M 383: Assignment 3

Nathan Stouffer

Exercises 2.2.4 — Problem 3

Problem. If x is a real number, show that there exists a Cauchy sequence of rationals $x_1, x_2, \ldots$ representing x such that $x_n < x$ for all n.

Proof.

Exercises 2.2.4 — Problem 7

Problem. Prove that $|x - y| \geq |x| - |y|$ for any real numbers x and y.

Proof. We must show that $|x - y| \geq |x| - |y|$ holds for all real numbers x and y. We take $|x + y| \leq |x| + |y|$ (the triangle inequality) to be true. Then let x and y be any real numbers. Certainly it is true that $|x| = |x - y + y|$ since $\mathbb{R}$ is an ordered field. Also, by the triangle inequality, $|x - y + y| \leq |x - y| + |y|$. Since $|x - y + y| = |x|$, we can say that $|x - y| + |y| \geq |x|$ which we can reorder to be $|x - y| \geq |x| - |y|$. So we have shown the desired inequality for all real numbers x and y.

Exercises 2.3.3 — Problem 1

Problem. Write out a proof that $lim_{k\to\infty}(x_k + y_k) = x + y$ if $lim_{k\to\infty}x_k = x$ and $lim_{k\to\infty}y_k = y$ for sequences of real numbers.

Proof.

Exercises 2.3.3 — Problem 3

Problem. Let $x_1, x_2, ...$ be a sequence of real numbers such that $|x_n| \leq 1/2^n$, and set $y_n = x_1 + x_2 + \cdots + x_n$. Show that the sequence $y_1, y_2, ...$ converges.

Proof. We must show that $y_1, y_2, ...$ converges. To this end, we introduce the following result: $1/n = \sum_{k=n}^{\infty} 1/2^k$ for all natural numbers. Let's now prove this result. First note that $x_{k+1} = 1/2 * x_k$.