pressure.

## Homework 9-4

---

### Imports

```
[1]: from thermostate import Q_, State
```

---

### Definitions

```
[2]: sub_1 = 'air'
sub_2 = 'water'

V = Q_(3.2e4, 'm**3')
T = Q_(27.0, 'degC')
p = Q_(1.0, 'bar')

R_bar = Q_(8.314462, 'J/mol/K')

omega = Q_(0.01, 'dimensionless')
```

---

### Problem Statement

A lecture hall having a volume of 32000 cubic meters contains moist air at 27 °C, 1 bar, and a humidity ratio of 0.01. Determine

1. the relative humidity,
2. the dew point temperature, in °C,
3. the mass of water vapor contained in the room, in kg.

### Solution

#### 1. Relative Humidity

$$\omega = 0.622 \left( \frac{p_v}{p - p_v} \right) \quad (1)$$

$$p_g = p(T_v, x_{sat}) \quad (2)$$

```
[3]: p_v = omega*p/(0.622+omega)
st_v = State(sub_2,p=p_v, T=T)
st_g = State(sub_2,T=st_v.T,x=Q_(1,"dimensionless"))
st_g = State(sub_2,T=T,x=Q_(1,"dimensionless"))
print(st_v.p/st_g.p)
```

0.44344974526988207 dimensionless

**Answer:** 44%

## 2. Dew Point Temperature (°C)

$$T_{DewPoint} = T(p_v, x_{sat}) \quad (3)$$

```
[4]: st_DP = State(sub_2, p=p_v, x=Q_(1, "dimensionless"))  
     print(st_DP.T.to("degC").round(2))
```

13.84 degC

**Answer:** 13.84 °C

## 3. Water Vapor Mass (kg)

$$pV = n\bar{R}T \quad (4)$$

```
[5]: n_v = p_v*V/(R_bar*T)  
     MW = Q_(18.0, 'g/mol') # molecular weight of water  
     m_v = n_v * MW  
     print(m_v.to("kg").round(2))
```

