Homework 2-1

Imports

[1]:
```
from thermostate import Q_, State
```

Definitions

[2]:
```
substance = 'water'

x_1 = Q_(0.0, 'dimensionless')
p_1 = Q_(1.0, 'bar')

p_2 = Q_(50.0, 'bar')
```

Problem Statement

Water as a saturated liquid at 1.0 bar enters a pump operating at steady state and is pumped isentropically to a pressure of 50.0 bar. Kinetic and potential energy effects are negligible. Determine the pump work input, in kJ/kg of water flowing, using:

1. Eq. 6.51(c): $\left(\frac{\dot{W}}{\dot{m}}\right)_{\text{int. rev.}} = v(p_1 - p_2)$
2. an energy balance and data from Tables A-3 and A-5 in the Appendix of your textbook.

Compare the results of Parts 1 and 2 and comment on any differences or similarities.

Solution

Part 1 - Use Eq. 6.51c

Eq. 6.51(c): $\left(\frac{\dot{W}}{\dot{m}}\right)_{\text{int. rev.}} = v(p_1 - p_2)$

[3]:
```
# Write your code here to solve the problem
# Make sure to write your final answer in the cell below.
st_1 = State(substance,p=p_1,x=x_1)
v = st_1.v
work_input = v*(p_1-p_2)
print(round(work_input.to("kJ/kg"),4))
```

```
-5.1115 kilojoule / kilogram
```

Answer: -5.1115$\frac{kJ}{kg}$

Part 2 - Use an energy balance and textbook tables

For this example, we will use the fact that our process is isentropic:

1. isentropic implies entropy of st_1 is the same as entropy of st_2
2. $\frac{\dot{W}_{cv}}{\dot{m}} = (h_1 - h_2)$

[4]:
```
# Write your code here to solve the problem
# Make sure to write your final answer in the cell below.
st_1 = State(substance,p=p_1,x=x_1)
st_2 = State(substance,p=p_2,s=st_1.s)
work_input = (st_1.h - st_2.h)
print(round(work_input.to("kJ/kg"),4))
```

```
-5.106 kilojoule / kilogram
```

Answer: -5.106$\frac{kJ}{kg}$

Part 3 - Compare Parts 1 and 2

Answer: The assumption that Eq. 6.51c makes in part 1 is assuming that the specific volume does not change. This is mostly true for water which is why it only differs from part 2 by one hundredth of a $\frac{kJ}{kg}$

Homework 2-2

Imports

[1]:
```
from thermostate import Q_, State
from numpy import sqrt
```

Definitions

[2]:
```
substance = 'air'

T_1 = Q_(1190.0, 'K')
p_1 = Q_(10.8, 'bar')

p_2 = Q_(5.2, 'bar')

p_3 = Q_(0.8, 'bar')

eta_t = Q_(0.85, 'dimensionless')
```

Problem Statement

Air enters the turbine of a jet engine at 1190.0 K, 10.8 bar, and expands to 5.2 bar. The air then flows through a nozzle and exits at 0.8 bar. Operation is at steady state, and the flow is adiabatic. The nozzle operates with no internal irreversibilities, and the isentropic turbine efficiency is 0.85. The air velocity is negligible at both the turbine's inlet and outlet, and the effects of potential energy can be neglected.

Assuming the ideal gas model for the air, determine the velocity of the air exiting the nozzle, in m/s.

