

Bucket problem

I started with the change in potential energy for a change in height Δh :

$$\Delta PE = \rho * g * A * h^2 - \rho * g * A * (h - \Delta h)^2 = \rho * g * A (h^2 - (h^2 - 2 * h * \Delta h + \Delta h^2))$$

We can simplify ΔPE to

$$\Delta PE = \rho * g * A (2 * h * \Delta h + \Delta h^2)$$

From here, we can note that the change in potential energy is equal to the kinetic energy leaving the bucket:

$$KE = (1/2) * \rho * A * \Delta h * v^2$$

Then, setting $\Delta PE = KE$, we get

$$\rho * g * A (2 * h * \Delta h + \Delta h^2) = (1/2) * \rho * A * \Delta h * v^2$$

$$2 * g * h * \Delta h + g * \Delta h^2 = (1/2) * \Delta h * v^2$$

$$4 * g * h + g * \Delta h = v^2$$

As we let Δh approach 0, we just get

$$v^2 = 4 * g * h$$

I can't seem to find where I went wrong, let me know what you see. Thanks
Tomas!