365.2 kilogram

**Answer:** 365.2 kg

## Homework 9-5

---

### Imports

```
[1]: from thermostate import Q_, State
```

---

### Definitions

```
[2]: sub_v = 'water'
sub_a = 'air'

T_1 = Q_(28.0, 'degC')
p_1 = Q_(1.0, 'bar')
phi_1 = Q_(0.7, 'dimensionless')

Qdot_out = Q_(11.0, 'refrigeration_tons')

phi_2 = Q_(1.0, 'dimensionless')

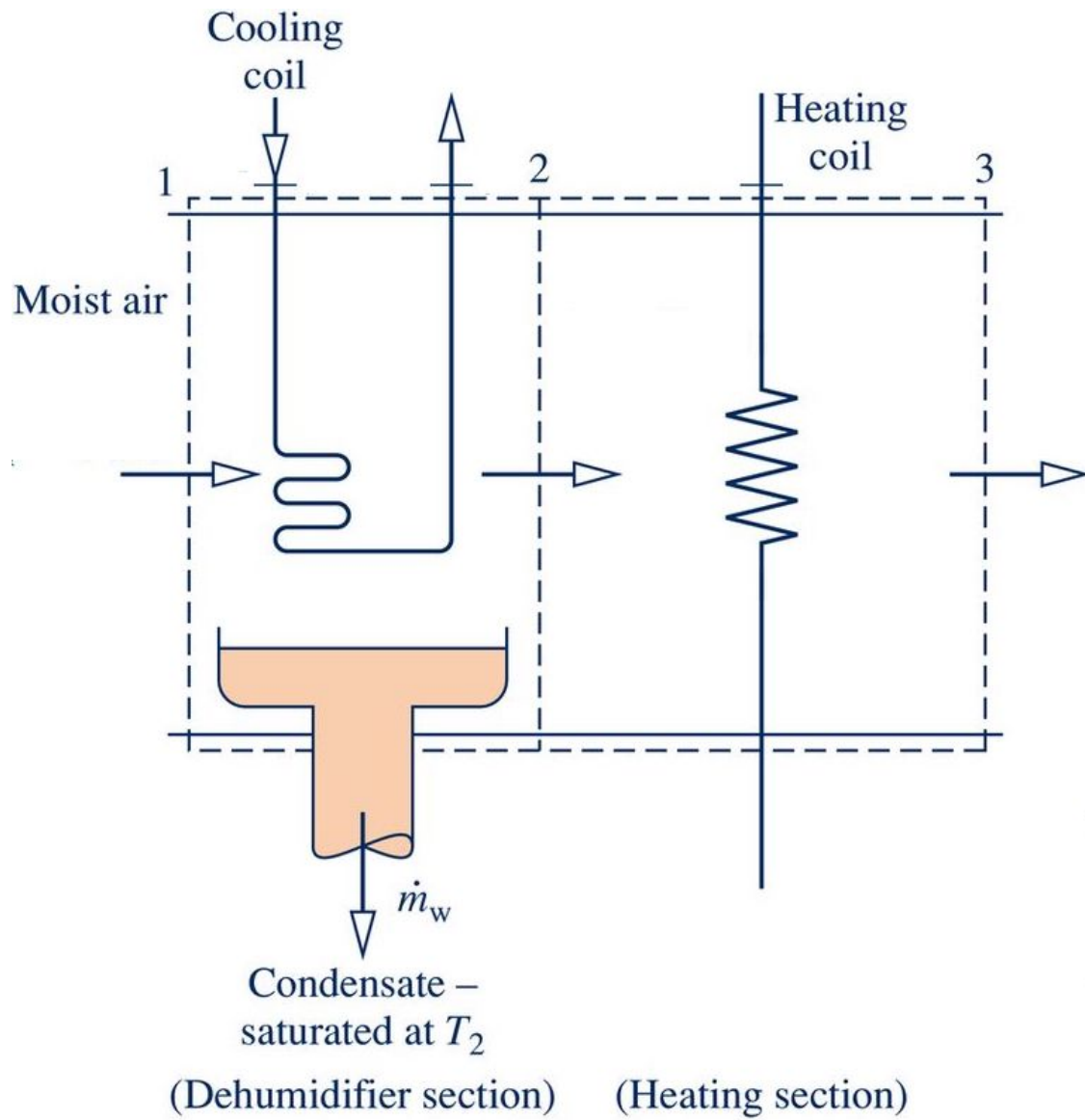
T_3 = Q_(24.0, 'degC')
p_3 = Q_(1.0, 'bar')
phi_3 = Q_(0.4, 'dimensionless')
```

---

### Problem Statement

An air conditioner operating at steady state takes in moist air at 28.0 °C, 1 bar, and 70 % relative humidity. The moist air first passes over a cooling coil in the dehumidifier unit and some water vapor is condensed. The rate of heat transfer between the moist air and the cooling coil is 11 refrigeration tons. Saturated moist air and condensate streams exit the dehumidifier at the same temperature. The condensed water is discarded, while the remiaining moist air is directed to the heating unit where it is heated to 24 °C, 1 bar and 40 % relative humidity. Neglecting kinetic and potential energy effects, determine

1. the temperature of the moist air and condensate exiting the dehumidifier at State 2, in °C,
2. the rate water is condensed and removed from the dehumidifier, in kg/min
3. the rate of heat transfer to the air passing through the heating unit, in kW.



