Fluid dynamics

What is a fluid?

- There is no good definition
- Fluids are gasses and liquids, taking the shape of their container

Absolute vs gage pressure

$P_{gage} = P_{abs} - P_{ambient}$

what engineers use

local

Fluids have a resistance to flowing

some fluids have more viscosity, resistance in flow

Warmer liquids are less viscous

Warmer gasses are more viscous

colide more and ∴ stick more

sheer thinning vs sheer thickening fluids

Resistance
stress

and ~~flowing~~
~~deformation~~
~~velocity gradient~~
rate of strain → differential fluid motion

linear stress-strain relationship, Newtonian

nonlinear relationship, non newtonian

μ proportionality coeff.

coeff. of viscocity

"dynamic viscocity" $\frac{N \cdot s}{m^2}$ or pa·s

$$\frac{\partial}{\partial t}(\text{Momentum}) = \overbrace{\vec{\nabla} \cdot}^{\text{divergence}} \vec{\tau} + \text{"other stuff"}$$

divide by dencity

stress

$$\frac{\partial}{\partial t}(\text{velocity}) = \frac{1}{\rho} \vec{\nabla} \cdot \vec{\tau} = \vec{\nabla} \cdot \left(\frac{\vec{\tau}}{\rho} \right) = \nabla \cdot \left(\frac{\mu \text{ "strain"}}{\rho} \right)$$

?

ν : kenimatic viscocity $m^2 s$

shear stress:

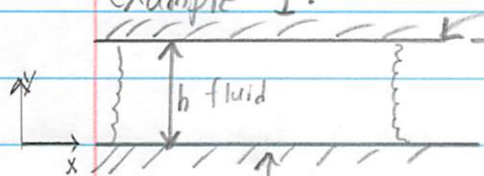
$$\tau = \mu \frac{\partial u}{\partial y}$$ ← fluid velocity

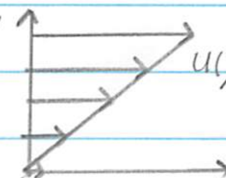
*μ must be positive

when τ is positive 

when τ is negative 

example 1:





Given: $\mu = 0.01$ Pa·s

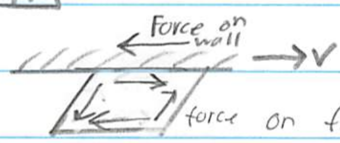
$h = 0.2$ m $V = 4$ m/s

Find the force on each plate. Area = $2m^2$

$$\tau = \mu \frac{\partial u}{\partial y} = \mu \cdot \frac{V}{h} = 0.2 \text{ Pa}$$

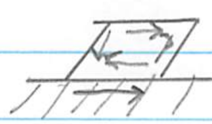
$$F = \tau A = \boxed{0.4N}$$

Top wall



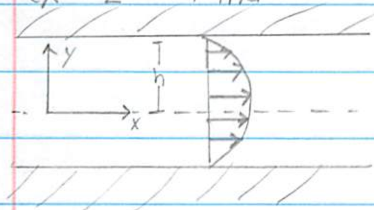
$F_{top\ wall} = 0.4$ to the left

Bottom wall



0.4 to the right

ex 2: Find τ in the fluid



$$u(y) = \frac{3V_0}{2}\left(1 - \left(\frac{y}{h}\right)^2\right)$$

$$\tau = \mu \frac{\partial u}{\partial y} = \mu\left(\frac{3V_0}{2}\left(-2 \cdot \frac{y}{h} \cdot \frac{1}{h}\right)\right) = \boxed{\frac{-3\mu V_0 y}{h^2}}$$

Fluid statics

no realitive motion between particles of the fluid

Pressure is a kind of stress

It does not depend on direction

Pascal's law: pressure is the isotropic part of the stress tensor

Pressure is a scalar

$$p = \frac{F}{A}$$

Eq of pressure

$$\nabla p = -\rho \vec{a}_t$$

pressure gradient

total fluid acceleration

$$\left.\begin{aligned} \nabla p &= \frac{\partial P}{\partial x}\hat{\imath} + \frac{\partial P}{\partial y}\hat{\jmath} + \frac{\partial P}{\partial z}\hat{k} \\ \vec{a}_t &= a_x\hat{\imath} + a_y\hat{\imath} + a_z\hat{k} \end{aligned}\right\} \text{cartesian}$$

Solving the pressure equation gives $p(x,y,z) \rightarrow F(x,y,z)$

Special pressure equation cases

case 1a: no motion and incompressible fluid

$\vec{a}_{t,ext} = 0$

$\frac{\partial P}{\partial x} = -\rho \cdot 0 \quad = 0$

$\frac{\partial P}{\partial y} = -\rho \cdot 0 \quad = 0$

$\frac{\partial P}{\partial z} = -\rho \cdot 0 - \rho g = -\rho g$ $\qquad \frac{\partial P}{\partial z} = -\rho g$

$\int_{P_1}^{P_2} dP = \int_{z_1}^{z_2} -\rho g dz \Rightarrow \boxed{P_2 = P_1 + \rho g \overbrace{(z_1 - z_2)}^{h}}$

Case 1b: no motion but compressible (like gasses)

$\frac{\partial P}{\partial z} = -\rho(z) g \qquad P_2 - P_1 = \int_{z_1}^{z_2} -\rho(z) g dz$

Case 2a: constant acceleration and incompressible

$\frac{\partial P}{\partial z} = -\rho g - \rho a$ ← acceleration on system

example: you have $\vec{a}_t = \langle 0, 3, 2 \rangle$ $\qquad \vec{\nabla} p = -\rho \vec{a}_t$

$\frac{\partial P}{\partial x} = 0$

$\frac{\partial P}{\partial y} = -\rho a_y$

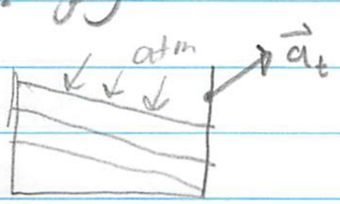
$\frac{\partial P}{\partial z} = -\rho a_z - \rho g$

Line of constant pressure, $dp = 0$

$dp = \frac{\partial P}{\partial y} dy + \frac{\partial P}{\partial z} dz = 0$

$-\rho a_y dy - \rho(a_z + g) dz = 0$

$\boxed{\frac{dz}{dy} = \frac{-a_y}{a_z + g}}$



Buoyancy:

$F_B = \overset{\text{fluid}}{\rho} g V_{submerged}$

Chapter 3: Bernoulli's equation

$P_1 + \frac{1}{2}\rho Vel_1^2 + \rho g z_1 = \text{Constant}$

P: pressure — ρ: density of fluid

Vel: Velocity — z: height

Assumptions to use equation:

1. no viscocity
2. Flow is steady
3. incompressible, (ρ = constant)
4. only valid along streamlines

Steady flow: for $\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$

the velocity field doesn't change in time

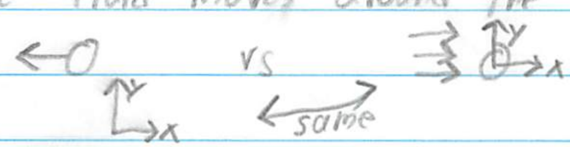
$$\frac{\partial \vec{V}}{\partial t} = \frac{\partial u}{\partial t}\hat{i} + \frac{\partial v}{\partial t}\hat{j} + \frac{\partial w}{\partial t}\hat{k} = \vec{0}$$

each individual $\frac{\partial}{\partial t}$ should be 0 too

Frame of Refrence

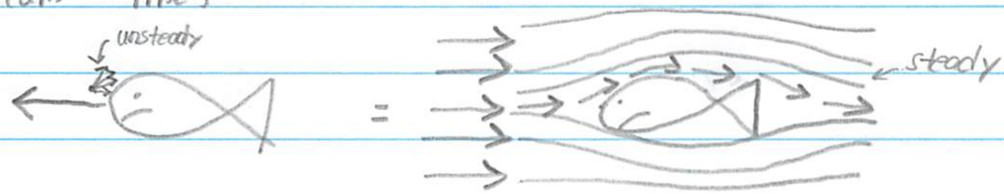
We like to take the frame of refrence of the fluid

- instead of moving through a fluid
- the fluid moves around the object



- unsteady flow can be steady in moving refrence frames

Stream lines

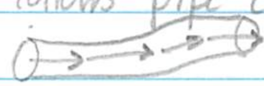


How the fluid moves around an object

Bernoulli only applies along the same streamline

How are streamlines given in problems?

- external flow is normally drawn
- internal flow follows pipe direction



Conservation of mass in flow

stuff going in must go out

$\dot{m}_1 = \dot{m}_2$

$\boxed{\rho Vel_1 A_1 = \rho Vel_2 A_2}$

$\vec{V}el_1$ and $\vec{V}el_2$ are normal to A_1 and A_2

$\vec{V}el_1$ and $\vec{V}el_2$ are the same over the whole area.

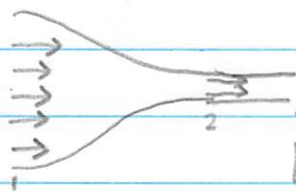
when incompressible, ρ = constant

$\vec{V}el_1 A_1 = \vec{V}el_2 A_2$

Volumetric flow rate: $Q = Vel\,A$ (m^3/s)

$\dot{M} = \rho\, Vel\, A = \rho Q$ (kg/s)

example 1: $\rho = 1000\ kg/m^3$



$V_1 = 10\ m/s$ $\quad V_2 = ?$

$A_1 = 1\ m^2$ $\quad A_2 = 0.5\ m^2$

$P_1 = 2 \cdot 10^5\ Pa$

$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$ ← 2 unknowns, p_2 and V_2

$\cancel{\rho} V_1 A_1 = \cancel{\rho} V_2 A_2$

plug it all in: $V_2 = 20\ m/s$, $p_2 = 0.5 \cdot 10^5\ Pa$

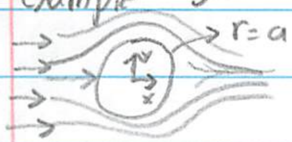
Bernoulli normal to streamlines

$$\frac{\partial P}{\partial n} = \frac{-\rho V^2}{R}$$ ← radius of streamline

$R \to \infty$ for straight stream lines

P = constant normal to straight stream lines

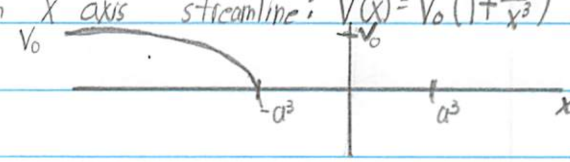
example 3:



V_0

P_0 cylinder with radius a

Velocity on x axis streamline: $V(x) = V_0\left(1 + \frac{a^3}{x^3}\right)$



$V = 0$ @ $x = -a^3$

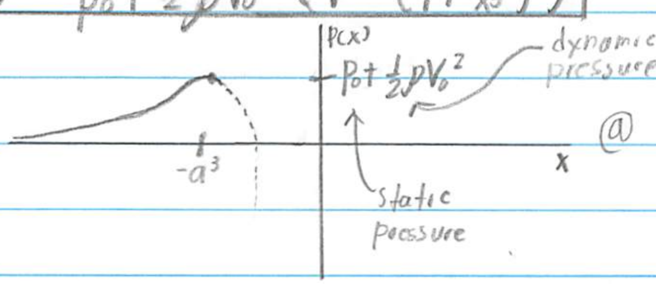
Find P(x) along the central streamline

$$p_0 + \frac{1}{2}\rho V_0^2 = p(x) + \frac{1}{2}\rho V^2(x)$$

$$p(x) = p_0 + \frac{1}{2}\rho V_0^2 - \frac{1}{2}\rho V^2(x)$$

$$\hookrightarrow \left(V_0\left(1 + \frac{a^3}{x^3}\right)\right)^2$$

$$\boxed{p(x) = p_0 + \frac{1}{2}\rho V_0^2\left(1 - \left(1 + \frac{a^3}{x^3}\right)^2\right)}$$



stagnation point @ $x = -a^3$

$V = 0$, $p = max = p_0 + \frac{1}{2}\rho V_0^2$

Eulerian and Lagrangian flow:

Lagrangian: track a mass of fluid as it flows

Fulerion: View fluid properties of a specific location

We ~~generally~~ only use Eulerian

Streamlines, streaklines, and Pathlines:

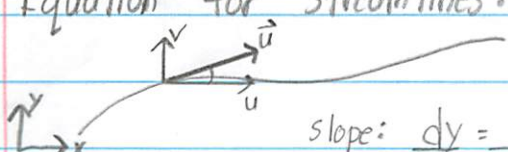
Pathlines: trajectory of a specific mass of fluid

