Homework 7-1

Imports

[1]:
```python
from thermostate import Q_, State
```

Definitions

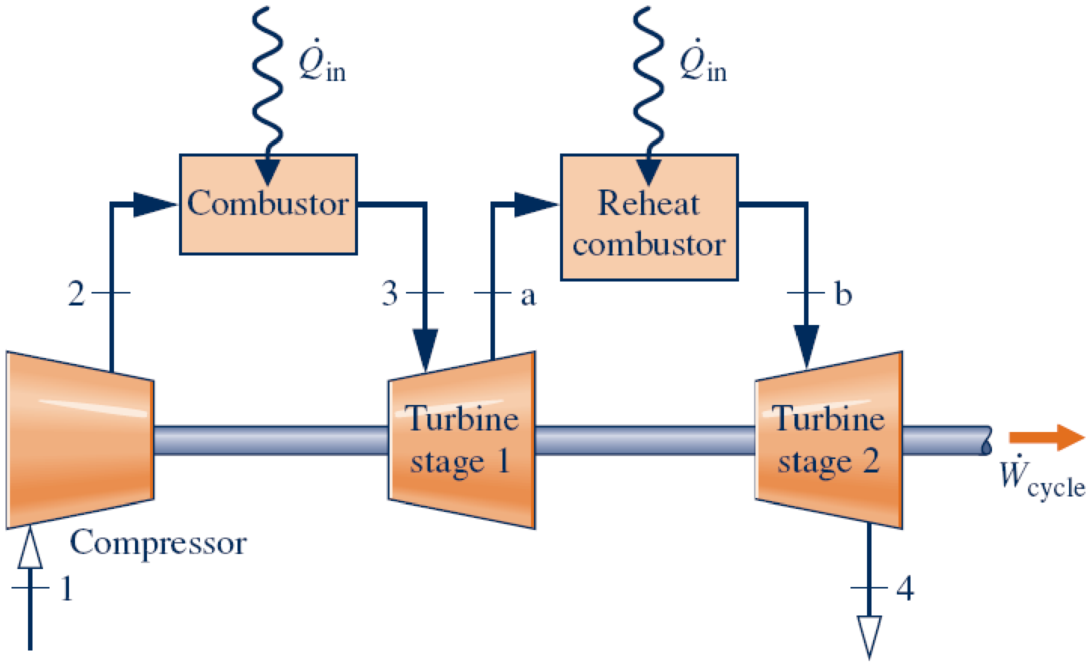
[2]:
```python
substance = 'air'

p_1 = Q_(100, 'kPa')
T_1 = Q_(300, 'K')

p2_p1 = Q_(10, 'dimensionless')

T_3 = T_b = Q_(1400, 'K')

p_a = p_b = Q_(300, 'kPa')
```

Problem Statement

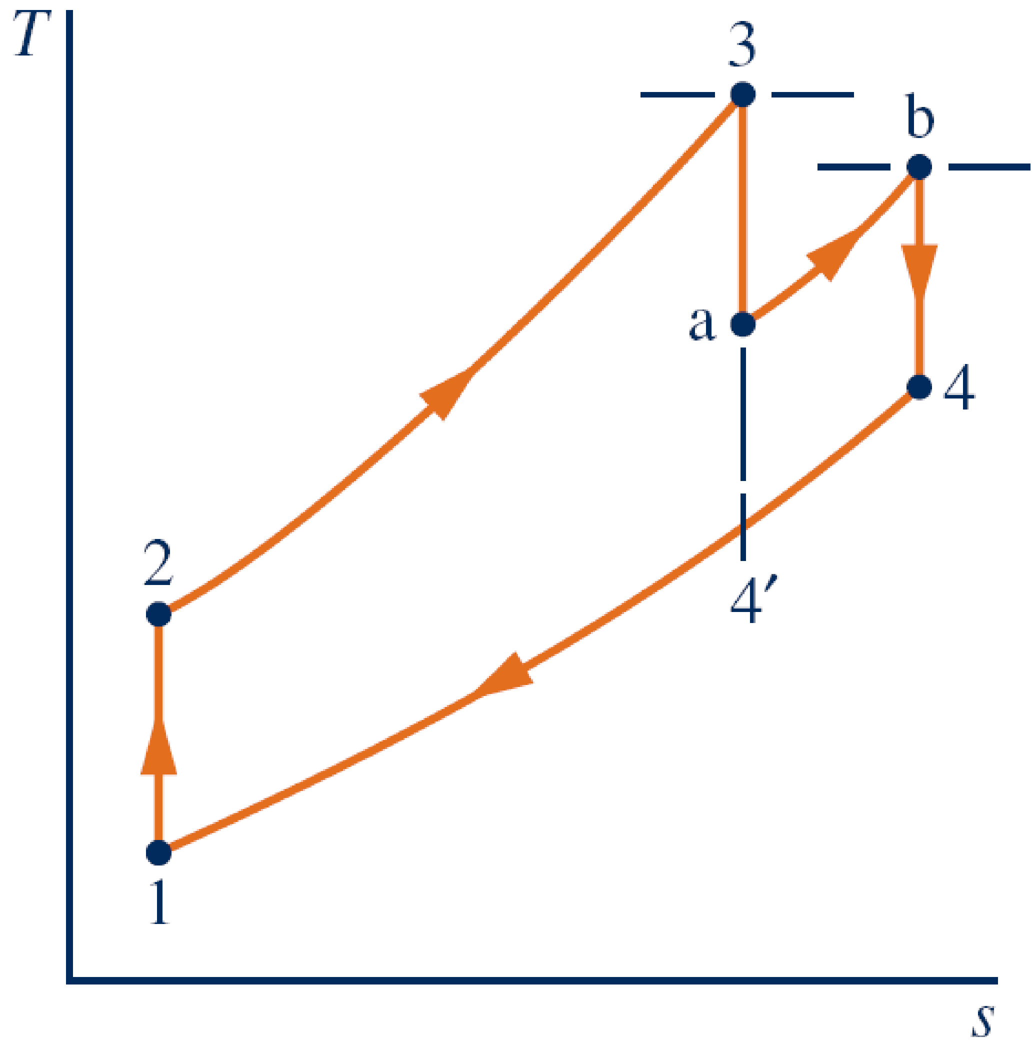
The system diagram below shows a two-stage gas turbine power cycle with a reheater operating at steady state. The $T-s$ diagram shows that the compressor and turbine processes are isentropic (i.e. this is an ideal cycle).

Air enters the compressor at 100 kPa, 300 K. The compressor pressure ratio is 10. The temperature at the inlet to the first turbine stage is 1400 K. The expansion takes places isentropically in two stages, with reheat to 1400 K between the stages at a constant pressure of 300 kPa.

