M 383: Assignment 4

Nathan Stouffer

Exercises 3.1.3 — Problem 1

Problem. Compute the sup, inf, limsup, liminf, and all the limit points of the sequence $x_1, x_2, \ldots$ where $x_n = 1/n + (-1)^n$.

Proof.

Exercises 3.1.3 — Problem 2

Problem. If a bounded sequence is the sum of a monotone increasing and a monotone decreasing sequence ($x_n = y_n + z_n$ where $\{y_n\}$ is monotone increasing and $\{z_n\}$ is monotone decreasing), does it follow that the sequence converges? What if $\{y_n\}$ and $\{z_n\}$ are bounded?

Proof.

Exercises 3.1.3 — Problem 4

Problem. Prove $\sup(A \cup B) \geq \sup A$ and $\sup(A \cap B) \leq \sup A$.

Proof. We begin by proving that $\sup(A \cup B) \geq \sup A$. To this end, suppose that the opposite were true: that $\sup(A \cup B) < \sup A$. By definition of sup, we know $\sup A = x$ where x is the smallest extended real number satisfying $a \leq x$ for all $a \in A$. We also know $\sup(A \cup B) = z$ where z is the smallest extended real number satisfying $a \leq z$ for all $a \in A$ and $b \leq z$ for all $b \in B$. From this, we conclude that $\sup(A \cup B) < \sup A \iff z < x$. We also know $a \leq z$ for all a, so it must be true that $a \leq z < x$ for all a. But we have just showed that x is not the smallest extended real number satisfying $a \leq x$ for all a, a contradiction! So it must be true that $\sup(A \cup B) \leq \sup A$.

Now we show that $\sup(A \cap B) \leq \sup A$.

Exercises 3.1.3 — Problem 6

Problem. Is every subsequence of a subsequence of a subsequence also a subsequence of the sequence?

Proof.

Exercises 3.1.3 — Problem 9

Problem. Can there exist a sequence whose set of limit points is exactly $1, 1/2, 1/3, ...$?

Proof.