The design process:

Identification of need → Definition of problem → Synthesis → Analysis and optimization → Evaluation → Presentation

* you can always be sent back to other steps

Uncertainty

what we don't know about enviroment, material properties and dimensions

Aleatoric uncertainty: natural randomness

Epistemic uncertainty: things we theoretically could know but we don't because it's impractical

Design factor:

material property

n_d = loss-of-function parameter / max allowable parameter

ex: max load ± 20%, Failure load ± 15%.

If load causing nominal failure is 2000 lbf, what is n_d and max allow.

material → L.o.F. load = load ± 0.15·load

applied → max allow = max ± 0.2·max

$$n_d = \frac{1/0.85}{1/1.2} = 1.41$$

max allow = 2000/1.41 = 1418 lbf

ex: tension axiel load P=2000 lbf. $\sigma = P/A$. material strength = 24 ksi. $n_d = 3$. What diameter rod should we use.

$$\sigma = \frac{P}{\pi d^2/4} \qquad n_d = \frac{S}{\sigma}$$

S ← strength of material

σ ← max stress we will put on it

$$\frac{S}{n_d} = \frac{4P}{\pi d^2}$$

$$d = 0.564 \text{ in} \Rightarrow \frac{5}{8} \text{ in}$$

n = when $d = \frac{5}{8}$ = 3.68 → F.S.

Probability of failure. p_f: instances of failure per total instances

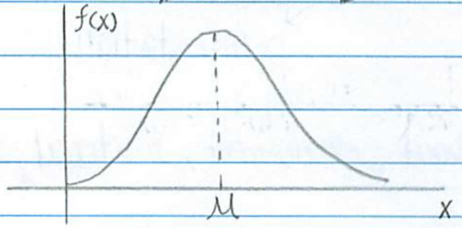
PDF: distribution of events

Gaussian (Normal) Distribution:

$$f(x) = \frac{1}{\hat{\sigma}_x \sqrt{2\pi}} \exp\left[\frac{-1}{2}\left(\frac{x-\mu_x}{\hat{\sigma}_x}\right)^2\right]$$

mean

standard deviation



$$p_f = \int_{-\infty}^{z_x} f(x)\,dx \quad \begin{cases} a & z_a < 0 \\ 1-a & z_a \geq 0 \end{cases}$$

$$z = \frac{x-\mu_x}{\hat{\sigma}_x}$$

ex: 250 rods with strength $\bar{s} = 45$ ksi, $\hat{\sigma}_s = 5$ ksi

a. How many have $s \leq 39.5$ ksi

b. How many $39.5 \leq s \leq 59.5$

transfor to normal distribution: $z = \frac{x-\mu_x}{\hat{\sigma}_x}$

$$z_{39.5} = \frac{s-\bar{s}}{\hat{\sigma}_x} = \frac{39.5-45}{5} = -1.1$$

From table, get $\Phi(-1.1) = 0.1357$
A-10

$250 \cdot 0.1357 = \boxed{34 \text{ rods}}$

$z_{59.5} = \frac{59.5-45}{5} = 2.9$. since positive $1-\Phi(-2.9) = 0.99813$

$P(39.5 \leq s \leq 59.5) = 0.99813 - 0.1357 = 0.86243$

$\approx \boxed{216 \text{ rods}}$

Discreate mean:

$$\bar{X} = \frac{1}{N}\sum_{i=1}^{K} f_i x_i$$

x_i: value of event

f_i: number of times event occurs

Discreate standard divation:

$$S_x = \sqrt{\frac{\sum_{i=1}^{K} f_i x_i^2 - N\bar{x}^2}{N-1}}$$

Reliability

Series system: if one thing fails it all fails

$$\text{Reliability} = \prod_{i=1} R_i$$

Probability Density Function

$\bar{\sigma} = \mu_\sigma \qquad \bar{S} = \mu_S$

average D.F. $\bar{n}_d = \mu_S / \mu_\sigma$

marginal safety: $m = S - \sigma$

Coefficence of varience: $C_\sigma = \hat{\sigma}_\sigma / \mu_\sigma$, $C_S = \hat{\sigma}_S / \mu_S$

D.F. using C: $$\bar{n}_d = \frac{1 \pm \sqrt{1-(1-z^2 C_S^2)(1-z^2 C_\sigma^2)}}{1 - z^2 C_S^2}$$

+ for $R > 0.5$

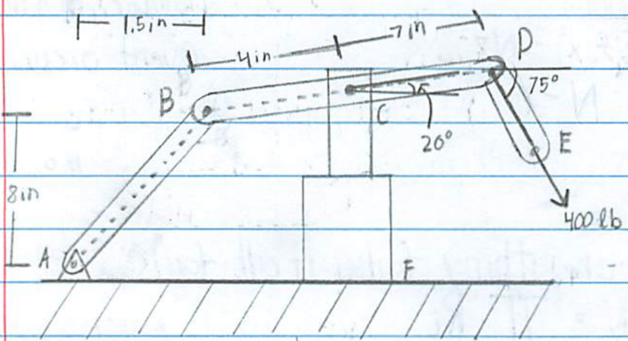
- for $R \leq 0.5$

$$z = -\left(\frac{\bar{n}_d - 1}{\sqrt{\bar{n}_d C_S^2 + C_\sigma^2}}\right)$$

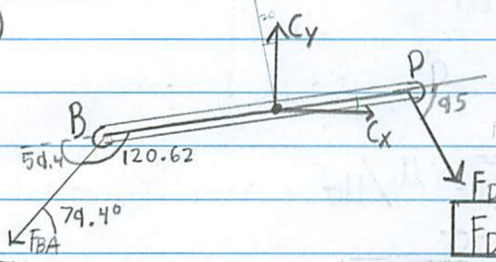
Homework #0 Nathan Stenseng

1.

Members BD and AB are $5/8$" thick (out of plane)
Member AB is 2" wide.
Bolt C is $3/8$" diameter.
(a) make an FBD for BD
(b) is BD experiencing bending, axial or both.
(c) Determine τ_{ave} in bolt C and τ at C in BD
(d) Forces on AB?
(e) σ_{ave} in AB



(a)



$\boxed{F_{DE} = 400 \text{ lb}}$

$\sum F_x:\ -F_{BA} \cdot \cos(59.4°) + C_x \cos(20°) + F_{BD}\cos(95) = 0$

$\sum M_c:\ F_{BA} \sin(59.4) = F_{DE} \sin(95)$

$\boxed{F_{BA} = 462.95 \text{ lb}}$ $\boxed{C_x = 288.03 \text{ lb}}$

$\sum F_y:\ -F_{BA}\sin(59.4°) + C_y \cos(20°) - F_{DE}\sin(95°) = 0$

$\boxed{C_y = 939.73 \text{ lb}}$

(b) member BD experiences axial loads from the forces pulling apart and pushing togeter, the cos components of the forces, and bending from the sin components of the forces.

