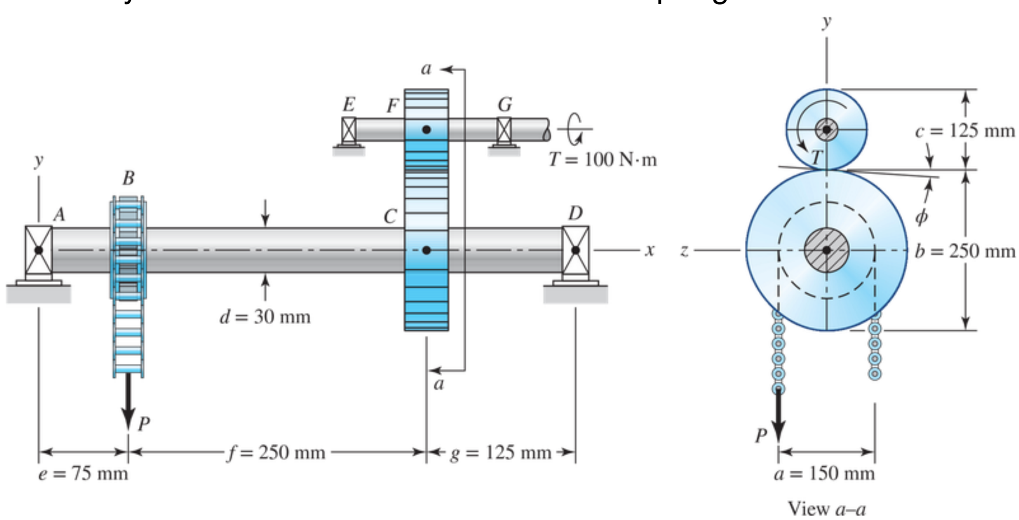
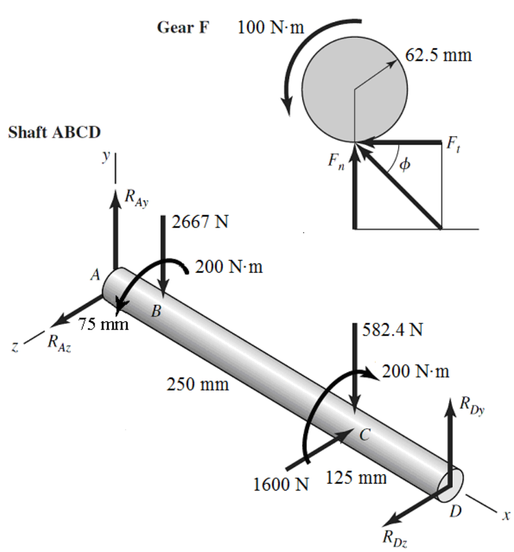
ME3227-001 Design of Machine Elements (Norato)
Final Exam (15 points / up to 21 points with bonus problems)

You can answer the bonus questions marked with a star to get 2 extra points each.

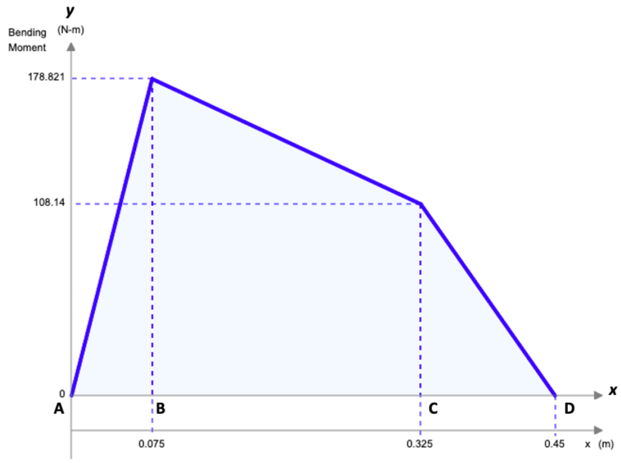
Consider the system shown in the figure below. A torque of $T = 100$ N•m is applied to the input shaft EFG, which runs at a constant angular speed of 100 rev/min. Gear *F*, in turn, transmits torque to shaft *ABCD* through gear *C*. The shaft then drives the sprocket *B*, transmitting a force *P* through the chain, as shown in the figure. The pitch diameters of sprocket *B*, gear *C* and gear *F* are 150, 250 and 125 mm, respectively. The contact force between gears *C* and *F* is transmitted through a pressure angle $\phi = 20°$. Shaft *ABCD* is made of 1018 CD steel. The bearings at *A* and *D* are spherical ball bearings. Gear *C* and sprocket *B* are each held in place on shaft *ABCD* through a square key, with static stress concentration factors $K_t = 2.14$ and $K_{ts} = 3.0$, and a ratio $\frac{r}{d} = 0.02$, where *r* is the key size and *d* the shaft diameter. The spur gear *C* has 30 teeth.

