# Homework 10-1

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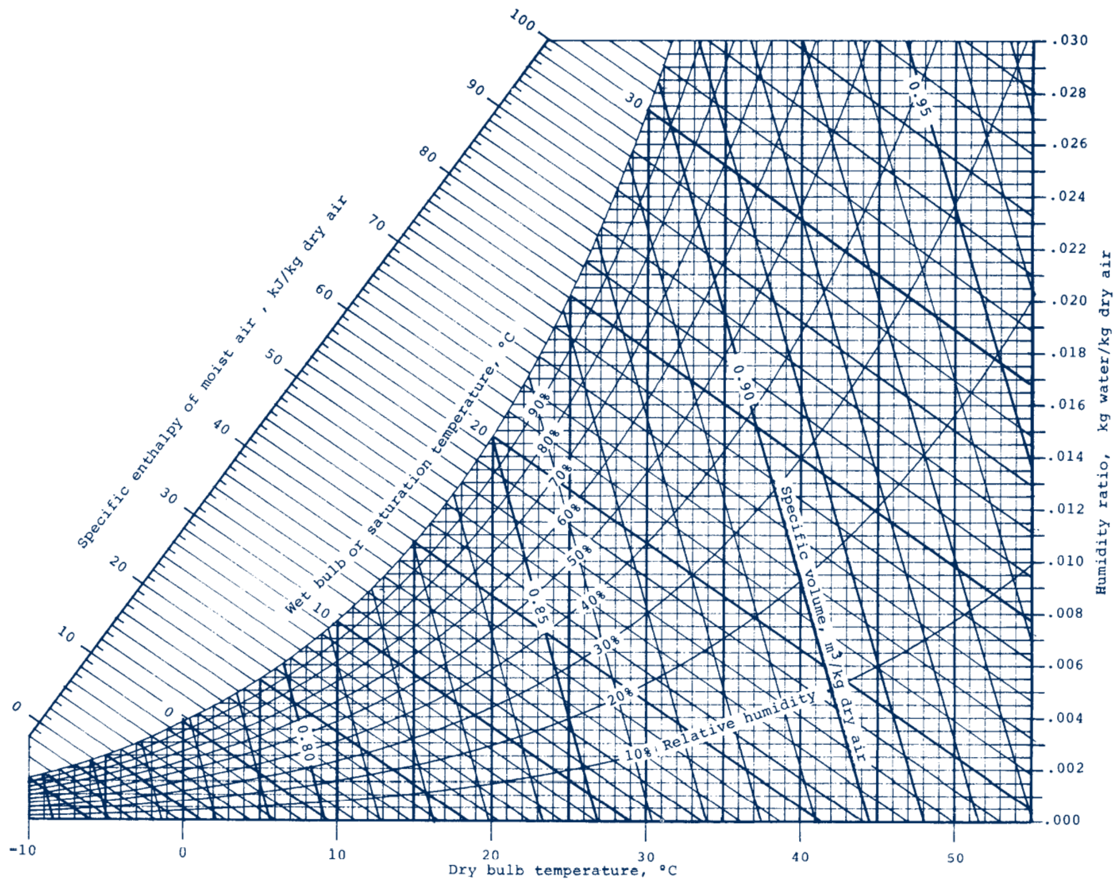
## Problem Statement

For this problem, just explain your reasoning in the Markdown cell below. You can use approximate values from the chart below, and you don't need to show your calculations.

A fixed amount of air initially at 52 °C, 1 atm, and 10% relative humidity is cooled at constant pressure to 15 °C.

1. Using the psychrometric chart, determine whether condensation occurs. (Figure A-9E in your textbook is a higher quality version of the chart below.) If there is condensation, determine the amount of water condensed, in **kg of water** per **kg of dry air**. If there is no condensation, determine the relative humidity at the final state.




## Solution

### Part 1: Determine whether condensation occurs. Then find either amount of water condensed, or final relative humidity.

Cooling at constant pressure means that the humidity ratio is also constant because $\omega = 0.622\frac{p_v}{p-p_v}$. Our first state will be the point $(T_{DB}, \phi) = (52^oC, 10\%)$. At this point, $\omega = 0.0085$. Now the next point we will look at is $(T_{DB}, \omega) = (15^oC, 0.0085)$. This has a realitive humidity of $\phi \approx 58\%$, not producing condensate.

