Exercises 1.1.3 — Problem 2c

Problem. Begin with the statement "Multiplication of integers is associative." Rewrite the state with explicit quantifiers. Then form the negation of the statement. Finally, recast the negation in a form similar to the original statement.

Proof. The statement "Multiplication of integers is associative" can be rewritten with explicit qunatifiers as

$$\forall a, b, c \in \mathbb{Z}, a * (b * c) = (a * b) * c$$

The negation of the statement is

$$\exists a, b, c \in \mathbb{Z} \text{ such that } a * (b * c) \neq (a * b) * c$$

We can rewrite the negation (in words) as "Multiplication of integers is not always associative."