Advanced Algorithms, Homework 1

Nathan Stouffer

Due: 27 August 2020

CSCI 432 Problem 1-1

Collaborators: *n/a*

Answer the following questions:

1. What is your elevator pitch? Describe yourself in 1-2 sentences.
2. What was your favorite CS class so far, and why?
3. What was your least favorite CS class so far, and why?
4. Why are you interested in taking this course?
5. What is your biggest academic or research goal for this semester (can be related to this course or not)?
6. What do you want to do after you graduate?
7. What was the most challenging aspect of your coursework last semester after the university transitioned to online?
8. What went well last semester for you after the university transitioned to online?

Answer

1. I am a senior in math and computer science. I really enjoy thinking, running, and skiing.
2. So far, my favorite computer science class has been computer graphics. I really liked how much math and programming there was. The math appealed to the side of me that enjoys thinking about problems and the programming appealed to the side of me that enjoys building solutions.
3. So far, my least favorite computer science class was computer architecture. I did not find the content very exciting. I prefer using the computer at higher level than messing around with resistors.
4. I am interested in taking advanced algorithms because I think it is a cool area. I like coming up with clean, fast solutions to problems and I think this will be an opportunity to do so. Another reason that I am interested in taking this course is because there are a lot of cool applications!
5. This is more of a year long goal, but I would like to publish a paper this year.
6. I am unsure of what I would like to do after I graduate. I am considering graduate school but I am also thinking about working in industry.
7. After school went online, the most challenging portion of my coursework was completing quality work in every one of my courses. It was sometimes difficult to stay motivated in courses that I was less interested in.
8. After going online, I was very good at staying organized and on top of my schoolwork.

CSCI 432 Problem 1-2

Collaborators: *n/a*

Please do the following:

1. Write this homework in LaTex. Note: if you have not used LaTex before and this is an issue for you, please contact the instructor or TA.
2. Update your photo on D2L to be a recognizable headshot of you.
3. Sign up for the class discussion board.

Answer I have completed the above tasks.

CSCI 432 Problem 1-3

Collaborators: *n/a*

In this class, please properly cite all resources that you use. To refresh your memory on what plagiarism is, please complete the plagiarism tutorial found here: `http://www.lib.usm.edu/plagiarism_tutorial`. If you have observed plagiarism or cheating in a classroom (either as an instructor or as a student), explain the situation and how it made you feel. If you have not experienced plagiarism or cheating or if you would prefer not to reflect on a personal experience, find a news article about plagiarism or cheating and explain how you would feel if you were one of the people involved.

Answer I saw cheating while in my high school math class. Some students copied their homework answers from other students. I felt rather indifferent about their cheating. I remember thinking that they were only cheating themselves and that it would catch up with them eventually.

CSCI 432 Problem 1-4

Collaborators:

Prove the following statement: Every tree with one or more nodes/vertices has exactly $n-1$ edges.

Answer We must show that every tree with one or more nodes has exactly $n-1$ edges. We take a tree to be a connected, undirected graph with no cycles. We will show this by using induction.

Base Case: Let's begin with the base case of a tree with a single node. Such a tree has no edges. Does our claim hold? Certainly: $n-1 = 1-1 = 0$. So the base case is proved.

Inductive assumption: We now assume that there exists some $k \in \mathbb{N}$ such that every tree with k nodes has $k-1$ edges.

Inductive step: Now can we show that every tree with $k+1$ nodes has k edges? Let's begin with a tree T_k with k nodes. From our inductive assumption, we know that T_k has $k-1$ edges. We then add one node to T_k to form T_{k+1}.

When adding node $k+1$, we are constrained to add exactly one edge. We must add a non-zero number of edges because T_{k+1} must be connected. But we cannot add more than one ege since T_k is connected and adding more than one edge would form a cycle, meaning T_{k+1} would not be a tree. So we can only add one edge when constructing T_{k+1}. So T_{k+1} must have $k-1+1 = k$ edges, which is what we needed to show in the inductive step.

Thus, by induction, we have shown that every tree with one or more nodes must have exactly $n-1$ edges.

CSCI 432 Problem 1-5

Collaborators:

Use the definition of big-O notation to prove that $f(n) = n^2 + 3n + 2$ is $O(n^2)$.

Answer To show that $f(n) = n^2 + 3n + 2 = O(n^2)$, we must show the existence of $n_0 \in \mathbb{N}$ and $c \in \mathbb{R}$ with $c > 0$ such that $f(n) \leq cn^2$ for all $n \geq n_0$. Towards finding such constants, we observe that

$$f(n) = n^2 + 3n + 2 \leq n^2 + 3n^2 + 2n^2 = 6n^2 \quad \forall n \in \mathbb{N}$$

We choose $n_0 = 1$ and $c = 6$. Thus, $f(n) = n^2 + 3n + 2$ is $O(n^2)$.

CSCI 432 Problem 1-6

Collaborators:

Consider the RightAngle algorithm on page 8 of the textbook.

1. When we design an algorithm, we design the algorithm to solve a problem or answer a question. What is the problem that this algorithm solves?
2. Prove that the algorithm terminates.

Answer

1. Given a line l and a point $p \in l$, the RightAngle algorithm draws a line that passes through p and is perpendicular to l.
2. We must now show that RightAngle terminates. I found that RightAngle terminates in all but one case.

   Case $A \neq P$: In this case, RightAngle terminates if each of its steps terminates. Steps one certainly terminates since it just a selection of a point on a line. Step two terminates because $P \neq A$ so Circle(P,A) can be drawn and there are two intersection points with l. Step three terminates since both circles can be drawn and they have two intersection points (because they are circles of radius r with distance between centers of r). Step four terminates because drawing a line between two distinct points is always possible.

   Case $A = P$: This case cannot terminate because Circle(P,P) cannot be drawn.

CSCI 432 Problem 1-7

Collaborators:

Consider the following statement: If a and b are both even numbers, then ab is an even number.

1. What is the definition of an odd number?
2. What is the definition of an even number?
3. What is the contrapositive of this statement?
4. What is the converse of this statement?
5. Prove this statement.

Answer We now give answers to the above questions.

1. The following is the definition of an odd number. If an integer n can be written as $n = 2 * k + 1$ for some integer k, then n is said to be odd.
2. The following is the definition of an even number. If an integer n can be written as $n = 2 * k$ for some integer k, then n is said to be even.
3. The contrapositive of "If a and b are both even numbers, then ab is an even number" is "If ab is an odd number, then either a or b must be odd."
4. The converse of "If a and b are both even numbers, then ab is an even number" is "If ab is an even number, then a and b are both even numbers."
5. We now prove the statement "If a and b are both even numbers, then ab is an even number." We prove this directly.

   Since a and b are both even, there exist integers k, j such that $a = 2k$ and $b = 2j$. By substitution, $ab = (2k)(2j) = 2(2kj)$. Let $n = 2kj$, we know that $n \in \mathbb{Z}$ by closure of integers with multiplication. We also know that $2n$ is even and that $2n = ab$. Since $2n$ is even, ab must be even as well. So, we have shown that given two even numbers a and b, their product ab must also be even.