Solution

Part 1 - Determine air exit velocity

Important equations:

1. $(h_3 - h_2) + \frac{1}{2}(v_{el3}^2 - v_{el2}^2) = 0 \Rightarrow v_{el3} = \sqrt{v_{el2} - 2(h_3 - h_2)}$
2. $s_2 = s_3$
3. $\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_t(h_1 - h_{2s})$

$h_{2s}$is the enthalpy of state 2 assuming no change in entropy while h_2 is the real enthalpy from eq. 3

We can assume v_{el2} is 0, neglegable

[3]:
```
# Write your code here to solve the problem
# Make sure to write your final answer in the cell below.
substance = "air"
st_1 = State(substance, T=T_1, p=p_1)
st_2s = State(substance, p=p_2, s=st_1.s)
h_2 = st_1.h - eta_t * (st_1.h - st_2s.h)
st_2 = State(substance, p=p_2, h=h_2)
st_3 = State(substance, p=p_3, s=st_2.s)
vel3 = sqrt(0-2*(st_3.h-st_2.h))
print(round(vel3.to("m/s"),4))
```

```
934.2466 meter / second
```

Answer: $934.2466 \frac{m}{s}$

Homework 2-3

Imports

[1]:
```
from thermostate import Q_, State
```

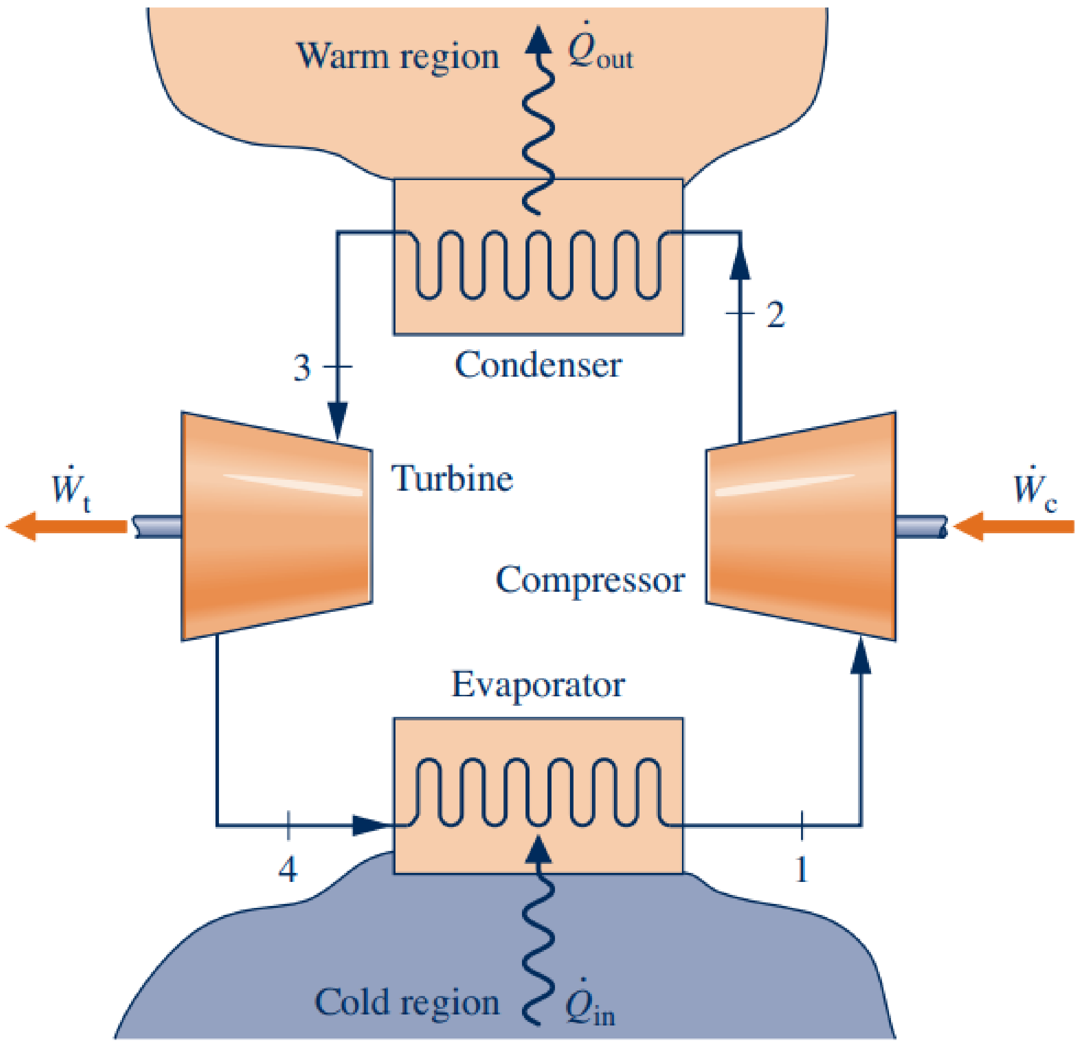
Definitions

[2]:
```
substance = 'ammonia'

T_1 = Q_(10.0, 'degF')

T_2 = Q_(120.0, 'degF')
x_2 = Q_(1.0, 'dimensionless')

x_3 = Q_(0.0, 'dimensionless')
```

Problem Statement

As shown in the figure below, a Carnot heat pump cycle operates at steady state with ammonia as the working fluid. The condenser temperature is 120.0 °F, with saturated vapor entering and saturated liquid leaving. The evaporator temperature is 10.0 °F.

1. Determine the heat transfer and work for each of the processes, in BTU/lb of ammonia flowing.
2. Evaluate the coefficient of performance, γ, of the heat pump cycle.
3. Evaluate the coefficient of performance, β, of a Carnot refrigeration cycle operating with the same states and processes.



