# M 472 – Homework 1 – Complex numbers

Nathan Stouffer

Due on January 20 on Gradescope

1. Perform the following calculations and express the answer in the form $x + iy$.
   (a) $(3-2i) - i(4+5i)$:
   $(3-2i) - i(4+5i) = (3-2i) + (5-4i) = (3+5) + i(-2-4) = 8 - 6i$

   (b) $(7-2i)(5+3i)$:
   $(7-2i)(5+3i) = (35+6) + i(-10+21) = 41 + 11i$

   (c) $(i-1)^3$:
   $(i-1)^3 = (i-1)(i-1)(i-1) = (-1+1-i-i)(i-1) = -2i(i-1) = 2+2i$

   (d) $\dfrac{1+2i}{3-4i} - \dfrac{4-3i}{2-i}$:
   $\dfrac{1+2i}{3-4i} - \dfrac{4-3i}{2-i} = \dfrac{1+2i}{3-4i}\dfrac{3+4i}{3+4i} - \dfrac{4-3i}{2-i}\dfrac{2+i}{2+i} = \dfrac{3-8+6i+4i}{3^2+4^2} - \dfrac{8+3-6i+4i}{2^2+1^2} =$
   $\dfrac{-5+10i}{25} - \dfrac{11-2i}{5} = \dfrac{-1+2i}{5} + \dfrac{-11+2i}{5} = \dfrac{-12+4i}{5} = -12/5 + i4/5$

2. Find the principal argument $\operatorname{Arg} z$ for
   (a) $z = \dfrac{-3}{4+4i}$
   (b) $z = \left(-\sqrt{3}+i\right)^5$

3. Find the square roots of
   (a) $z = 4i$
   (b) $z = -1 + \sqrt{3}i$

   and express them in rectangular coordinates.

4. Find and sketch the three cube roots of $z = -2 + 2i$. (Hint: They should form the vertices of an equilateral triangle centered at zero.)

5. Assuming that $|z_3| \neq |z_4|$, show that

$$\frac{\operatorname{Re}(z_1+z_2)}{|z_3+z_4|} \leq \frac{|z_1|+|z_2|}{||z_3|-|z_4||}.$$

6. Use de Moivre's formula to derive the trigonometric identities

$$\cos(3\theta) = \cos^3\theta - 3\cos\theta\,\sin^2\theta,$$
$$\sin(3\theta) = 3\cos^2\theta\,\sin\theta - \sin^3\theta.$$

7. In each case, sketch the set of points determined by the given condition.

(a) $|z - 2 + i| \leq 1$

(b) $|2z + 3| > 4$

(c) $|z - 1| < |z + i|$

(d) $(\operatorname{Re} z)(\operatorname{Im} z) > 1$

(e) $0 < \arg z < \pi/4$