CSCI 338 Computer Science Theory

Assignment 1 (8 marks)

Question 1 (1 marks)

Prove that $1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{1}{3}n(4n^2 - 1)$.

Question 2 (1 marks)

Given a planar graph $P = (V, E)$, we have Euler's formula: $|V| + |F| - |E| = 2$, where F is the set of faces of P and E is the set of edges of P. Let $|V| = n$, where V is the set of vertices of P. Prove that $|F|$ is at most $2n$.

Question 3 (2 marks)

Prove that in any simple graph there is a path from any vertex of odd degree to some other vertex of odd degree.

Question 4 (2 marks)

A fully binary tree T is a tree such that all internal nodes have two children. Prove that a fully binary tree with n internal nodes in total has $2n + 1$ nodes.

Question 5 (2 marks)

Given an undirected graph $G = (V, E)$, the breadth-first-search starting at $v \in V$ ($bfs(v)$ for short) is to generate a shortest path tree starting at vertex $v \in V$. The diameter of G is the longest of all shortest paths $\delta(u, v), u, v \in V$.

When G is a tree, the following algorithm is proposed to compute the diameter of G.

1. Run $bfs(w), w \in V$, and compute the vertex $x \in V$ furthest from w.
2. Run $bfs(x)$ and compute the vertex $y \in V$ furthest from x.
3. Return $\delta(x, y)$ as the diameter of G.

Prove that this algorithm is correct; i.e., $\delta(x, y)$ is in fact the longest among all the shortest paths between $u, v \in V$.

Date Due: **Tuesday, Jan 31, 2017, at 11:30pm.** Submissions must be scanned and submitted through D2L in the folder `Assignment 1`, in a pdf file like `last_first_assign1.pdf` (e.g., `zhu_binhai_assign1.pdf`). No late assignment will be accepted. It is recommended that you use `Latex` to compose your solution, unless you are sure that your handwriting is decent.