Solution

Part 1 - The work and heat transfer for each process

Important equations:

1. $\frac{\dot{Q}_{2-3}}{\dot{m}} = (h_2 - h_3)$
2. $\frac{\dot{Q}_{4-1}}{\dot{m}} = (h_4 - h_1)$
3. $\frac{\dot{W}_{1-2}}{\dot{m}} = (h_1 - h_2)$
4. $\frac{\dot{W}_{3-4}}{\dot{m}} = (h_3 - h_4)$

Carnot Cycle Assumption:

1. $T_2 = T_3$ and $T_4 = T_1$
2. no entropy production, all processes are reversible

3. Turbine and pump do not give off heat

[3]:
```
# Write your code here to solve the problem
# Make sure to write your final answer in the cell below.
st_2 = State(substance, T=T_2, x=x_2)
st_3 = State(substance, T=T_2, x=x_3)
st_4 = State(substance, T=T_1, s=st_3.s)
st_1 = State(substance, T=T_1, s=st_2.s)
q_23 = st_2.h-st_3.h
q_41 = st_4.h-st_1.h
w_12 = st_1.h-st_2.h
w_34 = st_3.h-st_4.h
print(round(w_12.to("BTU/lb"),4))
print(round(q_23.to("BTU/lb"),4))
print(round(w_34.to("BTU/lb"),4))
print(round(q_41.to("BTU/lb"),4))
```

```
-100.1646 btu / pound
454.2431 btu / pound
13.966 btu / pound
-368.0445 btu / pound
```

Answer:
$(\frac{\dot{W}_C}{\dot{m}}) = 100.1646\frac{BTU}{lb}$ into the system
$(\frac{\dot{Q}_{out}}{\dot{m}}) = 454.2431\frac{BTU}{lb}$ out of the system
$(\frac{\dot{W}_t}{\dot{m}}) = 13.966\frac{BTU}{lb}$ out of the system
$(\frac{\dot{Q}_{in}}{\dot{m}}) = 368.0445\frac{BTU}{lb}$ into the the system

Part 2 - The COP of the heat pump cycle

Important equations:

1. $\gamma = \frac{Q_H}{Q_H - Q_C}$

[4]:
```
# Write your code here to solve the problem
# Make sure to write your final answer in the cell below.
gamma = q_23/(q_23-(-q_41))
print(round(gamma,4))
```

```
5.2697 dimensionless
```

Answer: $\gamma = 5.2697$

Part 3 - The COP of the refrigeration cycle

Important equations:

1. $\beta = \frac{Q_C}{Q_H - Q_C}$

[5]:
```
# Write your code here to solve the problem
# Make sure to write your final answer in the cell below.
beta = -q_41/(q_23-(-q_41))
print(round(beta,4))
```

```
4.2697 dimensionless
```

Answer: $\beta = 4.2697$

Homework 2-4

Imports

[1]:
```
from thermostate import Q_, State
```

Definitions

[2]:
```
substance = 'air'

mdot = Q_(5.8, 'kg/s')

p_1 = Q_(1, 'bar')
T_1 = Q_(300, 'K')

p_2 = Q_(10, 'bar')

p_3 = Q_(10, 'bar')
T_3 = Q_(1400, 'K')

p_4 = Q_(1, 'bar')

eta_c = Q_(0.8, 'dimensionless')
eta_t = Q_(0.9, 'dimensionless')
```

Problem Statement

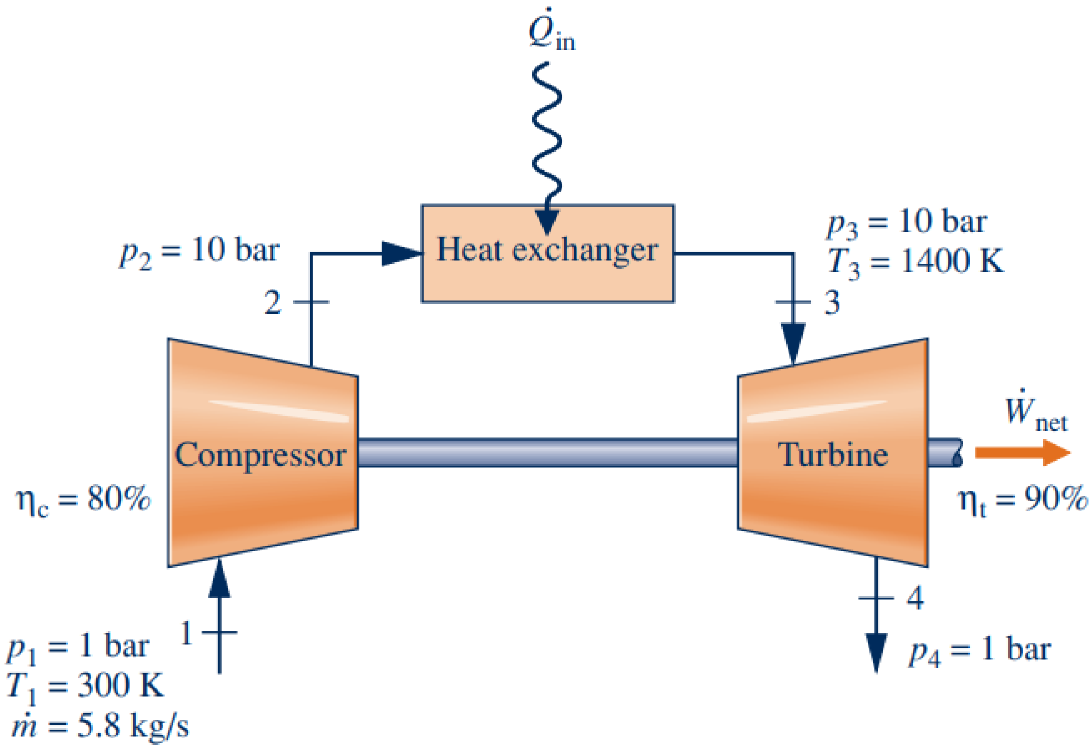
Recently promoted Commander Jordan Forge, chief engineer of the USS Enterskies, is trapped on the surface of a planet with high volcanic activity. His launch vehicle has been destroyed, but he thinks he has managed create a power system for the transporter using only a mirror, some string, and a spare power system, shown in the figure below.