1. For each of the components (compressor, heat exchangers, turbines) in the system, what thermodynamic properties remain constant?
2. Set all the states in the system using Thermostate (can skip this part if you do HW by hand).
3. Calculate the compressor work per unit mass of air flowing, in kJ/kg.
4. Calculate the turbine work per unit mass of air flowing for each of the two turbines separately, in kJ/kg.
5. Calculate the rate of heat transfer input per unit mass of air flowing in the combustor, in kJ/kg.
6. Calculate the rate of heat transfer input per unit mass of air flowing in the reheat combustor, in kJ/kg. **Note: despite using the same symbol, $\dot{Q}_{in}$ in the system diagram below, this is not necessarily equal to the value found in Part 5**.
7. Calculate the cycle thermal efficiency.



$\dot{Q}_{in}$
$\dot{Q}_{in}$
Combustor
Reheat combustor
2
3
a
b
Turbine stage 1
Turbine stage 2
$\dot{W}_{cycle}$
Compressor
1
4



Solution

Part 1: What's constant in each component?

Answer: In the compressor, we know enthalpy is constant.
In both heat exchangers, pressure is constant.
In both turbines, enthalpy is also constant.

Part 2: Set All the States Using Thermostate (Skip if Doing by Hand)

Fixing States:

State	1	2
1	p_1	T_1
2	p_2	$s_2 = s_1$
3	$p_3 = p_2$	T_3
a	p_a	$s_a = s_3$
b	$p_b = p_a$	$T_b = T_3$
4	$p_4 = p_1$	$s_4 = s_b$

[3]:
```
st_1 = State(substance, p=p_1, T=T_1)
st_2 = State(substance, p=p2_p1*p_1, s=st_1.s)
st_3 = State(substance, p=st_2.p, T=T_3)
st_a = State(substance, p=p_a, s=st_3.s)
st_b = State(substance, p=p_b, T=T_b)
st_4 = State(substance, p=p_1, s=st_b.s)
```

Answer: Does anybody actually do this by hand?

Part 3: Compressor Work per Unit Mass of Air Flowing, in kJ/kg

Important Equations:

1. $\frac{\dot{W}_c}{\dot{m}} = h_1 - h_2$

[14]:
```
wdot_c = st_1.h - st_2.h
print(wdot_c.to("kJ/kg").round(3))
```

```
-280.151 kilojoule / kilogram
```

Answer: 280.151 kJ/kg into the system

Part 4: Turbine Work per Unit Mass of Air Flowing for Each of the Two Turbines Separately, in kJ/kg

Important Equations:

1. $\frac{\dot{W}_{t1}}{\dot{m}} = h_3 - h_a$
2. $\frac{\dot{W}_{t2}}{\dot{m}} = h_b - h_4$

[15]:
```
wdot_t1 = st_3.h - st_a.h
wdot_t2 = st_b.h - st_4.h
print(wdot_t1.to("kJ/kg").round(3))
print(wdot_t2.to("kJ/kg").round(3))
```

```
420.301 kilojoule / kilogram
387.857 kilojoule / kilogram
```

Answer: Turbine 1 produced 420.301 kJ/kg of work
Turbine 2 produced 387.857 kJ/kg of work

Part 5: Rate of Heat Transfer Input per Unit Mass of Air Flowing in the Combustor, in kJ/kg

Important Equations:

1. $\frac{\dot{Q}_{Combustor}}{\dot{m}} = h_3 - h_2$

[16]:
```
qdot_combust = st_3.h-st_2.h
print(qdot_combust.to("kJ/kg").round(3))
```

```
936.085 kilojoule / kilogram
```

Answer: 936.085 kJ/kg

Part 6: Rate of Heat Transfer Input per Unit Mass of Air Flowing Into the Reheat Combustor, in kJ/kg

Important Equations:

1. $\frac{\dot{Q}_{Reheat}}{\dot{m}} = h_b - h_a$

[17]:
```
qdot_reheat = st_b.h-st_a.h
print(qdot_reheat.to("kJ/kg").round(3))
```

```
419.728 kilojoule / kilogram
```

Answer: 419.728 kJ/kg

Part 7: Cycle Thermal Efficiency

Important Equations:

1. $\eta = \frac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}}$

[18]:
```
wdot_cycle = wdot_t1 + wdot_t2 + wdot_c
qdot_in = qdot_combust + qdot_reheat
eta = wdot_cycle/qdot_in
print(eta.round(5))
```

```
0.38944 dimensionless
```

Answer: 38.944%

Homework 7-2

Imports

[1]:
```
from thermostate import Q_, State, units
from numpy import linspace, zeros_like
import matplotlib.pyplot as plt

import warnings
from pint.errors import UnitStrippedWarning
warnings.simplefilter(action='ignore', category=UnitStrippedWarning)
```

Definitions

[2]:
```
substance = 'air'

p_1 = Q_(1, 'bar')
T_1 = Q_(310, 'K')

p2_p1 = Q_(12, 'dimensionless')

T_4 = T_6 = Q_(1100, 'K')

p_5 = p_6 = Q_(6, 'bar')

p_7 = Q_(1, 'bar')

eta_c = Q_(0.8, 'dimensionless')
eta_t = Q_(0.85, 'dimensionless')
eta_reg = Q_(0.8, 'dimensionless')
```

Problem Statement

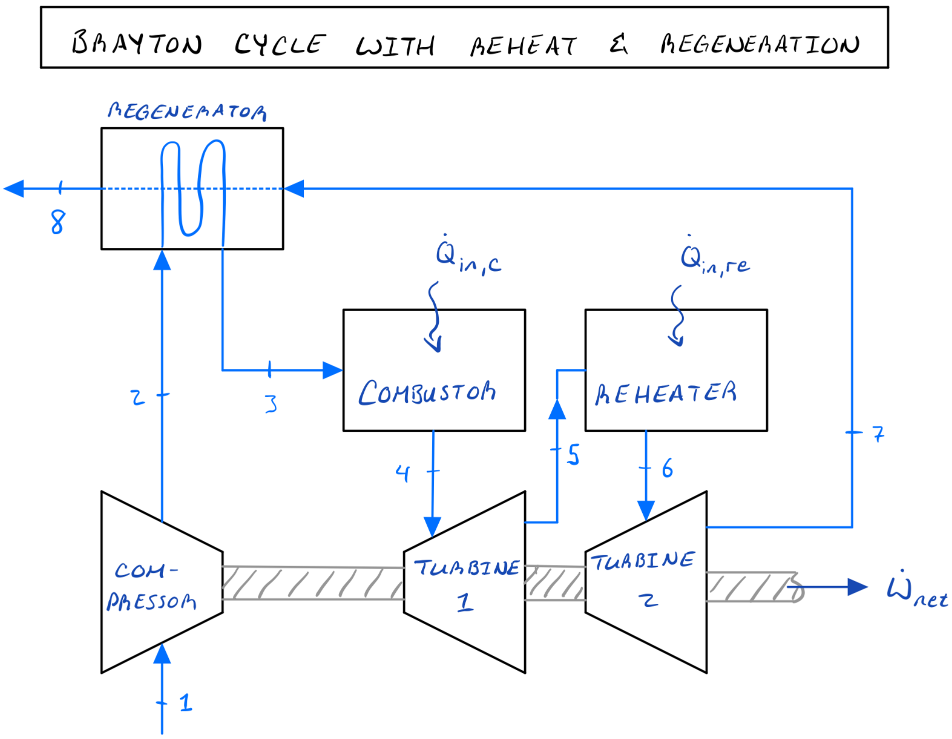
An air-standard Brayton cycle with reheat and regeneration operates at steady state as shown in the system diagram below.

