Review of Differentiation

Rules

1. **Constant:** $\frac{d}{dx}c = 0$
2. **Constant multiple:** $\frac{d}{dx}c\,f(x) = c\,f'(x)$
3. **Sum:** $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$
4. **Product:** $\frac{d}{dx}f(x)g(x) = f(x)g'(x) + g(x)f'(x)$
5. **Quotient:** $\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$
6. **Chain:** $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$
7. **Power:** $\frac{d}{dx}x^n = nx^{n-1}$
8. **Power:** $\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}g'(x)$

Functions

Trigonometric:

9. $\frac{d}{dx}\sin x = \cos x$
10. $\frac{d}{dx}\cos x = -\sin x$
11. $\frac{d}{dx}\tan x = \sec^2 x$
12. $\frac{d}{dx}\cot x = -\csc^2 x$
13. $\frac{d}{dx}\sec x = \sec x \tan x$
14. $\frac{d}{dx}\csc x = -\csc x \cot x$

Inverse trigonometric:

15. $\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$
16. $\frac{d}{dx}\cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$
17. $\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$
18. $\frac{d}{dx}\cot^{-1} x = -\frac{1}{1+x^2}$
19. $\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$
20. $\frac{d}{dx}\csc^{-1} x = -\frac{1}{|x|\sqrt{x^2-1}}$

Hyperbolic:

21. $\frac{d}{dx}\sinh x = \cosh x$
22. $\frac{d}{dx}\cosh x = \sinh x$
23. $\frac{d}{dx}\tanh x = \operatorname{sech}^2 x$
24. $\frac{d}{dx}\coth x = -\operatorname{csch}^2 x$
25. $\frac{d}{dx}\operatorname{sech} x = -\operatorname{sech} x \tanh x$
26. $\frac{d}{dx}\operatorname{csch} x = -\operatorname{csch} x \coth x$

Inverse hyperbolic:

27. $\frac{d}{dx}\sinh^{-1} x = \frac{1}{\sqrt{x^2+1}}$
28. $\frac{d}{dx}\cosh^{-1} x = \frac{1}{\sqrt{x^2-1}}$
29. $\frac{d}{dx}\tanh^{-1} x = \frac{1}{1-x^2}$
30. $\frac{d}{dx}\coth^{-1} x = \frac{1}{1-x^2}$
31. $\frac{d}{dx}\operatorname{sech}^{-1} x = -\frac{1}{x\sqrt{1-x^2}}$
32. $\frac{d}{dx}\operatorname{csch}^{-1} x = -\frac{1}{|x|\sqrt{x^2+1}}$

Exponential:

33. $\frac{d}{dx}e^x = e^x$
34. $\frac{d}{dx}a^x = a^x(\ln a)$

Logarithmic:

35. $\frac{d}{dx}\ln|x| = \frac{1}{x}$
36. $\frac{d}{dx}\log_a x = \frac{1}{x(\ln a)}$

Integral defined:

37. $\frac{d}{dx}\int_a^x g(t)\,dt = g(x)$
38. $\frac{d}{dx}\int_a^b g(x,t)\,dt = \int_a^b \frac{\partial}{\partial x}g(x,t)\,dt$