# Homework 10-2

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### Imports

[1]:
```
from thermostate import Q_, State
```

---

### Definitions

[2]:
```
sub_v = 'water'
sub_a = 'air'

MW_v = Q_(18.02, 'g/mol')
MW_a = Q_(28.97, 'g/mol')

T_1 = Q_(90.0, 'degF')
p_1 = Q_(14.7, 'psi')
phi_1 = Q_(0.75, 'dimensionless')
Vdot_1 = Q_(100.0, 'ft**3/min')

Wdot_cv = Q_(-15.0, 'hp')

p_2 = Q_(100.0, 'psi')
T_2 = Q_(400.0, 'degF')

p_3 = p_2
T_3 = Q_(100.0, 'degF')
phi_3 = Q_(1.0, 'dimensionless')

# you can use these to calculate saturated states
x_f = Q_(0.0, 'dimensionless')
x_g = Q_(1.0, 'dimensionless')
```

---

### Problem Statement

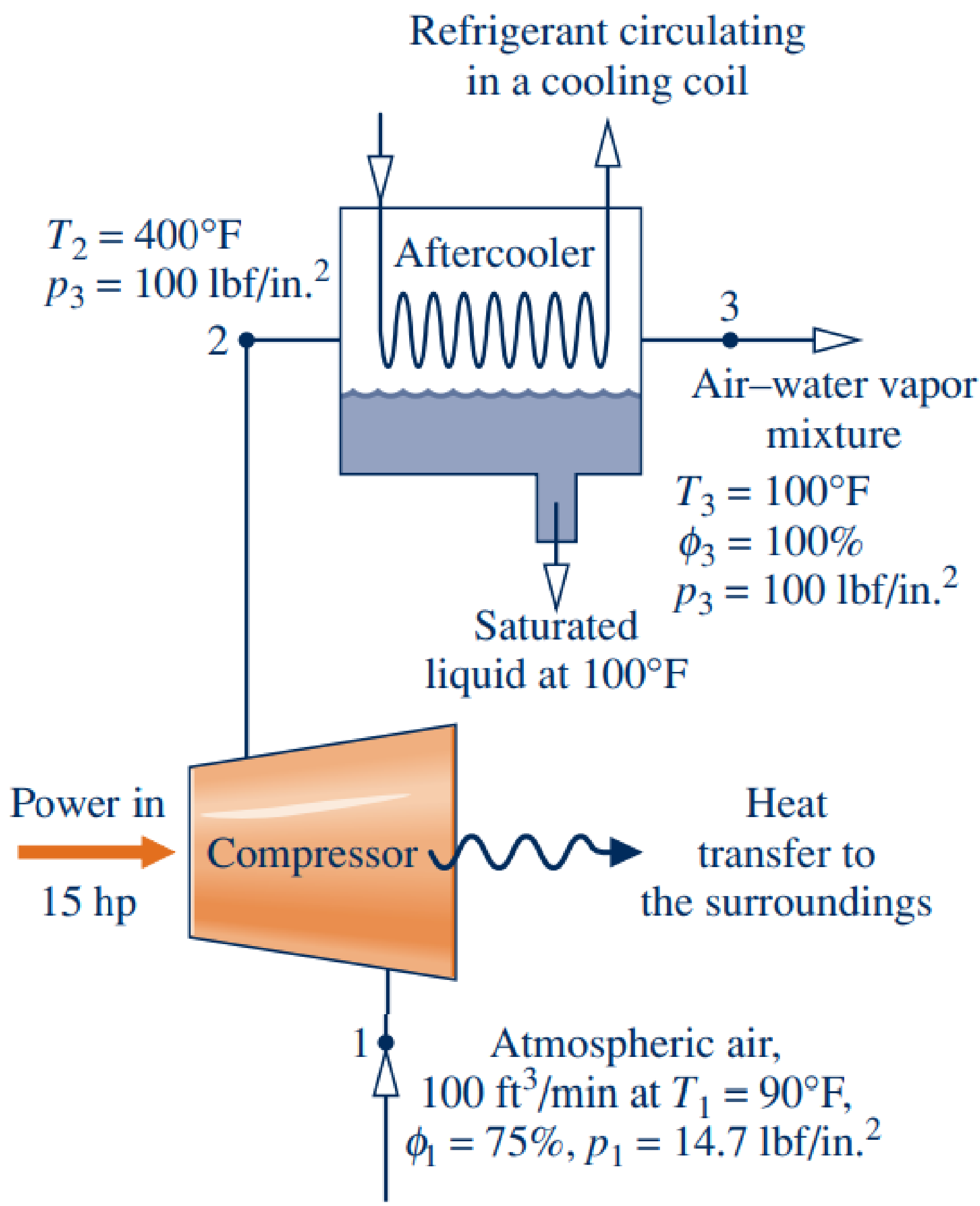
A compressor can be designed with an aftercooler as shown below.

Atmospheric air at 14.7 psi, 90 °F, and 75% relative humidity enters the compressor with a volumetric flow rate of 100 cubic feet per minute. The compressor power input is 15 hp.

The moist air exiting the compressor at 100 psi, 400 °F flows through the aftercooler, where it is cooled at constant pressure, exiting saturated at 100 °F. Condensate also exits the aftercooler at 100 °F.

For steady-state operation and negligible kinetic and potential energy effects, determine

1. the rate of heat transfer from the compressor to its surroundings, in Btu/min,
2. the mass flow rate of the condensate leaving the aftercooler, in lb/min,
3. the rate of heat transfer from the moist air to the refrigerant circulating in the cooling coil, in tons of refrigeration.