Air enters the compressor at 1 bar, 310 K, and the compressor pressure ratio is 12. The inlet temperature at both turbine stages is 1100 K. The reheater pressure is 6 bar. The final turbine outlet pressure is 1 bar.

The compressor operates with an isentropic efficiency of 80%, the turbines both operate with an isentropic efficiency of 85%, and the regenerator effectiveness is 80%.

1. Find the net cycle work per unit mass flow rate, in kJ/kg.

2. Find the total rate of heat transfer into the system per unit mass flow rate, in kJ/kg.
3. Calculate the cycle's thermal efficiency.
4. Find the cycle's back-work-ratio.
5. Briefly describe how the flow in the turbines gets converted to electrical power.
6. For reheater pressures from 1.1 bar to 10.0 bar, plot the cycle's thermal efficiency, the net work output per unit mass flow rate, and the heat transfer input per unit mass flow rate.
7. Briefly discuss trends in the plot in Part 6.



Solution

Fixing States:

State	1	2
1	p_1	T_1
2s	p_2	$s_{2s} = s_1$
2	p_2	h_2
3	$p_3 = p_2$	h_3
4	$p_4 = p_3$	T_4
5s	p_5	$s_{5s} = s_4$
5	p_5	h_5

	State	1	2
6		$p_6 = p_5$	$T_6 = T_4$
7s		p_7	$s_{7s} = s_6$
7		p_7	h_7

Efficency Equations:

1. $h_2 = (h_{2s} - h_1)/\eta_c + h_1$
2. $h_5 = \eta_t(h_{5s} - h_4) + h_4$
3. $h_7 = \eta_t(h_{7s} - h_6) + h_6$
4. $h_3 = (h_7 - h_2)/\eta_{reg} + h_2$

[3]:
```
st_1 = State(substance, p=p_1, T=T_1)
st_2s = State(substance, p=p2_p1*p_1, s=st_1.s)
h_2 = (st_2s.h - st_1.h)/eta_c + st_1.h
st_2 = State(substance, p=st_2s.p, h=h_2)

st_6 = State(substance, p=p_6, T=T_6)
st_7s = State(substance, p=p_7 , s=st_6.s)
h_7 = eta_t*(st_7s.h - st_6.h) + st_6.h
st_7 = State(substance, p=p_7, h=h_7)

h_3 = (st_7.h - st_2.h)/eta_reg + st_2.h
st_3 = State(substance, p=st_2.p, h=h_3)
st_4 = State(substance, p=st_3.p, T=T_4)
st_5s = State(substance, p=p_5, s=st_4.s)
h_5 = eta_t*(st_5s.h - st_4.h) + st_4.h
st_5 = State(substance, p=p_5, h=h_5)
```

Part 1: Net Cycle Work per Unit Mass Flow Rate, in kJ/kg

Important Equations:

1. $\frac{\dot{W}_{cycle}}{\dot{m}} = \sum \frac{\dot{W}}{\dot{m}}$
2. $\frac{\dot{W}_c}{\dot{m}} = h_1 - h_2$
3. $\frac{\dot{W}_{t1}}{\dot{m}} = h_4 - h_5$
4. $\frac{\dot{W}_{t2}}{\dot{m}} = h_6 - h_7$

[4]:
```
wdot_c = st_1.h - st_2.h
wdot_t1 = st_4.h - st_5.h
wdot_t2 = st_6.h - st_7.h
wdot_cycle = wdot_c + wdot_t1 + wdot_t2
print(wdot_cycle.to("kJ/kg").round(3))
```

```
156.42 kilojoule / kilogram
```

Answer: 156.42 kJ/kg

Part 2: Total Rate of Heat Transfer Into the System per Unit Mass Flow Rate, in kJ/kg

Imporant Equations:

1. $\frac{\dot{Q}_{in,c}}{\dot{m}} = h_4 - h_3$
2. $\frac{\dot{Q}_{in,re}}{\dot{m}} = h_6 - h_5$

[5]:
```
qdot_inc = st_4.h-st_3.h
qdot_inre = st_6.h-st_5.h
qdot_in = qdot_inc + qdot_inre
print(qdot_in.to("kJ/kg").round(3))
```

```
542.247 kilojoule / kilogram
```

Answer: 542.247 kJ/kg

Part 3: Cycle Thermal Efficiency

Important Equations:

1. $\eta = \frac{\dot{W}_{cycle}/\dot{m}}{\dot{Q}_{in}/\dot{m}}$

[6]:
```
eta = wdot_cycle/qdot_in
print(eta.round(5))
```

```
0.28847 dimensionless
```

Answer: 28.847%

Part 4: Cycle Back-Work Ratio

Important Equations:

1. $\text{BWR} = \frac{\dot{W}_c/\dot{m}}{\sum \dot{W}_t/\dot{m}}$

[7]:
```
BWR = (wdot_c)/(wdot_t1+wdot_t2)
print(BWR.round(5))
```

```
-0.71969 dimensionless
```

Answer: 71.969%

Part 5: How Does Turbine Work Turn Into Electrical Power?

Answer: The rotational work of the turbine is often used to spin the motor. The spinning motor is filled with electromagnets and as the fields are flipped, an electrical current is induced.

Part 6: Plot Thermal Efficiency, Work, and Heat Transfer for Varying Reheater Pressures

