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Distance-based functions for image comparison

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Abstract

The interest in digital image comparison is steadily growing in the computer vision community. The definition of a suitable comparison measure for non-binary images is relevant in many image processing applications. Visual tasks like segmentation and classification require the evaluation of equivalence classes. Measures of similarity are also used to evaluate lossy compression algorithms and to define pictorial indices in image content based retrieval methods. In this paper we develop a distance-based approach to image similarity evaluation and we present several image distances which are based on low level features. The sensitivity and effectiveness are tested on real data.

Keywords: Similarity; Image comparison; Distance function; Image analysis

1. Introduction

Estimation of image similarity is an important problem of image analysis. Measures of similarity between two images are useful for the comparison of algorithms devoted to noise reduction, image matching, image coding and restoration. Visual tasks are often based on the evaluation of similarities between *image-objects* represented in an appropriate feature space.

Content-based query systems may process a query on the basis of a classification procedure, that assigns the unknown to the closest available prototype. The performance of the whole query system depends on the definition of suitable similarity measure based on Image Distance Functions (IDFs) (Danielson, 1980; Russ and Russ, 1989). Image segmentation and classification require the evaluation of equivalence classes. Measures of similarity are also used to evaluate compression algorithms (Wilson et al., 1997), to define pictorial indices in image retrieval methods and to compare image restoration methods (Zamperoni and Starovoitov, 1996).

Image comparison is often performed by computing a *correlation function*, the *root of the mean square-error* or measures of the *signal-to-noise ratio*. The last approach is applicable only if there is enough knowledge of the image content. In the case of binary images, the comparison problem is much simpler; in fact the image content is easily extracted by assuming the set of black pixels as objects, and the remaining pixels as the background. In (Klette and Zamperoni, 1987) several measures of correspondence between binary images were described and compared. The authors

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have shown that distance-based measures perform better comparison of binary images than measures based on set memberships. Baddeley (1992) presented a new *error metric* for binary images. He calculated distances from every point of a two-dimensional image space to the nearest object pixel for both images.

If we operate on gray scale or color images, there are two basic means of comparison: (1) to extract some objects of interest by thresholding, segmentation, edge and shape detection, and then to compare the objects; (2) to compare images as whole entities. The first method leads to high level image recognition, while the second leads to low level image analysis. In both cases the choice of the suitable feature space (type and number of feature parameters) is a critical and not fully solved problem.

Global features directly derived from gray levels (e.g. first and second order statistics, color) can give a coarse indication of image similarity. However, they may produce unstable indications, because quite different images may have similar histograms. On the other hand, *structural* features (e.g. edges, skeleton, medial axis, convex hull, object symmetry) are very sensitive to the level and kind of noise in the image; moreover their estimation depends on the operators applied. For example, Mokadem et al. (1996) and Ghorbel (1994) developed an invariant shape distance for comparing geometric objects extracted from images. The distance was based on Fourier coefficients. In (Moghaddam et al., 1996) a method, invariant to affine transformations, is also proposed.

Distance functions, combining global and structural information, seem to be more adequate to characterize low-level similarity of images. Several distance-based ideas were developed for a measure design for gray-scale image comparison. Huttenlocher et al. (1993) and Dubuisson and Jain (1994) have developed object similarity measures, which were based on the Hausdorff metric. The new distance-based measures (mainly non-metrics) were applied to edge matching of binarized gray level images. Distance-based measures were tested for image segmentation by Kara Falah and Bolon (1992). Di Gesù (1994) proposed a new version of the *Mahalanobis distance* in order to cluster homogeneous pixels for image segmentation. Pal and Majumder (1986) considered similarity measure based on *fuzzy* metrics defined in a feature space. Tegolo (1994) partitioned two images D_1 and D_2 in $n \times m$ sub-images, and calculated the generalized invariant moments for each sub-image. The $\mathrm{IDF}(D_1,\ D_2)$ was defined as *the Euclidean distance* between vectors containing the list of invariant moments related to the images. It was applied in a content-based query system.

