Exercises 7.6.3 — Problem 2

Problem. If $|f_n(x) - f_n(y)| \leq M|x-y|^\alpha$ for some fixed M and $\alpha > 0$ and all x, y in a compact interval, show that $\{f_n\}$ is uniformly equicontinuous.

Proof. A sequence is uniformly equicontinuous if for all $1/m$ there exists a $1/k$ such that $|x-y| < 1/k$ implies $|f_n(x) - f_n(y)| < 1/m$ for every f_n in the sequence. If we are given $1/m$, choose $1/k = (1/Mm)^{1/\alpha}$, then we have $|f_n(x) - f_n(y)| \leq M|x-y|^\alpha \leq M1/k^\alpha = M((1/Mm)^{1/\alpha})^\alpha = M/Mm = 1/m$ for $|x-y| < 1/k$ as desired.

□