**Solution**

**Part 1: Rate of Heat Transfer Out of the Compressor (Btu/min)**

$$\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_{a3} \qquad \dot{m}_{v1} = \dot{m}_{v2} \qquad \omega_1 = \omega_2 \tag{1) (2) (3}$$

$$\phi_1 = \frac{p_{v1}}{p(T_1, x_{sat})} \tag{4}$$

$$\omega_1 = 0.622\frac{p_{v1}}{p_1 - p_{v1}} = \frac{m_{v1}}{m_{a1}} \tag{5}$$

$$p_1 = p_{a1} + p_{v1} \tag{6}$$

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + \sum m_{in}h_{in} - \sum m_{out}h_{out} \tag{7a}$$

$$0 = \dot{Q}_{cv} - \dot{W}_{cv} + m_{a1}h_{a1} + m_{v1}h_{v1} - m_{a2}h_{a2} - m_{v2}h_{v2} \tag{7b}$$

$$0 = \frac{\dot{Q}_{cv}}{\dot{m}_{a1}} - \frac{\dot{W}_{cv}}{\dot{m}_{a1}} + h_{a1} + \omega_1 h_{v1} - h_{a2} - \omega_2 h_{v2} \tag{7c}$$

$$v_{a1}\dot{m}_{a1} + v_{v1}\dot{m}_{v1} = \dot{V}_1 \tag{8a}$$

$$v_{a1} + v_{v1}\omega_1 = \frac{\dot{V}_1}{\dot{m}_{a1}} \tag{8b}$$

[3]:
```
#eq 4
p_v1 = phi_1 * State(sub_v, T=T_1, x=x_g).p
st_v1 = State(sub_v, p=p_v1, T=T_1)
p_v1 = st_v1.p
h_v1 = st_v1.h
#eq 5
omega_1 = omega_2 = 0.622*(p_v1/(p_1-p_v1))
#eq 6
p_a1 = p_1-p_v1
st_a1 = State(sub_a, p=p_a1, T=T_1)
h_a1 = st_a1.h
#eq 8b
#mdot_a = Vdot_1/(st_a1.v+st_v1.v*omega_1)
mdot_a = Vdot_1/st_a1.v
#eq 5
p_v2 = omega_2*p_2/(omega_2+0.622)
st_v2 = State(sub_v, p=p_v2, T=T_2)
h_v2 = st_v2.h
#eq 6
p_a2 = p_2-p_v2
st_a2 = State(sub_a, p=p_a2, T=T_2)
h_a2 = st_a2.h
#eq 7c
```

```python
Qdot = -mdot_a*(-Wdot_cv/mdot_a + h_a1 + omega_1*h_v1 - h_a2 - omega_2*h_v2)
print(Qdot.to("Btu/min"))
```

```
-91.79919045139117 btu / minute
```

**Answer:** 363.78 Btu/min out of the compressor

#### Part 2: Mass Flow Rate of the Condensate (lb/min)

$$\dot{m}_{v2} = \dot{m}_{v3} + \dot{m}_{w3} \tag{9}$$

$$\omega_2 = \omega_3 + \frac{\dot{m}_{w3}}{\dot{m}_a} \tag{10}$$

[4]:
```python
#eq 4
st_v3 = State(sub_v, T=T_3, x=x_g)
p_v3 = st_v3.p
h_v3 = st_v3.h
#eq 5
omega_3 = 0.622*(p_v3/(p_3-p_v3))
#eq 10
mdot_w3 = mdot_a*(omega_2-omega_3)
print(mdot_w3.to("lb/min"))
```

```
0.11861011618398638 pound / minute
```

**Answer:** 0.059 lb/min

#### Part 3: Rate of Heat Transfer Out of the Moist Air (refrigeration tons)

Use `refrigeration_tons` as output units.

$$0 = \dot{Q}_{cv} + \sum m_{in}h_{in} - \sum m_{out}h_{out} \tag{11a}$$

$$0 = \dot{Q}_{cv} + \dot{m}_{a2}h_{a2} + \dot{m}_{v2}h_{v2} - \dot{m}_{a3}h_{a3} - \dot{m}_{v3}h_{v3} - \dot{m}_{w3}h_{w3} \tag{11b}$$

$$0 = \frac{\dot{Q}_{cv}}{\dot{m}_a} + h_{a2} + \omega_2 h_{v2} - h_{a3} - \omega_3 h_{v3} - \frac{\dot{m}_{w3}}{\dot{m}_a}h_{w3} \tag{11c}$$

[5]:
```python
h_w3 = State(sub_v, T=T_3, x=x_f).h
#eq 6
p_a3 = p_3 - p_v3
h_a3 = State(sub_a, p=p_a3, T=T_3).h
```

```
#eq 11c
Qdot = -mdot_a*(h_a2 + omega_2*h_v2 - h_a3 - omega_3*h_v3 - h_w3* mdot_w3/
↪mdot_a)
print(Qdot.to("refrigeration_tons").round(2))
```

```
-3.27 refrigeration_ton
```