Streak lines: all particles in the flow that have previously passed through a common point

streamlines: lines tangent to velocity vectors

These lines are all the same in steady fluid flow only

Equation for streamlines:



slope: $\frac{dy}{dx} = \frac{v}{u}$ in 2D space

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$ in 3D space

example: find streamline for field $u(x,y) = \frac{-V_0}{\ell}x$, $v(x,y) = \frac{-V_0}{\ell}y$

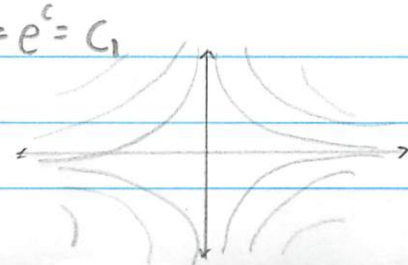
$$\frac{dy}{dx} = \frac{\frac{V_0}{\ell}x}{\frac{-V_0}{\ell}y} = \frac{-x}{y} \Rightarrow \int \frac{1}{y}\,dy = \int \frac{-1}{x}\,dx$$

$$\ln(y) + \ln(x) = C$$

$$\ln(xy) = C$$

$$xy = e^{c} = C_1$$

Different constants give



Acceleration fields in a fluid

$$\vec{a} = \frac{D\vec{V}}{Dt}$$

where D is the material derivative

$$\frac{D\vec{f}}{Dt} = \frac{\partial \vec{f}}{\partial t} + (\vec{V}\cdot\vec{\nabla})(\vec{f})$$

For cartesian 3D:

$$\vec{a} = \frac{D\vec{V}}{Dt} = \frac{\partial}{\partial t}\langle u,v,w\rangle + \left(\langle u,v,w\rangle \cdot \langle \tfrac{\partial}{\partial x}, \tfrac{\partial}{\partial y}, \tfrac{\partial}{\partial z}\rangle\right)\langle u,v,w\rangle$$

$$\begin{cases} a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \\ a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \\ a_z = \underbrace{\frac{\partial w}{\partial t}}_{\text{Steady/unsteady}} + \underbrace{u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}}_{\text{Convection}} \end{cases}$$

Reynolds Transport Theorem

Extensive property = Mass • Intensive property

ex: $B = m \cdot b$

mass: $m = m \cdot 1$

momentum: $m\vec{u} = m \cdot \vec{u}$

KE: $\frac{1}{2}mV^2 = m \cdot \frac{1}{2}V^2$

amount of extensive property B in a volume $\forall$

$$B = \int_{\forall} \rho b \, d\forall$$

amount of stuff through a surface

assume $\hat{n}$ point outward is positive

$$\dot{B} = \int_s \rho b \vec{V}\cdot\hat{n}\, ds$$

Flux example

mass flux through Area A with normal flow $\vec{v}$

$$\dot{m} = \int_A \rho b \vec{V} \cdot \hat{n} \, ds$$

$v\hat{\imath} \cdot \hat{\imath}$

$$= \rho v A$$

ex 2: mass flux

$$\vec{V} = u\hat{\imath} + v\hat{\jmath}$$

$$\vec{n} = \hat{\imath}$$

$$\dot{m} = \int_A \rho \underset{1}{b} \vec{V} \cdot \hat{n} \, ds$$

the intensive property (b) for mass is 1

$$= \rho u A$$

ex 3: find x component of momentum through ex 2

$$\dot{X}_{mom} = \int_A \rho \underset{u}{b} \vec{V} \cdot \hat{n} \, ds = \rho u^2 A$$

"Accounting" for fluid dynamics:

in a control volume

Reynolds transport theorem

$$\frac{\partial B_{cv}}{\partial t} + \int_{\underset{\text{control surface}}{cs}} \rho b \vec{V} \cdot \hat{n} \, ds = 0$$

change of extensive property in a control volume and flux of intensive property = 0

Chapter 1-3, Midterm review

Pressure $\xleftrightarrow{\text{Bernoulli}}$ Velocity

Statics $\rightarrow$ Pressure

$\Delta P = -\rho g \Delta z$

Pressure $\xrightarrow{F = A \cdot \text{stress}}$ Force

Bouancy $\rightarrow$ Force

Bouancy $\rightarrow$ Floating, submerged

$\dot{V}_{in} = \dot{V}_{out} \rightarrow \dot{M}_{in} = \dot{M}_{out} \rightarrow$ Velocity

Practice Midterm

1. $\vec{V} = -\frac{V_0}{\ell} x \hat{\imath} + \frac{V_0}{\ell} y \hat{\jmath}$, is the flow steady or unsteady

$$\frac{\partial \vec{V}}{\partial t} = 0\hat{\imath} + 0\hat{\jmath} = 0 \therefore \text{steady}$$

2. How many balloons does it take to fly if your mass = 82 kg and Volume = $0.08\ m^3$, $\rho_{atm} = 1.4\ kg/m^3$, $M_{balloon} = 0.1\ kg$, $V_{balloon} = 1.2\ m^3$.

Minimum amount: $F_b = F_g$

$$F_g = (m_{Human} + b m_b) g$$

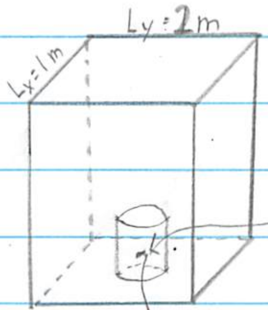
$$F_b = \rho g b V$$

$$b \rho g V_b = mg + b m_b g$$

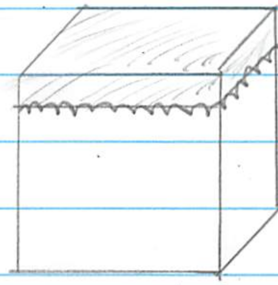
$$b(\rho g V_b - m_b g) = mg$$

$$b = \frac{mg}{\rho g V_b - m_b g} = 51.9 \text{ balloons} \Rightarrow 52 \text{ ballons}$$

3. There is a cup of water in an elevator and then the elevator accelerates downwords.



$g = -10\hat{k}\ \frac{m}{s^2}$



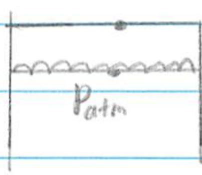
$P_{atm} = 10^5\ Pa$

$\vec{a} = 0$ $\quad V = 4m^3$ $\qquad\qquad \vec{a} = -12\hat{k}\ m/s^2$

a) What is the height of the water as the elevator drops?

$4 = 1 \cdot 2 \cdot h \quad \therefore \quad h = 2\,m$

b) what is the pressure at the top of the elevator?



$\text{pressure} = 10^5 \frac{N}{m^2} + 1000 \frac{kg}{m^3} \cdot 2 \frac{m}{s^2} \cdot 2m = \boxed{104\ kPa}$

c) What is the force on the ceiling?

$p = \frac{F}{A}$ $\quad A \leftarrow 2m^2$

$p = p_{up} - p_{down}$

$= 104\,kPa - 100\,kPa = 4\,kPa$

$F = 8000\ N = 8\ kN$

control Volume analysis of fluid flow

Momentum:

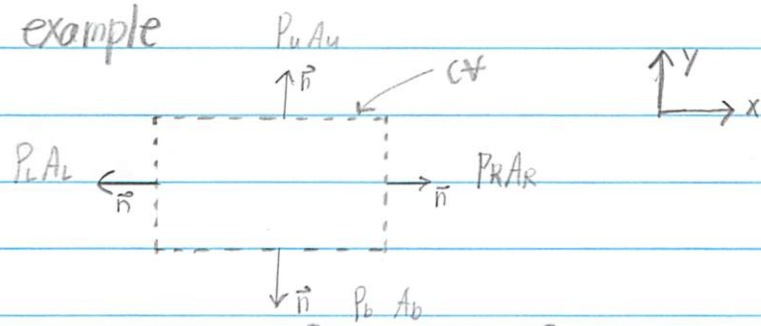
$$\frac{\partial}{\partial t}\int_{CV} \vec{V}\rho \, d\forall + \int_{CS} \vec{V}\rho \, \vec{V}\cdot\vec{n}\, ds = \sum \vec{F}$$

other forces and stresses that add to momentum

$\vec{V}$ because $\vec{V}$ is the intensive property of Momentum

$\vec{M}_{om} = m\vec{V}$

2D example

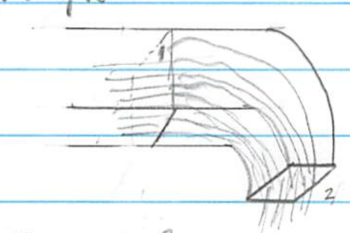


other ext forces

$$\text{Momentum-x}: \frac{\partial}{\partial t}\int_{CV} u\rho \, d\forall + \int_{CS} u\rho \, \vec{V}\cdot\vec{n}\, ds = P_L A_L - P_R A_L + F_x$$

$$\text{Momentum-y}: \frac{\partial}{\partial t}\int_{CV} v\rho \, d\forall + \int_{CS} v\rho \, \vec{V}\cdot\vec{n}\, ds = P_b A_b - P_u A_u + F_y$$

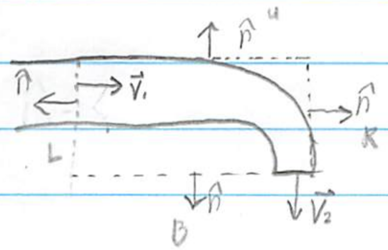
example



Determine the force to keep the Pipe in place. Assume ρ is constant, Steady flow, $\vec{V}$ is uniform.

$A_1 = 0.01\, m^2$ $\quad$ $A_2 = 0.0025\, m^2$ $\quad$ $P_2 = P_{atm} = 100$ kPa

$P_1 = 101$ kPa $\quad$ $V_2 = 16\, m/s \downarrow$ $\quad$ $\rho = 1.4\, kg/m^3$



$\vec{V}_1 = \langle u_1, 0, 0 \rangle$

$\vec{V}_2 = \langle 0, -V_2, 0 \rangle = \langle 0, -16, 0 \rangle$

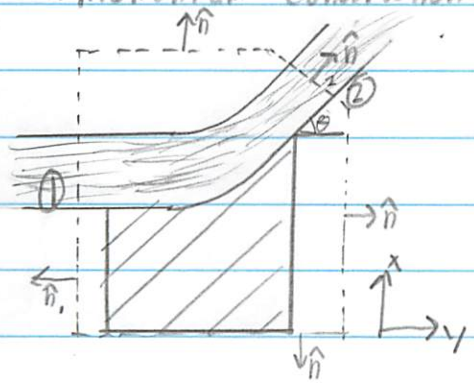
no, unsteady term: mass conservation: $\int_{CS} \rho \vec{V}\cdot\vec{n}\, ds = 0$

$$= \int_L \rho \vec{V}\cdot\vec{n}_L\, ds + \int_R \rho \vec{V}\cdot\vec{n}_R\, ds$$

$$= \int_{A_1} \rho \langle u_1, 0, 0\rangle \cdot \langle -1, 0, 0\rangle dA_1 + \int_{A_2} \rho \langle 0, -16, 0\rangle \cdot \langle 0, -1, 0\rangle dA$$

Procedure for solving problems

1. Draw CV
2. Draw $\hat{n}$ out
3. Write V's as $\vec{V}$
4. mass conservation
5. momentum conservation



$V_1 = 10 m/s$ $\quad V_2 = ?$

$A_1 = 2 m^2$ $\quad A_2 = A_1$

Find F_x and F_y on the vane.

step 1: draw cv.