The power system operates at steady state, and consists of three components in series: an air compressor, a heat exchanger, and a turbine. Air enters the compressor with a mass flow rate of 5.8 kg/s, at 1 bar, 300 K, and exits at 10 bar. The mirror reflects sunlight into the air via a heat exchanger, and the air exits the heat exchanger at 10 bar, 1400 K. The air exits the turbine at 1 bar. The compressor efficiency is 80 percent, and the turbine efficiency is 90 percent.

Air can be modeled as an ideal gas, and kinetic and potential energy effects can be neglected. Determine, in kW,

1. the power required by the compressor,
2. the power developed by the turbine,
3. the *net* power output of the overall power system.

If Jordan's transporter requires 1500 kW to beam him back up to the Enterskies, can he make a safe escape from the volcanos?



Solution

Part 1 - Power required by compressor

Important equations:

1. $\eta_c = \frac{h_{2s}-h_1}{h_2-h_1} \Rightarrow h_2 = \frac{h_{2s}-h_1}{\eta_c} + h_1$
2. $\dot{W}_{1-2} = \dot{m}(h_1 - h_2)$

[3]:
```
# Write your code here to solve the problem
# Make sure to write your final answer in the cell below.
st_1 = State(substance, p=p_1, T=T_1)
st_2s = State(substance, p=p_2, s=st_1.s)
h_2 = (st_2s.h-st_1.h)/eta_c + st_1.h
st_2 = State(substance, p=p_2, h=h_2)
power_12 = mdot * (st_1.h-st_2.h)
print(round(power_12.to("kW"),4))
```

```
-2031.0924 kilowatt
```

Answer: 2031.0924 kW into the compressor

Part 2 - Power developed by turbine

Important equations:

1. $\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} \Rightarrow h_4 = (h_{4s} - h_3) \cdot \eta_t + h_3$
2. $\dot{W}_{3-4} = \dot{m}(h_3 - h_4)$

[4]:
```python
# Write your code here to solve the problem
# Make sure to write your final answer in the cell below.
st_3 = State(substance, p=p_3, T=T_3)
st_4s = State(substance, p=p_4, s=st_3.s)
h_4 = (st_4s.h-st_3.h)*eta_t + st_3.h
st_4 = State(substance, p=p_4, h=h_4)
power_34 = mdot * (st_3.h-st_4.h)
print(round(power_34.to("kW"),4))
```

```
3693.7747 kilowatt
```

Answer: 3693.7747 kW out from the turbine

Part 3 - Net power output of the system

$\dot{W}_{net} = \dot{W}_{out} - \dot{W}_{in}$

[5]:
```python
power_net = power_34-(-power_12)
print(round(power_net.to("kW"),4))
```

```
1662.6824 kilowatt
```

Answer: 1662.6824 kW

Part 4 - Can Jordan escape?

[6]:
```python
power_net >= Q_(1500, "kW")
```

[6]:
```
True
```

Answer: Commander Jordan Forge CAN escape!

Homework 2-5

Problem Statement

Do this problem by hand.

In certain components of propulsion engines, such as nozzles, compressors, and turbines, the residence time of the fluid within the component is so short that the heat transfer per unit mass of fluid is negligible compared to the specific work or the change in kinetic energy.

For such adiabatic flows, the $T-s$ diagram immediately indicates the range of possible states that can be reached from any initial point (since $ds \geq 0$).

Show that on a temperature-entropy graph, constant-pressure lines for ideal gases must be concave upward, given that C_p must be positive. Show that lines of constant volume must also be must also be concave upward and have slopes that, at any given pair of T and s values, are steeper than those for constant pressure lines. What shape will constant-enthalpy lines have on the $T-s$ graph?

Hint: Take the derivatives of the ds equations for ideal gases found in your textbook.

Solution

Part 1 - Show Constant Pressure Lines Concave Upward

In the markdown cell below, either type your derivation using Markdown or include a high-quality picture/scan of your handwritten derivation.

