Homework 3

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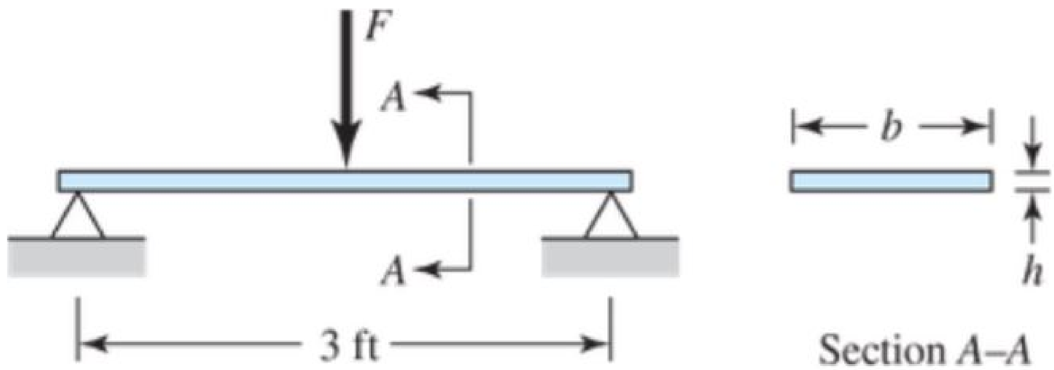
ME3227-001 - Spring 2021

Problem 4-22

Illustrated is a rectanguler steel bar with simple supports at the ends and loaded by a force F at the middle; the bar is to act as a spring. The ratio of the width of the thickness is to be about $b = 10h$, and the desired spring scale is 1800 lbf/in.

(a) Find a set of cross-section dimensions, using preferred fractional sizes from Table A-17.

(b) What deflections would cause a permanent set in the spring if this is estimated to occur at a normal stress of 60 kpsi.



Solution

Part (a)

The most important equation here is:

$$k = \frac{F}{y} \tag{1}$$

We also know that for a simply supported beam with central loading, the deflection is:

$$y_{max} = \frac{Fl^3}{48EI} \tag{2}$$

Plugging eq.(2) into eq.(1) yields:

$$k = \frac{48EI}{l^3} \tag{3}$$

But we want to know h which is burried in our I varible. We know:

$$I = \frac{bh^3}{12} \tag{4}$$

$$I = \frac{10h^4}{12} \tag{5}$$

Plugging eq.(5) into eq.(3) and rearranging yields:

$$h = \left(\frac{12kl^3}{480E}\right)^{\frac{1}{4}} \tag{6}$$

Plugging in our values yields:

[1]:
```python
from thermostate import Q_
k = Q_(1800, "lbf/in")
l = Q_(3, "ft")
E = Q_(27.6, "Mpsi")

h = ((12*k*l**3)/(480*E))**(1/4)
print(h.to("in").round(3))
```

```
0.525 inch
```

We see that $h = 0.525$ inch ideally so $b = 5.25$ inch ideals too, but looking at Table A-17, we see we can only order steel in $\frac{9}{16}$ inch for h and $5\frac{1}{4}$ inch for b. This would mean the cross section we order is:

$5\frac{1}{4}" \times \frac{9}{16}"$

Part (b)

We know also from our superposition table that the maximum moment will occur at the center of the beam, and the highest moment will give us the highest normal stress:

$$\sigma_{max} = \frac{Mc}{I} \tag{7}$$

$$M_{max} = \frac{F}{2}l \tag{8}$$

$$c = \frac{h}{2} \tag{9}$$

But we arn't asked for what force would cause this maximum σ to be reached, we are asked to find what deflection would cause this. We know from eq.(1) the:

$$F = ky \tag{10}$$

So plugging this into eq.(7) and rearranging yeilds:

$$y = \frac{4I\sigma_{max}}{lkh} \tag{11}$$

And then we will plug into this equaiton:

[2]:
```
sigma_max = Q_(60, "kpsi")
I = 10*h**4/12
y = 4*sigma_max*I/(k*l*h)
print(y.to("in").round(3))
```

```
0.447 inch
```

$y = 0.447$ in

Problem 4-44

A flat-bed trailer is to be designed with a curvature such that when loaded to capacity, the trailer bed is flat. The load capacity is to be 3000 lfb/ft between the axles, which are 25 ft apart. and the second-area moment of the steel structure is $I = 4885$ in^4. Determine the equation for the curvature of the unloaded bed and the maximum height of the bed relative to the axles.

Solution

First, we will look at the curvature caused by the load if the bed was initially flat.

We know that curvature, $\frac{1}{\rho}$, is defined by:

$$\frac{1}{\rho} = \frac{M}{EI} \tag{1}$$

Using the superposition tables in the book we know that a uniform load on simple supports will have:

$$M = \frac{wx}{2}(l - x) \tag{2}$$

This means our equations of curvature is:

$$\frac{1}{\rho} = \frac{-wx^2 + wlx}{2EI} \tag{3}$$

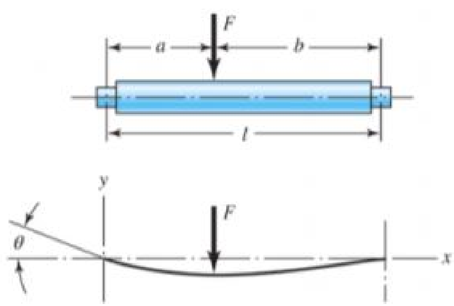
If we know that if this is the resulting curvature of a flat bed, then if we curve the bed opposite to this, we will get a flat bed after being loaded. This means our inital curvature equation will be:

$$\boxed{\frac{1}{\rho} = \frac{wx^2 - wlx}{2EI}}$$

Problem 4-45

The designer of a shaft usually had a slope constraint imposed by the bearings used. This limit will be denoted as ξ. If the sharft shown in the figure is to have a uniform diameter d except in the locality of the bearing mounting, it can be apporximated as a uniform beam with simple supports. Show the mimumum diameters to meet the slope constraint at the left and right bearings are, respectivly,

$$d_L = \left| \frac{32Fb(l^2 - b^2)}{3\pi El\xi} \right|^{\frac{1}{4}} \qquad d_R = \left| \frac{32Fa(l^2 - a^2)}{3\pi El\xi} \right|^{\frac{1}{4}}$$



Solution

For the bearings we are using, we will say that the slope constraint is as follows:

$$\theta_{max} = \xi \tag{1}$$

Using superposition tables for a simply supported intermediate load lets us know that:

$$\theta_L = \frac{-Fab(l+b)}{6EIl} \tag{2}$$

$$\theta_R = \frac{Fab(L+a)}{6EIl} \tag{3}$$

We are asked for the diameter of the rod which isn't visible yet. But we know that varible is baked into I. We can say:

$$I = \frac{\pi d^4}{64} \tag{4}$$

Plugging this into eq.(2) and and eq.(3), and replacing θ with ξ almost gets us there with:

$$d_L = \left(\frac{-32Fab(l+b)}{3\pi El\xi} \right)^{\frac{1}{4}} \tag{5}$$

$$d_R = \left(\frac{-32Fab(l+a)}{3\pi El\xi} \right)^{\frac{1}{4}} \tag{6}$$

The last substitution we will make is:

$$b = l - a \tag{7}$$

$$a = l - b \tag{8}$$

Plugging eq.(8) into eq.(5) and eq.(7) into eq.(6) yields:

$$\boxed{d_L = \left(\frac{-32Fb(l^2-b^2)}{3\pi El\xi} \right)^{\frac{1}{4}} \qquad d_R = \left(\frac{32Fa(l^2-a^2)}{3\pi El\xi} \right)^{\frac{1}{4}}}$$

Problem 5-38

A 1020 CD steel shaft is to transmit 20 hp while rotating at 1750 rpm. Determine the minimum diameter for the shaft to provide a minimum factor of safety of 3 based on the maximum-shear-stress theory.

Solution

According to maximum-shear-stress theory (MSS), the yielding stress in shearing(S_{sy}) is:

$$S_{sy} = \frac{S_y}{2} \tag{1}$$

Where S_y is the yield stress of a material in a pure tension test. To appy our factor of safety, we will say:

$$\tau_{max} = \frac{S_{sy}}{n} \tag{2}$$

Where n is our factor of safety. For a bar under pure torsion, we can also say:

$$\tau_{max} = \frac{Tr}{J} \tag{3}$$

We are going to want to find a design specification d so we will rearrange eq.(3), and plug in $J = \frac{\pi d^4}{32}$ to get:

$$\tau_{max} = \frac{16T}{\pi d^3} \tag{4}$$

Now combinding eq.(4) and eq.(2) and then rearranging gives us:

$$d = \left(\frac{16nT}{\pi S_{sy}}\right)^{\frac{1}{3}} \tag{5}$$

Our given information only provides us with n. We can find T knowing that:

$$H = T\omega \tag{6}$$

And S_y can be found from Table A-22, *Results of Tensile Test of Some Metals* where it says for 1020 Steel, $S_y = 42.0$ kpsi.

Plugging in these values yields:

[1]:
```
from thermostate import Q_
from math import pi
```

```
n = 3
H = Q_(20, "horsepower")
omega = Q_(1750, "rpm")
T = (H/omega).to("kip*in")
S_y = Q_(42, "kpsi")
S_sy = S_y/2

d = ((16*n*T)/(pi*S_sy))**(1/3)
print(d.to("in"))
```

```
0.8062315294092228 inch
```

$d = 0.8in$

Problem 5-39

A 30-mm-diameter shaft, made of AISI 1018 HR steel, transmits 10 kW of power while rotating at 200 rev/min. Assume any bending momets present in the shaft to be negligibly small compared to teh torque. Determine the static factor of safety base on

(a) the maximum-shear-stress failure theory

(b) the distortion-energy gailure theory

Solution

Part (a)

Since the bar is under oure torsion we know that:

$$\tau = \frac{Tr}{J} \tag{1}$$

We also know that according to maximum-shear-stress (MSS) theory:

$$S_{sy} = \frac{S_y}{2} \tag{2}$$

We can say the factor of safety is as follows:

$$n = \frac{S_{sy}}{\tau} \tag{3}$$

We know that $T = \frac{H}{\omega}$ and $S_y = 220$ MPa from Table A-20. Plugging the following equations in with our given information yields:

[1]:
```python
from thermostate import Q_
from numpy import pi

H = Q_(10, "kW")
omega = Q_(200, "rpm")
r = Q_(15, "mm")
T = H/omega
J = pi*(2*r)**4/32
tau = T*r/J
S_y = Q_(220, "MPa")
S_sy = S_y/2
n = S_sy/tau
print(n.to("dimensionless"))
```

```
1.2213635446348081 dimensionless
```

Factor of Saftey: $n = 1.22$