Measures of Fluid Mass and Weight

Chapter 1.4

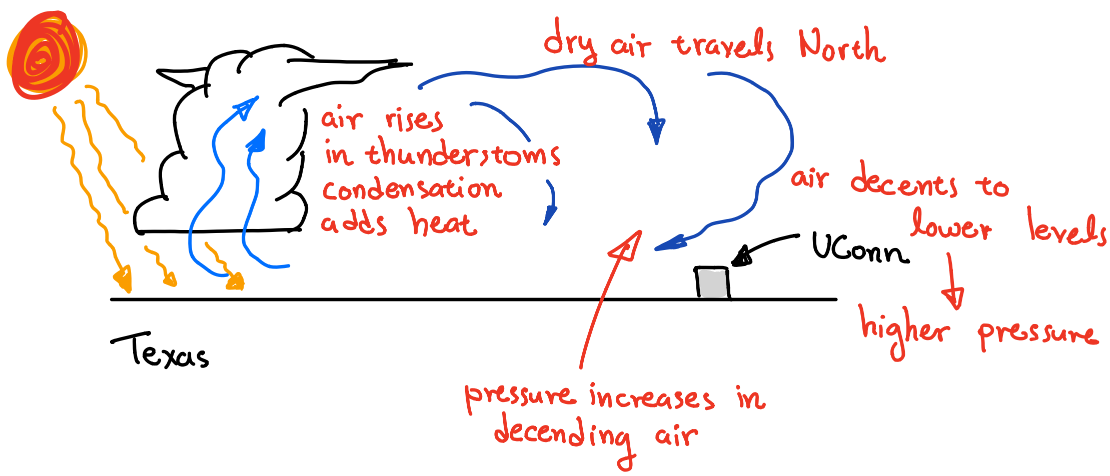
- Density ρ $\frac{\text{mass}}{\text{volume}}$ $\frac{kg}{m3}$

- Specific weight $\gamma = \rho g$ $\frac{\text{weight}}{\text{volume}}$ $\frac{N}{m3}$

- Specific gravity $SG = \frac{\rho}{\rho_{H_2O\ @4°C}}$ No units, non-dimensional

Ideal gas equation of state

- Real life example: atmospheric motion (simplified discussion)



relation between p and T? Such a relation is called equation of state

Ideal gas law $\frac{p}{\rho} = RT$

absolute pressure

absolute Temperature

specific gas constant

density

Idiabatic compression: $\frac{T}{p^{\frac{K-1}{K}}} = \text{const} \Rightarrow T = \text{const}\; p^{\frac{K-1}{K}}$

$$K = \frac{C_p}{C_v} \simeq 1.4 \text{ for air}$$

so... if $p \uparrow$ then $T \uparrow$

Absolute and Gage Pressure

Absolute pressure means pressure w.r.t. vaccum $\quad p=0$ means vaccume

Similar to absolute temperature

Gage pressure means pressure w.r.t. to the ambient (atmospheric) pressure

$$P_{gage} = P_{absolute} - P_{atmospheric}$$

Local "ambient" pressure

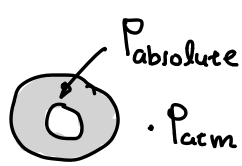
not on top of mount Everest, where you are!

- When to use what...

If doing thermodynamics (e.g. ideal gas equation) use absolute

Gage is often useful to compute stress on material

Example: Car tire



$P_g = P_{ab} - P_{atm}$: tire inflation pressure

$$P_{gage,beach} < P_{gage,mountain}$$

Viscosity   Chapter 1.6

Fluids have a resistance to flowing

↓ stress

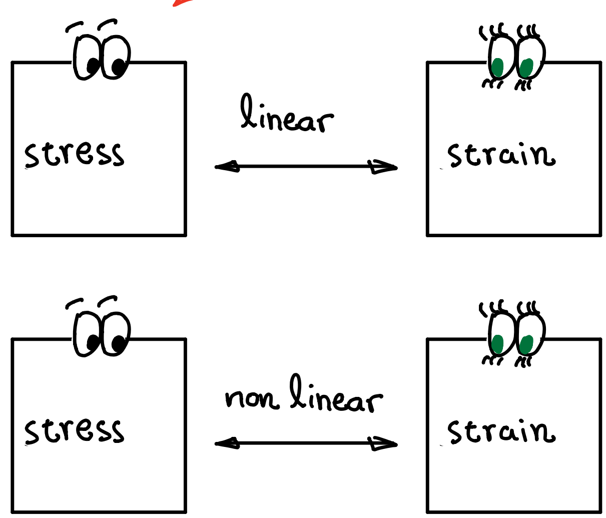
~~deformation~~ ×

~~velocity gradient~~ ×

rate of strain ✓ ← this means differential fluid motion

Question: What is the character of the resistance to flowing?

What is their relation?



Newtonian Fluid
- air
- water

Non-newtonian Fluid
- blood
- paint

Newtonian Fluid

$$\boxed{\text{stress}} = \mu \boxed{\text{strain}}$$

coefficient of viscosity $\rightarrow$ units $\dfrac{\text{force} * \text{time}}{(\text{length})^2} = \dfrac{Ns}{m^2} = Pa\,s$

or the dynamic viscosity

Another viscosity coefficient

$$\underbrace{\frac{\partial}{\partial t}(\text{momentum})}_{\text{rate of change of momentum}} = \nabla\cdot\tau + \text{other terms}$$

divide by density

$$\frac{\partial}{\partial t}(\text{velocity}) = \frac{1}{\rho}\nabla\cdot\tau + \frac{1}{\rho}(\text{other terms})$$

