M 384: Assignment 1

Nathan Stouffer

Exercises 6.1.5 — Problem 3

Problem. Derive the integration of the derivative theorem from the differentiation of the integral theorem.

Proof.

Exercises 6.1.5 — Problem 4

Problem. Prove the integral mean value theorem: if f is continuous on $[a, b]$ then there exists y in (a, b) such that $\int_a^b f(x)dx = (b - a)f(y)$.

Proof.

Exercises 6.1.5 — Problem 8

Problem. Let f be a C^1 function on the line, and let $g(x) = \int_0^1 f(xy)y^2 dy$. Prove that g is a C^1 function and establish a formula for $g'(x)$ in terms of f.

Proof.

Exercises 6.1.5 — Problem 10

Problem. For a continuous, positive function $w(x)$ on $[a, b]$, define the weighted average operator A_w to be

$$A_w(f) = \int_a^b f(x)w(x)dx / \int_a^b w(x)dx$$

for continous functions f. Prove that A_w is linear and lies between the maximum and minimum values of f.

Proof.