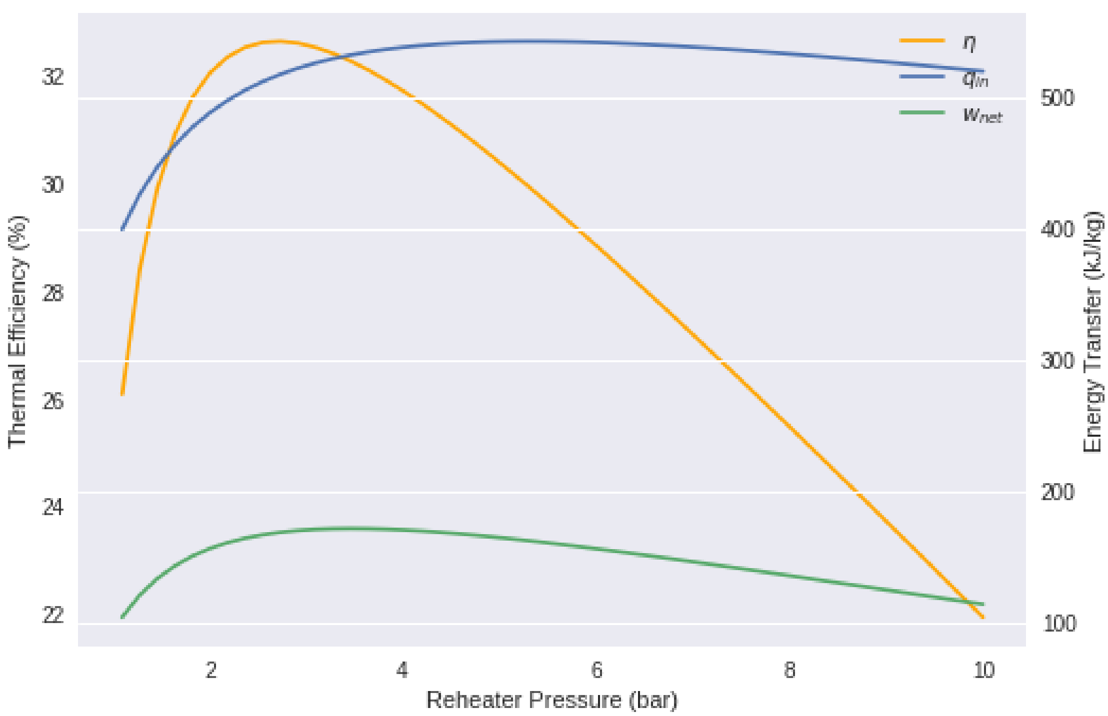
Use the code below, and fill in the function with your code from above. Be sure that your `eta`, `w_net`, and `q_in` variables are named the same as in the `return` line!

```
def calc_eta_w_q(p_re):

    p_5 = p_6 = p_re

    # TODO: FILL ME IN!

    w_net = w_net.to('kJ/kg')
    q_in = q_in.to('kJ/kg')
    eta = eta.to('percent')

    return eta, w_net, q_in


p_re_vals = linspace(1.1, 10)*units.bar
eta_vals = zeros_like(p_re_vals)*units.percent
w_net_vals = zeros_like(p_re_vals)*units.kJ/units.kg
q_in_vals = zeros_like(p_re_vals)*units.kJ/units.kg

for i, p_re in enumerate(p_re_vals):
    eta_vals[i], w_net_vals[i], q_in_vals[i] = calc_eta_w_q(p_re)

plt.style.use('seaborn')
fig, eta_ax = plt.subplots()
eta_ax.plot(p_re_vals, eta_vals, color='orange', label='$\eta$')
eta_ax.set_xlabel('Reheater Pressure (bar)')
eta_ax.set_ylabel('Thermal Efficiency (%)')
eta_ax.grid()

energy_ax = eta_ax.twinx()
energy_ax.plot(p_re_vals, q_in_vals, label='$q_{in}$')
energy_ax.plot(p_re_vals, w_net_vals, label='$w_{net}$')
energy_ax.set_ylabel('Energy Transfer (kJ/kg)')
```

```
lines, labels = eta_ax.get_legend_handles_labels()
lines2, labels2 = energy_ax.get_legend_handles_labels()
eta_ax.legend(lines + lines2, labels + labels2, loc='best');
```

[8]:
```
def calc_eta_w_q(p_re):

    p_5 = p_6 = p_re


    st_1 = State(substance, p=p_1, T=T_1)
    st_2s = State(substance, p=p2_p1*p_1, s=st_1.s)
    h_2 = (st_2s.h - st_1.h)/eta_c + st_1.h
    st_2 = State(substance, p=st_2s.p, h=h_2)

    st_6 = State(substance, p=p_6, T=T_6)
    st_7s = State(substance, p=p_7 , s=st_6.s)
    h_7 = eta_t*(st_7s.h - st_6.h) + st_6.h
    st_7 = State(substance, p=p_7, h=h_7)

    h_3 = (st_7.h - st_2.h)/eta_reg + st_2.h
    st_3 = State(substance, p=st_2.p, h=h_3)
    st_4 = State(substance, p=st_3.p, T=T_4)
    st_5s = State(substance, p=p_5, s=st_4.s)
    h_5 = eta_t*(st_5s.h - st_4.h) + st_4.h
    st_5 = State(substance, p=p_5, h=h_5)


    wdot_c = st_1.h - st_2.h
    wdot_t1 = st_4.h - st_5.h
    wdot_t2 = st_6.h - st_7.h
    w_net = wdot_c + wdot_t1 + wdot_t2


    qdot_inc = st_4.h-st_3.h
    qdot_inre = st_6.h-st_5.h
    q_in = qdot_inc + qdot_inre


    eta = w_net/q_in


    w_net = w_net.to('kJ/kg')
    q_in = q_in.to('kJ/kg')
    eta = eta.to('percent')

    return eta, w_net, q_in
```

```
p_re_vals = linspace(1.1, 10)*units.bar
eta_vals = zeros_like(p_re_vals)*units.percent
w_net_vals = zeros_like(p_re_vals)*units.kJ/units.kg
q_in_vals = zeros_like(p_re_vals)*units.kJ/units.kg

for i, p_re in enumerate(p_re_vals):
    eta_vals[i], w_net_vals[i], q_in_vals[i] = calc_eta_w_q(p_re)

plt.style.use('seaborn')
fig, eta_ax = plt.subplots()
eta_ax.plot(p_re_vals, eta_vals, color='orange', label='$\eta$')
eta_ax.set_xlabel('Reheater Pressure (bar)')
eta_ax.set_ylabel('Thermal Efficiency (%)')
eta_ax.grid()

energy_ax = eta_ax.twinx()
energy_ax.plot(p_re_vals, q_in_vals, label='$q_{in}$')
energy_ax.plot(p_re_vals, w_net_vals, label='$w_{net}$')
energy_ax.set_ylabel('Energy Transfer (kJ/kg)')

lines, labels = eta_ax.get_legend_handles_labels()
lines2, labels2 = energy_ax.get_legend_handles_labels()
eta_ax.legend(lines + lines2, labels + labels2, loc='best');
```



Part 7: Discuss Plot Trends

Answer: We see in this graph that there is a clear peak thermal efficency. This is because as reheat pressure increases, net work goes down faster than the input heat, lowering the thermal efficency. The reason net work goes down as reheat pressure goes up is because that higher pressure is still limited by the max temperature T_4 so the extra pressure is wasted.

Homework 7-3

Imports

[1]:
```python
from thermostate import Q_
from numpy import sqrt
```

Definitions

[2]:
```python
substance = 'air'

p_1 = Q_(1, 'bar')
T_1 = Q_(290, 'K')

p2_p1 = Q_(16, 'dimensionless')

T_d = Q_(290, 'K')

k = Q_(1.4, 'dimensionless')  # specific heat ratio
R_air = Q_(287.05, 'J/kg/K')
```

Problem Statement

Air enters a two-stage compressor operating at steady state at 1 bar, 290 K. The overall pressure ratio across the stages is 16, and each stage operates isentropically. Intercooling occurs at the pressure that minimizes total compressor work. Air exits the intercooler at 290 K. Assume ideal gas behavior, with $k = 1.4$, and that the compressors operate ideally.

