

Exercises 10.1.5 — Problem 13

Problem. Compute df for the following functions.

- a. $f : \mathbb{R}^2 \rightarrow \mathbb{R}^1$ $f(x_1, x_2) = x_1 e^{x_2}$
- b. $f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ $f(x_1, x_2, x_3) = (x_3, x_2)$
- c. $f : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ $f(x_1, x_2) = (x_1, x_2, x_1 x_2)$

Proof.

- a. Since $f : \mathbb{R}^2 \rightarrow \mathbb{R}^1$ we know that df is a 1×2 matrix.

$$df = \begin{bmatrix} \frac{\partial f}{\partial x_1} & \frac{\partial f}{\partial x_2} \end{bmatrix} = \begin{bmatrix} e^{x_2} & x_1 e^{x_2} \end{bmatrix}$$

- b. Since $f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ we know that df is a 2×3 matrix.

$$df = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

- c. Since $f : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ we know that df is a 3×2 matrix.

$$df = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ x_2 & x_1 \end{bmatrix}$$

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