

## Exercises 7.4.5 — Problem 7

*Problem.* Compute the power-series expansion of the  $f(x) = x^2/(1 - x^2)$  about  $x = 0$ .

*Proof.* We wish to find the power series expansion for  $x^2/(1 - x^2)$  about  $x = 0$ . Let  $u = x^2$ . Then

$$\frac{x^2}{1 - x^2} = \frac{u}{1 - u} = u \frac{1}{1 - u}$$

Now we know the power series for  $1/(1 - u) = \sum_{n=0}^{\infty} u^n$  so

$$u \frac{1}{1 - u} = u \sum_{n=0}^{\infty} u^n = \sum_{n=0}^{\infty} u^{n+1}$$

Since, we have  $u = x^2$ ,  $\sum_{n=0}^{\infty} u^{n+1} = \sum_{n=0}^{\infty} (x^2)^{n+1} = \sum_{n=0}^{\infty} x^{2n+2}$  so we have the power expansion for  $f(x) = x^2/(1 - x^2)$  about  $x = 0$ . Note this is valid only for  $|u| = |x^2| = x^2 < 1$ .

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