**Answer:** 1.63 refrigeration tons out of the moist air

# Homework 10-3

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## Imports

[1]:
```
from thermostate import Q_, State, units
import numpy as np
import matplotlib.pyplot as plt

import warnings
from pint.errors import UnitStrippedWarning
warnings.simplefilter(action='ignore', category=UnitStrippedWarning)
```

---

## Definitions

[2]:
```
sub_a = 'air'
sub_v = 'water'

# use these for calculating saturated states
x_f = Q_(0.0, 'dimensionless')
x_g = Q_(1.0, 'dimensionless')

# stream 1 properties
T_1 = Q_(60.0, 'degF').to('degR')
p_1 = Q_(1.0, 'atm')
phi_1 = Q_(0.3, 'dimensionless')

# stream 2 properties
T_2 = Q_(90.0, 'degF').to('degR')
p_2 = Q_(1.0, 'atm')
phi_2 = Q_(0.8, 'dimensionless')

# stream 3 properties
p_3 = Q_(1.0, 'atm')
```

---

## Problem Statement

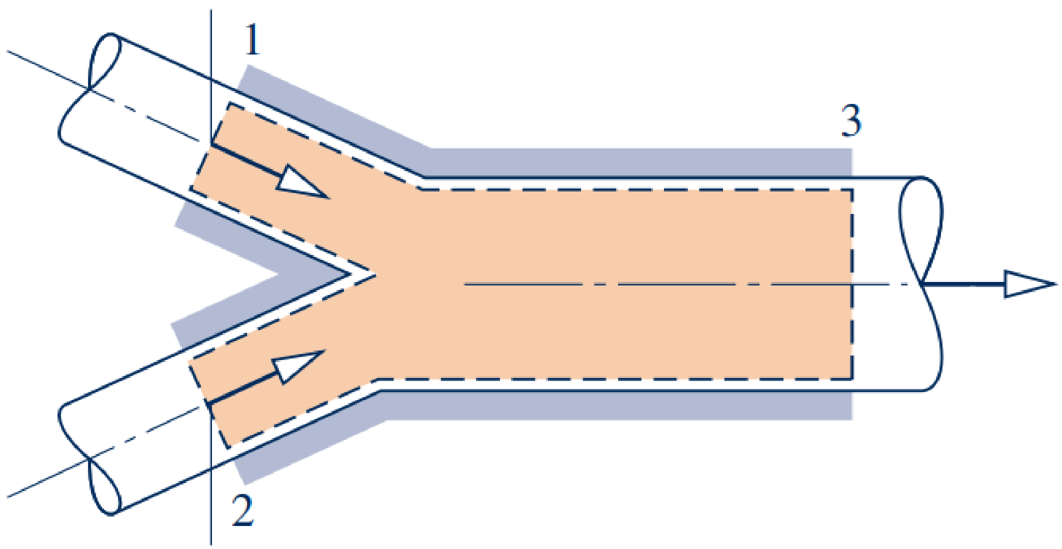
Commander Datum of the USS Enterskies is an android, and has taken a Cromulan disruptor blast to the chest, damaging his positronic cooling system. He's beginning to overheat, risking permanent damage. Lt. Cdr. Jordan Forge needs to design a cooling system which can provide air at 65 °F in order to cool down Datum.

Forge can divert air streams from two plumbing systems. Stream 1 provides air at 60 °F, 1 atm, 30% relative humidity. Stream 2 provides air at 90 °F, 1 atm, 80% relative humidity. He can mix the streams adiabatically.

A single stream (stream 3) exits the mixing chamber at temperature $T_3$ and 1 atm. The system operates at steady state and kinetic and potential energy effects are negligible.

Denote the ratio of dry air mass flow rates as $r = \dot{m}_{a1}/\dot{m}_{a2}$.

1. Determine $T_3$, in °F, for $r = 2$.
2. Plot $T_3$, in °F, for $r$ values from 0 to 10.
3. What ratio $r$ does Forge need in order to cool Datum with air at exactly 65 °F?




## Solution

### Part 1: Calculate $T_3$ (°F) for $r = 2$

$$\phi_1 = \frac{p_{v1}}{p(T_1, x_{sat})} \tag{1}$$

$$\omega_1 = 0.622\frac{p_{v1}}{p_1 - p_{v1}} = \frac{m_{v1}}{m_{a1}} \tag{2}$$

$$p_1 = p_{a1} + p_{v1} \tag{3}$$

$$\dot{m}_{a1} + \dot{m}_{a2} = \dot{m}_{a3} \tag{4a}$$

$$r + 1 = \frac{\dot{m}_{a3}}{\dot{m}_{a2}} \tag{4b}$$

$$\dot{m}_{v1} + \dot{m}_{v2} = \dot{m}_{v3} \tag{5a}$$

$$\dot{m}_{a1}\omega_1 + \dot{m}_{a2}\omega_2 = \dot{m}_{a3}\omega_3 \tag{5b}$$

$$r\omega_1 + \omega_2 = (r+1)\omega_3 \tag{5c}$$

$$\sum m_{in}h_{in} = \sum m_{out}h_{out} \tag{6a}$$

$$\dot{m}_{a1}h_{a1} + \dot{m}_{v1}h_{v1} + \dot{m}_{a2}h_{a2} + \dot{m}_{v2}h_{v2} = \dot{m}_{a3}h_{a3} + \dot{m}_{v3}h_{v3} \tag{6b}$$