Proof:

Equation 6.18 from the book states: $s(T_2,p_2) - s(T_1,p_1) = \int_{T_1}^{T_2} c_p(T)\frac{dT}{T} - Rln(\frac{p_2}{p_1})$

We will first claim $p_1 = p_2$ so pressure is constant and just looking at a small temperature interval so $c_p(t) = c_p$, now: $s_2(T_2) - s_1(T_1) = c_p ln(\frac{T_2}{T_1}) - Rln(1)$

Since the graph of T-s has s as the independent varible we'll also rearrange the equation: $T_2 - T_1 = exp[\frac{s_2-s_1}{c_p}] \Rightarrow T(s) = exp[\frac{s-s_o}{c_p}] + T_o$where $s \geq s_o$

Lastly to see the concavity of the line, we will take 2 derivatives of the function:$\frac{dT}{ds} = \frac{1}{c_p}exp[\frac{s-s_o}{c_p}], \frac{d^2T}{ds^2} = \frac{1}{c_p^2}exp[\frac{s-s_o}{c_p}]$

As a nonrigourous way of seeing this, we can approximate the equation as: $\frac{d^2T}{ds^2} = \frac{1}{(+)}e^{(+)}$ where $(+)$ is just some positive number

we know c_p is always postive and we already stated $s > s_o$ so $\frac{s-s_o}{c_p}$ will always be positive too

So that means the second dervitive of a constant pressure T-s line will aways be positive therfore the constant pressure line will concave upward

Part 2 - Show Constant Volume Lines Concave Upward

In the markdown cell below, either type your derivation using Markdown or include a high-quality picture/scan of your handwritten derivation.

Proof:

Equation 6.17 from the book states: $s(T_2, v_2) - s(T_1, v_1) = \int_{T_1}^{T_2} c_v(T)\frac{dT}{T} + Rln(\frac{v_2}{v_1})$

We will first claim $v_1 = v_2$ so volume is constant and just looking at a small temperature interval so $c_v(t) = c_v$, now: $s_2(T_2) - s_1(T_1) = c_v ln(\frac{T_2}{T_1}) + Rln(1)$

Since the graph of T-s has s as the independent varible we'll also rearrange the equation: $T_2 - T_1 = exp[\frac{s_2 - s_1}{c_v}] \Rightarrow T(s) = exp[\frac{s - s_o}{c_v}] + T_o$where $s \geq s_o$

Lastly to see the concavity of the line, we will take 2 derivatives of the function:$\frac{dT}{ds} = \frac{1}{c_v}exp[\frac{s-s_o}{c_v}], \frac{d^2T}{ds^2} = \frac{1}{c_v^2}exp[\frac{s-s_o}{c_v}]$

As a nonrigourous way of seeing this, we can approximate the equation as: $\frac{d^2T}{ds^2} = \frac{1}{(+)}e^{(+)}$ where $(+)$ is just some positive number

we know c_v is always postive and we already stated $s > s_o$ so $\frac{s-s_o}{c_v}$ will always be positive too

So that means the second dervitive of a constant volume T-s line will aways be positive therfore the constant volume line will concave upward

Now we will compare the constant volume lines to constant pressure lines. To prove the constant volume lines have a steeper slops than constant pressure lines, we will compare their first derivatives:

$$\frac{1}{c_v}exp[\frac{s-s_o}{c_v}] \geq \frac{1}{c_p}exp[\frac{s-s_o}{c_p}]$$

Splitting each equation into 2 parts, first we know $\frac{1}{c_v} \geq \frac{1}{c_p}$ since $c_v \leq c_p$. We can also know that $e^{[\frac{s-s_o}{c_v}]} \geq e^{[\frac{s-s_o}{c_p}]}$ for the same reason. Since the derivative of the constant volume equation is larger than the derivative for the constant pressure equation, the constant volume line must be steeper.