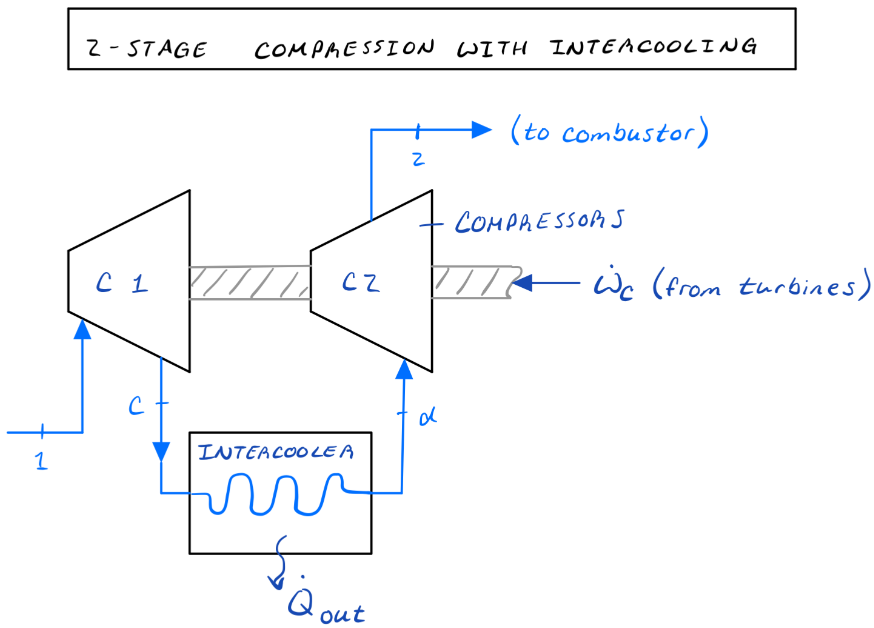
It can be shown that the intercooler pressure which minimizes the compressor work is given by

$$p_i = \sqrt{p_1 p_2 \left(\frac{T_d}{T_1}\right)^{k/(k-1)}}$$

(see Example 9.10 in the textbook for the derivation).

1. Calculate the intercooler pressure which minimizes total compressor work, in bar.
2. For an ideal gas, what is the relationship between specific enthalpy and temperature? (Just give the equation.)
3. Calculate the total work per unit mass flow rate required for the compressors, in kJ/kg.

You do not need to use Thermostate's `State` class for this problem.



Solution

Part 1: Intercooler Pressure to Minimize Total Compressor Work, in bar

Important Equations;

1. $p_i = \sqrt{p_1 p_2 \left(\frac{T_d}{T_1}\right)^{k/(k-1)}}$

[3]:
```
p_2 = p2_p1 * p_1
p_i = sqrt(p_1*p_2*(T_d/T_1)**(k/(k-1)))
print(p_i)
```

```
4.0 bar
```

Answer: 4 bar

Part 2: Relationship Between Specific Enthalpy and Temperature for an Ideal Gas

Answer: $\Delta h = \int_{T_1}^{T_2} c_p(T)dT$
If $c_p(T)$ is constant then $\Delta h = c_p \Delta T$

Part 3: Compressor Work per Unit Mass Flow Rate, in kJ/kg

Important Equations:

1. $\frac{\dot{W}_{c1}}{\dot{m}} = h_1 - h_c = c_p(T_1 - T_c)$
2. $\frac{\dot{W}_{c2}}{\dot{m}} = h_d - h_2 = c_p(T_d - T_2)$
3. $T_c = T_1(\frac{p_i}{p_1})^{\frac{k-1}{k}}$
4. $T_2 = T_d(\frac{p_2}{p_i})^{\frac{k-1}{k}}$
5. $c_p = \frac{kR}{k-1}$

[6]:
```
kp = (k-1)/k
p_2 = p_1*p2_p1
T_c = T_1*(p_i/p_1)**kp
T_2 = T_d*(p_2/p_i)**kp
c_p = k*R_air/(k-1)
wdot_c1 = c_p*(T_1-T_c)
wdot_c2 = c_p*(T_d-T_2)
wdot_in = wdot_c1+wdot_c2
print(wdot_in.to("kJ/kg").round(2))
```

```
-283.19 kilojoule / kilogram
```

Answer: 283.19 kJ/kg into the system

Homework 7-4

Imports

[1]:
```
from thermostate import Q_, State
```

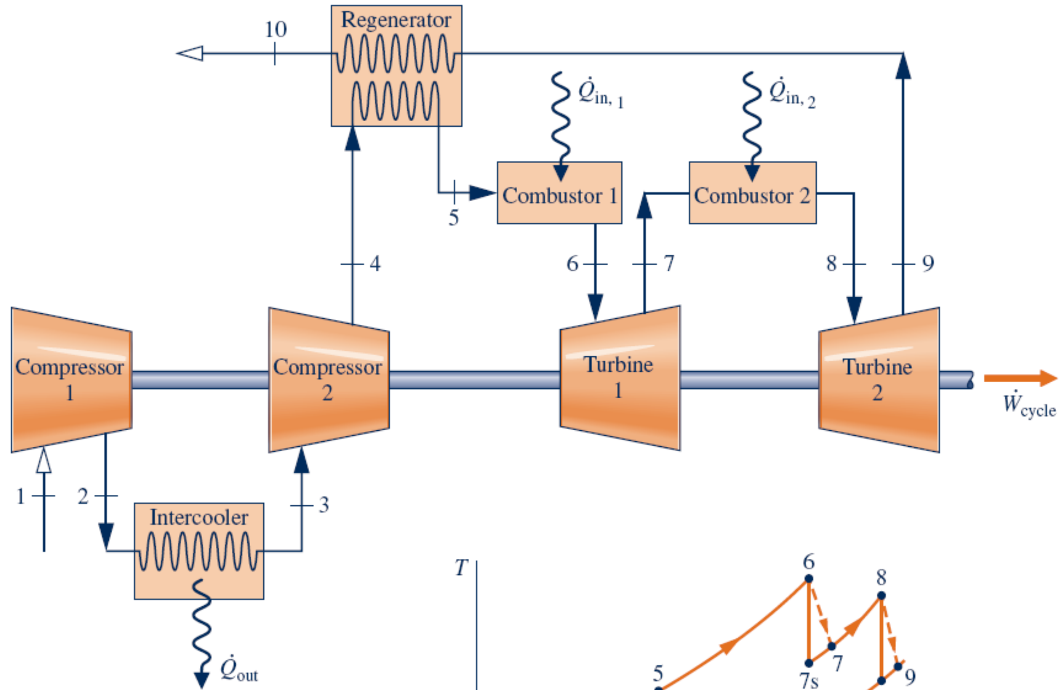
Definitions

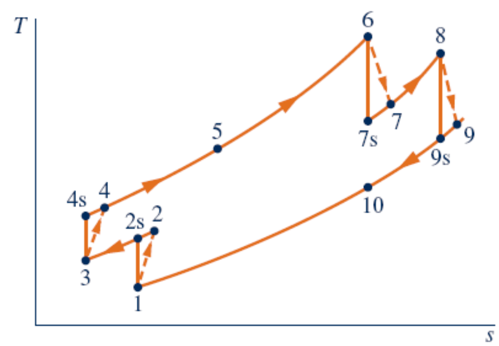
[2]:
```
substance = 'air'

p_1 = Q_(1, 'bar')
T_1 = Q_(310, 'K')

p_ratio = Q_(2, 'dimensionless')

T_3 = Q_(310, 'K')

T_6 = Q_(1100, 'K')

T_8 = Q_(1100, 'K')

eta_c = Q_(0.8, 'dimensionless')
eta_t = Q_(0.85, 'dimensionless')
eta_reg = Q_(0.8, 'dimensionless')
```