(c) $F_c = \sqrt{C_x^2 + C_y^2} = 982.88 \text{ lb}$

$\tau = \dfrac{F_c}{A_c}$ $A = \dfrac{\pi(3/8)^2}{4}$ $\therefore$ $\boxed{\tau_{ave} = 8.899 \text{ ksi}}$ bolt

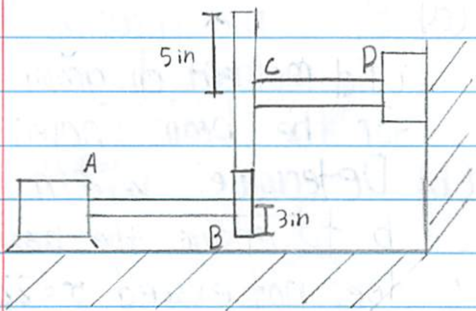
Bearing stress: $\sigma = \dfrac{P}{A}$ $P = F_c$
$A = t \cdot d$

$\boxed{\sigma = 4.193 \text{ ksi}}$

(d) Member AB experiences only axial loadins at both ends

(e) $\sigma = \dfrac{F}{A} = \dfrac{462.95}{\frac{5}{8}" \cdot 2"}$ $\boxed{\sigma = 370.36 \text{ psi}}$

2.



Transmission A has 16 hp at 1260 rpm to tool D.
(a) Determine torque on AB and CD
(b) Are these shafts bending
(c) Are they in axial load
(d) What is the smallest diameter of each shaft if $\tau_{max} = 8ksi$

(a) $T = \frac{P}{2\pi f}$ $\quad f = \frac{1260}{min} \cdot \frac{1 min}{60 sec} = 21 Hz$

$$16hp = \frac{6600 \, in \cdot lb/s}{1 \, hp} = 105.6 \frac{kip \cdot in}{s}$$

$T = \frac{105.6}{42\pi} = \boxed{0.8 \, kip \cdot in = T_{AB}}$

$\frac{T_{AB}}{T_{CD}} = \frac{r_B}{r_C}$ $\quad \boxed{T_{CD} = 1.33 \, kip \cdot in}$

(b) the shaft is bending an angle of γ along the longitudinal axis, one end of each shaft would rotate γ less than the other. This is from the modulous of rigidity.

(c) there are no axial loads because no forces are pushing along the axis of either beam

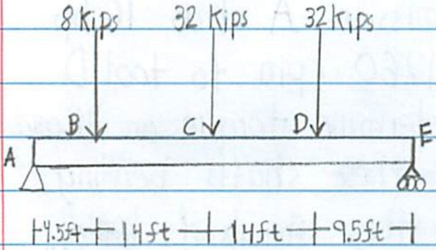
(d) $\tau = \frac{Tc}{J}$

$8 = \frac{0.8 \cdot 3}{\frac{\pi}{32} d^4}$

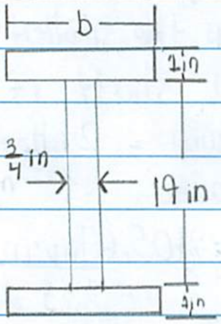
$8 = \frac{1.33 \cdot 5}{\frac{\pi}{32} d^4}$

$\boxed{d_{AB} = 1.32 \, in \quad d_{CD} = 1.71 \, in}$

3.



(a) Plot the shearing and moment diagrams for the beam shown
(b) Determine width b to ensure the beam does not exceed $\sigma = 22$ ksi

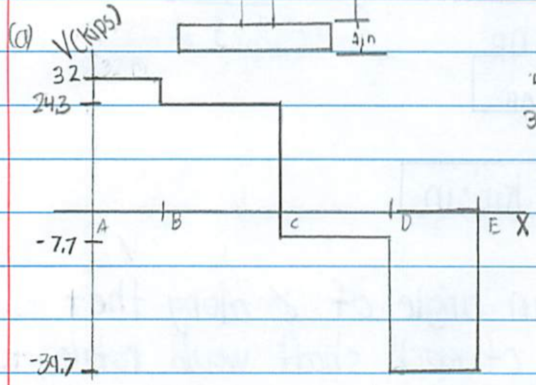


$\Sigma F_y: A_y + E_y = 8 + 32 + 32$

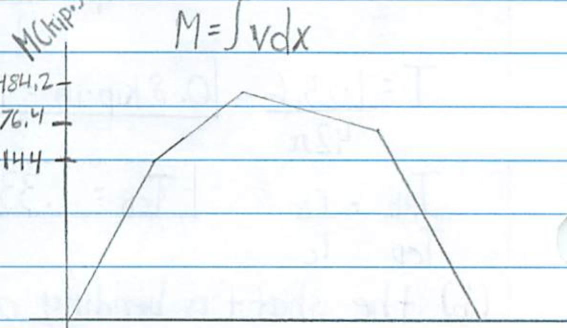
$\Sigma M_A: -8\cdot(4.5) - 32(18.5) - 32(32.5) + E_y(42) = 0$

$E_y = 39.7$ Kips $\quad A_y = 32.3$ kips

(a)



$M = \int V dx$



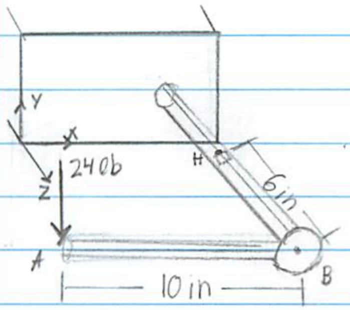
(b) $\sigma = \dfrac{Mc}{I_{xx}}$ where M is the max moment and c is the furthest distance from x-x

$$I_{xx} = \frac{\left(\frac{3}{4}\cdot 19^3\right)}{12} + 2\left(\frac{b\cdot 1^3}{12} + b\cdot 1\cdot(10^2)\right)$$

$$= \frac{6859}{16} + b\left(\frac{2}{12} + 10^2\right)$$

$$b = \left(\frac{484.2\cdot 12\cdot 10.5}{\sigma} - \frac{6859}{16}\right) \Big/ \left(\frac{2}{12} + 10^2\right)$$

$\boxed{b = 11.7 \text{ in}}$

4. 

