M 431: Assignment 2

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Problem. If G is a group in which $a^2 = e$ for all $a \in G$, show that G is abelian.

Proof.

$\square$

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Problem. If G is a finite group of even order, show that there must be an element $a \neq e$ such that $a = a^{-1}$.

Proof.

□

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Problem. If G is the dihedral group of order $2n$ as defined in Example 10, prove that

(a) If n is odd and $a \in G$ is such that $a * b = b * a$ for all $b \in G$, then $a = e$.

(b) If n is even, show that there is an $a \in G$, $a \neq e$, such that $a * b = b * a$ for all $b \in G$.

(c) If n is even, find all the elements $a \in G$ such that $a * b = b * a$ for all $b \in G$.

Proof.

$\square$

Custom Problem

Problem. Find the order of the group $Sym(\mathbb{Q})$ of the (rigid) symmetries of the cube $Q := [-1, 1]^3 \subset \mathbb{R}^3$. The task is not to just get the number but to organize all the possible symmetries and build a narrative that gets one to the answer in a rigorous way resting on few, easy to grasp geometric facts.

Proof.

$\square$