Problem Statement

An air-standard Brayton cycle with intercooling, reheat, and regeneration operates at steady state as shown in the system diagram below. The inlet temperature to both compressors is 310 K, and the inlet pressure to the first compressor is 1 bar. The inlet temperature to both turbines is 1100 K. The pressure ratio across each compressor and turbine stage is 2. Each compressor stage has an isentropic efficiency of 80%, and each turbine stage has an isentropic efficiency of 85%, and the regenerator has an effectiveness of 80%.

1. Calculate the thermal efficiency of the cycle.





Solution

Fixing States:

State	1	2
1	p_1	T_1
2s	p_2	$s_{2s} = s_1$
2	p_2	h_2
3	$p_3 = p_2$	T_3
4s	p_4	$s_{4s} = s_3$
4	p_4	h_4
5	$p_5 = p_4$	h_5
6	$p_6 = p_5$	T_6
7s	p_7	$s_{7s} = s_6$
7	p_7	h_7
8	$p_8 = p_7$	T_8
9s	p_9	$s_{9s} = s_8$
9	p_9	h_9

Efficency Equations:

1. $h_2 = (h_{2s} - h_1)/\eta_c + h_1$
2. $h_4 = (h_{4s} - h_3)/\eta_c + h_3$
3. $h_5 = (h_9 - h_4)\eta_{reg} + h_4$
4. $h_7 = \eta_t(h_{7s} - h_6) + h_6$
5. $h_9 = \eta_t(h_{9s} - h_8) + h_8$

[3]:
```
st_1 = State(substance, p=p_1, T=T_1)
p_2 = p_1*p_ratio
st_2s = State(substance, p=p_2, s=st_1.s)
h_2 = (st_2s.h-st_1.h)/eta_c + st_1.h
st_2 = State(substance, p=p_2, h=h_2)
st_3 = State(substance, p=p_2, T=T_3)
p_4 = st_3.p*p_ratio
st_4s = State(substance, p=p_4, s=st_3.s)
h_4 = (st_4s.h-st_3.h)/eta_c + st_3.h
st_4 = State(substance, p=p_4, h=h_4)

st_6 = State(substance, p=p_4, T=T_6)
p_7 = st_6.p/p_ratio
st_7s = State(substance, p=p_7, s=st_6.s)
h_7 = eta_t*(st_7s.h-st_6.h)+st_6.h
st_7 = State(substance, p=p_7, h=h_7)
st_8 = State(substance, p=p_7, T=T_8)
p_9 = st_8.p/p_ratio
st_9s = State(substance, p=p_9, s=st_8.s)
h_9 = eta_t*(st_9s.h-st_8.h)+st_8.h
st_9 = State(substance, p=p_9, h=h_9)

h_5 = (st_9.h-st_4.h)*eta_reg + st_4.h
st_5 = State(substance, p=p_4, h=h_5)
```

Part 1: Cycle Thermal Efficiency

Important Equations:

1. $\eta = \frac{\dot{W}_{net}/\dot{m}}{\dot{Q}_{in}/\dot{m}}$
2. $\frac{\dot{W}_{c1}}{\dot{m}} = h_1 - h_2$
3. $\frac{\dot{W}_{c2}}{\dot{m}} = h_3 - h_4$
4. $\frac{\dot{W}_{t1}}{\dot{m}} = h_6 - h_7$
5. $\frac{\dot{W}_{t2}}{\dot{m}} = h_8 - h_9$
6. $\frac{\dot{Q}_{in1}}{\dot{m}} = h_6 - h_5$

7. $\frac{\dot{Q}_{in2}}{\dot{m}} = h_8 - h_7$

[4]:
```
wdot_c1 = st_1.h-st_2.h
wdot_c2 = st_3.h-st_4.h
wdot_t1 = st_6.h-st_7.h
wdot_t2 = st_8.h-st_9.h
qdot_in1 = st_6.h-st_5.h
qdot_in2 = st_8.h-st_7.h
wdot_net = wdot_c1 + wdot_c2 + wdot_t1 + wdot_t2
qdot_in = qdot_in1 + qdot_in2
eta = wdot_net/qdot_in
print(eta)
```

```
0.3717311304357898 dimensionless
```

Answer: 37.17%

Homework 7-5

Imports

[1]:
```
from thermostate import Q_
```

Definitions

[2]:
```
# ----- GAS TURBINE CYCLE -----
sub_1 = 'air'

Qdot_in = Q_(50, 'MW')

p_1 = Q_(1, 'bar')
T_1 = Q_(25, 'degC')
h_1 = Q_(298.2, 'kJ/kg')

p_2 = Q_(14, 'bar')
h_2 = Q_(691.4, 'kJ/kg')

p_3 = p_2
T_3 = Q_(1250, 'degC')
h_3 = Q_(1663.9, 'kJ/kg')

p_4 = Q_(1, 'bar')
h_4 = Q_(923.2, 'kJ/kg')

p_5 = p_4
T_5 = Q_(200, 'degC')
h_5 = Q_(475.3, 'kJ/kg')

# ----- VAPOR POWER CYCLE -----
sub_2 = 'water'

p_6 = Q_(125, 'bar')
h_6 = Q_(204.5, 'kJ/kg')

p_7 = p_6
T_7 = Q_(500, 'degC')
h_7 = Q_(3341.8, 'kJ/kg')

p_8 = Q_(0.1, 'bar')
h_8 = Q_(2175.6, 'kJ/kg')
```