(a) Draw a mohr's circle for the stresses on point H

(b) Knowing the shaft has a diameter at H of 3/4", determine principle stresses and max τ

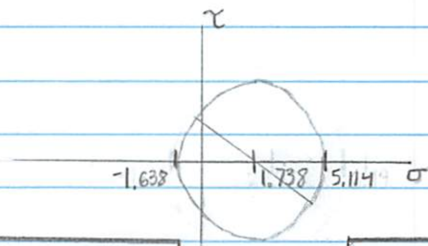
$$\tau_{xy} = \frac{Tr}{J} \qquad \sigma_z = \frac{Mc}{I}$$

$$\tau_{xy} = \tau_{yx} = \frac{(24 \cdot 10)(3/8)}{\frac{\pi}{32}(3/4)^4} = 2.897 \text{ ksi}$$

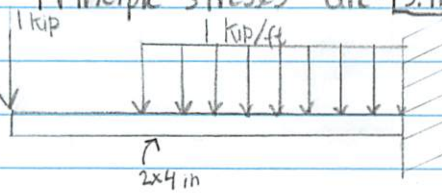
$$\sigma_z = \frac{(24 \cdot 6)(3/8)}{\frac{\pi}{4}(3/8)^4} = 3.476 \text{ ksi}$$

$$\sigma_{ave} = \frac{\sigma_z + 0}{2} = 1.738$$

$$R = \sqrt{\left(\frac{\sigma_z - 0}{2}\right)^2 + \tau_{xy}^2} = 3.38$$



Principle stresses are: $\boxed{5.114 \text{ ksi and } -1.638 \text{ ksi.}}$ τ_{max} is $\boxed{3.38 \text{ ksi}}$

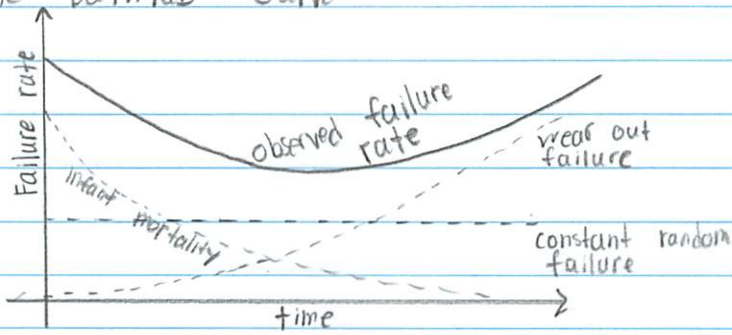
5. 

Use superposition to calculate slope and deflection

$$\theta = \frac{1}{EI}\left(\frac{-1 \cdot 36^3}{6}\right) + \frac{1}{EI}\left(\frac{-1 \cdot 60^2}{2}\right) = \boxed{-0.04489 \text{ rad}}$$

$$y = \frac{1}{EI}\left(\frac{-1000 \cdot (60)^3}{3} + \frac{-1000 \cdot (36)^4}{8} + \frac{-1000 \cdot 36^3}{6} \cdot 24\right) = \boxed{-2.196 \text{ in}}$$

The Bathtub Curve:



observed is the sum of dashed lines

Dimensions and tolorences:

Nominal size: What we expect 150 cm

Limits: max and min dimensions 149 cm, 151 cm

Tolerances: Difference between limits 2cm

Bilateral tolerance: 50 ± 2 cm

unilateral: $1^{+0,04}_{-0,01}$ cm

Clearence: Difference in size between 2 objects

interference is the opposite

Materials:

Tensile test: look at posions ratio

$\sigma = \frac{P}{A_o}$ $\quad\quad \varepsilon = \frac{\Delta l}{l_o}$

$\sigma = E\varepsilon$ for linear stress-strain

when deforming, Area changes

True stress: $\tilde{\sigma} = \sigma\left(\frac{A_o}{A}\right)$ $\quad\quad E \leftrightarrow$ stiffness

True strain: $\int_{l_o}^{l} dl/l_o = \ln(l/l_o) = \tilde{\varepsilon}$

In plastic zone: Volume is constant

$\therefore A_o l_o = Al = \text{constant}$

$\frac{A_o}{A} = \frac{l}{l_o} = 1+\varepsilon, \quad \varepsilon = \frac{A_o - A}{A}$

$\tilde{\varepsilon} = \ln(1+\varepsilon)$

$\tilde{\sigma} = \sigma(1+\varepsilon)$

Compression strength, Tortion strength, Tensile stress

$\tau_{max} = \frac{Gr}{l_o}\theta$

$= \frac{Tr}{J}$

Resilience:

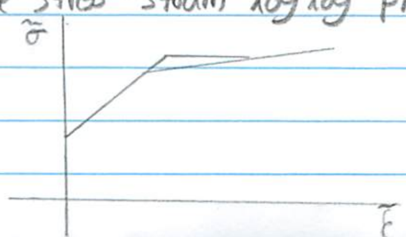
$u_R \approx \int_0^{\varepsilon_y} \sigma \, d\varepsilon$

if elastic linear: $u_R = \frac{1}{2} S_y \varepsilon_y = \frac{S_y^2}{2E}$

Toughness: Area under σ-ε all the way to fracture

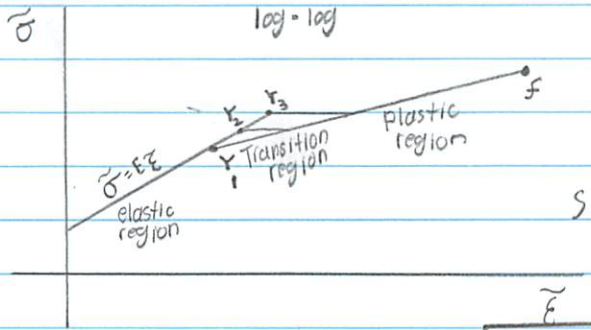
$u_T = \int_0^{\varepsilon_f} \sigma \, d\varepsilon \approx \left(\frac{S_y + S_{ut}}{2}\right)\varepsilon_f$

True stress-strain log log plot



made through testing

Rambery-Osgood relationship



$\tilde{\varepsilon} = \ln(1+\varepsilon)$ always

$\tilde{\sigma} = \sigma(1+\varepsilon)$

slope $= \dfrac{\log(y_2/y_1)}{\log(x_1/x_2)} = m$ also on table A-22

plastic region : $\boxed{\tilde{\sigma} = \sigma_0 \tilde{\varepsilon}^{m}}$ m is slop on log-log

Derived from: $\log\tilde{\sigma} = \log\sigma_0 + m\log\tilde{\varepsilon}$

σ_0 is strength coefficent: when $\tilde{\varepsilon}=1$ and $m\log(1)=0$

$m = \tilde{\varepsilon}_u$ Table A-22

Total strian: $\varepsilon = \varepsilon_p + \varepsilon_e$

$$\tilde{\varepsilon} = \frac{\tilde{\sigma}}{E} + \left(\frac{\tilde{\sigma}}{\sigma_0}\right)^{1/m}$$, rearranged above equation

Reduction in area: Ductility

$$R = \frac{A_0 - A_f}{A_0} = 1 - \frac{A_f}{A_0}$$ ← final, ← initial

Hardness: resistence to penitration

Rockwell: 3 scales for different cases

Brinell: Hardness number H_B: $\dfrac{\text{applied load}}{\text{spherical SA indentation}}$

linear relationship between S_u and H_B

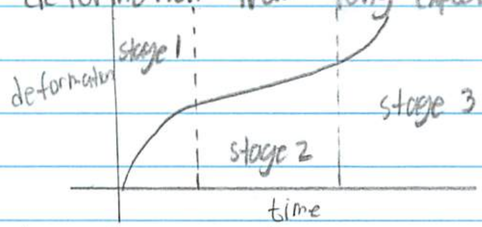
ex: $S_{u\,min} = 20$ksi, what should H_B be

