M 431: Assignment 8

Nathan Stouffer

Page 87 — Problem 2

Problem. Let G be the group of all real-valued functions on the unit interval $[0, 1]$, where we define, for $f, g \in G$, addition by $(f + g)(x) = f(x) + g(x)$ for every $x \in [0, 1]$. If $N = \{f \in G \mid f(1/4) = 0\}$, prove that $G/N \cong$ the real numbers under $+$.

Proof.

$\square$

3rd Iso Thm Example

Problem. Identify and illustrate with pictures the three quotient groups in the 3rd isomorphism theorem instantiated for $G = \mathbb{R} \times \mathbb{Z}_2$, $N = \mathbb{Z} \times \mathbb{Z}_2$, and $K = 2\mathbb{Z} \times \{0\}$.

Proof.

$\square$

2rd Iso Thm Example

Problem. Identify and illustrate with pictures all the groups involved in the 2nd isomorhpism theorem isntantiated for $G = \mathbb{R} \times \mathbb{R}$, $N = \mathbb{Z} \times \mathbb{R}$, $H = \mathbb{R} \times \{0\}$. In particular, draw the cosets making up the quotient groups and recognize the groups as familiar concrete groups.

Proof.

$\square$

Page 96 — Problem 5

Problem. Let G be a finite group, $N_1, N_2, ..., N_k$ normal subgroups of G such that $G = N_1 N_2 \cdots N_k$ and $|G| = |N_1||N_2| \cdots |N_k|$. Prove that G is the direct product of $N_1, N_2, \ldots, N_k$.

Proof.

$\square$

Page 96 — Problem 6

Problem. Let G be a group, $N_1, N_2, \ldots, N_k$ normal subgroups of G such that:

1. $G = N_1 N_2 \cdots N_k$
2. For each i, $N_i \cap (N_1 N_2 \cdots N_{i-1} N_{i+1} \cdots N_k) = (e)$

Prove that G is the direct product of $N_1, N_2, \ldots, N_k$

Proof.

□