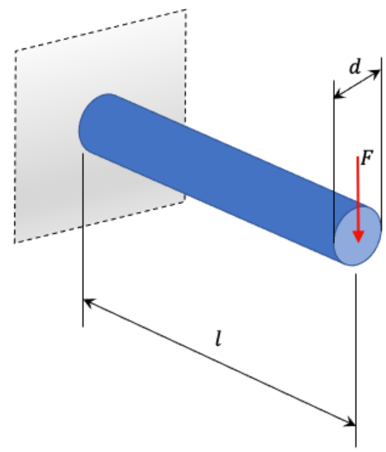
ME3227-001 Design of Machine Elements (Norato)
Midterm 1 (15 points)

All the questions without a star (four questions per problem) are <u>mandatory</u>. You can answer the bonus question marked with a star (one question per problem) to get extra credit *only if* you have answered all the other four mandatory questions.

Problem 1 (5 points)

A cantilever beam of length $l = 16$ in and with a circular cross section of diameter $d = 1$ in was designed to withstand a force F that follows a Gaussian distribution with a nominal value $\mu_F = 90$ lbf and a standard deviation $\sigma_F = 20$ lbf. The beam was originally designed of an aluminum available in a particular geographic region (Market A), with yield strength following a Gaussian distribution that has a mean $\mu_S = 20$ ksi and a standard deviation $\sigma_S = 2$ ksi. The beam is now going to be produced in another geographic region (Market B), but the same aluminum alloy is not available in that region. Instead, another aluminum alloy with $\mu_S = 22$ ksi and $\sigma_S = 5$ ksi will be used as a replacement.



In the following, assume the tolerances in the beam dimensions are tight and they have a negligible effect in the variability of the beam strength in comparison to the effect caused by the variability of the load magnitude and the material yield strength. Also, assume the variability of the orientation and point of application of the force is negligible as well. Round all your answers to two decimals.

a) Where does the largest stress occur? And what are the nominal value and standard deviation of the stress at that location? (Consider only bending).

b) What is the deterministic factor of safety in each of the two markets? What design is safer in terms of this factor?

c) What is the reliability of the design in each of the two markets? What design is more reliable? (*Hint*: recall the reliability corresponds to the probability that the margin of safety is larger than zero).

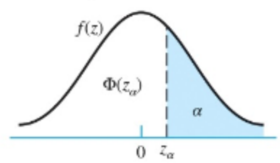
d) Is the design that is safer in terms of the factor of safety also the most reliable of the two? Why or why not?

e) * In terms of potential warranty costs for the beam design, which design method (deterministic or stochastic) would you chose as a designer and why?

Use the Table below if needed.

$$\Phi(z_\alpha) = \int_{-\infty}^{z_\alpha} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) du$$

$$= \begin{cases} \alpha & z_\alpha \le 0 \\ 1-\alpha & z_\alpha > 0 \end{cases}$$



Z_α	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3238	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823

Problem 2 (5 points)

The following table (which is an excerpt of Table A-21 in the book), lists some mean mechanical properties of heat-treated AISI 1030 steel. The modulus of elasticity of carbon steel is E = 207 GPa and its density is 7807 kg/m^3.

1 AISI No.	2 Treatment	3 Temperature °C (°F)	4 Tensile Strength MPa (kpsi)	5 Yield Strength, MPa (kpsi)	6 Elongation, %	7 Reduction in Area, %	8 Brinell Hardness
1030	Q&T*	205 (400)	848 (123)	648 (94)	17	47	495
	Q&T*	315 (600)	800 (116)	621 (90)	19	53	401
	Q&T*	425 (800)	731 (106)	579 (84)	23	60	302
	Q&T*	540 (1000)	669 (97)	517 (75)	28	65	255
	Q&T*	650 (1200)	586 (85)	441 (64)	32	70	207
	Normalized	925 (1700)	521 (75)	345 (50)	32	61	149
	Annealed	870 (1600)	430 (62)	317 (46)	35	64	137

a) Which heat treatment gives a higher resilience, normalizing or annealing? Explain your answer.

