M 384: Assignment 5

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Problem. If f is analytic in a neighborhood of x_0 and $f(x_0) = 0$, show that $f(x)/(x - x_0)$ is analytic in the same neighborhood.

Proof.

Problem. Prove that if f(x) is analytic on (a,b), then $F(x)=\int_c^x f(t)dt$ is also analytic on (a,b), where c is any point in (a,b).

Proof.

Problem. Compute the power-series expansion of the $f(x) = x^2/(1-x^2)$ about x = 0.

Proof.

Problem. Compute the radius of convergence of the following power series:

a.
$$\sum n^4/n!x^n$$

b. $\sum \sqrt{n}x^n$
c. $\sum n^22^nx^n$

b.
$$\sum \sqrt{n}x^n$$

c.
$$\sum n^2 2^n x^n$$

Proof.