M 384: Assignment 7

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Exercises 7.6.3 — Problem 2

Problem. If $|f_n(x) - f_n(y)| \leq M|x-y|^\alpha$ for some fixed M and $\alpha > 0$ and all x, y in a compact interval, show that $\{f_n\}$ is uniformly equicontinuous.

Proof.

□

Exercises 7.6.3 — Problem 5

Problem. Give an example of a sequence that is uniformly equicontinuous but not uniformly bounded.

Proof.

□

Exercises 7.6.3 — Problem 6

Problem. Prove that the family of all polynomials of degree $\leq N$ with coefficients in $[-1, 1]$ is uniformly bounded and uniformly equicontinuous on any compact interval.

Proof.

□

Exercises 7.6.3 — Problem 9

Problem. Give an example of a uniformly bounded and uniformly equicontinuous sequence of functions on the whole line that does not have any uniformly convergent subsequences.

Proof.

□