Homework 4

Nathan Stenseng

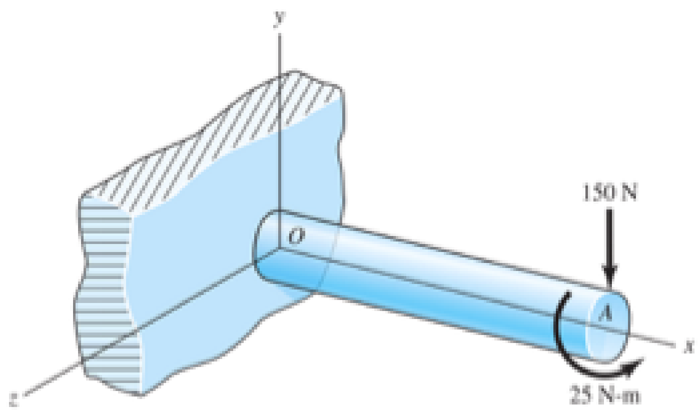
ME3227-001 - Spring 2021

Problem 5-49

Cantileverd rod OA is 0.5 m long, and made from AISI 1010 hot-rolled steel. A constant force and torque are applied as shown. Determine the minimum diameter, d, for the rod that will achieve a minimum static factor of safety of 2

(a) using the maximum-shear-stress

(b) using the distortion-energy failure theory



Solution

First we will need to find the maximum stress on the rod. Due to the point load, the largest bending stress is located along the outer most fiber, and similarly due to the torsion, the largest shear stress is in the outer most fiber. These two stresses are as follows:

$$\sigma_x = \frac{Mr}{I} \tag{1}$$

$$\tau_{zx} = \frac{Tr}{J} \tag{2}$$

Plugging in the forces and using our cross-section to calculate inertias gives:

$$\sigma_x = \frac{4 \cdot 75}{\pi r^3} \tag{3}$$

$$\tau_{zx} = \frac{2 \cdot 25}{\pi r^3} \tag{4}$$

Using a mhor circle gives us maximum stress values of:

$$center = (\frac{150}{\pi r^3}, 0) \tag{5}$$

$$Radius = \sqrt{\left(\frac{150}{\pi r^3}\right)^2 + \left(\frac{50}{\pi r^3}\right)^2} \tag{6}$$

$$\sigma_{max} = \frac{150}{\pi r^3} + Radius \tag{7}$$

$$\tau_{max} = Radius \tag{8}$$

Simplifying these equations gives:

$$\sigma_1 = \frac{1}{r^3}\left(\frac{150 + \sqrt{150^2 + 50^2}}{\pi}\right) \tag{9a}$$

$$\sigma_2 = \frac{1}{r^3}\left(\frac{150 - \sqrt{150^2 + 50^2}}{\pi}\right) \tag{9b}$$

$$\tau_{max} = \frac{1}{r^3}\left(\frac{\sqrt{150^2 + 50^2}}{\pi}\right) \tag{10}$$

From *Table A-20*, we see that 1010 hot-rolled steel has an yielding strength of 180 MPa.

Part (a)

Using the maximum-shear-stress theory, we will assume the ultimate shear stress will be:

$$S_{sy} = \frac{S_y}{2} \tag{11}$$

Now plugging our yielding strength S_y into equation 9 and S_{sy} into equation 10, will give us two radii for our rods, and we will choose the larger of the two. First though, we will half the max stresses because we want a factor of safety of 2.

[1]:
```
from thermostate import Q_,units
from numpy import pi
from numpy import sqrt

S_y = (180 * units.MPa)/2
S_sy = (S_y/2)

M = 150*units.N*units.m
T = 50*units.N*units.m

r = ((sqrt(M**2+T**2))/(pi*S_sy))**(1/3)
print(r.to("mm"))
```

```
print("Diameter is ", (r*2).to("mm").round(2))
```

```
10.380123945117276 millimeter
Diameter is  20.76 millimeter
```

According to MSS, our rod will need a diameter of

$d = 20.76$ mm

Part (b)

For this part, we will have the same tensile yeild strength but change the shear yeilding strength to:

$$S_y = \sigma' \tag{12}$$

Where:

$$\sigma' = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2} \tag{13}$$

[2]:
```
S_y = (180 * units.MPa)/2



s_1 = (M+sqrt(M**2+T**2))/pi
s_2 = (M-sqrt(M**2+T**2))/pi

r = ((sqrt(s_1**2+s_1*s_2+s_2**2))/(S_y))**(1/3)
print(r.to("mm"))

print("Diameter is ", (r*2).to("mm").round(2))
```

```
10.246121340686967 millimeter
Diameter is  20.49 millimeter
```

According to MSS, our rod will need a diameter of

$d = 20.49$ mm

Problem 5-54

Determine the mimimum factor of safety for yielding for the loading in problem 3-83. Use both the maximum-shear-stress and distortion energy theories and compare the results. The material is 1018 CD steel.