Wilson et al. (1997) extended the idea of *Baddeley's error metric*. However, computation of the new measure, which is used on gray scale images, is time consuming. The same authors tested also the *Sobolev norm* for image comparison purposes. It is calculated in the frequency domain by *the Fourier transform* of compared images. However, the Sobolev norm was not normalized and the results of its application were similar to those obtained by the simpler well-known *root-mean-squared error*.

In this paper we present new versions of IDFs based on *local distances* combining intensity and structural image features in different ways. The functions may also be named *hybrid* IDFs. They are implemented for the direct comparison of non-binary images without calculating visible features like edges or shapes.

Our experiments demonstrate the benefits of the new IDFs. The results indicate that better sensitivity in comparison of similar digital images can be reached by using the distance-based measures.

The paper is organized as follows. Section 2 describes the IDFs introduced. Section 3 illustrates part of our experimental results performed for real images. Final remarks are given in Section 4.

2. Image distance functions

A digital image $\boldsymbol{A}$ is a discrete function defined in a lattice domain D of size $N \times N$ and taking values in the set of gray levels $\{0, 1, \ldots, G\}$. Here, without losing generality, we consider an image $\boldsymbol{A}$ as a set of pixels $\{A_{ij}\}$, where every pixel is defined by its spatial coordinates (i, j) and gray value a_{ij}, i.e., a pixel is a point $A_{ij} = (i, j, a_{ij})$ in the 3-d space.

Four new versions of IDFs are described in this section. The first three are based on the distances between pixels of two given images $\boldsymbol{A} = \{A_{ij}\}$ and $\boldsymbol{B} = \{B_{ij}\}$ considered as points in 3-d space. The last function is a distance between patterns presented by local symmetry features calculated for every pixel of the compared images.

Images of equal size and gray-level scale may be compared by the formulae given below.

2.1. The Hausdorff-based distance

The normalized Hausdorff metric, HG, adopted for the direct comparison of gray-scale images $\boldsymbol{A}$ and $\boldsymbol{B}$, calculate the maximal inter pixel distance between the images,

$$\mathrm{HG}(\boldsymbol{A}, \boldsymbol{B}) = \max_{i,j}\{d(A_{ij}, \boldsymbol{B}), d(B_{ij}, \boldsymbol{A})\}, \tag{1}$$

where d is a metric defined in 3-d digital space to calculate distance from a pixel A_{ij} to the image $\boldsymbol{B}$,

$$d(A_{ij}, \boldsymbol{B}) = \min_{B_{lm}\in \boldsymbol{B}}\{d(A_{ij}, B_{lm})\}. \tag{2}$$

The same metric is used to calculate distance from a pixel B_{ij} to the image $\boldsymbol{A}$,

$$d(B_{ij}, \boldsymbol{A}) = \min_{A_{lm}\in \boldsymbol{A}}\{d(B_{ij}, A_{lm})\}. \tag{3}$$

In experiments we used *the chess* and *the city-block metrics* as the function d,

$$d^{\text{city}}(A_{ij}, B_{lm}) = (|i-l| + |j-m|)/N + |a_{ij} - b_{lm}|/G,$$

$$d^{\text{chess}}(A_{ij}, B_{lm}) = \max\{|i-l|/N, |j-m|/N, |a_{ij} - b_{lm}|/G\},$$

because they are faster for calculations. The complexity of the HG computation is $\mathrm{O}(N^4)$.

2.2. The local distance based function

The averaged distance, AD, calculates the square root from averaged local distances,

$$\mathrm{AD}(\boldsymbol{A}, \boldsymbol{B}) = \sqrt{\frac{1}{2(N-2W)}\sum_{i,j=W}^{N-W}\sqrt{d^2(A_{ij}, \boldsymbol{B}_{W_{ij}}) + d^2(B_{ij}, \boldsymbol{A}_{W_{ij}})}}, \tag{4}$$

where $\boldsymbol{A}_{W_{ij}}$ and $\boldsymbol{B}_{W_{ij}}$ are sub-images of the images $\boldsymbol{A}$ and $\boldsymbol{B}$ limited in the spatial domain D by a digital square of size $2W+1$ centered in the point $(i,j) \in D$; in our experiments the distance function d has been computed as in the formulae (2) and (3).