Step 2: $\hat{n}_1 = \langle -1, 0, 0 \rangle$ $\quad \hat{n}_2 = \langle \cos\theta, \sin\theta, 0 \rangle$

step 3: $\vec{V}_1 = \langle 10, 0, 0 \rangle$ $\quad \vec{V}_2 = \langle V_2\cos\theta, V_2\sin\theta, 0 \rangle = \langle 10\cos\theta, 10\sin\theta, 0 \rangle$

Step 4: $A_1 V_1 = A_2 V_2 \quad \therefore \quad V_1 = V_2$

step 5:

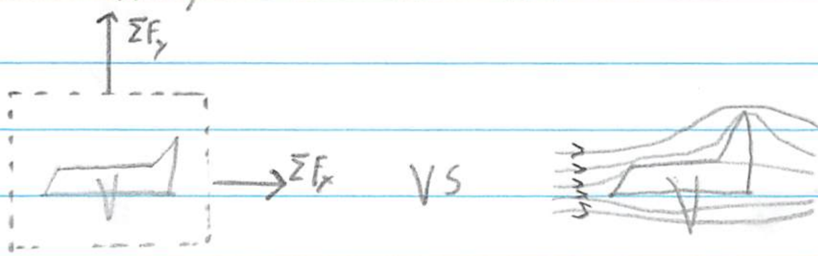
$$X_{momentum}: \overbrace{\cancel{\frac{\partial}{\partial t}\int_{CV} u\rho\, dV}}^{=0 \text{ steady}} + \int_{CS} u\rho\,\vec{V}\cdot\hat{n}\, ds = F_x$$

$$\Rightarrow \int_{A_1} u_1 \rho\, \vec{V}_1\cdot\hat{n}_1\, ds + \int_{A_2} u_2\rho\,\vec{V}_2\cdot\hat{n}_2\, ds = F_x$$

$$\Rightarrow 10\rho(-10)\int_{A_1} ds + 10\cos(\theta)\rho\, 10\int_{A_2} ds = F_x$$

$$\Rightarrow -10^2\rho A_1 + 10^2\rho\cos(\theta) A_2 = F_x$$

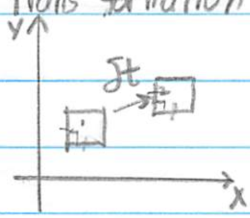
Differential Analysis of Fluid Flow



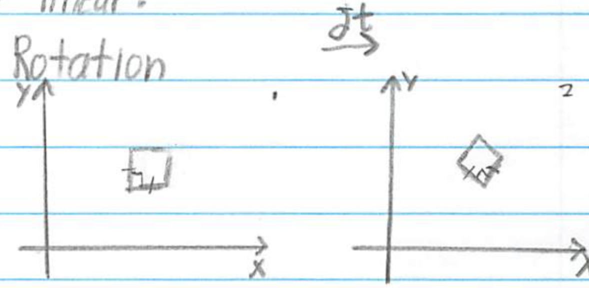
Describe fluid flow with a "fluid element"

Infintesimal element motion is only linear:

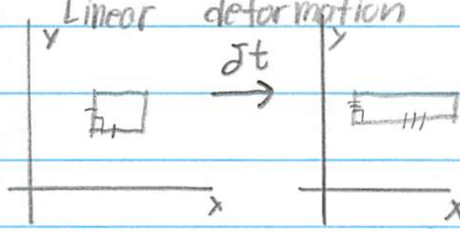
Trans formation



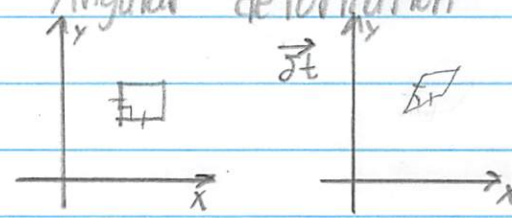
Rotation



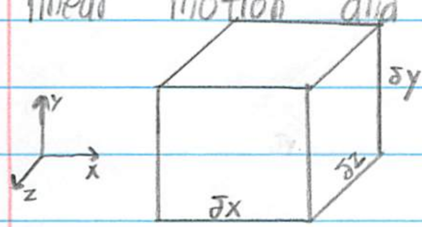
Linear deformation



Angular deformation

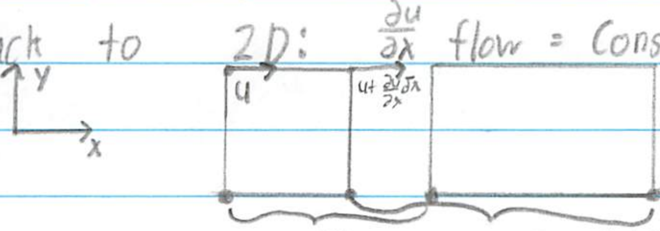


linear motion and deformation



undeformed volume : $\delta x \delta y \delta z$

back to 2D: $\frac{\partial u}{\partial x}$ flow = Constant. u linearly increases with x



$u\delta t$ $\quad$ $u\delta t + \frac{\partial u}{\partial x}\delta x\,\delta t = \left(u + \frac{\partial u}{\partial x}\delta x\right)\delta t$

translation $\quad$ linear deformation

new length $= \underbrace{\left(u + \frac{\partial u}{\partial x}\delta x\right)\delta t - u\delta t}_{\text{increase of } \delta x} + \underbrace{\delta x}_{\text{original } \delta x}$

new volume $= \delta x \delta y \delta z + \frac{\partial u}{\partial x}\delta x \delta y \delta z\, \delta t$

rate of change of volume per unit volume:

$$\frac{\frac{d}{dt}\left(\delta \forall\right)}{\forall_0}$$

For our case: $= \frac{\frac{d}{dt}\left(\delta x \delta y \delta z + \frac{\partial u}{\partial x}\delta x \delta y \delta z \delta t - \delta x \delta y \delta z\right)}{\delta x \delta y \delta z}$

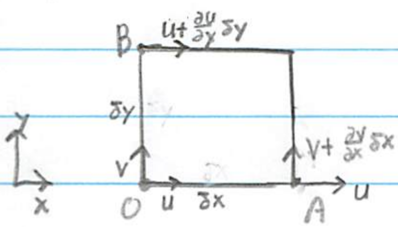
$$= \frac{d}{dt}\frac{\partial u}{\partial x}\delta t$$

$$\lim_{\delta t \to 0} = \frac{\partial u}{\partial x}$$

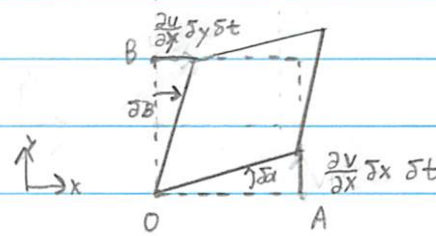
Volumetric dialation rate for 3D

$$\frac{\frac{d}{dt}(\delta \forall)}{\forall_0} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \vec{\nabla} \cdot \vec{V} = \operatorname{div}(\vec{V})$$

Angular Motion and Deformation



$\frac{\partial u}{\partial y} = \text{constant}$ and $\frac{dv}{dx} = \text{constant}$



$$\tan(\delta\alpha) = \frac{\frac{\partial v}{\partial x}\delta x \delta t}{\delta x} \approx \delta\alpha = \frac{\partial v}{\partial x}\delta t$$

$$\delta\beta = \frac{\partial u}{\partial x}\delta t$$

$$\omega_{OA} = \lim_{\delta t \to 0}\frac{\delta\alpha}{\delta t} = \lim_{\delta t \to 0}\frac{\frac{dv}{dx}\delta t}{\delta t} = \frac{\partial v}{\partial x}$$

$$\omega_{OB} = \frac{\partial u}{\partial y}$$

We can average b.c. we assume it is all linear

Total rotation: $\omega_z = \frac{1}{2}(\omega_{OA} + \omega_{OB}) = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

rotation out of the plane

$\therefore \omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$, $\omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$

rotation vector : $\vec{\omega} = \langle \omega_x, \omega_y, \omega_z \rangle = \frac{1}{2}\vec{\nabla} \times \vec{V}$

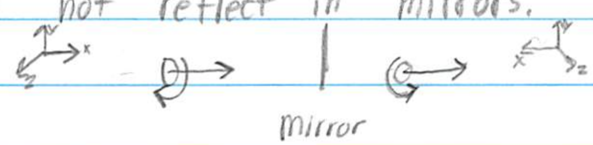
Vorticity : $\vec{\zeta} = 2\vec{\omega} = \vec{\nabla} \times \vec{V} = \text{curl}(\vec{V})$

in cartesian only — in general coordinate systems

Types of flow

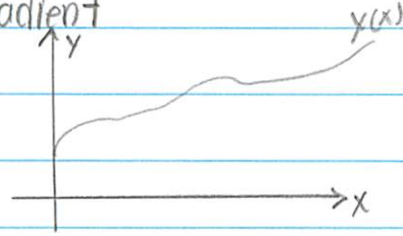
$\vec{\zeta} = \vec{0}$: irrotational flow $\vec{\zeta} \neq \vec{0}$: rotational flow

* rotational vectors are not real vectors because they do not reflect in mirrors.



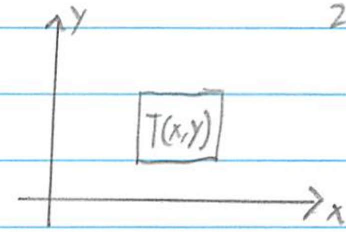
so actually we call it a tensor since it doesn't obey this vector mirroring property. We can also call it a pseduovector

Gradient



2D, 1 varible

$\frac{dy}{dx}$



2D, 2 varible

$\vec{\nabla} T = \langle \frac{\partial T}{\partial x}, \frac{\partial T}{\partial y} \rangle$

For fluids: $\vec{V} = \langle u(x,y,z), v(x,y,z), w(x,y,z) \rangle$

$$\vec{\nabla}\vec{V} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{bmatrix}$$ This is a tensor

Velocity gradient tensor

$$\vec{\nabla}\vec{V} = \frac{1}{2}\Big(\underbrace{\vec{\nabla}\vec{V} + \vec{\nabla}\vec{V}^T}_{\text{symetric}}\Big) + \frac{1}{2}\Big(\underbrace{\vec{\nabla}\vec{V} - \vec{\nabla}\vec{V}^T}_{\text{anti semetric}}\Big)$$

$$= \underset{\text{rate of stress tensor}}{S} + \underset{\text{rotation tensor}}{\underline{\Omega}}$$

$$\text{trace}(\vec{\nabla}\vec{V}) = \text{trace}(S) = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \text{div}(\vec{V})$$

↖ sum of diagonol element

Mass conservation (continuity)

previously we saw $\frac{\partial}{\partial t}\int_{cv} \rho \, d\forall + \int_{cs} \rho \vec{V} \cdot \hat{n} \, ds = 0$

$$+ \int_{cv} \text{div}(\rho\vec{V}) \, dV = 0$$

← divergence theorem

$$\int_{cv} \left[\frac{\partial}{\partial t}\rho + \text{div}(\rho\vec{V})\right] d\forall = 0$$

$$\therefore \frac{\partial \rho}{\partial t} + \text{div}(\rho\vec{V}) = 0$$

$$\underbrace{\frac{\partial \rho}{\partial t}}_{=0} + \underbrace{(\vec{\nabla}\rho)}_{=0} \cdot \vec{V} + \rho \, \text{div}(\vec{V}) = 0$$

for steady

$$\rho \, \text{div}(\vec{V}) = 0$$

$$\boxed{\text{div}(\vec{V}) = 0}$$

$$m a = \sum F$$

Momentum equation (Navier-stokes)

Fluid accelloration = internal forces + external forces

$$\rho \frac{D\vec{V}}{Dt} = \rho\vec{g} + \mathrm{div}(T_{ij})$$

stress tensor

$$\frac{\partial \vec{V}}{\partial t} + \mathrm{div}(\vec{V})\vec{V} = \vec{g} + \frac{1}{\rho}\mathrm{div}(T_{ij})$$

$$\text{pressure} = \frac{-1}{3}\,\text{trace}(T_{ij}) = \frac{-1}{3}(T_{11}+T_{22}+T_{33})$$

$$T_{ij} = \begin{bmatrix} -p & 0 & 0 \\ 0 & -p & 0 \\ 0 & 0 & -p \end{bmatrix} + \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix}$$

isotropic stress deviatoric stress

~~Recall~~ for newtonian fluid

$$\tau = \frac{-2}{3}\mu\,\underbrace{\mathrm{div}(\vec{V})}_{=0 \text{ for incompessible fluids}}\,I_{ij} + 2\mu S_{ij}$$

$$\frac{\partial \vec{V}}{\partial t} + \mathrm{div}(\vec{V})\vec{V} = \frac{-1}{\rho}\vec{\nabla}p + \underbrace{\frac{1}{\rho}\mathrm{div}(2\mu S_{ij})}_{\nu(\vec{\nabla})^2\vec{V}} + \vec{g}$$

$\nu = \mu/\rho$ kinimatic viscocity

Euler equations:

Navier stokes without the viscious terms

in 2D:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{-1}{\rho}\frac{\partial p}{\partial x}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \frac{-1}{\rho}\frac{\partial p}{\partial y}$$

Streamfunction $\Psi(x,y)$

where: $u(x,y) = \frac{\partial \Psi(x,y)}{\partial y}$

$v(x,y) = -\frac{\partial \Psi(x,y)}{\partial x}$

in 2D, $\Psi(x,y)$ is a scaler that can generate a vector field

Ψ is constant on streamlines

$\Delta\Psi$ is the volumetric flow rate

Velocity Potential φ

Assumptions:

- incompessible
- inviscid
- irrotational $\vec{\zeta} = \vec{0}$ everywhere

where $\varphi(t,x,y,z)$ defined by $\vec{V} = \vec{\nabla}\varphi$

$$u = \frac{\partial \varphi}{\partial x} \qquad v = \frac{\partial \varphi}{\partial y} \qquad w = \frac{\partial \varphi}{\partial z}$$

continuity is then

$$\vec{\nabla} \cdot \vec{V} = 0$$

$$\vec{\nabla} \cdot \vec{\nabla}\varphi = 0 = \nabla^2 \varphi = 0$$

For 2D: $\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0$

similarly: $\zeta_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$

$$-\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2} = 0 \quad \therefore \quad \nabla^2 \psi = 0$$