3-83 review

$\sigma_1 = 36.68$ kpsi

$\sigma_2 = -1.47$ kpsi

$\tau_{max} = 19.07$ kpsi

Solution

According to *Table A-20*, 1018 CD steel has a yielding strength of 54 kpsi.

Maximum-shear-stress theory states that:

$$S_{sy} = \frac{S_y}{2}$$

Now we will check the principle shear stresses against the shear yeilding stresses to determine the factor of safety.

[1]:
```
from thermostate import Q_, units

S_y = Q_(54, "kpsi")
S_sy = S_y/2

sigma_1 = Q_(36.68, "kpsi")
sigma_2 = Q_(-1.47, "kpsi")
tau_max = Q_(19.07, "kpsi")

n = S_sy/tau_max
print(n)
```

```
1.415836392239119 dimensionless
```

This shows that we will have a shear failure with a factor of safety of:

$\boxed{n = 1.42}$

In order to use the distortion energy failure theory, we will first need to calculate the Von Mises stress:

$$\sigma' = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$$

With this, our factor of safety will be:

$$n = \frac{S_y}{\sigma'}$$

[2]:
```
from numpy import sqrt

sigma_p = sqrt(sigma_1**2 - sigma_1*sigma_2 + sigma_2**2)
n = S_y/sigma_p
print(n)
```

```
1.4424366861077806 dimensionless
```

$n = 1.44$

Problem 5-97

A plate 100 mm wide, 200 mm long, and 12 mm thick is loaded in tension in the direction of the length. The plate contains a crack as shown in *Figure 5-26* with the crack length of 16 mm. The material is steel with $K_{Ic} = 80$ MPa$\cdot\sqrt{m}$, and $S_y = 950$ MPa. Determine the maximum possible load that can be applied before the plate:

(a) yeilds

(b) has uncontrollable crack growth

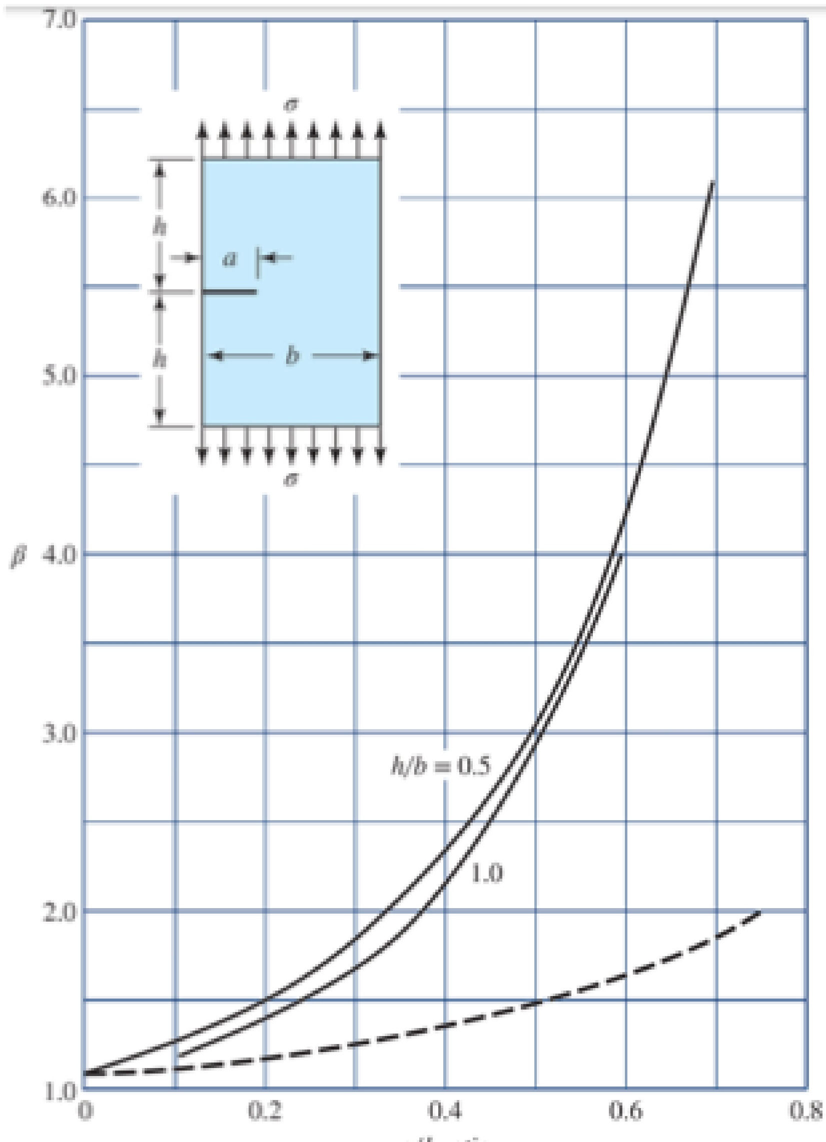


Figure 5–26 Plate loaded in longitudinal tension with a crack at the edge; for the solid curve there are no constraints to bending; the dashed curve was obtained with bending constraints added.

Solution

Part (a)

For part a, we just want to find yeilding force, so we will look at the thinnest portion of the plate, where the crack is.

$$\sigma_y = \frac{F}{A_{min}} \tag{1}$$

$$A = t(w - c_l) \tag{2}$$

Where t is plate thickness, w is the original plate width, and $w - c$ is the plate width minus the crack. Plugging these values in gives a force required for yeilding of:

[1]:
```
from thermostate import Q_, units

S_y = Q_(950, "MPa")

t = Q_(12, "mm")
w = Q_(100, "mm")
l = Q_(200, "mm")

c_l = Q_(16, "mm")

A = t*(w-c_l)
F = S_y*A
print(F.to("kN"))
```

```
957.6 kilonewton
```

The force required to begin yeilding is:

$$\boxed{F = 957.6 \text{ kN}}$$

Part (b)

Uncontrollable crack grown occurs when :

$$K_{Ic} = K_I \tag{3}$$

Also knowing the geometry of the crack we can find:

$$K_I = \beta\sigma\sqrt{\pi a} \tag{4}$$

Viewing the figure in the problem statement we will say:

$a = 16$ mm

$b = 100$ mm

$h = 100$ mm

$h/b = 1$

$a/b = 0.16$

$\beta \approx 1.3$

Now the only difference is:

$$\sigma = \frac{F}{A_{plate}}$$

Rearranging equation 4 yeilds:

$$F = \frac{K_{Ic} A_{plate}}{\beta\sqrt{\pi a}}$$

[2]:
```python
K_Ic = Q_(80, "MPa*m**(0.5)")

A = t*w
a = c_l
beta = 1.3
from numpy import pi
from numpy import sqrt

F = K_Ic*A/(beta*sqrt(pi*a))
print(F.to("kN"))
```

```
329.3767599051271 kilonewton
```

The force required to begin uncontrollable crack propagation is:

$$\boxed{F = 329.4 \text{ kN}}$$

Problem 6-10

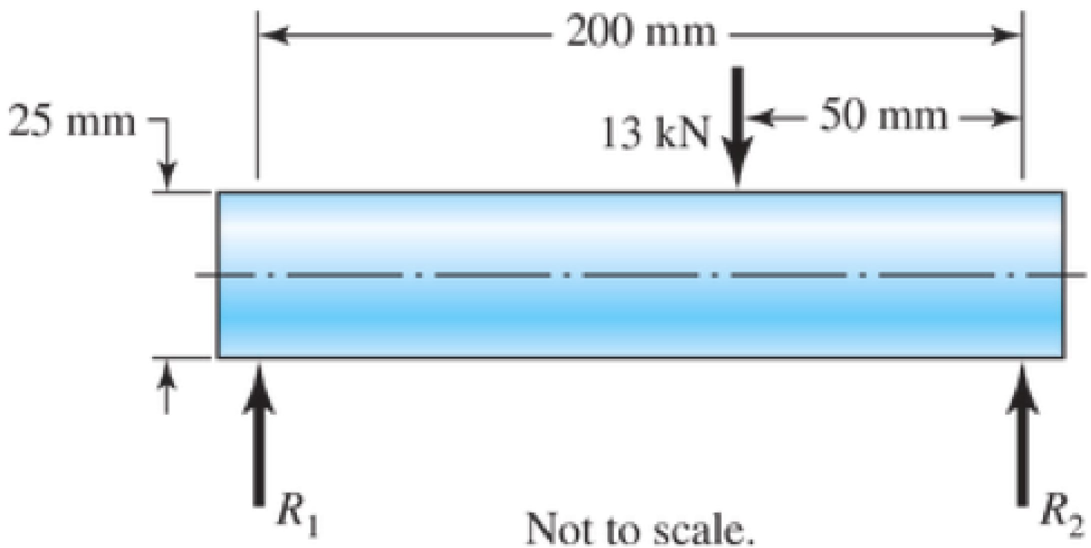
A rotating shaft of 25-mm diameter is simply supported by bearing reaction force R_1 and R_2. The shaft is loaded with a transverse load of 13 kN as shown in the figure. The shaft is made from AISI 1045 hot-rolled steel. The surface has been machined. Determine