Part 3 - Determine Shape of Constant Enthalpy Lines

In the markdown cell below, either type your derivation using Markdown or include a high-quality picture/scan of your handwritten derivation.

For ideal gasses, we know that enthalpy is only a function of one varible, Temperature. Constant enthalpy implies $h_1 = h_2 \Rightarrow \Delta h = 0$

We also know $\Delta h = \int_{T_1}^{T_2} c_p(T)dT$

If $0 = \int_{T_1}^{T_2} c_p dT$ then $T_1 = T_2$

This means a line on a T-s diagram would be with a constant temperature, looking like a straight horizontal line.

Homework 2-6

Problem Statement

Do this problem by hand.

A new energy-conversion machine has been proposed that has two inflows of air. At one inlet port, 10 kg/s enters at 10 MPa and 75 °C. At the other inlet, 2 kg/s enters at 2 MPa and 1000 °C. There is a single outlet stream discharging to the atmosphere at 0.1 MPa.

No fuel enters the machine, nor are there any other inflows or outflows of material. The casing around the machine is well-insulated (so that the heat transfer between the machine and the surrounding air is negligible). Changes in kinetic and potential energy are insignificant.

Assume for the air that $k = 1.4$ and $c_p = 1.0035$ kJ/kg K.

What is the maximum power that the machine could produce in steady flow if the exhaust pressure were 0.1 MPa?

Solution

The ultimate energy balence we will be trying to solve is as follows: $0 = -\dot{W}_{cv} + \dot{m}_1(h_1) + \dot{m}_2(h_2) - (\dot{m}_e)(h_e)$

Maximum power production, maximun efficency, also makes us want to assume entropy production, $\dot{\sigma} = 0$ in: $0 = (\dot{m}_1 s_1) + (\dot{m}_2 s_2) - (\dot{m}_e s_e) + \dot{\sigma}$

Lastly, conservation of mass at steady state tells us: $\dot{m}_1 + \dot{m}_2 = \dot{m}_e$

- $\dot{m}_1 = 10\frac{kg}{s}$
- $\dot{m}_2 = 2\frac{kg}{s}$
- $\dot{m}_3 = 12\frac{kg}{s}$

We can rearrange our entropy equation now to say: $0 = \dot{m}_1(s_1 - s_e) + \dot{m}_2(s_2 - s_e)$

Now lets look into each inlet/outlet to see the properties of the air passing through:

State	$T(K)$	$h(\frac{kJ}{kg})$
1	348.15	348.63
2	1273.15	1364.126
e	124.95	124.657

We will have to use interpolation to get most of these states with the general interpolation equation being:

$$y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1$$

$h_1 = \frac{350.49 - 340.42}{350 - 340}(348.15 - 340) + 340.42$

$h_2 = \frac{1372.24-1348.55}{1280-1260}(1273.15-1260)+1348.55$

At this point I ran into a little bit of a road block, So I went though our knowns and unknowns. $0 = -\dot{W}_{cv} + \dot{m}_1(h_1) + \dot{m}_2(h_2) - (\dot{m}_e)(h_e)$ is 1 equation with 2 unknowns, h_e and $\dot{W}_{cv}$ so it is unsolvable.

$0 = \dot{m}_1(s_1 - s_e) + \dot{m}_2(s_2 - s_e)$ is 1 equation with 3 unknowns, s_1, s_2, s_e so it is also unsolvable. But there is another way to write it.

$0 = \dot{m}_1(s_1(T_1, p_1) - s_e(T_e, p_e)) + \dot{m}_2(s_2(T_2, p_2) - s_e(T_e, p_e))$

And since we are given so may pressure values and temperature values in the problem statment, this 1 equation only has 1 unknown, T_e.