Table A-24

$$H_B = \frac{S_u + 12.5}{0.23} = 141$$

creep:

deformation from long exposure to load and high temp

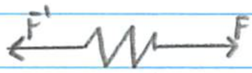


Ashby Plots: Density vs E Fig 2-24

lets you choose stiffest and lightest

specific Modulus: E/ρ

Spring constant for a bar



$$\sigma = E\varepsilon$$

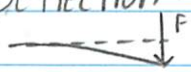
$$\frac{F}{A} = E \cdot \frac{\delta}{L}$$

$$F = \frac{AE\delta}{L}$$

$F = k\delta$ ← displacement

$$\boxed{k = \frac{F}{\delta} = \frac{AE}{L}}$$

Deflection stiffness: $\Delta y = \dfrac{F\ell^3}{3EI}$



$$k = \frac{F}{\Delta y} = \frac{3EI}{\ell^3} \qquad I = \frac{A^2}{4\pi}$$

$$A = \left(\frac{4\pi k \ell^3}{3E}\right)^{1/2}$$

performance metric: mass $= A\ell\rho$

seperate: $m = 2\sqrt{\pi/3}\,(k)^{1/2} \cdot \ell^{5/2} \cdot \dfrac{\rho}{E^{1/2}}$

3 terms $f_1(k) \cdot f_2(\ell) \cdot f_3(M)$

where $M = \dfrac{E^{1/2}}{\rho}$

find best M in Ashby plot

Homework 1 Nathan Stenseng

1-12 A solid circular rod of diameter d experiences bending of $M = 1000\ lbf\cdot in.$ inducing $\sigma = 32M/(\pi d^3)$. Using material strength $S = 25\ ksi$ and $n_d = 2.5$, Determine minimum rod diameter, d. Use Table A-17 to find a size and calculate a factor of safety.

$$n_d = \frac{S}{\sigma} \Rightarrow 2.5 = \frac{25\ ksi}{32M}\cdot \pi d^3$$

$$*\ M = 1\ kip\cdot in$$

$$d = \left(\frac{2.5\cdot 32\cdot 1}{25\cdot\pi}\right)^{1/3} = \boxed{1.006\ in = d}$$

rounding up to nearist size is 1.25 in

$$F.S. = \frac{S}{\sigma} \Rightarrow F.S. = \frac{25}{32\cdot 1}\cdot\pi(1.25)^3 = \boxed{4.79 = F.S}$$

1-13 A fatigue test is performed where rods experience a tensile then equal compressive loading per cycle. The cycles to failure experienced by 69, 5160H steel 1.5in Hexagonal bars are:

L	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210
f	2	1	3	5	8	12	6	10	8	5	2	3	2	1	0	1

where L is thousand of cycles till failure and f is amount of rods.

(a) Estimate the mean and standard diviation

(b) Assuming the distribution is normal, how many specimen fail before 115 kcycles.

(a) $$\text{mean} = \bar{x} = \frac{1}{N}\sum_{i=1}^{k} f_i\cdot x_i = \boxed{122.9\ kcycles = \bar{x}}$$

$$\text{standard diviation} = \sqrt{\frac{\Sigma f_i\cdot x_i^2 - N\bar{x}^2}{N-1}} = \boxed{30.298\ kcycles = S_x}$$

(b) $$z = \frac{x-\bar{x}}{S_x} = \frac{115-122.9}{30.298} = -0.267$$

From Table A10, $\Phi(-0.26) = 0.39743$

$$0.39743\cdot 69 = 27.42\ rods$$

$$= \boxed{27\ rods}$$

1-18 $\tau_{max} = 16T/(\pi d^3)$ for a solid round bar. A round, cold-drawn 1018 steel rod is subjected to a mean torque of $\bar{T} = 1.5\,kN\cdot m$ with standard divation $\hat{\sigma}_T$ of $145\,N\cdot m$. The rods mean shear yield is $\bar{S}_{sy} = 312\,MPa$ with standard divation $\hat{\sigma}_{S_{sy}} = 23.5\,MPa$. Assume normal distribution. What $\bar{n}_d$ would have reliability of 99%. Determine the corrisponding diameter of rod.

$C_s = \frac{\hat{\sigma}_s}{\bar{S}}$ $\quad C_\tau = \frac{\hat{\sigma}_\tau}{\bar{\tau}}$ looking at table A-10.

we see 0.01 would be $z = -2.33$

$= 0.075 \quad C_\tau = \frac{\cancel{16}\hat{\sigma}_T/\cancel{(\pi d^3)}}{\cancel{16}\bar{T}/\cancel{(\pi d^3)}} = \frac{\hat{\sigma}_T}{\bar{T}} = 0.0967$

$\bar{n}_d = \frac{1 \pm \sqrt{1-(1-z^2C_s^2)(1-z^2C_\tau^2)}}{1-z^2C_s^2} = 1.323$

$\bar{n}_d = \frac{\bar{S}}{\bar{\tau}} \Rightarrow 1.323 = \frac{312\,MPa}{1500\cdot 16\,N\cdot m} \cdot \pi d^3 = d = 3.187\cdot 10^{-2}\,m = \boxed{31.87\,cm}$

$\frac{\cancel{N}/m^2 \cdot 10^6}{\cancel{N}\cdot m} = \left(\frac{10^6}{m^3}\right)^{1/3} = \frac{10^2}{m}$

1-20 A rod under axial load a will experience σ_a. Under bending load by, the top fiber experiences σ_b. $\sigma_{max} = \sigma_a + \sigma_b$.
$\bar{\sigma}_a = 90\,MPa$, $\hat{\sigma}_{\sigma_a} = 8.4\,MPa$, $\bar{\sigma}_b = 383\,MPa$, $\hat{\sigma}_{\sigma_b} = 22.3\,MPa$. The steel rod has yeilding strength $\bar{S}_y = 553\,Mpa$, $\hat{\sigma}_{sy} = 42.7\,Mpa$.

$\bar{n}_d = \frac{\bar{S}_y}{\bar{\sigma}_{max}} = \boxed{1.169 = \bar{n}_d}$ $\quad z = \frac{-(\bar{n}_d - 1)}{\sqrt{\bar{n}_d^2 C_s^2 + C_{\sigma_{max}}^2}} = -1.52$

$\bar{\sigma}_{max} \leftarrow \bar{\sigma}_a + \bar{\sigma}_b$

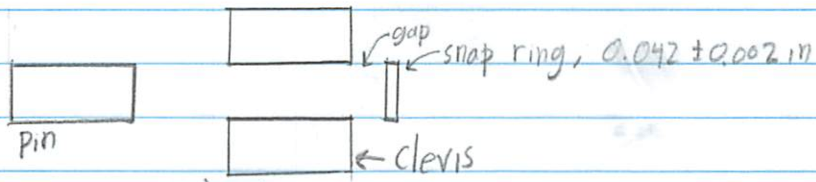
$C_s = \frac{\hat{\sigma}_{sy}}{\bar{S}_{sy}} = 0.077 \quad C_{\sigma max} = \frac{\cancel{\hat{\sigma}_a + \hat{\sigma}_b}\ \sqrt{\hat{\sigma}_a^2 + \hat{\sigma}_b^2}}{\bar{\sigma}_a + \bar{\sigma}_b} = 0.0649$

