M 431: Assignment 12

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Page 139 — Problem 3

Problem. Let p be an odd prime and let $1 + 1/2 + \cdots + 1/(p-1) = a/b$ where $a, b \in \mathbb{Z}$. Show that $p \mid a$.

Proof.

□

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Problem. In example 3, show that $M = \{x(2+i) \mid x \in R\}$ is a maximal ideal of R.

Proof.

$\square$

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Problem. In Example 3, show that $R/M \cong \mathbb{Z}_5$.

Proof.

□

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Problem. In Example 3, show that $R/I \cong \mathbb{Z}_5 \oplus \mathbb{Z}_5$.

Proof.

$\square$

Page 163 — Problem 1

Problem. If F is a field, show that the only invertible elements in $F[x]$ are the nonzero elements of F.

Proof.

$\square$

Page 163 — Problem 3

Problem. Find the greatest common divisor of the following polynomials over $\mathbb{Q}$, the field of rational numbers.

(a) $x^3 - 6x + 7$ and $x + 4$
(b) $x^2 - 1$ and $2x^7 - 4x^5 + 2$
(c) $3x^2 + 1$ and $x^6 + x^4 + x + 1$
(d) $x^3 - 1$ and $x^7 - x^4 + x^3 - 1$

Proof.

$\square$