Brief Table of Integrals

1. $\int u^n\,du = \frac{u^{n+1}}{n+1} + C,\ n \neq -1$
2. $\int \frac{1}{u}\,du = \ln|u| + C$
3. $\int e^u\,du = e^u + C$
4. $\int a^u\,du = \frac{1}{\ln a}a^u + C$
5. $\int \sin u\,du = -\cos u + C$
6. $\int \cos u\,du = \sin u + C$
7. $\int \sec^2 u\,du = \tan u + C$
8. $\int \csc^2 u\,du = -\cot u + C$
9. $\int \sec u \tan u\,du = \sec u + C$
10. $\int \csc u \cot u\,du = -\csc u + C$
11. $\int \tan u\,du = -\ln|\cos u| + C$
12. $\int \cot u\,du = \ln|\sin u| + C$
13. $\int \sec u\,du = \ln|\sec u + \tan u| + C$
14. $\int \csc u\,du = \ln|\csc u - \cot u| + C$
15. $\int u \sin u\,du = \sin u - u\cos u + C$
16. $\int u \cos u\,du = \cos u + u\sin u + C$
17. $\int \sin^2 u\,du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$
18. $\int \cos^2 u\,du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$
19. $\int \tan^2 u\,du = \tan u - u + C$
20. $\int \cot^2 u\,du = -\cot u - u + C$
21. $\int \sin^3 u\,du = -\frac{1}{3}\left(2 + \sin^2 u\right)\cos u + C$
22. $\int \cos^3 u\,du = \frac{1}{3}\left(2 + \cos^2 u\right)\sin u + C$
23. $\int \tan^3 u\,du = \frac{1}{2}\tan^2 u + \ln|\cos u| + C$
24. $\int \cot^3 u\,du = -\frac{1}{2}\cot^2 u - \ln|\sin u| + C$
25. $\int \sec^3 u\,du = \frac{1}{2}\sec u \tan u + \frac{1}{2}\ln|\sec u + \tan u| + C$
26. $\int \csc^3 u\,du = -\frac{1}{2}\csc u \cot u + \frac{1}{2}\ln|\csc u - \cot u| + C$
27. $\int \sin au \sin bu\,du = \frac{\sin(a-b)u}{2(a-b)} - \frac{\sin(a+b)u}{2(a+b)} + C$
28. $\int \cos au \cos bu\,du = \frac{\sin(a-b)u}{2(a-b)} + \frac{\sin(a+b)u}{2(a+b)} + C$
29. $\int e^{au} \sin bu\,du = \frac{e^{au}}{a^2+b^2}(a\sin bu - b\cos bu) + C$
30. $\int e^{au} \cos bu\,du = \frac{e^{au}}{a^2+b^2}(a\cos bu + b\sin bu) + C$
31. $\int \sinh u\,du = \cosh u + C$
32. $\int \cosh u\,du = \sinh u + C$
33. $\int \operatorname{sech}^2 u\,du = \tanh u + C$
34. $\int \operatorname{csch}^2 u\,du = -\coth u + C$
35. $\int \tanh u\,du = \ln(\cosh u) + C$
36. $\int \coth u\,du = \ln|\sinh u| + C$
37. $\int \ln u\,du = u\ln u - u + C$
38. $\int u \ln u\,du = \frac{1}{2}u^2 \ln u - \frac{1}{4}u^2 + C$
39. $\int \frac{1}{\sqrt{a^2-u^2}}\,du = \sin^{-1}\frac{u}{a} + C$
40. $\int \frac{1}{\sqrt{a^2+u^2}}\,du = \ln\left|u + \sqrt{a^2+u^2}\right| + C$
41. $\int \sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$
42. $\int \sqrt{a^2+u^2}\,du = \frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2}\ln\left|u + \sqrt{a^2+u^2}\right| + C$
43. $\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$
44. $\int \frac{1}{a^2-u^2}\,du = \frac{1}{2a}\ln\left|\frac{a+u}{a-u}\right| + C$

***Note*:** Some techniques of integration, such as integration by parts and partial fractions, are reviewed in the *Student Resource Manual* that accompanies this text.

Table of Laplace Transforms

$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$
1. 1	$\frac{1}{s}$
2. t	$\frac{1}{s^2}$
3. t^n	$\frac{n!}{s^{n+1}}$, n a positive integer
4. $t^{-1/2}$	$\sqrt{\frac{\pi}{s}}$
5. $t^{1/2}$	$\frac{\sqrt{\pi}}{2s^{3/2}}$
6. t^α	$\frac{\Gamma(\alpha+1)}{s^{\alpha+1}}$, $\alpha > -1$
7. $\sin kt$	$\frac{k}{s^2+k^2}$
8. $\cos kt$	$\frac{s}{s^2+k^2}$
9. $\sin^2 kt$	$\frac{2k^2}{s(s^2+4k^2)}$
10. $\cos^2 kt$	$\frac{s^2+2k^2}{s(s^2+4k^2)}$
11. e^{at}	$\frac{1}{s-a}$
12. $\sinh kt$	$\frac{k}{s^2-k^2}$
13. $\cosh kt$	$\frac{s}{s^2-k^2}$
14. $\sinh^2 kt$	$\frac{2k^2}{s(s^2-4k^2)}$
15. $\cosh^2 kt$	$\frac{s^2-2k^2}{s(s^2-4k^2)}$
16. te^{at}	$\frac{1}{(s-a)^2}$
17. $t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$, n a positive integer
18. $e^{at}\sin kt$	$\frac{k}{(s-a)^2+k^2}$
19. $e^{at}\cos kt$	$\frac{s-a}{(s-a)^2+k^2}$
20. $e^{at}\sinh kt$	$\frac{k}{(s-a)^2-k^2}$
21. $e^{at}\cosh kt$	$\frac{s-a}{(s-a)^2-k^2}$
22. $t\sin kt$	$\frac{2ks}{(s^2+k^2)^2}$
23. $t\cos kt$	$\frac{s^2-k^2}{(s^2+k^2)^2}$
24. $\sin kt + kt\cos kt$	$\frac{2ks^2}{(s^2+k^2)^2}$
25. $\sin kt - kt\cos kt$	$\frac{2k^3}{(s^2+k^2)^2}$
26. $t\sinh kt$	$\frac{2ks}{(s^2-k^2)^2}$
27. $t\cosh kt$	$\frac{s^2+k^2}{(s^2-k^2)^2}$
28. $\frac{e^{at}-e^{bt}}{a-b}$	$\frac{1}{(s-a)(s-b)}$
29. $\frac{ae^{at}-be^{bt}}{a-b}$	$\frac{s}{(s-a)(s-b)}$
30. $1-\cos kt$	$\frac{k^2}{s(s^2+k^2)}$
31. $kt-\sin kt$	$\frac{k^3}{s^2(s^2+k^2)}$
32. $\frac{a\sin bt - b\sin at}{ab(a^2-b^2)}$	$\frac{1}{(s^2+a^2)(s^2+b^2)}$
33. $\frac{\cos bt - \cos at}{a^2-b^2}$	$\frac{s}{(s^2+a^2)(s^2+b^2)}$
34. $\sin kt\sinh kt$	$\frac{2k^2s}{s^4+4k^4}$
35. $\sin kt\cosh kt$	$\frac{k(s^2+2k^2)}{s^4+4k^4}$
36. $\cos kt\sinh kt$	$\frac{k(s^2-2k^2)}{s^4+4k^4}$
37. $\cos kt\cosh kt$	$\frac{s^3}{s^4+4k^4}$
38. $\sin kt\cosh kt + \cos kt\sinh kt$	$\frac{2ks^2}{s^4+4k^4}$
39. $\sin kt\cosh kt - \cos kt\sinh kt$	$\frac{4k^3}{s^4+4k^4}$
40. $\sinh kt - \sin kt$	$\frac{2k^3}{s^4-k^4}$

