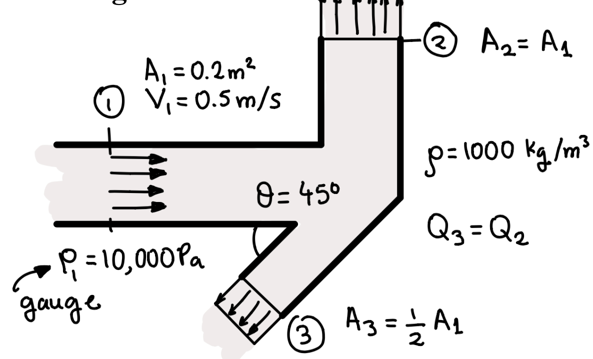
1. Water with density $\rho = 1000\,\frac{\text{kg}}{\text{m}^3}$ is entering the pipe of figure 3 at Station 1 and exiting to the environment at Stations 2 and 3. As Station 1, the uniform velocity is V_1 = 0.5 m/s, area A_1 = 0.2 m^2, and gauge pressure p_1 = 10,000 Pa. The flow exits vertically at Station 2 and at an angle of 45° at Station 3. At Station 2 the *cross-sectional* area is $A_2 = A_1$ and at Station 3 the *cross-sectional* area is $A_3 = A_1/2$. The flow rate at Station 2 is the same as Station 3. Assuming steady, incompressible flow, and uniform velocity profiles at all stations:

   a. Find the uniform velocity V_2 at Station 2. [5 points]
   b. Find the uniform velocity V_3 at Station 3. [5 points]
   c. The horizontal component of the force on the pipe section after Station 1. [15 points]
   d. The vertical component of the force on the pipe section after Station 1. [15 points]

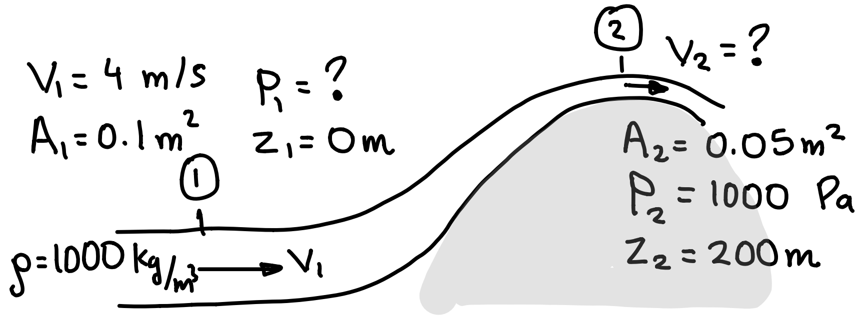
Figure 3



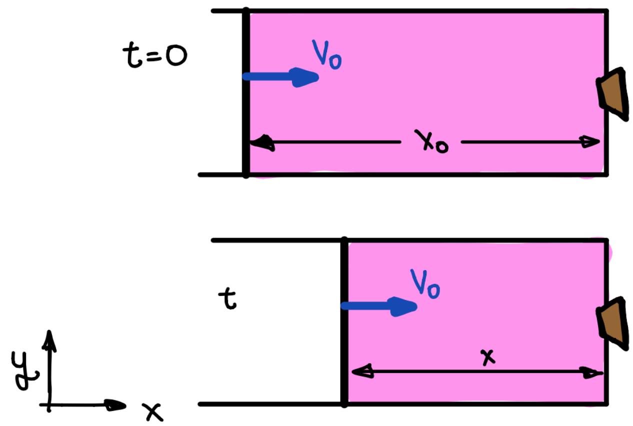
You must *clearly* draw the control volume. All your calculations must be *consistent* with the control volume.

2. Water, $\rho_{\text{water}} = 1000\,\frac{\text{kg}}{\text{m}^3}$, is flowing in a pipe over a hill from (1) to (2) as in the figure. The pipe at station (1) has area $A_1 = 0.1\ \text{m}^2$. At station (2) the area is $A_2 = 0.05\ \text{m}^2$. The uniform flow velocity at station (1) is $V_1 = 4\,\frac{\text{m}}{\text{s}}$. Station (2) is 200 m higher than station (1). The acceleration of gravity is $g = 10\ \frac{\text{m}}{\text{s}^2}$. Assuming steady, incompressible, inviscid flow, and uniform velocity at (1) and (2):

   a. Find the velocity V_2 at (2) [10 points]
   b. Find the pressure p_1 at (1). [10 points]