$\Phi(-1.52) = 0.06426$

$R = 1 - 0.06426 = 0.9357$

$\boxed{R = 93.57\%}$

1-23 A pin has dimensions $a \pm t_a$. A clevis has a thickness 1.500 ± 0.005 in. The designer determines a gap between 0.004 and 0.05 in will be satisfactory. Determine a and t_a



$\bar{a} = 1.5 + 0.027 + 0.042 = 1.569$ in

$t_w = 0.023 = t_a + t_c + t_s$

$t_a = 0.016$

$\boxed{\text{pin} : 1.569 \pm 0.016 \text{ in}}$

chapter 3. load and stress analysis

$\Sigma F = 0 \quad \Sigma M = 0$ in equilbrium

load intencity: F/m : distributed load : $q(x)$

$$V = \frac{dM}{dx} \qquad \frac{dV}{dx} = q(x)$$

Combinding Moments from two planes:

$$M = \sqrt{M_y^2 + M_z^2}$$

it will be at angle in y-z plane

cartesian stress:

$\sigma_x \quad \tau_{xy} \quad \tau_{xz}$

x face normal to x axis, y direction

stress is a tensor

$$\sigma = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \qquad \vec{T}^{(n)} = \vec{n} \cdot \vec{\sigma}$$

Midterm 1 practice

2-14 1040 hot-rolled steel is heat treated to S_u of 100 ksi. What is its Brinell hardness

Steel: $S_u = 0.5\, H_B$ ksi

$100 = 0.5\, H_B$ $\quad\quad H_B = 200$

2-25 Use Ashby chart to find best tensile force rod.

min: $\frac{\rho}{E}$ $\therefore$ max $\frac{E}{\rho}$ $\quad\quad$ max: $\frac{E}{\rho}$

2-30 prove axial load is S/ρ

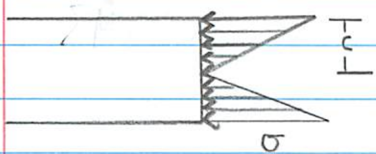
axial: $\sigma = E\varepsilon$

$S = E\varepsilon$ $\quad\quad \varepsilon = \frac{\delta l}{l}$ $\quad\quad$ preformance matrix: $m = Al\rho$

$S = \frac{E\,\delta l}{l}$

$l = \frac{E\,\delta l}{S}$ $\quad\quad m = A\,E\delta l \cdot \left(\frac{\rho}{S}\right)$

Back to class: Transverse shear stress



$\tau = \frac{VQ}{Ib}$ $\quad\quad \sigma = \frac{Mc}{I}$

Plane-stress transform

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos(2\varphi) + \tau_{xy}\sin(2\varphi)$$

$$\tau = \frac{-(\sigma_x - \sigma_y)}{2}\sin(2\varphi) + \tau_{xy}\cos(2\varphi)$$

Mohr's circle:

Center $= (\sigma, \tau) = \left((\sigma_x + \sigma_y)/2,\ 0\right)$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

(−) for ccw τ

(+) cw τ

Elastic strain:

Hooks law: $\sigma = E\varepsilon$

$\varepsilon_x = \frac{\sigma_x}{E}$ $\qquad$ $\varepsilon_y = \varepsilon_z = -\nu\frac{\sigma_x}{E}$ ← poisson's ratio

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu(\sigma_y + \sigma_z)\right)$$

shear: $\tau = G\gamma$ $\qquad$ $E = 2G(1+\nu)$ ← linear homogenous isotropic

Stresses in bending:

positive bending 

negative bending 

$\sigma_x = \frac{-My}{I}$ $\qquad$ compression on top fiber which is why their is a negative

Recall Mohr's circle stuff

2 plane bending:

$$\sigma_x = \frac{-M_z y}{I_z} + \frac{M_y z}{I_y}$$

Don't pay much attention to sign, just use logic

For circles, I's are the same

Shear stresses in bending

$\tau = \frac{VQ}{Ib}$ ← 1st moment area; b ← thickness; τ: max top Fiber

$$\tau = \frac{V}{Ib}\int_{y_1}^{c} y\,dA$$

look up table 3-2 for common beams

Transverse shear vs bending stress

look at y/c: $y/c = 0$ is neutral axis. $y/c = 1$ is at $y = c$

see that vs beam L/h

τ is importaint when $L/h \ll 10$

From mohr you find: $\tau_{max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$ ← transvere shear

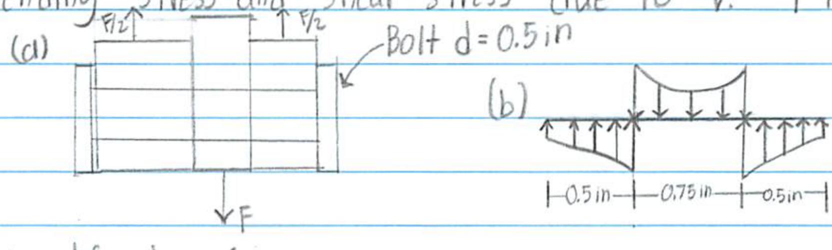
Torsion:

$\theta = \frac{Tl}{GJ}$ $\qquad$ angle of twist

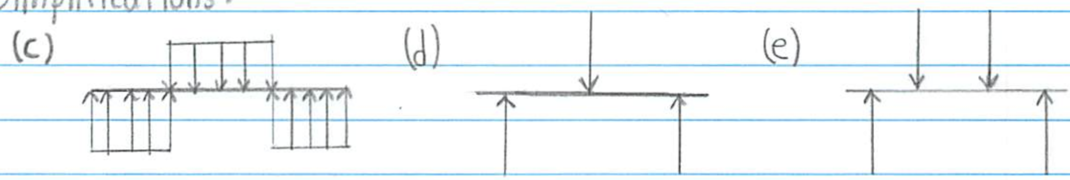
$\tau_{max} = \frac{Tr}{J}$ $\qquad$ only round crossection

Homework 2 Nathan Stenseng

3-41 A Knuckle joint carries a load of F. There are deflections which the loading pattern shown below. This loading can simplify 3 ways, also shown below. Estimate max bending stress and shear stress due to V. F=1000 lbf



simplifications:



Torsion equations are only for round cross-sections

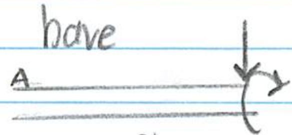
Power:

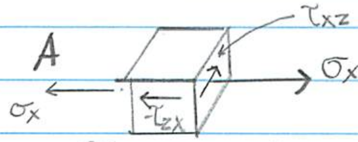
$$H = T\omega$$

Torsion moment is a bending moment in line with the element

3D tensors:

If we have





Bending moment produces σ_x

No transverse shear

because top fiber

τ_{xz} from torsion

τ_{zx} to keep equilibrium

Stress Concentration:

k_t: localized stress increase near holes

Graphs: A-10 and A-16

Techniques to reduce localized stress.