$f(t)$	$\mathscr{L}\{f(t)\} = F(s)$
41. $\cosh kt - \cos kt$	$\dfrac{2k^2 s}{s^4 - k^4}$
42. $J_0(kt)$	$\dfrac{1}{\sqrt{s^2 + k^2}}$
43. $\dfrac{e^{bt} - e^{at}}{t}$	$\ln \dfrac{s - a}{s - b}$
44. $\dfrac{2(1 - \cos kt)}{t}$	$\ln \dfrac{s^2 + k^2}{s^2}$
45. $\dfrac{2(1 - \cosh kt)}{t}$	$\ln \dfrac{s^2 - k^2}{s^2}$
46. $\dfrac{\sin at}{t}$	$\arctan\left(\dfrac{a}{s}\right)$
47. $\dfrac{\sin at \cos bt}{t}$	$\dfrac{1}{2}\arctan\dfrac{a + b}{s} + \dfrac{1}{2}\arctan\dfrac{a - b}{s}$
48. $\dfrac{1}{\sqrt{\pi t}} e^{-a^2/4t}$	$\dfrac{e^{-a\sqrt{s}}}{\sqrt{s}}$
49. $\dfrac{a}{2\sqrt{\pi t^3}} e^{-a^2/4t}$	$e^{-a\sqrt{s}}$
50. $\operatorname{erfc}\left(\dfrac{a}{2\sqrt{t}}\right)$	$\dfrac{e^{-a\sqrt{s}}}{s}$
51. $2\sqrt{\dfrac{t}{\pi}} e^{-a^2/4t} - a \operatorname{erfc}\left(\dfrac{a}{2\sqrt{t}}\right)$	$\dfrac{e^{-a\sqrt{s}}}{s\sqrt{s}}$

$f(t)$	$\mathscr{L}\{f(t)\} = F(s)$
52. $e^{ab} e^{b^2 t} \operatorname{erfc}\left(b\sqrt{t} + \dfrac{a}{2\sqrt{t}}\right)$	$\dfrac{e^{-a\sqrt{s}}}{\sqrt{s}(\sqrt{s} + b)}$
53. $-e^{ab} e^{b^2 t} \operatorname{erfc}\left(b\sqrt{t} + \dfrac{a}{2\sqrt{t}}\right) + \operatorname{erfc}\left(\dfrac{a}{2\sqrt{t}}\right)$	$\dfrac{b e^{-a\sqrt{s}}}{s(\sqrt{s} + b)}$
54. $e^{at} f(t)$	$F(s - a)$
55. $\mathscr{U}(t - a)$	$\dfrac{e^{-as}}{s}$
56. $f(t - a)\mathscr{U}(t - a)$	$e^{-as} F(s)$
57. $g(t)\mathscr{U}(t - a)$	$e^{-as}\,\mathscr{L}\{g(t + a)\}$
58. $f^{(n)}(t)$	$s^n F(s) - s^{(n-1)} f(0) - \cdots - f^{(n-1)}(0)$
59. $t^n f(t)$	$(-1)^n \dfrac{d^n}{ds^n} F(s)$
60. $\displaystyle\int_0^t f(\tau) g(t - \tau)\, d\tau$	$F(s)G(s)$
61. $\delta(t)$	1
62. $\delta(t - t_0)$	e^{-st_0}