ME 3250 Section 2 – Midterm #2

1. Pratt & Whitney makes the F119A turbofan engine for the F-22 Raptor. The F119 has a cool feature: it has thrust vectoring. Because of this new feature, Pratt engineers need to perform a new calculation of the forces on the engine's test stand. Let's help them figure this out!

   The engine area at the inlet (location 1) is $A_1 = 2$ m^2. The nozzle (location 2) has a rectangular cross section with height $h = 0.5$ m and depth $d = 0.8$ m.

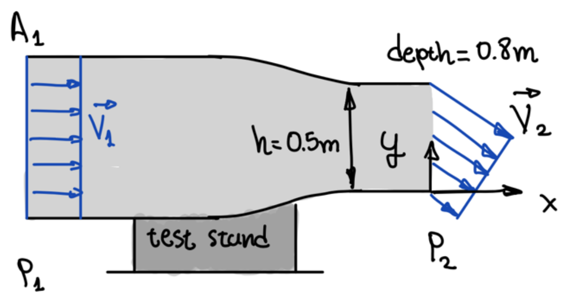
   At the inlet, the velocity is uniform and along the horizontal direction with magnitude V_1.
   At the nozzle, the velocity is $\overrightarrow{V_2} = 1000\, y\, \hat{\imath} - 20\, \hat{\jmath} \frac{\mathrm{m}}{\mathrm{s}}$. See coordinate system in figure.

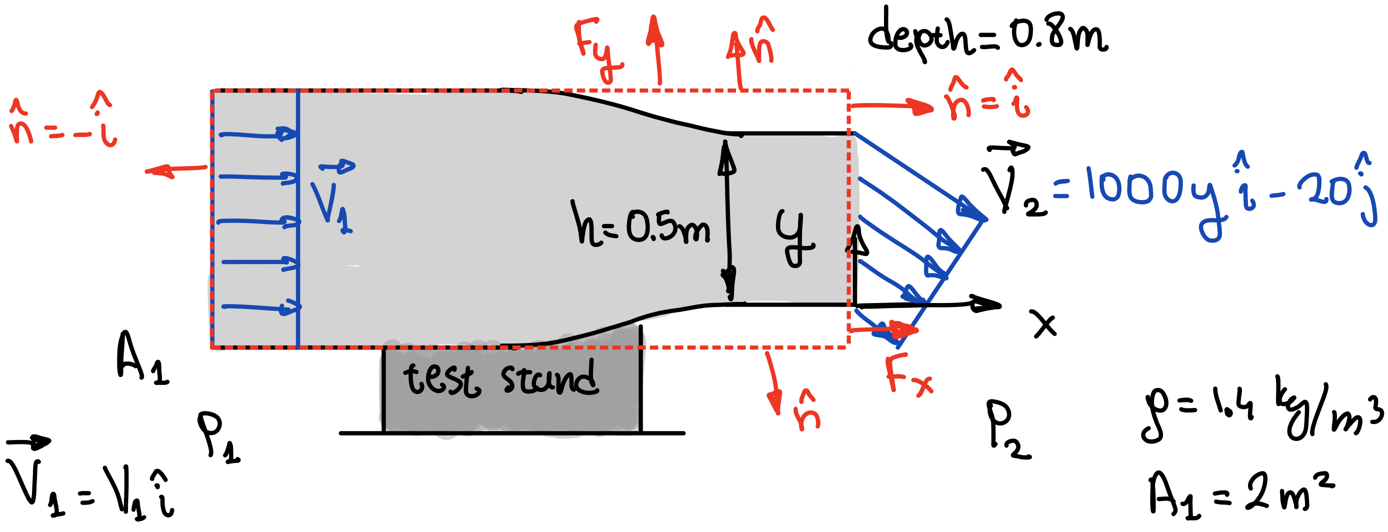
   At the inlet, the gage pressure is $p_1 = -1000$ Pa, and at the nozzle the gage pressure is zero $p_2 = 0$.

   Density is $\rho = 1.4$ kg/m^3 at both inlet and nozzle. The flow is steady.

   a. Draw the control volume. [10 points]
   b. Draw the normal vectors on the control volume. [10 points]
   c. Find the inlet velocity V_1? [20 points]
   d. Find the horizontal force required to hold the test stand. [30 points]
   e. Find the vertical force required to hold the test stand. [20 points]

   All your calculations must be *consistent* with the control volume and normal vectors.