$$\dot{m}_{a1}h_{a1} + \dot{m}_{a1}\omega_1 h_{v1} + \dot{m}_{a2}h_{a2} + \dot{m}_{a2}\omega_2 h_{v2} = \dot{m}_{a3}h_{a3} + \dot{m}_{a3}\omega_3 h_{v3} \tag{6c}$$

$$r(h_{a1} + \omega_1 h_{v1}) + h_{a2} + \omega_2 h_{v2} = (r+1)(h_{a3} + \omega_3 h_{v3}) \tag{6d}$$

[3]:

```
r=2
#Setting Stream 1 States
#eq 1
p_v1 = phi_1 * State(sub_v, T=T_1, x=x_g).p
st_v1 = State(sub_v, p=p_v1, T=T_1)
h_v1 = st_v1.h
#eq 2
omega_1 = 0.622*(p_v1/(p_1-p_v1))
#eq 3
p_a1 = p_1 - p_v1
st_a1 = State(sub_a, p=p_a1, T=T_1)
h_a1 = st_a1.h

#Setting Stream 2 States
#eq 1
p_v2 = phi_2 * State(sub_v, T=T_2, x=x_g).p
st_v2 = State(sub_v, p=p_v2, T=T_2)
h_v2 = st_v2.h
#eq 2
omega_2 = 0.622*(p_v2/(p_2-p_v2))
#eq 3
p_a2 = p_2 - p_v2
st_a2 = State(sub_a, p=p_a2, T=T_2)
h_a2 = st_a2.h

#eq 5c
omega_3 = (r*omega_1 + omega_2)/(r + 1)
#eq 2
p_v3 = omega_3*p_3/(omega_3+0.622)
#eq 3
```

```
p_a3 = p_3 - p_v3
```

[4]:
```
def calc_diff(T_3_val):
    T_3 = T_3_val

    h_v3 = State(sub_v, p=p_v3, T=T_3).h
    h_a3 = State(sub_a, p=p_a3, T=T_3).h
    #eq 6d
    diff = r*(h_a1 + omega_1*h_v1) + h_a2 + omega_2*h_v2 - (r + 1)*(h_a3 +␣
 ↪omega_3*h_v3)
    return diff

T_3_vals_best = np.linspace(70, 71, 100)*units.degF
diff_vals = np.zeros_like(T_3_vals_best)*units.BTU/units.lb

for i, T_3 in enumerate(T_3_vals_best):
    diff_vals[i] = calc_diff(T_3)

diff_vals = abs(diff_vals)
minimum = min(diff_vals)
print(minimum)
location = np.where(diff_vals == minimum)
T_3_best = T_3_vals_best[location]
print(T_3_best)
```

```
0.00021083033889906862 btu / pound
[70.25252525] degF
```

