M 431: Assignment 9

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Problem. Prove that a group of order 35 is cyclic.

Proof.

□

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Problem. If G is a group with subgroups A, B of orders m, n, respectively, where m and n are relatively prime, prove that the subset of G, $AB = \{ab \mid a \in A, b \in B\}$, has mn distinct elements.

Proof.

$\square$

Conjugacy Stabilizers

Problem. Suppose that $K, H \leq G$ and H is normal. Let $[a]$ stand for the $K-$conjugacy class of a: $[a] := \{bab^{-1} \mid b \in K\}$. Introduce the stabilizer of a: $Stab(a) := \{b \in K \mid bab^{-1} = a\}$.

(a) Show that, for any $a' \in [a]$, $Stab(a')$ and $Stab(a)$ are related by conjugation, $\exists k \in K$ where

$$Stab(a') = k\, Stab(a)\, k^{-1}$$

Conclude that $|Stab(a)| = |Stab(a')|$.
(b) Use part (a) to show that following formula for the cardinality of $[a]$:

$$\#[a] = \frac{|K|}{|Stab(a)|}$$

Proof.

□

Abelinan Classification

Problem. List all abelian isomorphism classes with order 108.

Proof.

□

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Problem. Let G be an abelina group of order p^n, p a prime, and let $a \in G$ have maximal order. Show that $x^{o(a)} = e$ for all $x \in G$.

Proof.

$\square$