increase radius

Temperature effects:

$$\varepsilon = \alpha(\Delta T)$$

Chapter 4: Deflection and stiffness

Treating things as springs:

$$k = \frac{F}{y} \qquad k = \frac{AE}{l} \quad \text{because } \delta = y = \frac{Fl}{AE}$$

For torsion:

$$k = \frac{T}{\theta} = \frac{GJ}{l}$$

Curvature:

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{d^2y}{dx^2} \qquad \theta = \frac{dy}{dx}$$

singularity functions

$$\langle x-a \rangle^n = (x-a)^n \cdot U(x-a) \quad \text{if } n \geq 0$$

$$\langle x-a \rangle^{-1} = \delta(x-a)$$

Strain energy:

energy stored in elasic material

$$U = \frac{F}{2y} = \frac{F^2}{2k}$$

if $k = AE/l$ (Tension / compression)

$$U = \frac{F^2 l}{2AE} \quad \text{or} \quad U = \int \frac{F^2}{2AE}\,dx \quad \text{for nonuniform bars}$$

if $k = GJ/l$ (Torsion)

$$U = \frac{T^2 l}{2GJ} \quad \text{or} \quad U = \int \frac{T^2}{2GJ}\,dx$$

Direct shear loading

$$U = \int \frac{F^2}{2AG}\,dx$$

Bending

$$U = \int \frac{M^2}{2EI}\,dx = \frac{M^2 l}{2EI}$$

* integrate if M is $M(x)$

Transverse shear

$$U = \int \frac{CV^2}{2AG}\,dx$$

C is from table 4.1 based off shape

Castigliano's Theorem:

$dU = y\,dF \qquad \therefore \; y = \frac{dU}{dF}$

$\delta_i = \frac{\partial U}{\partial F_i}$ direction i

$\theta_i = \frac{\partial U}{\partial M_i}$

deflection under bending

$U = \int \frac{M^2}{2EI} dx$

chain rule

$\delta = \frac{\partial}{\partial F_i} \int \frac{M^2}{2EI} dx = \int \frac{2M \cdot \frac{\partial M}{\partial F_i}}{2EI} dx = \int \frac{1}{EI}\left(M \frac{\partial M}{\partial F_i}\right) dx$

Chapter 5: Failure from static loading

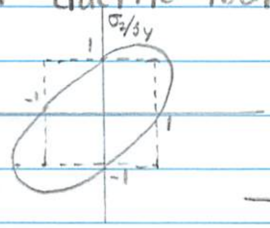
static strength

$$n = \frac{\text{strength}}{\text{stress}} = \frac{S}{\sigma}$$

This is uniaxial, what about multiaxial

Maximum Principle normal stress theory:

- look at σ_1 and compare it to strength yielding
- Not good for ductile materials



inside --- is safe according to theory

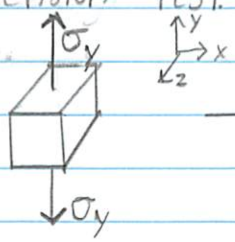
— Oval is experimental failure data

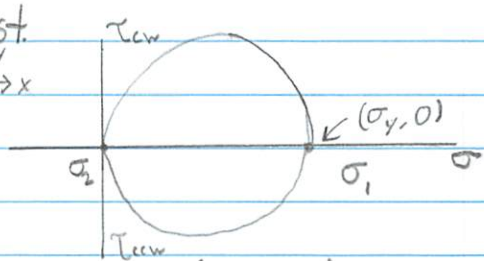
· if an element is under tension ($+\sigma$) and compression (σ is negative) this theory does not work

Maximum shear stress theory

· We see yielding when max shear exseeds max shear stress in tension test.

Tension test: Find's S_{yield}





So in a tensile test: $\tau_{max} = S_y/2$

@ 45° from principle

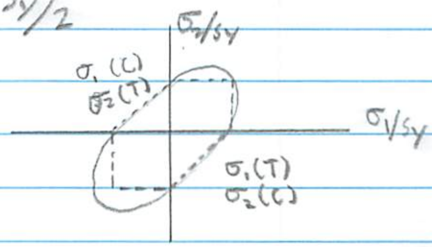
Center $= (\sigma/2, 0)$

$R = \sigma_1/2$

Theory: element fails when $\tau_{max} > S_y/2$ ← from tension test

$$\frac{\sigma_1 - \sigma_3}{2} \leq \frac{S_y}{2} \quad \text{is good}$$

$$n = \frac{S_y}{\sigma_1 - \sigma_3}$$



Distortion Energy (DE) Failure theory:

Failure is not just by stress but it's because of distortion strain energy

von Mises stress:

$$\sigma' = \sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}}$$ This is for principle stresses

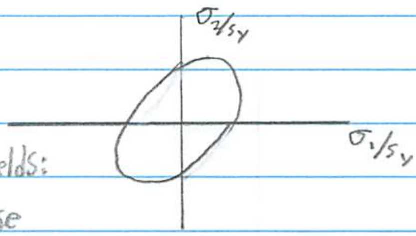
if $\sigma' > S_y$, we will see yielding

For plane stress

$$\sigma' = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$$

agian, failure is $\sigma' > S_y$

plot yields: this ellipse



for pure shear: $S_{xy} = 0.577 S_y = S_y/\sqrt{3}$

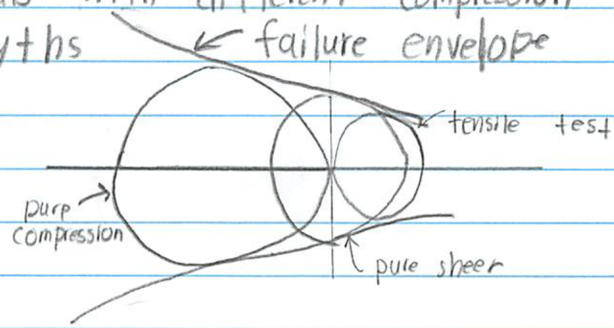
Von Mises for general stress loads:

$$\sigma' = \frac{1}{\sqrt{2}} \cdot \sqrt{(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2)}$$

$n = S_y/\sigma'$

Mohr theory:

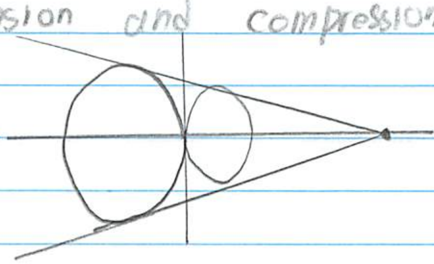
Materials with different compression and tension strengths

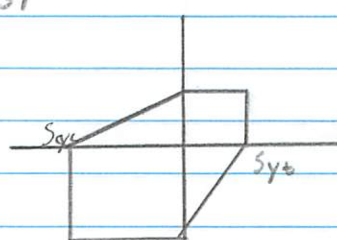


Coulomb-Mohr:

only use tension and compression test

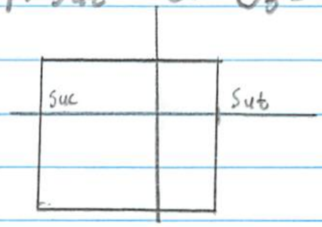
$$\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = 1$$