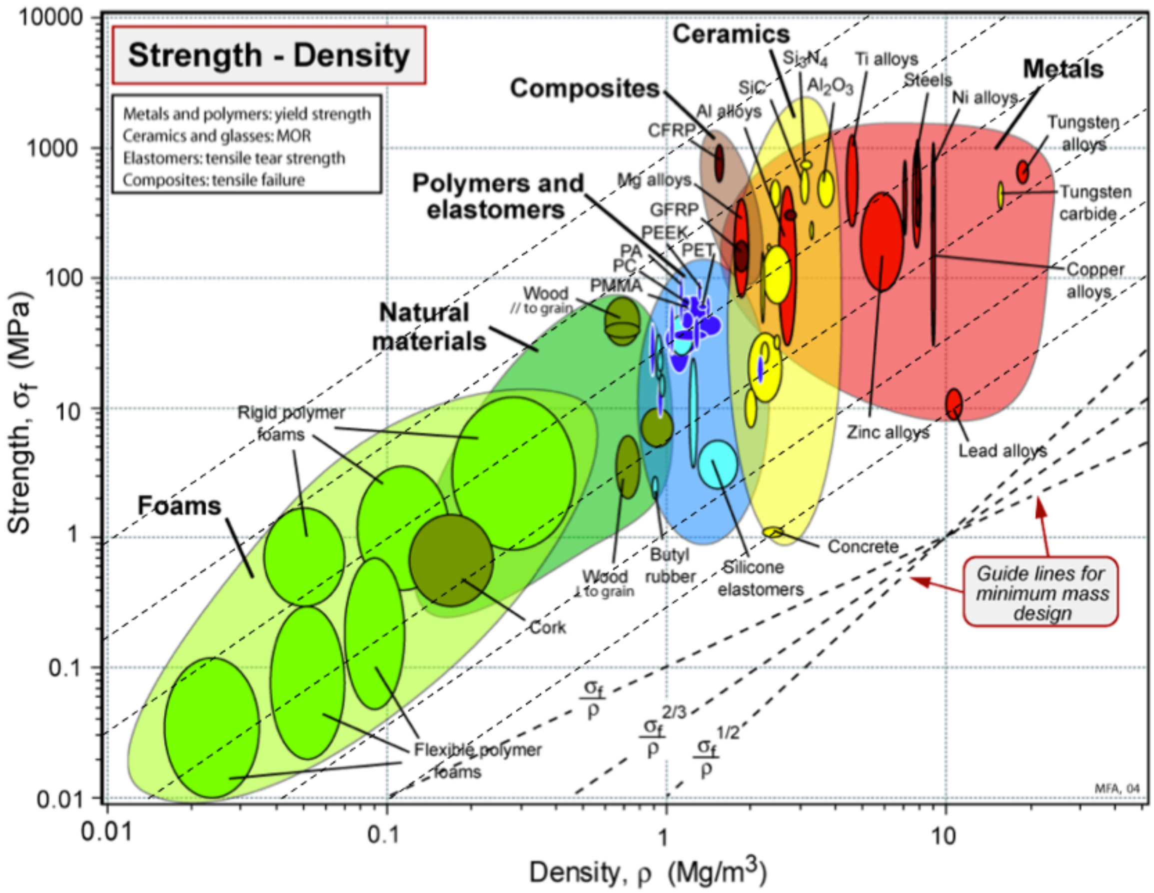
b) Does the heat treatment affect the relationship between Brinell hardness and tensile strength? Explain your answer. (You can pick only three steels to verify this).

c) What Q&T material is tougher: the one tempered at 205 °C or the one tempered at 650 °C?

Hint: remember that the true strain can be computed as $\tilde{\varepsilon} = \ln(A_0/A)$ and that the reduction in area is $R = 1 - A_f/A_0$, and so $\tilde{\varepsilon}_f = \ln(1/(1-R))$. From this and the fact that $\tilde{\varepsilon} = \ln(1-\varepsilon)$, we have $\varepsilon_f = R/(1-R)$.

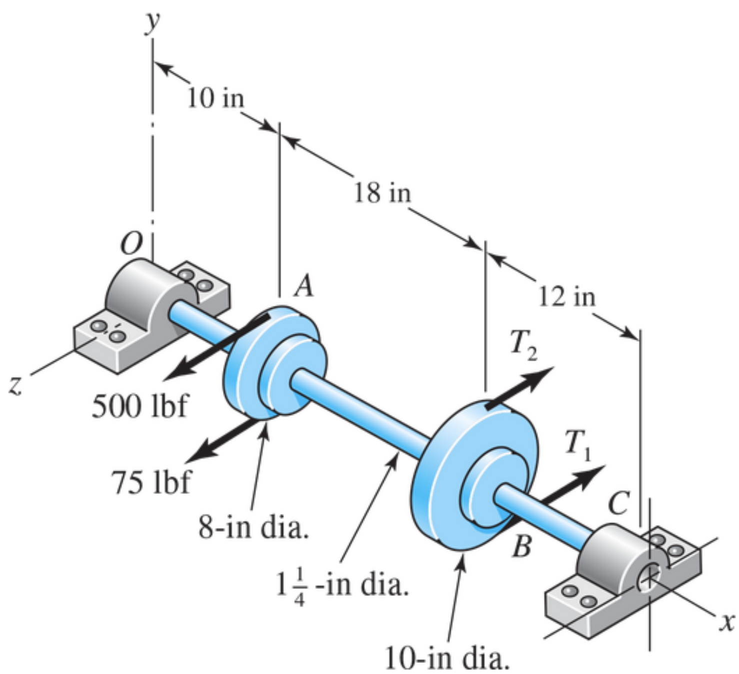
d) Suppose you are using this steel to design the pressure vessel containing an inert gas. In this case, strength is required to withstand the hoop and longitudinal stresses caused by the internal pressure; however, as a matter of safety, it is also desired that if the pressure increases drastically over its normal operating range the vessel deforms substantially before fracturing. Is there a tradeoff between yield strength and toughness as a function of tempering temperature? And if so, would this trade-off be a consideration in your design?

e) * Now consider you are trying to design a component subject to bending. Your goal in designing this component is to minimize its weight while ensuring the component is sufficiently strong. You consider two materials: steel AISI 1030 Q&T (at a tempering temperature of 205 °C) and aluminum 7075-T6 with a yield strength of 542 MPa and a density of 2700 kg/m^3. Considering the Ashby plot for strength shown below, which material choice will allow you to minimize the component's weight? Explain your answer.



Problem 3 (5 points)

Consider the countershaft shown in the figure, which carries two V-belt pulleys. Pulley *A* receives power from a motor through a belt with the belt tensions shown. The power is transmitted through the shaft and delivered to the belt on pulley *B*. Assume the belt tension on the loose side at *B* is 15 percent of the tension on the tight side.



a) Determine the tensions in the belt on pulley *B*, assuming the shaft is running at constant speed.
b) Find the magnitudes of the reaction forces on the bearings at *C* and *O*. Note that the inner race of the bearings allows for some angular misalignment, and so they act as simple supports (i.e., they do not constraint the angular deformation of the shaft).
c) Draw shear-force and bending-moment diagrams for the shaft.
d) Does this shaft experience bending moments about two axis or only one? What is the torsional moment between *A* and *O* and between *B* and *C*? Explain your answer.
e) * Is this system statically determinate or indeterminate? Explain your answer.