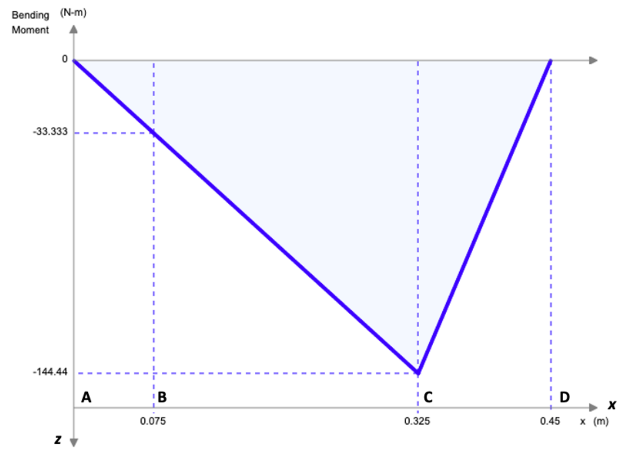


Free-body diagrams of shaft *ABCD* and gear *F* are shown below:



Through equilibrium, it has been determined that the reaction forces are $R_{A_y} = 2384$ N, $R_{A_z} = 444$ N, $R_{D_y} = 865.1$ N and $R_{D_z} = 1156$ N. The moment diagrams in the *xy* and *xz* planes are shown below:





In case the numbers above are not legible, the moments at points *B* and *C* above are:

Point	Moment in *xy*-plane (N•m)	Moment in *xz*-plane (N•m)
B	178.821	-33.333
C	108.14	-144.44

a) Which point will experience the largest stress, *B* or *C*? Why? (3 points)
b) Determine the static factor of safety for yielding at the critical point (*B* or *C*) using both the maximum shear stress (MSS) and distortion energy (DE) criteria. Is the shaft safe in terms of yielding? (5 points)
c) Determine the fatigue factor of safety for this shaft using the Goodman criterion. (7 points)
d) * Use superposition to compute the deflection of shaft *ABCD* at the gear *C* and determine whether the deflection exceeds the maximum allowable deflection for the spur gear at *C*. Use 207 GPa for the modulus of steel. (2 points)
e) * Using superposition, compute the slope at gears *A* and *D* and determine if the slope exceeds the maximum allowable slope for the bearings. Use 207 GPa for the modulus of steel. (2 points)
f) * Compute the maximum bending stress for the tooth of gear *C* using the Lewis equation (including the dynamic factor). The gear teeth are milled. The face width of 4 times the circular pitch. The shaft turns at 100 rev/min. (2 points)

