Advanced Algorithms, Homework 3

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due: 17 September 2020

This homework assignment is due on 3 September 2020, and should be submitted as a single PDF file to to Gradescope.

General homework expectations:

- Homework should be typeset using LaTex.
- Answers should be in complete sentences and proofread.
- This homework can be submitted as a group.

CSCI 432 Problem 3-1

Collaborators: *n/a*

Work in a group of size ≥ 2. Explain your strategy for working in a group.

Answer TODO: your answer goes between these lines

CSCI 432 Problem 3-2

Collaborators: *Nathan Stouffer and Kevin Browder*

Your group should make at least five contributions to the Piazza board. A contribution can be either asking a relevant question, responding to another student's question, responding to an instructor's question, or choosing a question from Chapter 1 and attempting to solve it, then describing where you get stuck in answering it.

Answer Our groups contributions are:

1. (TODO: state the problem number, name of poster, and date/time). TODO: copy the post here.
2. (TODO: state the problem number, name of poster, and date/time). TODO: copy the post here.
3. (TODO: state the problem number, name of poster, and date/time). TODO: copy the post here.
4. (TODO: state the problem number, name of poster, and date/time). TODO: copy the post here.
5. (TODO: state the problem number, name of poster, and date/time). TODO: copy the post here.

CSCI 432 Problem 3-3

Collaborators: *Nathan Stouffer and Kevin Browder*

Give the algorithm for binary search, using a for loop and no recursion.

1. Describe the problem in your own words, including describing what the input and output is.
2. Describe, in paragraph form, the algorithm you propose.
3. Provide this algorithm in the algorithm environment.
4. Use a decrementing function to prove that the loop terminates.
5. What is the loop invariant? Provide the proof.

Answer TODO: your answer goes between these lines

CSCI 432 Problem 3-4

Collaborators: *Nathan Stouffer and Kevin Browder*

Chapter 2, Problem 1b (Generalized SubsetSum).

1. Describe the problem in your own words, including describing what the input and output is.
2. Describe, in paragraph form, the algorithm you propose.
3. Provide this algorithm in the algorithm environment.
4. What is the runtime of your algorithm?
5. Prove partial correctness (that if your algorithm terminates, it is correct).

Answer TODO: your answer goes between these lines

CSCI 432 Problem 3-5

Collaborators: *Nathan Stouffer and Kevin Browder*

Describe two different data structures that you can use to store a graph. Please give a complete description (i.e., a response of "an array" will not suffice).

Answer TODO: your answer goes between these lines

CSCI 432 Problem 3-6

Collaborators: *Nathan Stouffer and Kevin Browder*

Walk through the exponential time Longest Increasing Subsequence (LIS) algorithm on page 108 for the input: $[1, 7, 6, 11, 3, 11]$.

Walk through the algorithm using the Dynamic Programming algorithm present in Section 3.6.

Answer TODO: your answer goes between these lines

CSCI 432 Problem 3-7

Collaborators: *Nathan Stouffer and Kevin Browder*

What is the closed form of the following recurrence relations? Use Master's theorem to justify your answers:

1. $T(n) = 16T(n/4) + \Theta(n)$
2. $T(n) = 2T(n/2) + n \log n$
3. $T(n) = 6T(n/3) + n^2 \log n$
4. $T(n) = 4T(n/2) + n^2$
5. $T(n) = 9T(n/3) + n$

Note: we assume that $T(1) = \Theta(1)$ whenever it is not explicitly given.

Answer TODO: your answer goes between these lines

CSCI 432 Problem 3-8

Collaborators: *Nathan Stouffer and Kevin Browder*

The skyline problem: You are in Camden, NJ waiting for the ferry across the river to get into Philadelphia, and are looking at the skyline. You take a photo, and notice that each building has the silhouette of a rectangle. Suppose you represent each building b as a triple $(x_b^{(1)}, x_b^{(2)}, y_b)$, where the building can be seen from $x_b^{(1)}$ to $x_b^{(2)}$ horizontally and has a height of y_b. Let `rect(b)` be the set of points inside this rectangle (including the boundary). Let `buildings` be a set of n such triples representing buildings. Design an algorithm that takes `buildings` as input, and returns the skyline, where the skyline is a sequence of (x, y) coordinates defining $\cup_{b \in \texttt{buildings}} \texttt{rect}(b)$. The output should start with $(\min_b x_b^{(1)}, 0)$ and end with $(\max_b x_b^{(1)}, 0)$.

1. Describe the problem in your own words, including describing what the input and output is.
2. Describe, in paragraph form, the algorithm you propose.
3. Provide this algorithm in the algorithm environment.
4. What is the runtime of your algorithm? If you do not know, either give the tightest bounds you know, or provide a decrementing function to show that it does terminate.
5. Prove partial correctness (that if your algorithm terminates, it is correct).

Answer TODO: your answer goes between these lines