M 384: Assignment 9

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Problem 1

Problem. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x, y) = \begin{cases} \frac{xy(x^2 - x^2)}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

(a) Show that $\partial f / \partial x$, $\partial f / \partial y$ exist for all $(x, y) \in \mathbb{R}^2$

(b) Show that both $\frac{\partial^2 f(0,0)}{\partial x \partial y}$ and $\frac{\partial^2 f(0,0)}{\partial y \partial x}$ exist but $\frac{\partial^2 f(0,0)}{\partial x \partial y} \neq \frac{\partial^2 f(0,0)}{\partial y \partial x}$

Proof.

Problem 2

Problem. For any $x = (x_1, \ldots, x_n) \in \mathbb{R}^n$ and any multi-index $\alpha = (\alpha_1, \ldots, \alpha_n)$, prove that $|x^\alpha| \leq |x|^{|\alpha|}$ where $x^\alpha = x_1^{\alpha_1} \cdots x_n^{\alpha_n}$, $|x| = \sqrt{x_1^2 + \cdots + x_n^2}$, and $|\alpha| = \alpha_1 + \cdots + \alpha_n$

Proof.