$$\nabla\cdot\frac{1}{\rho}\boxed{\text{stress}}$$

$$\nabla\cdot\frac{\mu}{\rho}\boxed{\text{strain}}$$

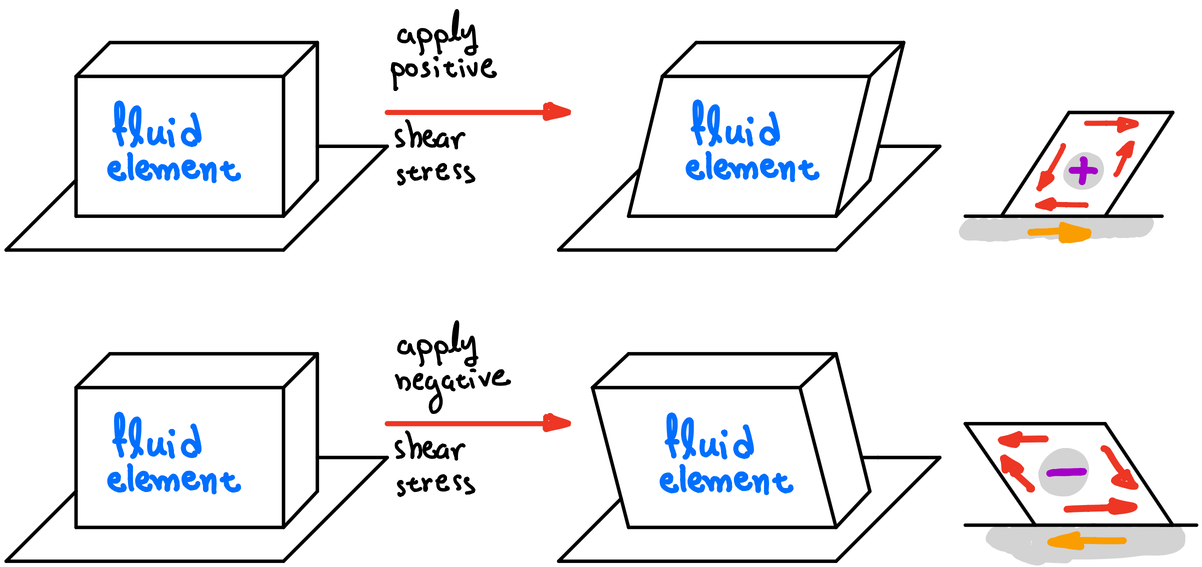
$\nu = \frac{\mu}{\rho}$: kinematic viscosity

Units : velocity * length

diffusivity coefficient of momentum

$$\frac{\partial}{\partial t}(\text{velocity}) = \nu\,\nabla\cdot\boxed{\text{strain}} + \frac{1}{\rho}(\text{other terms})$$

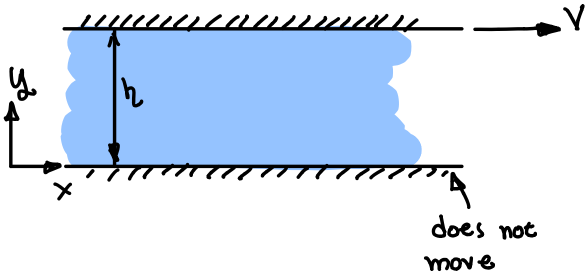
"Direction" of shear stress

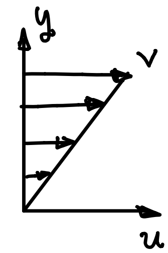


Important relations

- stress = μ * rate of strain
- force = area * stress
- torque = force * length

Example 1:





$$u(y) = \frac{y}{h} V$$

Given : $u(y)$, μ

Find : τ , Force on area = $2\,m^2$ top and bottom

$\mu = 0.01$ Pa s
$h = 0.2$ m
$V = 4$ m/s

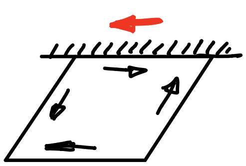
Shear stress $\underbrace{\tau = \mu \frac{\partial u}{\partial y}}$ special form for one-dimensional case

$$\tau = \mu \frac{\partial}{\partial y}\left(\frac{y}{h} V\right) = \frac{\mu}{h} V \quad \frac{N}{m^2} \text{ or Pa}$$

$$\tau = \frac{0.01}{0.2} \times 4 = 0.2 \text{ Pa}$$ ← positive

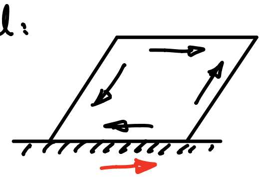
Force = $\tau \times$ area = 0.2×2 = 0.4 N

top wall:



0.4 N to the left

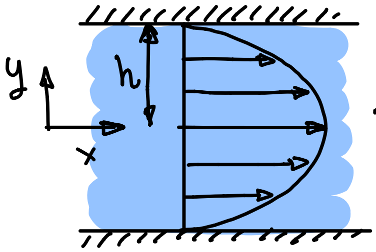
bottom wall:



0.4 N to the right

Example 2:

(example 1.5 of text book)



$$u(y) = \frac{3V}{2}\left[1 - \left(\frac{y}{h}\right)^2\right]$$

Given $u(y)$ <u>find</u> shear stress τ

$$\tau = \mu \frac{\partial u}{\partial y} = \mu \frac{\partial}{\partial y}\left(\frac{3V}{2}\left[1 - \frac{y^2}{h^2}\right]\right) = -\mu \frac{3Vy}{h^2}$$

stress depends on location

Stress in center $\tau(y=0) = 0$

bottom wall $\tau(y=-h) = -\mu \frac{3V(-h)}{h^2} = \frac{3\mu V}{h}$

top wall $\tau(y=+h) = -\mu \frac{3Vh}{h^2} = -\frac{3\mu V}{h}$