M 384: Assignment 6

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Exercises 7.5.5 — Problem 7

Problem. If f is C^1 on [a,b] prove that there exists a cubic polynomial P such that f-P and its first derivative vanish at the endpoints of the interval.

Proof.

Exercises 7.5.5 — Problem 9

Problem. If f(c) = 0 for some point c in (a, b), prove that the polynomials approximating f on [a, b] may be taken to vanish at c.

Proof.

Exercises 7.5.5 — Problem 14

Problem.

a. For $c_m = \int_{-1}^{1} (1-x^2)^m dx$, obtain the identity $c_m = c_{m-1} - (1/2m)c_m$ by integration by parts. b. Show that

$$c_m = 2 \frac{2 * 4 * 6 * \cdots * (2m)}{3 * 5 * 7 * \cdots * (2m+1)} = \frac{2(2^m m!)^2}{2m+1}!$$

Proof.