Max-normal-stress for Brittle materials

failure if: $\sigma_1 > S_{ut}$ or $\sigma_3 \le -S_{uc}$



use ultimate, not yield for brittle materials

Coulomb-Mohr works for brittle too but with ultimate instead of yielding

Chapter 6: Fatigue Failure Resulting from Varible loading

Fatigue:

- Better steel led to structures with more cyclic loading for longer
- realized static loading was an overestimate after time
- Due to crack nucleation and crack propigation

  nucleation at:

  - high stress
  - Geometric changes

3 stages

I • Micro crack due to cyclic plastic deformation

II • Becomes macro-crack that opens and closes (Beach mark)

III • crack propogates

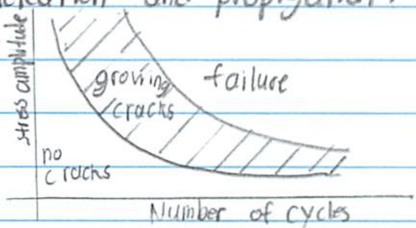
Nucleation:

- localized plastic strain
- slip planes (atoms slip past eachother) Max sheer plane
  - groups are called slip bands
- likely to occur at surface

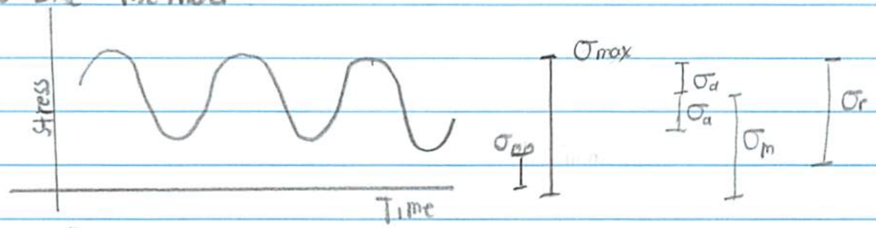
High-cycle fatige onsets after 10,000 cycles (stress life)

low-cycle fatigue short life (strain life) — (fatigue method

nucleation and propigation:



Stress-Life method:



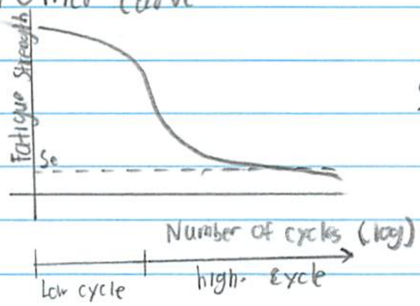
Doesn't have to be a smooth curve

Repeated stress: $\sigma_{min} = 0$

Completly reversed stress: $\sigma_m = 0$

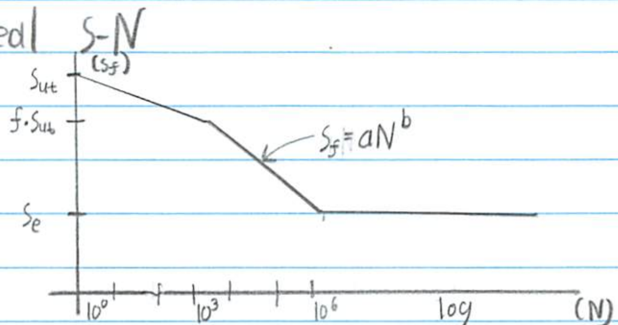
The S-N Diagram

Wöhler curve



S_e is endurance limit

Ideal S-N



estimate endurance limit S_e (All experimental)

$$S_e' = \begin{cases} 0.5 \cdot S_{ut} & S_{ut} \leq 200\ \text{kpsi}\ (1400\ \text{Mpa}) \\ 100\ \text{kpsi} & S_{ut} > 200\ \text{kpsi} \\ 700\ \text{Mpa} & S_{ut} > 1400\ \text{MPa} \end{cases}$$

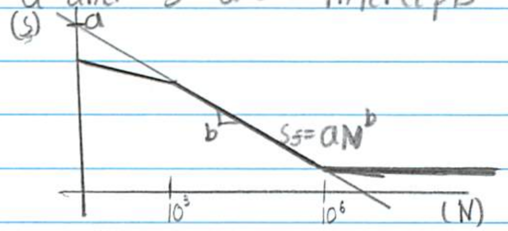
f: for steel

$$f = \begin{cases} 1.06 - 2.8(10^{-3}) S_{ut} + 6.9(10^{-6}) S_{ut}^2 & 70 < S_{ut} < 200\ \text{kpsi} \\ 1.06 - 4.1(10^{-4}) S_{ut} + 1.5(10^{-7}) S_{ut}^2 & 500 < S_{ut} < 1400\ \text{MPa} \end{cases}$$

Basquin equation:

$S_f = aN^b$ or $N = (S_f/a)^{1/b}$

a and b are intercepts and slope on S-N graph



$$a = \frac{(f \cdot S_{ut})^2}{S_e}$$

$$b = \frac{-1}{3} \log\left(\frac{f \cdot S_{ut}}{S_e}\right)$$

For steel

if fully reversible stress σ_{ar}

$\sigma_{ar} = \sigma_f' (2N)^b$ ← fatigue strength exponent

↳ fatigue strength coefficent

values are tabulated

Endurance limit modifying factors

$S_e = k_a k_b k_c k_d k_e S_e'$

Marin factors:

not same as basquin

k_a = surface factor: Figure 6-24 or Table 6-2 ($k_a = a S_{ut}^b$)

k_b = size factor

k_c = load factor

k_D = temperature factor

k_e = reliability factor

Round/rotating

$$k_b = \begin{cases} 0.879 d^{-0.107} & 0.3 \le d \le 2 \text{ in} \\ 0.91 d^{-0.157} & 2 < d \le 10 \text{ in} \\ 1.24 d^{-0.107} & 7.62 \le d \le 51 \text{ mm} \\ 1.51 d^{-0.157} & 51 < d \le 254 \text{ mm} \end{cases}$$

if $d \le 0.3$ in $k_b = 1$

for axial load, $k_b = 1$

not round rods: Table 6-3

↳ or not rotating

K_c: load factor

$$K_c \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsional} \end{cases}$$

only if this is the pure loading

K_d: temp factor (S_T opperating temp, S_{RT} room temp)

