## Exercises 3.1.3 — Problem 2

*Problem.* If a bounded sequence is the sum of a monotone increasing and a monotone decreasing sequence ($x_n = y_n + z_n$ where $\{y_n\}$ is monotone increasing and $\{z_n\}$ is monotone decreasing), does it follow that the sequence converges? What if $\{y_n\}$ and $\{z_n\}$ are bounded?

*Proof.* Suppose we have a monotone increasing sequence $\{y_n\}$ and a monotone decreasing sequence $\{z_n\}$. Is their bounded sum $\{x_n\} = \{y_n + z_n\}$ necessarily convergent? No. Consider such sequences $\{y_n\} = 1, 2, 2, 3, 3, 4, 4, ...$ and $\{z_n\} = -1, -1, -2, -2, -3, -3, -4, ...$ with sum $\{x_n\} = 0, 1, 0, 1, 0, 1, 0, ...$ which has no limit.

If we require that $\{y_n\}$ and $\{z_n\}$ are bounded, then we can claim that $\{x_n\}$ converges. This is because Theorem 3.1.2 says that any bounded, monotone increasing sequence converges (and there is an analogous result for bounded, monotone decreasing sequences). So $\{y_n\}$ and $\{z_n\}$ converge and $\{x_n\} = \{y_n + z_n\}$ is just the sum of two converging sequences, so $\{x_n\}$ must also converge.