ME 3227 Final Project

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Introduction:

In this project, I will be analyzing the stresses on the E^bkey shaft of a tenor saxophone. This is one of the most common components on a saxophone to get stuck which is why I choose it for my analysis. We will be treating the key as a main shaft with simple supports and two other plates normal to the shaft which have vertical forces on them, imparting a torque on the shaft. In reality, one of the forces will be a figure pressing down on the button, the other force will be an equal and opposite force as the key lifts up and reaches a stopper. There is also an additional continues spring acting on the shaft to ensure when no forces are applied, the shaft is rotated, keeping the key shut.

Part I – Graphical Description

The element we are looking at is a shaft that is used to lift a key on a saxophone. A lever when pressed applies a torque, rotating the shaft. When the shaft rotates, on the opposite end it lifts a key which allows an opening for air to escape, changing the pitch of the saxophone.

Figure 1A (left) and 1B (right): Imagines of our element and its location on the saxophone. A saxophone has over 22 keys and in these figues it may be hard to distinguish our specific element, the E^bkey.

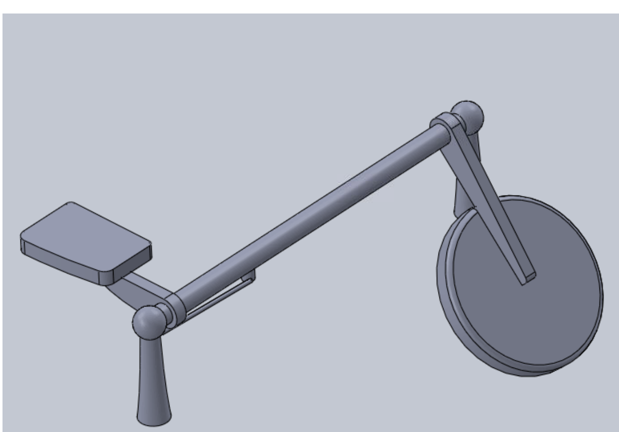


Figure 2: Solidworks model of our machine element. This allows an isolated depiction of the part away from the entire saxophone. It also better shows the spring, which provides a constant static loading on the part.