**Answer:** $70.25\ ^oF$

**Part 2: Plot $T_3$ for $0 \leq r \leq 10$**

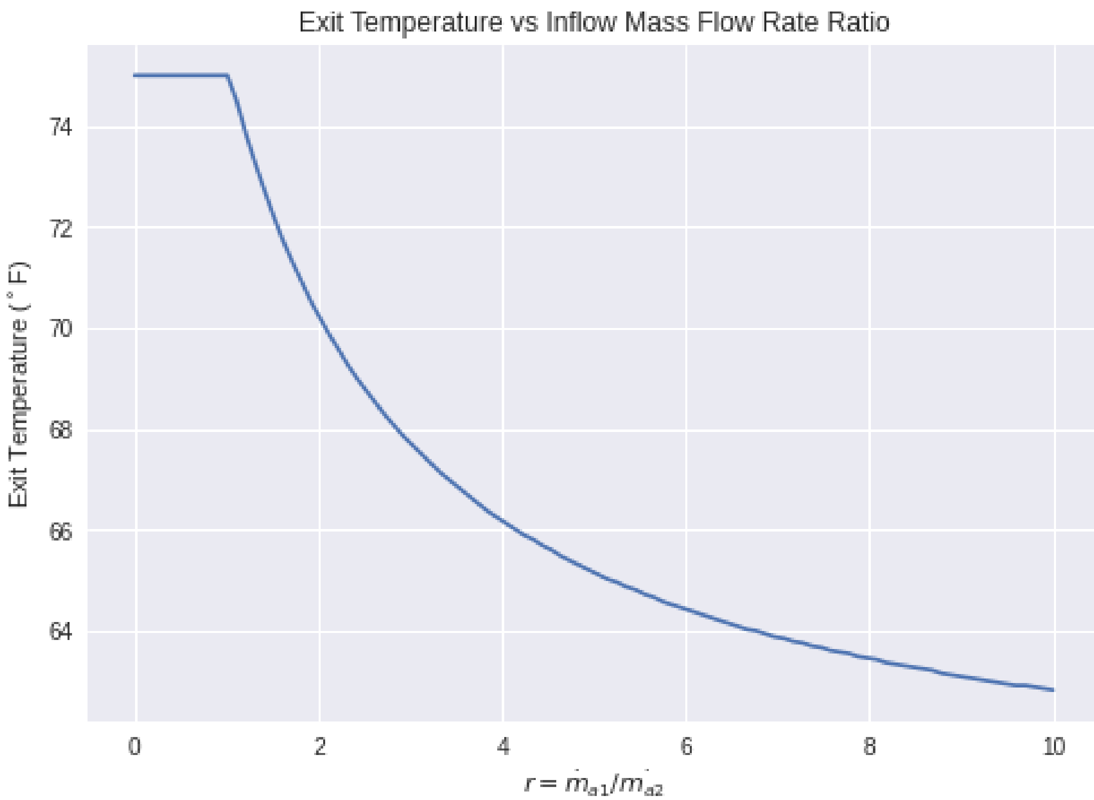
```
def calc_T_3(r):

    # FILL THIS IN

    return T_3.to('degF')

r_vals = np.linspace(0, 10)*units.dimensionless
T_3_vals = np.zeros_like(r_vals)*units.degF

for i, r in enumerate(r_vals):
    T_3_vals[i] = calc_T_3(r)

plt.style.use('seaborn')
plt.plot(r_vals, T_3_vals)
plt.title('Exit Temperature vs Inflow Mass Flow Rate Ratio')
```

```
plt.xlabel('$r = \dot{m}_{a1}/\dot{m_{a2}}$')
plt.ylabel('Exit Temperature ($^\circ$F)');
```

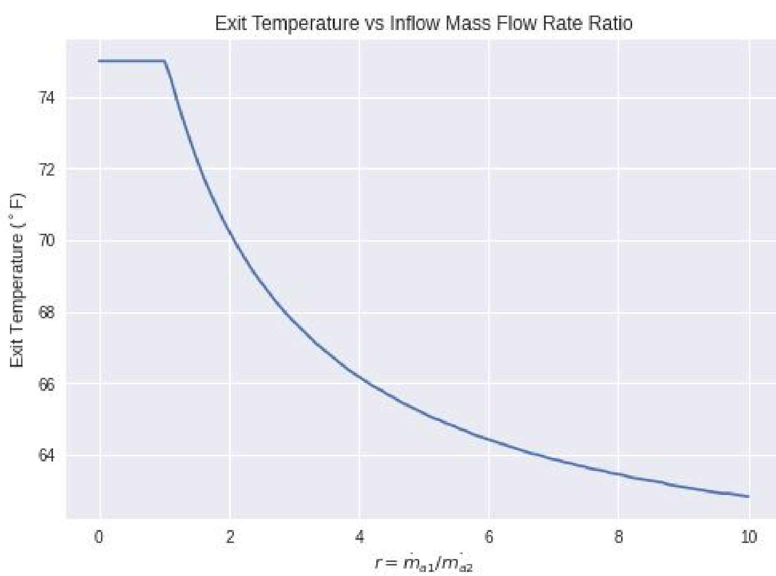
**Write your engineering model, equations, and/or explanation of your process here.**

