M 383: Assignment 7

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Exercises 4.1.5 — Problem 2

Problem. Let A be the set defined by the equations $f_1(x) = 0, f_2(x) = 1, ..., f_n(x) = 0$, where $f_1, ..., f_n$ are continuous functions defined on the whole line. Show that A is closed. Must A be compact?

Proof.

Exercises 4.1.5 — Problem 4

Problem. Show that the function $f(x) = x^\beta$ on $[0, 1]$ for $0 < \beta \leq 1$ satisfies a Holder condition of order α for $0 < \alpha \leq \beta$ but not for $\alpha > \beta$.

Proof.

Exercises 4.1.5 — Problem 7

Problem. Give an example of a continuous function with domain $\mathbb{R}$ such that the inverse image of a compact set is not compact.

Proof.

Exercises 4.1.5 — Problem 10

Problem. Show that a function that satisfies a Lipschitz condition is uniformly continuous.

Proof.