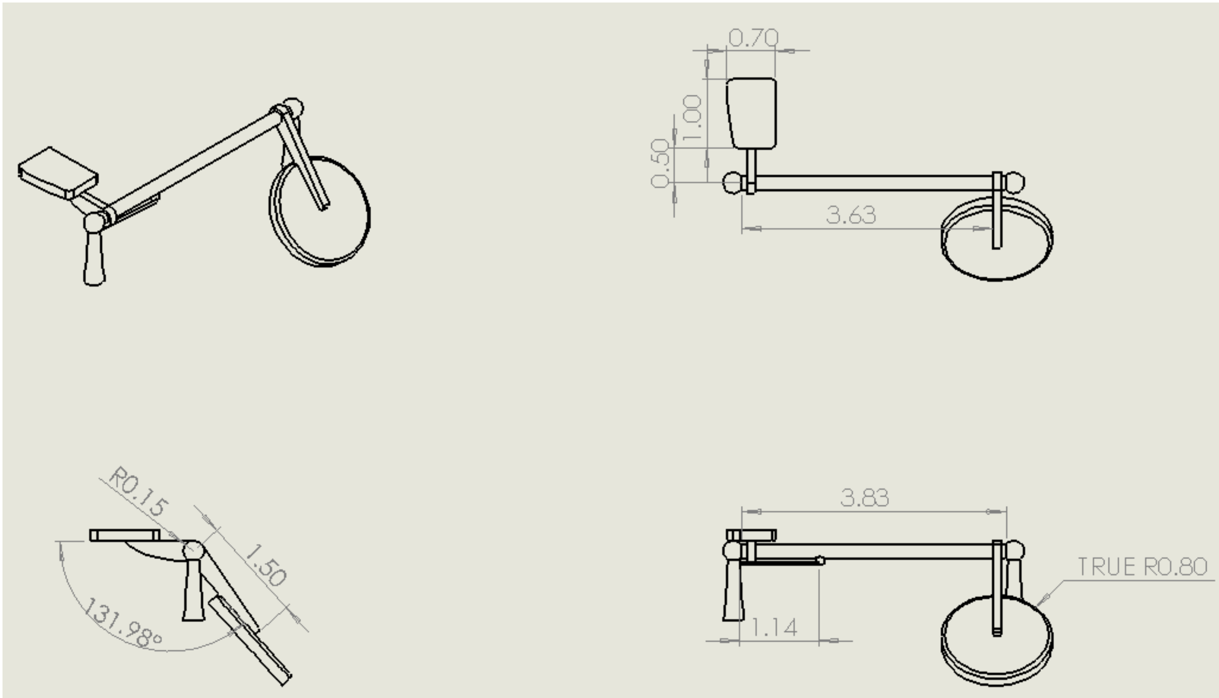


Figure 3: Engineering drawing of our machine element; this drawing provides measured values for the dimensions of the part.

The critical key dimensions for our purposes are the following. The shaft of our element has a diameter of 0.15 inches. We will assume the force of the figure pressing the lever is a point force located at the center of the button, being 1.0 inch away from the center of the shaft. Similarly, the reaction force of the key will be located at its center, 1.50 inches away from the center line of the shaft. Lastly, the spring which keeps the key down when not in use will be approximated as a torsional spring along the axis of the shaft.

Part II – Analysis

Saxophones were traditionally made of wood, which is why they are apart of the woodwind musical instrument family. Now though, Yamaha, the primary producer of musical instruments, casts and then assembles saxophones out of a brass alloy, 70% copper and 30% zinc, with some even having polished gold or silver plating.

	Tensile Strength S_{ut}	Yielding Strength S_y	Elastic Modulus E	Shear Modulus G	Poisson Ratio ν
Brass Alloy	52.2 Kpsi	20.3 Kpsi	15.2 Mpsi	5.66 Mpsi	0.34

Table 1: Material properties of the brass alloy our machine element is made of.

There are two loadings this element will experience. Case 1: when the key is not being played, and the torsional spring is causing the button to seal the whole not allowing air thought. Case 2: when the key is being played, a figure overcomes the torsional force, and the key is being lifted allowing air to flow through the hole. The only applied force, $F_{applied}$, is located at point E. In

Case 1, $F_{applied} = 0$ so the rotational torque at point B is providing a positive twist on the rod which causes point F to be restricted by the outer barriers of the hole. In Case 2, the pinky finger of the person playing the instrument applies a force at point E, overcoming the torsion at point B and rotating the rod until another barrier at point F restricts further movement. In both cases, there are also rolling bearings that create simple support reactions for the shaft OD.

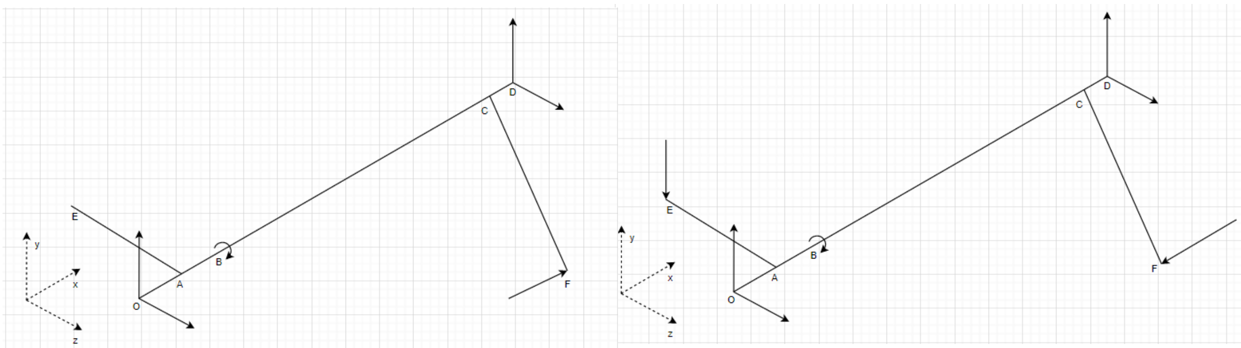


Figure 4 (left) and 5 (right): The free body diagrams of Case 1 and Case 2 respectively.