[10]:
```
def calc_T_3(r):

    #Setting Stream 1 States
    #eq 1
    p_v1 = phi_1 * State(sub_v, T=T_1, x=x_g).p
    st_v1 = State(sub_v, p=p_v1, T=T_1)
    h_v1 = st_v1.h
    #eq 2
    omega_1 = 0.622*(p_v1/(p_1-p_v1))
    #eq 3
    p_a1 = p_1 - p_v1
    st_a1 = State(sub_a, p=p_a1, T=T_1)
    h_a1 = st_a1.h

    #Setting Stream 2 States
    #eq 1
    p_v2 = phi_2 * State(sub_v, T=T_2, x=x_g).p
    st_v2 = State(sub_v, p=p_v2, T=T_2)
    h_v2 = st_v2.h
    #eq 2
    omega_2 = 0.622*(p_v2/(p_2-p_v2))
    #eq 3
    p_a2 = p_2 - p_v2
    st_a2 = State(sub_a, p=p_a2, T=T_2)
    h_a2 = st_a2.h

    #eq 5c
    omega_3 = (r*omega_1 + omega_2)/(r + 1)
    #eq 2
    p_v3 = omega_3*p_3/(omega_3+0.622)
    #eq 3
    p_a3 = p_3 - p_v3

    def calc_diff(T_3_val):
        T_3 = T_3_val

        h_v3 = State(sub_v, p=p_v3, T=T_3).h
        h_a3 = State(sub_a, p=p_a3, T=T_3).h
        #eq 6d
        diff = r*(h_a1 + omega_1*h_v1) + h_a2 + omega_2*h_v2 - (r + 1)*(h_a3 +
 ↪omega_3*h_v3)
        return diff
```

```
    T_3_vals_best = np.linspace(60, 75, 500)*units.degF
    diff_vals = np.zeros_like(T_3_vals_best)*units.BTU/units.lb

    for i, T_3 in enumerate(T_3_vals_best):
        diff_vals[i] = calc_diff(T_3)

    diff_vals = abs(diff_vals)
    minimum = min(diff_vals)
    location = np.where(diff_vals == minimum)
    T_3_best = T_3_vals_best[location]

    return T_3_best.to('degF')




r_vals = np.linspace(0, 10, 100)*units.dimensionless
T_3_vals = np.zeros_like(r_vals)*units.degF

for i, r in enumerate(r_vals):
    T_3_vals[i] = calc_T_3(r)

plt.style.use('seaborn')
plt.plot(r_vals, T_3_vals)
plt.title('Exit Temperature vs Inflow Mass Flow Rate Ratio')
plt.xlabel('$r = \dot{m}_{a1}/\dot{m_{a2}}$')
plt.ylabel('Exit Temperature ($^\circ$F)');
```



Exit Temperature vs Inflow Mass Flow Rate Ratio
Exit Temperature (° F)
74
72
70
68
66
64
0
2
4
6
8
10
$r = \dot{m}_{a1}/\dot{m}_{a2}$

Exit Temperature vs Inflow Mass Flow Rate Ratio
Exit Temperature (° F)
74
72
70
68
66
64
0
2
4
6
8
10
$r = \dot{m}_{a1}/\dot{m}_{a2}$

**Part 3: What $r$ does Forge need?**

[24]:
```
location = np.where(T_3_vals.round(1) == Q_(65, "degF"))
print(r_vals[location])
```

```
[5.15151515 5.25252525] dimensionless
```

**Answer:** An ideal r value for forge to have 65 $^o$F cooling air in stream 3 will be between 5.15 < r < 5.25

# Homework 10-4

---

### Imports

[1]:
```
from thermostate import Q_, State
```

---

### Definitions

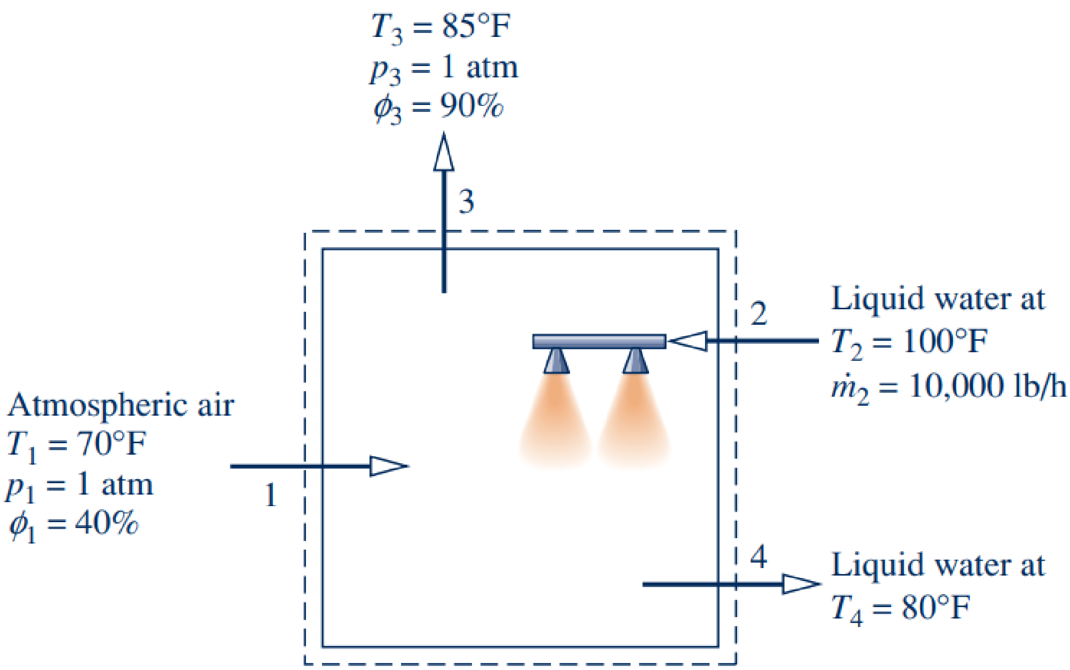
[2]:
```
sub_a = 'air'
sub_v = 'water'

T_1 = Q_(70.0, 'degF')
p_1 = Q_(1.0, 'atm')
phi_1 = Q_(0.4, 'dimensionless')

T_2 = Q_(100.0, 'degF')
mdot_w2 = Q_(10000.0, 'lb/hr')

T_3 = Q_(85.0, 'degF')
p_3 = Q_(1.0, 'atm')
phi_3 = Q_(0.9, 'dimensionless')

T_4 = Q_(80.0, 'degF')

# you can use these for saturated states
x_f = Q_(0.0, 'dimensionless')
x_g = Q_(1.0, 'dimensionless')
```