To avoid edge effects we tested $(N-2W)^2$ central pixels of any image. It follows that the complexity of the measure AD is $\mathrm{O}((N-W)^2(2W)^2)$.

2.3. The global feature based distance

The global distance, GD, combines spatial and intensity pixel features in another way:

$$\mathrm{GD}(\boldsymbol{A}, \boldsymbol{B}) = \frac{2}{N^2(N^2-1)}\sum_{A_{ij}\in \boldsymbol{A}}\sum_{B_{lm}\in \boldsymbol{B}}\delta(A_{ij}, B_{lm}), \tag{5}$$

$$\delta(A_{ij}, B_{lm}) = \alpha d^D((i,j),(l,m)) + \beta \mathrm{GR}(a_{ij}, b_{lm}), \tag{6}$$

$$\mathrm{GR}(a_{ij}, b_{lm}) = \frac{|a_{ij} - b_{lm}|}{\max(a_{ij}, b_{lm})}, \tag{7}$$

the weighting of α and β are some coefficients such that $0 < \alpha, \beta < 1$ and $\alpha + \beta = 1$. They allow to weight the two components of δ. The function d^D is a digital metric defined in the 2-d image spatial domain D, divided by the diameter of the domain D with respect to this metric. If d^D is the city-block metric, the diameter equals $2N$. The function GR calculates the contrast between two compared pixels.

Direct computation of GD for images $\boldsymbol{A}$ and $\boldsymbol{B}$ is computationally time consuming with $\mathrm{O}(N^4)$. It can be approximated by the mean value of the same functions but calculated for smaller square sub-images $\boldsymbol{A}_{st} \subset \boldsymbol{A}$ and $\boldsymbol{B}_{st} \subset \boldsymbol{B}$ with size $n \times n$ where $n \ll N$,

$$\mathrm{GD}(\boldsymbol{A}, \boldsymbol{B}) = \frac{1}{n^2}\sum_{s=0}^{N/n}\sum_{t=0}^{N/n}\mathrm{GD}(\boldsymbol{A}_{st}, \boldsymbol{B}_{st}). \tag{8}$$

In our study the sub-images form a partition of the images $\boldsymbol{A} = \cup \boldsymbol{A}_{st}$ and $\boldsymbol{B} = \cup \boldsymbol{B}_{st}$. The indices (s,t) define the upper-left pixel position $(i,j) = (s \times n, t \times n)$ in the images $\boldsymbol{A}$ and $\boldsymbol{B}$. In this way the computational complexity becomes $\mathrm{O}(n^4)$ for every $\mathrm{GD}(\boldsymbol{A}_{st}, \boldsymbol{B}_{st})$ of n^2 pairs and totally $\mathrm{O}(N^2 n^2)$. In the experiments we used $n = 5, \alpha = 0.5, \beta = 0.5$.

2.4. The symmetry based distance

To get local symmetry features with respect to several axes, we applied *the Discrete Symmetry Transform*, $\boldsymbol{T}$, to the images $\boldsymbol{A}$ and $\boldsymbol{B}$ (Di Gesù and Valenti, 1996). The transform $\boldsymbol{T}$ of an image $\boldsymbol{A}$ gives a set of K axial moments computed in a digital disk C_R of radius R centered on each point (i, j) of the spatial domain D,

$$T_k^A(i,j) = \sum_{(i+r,j+s)\in C_R} \left| r\sin\left(\frac{k\pi}{N}\right) - s\cos\left(\frac{k\pi}{N}\right) \right| a_{i+rj+s}, \tag{9}$$

with $k = 0, 1, \ldots, k, \ldots, K-1$. The maximum value of K depends of the circle defined in the spatial domain D. After the application of the Discrete Symmetry Transform, the image $\boldsymbol{A}$ may be represented by a set of K feature matrices,

$$\boldsymbol{T}(\boldsymbol{A}) = (T_0^A, T_1^A, \ldots, T_k^A, \ldots, T_{K-1}^A).$$