For both cases, we will look at the static equilibrium equations which are as follows:

$$\Sigma F_y: O_y - F_E - F_F\, cos(48^o) + D_y = 0$$

$$\Sigma F_z: O_z - F_F sin(48^o) + D_z = 0$$

$$\Sigma M_x: -F_E r_{EA} + M_B + F_F r_{FC} = 0$$

$$\Sigma M_y: F_F sin(48^o) r_{OC} - D_z r_{OD} = 0$$

$$\Sigma M_z: -F_E r_{OA} - F_F cos(48^o) r_{OC} + D_y r_{OD} = 0$$

We will also assume that for Case 2, the force applied at point E to open the key provides a moment that is triple the force keeping it closed, M_B. A rough estimate for this is setting $M_B = 1\ lbf \cdot in$ and $F_{applied} = F_E = 3\ lbf$. Using this information to solve the equilibrium equations yields the following forces in each case:

	F_E (lbf)	M_B (lbf*in)	F_F (lbf)	O_y (lbf)	O_z (lbf)	D_y (lbf)	D_z (lbf)
Case 1	0	1	0.67	-0.02	-0.03	-0.42	-0.47
Case 2	3	1	1.33	2.77	0.05	1.12	0.94

Table 2: Applied and reaction forces experienced by our machine element.

With the loadings specified in *Table 2*, we can calculate the stresses we will experience on our beam through the loading cycle, from the loading in Case 1 to loading in Case 2. Our beam experiences both torsion and bending from the reaction forces so we will assume that the largest stress is located on the outer most fiber of the beam. Viewing the shear and moment diagrams below, we can say where along the rod the maximum stress occur.