---

### Problem Statement

Liquid water at 100 °F enters a cooling tower operating at steady state, and cooled water exits the tower at 80 °F. Other property data are shown in the figure below. No makeup water is provided. Determine

1. the mass flow rate of the entering atmospheric air, in lb/hr,
2. the rate at which water evaporates, in lb/hr,
3. the mass flow rate of the exiting liquid stream, in lb/hr.




## Solution

### Part 1: Mass Flow Rate of Entering Atmospheric Air (lb/hr)

$$\phi_1 = \frac{p_{v1}}{p(T_1, x_{sat})} \tag{1}$$

$$\omega_1 = 0.622\frac{p_{v1}}{p_1 - p_{v1}} = \frac{m_{v1}}{m_{a1}} \tag{2}$$

$$p_1 = p_{a1} + p_{v1} \tag{3}$$

$$\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a \tag{4}$$

$$\dot{m}_{v1} + \dot{m}_{w2} = \dot{m}_{v3} + \dot{m}_{w4} \tag{5a}$$

$$\frac{\dot{m}_{w4}}{\dot{m}_a} = \omega_1 + \frac{\dot{m}_{w2}}{\dot{m}_a} - \omega_3 \tag{5b}$$

$$\sum m_{in}h_{in} = \sum m_{out}h_{out} \tag{6a}$$

$$\dot{m}_a h_{a1} + \dot{m}_{v1}h_{v1} + \dot{m}_{w2}h_{w2} = \dot{m}_a h_{a3} + \dot{m}_{v3}h_{v3} + \dot{m}_{w4}h_{w4} \tag{6b}$$

$$h_{a1} + \omega_1 h_{v1} + \frac{\dot{m}_{w2}}{\dot{m}_a} h_{w2} = h_{a3} + \omega_3 h_{v3} + \frac{\dot{m}_{w4}}{m_a} h_{w4} \tag{6c}$$

$$h_{a1} + \omega_1 h_{v1} + \frac{\dot{m}_{w2}}{\dot{m}_a} h_{w2} = h_{a3} + \omega_3 h_{v3} + \omega_1 h_{w4} + \frac{\dot{m}_{w2}}{\dot{m}_a} h_{w4} - \omega_3 h_{w4} \tag{6d}$$

[3]:
```
#Setting state 1
#eq 1
p_v1 = phi_1 * State(sub_v, T=T_1, x=x_g).p
st_v1 = State(sub_v, p=p_v1, T=T_1)
h_v1 = st_v1.h
#eq 2
omega_1 = 0.622*(p_v1/(p_1-p_v1))
#eq 3
p_a1 = p_1 - p_v1
st_a1 = State(sub_a, p=p_a1, T=T_1)
h_a1 = st_a1.h

#Setting state 3
#eq 1
p_v3 = phi_3 * State(sub_v, T=T_3, x=x_g).p
st_v3 = State(sub_v, p=p_v3, T=T_3)
h_v3 = st_v3.h
#eq 2
omega_3 = 0.622*(p_v3/(p_3-p_v3))
#eq 3
p_a3 = p_3 - p_v3
st_a3 = State(sub_a, p=p_a3, T=T_3)
h_a3 = st_a3.h

#Water States
st_w2 = State(sub_v, T=T_2, x=x_f)
h_w2 = st_w2.h
st_w4 = State(sub_v, T=T_4, x=x_f)
h_w4 = st_w4.h

#eq 6d
mdot_a = (mdot_w2*h_w2 - mdot_w2*h_w4)/(h_a3 + omega_3*h_v3 + omega_1*h_w4 -
 ↪omega_3*h_w4 - h_a1 - omega_1*h_v1)
print(mdot_a)
```

```
9120.223208547166 pound / hour
```

**Answer:** 9120.22 lb/hr

**Part 2: Mass Flow Rate at which Water Evaporates (lb/hr)**

$$\dot{m}_{evaporate} = \dot{m}_{w2} - \dot{m}_{w4} \tag{7}$$

[4]:
```
#eq 5b
mdot_w4 = mdot_a*(omega_1 + mdot_w2/mdot_a - omega_3)
#eq 7
print(mdot_w2-mdot_w4)
```

```
158.45868089752184 pound / hour
```

**Answer:** 158.46 lb/hr

**Part 3: Mass Flow Rate of Exiting Liquid Water Stream (lb/hr)**

[5]:
```
print(mdot_w4.to("lb/hr"))
```

```
9841.541319102478 pound / hour
```

**Answer:** 9841.54 lb/hr