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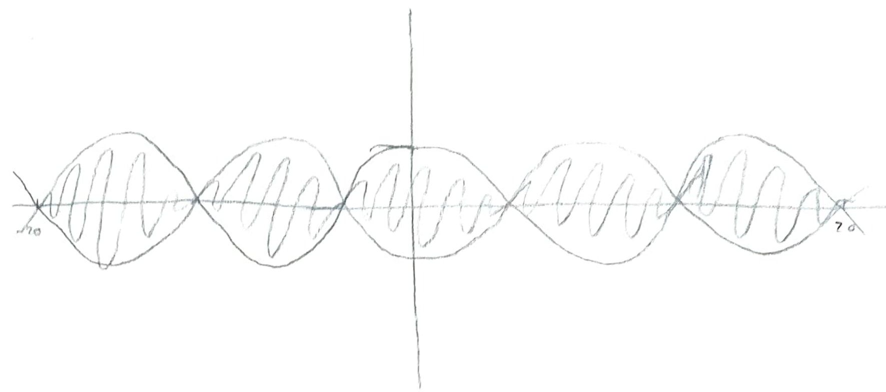
Dynamics
HW 4

4.2.2| Graph $x(t) = \sin 8t + \sin 9t$ for $-20 < t < 20$.



$$x(t) = \sin 8t + \sin 9t = 2\sin\left(\tfrac{17}{2}t\right)\cos\left(\tfrac{1}{2}t\right) = 2\cos\left(\tfrac{1}{2}t\right)\sin\left(\tfrac{17}{2}t\right)$$

(a) the period of the amplitude modulations looks to be 4π!

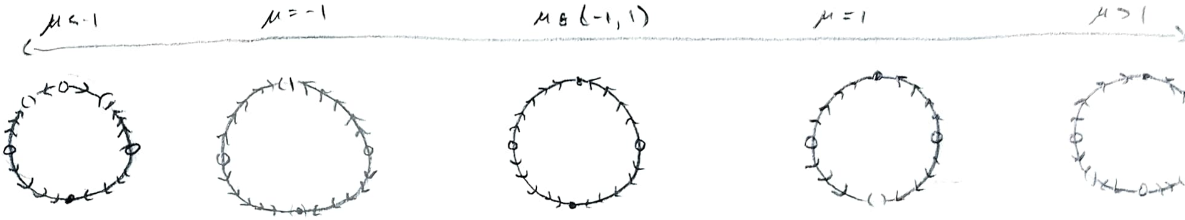
(b) solving analytically, $\sin(17/2\, t)$ oscillates quickly while $\cos(1/2\, t)$ oscillates slowly (which will be the amplitude modulation).

So $T = 2\pi/(1/2) = 2\pi \cdot 2 = 4\pi$

4.3.8| For $\dot{\theta} = \sin(2\theta)/(1+\mu\sin\theta)$ draw the phase portrait as a function of μ and classify the bifurcations

$\dot{\theta} = 0 \iff \sin(2\theta) = 0$ which occurs when $\theta = k\frac{\pi}{2}$ for some $k \in \mathbb{Z}$.
However, the phase portrait could also change when the denominator is 0 because that provides a discontinuity:

$\mu < -1$ $\quad$ $\mu = -1$ $\quad$ $\mu \in (-1, 1)$ $\quad$ $\mu = 1$ $\quad$ $\mu > 1$



"saddle node bifurcations" $\quad$ "saddle node"

4.4.4| If we add a torsional spring with opposing torque $-k\theta$ then our force eqn becomes

$$b\dot{\theta} + mgL\sin\theta = \Gamma - k\theta$$

(a) This equation does not have a well defined vector field on the circle. (consider $\theta = 0$ and $\theta = 2\pi$).