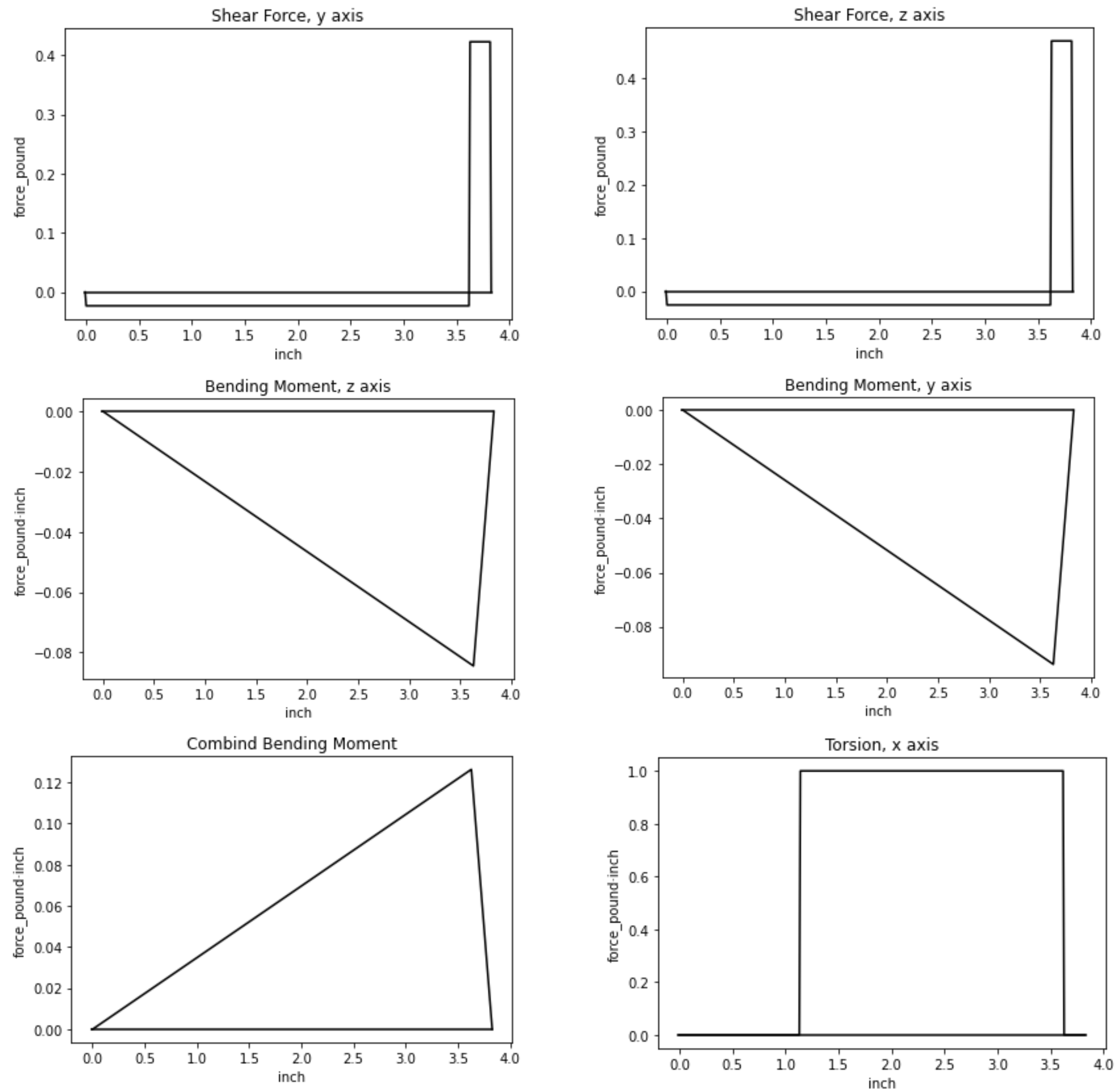


Figure 6: The shear and bending moment diagrams for both the y and z axis, combined bending moment diagram, as well as the torsional forces along the x axis for Case 1. In Case 1, the y and z axis diagrams are near similar because the only force is at a 48 º angle which means it basically applies equal forces to both axis.

Looking at the bottom two graphs of *Figure 6*, it is clear to see that our point of maximum stress is going to be located at 3.63 inches along the shaft, point C in the *Figure 4/5*. This point is where both the maximum bending moment and maximum torsional force occur.