The symmetry distance SD between two images is then calculated as a function of the features $\boldsymbol{T}(\boldsymbol{A})$ and $\boldsymbol{T}(\boldsymbol{B})$,

$$\mathrm{SD}(\boldsymbol{T}(\boldsymbol{A}), \boldsymbol{T}(\boldsymbol{B})) = \frac{1}{N^2 K} \sum_{i,j=0}^{N} \sum_{k=0}^{K} \mathrm{GR}(T_k^A(i,j), T_k^B(i,j)), \tag{10}$$

where GR is computed like in Eq. (7), but for real values of the transformed images. It can be easily shown that the SD is still a distance function. In experiments we used $R = 5$ and $K = 3$. The complexity of the measure SD is $\mathrm{O}(N^2 KC)$, where C is the number of points in the digital disk C_R.

2.5. Widely used measures

The above measures have been compared with *the normalized cross correlation ratio*, CO and *the root-mean-squared error*, SE, which is equal to the *normalized Euclidean metric* applied just to image gray values.

$$\mathrm{CO}(\boldsymbol{A}, \boldsymbol{B}) = 1 - \frac{\sum_{i,j=0}^{N} a_{ij} b_{ij}}{\sqrt{\sum_{i,j=0}^{N} a_{ij}^2 \sum_{i,j=0}^{N} b_{ij}^2}},$$
$$\mathrm{SE}(\boldsymbol{A}, \boldsymbol{B}) = \frac{1}{NG} \sqrt{\sum_{i,j=0}^{N} (a_{ij} - b_{ij})^2}. \tag{11}$$

Computational cost of these measures is $\mathrm{O}(N^2)$.

2.6. Properties

The functions HG, AD, GD and SD defined above satisfy some properties which are useful in image comparison applications. These properties are:

- *Normalization*. Their values are in the interval [0, 1]. This property allows to minimize effects due to different and uniform illumination.
- *Consistency*. Their values equal zero, when digital images compared are identical and equal one, when one image has constantly zero gray values, and the other one has uniform gray values set to the maximum G.
- *Symmetry*. They are symmetric functions.
- *Triangularity*. All of them, except AD and CO, satisfy the triangular inequality.
- *Complexity*. The lowest complexity of comparison functions equals $\mathrm{O}(N^2)$ and the highest one equals $\mathrm{O}(N^4)$, but the calculation time depends on the operations used.

It follows that functions HG, GD, SD and SE are metrics, while AD and CO are similarity functions. Nevertheless it must be noted that perception experts make a distinction between *geometric* and *perceptual* distances. For example, the triangular inequality property is not always satisfied in human perception as noted by Ashby and Perrin (1988). The violation of the symmetry property has also been noted in human perception.

To reduce the complexity of the HG and AD we halted calculation of local distances when they reached their minimal possible values. For example, when we use the chess metric d, then the minimal values of $d(A_{ij}, \boldsymbol{B})$ equal

$$d(A_{ij}, \boldsymbol{B}) = \min_{B_{lm}\in\boldsymbol{B}} \{\max\{|i-l|, |j-m|\}\} \leqslant W,$$

if $a_{ij} = b_{lm}$. After this improvement complexity of the AD and HG is about $O(N^2\gamma)$ and $\gamma \ll N^2$ for very similar images.