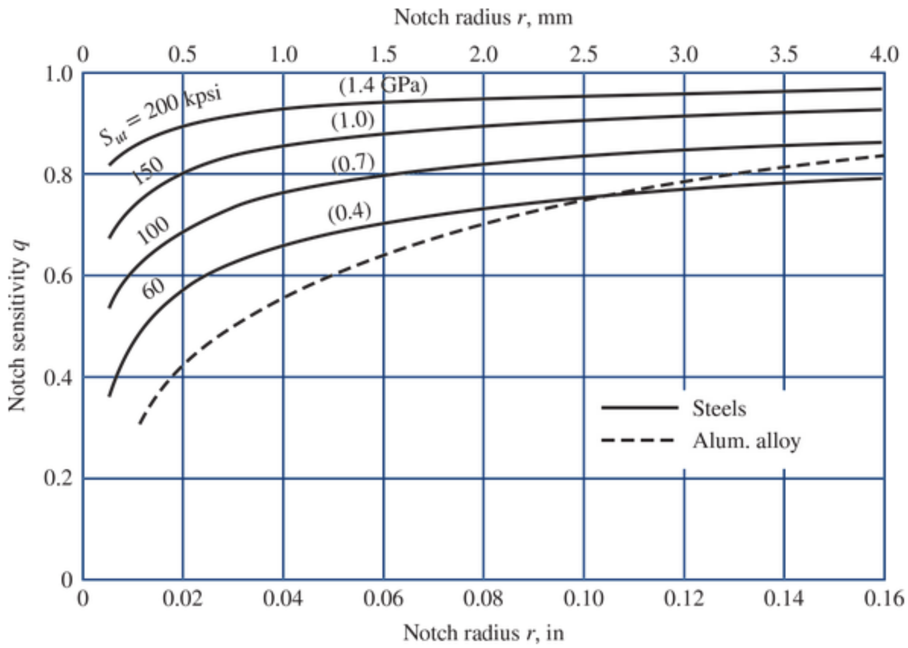
Table A–20 **Deterministic ASTM Minimum Tensile and Yield Strengths for Some Hot-Rolled (HR) and Cold-Drawn (CD) Steels**

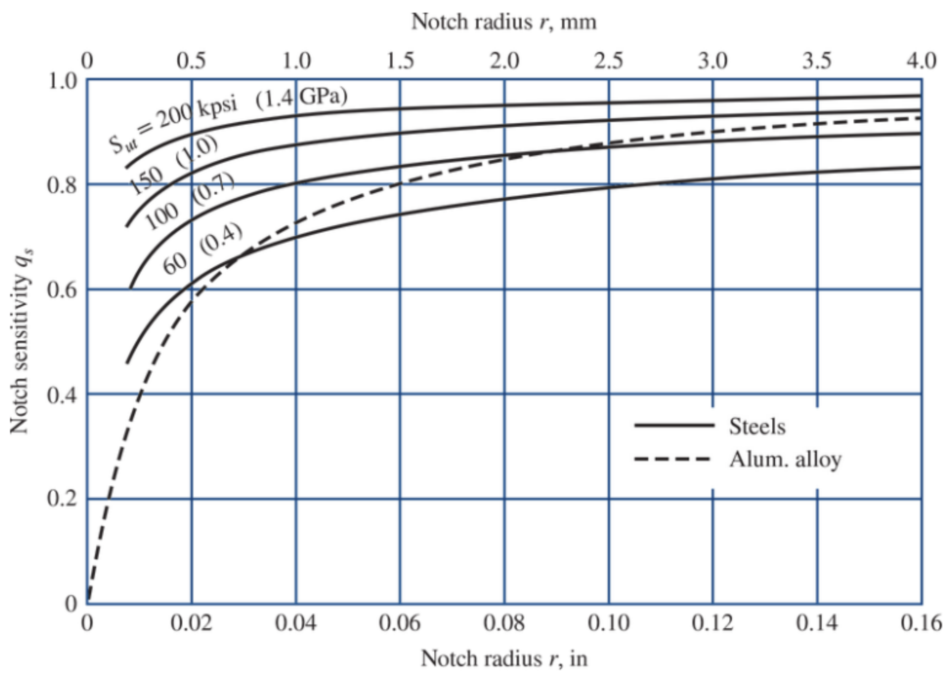
[The strengths listed are estimated ASTM minimum values in the size range 18 to 32 mm $\left(\frac{3}{4} \text{ to } 1\frac{1}{4}\text{in}\right)$. These strengths are suitable for use with the design factor defined in Sec. 1–10, provided the materials conform to ASTM A6 or A568 requirements or are required in the purchase specifications. Remember that a numbering system is not a specification.]