(a) the minimum static factor of safety based on yielding

(b) the endurance limit, adjusted as necessary with Marin factors

(c) the minimum fatigue factor of safety based on achieving infinate life

(d) If the fatigue factor of safety is less than 1, then estimate the life of the part in number of rotations.



Solution

Part (a)

First we will look at the static equilibrium equations to determine the reactions:

$$\Sigma F_y : R_1 + R_2 - 13 = 0 \tag{1}$$

$$\Sigma M : 13 \cdot 50 - R_1 \cdot 200 = 0 \tag{2}$$

Solving these equations gives us values of $R_1 = 3.25$ kN and $R_2 = 9.75$ kN.

For a shaft purley in bending:

$$\sigma_1 = \frac{M_{max} r}{I} \tag{3}$$

Where the max bending moment is $M_{max} = 487.5$ kN·mm.

Looking at *Table A-20*, AISI 1045 hot-rolled steel has a yielding strength of $S_y = 310$ MPa, and an ultimate tensile strength of $S_{ut} = 570$ MPa.

For our static loading condition, the factor of safety, n is defined by:

$$n = \frac{S_y}{\sigma_1} \tag{4}$$

[1]:
```
from thermostate import Q_, units
from numpy import pi
S_y = Q_(310, "MPa")
M = Q_(487.5, "kN * mm")
d = Q_(25, "mm")
r = d/2

sigma_1 = (M*r)/(pi * r**4 / 4)
print(sigma_1.to("MPa"))
n = S_y/sigma_1
print(n.to("dimensionless"))
```

```
317.80059036589665 megapascal
0.9754544497324077 dimensionless
```

The factor of safety based on yeilding is:

$n = 0.975$

Which means that static yielding does occur.

Part (b)

The estimated endurance limit is for $S_{ut} \leq 1,400$ MPa:

$$S'_e = 0.5 S_{ut}$$

The endurance limit modifiers are: Surface factor for machined surfaces:

$$k_a = 3.04 {S_{ut}}^{-0.217}$$

Size factor for round shafts with $7.62 \leq d \leq 51$ mm:

$$k_b = 1.24 d^{-0.107}$$

Load factor for bending:

$$k_c = 1$$

Temperature factor:

$$k_d = 1$$

Reliability factor for 50% reliability:

$$k_e = 1$$

[2]:
```
S_ut = Q_(570, "MPa")
S_ep = 0.5 * S_ut
k_a = (3.04*S_ut**(-0.217)).magnitude
k_b = (1.24*d**(-0.107)).magnitude
S_e = S_ep*k_a*k_b
print(S_e)
```

```
192.10368873356663 megapascal
```

$S_e = 192.1$ MPa

Part (c)

As we saw in the previous part, our max loading is $\sigma = 317.8$ MPa, which means we will not be in the infinite life regime.

The infinite life factor of saftey is:

[3]:
```
n = S_e/sigma_1
print(n.to("dimensionless"))
```

```
0.6044787031779579 dimensionless
```

$n = 0.6$

Part (d)

Since we are not in the infinite life range, we will use Basquin's equations for completly reversible loading to calculate our lifetime:

$$N = \left(\frac{\sigma_1}{a}\right)^{1/b} \tag{5}$$

$$a = \frac{(fS_{ut})^2}{S_e} \tag{6}$$

$$b = -\frac{1}{3}log\left(\frac{fS_{ut}}{S_e}\right) \tag{7}$$

$$f = 1.06 - 4.1(10^{-4})S_{ut} + 1.5(10^{-7})S_{ut}^2 \tag{8}$$

[4]:
```python
import numpy as np
f = 1.06 - 4.1*10**(-4)*S_ut.magnitude + 1.5*10**(-7)*(S_ut.magnitude)**2
a = (f*S_ut)**2/S_e
b = -1/3*np.log10(f*S_ut/S_e)
N = (sigma_1/a)**(1/b)
print(N.to("dimensionless"))
```

```
26133.53893700502 dimensionless
```

$N = 26,133$ Cycles

Problem 6-13

A solid square rod is centilevered at one end. The rod is 0.6 m long and supports a completely reversing transverse load at the other end of ossilating 2 kN. The material is AISI 1080 hot-rolled steel. If the rod must support this load for 10^4 cycles witha design factor of 1.5, what dimension should the square cross section have? Negect any stress concentrations at the support end.

