M 472 – Homework 1 – Complex numbers

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Due on January 20 on Gradescope

1. Perform the following calculations and express the answer in the form $x + iy$.
   (a) $(3 - 2i) - i(4 + 5i)$
   (b) $(7 - 2i)(5 + 3i)$
   (c) $(i - 1)^3$
   (d) $\dfrac{1 + 2i}{3 - 4i} - \dfrac{4 - 3i}{2 - i}$
2. Find the principal argument $\operatorname{Arg} z$ for
   (a) $z = \dfrac{-3}{4 + 4i}$
   (b) $z = \left(-\sqrt{3} + i\right)^5$
3. Find the square roots of
   (a) $z = 4i$
   (b) $z = -1 + \sqrt{3}i$

   and express them in rectangular coordinates.
4. Find and sketch the three cube roots of $z = -2 + 2i$. (Hint: They should form the vertices of an equilateral triangle centered at zero.)
5. Assuming that $|z_3| \neq |z_4|$, show that

$$\frac{\operatorname{Re}(z_1 + z_2)}{|z_3 + z_4|} \leq \frac{|z_1| + |z_2|}{||z_3| - |z_4||}.$$

6. Use de Moivre's formula to derive the trigonometric identities

$$\cos(3\theta) = \cos^3\theta - 3\cos\theta\,\sin^2\theta,$$
$$\sin(3\theta) = 3\cos^2\theta\,\sin\theta - \sin^3\theta.$$

7. In each case, sketch the set of points determined by the given condition.
   (a) $|z - 2 + i| \leq 1$
   (b) $|2z + 3| > 4$
   (c) $|z - 1| < |z + i|$
   (d) $(\operatorname{Re} z)(\operatorname{Im} z) > 1$
   (e) $0 < \arg z < \pi/4$