ψ and φ are the solution to the same pde but have different boundry conditions

we can actually show ψ and φ are orthoginal

the laplace equation is linear

$\therefore$ if $\nabla^2 \varphi_1 = 0$ and $\nabla^2 \varphi_2 = 0$ then $\nabla^2(\varphi_1 + \varphi_2) = 0$

We can create flow C by adding flow A and flow B

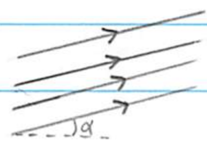
*note, we can also add velocity components but not pressures

Basic Flows: all centered about the origin

uniform flow: cartesian

$\varphi = Ux\cos(\alpha) + Uy\sin(\alpha)$

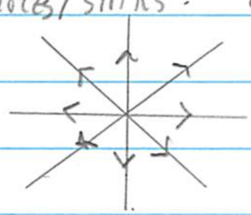
$\psi = Uy\cos(\alpha) + Ux\sin(\alpha)$



$u = U\cos(\alpha)$

$v = U\sin(\alpha)$

source/sinks: cylindrical



$\varphi = \frac{m}{2\pi}\ln(r)$ $\quad \psi = \frac{m}{2\pi}\theta$

m is the strength of the source/sink

$V_r = \frac{m}{2\pi r}$ $\quad V_\theta = 0$

Free vortex: cylindrical

$\varphi = \frac{\Gamma}{2\pi}\theta$ $\quad \psi = \frac{\Gamma}{2\pi}\ln(r)$

$\Gamma > 0$, ccw. $\quad \Gamma < 0$, cw

$V_r = 0$ $\quad V_\theta = \Gamma/2\pi r$

Doublet: cylindrical

$\varphi = \frac{K\cos(\theta)}{r}$ $\quad \psi = \frac{-K\sin(\theta)}{r}$

$V_r = \frac{-K\cos(\theta)}{r^2}$ $\quad V_\theta = \frac{-K\sin(\theta)}{r^2}$

Recall:

$u = \frac{\partial\varphi}{\partial x} = \frac{\partial\psi}{\partial y}$ $\quad v = \frac{\partial\varphi}{\partial y} = \frac{-\partial\psi}{\partial x}$

$V_r = \frac{\partial\varphi}{\partial r} = \frac{1}{r}\frac{\partial\psi}{\partial\theta}$ $\quad V_\theta = \frac{1}{r}\frac{\partial\varphi}{\partial\theta} = \frac{-\partial\psi}{\partial r}$

Circulation in fluids

$$\Gamma = \oint_{cs} \vec{V} \cdot d\vec{s}$$

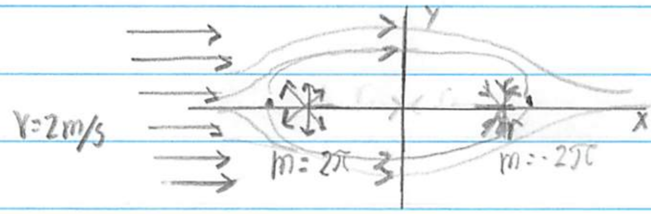
in potential flow:

$$\Gamma = \oint_{cs} \nabla\varphi \cdot d\vec{s}$$

For a free vortex

$$\Gamma = \int \frac{\Gamma}{2\pi r} r\,d\theta \Rightarrow \Gamma/2\pi \int_0^{2\pi} d\theta = r$$

example:



a. What is the velocity potential

$$\varphi = \varphi_{uniform} + \varphi_{sources} + \varphi_{sink}$$
$$= 2x + \ln(r_1) - \ln(r_2)$$

$$r_1 = \sqrt{(x-h)^2+(y-k)^2} \quad h \leftarrow -0.5,\ k \leftarrow 0 \qquad r_2 = \sqrt{(x-h)^2+(y-h)^2} \quad h \leftarrow 0.5,\ k \leftarrow 0$$

$$\varphi = 2x + \tfrac{1}{2}\ln\left((x+0.5)^2+y^2\right) - \tfrac{1}{2}\ln\left((x-0.5)^2+y^2\right)$$

b. What is the length of the oval.

2 end points are where velocity = 0, stagnation point

$\vec{V} = \vec{0}$ We will only look at x-component because y component is zero all along the x-axis

$$u = \frac{\partial\varphi}{\partial x} = 2 + \frac{1}{2}\frac{1}{(x+0.5)^2+y^2} \cdot 2(x+0.5) - \frac{1}{2}\frac{2(x-0.5)}{(x-0.5)^2+y^2} = 0$$

We know its on the x-axis so $y = 0$

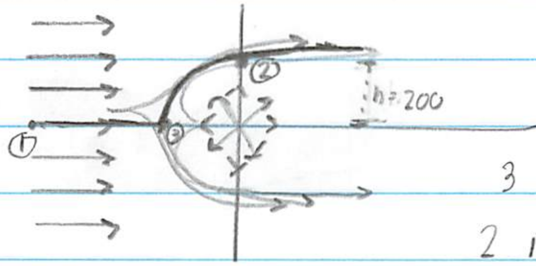
$$0 = 2 + \frac{1}{x+\frac{1}{2}} - \frac{1}{x-1/2} \qquad x = \frac{\sqrt{3}}{2} \text{ and } -\frac{\sqrt{3}}{2}$$

Oval length is $\sqrt{3}$ meters

Example 6.7

$U_1 = 40$ mph

$p_1 = 0$ gage



$$V = U_1^2\left(1 + 2\frac{b}{r}\cos\theta + \frac{b^2}{r^2}\right)$$

3 is at $(r=b, \theta=\pi)$

2 is at $(r=100, \theta=\pi/2)$

$h = \pi b \therefore b = \frac{200}{\pi}$

given relationship $\longrightarrow y_2 = \frac{\pi b}{2}$

a. What is V_2

b. What is p_2

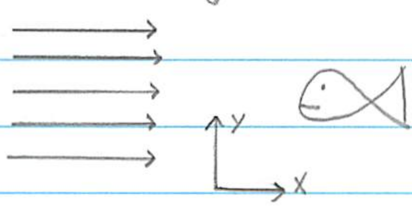
$$V(r, \pi/2) = U_1^2\left(1 + \frac{4}{\pi^2}\right) = 47.4 \text{ mph}$$

$$p_2 = p_1 + \tfrac{1}{2}\rho U_1^2 + \rho g \underbrace{y_1}_{0} - \tfrac{1}{2}\rho V_2^2 - \rho g y_2 = 0.0647 \text{ psi gage}$$

Dimensional Analysis

How to go from scaled wind tunnel $\rightarrow$ full scale life

Force on objects in a flow field



$F = \sigma A$

$\Downarrow$ stress tensor

$$\text{Force on shark} = \int_{CS} T_{ij} \cdot \hat{n}\, dA$$

Force on fish in $\hat{u}$ is called drag

3. Drag on a sphere (experimental)

Resistance points in direction of flow

Varibles:		Units
Drag force	F_D	ML/t^2
Diameter	d	L
Velocity	V	L/t
Density fluid	ρ	M/L^3
Viscosity	μ	M/TL

We have 5 varibles but only 3 dimensions (units) so lets see if we can simplify things instead of varrying 5 varibles

Bucking Pi theorem

A problem can be reduced to k-r non-dimensional groups

k: number of varibles

r: number of refrences dimensions

For our sphere and air:

$k=5$, $r=3$

$k-r = 5-3 = 2$ varibles needed

lets make 2 new varibles

- must be non-dimensional
- be combinations of previous varibles
- must use all varibles

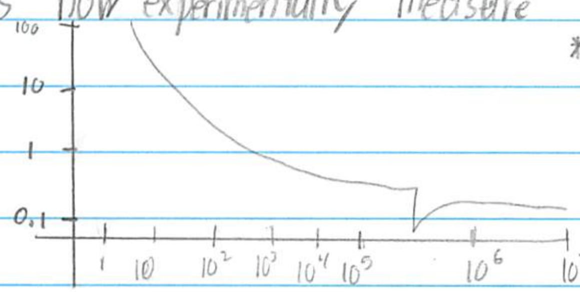
2 varibles: (there are many varible combinations for different uses)

Reynolds number: $Re = \dfrac{\rho d V}{\mu} = \dfrac{d \cdot V}{\nu}$ ← kinematic viscosity

Drag coefficent: $C_D = \dfrac{F_D}{\rho V^2 d^2}$ or $\dfrac{F_D}{\underbrace{\frac{1}{2}\rho V^2}_{KE} A_c}$ ← cross-section Area

lets now experimentally measure C_D vs. Re

*loglog scale



now we say: $F_D = \dfrac{1}{2}\rho V^2 A \; C_D(Re)$

$\rho d V/\mu$

C_D as a function of Re (Graph)

Mach number = $\dfrac{V}{\sqrt{\gamma R T}}$

$\gamma \approx 1.17$

ratio of specific heats

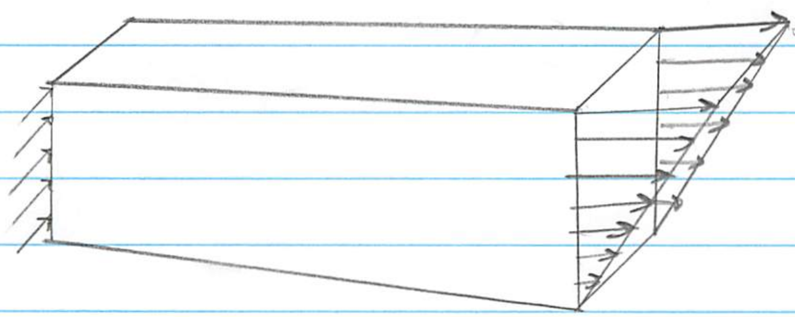
Midterm 2 review

problem 1: consider the flow below with depth $b = 2m$, $\rho = 1.4\ kg/m^3$

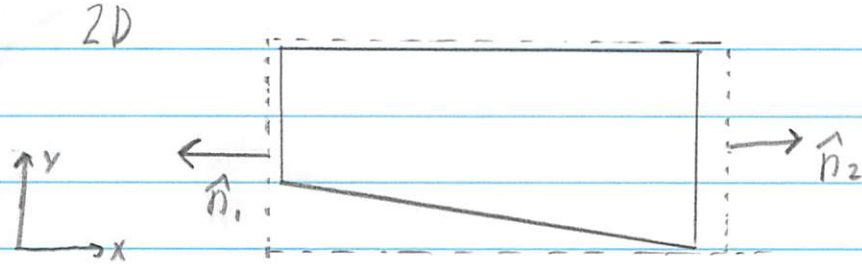
at 1: $\vec{V}_1$ is at an angle $\theta = 60°$. The velocity is uniform. $p_1 = 100$ Pa gage. pipe height $h = 1$ m.

at 2: $\vec{V}_2 = 2y\hat{\imath}$ m/s. $p_2 = 400$ pa gage, $h_2 = 2m$

a. Draw the control volume



2D



$$\vec{V}_1 = V_1 \cos\theta\, \hat{\imath} + V_1 \sin\theta\, \hat{\jmath}$$

mass: $\displaystyle\int_{cs} \rho \vec{V} \cdot \hat{n}\, ds = 0$

$$\int_1 \rho \vec{V}_1 \cdot \hat{n}_1 dA_1 + \int_2 \rho \vec{V}_2 \cdot \hat{n}_2 dA_2 = 0$$

$\vec{V}_1 \cdot \hat{n}_1$ $\qquad\qquad$ $\vec{V}_2 \cdot \hat{n}_2$

$\langle V_1 \cos\theta, V_2 \sin\theta \rangle \cdot \langle -1, 0 \rangle$ $\qquad\qquad$ $\langle 2y, 0 \rangle \cdot \langle 1, 0 \rangle$

$-V_1 \cos\theta$ $\qquad\qquad$ $2y$

$$-V_1 \cos(\theta)\, \rho \cdot 2 + \rho 2 \cdot y^2\big|_0^2 = 0$$

$$-2\rho \cos(\theta) V_1 + \rho 2 \cdot 4 = 0$$

$$V_1 = 4/\cos(\theta)$$

Momentum conservation:

$$\int_{cs} \vec{V} \rho \vec{v} \cdot \hat{n} \, ds = (p_1 A_1 - p_2 A_2)\hat{\imath} + F_{x\,pipe} + F_{y\,pipe}$$

$$\int_1 \vec{V}_1 \rho \vec{V}_1 \cdot \hat{n}_1 \, dA_1 + \int_2 \vec{V}_2 \rho \vec{V}_2 \cdot \hat{n}_2 \, dA_2 =$$

x comp:

$$\int_0^2 \int_1^2 \frac{4 \cancel{\cos(\theta)}}{\cancel{\cos}(\theta)} \rho \left(\frac{-4 \cancel{\cos(\theta)}}{\cancel{\cos}(\theta)} \right) dy dz + \int_0^2 \int_0^2 2y \rho \, 2y \, dy dz =$$

$$-16\rho \cdot 2 + 2 \int_0^2 4 \rho y^2 dy$$

$$-32\rho + \frac{8\rho}{3} \left(y^3 \right)_0^2$$

$$-32\rho + \frac{64\rho}{3} = p_1 \overset{\nearrow 200}{A_1} - p_2 \overset{\nearrow 1600}{A_2} + F_x$$

