M 384: Assignment 4

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Exercises 7.3.4 — Problem 2

Problem. Suppose $f_n \to f$ and the function f_n all satisfy the Lipschitz condition $|f_n(x) - f_n(y)| \leq M|x - y|$ for some constant M independant of n. Prove that f also satisfies the same Lipschitz condition.

Proof.

□

Exercises 7.3.4 — Problem 5

Problem. If $\lim_{n\to\infty} f_n = f$ and the functions f_n are all monotone increasing, must f be monotone increasing? What happens if f_n are all strictly increasing?

Proof.

□

Exercises 7.3.4 — Problem 6

Problem. Give an example of a sequence of continuous functions converging pointwise to a function with a discontinuity of the second kind.

Proof.

□

Exercises 7.3.4 — Problem 7

Problem. If $|f_n(x)| \leq a_n$ for all x and $\sum_{n=1}^{\infty} a_n$ convreges, prove that $\sum_{n=1}^{\infty} f_n(x)$ converges uniformly.

Proof.

□