We will now plug in the equations for $s_1(T_1, p_1) - s_e(T_e, p_e)$ and $s_2(T_2, p_2) - s_e(T_e, p_e)$, eq 6.22 in the book, and then we can solve for T_e.

$$0 = \dot{m}_1(c_p ln(\frac{T_1}{T_e}) - Rln(\frac{p_1}{p_e})) + \dot{m}_2(c_p ln(\frac{T_2}{T_e}) - Rln(\frac{p_2}{p_e}))$$

$$0 = \dot{m}_1 c_p ln(\frac{T_1}{T_e}) - \dot{m}_1 Rln(\frac{p_1}{p_e}) + \dot{m}_2 c_p ln(\frac{T_2}{T_e}) - \dot{m}_2 Rln(\frac{p_2}{p_e})$$

$$\dot{m}_1 Rln(\frac{p_1}{p_e}) + \dot{m}_2 Rln(\frac{p_2}{p_e}) = \dot{m}_1 c_p ln(\frac{T_1}{T_e}) + \dot{m}_2 c_p ln(\frac{T_2}{T_e})$$

$$\frac{\dot{m}_1 Rln(\frac{p_1}{p_e}) + \dot{m}_2 Rln(\frac{p_2}{p_e})}{c_p} = \dot{m}_1 ln(\frac{T_1}{T_e}) + \dot{m}_2 ln(\frac{T_2}{T_e})$$

$$\frac{\dot{m}_1 Rln(\frac{p_1}{p_e}) + \dot{m}_2 Rln(\frac{p_2}{p_e})}{c_p} = ln(\frac{T_1^{\dot{m}_1}}{T_e^{\dot{m}_1}}) + ln(\frac{T_2^{\dot{m}_2}}{T_e^{\dot{m}_2}})$$

$$\frac{\dot{m}_1 Rln(\frac{p_1}{p_e}) + \dot{m}_2 Rln(\frac{p_2}{p_e})}{c_p} = ln(\frac{T_1^{\dot{m}_1}}{T_e^{\dot{m}_1}} \cdot \frac{T_2^{\dot{m}_2}}{T_e^{\dot{m}_2}})$$

$$exp[\frac{\dot{m}_1 Rln(\frac{p_1}{p_e}) + \dot{m}_2 Rln(\frac{p_2}{p_e})}{c_p}] = \frac{T_1^{\dot{m}_1} T_2^{\dot{m}_2}}{T_e^{\dot{m}_1} T_e^{\dot{m}_2}}$$

$$T_e^{\dot{m}_1} T_e^{\dot{m}_2} = \frac{T_1^{\dot{m}_1} T_2^{\dot{m}_2}}{exp[\frac{\dot{m}_1 Rln(\frac{p_1}{p_e}) + \dot{m}_2 Rln(\frac{p_2}{p_e})}{c_p}]}$$

$$T_e^{\dot{m}_1 + \dot{m}_2} = \frac{T_1^{\dot{m}_1} T_2^{\dot{m}_2}}{exp[\frac{\dot{m}_1 Rln(\frac{p_1}{p_e}) + \dot{m}_2 Rln(\frac{p_2}{p_e})}{c_p}]}$$

$$T_e = (\frac{T_1^{\dot{m}_1} T_2^{\dot{m}_2}}{exp[\frac{\dot{m}_1 Rln(\frac{p_1}{p_e}) + \dot{m}_2 Rln(\frac{p_2}{p_e})}{c_p}]})^{\frac{1}{\dot{m}_1 + \dot{m}_2}} = 124.95K$$

Now assuming a constant c_p, we know $h_e - h_r = c_p[T_e - T_r] \Rightarrow h_e = 124.657\frac{kJ}{kg}$ where T_r and then subsequently h_r are the closest tablulated values used for a refrence.

Now we can use $0 = -\dot{W}_{cv} + \dot{m}_1(h_1) + \dot{m}_2(h_2) - (\dot{m}_e)(h_e)$ since it only has 1 varible now, $\dot{W}_{cv}$. And solving that we get

$$\dot{W}_{cv} = 4718.668kW$$