M 431: Assignment 6

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Homomorphisms between clocks

Problem. Find all homomorphisms from $\mathbb{Z}_{12}$ to $\mathbb{Z}_{15}$.

Proof.

$\square$

Page 74 — Problem 13

Problem. If G is a finite abelian group of order n and $\varphi : G \longrightarrow G$ is defined by $\varphi(a) = a^m$ for all $a \in G$, find the necessary and sufficient condition that φ be an isomorphism of G onto itself.

Proof.

$\square$

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Problem. If G is a group and $a \in G$, define $\sigma_a(g) = aga^{-1}$. We saw in Example 9 of this section that σ_a is an isomorphism of G onto itself, so $\sigma_a \in A(G)$, the group of all $1-1$ mappings of G (as a set) onto itself. Define $\psi : G \longrightarrow A(G)$ by $\psi(a) = \sigma_a$ for all $a \in G$. Prove that

(a) ψ is a homomorphism of G into $A(G)$.

(b) $\ker \psi = Z(G)$ the center of G.

Proof.

□

Heisenberg to plane

Problem. Find an epimorphism the Heisenberg group $\mathbb{H}_3(\mathbb{R})$ onto $\mathbb{R}^2$.

Proof.

$\square$

Klein group

Problem. Show that the group $Sym(R)$ where R is a rectangle that is not a square is isomorphic to the product $\mathbb{Z}_2 \times \mathbb{Z}_2$.

Proof.

$\square$