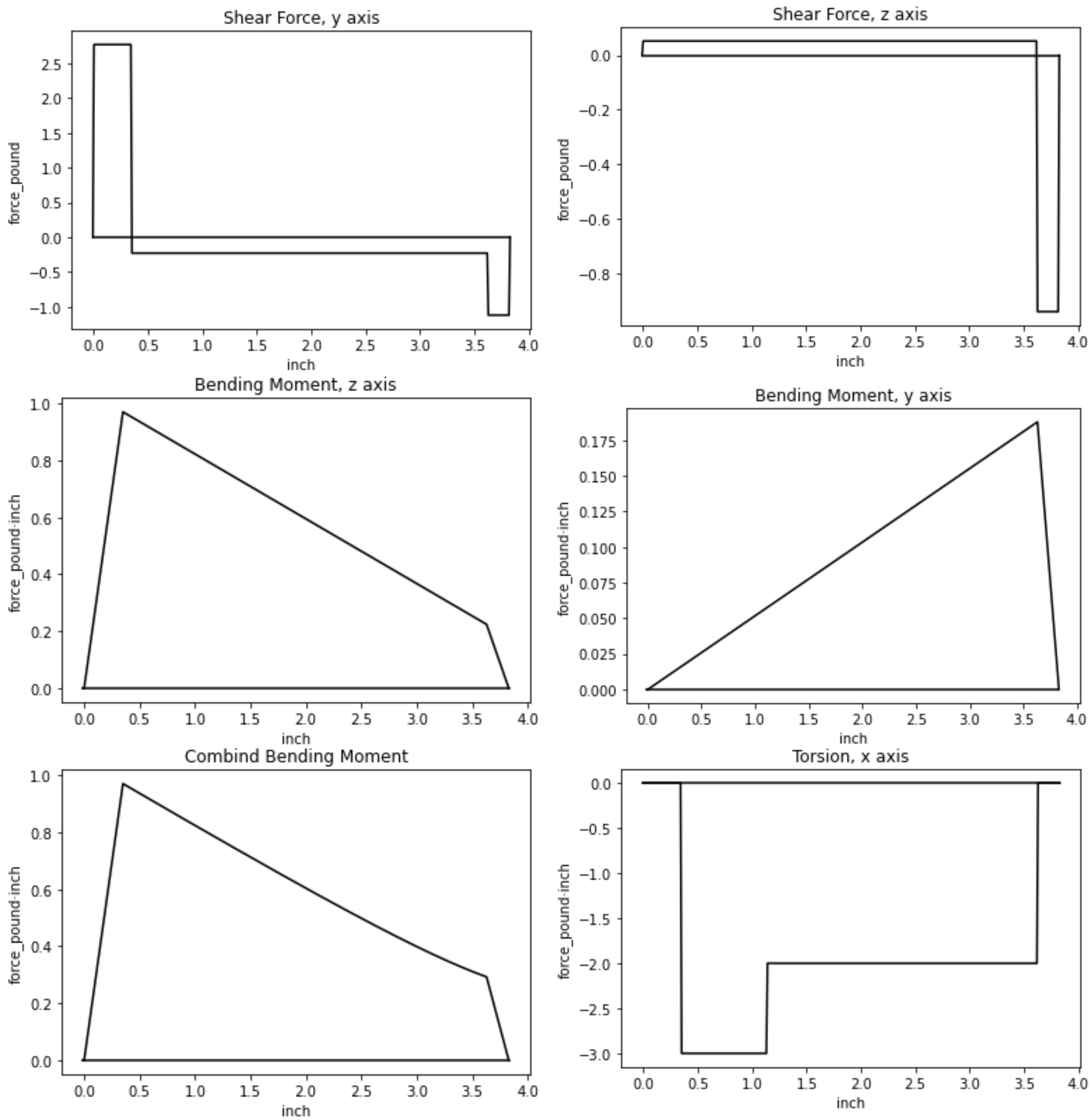


Figure 7 depicts the shear and bending moment diagrams for both the y and z axis, combined bending moment diagram, as well as the torsional forces along the x axis for Case 2.

Similar to in Case 1, the point of maximum bending moment and maximum torsion occur at the same point, but for Case 2 that is 0.35 inches along the shaft, point A in the *Figure 4/5*. The point of maximum bending and torsion are important because those are where the critical stresses occur. However, at point A, the change in loading from Case 1 to Case 2 are much greater than what we see for point C, which is our critical point in Case 1 static loading. We will say point A is the critical point for the static Case 2 loading and for our cyclical fatigue loadings and focus all further calculations at point A.

Now that we have determined our critical point along the shaft, we can calculate the stresses that the point undergo during cyclical loading from Case 1 to Case 2. For our bending and torsion forces, the stresses at the point are:

$$\sigma_x = \frac{Mc}{I}$$

$$\tau_{xz'} = \frac{Tr}{J}$$

Where M and T are the maximum moment and torque. $\tau_{xz'}$ is a shear stress which occurs normal to the x axis and in the direction of the z' axis. The y' and z' axis are axis defined to be inline and perpendicular with our combined bending moment respectively. The y' axis is the axis in the yz plane which the maximum combined bending moment happens about. The z' axis is normal to the y' axis. Plugging values into these equations yields *Table 3* which is shown below.

