Solutions

ME 3250 – Midterm #1

October 8, 2021

- Exam duration is 40 minutes
- The exam includes two problems
- Please write legibly and show all steps of your solution. If an answer "magically" appears and the grader cannot trace back the steps of your solution, you will not receive any credit.

1. Red October (again)

A submarine is fully submerged and floats at depth $h = 10$ m below the surface. The atmospheric (absolute) pressure at the surface is $p_{atm} = 10^5$ Pa.

For this problem, we will simplify the submarine and we will assume it is just a container that is partially filled with sea water with air at the top part of the tank. The bottom of the tank has an opening and sea water can go in or out. The density of water is $\rho_{water} = 1000\ \mathrm{kg/m^3}$. The acceleration of gravity is $g = 10\ \mathrm{m/s^2}$.

a. What is the absolute pressure at the depth $h = 10$ m where the submarine is floating? [5 points]

b. At depth $h = 10$ m, the submarine's tank is filled with air with density $\rho_{air} = 2.5\ \mathrm{kg/m^3}$ and volume V_{air} and water with volume $V_{water} = 200\ \mathrm{m^3}$. The submarine's hull has mass $M_{hull} = 4987.5$ kg and zero volume. Find the volume of air in the tank $V_{air} = ?$ [20 points]

c. Find the buoyancy force on the submarine when it is at depth $h = 10$ m. [10 points]

d. The submarine dives to depth $d = 100$ m. What is the absolute pressure at depth *d*? [5 points]

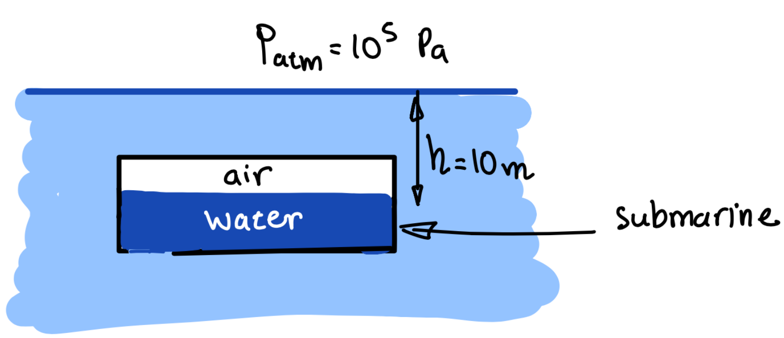
e. Assuming the air in the tank is an ideal gas. What is the density of the air in the tank at depth *d*? The pressure of the air in the tank is the same as the hydrostatic pressure at the submarine depth. Also, assume that as the submarine dives the air temperature does not change. [10 points]

f. What is the volume of the air in the tank when the submarine is as depth *d*? [10 points]

g. What is the density of the submarine at depth *d*? [10 points]

h. Did the buoyancy of the submarine change when it dived from depth *h* to depth *d*? [5 points]

i. Can the submarine float at depth *d*? [5 points]



a. $\Delta p = -\rho g \Delta z$ 3

$p_h - p_{atm} = -\rho_w g(-h)$ 1

$p_h = p_{atm} + \rho_w g h = 2\times10^5$ Pa absolute 1

b. Submarine floats => Buoyancy = Weigt

$\rho_w g V_{sub} = M_{sub} g$ 5

$\rho_w (V_{air} + V_w) = (M_{hull} + \rho_w V_w + \rho_{air} V_{air})$ 5×2

4 $V_{air} = \dfrac{M_{hull}}{\rho_w - \rho_{air}} = 5 \text{ m}^3$ 1

c. $B = \rho_w g V_{sub} = \rho_w g (V_{air} + V_w) = 2.05\times10^6$ N 5 3 1 1

d. $p_d = p_{atm} + \rho_w g d = 1.1\times10^6$ Pa absolute 3 1 1

e. $p_h = \rho_{air,h} R T_h$ 2 $\quad p_d = \rho_{air,d} R T_h$ same at both depths

4 $\rho_{air,d} = \rho_{air,h} \dfrac{p_h}{p_d} = 27.5$ kg/m³ 3 1

f. Air mass $= \rho_{air,h} V_{air,h} = \rho_{air,d} V_{air,d}$ 5

$V_{air,d} = \dfrac{\rho_{air,h}}{\rho_{air,d}} V_{air,h} = 0.454 \text{ m}^3$ 4 1

g. $\rho_{sub,d} = \dfrac{M_{sub}}{V_{sub}} = \dfrac{M_{hull} + \rho_{air,d} V_{air,d} + \rho_w V_{w,d}}{V_{sub}}$ 5