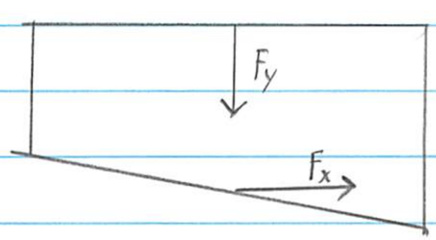
$$F_{x\,pipe} = 1385 \text{ N} = \boxed{1.39 \text{ kN}}$$

y comp:

$$\int \frac{4 \sin(\theta)}{\cos(\theta)} \rho (-4) \, dA_1 + \int_0^2 \int_0^2 0 \, dy dz =$$

$$-16\rho \left(\frac{\sin(60°)}{\cos(60°)} \right) 2 = F_y$$

$$-32\rho\sqrt{3} = \boxed{F_y = -77.6 \text{ N}}$$



Problem 2

spiral flow is modeled as a sourse with $m=1$ and free vortex with $\Gamma=-10$. Both the source and vortex are centered in the coordinate system.

a. What is the velocity potential of the system?

$$\phi = \frac{m}{2\pi}\ln(r) + \frac{\Gamma}{2\pi}\theta$$

$$\boxed{\phi = \frac{\ln(r)}{2\pi} - \frac{10\theta}{2\pi}}$$

b. Show that V only depends on r.

$$\vec{V} = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j}$$

This isn't useful so we should use cylindrical coordinates.

$$\vec{V} = \frac{\partial\phi}{\partial r}\hat{r} + \frac{\partial\phi}{\partial\theta}\cdot\frac{1}{r}\hat{\theta}$$

$$= \left(\frac{1}{2\pi r} - 0\right)\hat{r} + \left(0 - \frac{10}{2\pi}\right)\cdot\frac{1}{r}\hat{\theta}$$

$$\boxed{V = \sqrt{u_r^2 + u_\theta^2} \leftarrow \text{function of } r \quad \uparrow \text{function of } r}$$

c. Show that for from the center V = 0

$$V = \sqrt{\frac{1}{4\pi^2 r^2} + \frac{100}{4\pi^2 r^2}} = \frac{\sqrt{101}}{2\pi r}$$

$$\boxed{\lim_{r\to\infty}\frac{\sqrt{101}}{2\pi r} = 0}$$

d. find $p(r)$ knowing $\lim_{r \to \infty} p(r) = 57500$ Pa

$$p_\infty + \frac{1}{2}\rho V_\infty^2 = p(r) + \frac{1}{2}\rho V^2(r)$$

$$57500 + 0 = p(r) + \frac{1}{2}\rho\left(\frac{101}{4\pi^2 r^2}\right)$$

$$\boxed{p(r) = 57500 - \frac{101\rho}{8\pi^2 r^2}}$$

Problem 3: imagine compressing a syringe twice such that $V_1 > V_2 > V_3$.

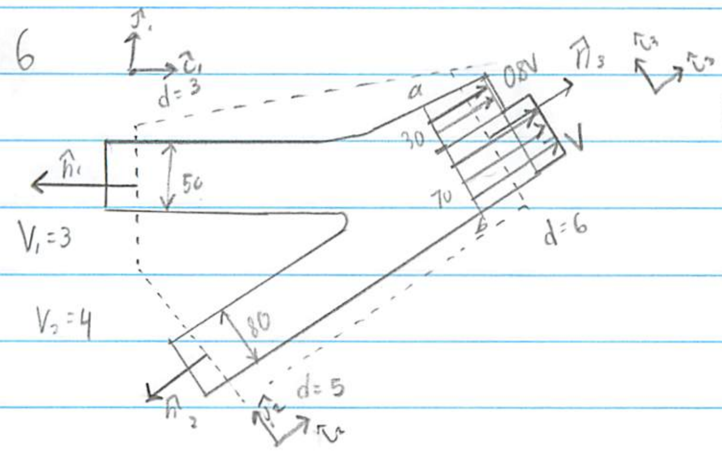
a. What is the sign of the volumetric dialation rate.

$\operatorname{div}(\vec{V}) > 0$ $\therefore$ positive

b. Why?

divergence measures the amount of stuff in a space. As the syringe is compressed, more stuff is in a smaller space.

5.16



Find V

Mass conservation

steady flow: $\int_{cs} \rho \vec{V} \cdot \hat{n}\, ds$

$$\int_1 \rho \vec{V}_1 \cdot \hat{n}_1 dA_1 + \int_2 \rho \vec{V}_2 \cdot \hat{n}_2 dA_2 + \int_{3a} \rho \vec{V}_{3a} \cdot \hat{n}_{3a} dA_{3a} + \int_{3b} \rho \vec{V}_{3b} \cdot \hat{n}_{3b} dA_{3b} = 0$$

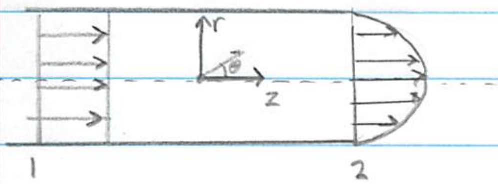
$\vec{V}_1 = 3\hat{\imath}_1$ $\quad$ $\vec{V}_2 = 4\hat{\imath}_2$ $\quad$ $\vec{V}_{3a} = 0.8V\hat{\imath}_3$ $\quad$ $\vec{V}_{3b} = V\hat{\imath}_3$

$\hat{n}_1 = -\hat{\imath}_1$ $\quad$ $\hat{n}_2 = -\hat{\imath}_2$ $\quad$ $\hat{n}_{3a} = \hat{\imath}_3$ $\quad$ $\hat{n}_{3b} = \hat{\imath}_3$

$$-3 \cdot 150 - 4 \cdot 400 + 0.8V \cdot 180 + V \cdot 420 = 0$$

$$\boxed{V = 3.635 \text{ ft/s}}$$

Circular pipe   given: $V_z(r) = V_0\left(1 - r^2/R^2\right)$



Find V_0

F_z

F_r

$\vec{V}_1 = V_1\hat{k}$ $\vec{V}_2 = V_2(r)\hat{k}$

$\hat{n}_1 = -\hat{k}$ $\hat{n}_2 = \hat{k}$

mass conservation:

$$\int_A \rho \vec{V}_1 \cdot \hat{n}_1 \, dA_1 + \int_0^{2\pi}\int_0^R \rho \vec{V}_2 \cdot \hat{n}_2 \, r\,dr\,d\theta = 0$$

$$-V_1 \pi R^2 + \pi V_0 R^2 + 2\pi \int_0^R -r^3/R^2 \cdot V_0 \, dr = 0$$

$$+ 2\pi \cdot \frac{-R^4}{4R^2}$$

$$-\pi \, R^2/2 \, V_0 = 0$$

$$\therefore \boxed{V_0 = 2V_1}$$

Momentum conservation

Axial: $\int_{cs} u_z \rho \vec{V}_2 \cdot \hat{n}_2 \, ds =$

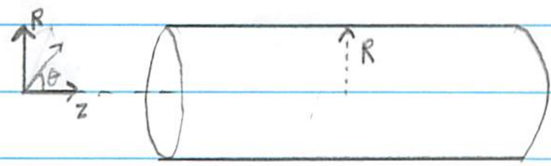
$$\rho \int_0^{2\pi}\int_0^R \left(2V_1 - 2V_1 r^2/R^2\right)^2 r\,dr\,d\theta = p_1 A_1 - p_2 A_2 + F_z$$

$$\cancel{2V_1 \rho \pi R^2 - 2V_1 \rho \cdot 2\pi \cdot R^2/4} = p_1 A_1 - p_2 A_2 + F_z$$

$$2V_1 \rho \pi R^2 \left(1 - \pi/2\right) = p_1 A_1 - p_2 A_2 + F_z$$

$F_r = 0$

Viscous flow in circular pipes



cylindrical coordinates r, θ, z

Viscous flow $\vec{V} = u_r \hat{r} + u_\theta \hat{\theta} + u_z \hat{k}$

$\vec{V}(t, r, \theta, z)$ and $p(t, r, \theta, z)$

so not steady

Assumptions for flow in our pipe

- steady flow $\frac{d}{dt} = 0$
- Straight lines along pipe $u_r = 0$
- No swirls $u_\theta = 0$
- Axisymmetric flow $\frac{\partial}{\partial \theta} = 0$
- Laminar flow (no turbulence) $Re < 1000$

Navier-stokes: After simplifing due to assumptions

Continuity:

$$\boxed{\frac{\partial u_z}{\partial z} = 0}$$

r-component:

$$\boxed{\frac{\partial p}{\partial r} = 0}$$

θ-component:

$$0 = 0$$

z-component:

=0 due to new continuity

$$\rho u_z \frac{\partial u}{\partial z} = -\frac{\partial p}{\partial z} + \mu \left[\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial u_z}{\partial r} \right) + \frac{\partial^2 u_z}{\partial z^2} \right]$$

$$\therefore \boxed{\mu \cdot \frac{1}{r} \cdot \frac{\partial}{\partial r}\left(r \frac{\partial u_z}{\partial r} \right) = \frac{\partial p}{\partial z}}$$

to extract u_z from our z equation

$$\int \frac{\partial}{\partial r}\left(r \frac{\partial u_z}{\partial r}\right) dr = \int \frac{r}{\mu}\frac{\partial P}{\partial z} dr$$

$$r\frac{\partial u_z}{\partial r} = \frac{r^2}{2\mu}\frac{\partial P}{\partial z} + C_1$$

$$\int \frac{\partial u_z}{\partial r} dr = \int \frac{r}{2\mu}\frac{\partial P}{\partial z} + \frac{C_1}{r} dr$$

$$u_z = \frac{r^2}{4\mu}\frac{\partial P}{\partial z} + C_1 \ln(r) + C_2$$

Boundry conditions to determine constants

- this $\ln(r)$ term $= -\infty$ as $r \to 0$ but we can't have infinite flow so $C_1 = 0$ to get rid of this term
- No-slip; the fluid touching the wall goes the walls speed "$u_z(R) = 0$"

$$0 = \frac{R^2}{4\mu}\frac{\partial P}{\partial z} + C_2 \quad \therefore \quad C_2 = \frac{-R^2}{4\mu}\frac{\partial P}{\partial z}$$

$$\boxed{u_z(r) = \frac{(r^2 - R^2)}{4\mu}\frac{\partial P}{\partial z}}$$

Volumetric flow rate

$$Q = VA$$

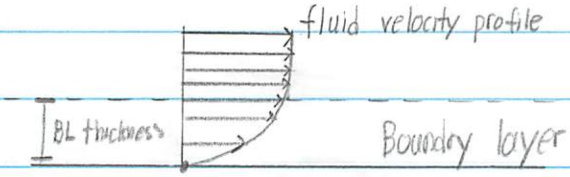
$$Q = \int u_z(r)\, dA = \int_0^{2\pi}\int_0^R u_z(r)\cdot r\,dr\,d\theta = \frac{-\pi R^4}{8\mu}\frac{\partial P}{\partial z}$$

$$\therefore \quad \frac{\partial P}{\partial z} = \frac{-8\mu Q}{\pi R^4} = \frac{\Delta P}{l}$$

^ linear

pressure drop: $\frac{(P_1 - P_2)}{l} = \frac{8\mu Q}{\pi R^4}$

Boundry layer



Velocity is zero due to no slip condition

Boundry thickness tends to be 1 mm - 1cm based on Re #

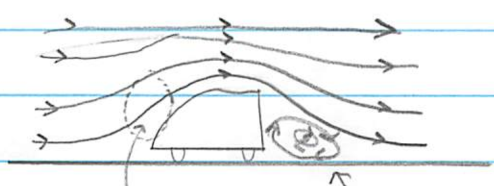
Drag force - boundry layer - flow separation

