

### Exercises 4.1.5 — Problem 7

*Problem.* Give an example of a continuous function with domain  $\mathbb{R}$  such that the inverse image of a compact set is not compact.

*Proof.* Consider the example I gave in Problem 2 where  $f$  is the constant function that takes every real number to 0. Certainly  $f$  is continuous and the compact set  $\{0\}$  has  $\mathbb{R}$  for an inverse image under  $f$ . Although  $\mathbb{R}$  is closed, it is not bounded so  $\mathbb{R}$  is not compact.