Solution

The maximum moment on the rod is 1.2 kN·m. With this, the maximum stress is:

$$\sigma_1 = \frac{Mc}{I} \tag{1}$$

Where for our square cross section:

$$I = \frac{a^4}{12}$$

$$c = a/2$$

And a is the side length of our square and our main design parameter.

Looking at *Table A-20*, AISI 1080 hot-rolled steel has an ultimate strength of $S_{ut} = 770$ MPa.

Since we want our element to last 10^4 cycles, we will use Basquin's equation:

$$S_f = aN^b \tag{2}$$

Where:

$$a = \frac{(fS_{ut})^2}{S_e}$$

$$b = -\frac{1}{3}log\left(\frac{fS_{ut}}{S_e}\right)$$

For steel:

$$f = 1.06 - 4.1(10^{-4})S_{ut} + 1.5(10^{-7})S_{ut}^2 \tag{3}$$

Our endurance limit is:

$$S_e' = 0.5S_{ut} \tag{4}$$

With Marin factors of:

$$k_a = 33.6S_{ut}^{-0.655}$$

$$k_b = \begin{cases} 1.24d^{-0.107} & 7.62 \leq d \leq 51mm \\ 1.51d^{-0.157} & 51 < d \leq 254mm \end{cases}$$

This means that our infinite life strength is:

$$S_e = k_a k_b S_e' \tag{5}$$

Rearranging our equations and specifically setting eq. 2 into eq. 1 yields:

$$\frac{6M}{a^3} = \frac{(fS_{ut})^2}{S_e} N^{log\left(\frac{fS_{ut}}{S_e}\right)^{-1/3}} \tag{6}$$

Plugging in our N value lets us us log rules to show:

$$N^{log(s)^h} = 10^{4log_{10}(s)^h} = 10^{log_{10}(s)^{4h}} = s^{4h}$$

Applying this to eq. 6:

$$\frac{6M}{a^3} = \frac{(fS_{ut})^2}{S_e} \left(\frac{fS_{ut}}{S_e}\right)^{-4/3} = (fS_{ut})^{2/3}(S_e)^{1/3} \tag{7}$$

Here we will need to split into two equations since the size factor is defined as a piecewise equations. Before we split the equation, we will collapse our constants into numbers to simplify our math.

[1]:
```
from thermostate import Q_, units

S_ut = Q_(770, "MPa")
M = Q_(1.2, "kN*m").to("kN*mm")
f = 1.06 - 4.1*10**(-4)*S_ut.magnitude + 1.5*10**(-7)*(S_ut.magnitude)**2

C = 6*M
print(C)
S_ep = 0.5 * S_ut
k_a = 33.6*S_ut.magnitude**(-0.655)
D = (f*S_ut)**(2/3)*(k_a*S_ep)**(1/3)
print(D)
```

```
7200.0 kilonewton * millimeter
409.15545609172 megapascal
```

The equation, collapsing together contants, can be simplified to:

$$\frac{C}{a^3} = Dk_b$$

Splitting this into our two possible size factor equations yeilds:

$$\begin{cases} \frac{C}{a^3} = D(1.24d_e^{-0.107}) \\ \frac{C}{a^3} = D(1.51d_e^{-0.157}) \end{cases}$$

Where for a square rod, effective diameter is:

$$d_e = 0.808a$$

Pluggin this in a rearranging yeilds:

$$\begin{cases} \frac{C}{1.24D*0.808^{-0.107}} = a^{2.893} \\ \frac{C}{1.51D*0.808^{-0.157}} = a^{2.843} \end{cases}$$

Solving for a in both cases yields:

[3]:
```
a = (C/(1.24*0.808**(-0.107)))**(1/2.893)
print(a.magnitude)

a = (C/(1.51*0.808**(-0.157)))**(1/2.843)
print(a.magnitude)
```

```
19.843407698614147
19.44087643291438
```

This means our rod should have a square cross section with side length:

$a = 19.84$ mm