Force on object = $\int_{surface} T_{ij} \cdot \hat{n}\, dA$

stress tensor

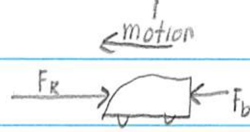
unit vector

$$= \Big[\underbrace{\int pressure \cdot \hat{n}\, dA}_{\text{form drag}} + \underbrace{\int T_{ij} \cdot \hat{n}\, dA}_{\text{skin friction}} \Big] \cdot \frac{\vec{V}}{\|V\|}$$



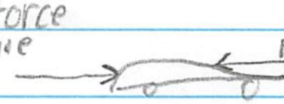
pressure behind car due to bernoulli: $\frac{\partial P}{\partial n} = \frac{-\rho V^2}{R}$

curvature of streamlines

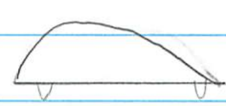


Flow seperation force

F_b is small due to low pressure

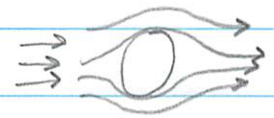


Boundry layar skin friction



$F = \tau A = \mu \frac{\partial u}{\partial y} A$

$Re < 5$

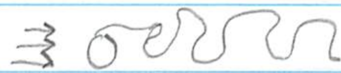


steady, no flow seperation

$5 < Re < 50$

: steady, flow seperation

$50 < Re < 150$



unsteady laminar

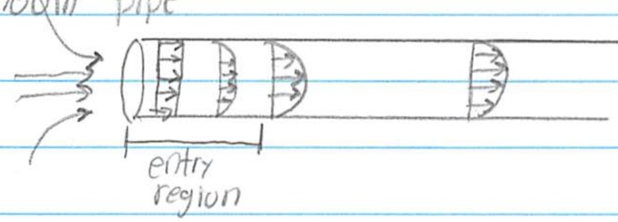
$150 < Re < 10^5$

turbulent flow

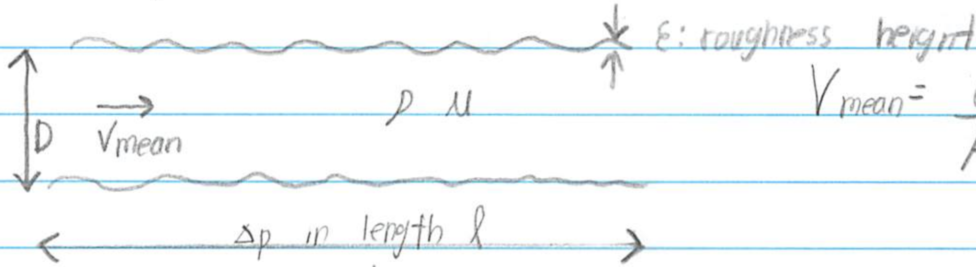
$10^5 < Re < 10^6$

turbulent wake

Ideal smooth pipe



Rough Pipe



$$V_{mean} = \frac{Q}{A} = \frac{Q}{\pi R^2}$$

parameters:		Dimensions
flow	V_{mean}	L/T
	Δp	M/T^2L
fluid	ρ	M/L^3
	μ	M/TL
geometry	D	L
	l	L
	ε	L

Pi-Theorem : $k - r = 7 - 3 = 4$

Dimensionless groups : $\dfrac{\Delta p}{\frac{1}{2}\rho V_{mean}^2}$, $Re = \dfrac{\rho V_{mean} D}{\mu}$

aspect ratio : $\dfrac{l}{D}$, $\dfrac{\varepsilon}{D}$: realitive roughness in general: $\frac{\Delta p}{\frac{1}{2}\rho V_{mean}^2} = f(Re, \ell/D, \varepsilon/D)$

We often want pressure drop per length though so we rearrange: $\underbrace{\frac{\Delta p}{\frac{1}{2}\rho V_{mean}^2} \cdot \frac{D}{\ell}}_{\text{friction factor } f} = f(Re, \varepsilon/D)$

$f = \text{function}(Re, \varepsilon/D) \rightarrow$ Moody diagram

laminar if $f = 64/Re$

pipe pressure: Remember, if you have viscosity, don't use Bernoulli

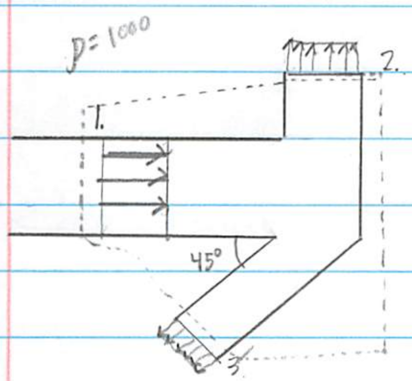
Smooth flow in pipes: $U_z(r) = \frac{1}{4\mu}\frac{\partial P}{\partial z}(r^2 - R^2)$

$Q = \frac{\pi R^4}{8\mu}\frac{\Delta p}{\ell}$

Laminar pipe flow

$V_{mean} = \frac{Q}{\pi R^2}$ $\qquad Re = \frac{\rho V_{mean} D}{\mu}$

Final review



$A_1 = 0.2 m^2 \quad V_1 = 0.5 m/s \quad p_1 = 10{,}000 Pa$ gauge

$A_2 = A_1 \quad\quad p_2 = 0$

$A_3 = \frac{1}{2} A_1 \quad Q_3 = Q_2 \quad p_3 = 0$

$\hat{n}_1 = -\hat{\imath} \quad \hat{n}_2 = \hat{\jmath} \quad \hat{n}_3 = -\frac{\sqrt{2}}{2}\hat{\imath} - \frac{\sqrt{2}}{2}\hat{\jmath}$

since flow is uniform (constant) $Q_1 = Q_2 + Q_3$

$A_1 V_1 = 2 Q_2 = 0.2 \cdot 0.5 \quad \therefore \quad Q_2 = Q_3 = 0.05 \; m^3/s$

$\therefore \; V_2 = 0.25 \; m/s \quad\quad \vec{V}_2 = 0.25 \hat{\jmath}$

$V_3 = 0.5 \; m/s \quad\quad \vec{V}_3 = \underbrace{-\frac{\sqrt{2}}{4}}_{u}\hat{\imath} - \underbrace{\frac{\sqrt{2}}{4}}_{v}\hat{\jmath}$

momentum conservation

X-direction

$$\iint_{A_1} u_1 \rho \, \vec{V}_1 \cdot \hat{n}_1 \, ds + \iint_{A_3} u_3 \rho \, \vec{V}_2 \cdot \hat{n}_2 \, ds = F_x + p_1 A_1 + p_3 A_3 \cos(\theta)$$

$$0.5\rho(-0.5)A_1 + \left(-\frac{\sqrt{2}}{4}\right)\rho\left(\frac{1}{2}\right) = F_x + 10{,}000 \cdot 0.2$$

$F_x = -2227 = 2.2$ kN to the left (pipe force exburted)

y-direction

$$0.25\,\rho(0.25)A_2 + \left(\frac{-\sqrt{2}}{4}\right)\rho\left(\frac{1}{2}\right) = F_y + \underbrace{p_3 A_3 - p_2 A_2}_{\to 0}$$

Ideal gas law: $\rho = P/RT$

Sheer stress:

$\tau = F/A \qquad \tau = \mu \frac{\partial u}{\partial y}$

Bouancy Force:

$F_b = \rho g V_{fluid}$

line of constant pressure:

$dp = 0 = \frac{\partial p}{\partial x} dx + \frac{\partial p}{\partial y} dy + \frac{\partial p}{\partial z} dz$

Bernoulli's Equation

- No Viscosity • Steady flow • incompressible • streamline
- $p + \frac{1}{2}\rho V^2 + \rho g z = C$

Material Derivative:

$\frac{D()}{Dt} = \frac{\partial()}{\partial t} + (\vec{V} \cdot \vec{\nabla})()$

$\vec{a} = \frac{D\vec{V}}{Dt}$

Extensive = mass • intensive => $B = m \cdot b$

Volumetric flow rate

$Q = \int v\, dA$

Finite control volume Analysis

- at steady state
  - Mass conservation

    $\int \rho \vec{V} \cdot \hat{n}\, ds = 0$

    - pipes: $\int_{in} \rho \vec{V}_{in} \cdot \hat{n}_{in} dA_{in} + \int_o \rho \vec{V}_o \cdot \hat{n}_o dA_o = 0$
  - Momentum conservation

    X-direction:

$\int u \rho \vec{V} \cdot \hat{n}\, ds = P_L A_L - P_R A_R + F_x$

y-direction:

$\int v \rho \vec{V} \cdot \hat{n}\, ds = \underbrace{P_b A_b - P_T A_T + F_y}_{\text{sum of forces on pipe}}$

Differential Analysis

Volumetric dialation: $\vec{\nabla} \cdot \vec{V}$

Vorticity: $\vec{\xi} = 2\,(\vec{\nabla} \times \vec{V})$

Naviar stokes:

$$\begin{cases} \vec{\nabla} \cdot \vec{V} = 0 \\ \frac{D\vec{V}}{Dt} = \frac{-1}{\rho} \vec{\nabla} p + \nu \vec{\nabla}^2 \vec{V} + \vec{g} \end{cases} \qquad \nu = \mu/\rho$$

Euler Equations: 2D Nariar stokes, no viscosity

$$\begin{cases} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \\ \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{-1}{\rho}\frac{\partial p}{\partial x} \\ \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \frac{-1}{\rho}\frac{\partial p}{\partial y} \end{cases}$$

Potential functions

- incompressible • inviscious • irrotational

Streamfunction $\psi(x,y)$

$u = \frac{\partial \psi}{\partial y} \qquad v = \frac{-\partial \psi}{\partial x}$ linearity rules apply

Velocity potential $\phi(x,y)$

$u = \frac{\partial \phi}{\partial x} \qquad v = \frac{\partial \phi}{\partial y}$

Cartesian → cylindrical

$r = \sqrt{(x-h)^2 + (y-k)^2} \qquad \theta = \tan^{-1}\left(\frac{y-k}{x-h}\right) \qquad - \iint r\, dr\, d\theta$

Dimensional Analysis

nondimensional groups: k - r

k: # of varibles   r: # of different units

$Re = \frac{\rho V D}{\mu} \qquad C_d = \frac{F_d}{\rho V^2 D^2}$

Boundry layar

Skin friction: $F = \tau A = \mu \frac{\partial u}{\partial y} A$

Viscous flow in pipes

Smooth pipes:

$u_z(r) = \frac{1}{4\mu}\frac{(P_1 - P_2)}{\ell}(R^2 - r^2)$

$Q = \int u_z\, dA = \frac{\pi R^4}{8\mu}\frac{(P_1 - P_2)}{\ell}$

Rough pipe:

Friction factor

$f = \frac{(P_1 - P_2)}{\frac{1}{2}\rho V_{mean}^2} \cdot \frac{D}{\ell}$

Moody diagram

f as a function of Re, $\frac{\varepsilon}{D}$ ← check units

Assumptions: $\frac{d()}{dt} = 0$

$u_r = 0, \quad u_\theta = 0, \quad \frac{\partial()}{\partial \theta} = 0, \quad Re < 1000$

