# Exercises 7.3.4 — Problem 6

Problem. Give an example of a sequence of continuous functions converging pointwise to a function with a discontinuity of the second kind.

Proof. We begin by defining a function with domain $[0, 1]$ and a discontinuity of the second kind at $x = 0$. For $x = 1/2k$, define $f(x) = 0$ and for $x = 1/(2k + 1)$ define $f(x) = 1$. For values in between an "even" and "odd" pair, linearly interpolate between 0 and 1. Finally define $f(0) = 0$. The left figure depicts the limit (as far as I drew it). Then on the right we have a depiction of an element of the sequence. For a
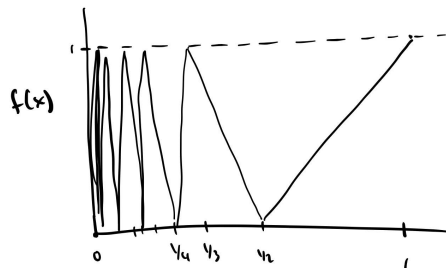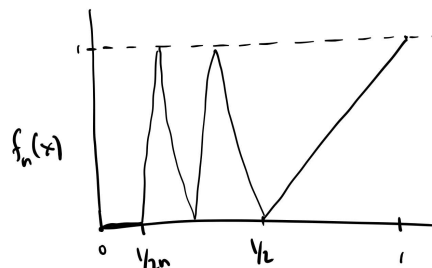
Limit function $f(x)$ Element of the sequence

function f_n in the sequence, we define $f_n(x) = 0$ for $x < 1/2n$ and defined as the limit for $x \geq 1/2n$. Certainly the sequence $f_1, f_2, \ldots$ converges pointwise to f on their common domain $[0, 1]$. Further, each f_n is continuous but the limit has a discontinuity of the second kind.

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