Exercises 7.4.5 — Problem 8

Problem. Compute the radius of convergence of the following power series:

a. $\sum n^4/n!x^n$

b. $\sum \sqrt{n}x^n$

c. $\sum n^2 2^n x^n$

Proof.

a. Here $1/R = \limsup_{n\to\infty}(n^4/n!)^{1/n} = \lim_{n\to\infty}(n^4)^{1/n} \lim_{n\to\infty}(1/n!)^{1/n}$. Since n^4 is a polynomial in n we know $\lim_{n\to\infty}(n^4)^{1/n} = 1$ and it was proven in the textbook that $\lim_{n\to\infty}(1/n!)^{1/n} = 0$. Since the RHS is 0, $R = +\infty$.

b. Again $1/R = \limsup_{n\to\infty}(\sqrt{n})^{1/n} = (\limsup_{n\to\infty} n^{1/n})^{1/2} = 1^{1/2} = 1$. So $R = 1$.

c. As always, $1/R = \limsup_{n\to\infty}(n^2 2^n)^{1/n} = \lim_{n\to\infty}(n^2)^{1/n} \lim_{n\to\infty}(2^n)^{1/n} = 1 \lim_{n\to\infty} 2 = 2$ which means $R = 1/2$.

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