Exercises 3.1.3 — Problem 6

Problem. Is every subsequence of a subsequence of a subsequence also a subsequence of the sequence?

Proof. Given a sequence $\{x_n\}$ with a subsequence $\{x'_n\}$ we must show that any $\{x''_n\}$ (a subsequence of $\{x'_n\}$) is also a subsequence of $\{x_n\}$. We know every element of $\{x'_n\}$ is an element of $\{x_n\}$ since $\{x'_n\}$ is obtained by crossing off elements of $\{x_n\}$. We also know that $\{x''_n\}$ is obtained by crossing off elements of $\{x'_n\}$. Then, we can obtain $\{x''_n\}$ by crossing off every element of $\{x_n\}$ that should not be in $\{x'_n\}$ and that should not be in $\{x''_n\}$. Therefore, $\{x''_n\}$ must also be a subsequence of $\{x_n\}$.