3. Experimental results

A set of experiments has been carried out to test the sensitivity of the new IDFs. For this purpose, a sample of about 100 real gray scale images has been used. Images were initially grouped by humans and then automatically compared. Test images used are digital photos with $G = 255$ from various pictorial databases. We summarize our results of just two experiments in this paper. Two sets of test images are shown in Figs. 1 and 2. The images are taken from the JACOB pictorial database (La Cascia and Ardizzone, 1996). Several images were acquired under different lighting conditions. One can easily see the similarity of several scenes presented in Figs. 1 and 2, but all digital images are different.

The goal of the first experiment was to test the sensitivity of the IDFs introduced in Section 2 in the case of small local changes in real images (shift, small rotations ($\leqslant 5°$), and small affine deformations). Fig. 1 displays three groups of very close frames presenting digital images with slightly changing positions of speaking people. We compared each of the nine images with the others. Usually, the studied IDFs demonstrate that every image of one row in Fig. 1 is more similar to other images from the same row than to the images from other rows. In Table 1 the values of all measures are evaluated with respect to the prototype image $a1$, which is the leftmost picture in Fig. 1. The function AD was calculated with $W = 7$ and $W = 10$ for images in Figs. 1 and 2, respectively. Based on the CO, we may make an incorrect conclusion that images $a1$ and $a3$, $a1$ and $c3$ (Fig. 1) have the same similarity, although other measures show larger difference between these image pairs. This test demonstrates better sensitivity of the distance-based measures applied to very similar digital images of the same scenes. Hence, these measures may be used for coarse image recognition.

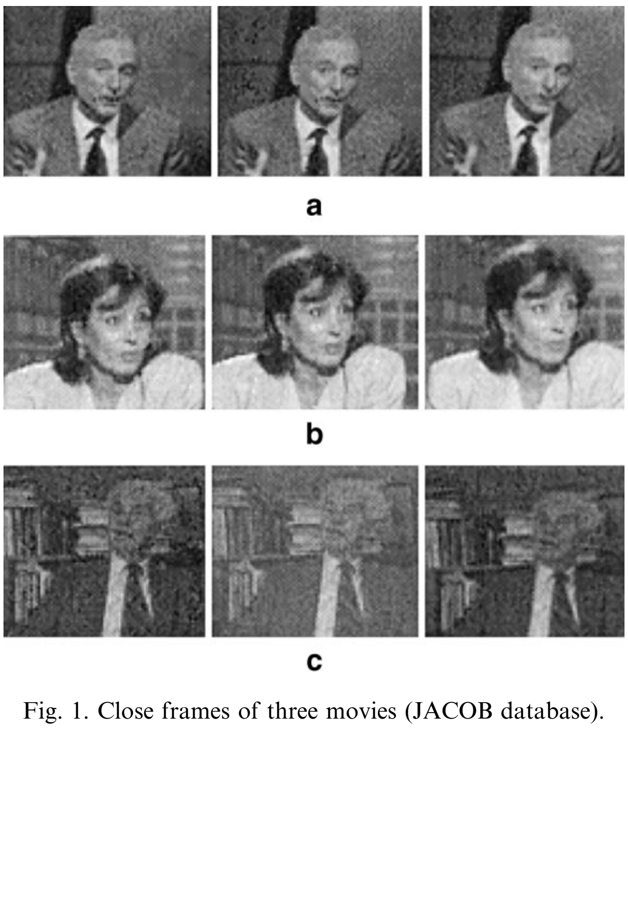

Fig. 1. Close frames of three movies (JACOB database).

The second experiment has been performed to measure the sensitivity to local and global changes in various scenes. We compared images data containing both similar and different scenes (landscapes, people and animals). Fig. 2 shows a set of 16 test images. Table 2 presents several experimental results when the studied measures were applied to the images. We grouped experimental results into two categories: neighbor and far frames of the same movie (from human point of view the pictures presented by these frames are similar) and frames from different movies. Note that all new measures reflect the similarity of the closest digital images (the neighbor frames of the same movie); for example, images 06 and 07, 12 and 13 in Fig. 2. The measure based on the SE considers images 11 and 15 more similar than images 12 and 13! That is the result of the distance calculation just in the intensity space. Images 01, 02 and 03 of Fig. 2 present three very separate frames of one movie. Human vision can recognize global similarity of these images, but only leftmost parts of the digital images have small locally similar areas. To analyze the similarity of such images by local methods one can apply a measure that combines both intensity and local structural information as in SD.

