

Exercises 2.2.4 — Problem 7

Problem. Prove that $|x - y| \geq |x| - |y|$ for any real numbers x and y .

Proof. We must show that $|x - y| \geq |x| - |y|$ holds for all real numbers x and y . We take $|x + y| \leq |x| + |y|$ (the triangle inequality) to be true. Then let x and y be any real numbers. Certainly it is true that $|x| = |x - y + y|$ since \mathbb{R} is an ordered field. Also, by the triangle inequality, $|x - y + y| \leq |x - y| + |y|$. Since $|x - y + y| = |x|$, we can say that $|x - y| + |y| \geq |x|$ which we can reorder to be $|x - y| \geq |x| - |y|$. So we have shown the desired inequality for all real numbers x and y .