M 472 – Homework 3

Cauchy-Riemann Equations, Elementary Functions

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Due Friday, February 12, on Gradescope

1. Assume that a function f is analytic in a domain D, and that f is real-valued. Show that f must be a constant function. (Hint: Use the Cauchy-Riemann equations.)

2. Find all values of z such that
   (a) $e^z = -1 + i$,
   (b) e^z is purely imaginary,
   (c) $|e^z| < e^2$.

3. Find $\operatorname{Log}(i^3)$ and $\operatorname{Log} i$, and show that $\operatorname{Log}(i^3) \neq 3 \operatorname{Log} i$.

4. (a) Find a complex number z such that $\operatorname{Log}(e^z) \neq z$.
   (b) For which complex numbers z does the equality $\operatorname{Log}(e^z) = z$ hold?

5. (a) Find the principal value of $(-1 + i)^i$.
   (b) Find all values of $(-1 + i)^i$.

6. Find all values z such that
   (a) $\sin z = 2$,
   (b) $\sin z = 2i$.