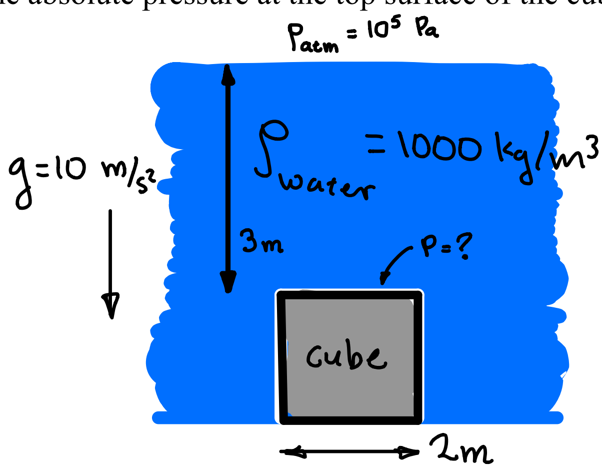
3. Air is compressed in a syringe as in the figure. The needle is blocked so air is compressed into to barrel without leaking. The plunger moves with velocity V_0 and at $t = 0$ is x_0 distance away from the end.



   a. During the compression the volumetric dilatation rate of the air inside the syringe is… (choose one) [2 points]
      i. Negative
      ii. Zero
      iii. Positive

   b. What is the dilatation rate? [8 points]

4. A cube with sides $L = 2$ m is submerged in water with density $\rho_{\text{water}} = 1000\,\frac{\text{kg}}{\text{m}^3}$. The top of the cube is 3 m below the surface of water. The atmospheric pressure is $p_{\text{atm}} = 10^5$ Pa. Gravity acceleration is $g = 10\,\frac{\text{m}}{\text{s}^2}$.
   a. What is the buoyancy force on the cube? [10 points]
   b. What is the absolute pressure at the top surface of the cube? [10 points]



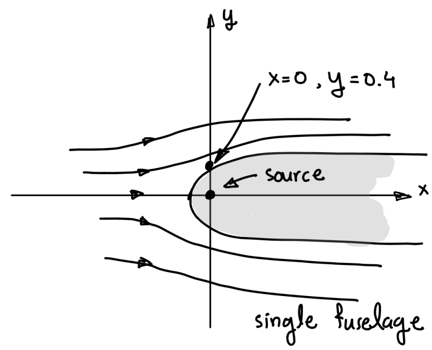
5. Some aircraft have two fuselages, instead of one, as in the figure below. We will show that the presence of a second fuselage can affect the flow and pressure distribution on the other fuselage.

a. In the first calculation, we will consider a single fuselage. We will model the flow of incompressible fluid with density $\rho = 1.2\ \mathrm{kg/m^3}$ using two-dimensional potential flow composed of a uniform flow with velocity potential:

$$\phi_{\text{uniform}}(x, y) = x$$

and a source located at $x = 0$ and $y = 0$, with velocity potential:

$$\phi_1(x, y) = \frac{\ln(x^2 + y^2)}{8}$$

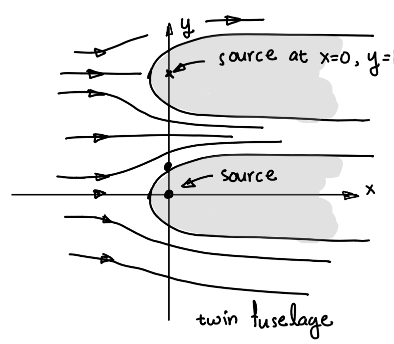


Find:

i. The velocity potential of the flow around a single fuselage [5pts]
ii. The expressions for the two velocity components $u(x,y)$ and $v(x,y)$ [5pts]
iii. The velocity components at $x = 0$ and $y = 0.4$ m. [5pts]
iv. The gage pressure at at $x = 0$ and $y = 0.4$ m. (Neglect gravity) [5pts]

b. In a second calculation, we consider two fuselages, by adding a second source at $x = 0$ and $y = 1$. The velocity potential of a second source is:

$$\phi_2(x, y) = \frac{\ln(x^2 + (y - 1)^2)}{8}$$



Find:

i. The velocity potential of the flow around the twin fuselage [5pts]
ii. The expressions for the two velocity components $u(x,y)$ and $v(x,y)$. [5pts]
iii. The velocity components at $x = 0$ and $y = 0.4$ m. [5pts]
iv. The gage pressure at at $x = 0$ and $y = 0.4$ m. (Neglect gravity) [5pts]

6. A rectangular tank with dimensions of width and depth equal to L (see figure) is initially filled with an incompressible fluid of density ρ and viscosity μ up to a height of H_0. The top of the tank is open to the atmosphere. At the bottom, the tank is connected to a circular pipe of length l and radius R. The fluid flows through the pipe and exits to the atmosphere. The acceleration of gravity is g.
   a. Find the expression for the height $h(t)$ of the fluid in the tank as a function of time. Assume steady, laminar, fully developed flow in the pipe. [15 pts]
   b. After how long will the tank empty? Can you explain your result? [5 pts]