Advanced Algorithms, Homework 1

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Due: 27 August 2020

CSCI 432 Problem 1-1

Collaborators: *n/a*

Answer the following questions:

1. What is your elevator pitch? Describe yourself in 1-2 sentences.
2. What was your favorite CS class so far, and why?
3. What was your least favorite CS class so far, and why?
4. Why are you interested in taking this course?
5. What is your biggest academic or research goal for this semester (can be related to this course or not)?
6. What do you want to do after you graduate?
7. What was the most challenging aspect of your coursework last semester after the university transitioned to online?
8. What went well last semester for you after the university transitioned to online?

Answer TODO: your answer goes between these lines

CSCI 432 Problem 1-2

Collaborators: *n/a*

Please do the following:

1. Write this homework in LaTex. Note: if you have not used LaTex before and this is an issue for you, please contact the instructor or TA.
2. Update your photo on D2L to be a recognizable headshot of you.
3. Sign up for the class discussion board.

Answer TODO: write a statement confirming you have completed these tasks.

CSCI 432 Problem 1-3

Collaborators: *n/a*

In this class, please properly cite all resources that you use. To refresh your memory on what plagiarism is, please complete the plagiarism tutorial found here: `http://www.lib.usm.edu/plagiarism_tutorial`. If you have observed plagiarism or cheating in a classroom (either as an instructor or as a student), explain the situation and how it made you feel. If you have not experienced plagiarism or cheating or if you would prefer not to reflect on a personal experience, find a news article about plagiarism or cheating and explain how you would feel if you were one of the people involved.

Answer TODO: your answer goes between these lines

CSCI 432 Problem 1-4

Collaborators:

Prove the following statement: Every tree with one or more nodes/vertices has exactly $n - 1$ edges.

Answer TODO: your answer goes between these lines

CSCI 432 Problem 1-5

Collaborators:

Use the definition of big-O notation to prove that $f(x) = n^2 + 3n + 2$ is $O(n^2)$.

Answer TODO: your answer goes between these lines

CSCI 432 Problem 1-6

Collaborators:

Consider the RightAngle algorithm on page 8 of the textbook.

1. When we design an algorithm, we design the algorithm to solve a problem or answer a question. What is the problem that this algorithm solves?
2. Prove that the algorithm terminates.

Answer TODO: your answer goes between these lines

CSCI 432 Problem 1-7

Collaborators:

Consider the following statement: If a and b are both even numbers, then ab is an even number.

1. What is the definition of an odd number?
2. What is the definition of an even number?
3. What is the contrapositive of this statement?
4. What is the converse of this statement?
5. Prove this statement.

Answer We now give answers to the above questions.

1. The following is the definition of an odd number. If an integer n can be written as $n = 2 * k + 1$ for some integer k, then n is said to be odd.
2. The following is the definition of an even number. If an integer n can be written as $n = 2 * k$ for some integer k, then n is said to be even.
3. The contrapositive of "If a and b are both even numbers, then ab is an even number" is "If ab is an odd number, then either a or b must be odd."
4. The converse of "If a and b are both even numbers, then ab is an even number" is "If ab is an even number, then a and b are both even numbers."
5. We now prove the statement "If a and b are both even numbers, then ab is an even number." We prove this directly.

   Since a and b are both even, there exist integers k, j such that $a = 2k$ and $b = 2j$. By substitution, $ab = (2k)(2j) = 2(2kj)$. Let $n = 2kj$, we know that $n \in \mathbb{Z}$ by closure of integers with multiplication. We also know that $2n$ is even and that $2n = ab$. Since $2n$ is even, ab must be even as well. So, we have shown that given two even numbers a and b, their product ab must also be even.