**Solution**

**1. Temperature of the Moist Air and Condensate at State 2**

$$\phi_3 = \frac{p_{v3}}{p(T_{v3}, x_{sat})} \quad (1)$$

$$p_1 = p_2 = p_3 \quad (2)$$

$$\omega_2 = \omega_3 \quad (3a)$$



$$0.622 \left( \frac{p_{v2}}{p_2 - p_{v2}} \right) = 0.622 \left( \frac{p_{v3}}{p_3 - p_{v3}} \right) \quad (3b)$$

```
[3]: p_g3 = State(sub_v,T=T_3,x=Q_(1,"dimensionless")).p
p_v3 = p_g3*phi_3 #1
st_v3 = State(sub_v,T=T_3,p=p_v3)
h_v3 = st_v3.h
p_2 = p_3 #2
p_v2 = p_v3*p_2/(p_3) #3b
st_v2 = State(sub_v, p=p_v2,x=Q_(1, "dimensionless"))
T_2 = st_v2.T
print(st_v2.T.to("degC"))
```

9.583247882556691 degC

**Answer: 9.58 °C**

## 2. Mass Flow Rate of Condensate

$$\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_{a3} \quad (4)$$

$$\dot{m}_{w2} = \dot{m}_{v1} - \dot{m}_{v2} \quad (5)$$

$$\frac{\dot{m}_{w2}}{\dot{m}_a} = \omega_1 - \omega_2 \quad (6)$$

$$0 = \dot{Q} + \sum \dot{m}_i h_i - \sum \dot{m}_e h_e \quad (7a)$$

$$0 = \dot{Q}_{out} + \dot{m}_{a1} h_{a1} + \dot{m}_{v1} h_{v1} - \dot{m}_{a2} h_{a2} - \dot{m}_{v2} h_{v2} - \dot{m}_{w2} h_{w2} \quad (7b)$$

$$0 = \frac{\dot{Q}_{out}}{\dot{m}_a} + h_{a1} + \omega_1 h_{v1} - h_{a2} - \omega_2 h_{v2} - (\omega_1 - \omega_2) h_{w2} \quad (7c)$$

$$\omega_1 = 0.622 \left( \frac{p_{v1}}{p_1 - p_{v1}} \right) \quad (8)$$

```
[6]: st_g1 = State(sub_v,T=T_1,x=Q_(1,"dimensionless"))
p_v1 = st_g1.p*phi_1 #1

p_a1 = p_1 - p_v1
st_a1 = State(sub_a,T=T_1,p=p_a1)
h_a1 = st_a1.h
```

```

p_a2 = p_2 - p_v2
st_a2 = State(sub_a,T=T_2,p=p_a2)
h_a2 = st_a2.h

st_v1 = State(sub_v,p=p_v1,T=T_1)
p_v1 = st_v1.p
h_v1 = st_v1.h

h_v2 = st_v2.h

st_w2 = State(sub_v,T=T_2,x=Q_(0, "dimensionless"))
h_w2 = st_w2.h

omega_1 = 0.622*(p_v1/(p_1-p_v1)) #8
omega_2 = 0.622*(p_v2/(p_2-p_v2)) #8

mdot_a = (Qdot_out)/
↪ (h_a1+omega_1*h_v1-h_a2-omega_2*h_v2-(omega_1-omega_2)*h_w2) #7c

mdot_w2 = (omega_1-omega_2)*mdot_a #6
print(mdot_w2.to("kg/min"))

```

0.514585141185144 kilogram / minute  
 282.73324788255667 kelvin  
 301.15 kelvin

**Answer:** 0.515 kg/min

### 3. Rate of Heat Transfer in the Heating Unit

$$0 = \dot{Q} + \dot{m}_{a2}h_{a2} + \dot{m}_{v2}h_{v2} - \dot{m}_{a3}h_{a3} - \dot{m}_{v3}h_{v3} \quad (7d)$$

$$0 = \frac{\dot{Q}}{m_a} + h_{a2} + \omega_2 h_{v2} - h_{a3} - \omega_3 h_{v3} \quad (7e)$$

```

[5]: omega_3 = 0.622*(p_v3/(p_3-p_v3)) #8

st_a3 = State(sub_a,T=T_3,p=(p_3-p_v3))
h_a3 = st_a3.h

Qdot = -mdot_a*(h_a2+omega_2*h_v2-h_a3-omega_3*h_v3) #7e
print(Qdot.to("kW"))

```

13.417434659913031 kilowatt

**Answer:** 13.42 kW