(b) Non-dimensionalize this equation

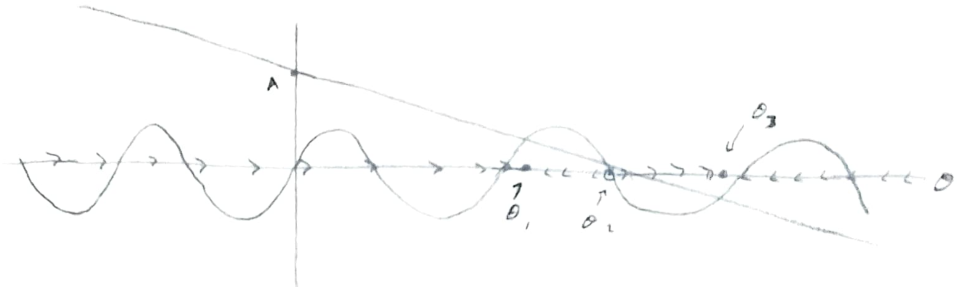
$$b\dot{\theta} = \Gamma - k\theta - mgL\sin\theta \to \frac{b}{mgL}\dot{\theta} = \frac{\Gamma}{mgL} - \frac{k}{mgL} - \sin\theta$$

let $A = \Gamma/mgL$ and $B = k/mgL$, then let $\tau = mgL/b\ t$ so that $\frac{d\theta}{d\tau}\frac{mgL}{b} = \dot{\theta}$
then our eqn simplifies to

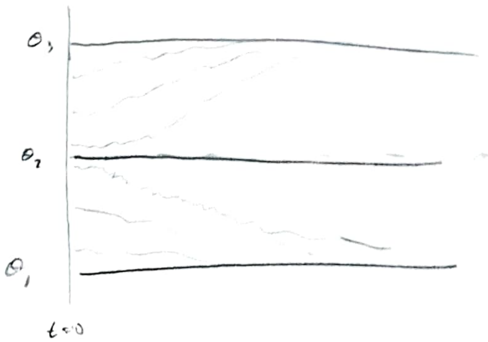
$$\frac{d\theta}{d\tau} = \theta' = A - B\theta - \sin\theta$$

(c) What does the pendulum do in the long run?

The system has fixed points where $A - B\theta = \sin\theta$ (for $\theta \in \mathbb{R}$)



The system has a number of fixed points θ_2 is unstable while θ_1 and θ_3 are stable. The system will most likely be attracted to θ_1 or θ_3 depending on the initial conditions.



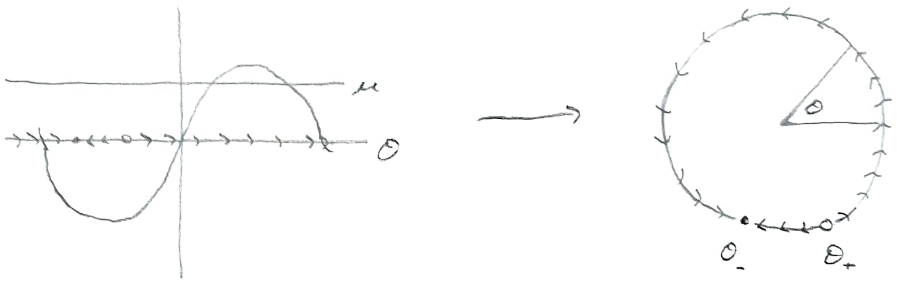
(d) Show that many bifurcations occur as k is varied from $(0, \infty)$. Classify the bifurcations

k affects $B = k/mgL$ which is the slope of the line in the graph from part c. Varying the slope will move where the line intersects $\sin\theta$. These bifurcations will be saddle nodes because they can disappear/appear. However, once they appear, they may disappear soon and vice versa.

4.5.3 | Let $\dot{\theta} = \mu + \sin\theta$ for μ close to 1.

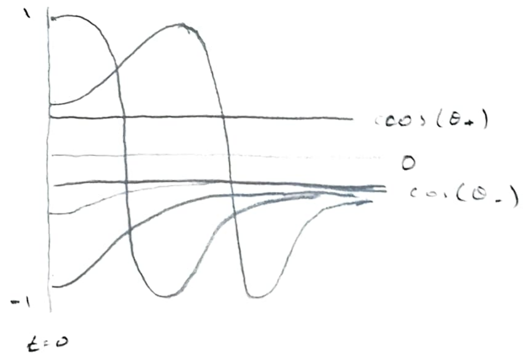
(a) Show that $\dot{\theta} = \mu + \sin\theta$ satisfies the conditions of an excitable system. What is the threshold/stable point

First note that we know $f(\theta)$ to be 2π periodic:



θ_- plays the role of the stable point while θ_+ is the threshold.

(b) $V(t) = \cos\theta(t)$. Sketch $V(t)$ for various initial conditions



sorry, it got a bit busy.