- cs = control surface
- You can split the control surface into multiple flat plans and add it together
- Momentum Conservation
  - $\frac{\partial}{\partial t}\int_{c\forall} \vec{V}\rho d\forall + \int_{cs} \vec{V}\rho\vec{V}\cdot\hat{n}ds = \sum F_{ext}$
    - F_{ext} is all external forces
    - Like pressure and stress and such
  - External Forces x direction (typically)
    - Since $F = pA$
    - $\sum F_{ext} = p_{left}A_{left} - p_{right}A_{right} + F_x$
  - External Forces y direction (typically)
    - $\sum F = p_{bottom}A_{bottom} - p_{top}A_{top} + F_y$
- Chapter 6: Differential Analysis of Fluid Flow
  - Volumetric Dilation
    - $div(\vec{V})$
      - $div(\vec{V}) = \vec{\nabla}\cdot\vec{V}$ watch your coordinate system tho
  - Rotation Vector $\vec{\omega}$
    - $\frac{1}{2}\, curl(\vec{V})$
      - $curl(\vec{V}) = \vec{\nabla}\times\vec{V}$ watch your coordinate system tho
  - Vorticity $\vec{\zeta}$
    - $curl(\vec{V})$ or $2\vec{\omega}$ $= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$
  - Gradient Tensor
    - $\vec{\nabla}\vec{V} = S_{ij} + \Omega_{ij}$
      - $S = \frac{1}{2}(\vec{\nabla}\vec{V} + (\vec{\nabla}\vec{V})^T)$
        - the rate of stress tensor
      - $\Omega = \frac{1}{2}(\vec{\nabla}\vec{V} - (\vec{\nabla}\vec{V})^T)$
        - Rotation tensor
  - $trace(\vec{\nabla}\vec{V}) = div(\vec{V})$
    - $trace$ is the sum of the diagonal components of a square matrix
  - Mass conservation
    - $\vec{\nabla}\cdot\vec{V} = 0$
  - "Newton's Second Law"
    - $ma = \sum F$
      - $ma = \rho\frac{D\vec{V}}{Dt}$
      - $F_{int} = \vec{\nabla}\cdot T_{ij}$
      - $F_{ext} = \rho\vec{g}$
  - Stress tensor
    - T_{ij} =isotropic tensor + deviatoric stress tensor
      - isotropic tensor = $-pI_{33}$
      - Where I_{ij} is an $i\times j$ identity matrix
      - deviatoric stress tensor = τ_{ij} the typical stress tensor from mechanics of materials
  - Deviatoric Stress Tensor
    - $\tau_{ij} = \frac{-2}{3}\mu(\vec{\nabla}\cdot\vec{V})I_{ij} + 2\mu S_{ij}$
      - $S_{ij} = 0$ in incompressible fluids
      - μ =dynamic viscosity
  - Naviar Stokes
    - $\vec{\nabla}\cdot\vec{V} = 0$
    - $\frac{\partial\vec{V}}{\partial t} + (\vec{V}\cdot\vec{\nabla})\vec{V} = \frac{-1}{\rho}\vec{\nabla}p + \nu\vec{\nabla}^2\vec{V} + \vec{g}$
      - $\nu = \frac{\mu}{\rho}$ kinematic viscosity
    - 2D Navier Stokes
      - Euler Equations
        - Naviar stokes without viscous terms
        - $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$
        - $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{-1}{\rho}\frac{\partial p}{\partial x}$
        - $\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \frac{-1}{\rho}\frac{\partial p}{\partial y}$
  - Potential functions
    - Linearity applies
      - Assumptions incompressible
      - Inviscious
      - Irrotational
    - Stream functions $\psi(x,y)$
      - $u(x,y) = \frac{\partial\psi(x,y)}{\partial y}$
      - $v(x,y) = \frac{-\partial\psi(x,y)}{\partial x}$
        - ψ is constant along streamlines
        - $\Delta\psi$ is volumetric flow rate
    - Velocity potential ϕ
      - $\vec{V} = \vec{\nabla}\phi$
        - $u = \frac{\partial\phi}{\partial x}$
        - $v = \frac{\partial\phi}{\partial y}$
  - Circulation
    - $\Gamma = \oint_{cs} \vec{V}\cdot d\vec{s}$

Polar: $\iint r\,dr\,d\theta$

$r = \sqrt{(x-h)^2 + (y-k)^2}$

$\theta = \tan^{-1}\left(\frac{(y-k)}{(x-h)}\right)$

$u_z(r) = \frac{1}{4\mu}\frac{(P_1-P_2)}{\ell}(R^2 - r^2)$

- Shear Stress
  - $\tau = \frac{F}{A}$
- Newtonian Fluid Shear Stress
  - $\tau = \mu \frac{\partial u}{\partial y}$
    - μ = dynamic viscosity
    - u = fluid velocity
    - y = the y direction
- $\frac{\partial}{\partial t}(Momentum) = \nabla \cdot \tau + "other\ stuff"$
  - Divide by density on both sides
- $\frac{\partial}{\partial t}(Velocity) = \nabla \cdot \frac{\tau}{\rho}$
  - Where τ is stress
- Buoyancy Force
  - $F_b = \rho g V = \gamma V$
    - ρ = the density of the fluid
    - V = the volume that is submerged
    - $\gamma = \rho g$ = specific weight
- Line of constant pressure
  - Multivariable Calculus review:
    - $dp = \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy + \frac{\partial p}{\partial z}dz = 0$
      - For constant pressure, derivative $dp = 0$
      - This come from the 2d equation:
        - $y - y_0 = \frac{dy}{dx}(x - x_0)$
        - $dy = \frac{dy}{dx}dx$
- Bernoulli's Equation
  - $p + \frac{1}{2}\rho V^2 + \rho g z = \text{Constant}$
    - p = pressure
    - ρ = fluids density
    - V = Velocity **magnitude**
    - $V = \sqrt{x^2 + y^2 + z^2}$
    - z = height
    - Assumptions
      - No Viscosity
      - Steady flow
      - Incompressible
      - Only along streamlines
- Steady State flow definition
  - $\frac{\partial \vec{V}}{\partial t} = \vec{0}$
- Conservation of mass flow
  - $\rho V_1 A_1 = \rho V_2 A_2$
    - V = Velocity
    - A = Pipe area
- Volumetric flow rate
  - $Q = VA = \int V dA$
- Mass flow rate
  - $\dot{M} = \rho V A = \rho Q$
- Chapter 4:Fluid Kinematics
- Equation for Streamlines
  - $\frac{dx}{u} = \frac{dy}{v} = \frac{dw}{z}$
    - For two dimensions:
      - $\frac{dy}{dx} = \frac{v}{u}$
      - Because stream line is always **tangent** to velocity field
- Acceleration
  - $\vec{a} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial t} + v\frac{\partial \vec{V}}{\partial t} + w\frac{\partial \vec{V}}{\partial t}$
    - $\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$
    - $\vec{V} = u\hat{\imath} + v\hat{\jmath} + w\hat{k}$
- Material Derivative
  - $\frac{D()}{Dt} = \frac{\partial()}{\partial t} + (\vec{V} \cdot \vec{\nabla})()$
    - Acceleration $\vec{a} = \frac{D\vec{V}}{Dt}$
- Extensive/Intensive relationship
  - $B = m \cdot b$
    - B is an extensive property
    - b is an intensive property
    - m = mass
- Amount of B in V
  - $B = \int \rho b d\forall$
    - $\forall$=Volume
    - b = intensive property
    - B = extensive property
- Flux of extensive property
  - $\dot{B} = \int_S \rho b\vec{V} \cdot \vec{n} dS$
    - S = a surface
    - b = intensive property
    - B = extensive property
    - $\vec{V}$ = velocity vector
    - $\vec{n}$ = unit vector normal to the surface
- Reynolds Transport Theorem
  - $\frac{\partial B_{cv}}{\partial t} + \dot{B}_{cs} = 0$
    - $\frac{\partial B_{cv}}{\partial t}$ = the unsteady term. It should be zero in steady flow
    - $\dot{B}$ = the fluid flux flowing through the surface
- Mass Conservation
  - $\frac{\partial}{\partial t}\int_{c\forall} \rho d\forall + \int_{cs} \rho\vec{V} \cdot \hat{n} ds = 0$
    - The first term is the unsteady term
    - $c\forall$=is the control Volume

- cs = control surface
- You can split the control surface into multiple flat plans and add it together

- Momentum Conservation
  - $\frac{\partial}{\partial t}\int_{c\forall} \vec{V}\rho d\forall + \int_{cs} \vec{V}\rho\vec{V}\cdot\hat{n}ds = \sum F_{ext}$
    - $F_{ext}$is all external forces
    - Like pressure and stress and such
  - External Forces x direction (typically)
    - Since $F = pA$
    - $\sum F_{ext} = p_{left}A_{left} - p_{right}A_{right} + F_x$
  - External Forces y direction (typically)
    - $\sum F = p_{bottom}A_{bottom} - p_{top}A_{top} + F_y$
- Chapter 6: Differential Analysis of Fluid Flow
- Volumetric Dilation
  - $div(\vec{V})$
    - $div(\vec{V}) = \vec{\nabla}\cdot\vec{V}$watch your coordinate system tho
- Rotation Vector $\vec{\omega}$
  - $\frac{1}{2}\,curl(\vec{V})$
    - $curl(\vec{V}) = \vec{\nabla}\times\vec{V}$watch your coordinate system tho
- Vorticity $\vec{\zeta}$
  - $curl(\vec{V})$ or $2\vec{\omega} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$
- Gradient Tensor
  - $\vec{\nabla}\vec{V} = S_{ij} + \Omega_{ij}$
    - $S = \frac{1}{2}(\vec{\nabla}\vec{V} + (\vec{\nabla}\vec{V})^T)$
      - the rate of stress tensor
    - $\Omega = \frac{1}{2}(\vec{\nabla}\vec{V} - (\vec{\nabla}\vec{V})^T)$
      - Rotation tensor
- $trace(\vec{\nabla}\vec{V}) = div(\vec{V})$
  - $trace$is the sum of the diagonal components of a square matrix
- Mass conservation
  - $\vec{\nabla}\cdot\vec{V} = 0$
- "Newton's Second Law"
  - $ma = \sum F$
    - $ma = \rho\frac{D\vec{V}}{Dt}$
    - $F_{int} = \vec{\nabla}\cdot T_{ij}$
    - $F_{ext} = \rho\vec{g}$
- Stress tensor
  - T_{ij} =isotropic tensor + deviatoric stress tensor
    - isotropic tensor = $-pI_{33}$
    - Where $I_{ij}$is an $i \times j$identity matrix
    - deviatoric stress tensor = τ_{ij} the typical stress tensor from mechanics of materials
- Deviatoric Stress Tensor
  - $\tau_{ij} = \frac{-2}{3}\mu(\vec{\nabla}\cdot\vec{V})I_{ij} + 2\mu S_{ij}$
    - $S_{ij} = 0$ in incompressible fluids
    - μ =dynamic viscosity
- Naviar Stokes
  - $\vec{\nabla}\cdot\vec{V} = 0$
  - $\frac{\partial\vec{V}}{\partial t} + (\vec{V}\cdot\vec{\nabla})\vec{V} = \frac{-1}{\rho}\vec{\nabla}p + \nu\vec{\nabla}^2\vec{V} + \vec{g}$
    - $\nu = \frac{\mu}{\rho}$kinematic viscosity
  - 2D Navier Stokes
    - Euler Equations
      - Naviar stokes without viscous terms
      - $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$
      - $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{-1}{\rho}\frac{\partial p}{\partial x}$
      - $\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \frac{-1}{\rho}\frac{\partial p}{\partial y}$
- Potential functions
  - Linearity applies
    - Assumptions incompressible
    - Inviscious
    - Irrotational
  - Stream functions $\psi(x,y)$
    - $u(x,y) = \frac{\partial\psi(x,y)}{\partial y}$
    - $v(x,y) = \frac{-\partial\psi(x,y)}{\partial x}$
      - $\psi$is constant along streamlines
      - $\Delta\psi$ is volumetric flow rate
  - Velocity potential ϕ
    - $\vec{V} = \vec{\nabla}\phi$
      - $u = \frac{\partial\phi}{\partial x}$
      - $v = \frac{\partial\phi}{\partial y}$
- Circulation
  - $\Gamma = \oint_{cs} \vec{V}\cdot d\vec{s}$

Polar: $\iint r\,dr\,d\theta$

$r = \sqrt{(x-h)^2 + (y-k)^2}$

$\theta = \tan^{-1}\left(\frac{(y-k)}{(x-h)}\right)$

$U_z(r) = \frac{1}{4\mu}\frac{(P_1 - P_2)}{\ell}(R^2 - r^2)$

Nathan
Stenseng

150/150 100%

ME 3250 Section 2 – Midterm #2

1. Pratt & Whitney makes the F119A turbofan engine for the F-22 Raptor. The F119 has a cool feature: it has thrust vectoring. Because of this new feature, Pratt engineers need to perform a new calculation of the forces on the engine's test stand. Let's help them figure this out!

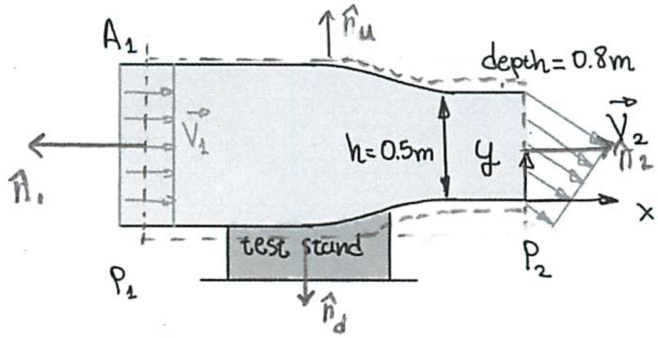
   The engine area at the inlet (location 1) is $A_1 = 2\ \text{m}^2$. The nozzle (location 2) has a rectangular cross section with height $h = 0.5$ m and depth $d = 0.8$ m.

   At the inlet, the velocity is uniform and along the horizontal direction with magnitude V_1.
   At the nozzle, the velocity is $\vec{V_2} = 1000\, y\, \hat{\imath} - 20\, \hat{\jmath}\, \frac{\text{m}}{\text{s}}$. See coordinate system in figure.

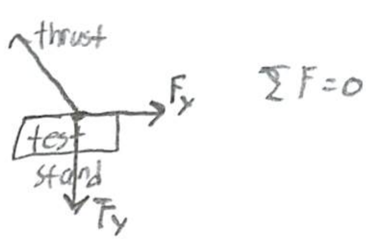
   At the inlet, the gage pressure is $p_1 = -1000$ Pa, and at the nozzle the gage pressure is zero $p_2 = 0$.