$V_{sub} = V_{air,h} + V_{w,h} = 205 \text{ m}^3$ 2

$V_{w,d} = V_{sub} - V_{air,d} = 204.54 \text{ m}^3$ 2

$\rho_{sub,d} = 1022$ kg/m³ 1

h. No; Buoyancy $= \rho_w g V_{sub}$ 3 2 — same at both depths. Buoyancy is the same but weight is more at "d" more mass in the submarine

i. No; $\rho_{sub,d} > \rho_w$ it will sink 3 2

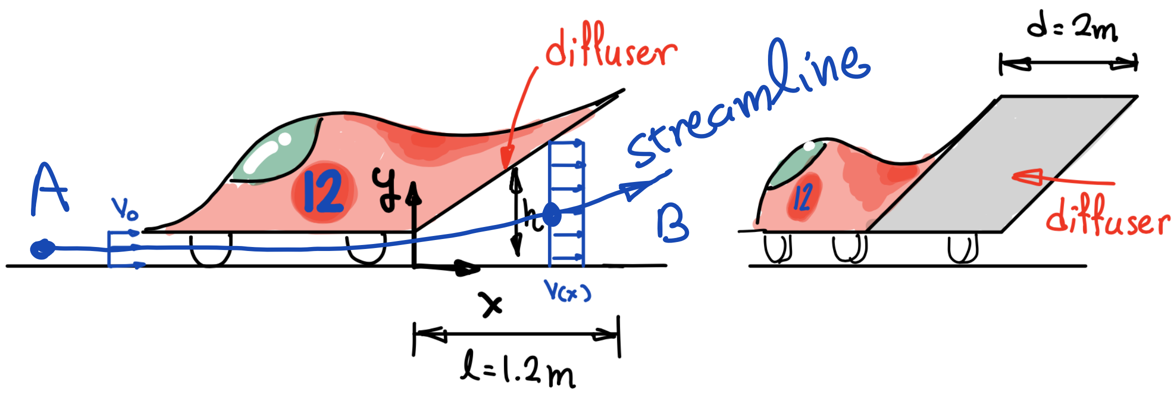
2. Tom's race car

Tom (which we met in Homework 1) has retired from football and started racing cars. After taking Fluid Dynamics I, he has an idea on how to improve his race car performance. He decided to add a "diffuser" to the back of his car. The diffuser has variable height following $h(x) = 0.1 + 0.5\,x$ m, based on the coordinate system of the figure. The length of the diffuser is $l = 1.2$ m and span of the diffuser is $d = 2$ m (see figure below).

We will assume steady, incompressible, inviscid flow. The air flow moves between the car's undercarriage and the road and then goes through the diffuser as in the image. The undercarriage is 0.1 m above the road. Assume that the flow is uniform and always in the x-direction. Neglect the car's wheels. The air density is $\rho = 1.5\ \mathrm{kg/m^3}$. The atmospheric (absolute) pressure is $p_{atm} = 10^5$ Pa.

For a car moving at $V_0 = 40\,\frac{\mathrm{m}}{\mathrm{s}}$, help Tom calculate:

a. What is the volumetric flow rate through the diffuser? [5 points]
b. Find the velocity magnitude $V(x)$ through the diffuser. [15 points]
c. Find the pressure variation as a function of x in the diffuser, $p(x) =$? Neglect any changes in height as the flow moves through the diffuser. [25 points]
d. Find the force in the y-direction on the diffuser surface. That is, the y-component of the force on the gray-shaded area in the figure below. [15 points]
e. Find the force in the x-direction on the diffuser surface. That is, the x-component the force on the gray-shaded area in the figure below. [10 points]



Helpful reminder: $\int \frac{dx}{(\alpha+x)^2} = -\frac{1}{\alpha+x}$

a. $Q = V_0 \, \text{Area} = V_0 \, h(0) \, d = 8 \ m^3/s$ 3 1 1

b. $Q = V(x) A(x) \rightarrow V(x) = \frac{Q}{A(x)} = \frac{8}{2h(x)} = \frac{4}{0.1+0.5x}$ 5 5 5 5

$$V(x) = \frac{8}{0.2+x} \ m/s$$

c. Bernoulli : Streamline $A \rightarrow B$

$$p_0 + \frac{1}{2}\rho V_0^2 = p(x) + \frac{1}{2}\rho V(x)^2$$ 15

$$p(x) = p_0 + \frac{1}{2}\rho\left(V_0^2 - \frac{64}{(0.2+x)^2}\right)$$ 10

d. $F_y = \int_0^d \int_0^l p(x) \, dx \, dz$ 6

$$= p_0 l d + \frac{1}{2}\rho V_0^2 l d - \frac{1}{2}\rho \, 64 d \int_0^l \frac{1}{(0.2+x)^2} dx$$ 4

$$= p_0 l d + \frac{1}{2}\rho V_0^2 l d + \frac{1}{2}\rho \, 64 d \left(\frac{1}{0.2+l} - \frac{1}{0.2}\right)$$ 1 1 2

$$= 1.88 \times 10^6 \ N$$ 1

e. $F_x = -0.5 F_y = 0{,}943 \times 10^6 \ N$ 10