	M (lbf*in)	T (lbf*in)	σ_x (psi)	$\tau_{xz'}$ (psi)
Case 1	0.126	0	47.5	0
Case 2	0.971	-3	366.3	-565.9

Table 3: The loads and stresses at the critical location of our rod, point A.

For fatigue calculations, we will use our von Mises stress because we have a combination of torsion and bending at our critical point. Our von Mises stress is defined by:

$$\sigma' = \sqrt{\sigma_x^2 + 3\tau_{xz'}^2}$$

We can use this simple equation because we do not need to account for any stress concentration factors on our smooth, polished shaft. Using the von Mises, we'll calculate our stress amplitudes, mean stress, stress range, and maximum and minimum stresses, all depicted in *Table 4*:

	σ_{min} (psi)	σ_{max} (psi)	σ_r (psi)	σ_m (psi)	σ_a (psi)
Case 1	47.5	1046	998.8	546.9	499.4

Table 4: Von Mises stresses

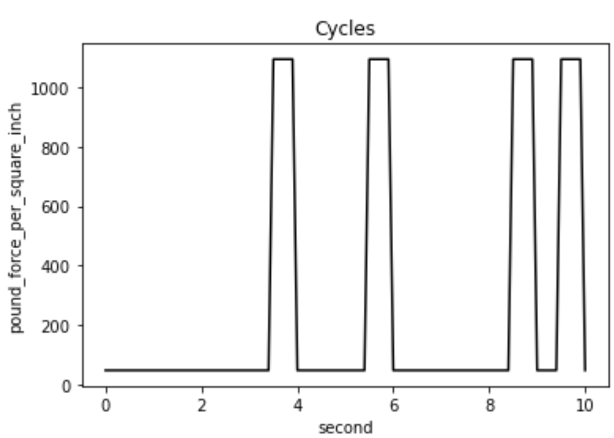


Figure 8: An example of the saxophone playing a song. Every few seconds the key gets pressed and we see our maximum stress, all other times we see our minimum stress.

Unlike for steel, brass does not have an endurance strength limit for infinite life. Instead, we will be using a modified Basquin equation that was found by the University of Illinois while testing brass poles fluctuating in the wind:

$$\sigma_r = 433N^{-0.153}$$

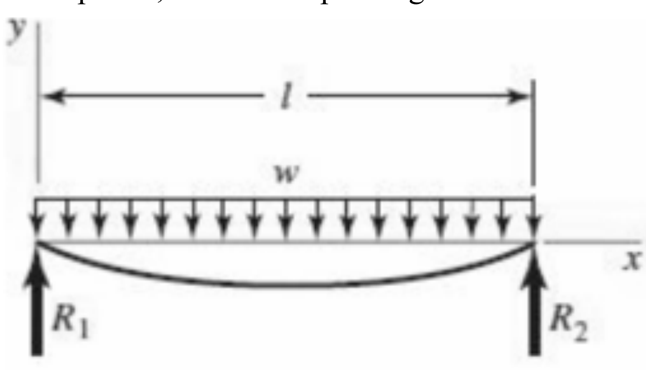
We will assume our brass alloy and loading conditions follow the same equation. Solving for N means we can expect our machine element to fail after 1.72×10^{17} cycles. We have no fatigue factor of safety since our material has no infinite life regime but for particle purposes, we have infinite life at 10^{17} cycles.

We will also look at our static loading from Case 2, our maximum stresses, to see what our statistic factor of safety is. We will say that our factor of safety is:

$$n = \frac{S_y}{\sigma'}$$

Plugging in our maximum von Mises stress, we get a static safety factor of 20.32. A factor of safety this large seems excessive for an instrument. This leads to think that the design of the shaft is not simply designed for the intended applied loadings but is more specialized for unintentional loadings.

