Exercises 7.6.3 — Problem 5

Problem. Give an example of a sequence that is uniformly equicontinuous but not uniformly bounded.

Proof. Consider the sequence $\{f_n(x)\}$ where we take $f_n(x) \equiv n$. Then $|f_n(x) - f_n(y)| = |n - n| = 0$ for all x, y yet there is no bound on the sequence.

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