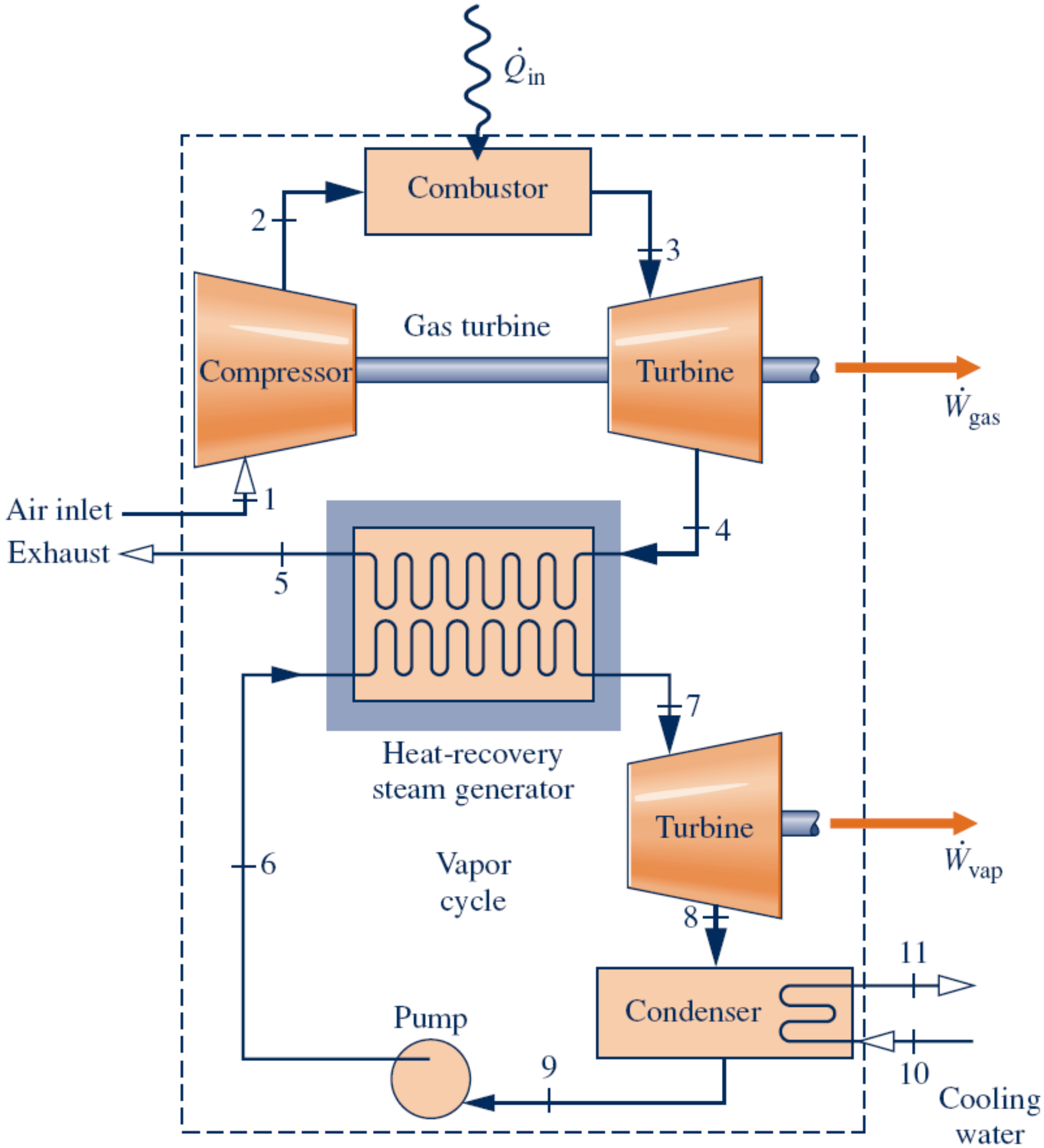
```python
p_9 = p_8
h_9 = Q_(191.8, 'kJ/kg')

T_10 = Q_(20, 'degC')
h_10 = Q_(84.0, 'kJ/kg')

T_11 = Q_(35, 'degC')
h_11 = Q_(146.7, 'kJ/kg')
```

Problem Statement

A combined gas turbine-vapor power plant operates at steady state as in the system diagram below. Important data at each state are given in the table below. Air receives heat input at a rate of 50 MW in the combustor of thee gas turbine plant. Except for the combustor and the condenser, all components operate adiabatically.

1. Find the mass flow rate of the air, steam, and cooling water, each in kg/s.
2. Find the net power developed by the gas turbine cycle and the vapor power cycle, separately, each in MW.
3. Find the thermal efficiency of the combined cycle.

You don't need to use Thermostate's `State` class for this problem.


$\dot{Q}_{in}$
Combustor
2
3
Gas turbine
Compressor
Turbine
$\dot{W}_{gas}$
Air inlet
1
4
Exhaust
5
7
Heat-recovery
steam generator
Turbine
$\dot{W}_{vap}$
6
Vapor
cycle
8
11
Condenser
Pump
9
10
Cooling
water

State	p (bar)	T (°C)	h (kJ/kg)
1	1	25	298.2
2	14	—	691.4
3	14	1250	1663.9
4	1	—	923.2
5	1	200	475.3
6	125	—	204.5
7	125	500	3341.8
8	0.1	—	2175.6
9	0.1	—	191.8
10	—	20	84.0
11	—	35	146.7

Solution

Part 1: Mass Flow Rates of Air, Steam, and Cooling Water, in kg/s

Important Equations:

1. $\dot{Q}_{in} = \dot{m}_{air}(h_3 - h_2) \quad \Rightarrow \quad \dot{m}_{air} = \frac{\dot{Q}_{in}}{h_3 - h_2}$
2. $\dot{m}_{air}h_4 + \dot{m}_{steam}h_6 = \dot{m}_{air}h_5 + \dot{m}_{steam}h_7 \quad \Rightarrow \quad \dot{m}_{steam} = \frac{\dot{m}_{air}(h_4 - h_5)}{h_7 - h_6}$
3. $\dot{m}_{steam}h_8 + \dot{m}_{cooling}h_{10} = \dot{m}_{steam}h_9 + \dot{m}_{cooling}h_{11} \quad \Rightarrow \quad \dot{m}_{cooling} = \frac{\dot{m}_{steam}(h_8 - h_9)}{h_{11} - h_{10}}$

[3]:
```
mdot_air = Qdot_in/(h_3-h_2)
print(mdot_air.to("kg/s").round(2))
mdot_steam = mdot_air*(h_4-h_5)/(h_7-h_6)
print(mdot_steam.to("kg/s").round(2))
mdot_cooling = mdot_steam*(h_8-h_9)/(h_11-h_10)
print(mdot_cooling.to("kg/s").round(2))
```

```
51.41 kilogram / second
7.34 kilogram / second
232.24 kilogram / second
```

Answer: Air: 51.41 kg/s
Steam: 7.34 kg/s
Cooling Water: 232.24 kg/s

Part 2: Net Power of Each Cycle Separately, Each in MW

Important Equations:

1. $\dot{W}_{cycle:gas} = \dot{m}_{air}((h_3 - h_4) + (h_1 - h_2))$
2. $\dot{Q}_{in:gas} = \dot{m}_{air}(h_3 - h_2)$
3. $\dot{W}_{cycle:vap} = \dot{m}_{steam}((h_7 - h_8) + (h_9 - h_6))$
4. $\dot{Q}_{in:vap} = \dot{m}_{steam}(h_7 - h_6)$

[5]:
```
Wdot_gas = mdot_air * ((h_3-h_4)+(h_1-h_2))
Qdot_gas = mdot_air * (h_3-h_2)
Wdot_vap = mdot_steam * ((h_7-h_8)+(h_9-h_6))
print(Wdot_gas.to("MW").round(2))
print(Wdot_vap.to("MW").round(2))
print(mdot_steam*(h_7-h_6))
print(mdot_air*(h_5-h_4))
```

```
17.87 megawatt
8.47 megawatt
23.028277634961437 megawatt
-23.028277634961437 megawatt
```