Personally, I have already replaced this specific component on my saxophone twice, but I haven't reached the fatigue failure expectation of 1.72×10^{17} cycles. However, there is another unintentional loading that this shaft often experiences. If a saxophone is placed backwards on a chair with an indentation for the sitter, it will be supported by the two sides of the indention, approximating a simply supported beam. But with our small shaft protruding along the outside of saxophone, it can end up taking the brunt of the reaction force from the chair.



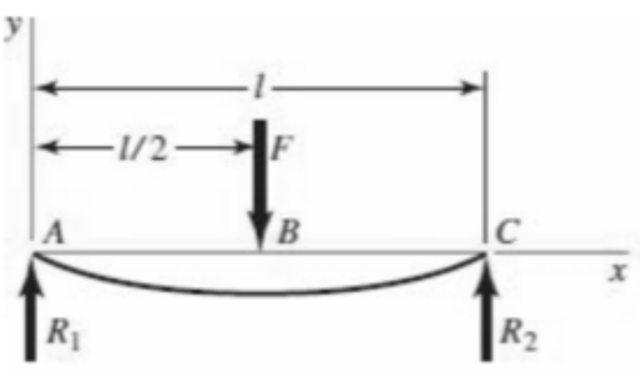


Figure 9 (left): The unintended loading where w is the distributed weight of the saxophone, assumed to be constant, and the two supports are the indented chair push back on the saxophone. Figure 10 (right) is an inverted depiction of what is happening at R_2 for Figure 9. Though Fig. 9 approximated the saxophone as having a flat edge, it often ends up being that the saxophone is placed directly on the rod we have been analyzing, such that the F in Fig. 10 is R_2 in Figure 9, and R_1 and R_2 in Fig. 10 are O_y and D_y from our Fig 4/5.

This loading is the likely reason many saxophonists need to replace this element on their instrument. If permeate deflection occurs in this rod, the sealing and opening of the key begins to fail and eventually the key will be slightly open all the time, allowing air out even when the key is not pressed. This means the saxophone will be out of tune.

Assuming our saxophone has a constant weight distribute, we can solve for the reaction in *Fig. 9.* With a length of 4.6 feet, our distributed load $w = 2.39$ lbf/ft our reaction in *Fig. 9* and our force in *Fig. 10* is $6.5\ lbf$. With that force being applied to the center of our machine element, we will expect a deflection following:

$$y_{max} = -\frac{Fl^3}{48EI}$$

Plugging in the dimension from *Fig. 3* and material properties from *Table 1,* we find that our maximum deflection is 0.003 inches. This deflection is enough to cause damage to the functionality of the saxophone which is often why musicians are told to be careful when placing down their instrument.

Conclusion:

In this project, I analyzed the E^bkey of a tenor saxophone. We saw that in regular use, the element can be cycled 1.72×10^{17}times and has a static factor of safety equal to 20.32. This led to looking into the forces that are likely to cause failure, placing the saxophone down incorrectly, and quantified that amount of deflection that comes from that.

References:

[1] How the Saxophone is Made: What metal is used to make a saxophone? - Musical Instrument Guide - Yamaha Corporation. (2021). YAMAHA, Making Waves. https://www.yamaha.com/en/musical_instrument_guide/saxophone/manufacturing/

[2] How saxophone is made - material, manufacture, making, history, used, parts, dimensions, composition, steps. (2021). How Products Are Made. http://www.madehow.com/Volume-6/Saxophone.html#:%7E:text=The%20most%20common%20type%20used,alloy%20workable%20at%20lower%20temperatures

[3] AZoM. (2013, July 11). Brass UNS C28000. AZoM.Com. https://www.azom.com/article.aspx?ArticleID=6380

[4] Rudd, Banas, Hahin. (2013). FATIGUE PERFORMANCE OF BRASS BREAKAWAY LIGHT POLE COUPLINGS. CIVIL ENGINEERING STUDIES Illinois Center for Transportation Series No. 12–023 UILU-ENG-2012-2026. Published. https://apps.ict.illinois.edu/projects/getfile.asp?id=3079