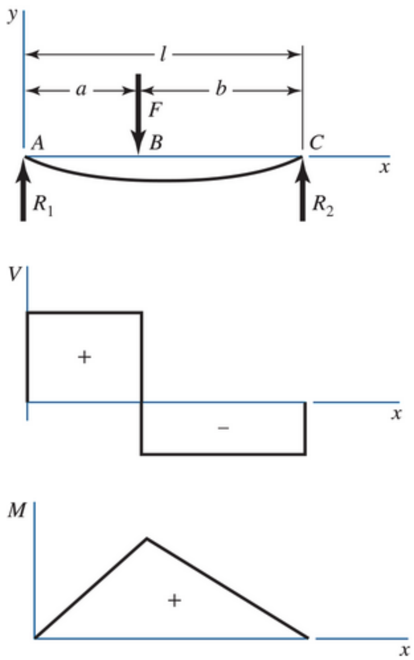
1	2	3	4	5	6	7	8
UNS No.	**SAE and/or AISI No.**	**Processing**	**Tensile Strength, MPa (kpsi)**	**Yield Strength, MPa (kpsi)**	**Elongation in 2 in, %**	**Reduction in Area, %**	**Brinell Hardness**
G10060	1006	HR	300 (43)	170 (24)	30	55	86
		CD	330 (48)	280 (41)	20	45	95
G10100	1010	HR	320 (47)	180 (26)	28	50	95
		CD	370 (53)	300 (44)	20	40	105
G10150	1015	HR	340 (50)	190 (27.5)	28	50	101
		CD	390 (56)	320 (47)	18	40	111
G10180	1018	HR	400 (58)	220 (32)	25	50	116
		CD	440 (64)	370 (54)	15	40	126
G10200	1020	HR	380 (55)	210 (30)	25	50	111
		CD	470 (68)	390 (57)	15	40	131
G10300	1030	HR	470 (68)	260 (37.5)	20	42	137
		CD	520 (76)	440 (64)	12	35	149

Table 6–2 **Curve Fit Parameters for Surface Factor, Equation (6–18)**

Surface Finish	Factor a		Exponent b
	S_{ut}, kpsi	S_{ut}, MPa	
Ground	1.21	1.38	−0.067
Machined or cold-drawn	2.00	3.04	−0.217
Hot-rolled	11.0	38.6	−0.650
As-forged	12.7	54.9	−0.758


Notch radius r, mm
0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0
S_{ut} = 200 kpsi
(1.4 GPa)
150
(1.0)
100
(0.7)
60
(0.4)
Steels
Alum. alloy
Notch sensitivity q
1.0 0.8 0.6 0.4 0.2 0
0 0.02 0.04 0.06 0.08 0.10 0.12 0.14 0.16
Notch radius r, in


Notch radius r, mm
0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0
S_{ut} = 200 kpsi (1.4 GPa)
150 (1.0)
100 (0.7)
60 (0.4)
Steels
Alum. alloy
Notch sensitivity q_s
1.0 0.8 0.6 0.4 0.2 0
0 0.02 0.04 0.06 0.08 0.10 0.12 0.14 0.16
Notch radius r, in



$$R_1 = \frac{Fb}{l} \qquad R_2 = \frac{Fa}{l}$$

$$V_{AB} = R_1 \qquad V_{BC} = -R_2$$

$$M_{AB} = \frac{Fbx}{l} \qquad M_{BC} = \frac{Fa}{l}(l - x)$$

$$y_{AB} = \frac{Fbx}{6EIl}(x^2 + b^2 - l^2)$$

$$y_{BC} = \frac{Fa(l - x)}{6EIl}(x^2 + a^2 - 2lx)$$

Table 7–2 Typical Maximum Ranges for Slopes and Transverse Deflections

Slopes	
Tapered roller	0.0005–0.0012 rad
Cylindrical roller	0.0008–0.0012 rad
Deep-groove ball	0.001–0.003 rad
Spherical ball	0.026–0.052 rad
Self-align ball	0.026–0.052 rad
Uncrowned spur gear	<0.0005 rad
Transverse Deflections	
Spur gears with $P < 10$ teeth/in	0.010 in
Spur gears with $11 < P < 19$	0.005 in
Spur gears with $20 < P < 50$	0.003 in

Table 14–2 **Values of the Lewis Form Factor Y (These Values Are for a Normal Pressure Angle of 20°, Full-Depth Teeth, and a Diametral Pitch of Unity in the Plane of Rotation)**

Number of Teeth	Y	Number of Teeth	Y
12	0.245	28	0.353
13	0.261	30	0.359
14	0.277	34	0.371
15	0.290	38	0.384
16	0.296	43	0.397
17	0.303	50	0.409
18	0.309	60	0.422
19	0.314	75	0.435
20	0.322	100	0.447
21	0.328	150	0.460
22	0.331	300	0.472
24	0.337	400	0.480
26	0.346	Rack	0.485