Answer: Gas cycle: 17.87 MW
Vapor cycle: 8.47 MW

Part 3: Thermal Efficiency of Combined Cycle

Important Equations:

1. $\eta = \frac{\dot{W}_{cycle:gas} + \dot{W}_{cycle:vap}}{\dot{Q}_{in}}$

[ ]:
```
eta = (Wdot_gas+Wdot_vap)/Qdot_gas
print(eta)
```

Answer: 52.67%

Homework 7-6

Imports

[1]:
```
from thermostate import Q_, State
from numpy import sqrt
```

Definitions

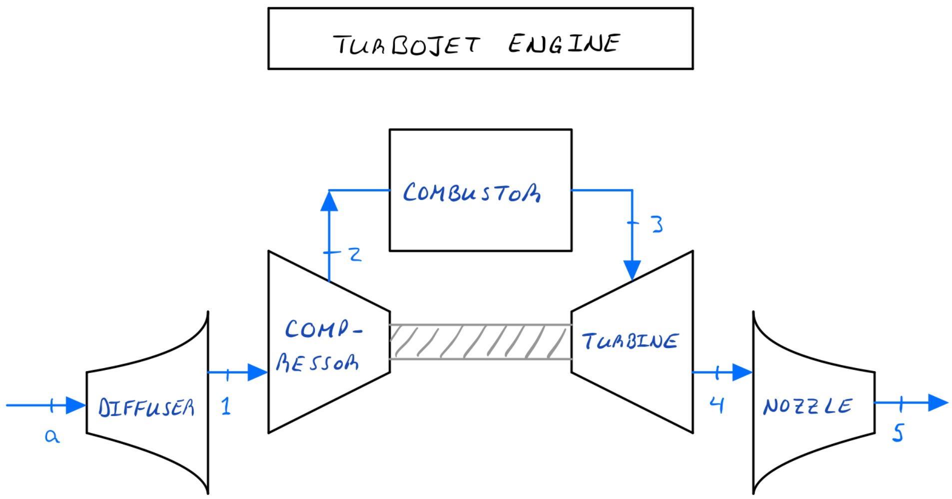
[2]:
```
substance = 'air'

mdot = Q_(25, 'kg/s')

p_a = Q_(26, 'kPa')
T_a = Q_(230, 'K')
Vel_a = Q_(220, 'm/s')

p2_p1 = Q_(11, 'dimensionless')

T_3 = Q_(1400, 'K')

p4_p3 = Q_(2.5, 'dimensionless')

p_5 = Q_(26, 'kPa')

eta_c = Q_(0.85, 'dimensionless')
eta_t = Q_(0.9, 'dimensionless')
```

Problem Statement

After getting sick of being stranded on planets all the time, the actor playing Commander Jordan Forge of the USS Enterprise has resigned from the show and begun working in the Star Worse universe, a cheap fan-made knockoff TV show, playing a character conveniently named Commander Jordan Forge of the USS Enterskies. However, in this universe, starships don't exist, so the USS Enterskies is an aircraft carrier. It's all very confusing, and legal proceedings regarding copyright infringement are in progress.

Forge is analyzing the aircraft whose system diagram is shown below. Air at 26 kPa, 230 K, and 220 m/s enters the turbojet engine in flight. The air mass flow rate is 25 kg/s. The compressor pressure ratio is 11, the turbine pressure ratio is 2.5, the turbine inlet temperature is 1400 K, and air exits the nozzle at 26 kPa. The diffuser and nozzle processes are isentropic, the compressor and turbine have isentropic efficiencies of 85% and 90%, respectively, and there is no pressure drop for flow through the combustor. Kinetic energy is negligible everywhere except at the diffuser inlet and the nozzle exit. On the basis of air-standard analysis, determine

1. the pressure (kPa) and temperature (K) at each of the states,
2. the rate of heat addition to the air passing through the combustor, in MW,
3. the velocity at nozzle exit, in m/s.



Solution

Part 1: Pressure (bar) and Temperature (K) at Each State

Important Equations:

1. $(h_1 - h_a) + \frac{1}{2}(Vel_1^2 - Vel_a^2) = 0 \qquad Vel_1 = 0\frac{m}{s}$

[6]:
```
st_a = State(substance, p=p_a, T=T_a)
h_1 = -0.5*(-Vel_a**2)+st_a.h
st_1 = State(substance, h=h_1, s=st_a.s)
p_2 = st_1.p*p2_p1
st_2s = State(substance, p=p_2, s=st_1.s)
h_2 = (st_2s.h-st_1.h)/eta_c + st_1.h
st_2 = State(substance, p=p_2, h=h_2)
st_3 = State(substance, p=p_2, T=T_3)
p_4 = st_3.p*p4_p3
st_4s = State(substance, p=p_4, s=st_3.s)
h_4 = eta_t*(st_4s.h-st_3.h)+st_3.h
st_4 = State(substance, p=p_4, h=h_4)
st_5 = State(substance, p=p_5, s=st_4.s)
```

```
print(st_a.p.to("kPa").round(2),st_a.T.to("K").round(2))
print(st_1.p.to("kPa").round(2),st_1.T.to("K").round(2))
print(st_2.p.to("kPa").round(2),st_2.T.to("K").round(2))
print(st_3.p.to("kPa").round(2),st_3.T.to("K").round(2))
print(st_4.p.to("kPa").round(2),st_4.T.to("K").round(2))
print(st_5.p.to("kPa").round(2),st_5.T.to("K").round(2))
```

```
26.0 kilopascal 230.0 kelvin
36.85 kilopascal 254.15 kelvin
405.32 kilopascal 544.96 kelvin
405.32 kilopascal 1400.0 kelvin
1013.29 kilopascal 1704.42 kelvin
26.0 kilopascal 684.99 kelvin
```

Answer:

a. 26.0 kPa 230.0 kelvin
1. 36.85 kPa 254.15 kelvin
2. 405.32 kPa 544.96 kelvin
3. 405.32 kPa 1400.0 kelvin
4. 1013.29 kPa 1704.42 kelvin
5. 26.0 kPa 684.99 kelvin

Part 2: Rate of Heat Transfer Into the Combustor, in MW

Important Equations:

1. $\dot{Q}_{in} = \dot{m}(h_3 - h_2)$

[4]:
```
Qdot_in = mdot*(st_3.h-st_2.h)
print(Qdot_in.to("MW").round(2))
```

```
24.16 megawatt
```

Answer: 24.16 MW

Part 3: Velocity at Nozzle Exit, in m/s

Imporant Equations:

1. $(h_5 - h_4) + \frac{1}{2}(Vel_5^2 - Vel_4^2) = 0 \qquad Vel_4 = 0\frac{m}{s}$

[5]:
```
Vel_5 = sqrt(-(st_5.h-st_4.h)*2)
print(Vel_5.to("m/s").round(2))
```

```
1542.16 meter / second
```

Answer: 1542.16 m/s