M 431: Assignment 7

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Page 74 — Problem 15

Problem. If G is any group, $N \trianglelefteq G$ and $\varphi : G \longrightarrow G'$ a homomorphism of G onto G', prove that the image, $\varphi(N)$, of N is a normal subgroup of G'.

Proof.

$\square$

Page 74 — Problem 16

Problem. If $N \trianglelefteq G$ and $M \trianglelefteq G$ and $MN = \{mn \mid m \in M, n \in \mathbb{N}\}$, prove that MN is a subgroup of G and that $MN \trianglelefteq G$.

Proof.

$\square$

Page 83 — Problem 3

Problem. If G is a group and $N \trianglelefteq G$, show that if $\overline{M}$ is a subgroup of G/N and $M = \{a \in G \mid Na \in \overline{M}\}$, then M is a subgroup of G and $N \subset M$.

Proof.

$\square$

Page 83 — Problem 4

Problem. If $\overline{M}$ in the previous problem is normal in G/N, show that the M defined is normal in G.

Proof.

$\square$

Page 87 — Problem 3

Problem. Let G be the group of nonzero real numbers under multiplication and let $N = \{1, -1\}$. Prove that $G/N \cong$ positive real numbers under multiplication.

Proof.

$\square$

Page 88 — Problem 6

Problem. If G is a group and $N \trianglelefteq G$, show that if $a \in G$ has finite order $o(a)$, then Na in G/N has finite order m where $m \mid o(a)$.

Proof.

$\square$