Exercises 7.5.5 — Problem 9

Problem. If $f(c) = 0$ for some point c in (a, b), prove that the polynomials approximating f on $[a, b]$ may be taken to vanish at c.

Proof. By the WAT, we gain a sequence of polynomials f_n that converges to f uniformly. Since uniform convergence implies pointwise convergence, we know $f_n(c) \to 0$ as $n \to \infty$, so the polynomials f_n vanish at c.

□