3.3.6

We are given the problem

$$\dot{x} = xy - 1$$
$$\dot{y} = x - y^3$$

and we are supposed to find/classify the fixed points and do some other stuff with the phase portrait.

I found my Jacobian to be $J = \begin{bmatrix} y & x \\ 1 & -3y^2 \end{bmatrix}$ and my equilibria points to be $(1, 1)$ and $(-1, -1)$. I think I have the case where $(x^*, y^*) = (1, 1)$ under control but I am struggling on the other equilibria.

If $(x^*, y^*) = (-1, -1)$, then $J_{(-1,-1)} = \begin{bmatrix} -1 & -1 \\ 1 & -3 \end{bmatrix}$ and the repeated eigenvalue is -2 with eigenvector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. So the linear system is a degenerate node because the eigenspace is 1 dimensional.

I am unsure if this transfers to the non linear system. On the one hand, it feels like this is inidicative of the nonlinear system since the real part of the eigenvector is greater than 0. But on the other hand, a degenerate case feels like it should not transfer to the nonlinear system because it is sensitive to changes in the eigenvalues.

Let me know what you think. Thanks Tomas!