$\rho = 1.4\ kg/m^3$

$A_1 = 2\,m^2$

$p_1 = -1000\ Pa$

$p_2 = 0$

CV: 10

$\hat{n}$: 10

Mass Conservation: 2

$$\cancel{\frac{\partial}{\partial t}\int_{cv} \rho\, dV} + \int_{cs} \rho\, \vec{V}\cdot\hat{n}\, dA = 0$$

0 steady flow

$$\int_{A_1} \rho\, \vec{V}_1\cdot\hat{n}\, dA + \int_{A_2} \rho\, \vec{V}_2\cdot\hat{n}\, dA = 0 \quad 2$$

$$\int_{A_1} \rho\,(V_1\hat{i})\cdot(-\hat{i})\, dA + \int_{A_2} \rho\,(1000y\,\hat{i} - 20\,\hat{j})\cdot\hat{i}\, dA = 0$$

$$-\rho V_1 A_1 + \rho d\, 1000 \int_0^h y\, dy = 0$$

$$V_1 = \frac{1000\, d\, h^2}{2A_1} = \frac{1000 \times 0.8 \times 0.5^2}{2 \times 2} = 50\ m/s$$

x-momentum conservation

$$\cancel{\frac{\partial}{\partial t}\int_{cv} u\rho\, d\forall} + \int_{cs} u\,\rho\, \vec{V}\cdot\hat{n}\, dA = \sum F_x$$

0 steady flow

$$\int_{A_1} V_1\, \rho\, V_1\hat{i}\cdot(-\hat{i})\,dA + \int_{A_2} 1000y\,\rho\left(1000y\hat{i}-20\hat{j}\right)\cdot\hat{i}\,dA = \underbrace{\underset{=A_1}{A_{left}}\,\underset{=P_1}{P_{left}} - A_{right}P_{right}}_{} + F_x$$

$$-\rho A_1 V_1^2 + \rho\, 1000^2 d \int_0^h y^2\,dy = A_1 P_1 - A_{right}\underset{=0 \text{ gage}}{P_{right}} + F_x$$

$$F_x = -A_1P_1 - \rho A_1 V_1^2 + \rho\, 1000^2 d\, \frac{h^3}{3}$$

$$= -2\times(-1000) - 1.4\times 2\times 50^2 + 1.4\times 1000^2\times 0.8\times \frac{0.5^3}{3}$$

$$= 41{,}667 \text{ N}$$

y-momentum conservation

$$\cancel{\frac{\partial}{\partial t}\int_{cv} v\rho\, d\forall} + \int_{cs} v\,\rho\, \vec{V}\cdot\hat{n}\, dA = \sum F$$

0 steady flow

$$\int_{A_1} 0\, \rho\, V_1\hat{i}\cdot(-\hat{i})\,dA + \int_{A_2} (-20)\,\rho\left(1000y\hat{i}-20\hat{j}\right)\cdot\hat{i}\,dA = F_y$$

no pressure terms

$$-20\times 1000\,\rho\, d\int_0^h y\,dy = F_y$$

$$F_y = -20\times 1000\,\rho\, d\,\frac{h^2}{2}$$

$$F_y = -20\times 1000\times 1.4\times 0.8\,\frac{0.5^2}{2} = -2800 \text{ N}$$

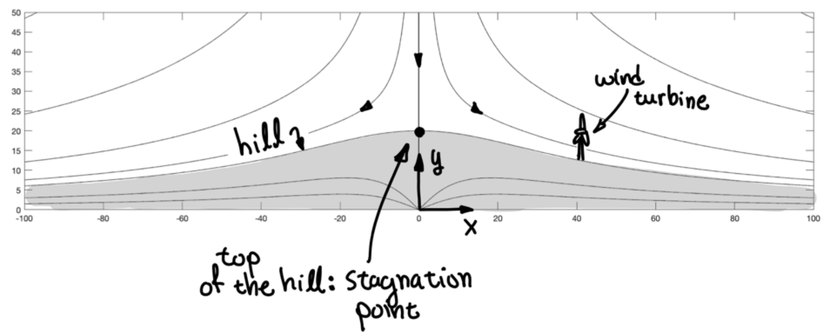
2. Eversource is building wind farms on Connecticut's rolling hills. Eversource's engineers are interested in a flow over the hill as in the Figure below. We will model the flow as a potential flow that can be constructed using the superposition of a "stagnation flow" with velocity potential:

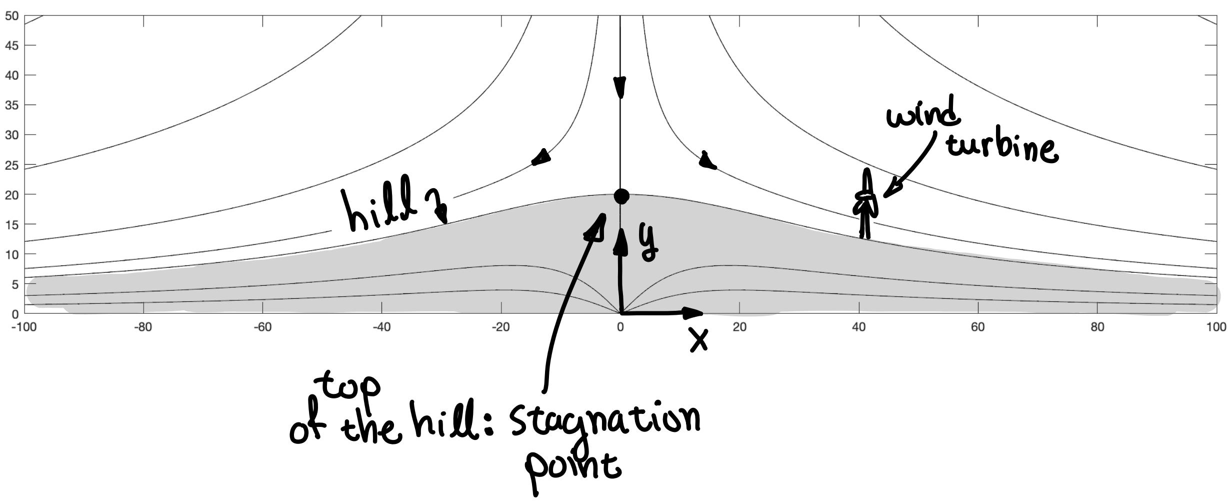
$$\phi_{st}(x, y) = x^2 - y^2$$

and a source of strength m located at $x = 0$ and $y = 0$, with velocity potential:

$$\phi_{sr}(x, y) = m \ln(x^2 + y^2)$$

   a. What is the velocity potential of the flow descending over the hill? [10 points]
   b. Determine the source strength m if the hill height is 20 m. [20 points]
   c. Find the velocity vector at location at $x = 40$ m and $y = 20$ m (you need to find both x and y components of the velocity) [20 + 10 points]





$\varphi_{st}(x,y) = x^2 - y^2$

$\varphi_{sr}(x,y) = m \ln(x^2+y^2)$

5 5

$\varphi_{flow}(x,y) = \varphi_{st} + \varphi_{sr} = x^2 - y^2 + m \ln(x^2+y^2)$

On y-axis only v velocity component is not zero

so at stagnation $v(0, y=20) = 0$ 5

$v(x,y) = \dfrac{\partial \varphi_{flow}}{\partial y} = -2y + \dfrac{2ym}{x^2+y^2}$ 5 set $v(0,20) = 0$ to find m: 5

$$-2\times 20 + \frac{2\times 20\, m}{20^2} = 0 \quad \Rightarrow m = 20^2 = 400$$ 5

Flow field using $m = 400$:

$$v(x,y) = -2y + \frac{800\, y}{x^2+y^2}$$ 10

$$u(x,y) = \frac{\partial \varphi_{flow}}{\partial x} = 2x + \frac{800\, x}{x^2+y^2}$$ 5

at location $x = 40\,m$ $y = 20\,m$

$$u(40,20) = 2\times 40 + \frac{800\times 40}{40^2+20^2} = 96 \text{ m/s}$$ 10

$$v(40,20) = -2\times 20 + \frac{800\times 20}{40^2+20^2} = -32 \text{ m/s}$$ 5

$$\vec{V}(40,20) = 96\,\hat{i} - 32\,\hat{j}$$