$K_d = S_T/S_{RT}$ if no known data on material

$$S_T/S_{RT} = 0.98 + 3.5(10^{-4})T_{°F} - 6.3(10^{-7})T_{°F}^2$$

$$S_T/S_{RT} = 0.99 + 5.9(10^{-4})T_{°C} - 2.1(10^{-6})T_{°C}^2$$

or fig. 2-17

if we know strenghth at our operating temp, $K_d = 1$

and $S_{UT} = S_T/S_{RT} \cdot S_{UT\,R}$

K_e: reliability factor

Table 6-4

$K_e = 1 - 0.08 z_a$

} normal distribution assumption

Stress Concentration and Notch Sensitivity

$$K_f = \frac{\text{strenght of notch-free}}{\text{strength with notchs}}$$

* Fatigue Strength

same idea but different than static stress consentration

$\sigma_{max} = K_f \sigma_0$ $\qquad$ $\tau_{max} = K_{fs} \tau_0$

increases stress the component is seeing

notch sensitivity: K_t is static consentration factor

$$q = \frac{K_f - 1}{K_t - 1}$$

no sensitivity ($q = 0$) $K_f = 0$

fully sensitive ($q = 1$) $K_f = K_t$

q is a material property

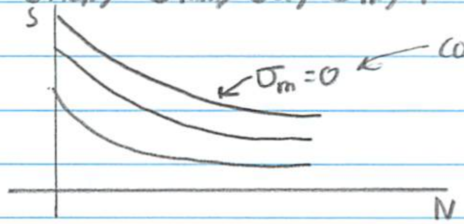
you can look at charts for q or

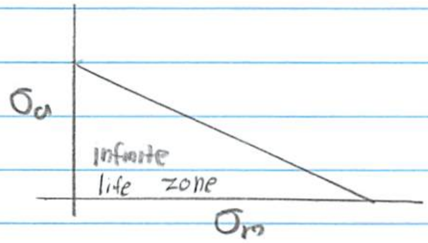
$$q = \frac{1}{1 + \sqrt{a}/\sqrt{r}}$$

$\sqrt{a}$: Neuber constant

Fluctuating Stresses

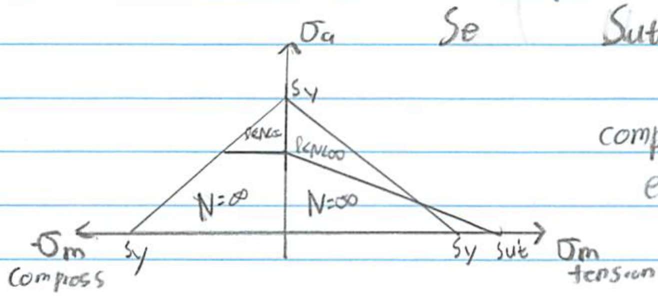
σ_{max}, σ_{min}, σ_a, σ_m, R





$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = 1$$ Goodman line

fatigue factor of saftey $\frac{n_f \sigma_a}{S_e} + \frac{n_f \sigma_m}{S_{ut}} = 1$



compressive mean doesn't effect failure

Example 6-12

Combind loads:

compute and use von Mises stress
max sure K_f are specified for each load
for bending and torsion:

$$\sigma' = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$$

$$\sigma'_a = \sqrt{(K_f \sigma_x)^2 + 3(K_{fs} \tau_{xy})^2}$$

example 6-17

Fatigue failure criteria:
several failure criteria methods

Already discussed Goodman line

- Failure: $\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = 1$
- Design: $n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}\right)^{-1}$
- this is conservitived, (+) $\sigma_m > 0$

Morrow

- $\frac{\sigma_a}{S_e} + \frac{\sigma_m}{\tilde{\sigma}_f} = 1$ — $\tilde{\sigma}_f$ is true fracture strength
- $n_f = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{\tilde{\sigma}_f}\right)^{-1}$

if: HB < 500
$\tilde{\sigma}_f = S_{ut} + 50$ kpsi
or $S_{ut} + 345$ MPa

Gerber

- $\frac{\sigma_a}{S_e} + \left(\frac{\sigma_m}{S_{ut}}\right)^2 = 1$
- $n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2\left(\frac{\sigma_a}{S_e}\right)\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\,\right]$
- $\sigma_m \geq 0$
- good fit to data but may overestimate

Soderberg

- $\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = 1$
- $n = \left(\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y}\right)^{-1}$ Ultra conservitive

ASME

- $\left(\frac{\sigma_a}{S_e}\right)^2 + \left(\frac{\sigma_m}{S_y}\right)^2 = 1$ sometimes conservitive
- $n_f = \left(\left(\frac{\sigma_a}{S_e}\right)^2 + \left(\frac{\sigma_m}{S_y}\right)^2\right)^{-1/2}$ $\sigma_m \geq 0$

Smith-Watson-Topper (SWT)

- Theory based
- $S_e = \sqrt{\sigma_{max}\sigma_a} = \sqrt{(\sigma_m+\sigma_a)\sigma_a}$ and $< S_y$
- $n_f = \dfrac{S_e}{\sqrt{(\sigma_m+\sigma_a)\sigma_a}}$

Combind loadings

Find von Mises stress

use that as our stress and use

σ_a' σ_m' σ_{max}' ect

$K_c = 1$ always

Gears

$P = \dfrac{N}{d}$ diametrial pitch (U.S.) teeth/diameter

$m = \dfrac{d}{N}$ module (metric) diameter/teeth

$p = \dfrac{\pi d}{n}$ circular pitch

Conjugate Action: angular velocity of 2 things touching is equal. This point determines the pitch circle.

Interference: when there is no conjugate action

if two sizes, Gear is bigger, pinion is smaller

Analysis

assume each tooth is a cantilever beam

Shafts and shaft components

Deflection is due to geometry mainly $\frac{1}{EI}$

Stress is controlled by geometry

Strength is purley material property

Common material

Steel, low carbon 1020-1050 AISI

cold drawn for $d<3in$

↖ hot roll for $d>3in$

requires machining if HR

Axial layout

should avoid cantilever components

everything between 2 gears

They should be as short as possible to minimize deflection

Have 2 supports at most to simplify assembly

Transferring tourque from gear to shaft

Key acts as weakest link to fail before gear or shaft

$$\sigma_{max} = \sqrt{(\sigma_m+\sigma_a)^2+3(\tau_m+\tau_a)^2}$$

$$\approx \sigma'_a+\sigma'_m$$

Designing

find better diameter with less deflection

$$d_{new} = d_{old}\left|\frac{n_d\, y_{old}}{y_{all}}\right|^{1/4}$$

Gears analysis

Lewis equation

$$\sigma = \frac{W^t P}{F y} \qquad y = \frac{Y}{\pi}$$ ← lewis form factor

Velocity form factor $\quad K_v$ constant

$$\sigma = \frac{K_v W^t P}{F y}$$

2 failures σ or σ_c

2 safety factors

S_F and S_H

Fatigue for gears

$$K_f = H + \left(\frac{t}{r}\right)^L \left(\frac{t}{l}\right)^M$$

$$H = 0.34 - 0.4583662\phi$$

$$L = 0.316 - 0.4583662\phi$$

$$M = 0.29 + 0.4583662\phi$$

$$r = \frac{(b - r_f)^2}{(d/2) + b - r_f}$$

AGMA equations

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{U.S.} \\ W^t K_o K_v K_s \dfrac{1}{b m_t} \dfrac{K_H K_B}{Y_J} & \text{S.I.} \end{cases}$$

$$J = \frac{Y}{K_f m_N}$$ or Fig. 14-6

$$m_N = \frac{p_N}{0.95Z}$$

Summary Fig. 14-17 and Fig. 14-18