M 431: Assignment 3

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Problem. Let G be a finite, nonempty set with an operation $*$ such that:

1. G is closed under $*$
2. $*$ is associative
3. Given $a, b, c \in G$ with $a * b = a * c$, then $b = c$
4. Given $a, b, c \in G$ with $b * a = c * a$, then $b = c$

Prove that G must be a group under $*$.

Proof.

$\square$

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Problem. Let S_3 be the symmetric group of degree 3. Find all the subgroups of S_3.

Proof.

$\square$

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Problem. Prove that a cyclic group is abelian.

Proof.

$\square$

Heisenberg group problem

Problem. Recall the general linear group $\mathbb{GL}_3(\mathbb{R})$ of 3×3 invertible matrices with real entries (taken with the matrix product). Verify that the following subset, called the Heisenberg group, is a subgroup of $\mathbb{GL}_3(\mathbb{R})$:

$$\mathbb{H}_3(\mathbb{R}) := \left\{ \begin{bmatrix} 1 & x & z \\ 0 & 1 & y \\ 0 & 0 & 1 \end{bmatrix} \mid x, y, z \in \mathbb{R} \right\}$$

Proof.

$\square$

Cube subgroups problem

Problem. Recall the group $Sym(Q)$ of the rigid symmetries of the cube $Q := [-1, 1]^3$ in $\mathbb{R}^3$. Describe in words/pictures the following:

a subgroup of order 4
a subgroup of order 12
a subgroup of order 3
a subgroup of order 6
a subgroup of order 8

Proof.

□