   Density is $\rho = 1.4\ \text{kg/m}^3$ at both inlet and nozzle. The flow is steady.
   a. Draw the control volume. [10 points]
   b. Draw the normal vectors on the control volume. [10 points]
   c. Find the inlet velocity V_1? [20 points]
   d. Find the horizontal force required to hold the test stand. [30 points]
   e. Find the vertical force required to hold the test stand. [20 points]

   All your calculations must be *consistent* with the control volume and normal vectors.



$\hat{n}_1 = -\hat{\imath}$
$\hat{n}_2 = \hat{\imath}$



$\Sigma F = 0$

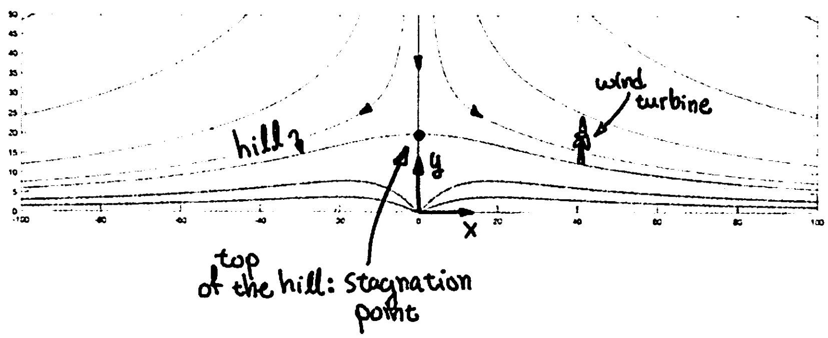
2. Eversource is building wind farms on Connecticut's rolling hills. Eversource's engineers are interested in a flow over the hill as in the Figure below. We will model the flow as a potential flow that can be constructed using the superposition of a "stagnation flow" with velocity potential:

$$\phi_{st}(x,y) = x^2 - y^2$$

and a source of strength m located at $x = 0$ and $y = 0$, with velocity potential:

$$\phi_{sr}(x,y) = m\ln(x^2 + y^2)$$

a. What is the velocity potential of the flow descending over the hill? [10 points]
b. Determine the source strength m if the hill height is 20 m. [20 points]
c. Find the velocity vector at location at $x = 40$ m and $y = 20$ m (you need to find both x and y components of the velocity) [20 + 10 points]



c. conservation of mass

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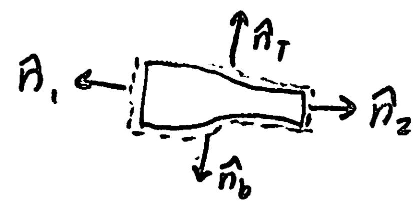


Steady state

$$\int_{cs} \rho \vec{V} \cdot \hat{n}\, ds = 0$$

$$\int_1 + \int_2 + \overset{\to 0}{\int_T} + \overset{\to 0}{\int_b} = 0$$

$$\int_1 \cancel{\rho} \vec{V}_1 \cdot \hat{n}_1\, dA + \int_0^{0.8}\int_0^{0.5} \cancel{\rho} \vec{V}_2 \cdot \hat{n}_2\, dy\, dz = 0$$

$\vec{V}_1 = V_1 \hat{i}$

$\hat{n}_1 = -\hat{i}$

$\langle 1000y, -20 \rangle \cdot \langle 1, 0 \rangle$

$1000y$

$$-V_1 \cdot 2 + 0.8 \left[1000 \frac{y^2}{2}\right]_0^{0.5} = 0$$

$$-2V_1 + 100 = 0$$

$$\boxed{V_1 = 50 \text{ m/s}}$$

d. x comp:

$$\int_{cs} u \rho \vec{V} \cdot \hat{n}\, ds = P_1 A_1 - P_2 A_2 + F_x$$

$$\int_1 V_1 \rho (-V_1)\, dA + \int_0^{0.8}\int_0^{0.5} 1000y \cdot \rho \cdot 1000y\, dy\, dz =$$

$$-\rho V_1^2 \cdot 2 + 0.8 \cdot \rho \cdot 1000^2 \int_0^{0.5} y^2 dy =$$

$$\frac{y^3}{3}$$

$\rho = 1.4$

$$-\rho V_1^2 \cdot 2 + \rho \cdot 8 \cdot 10^5 \cdot \frac{0.125}{3} = -1000 \cdot 2 - 0 + F_x$$

$$\boxed{F_x = 41.67 \text{ kN}}$$

e. y comp.

$$\int_{cs} V \cdot \rho \vec{V} \cdot \hat{n}\, ds = \cancel{P_{atm} A_{bot}} - \cancel{P_{atm} A_{top}} + F_{y}$$

$$0 + \int_0^{0.8}\int_0^{0.5} -20 \cdot \rho \cdot \langle 1000y, -20 \rangle \cdot \langle 1, 0 \rangle \, dy\,dx = F_y$$

$$0.8\rho \cdot -20 \int_0^{0.5} 1000\,y\,dy = F_y$$

$$1000\frac{(0.5)^2}{2}$$

$$\boxed{F_y = -2.8 \text{ kN}}$$

2. $\phi_{hill} = \phi_{str} + \phi_{sr}$

$$= \boxed{x^2 - y^2 + m\ln(x^2+y^2)}$$

b. @ $(0,20)$ $\vec{V} = \langle 0, 0 \rangle$

$$\frac{\partial \phi}{\partial x} = 2x + \frac{m}{x^2+y^2} \cdot 2x = u$$

$$\frac{\partial \phi}{\partial y} = -2y + \frac{m}{x^2+y^2} \cdot 2y = v$$

$$0 = 0 + \frac{m}{0+20^2} \cdot 0$$

$$0 = -2 \cdot 20 + \frac{m}{0+20^2} \cdot 2 \cdot 20$$

$$0 = -40 + \frac{m}{400} \cdot 40$$

$$40 = \frac{m}{10} \quad \therefore \boxed{m = 400}$$

e. y comp.

$$\int_{cs} V \cdot \rho \vec{V} \cdot \vec{n} \, ds = \cancel{P_{atm} A_{bot}} - \cancel{P_{atm} A_{top}} + F_y$$

$$0 + \int_0^{0.8} \int_0^{0.5} -20 \cdot \rho \cdot \langle 1000y, -20 \rangle \cdot \langle 1, 0 \rangle \, dy dx = F_y$$

$$0.8 \rho \cdot -20 \int_0^{0.5} 1000 y \, dy = F_y$$

$$1000 \frac{(0.5)^2}{2}$$

$$\boxed{F_y = -2.8 \text{ kN}}$$

2. $\phi_{hill} = \phi_{str} + \phi_{sr}$

$$= \boxed{x^2 - y^2 + m \ln (x^2 + y^2)}$$

b. @ $(0, 20)$ $\vec{V} = \langle 0, 0 \rangle$

$$\frac{\partial \phi}{\partial x} = 2x + \frac{m}{x^2 + y^2} \cdot 2x = u$$

$$\frac{\partial \phi}{\partial y} = -2y + \frac{m}{x^2 + y^2} \cdot 2y = v$$

$$0 = 0 + \frac{m}{0 + 20^2} \cdot 0$$

$$0 = -2 \cdot 20 + \frac{m}{0 + 20^2} \cdot 2 \cdot 20$$

$$0 = -40 + \frac{m}{400} \cdot 40$$

$$40 = \frac{m}{10} \quad \therefore \quad \boxed{m = 400}$$

Nathan Stenseng

c. $\vec{V} = \langle u, v \rangle$

$$u = 2x + \frac{400}{x^2+y^2} \cdot 2x$$

$$v = -2y + \frac{400}{x^2+y^2} \cdot 2y$$

@ (40, 20)

$$u = 2 \cdot 40 + \frac{400}{40^2+20^2} \cdot 2 \cdot 40 = 96 \text{ m/s}$$

$$v = -2 \cdot 20 + \frac{400}{40^2+20^2} \cdot 2 \cdot 20 = -32 \text{ m/s}$$

$$\boxed{\vec{V} = (96\hat{\imath} - 32\hat{\jmath}) \text{ m/s}}$$ @ (40, 20)

	Gage Pressure	$P_{gage} = P_{abs} - P_{ambient}$	P_{abs} = absolute pressure $P_{ambient}$ = local pressure P_{gage} = what engineers measure
1.6	Specific Weight	$\gamma = \rho g$	
1.7	Specific Gravity	$SG = \frac{\rho}{\rho_{H_2O\,at\,4°C}}$	
1.8	Ideal Gas Law	$\rho = \frac{p}{RT}$	p = absolute pressure R = a substance specific gas constant T = absolute temperature
	Shear Stress	$\tau = \frac{F}{A}$	
1.9	Newtonian Fluid Shear Stress	$\tau = \mu \frac{\partial u}{\partial y}$	μ = dynamic viscosity u = fluid velocity y = the y direction

- $\frac{\partial}{\partial t}(Momentum) = \nabla \cdot \tau +$ "*other stuff*"
  - Divide by density on both sides
- $\frac{\partial}{\partial t}(Velocity) = \nabla \cdot \frac{\tau}{\rho}$
  - Where τ is stress

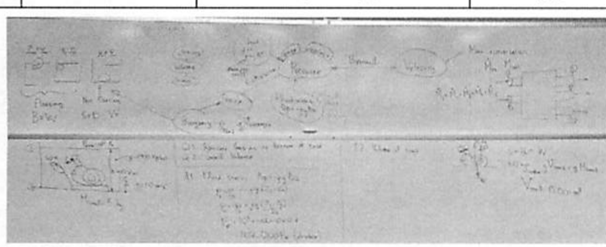
	Equation of Pressure Gradient	$\nabla p = -\rho \vec{a}_i$	$\vec{\nabla} p = \frac{\partial p}{\partial x}\hat{i} + \frac{\partial p}{\partial y}\hat{j} + \frac{\partial p}{\partial z}\hat{k}$ $\vec{a}_i$ = acceleration
1a	Special Pressure Equation Cases 1a: No motion and incompressible	$\frac{\partial p}{\partial x} = 0$ $\frac{\partial p}{\partial y} = 0$ $\frac{\partial p}{\partial z} = -\rho g$	$\vec{a}_{i,x,y} = 0$ External forces are 0 and fluid is incompressible $-\rho g$ is from gravity

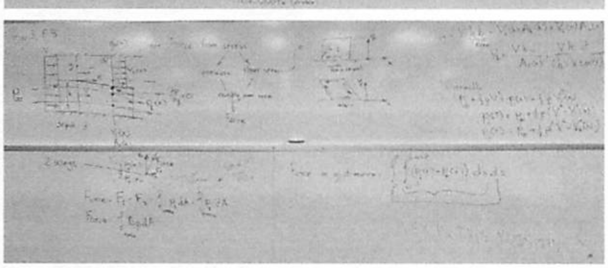
$\Delta p = \rho g \Delta z$

1b	Special Pressure Equation Cases 1b: No motion and compressible	$\frac{\partial p}{\partial z} = -\rho(z)g$	Most gasses are this way Density is a function of position
2a	Special Pressure Equation Cases 2a: Constant acceleration and incompressible	$\frac{\partial p}{\partial x} = -\rho a_x$ $\frac{\partial p}{\partial y} = -\rho a_y$ $\frac{\partial p}{\partial z} = -\rho g - \rho a_z$	$\vec{a}_i = \langle a_x, a_y, a_z \rangle$
2.22	Buoyancy Force	$F_b = \rho g V = \gamma V$	ρ = the density of the fluid V = the volume that is submerged $\gamma = \rho g$ = specific weight
	Multivariable Calculus review: Line of constant pressure	$dp = \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy + \frac{\partial p}{\partial z}dz = 0$	For constant pressure, derivative $dp = 0$ This come from the 2d equation: $y - y_0 = \frac{dy}{dx}(x - x_0)$ $dy = \frac{dy}{dx}dx$

3.7	Bernoulli's Equation	$p + \frac{1}{2}\rho V^2 + \rho g z =$ Constant Assumptions: 1. No Viscosity, 2. Steady flow, 3. Incompressible, 4. Only along streamlines	p = pressure ρ = fluids density V = Velocity **magnitude** $V = \sqrt{x^2 + y^2 + z^2}$ z = height
	Steady State flow definition	$\frac{\partial \vec{V}}{\partial t} = 0$	
	Conservation of mass flow	$\rho V_1 A_1 = \rho V_2 A_2$	V = Velocity A = Pipe area
	Volumetric flow rate	$Q = VA$	$\dot{M}$ = mass flow rate

		$\dot{M} = \rho V A = \rho Q$	Q = volumetric flow rate





Positive shear

Negative Shear