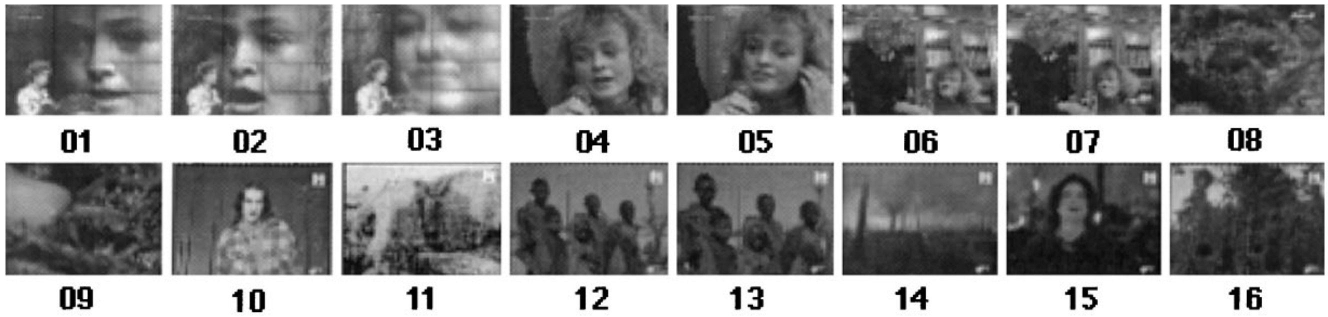

Fig. 2. Close, separate and different television frames (JACOB database).

Table 1
Comparison of the image $a1$ with other images presented in Fig. 1

Image	HG	AD	GD	SD	CO	SE
$a2$	0.05	0.09	0.15	0.10	0.03	0.08
$a3$	0.04	0.09	0.19	0.20	0.06	0.06
$b1$	0.39	0.24	0.31	0.36	0.13	0.41
$b2$	0.41	0.26	0.23	0.36	0.11	0.43
$b3$	0.38	0.26	0.30	0.38	0.12	0.48
$c1$	0.45	0.18	0.40	0.43	0.09	0.65
$c2$	0.36	0.17	0.32	0.42	0.06	0.68
$c3$	0.50	0.18	0.33	0.43	0.08	0.60

Table 2
Comparison of close, far and different frames shown in Fig. 2

Measure	Similar scenes					Different scenes				
	Close frames		Far frames			Different frames				
	06,07	12,13	01,02	01,03	02,03	12,14	13,14	05,08	08,09	11,15
HG	0.06	0.11	0.52	0.73	0.73	0.40	0.39	0.43	0.28	0.60
AD	0.09	0.10	0.21	0.24	0.27	0.19	0.19	0.17	0.17	0.59
GD	0.20	0.21	0.25	0.24	0.23	0.27	0.27	0.35	0.30	0.69
SD	0.34	0.30	0.31	0.30	0.25	0.35	0.36	0.40	0.50	0.40
CO	0.06	0.07	0.07	0.13	0.12	0.10	0.10	0.12	0.14	0.11
SE	0.07	0.23	0.66	0.49	0.51	0.63	0.60	0.58	0.22	0.15

All measures give bigger values for different frames. A threshold value has been evaluated from a training set to decide the similarity between images. However, this must be done for each IDF separately, because in spite of the normalization property in the interval [0, 1], it is not possible to fix a unique discrimination threshold value.

Correlation functions and the root-mean-squared error are less sensitive measures for comparison of such kind of images. For example, CO(01, 03) = 0.13, but CO(11, 15) = 0.11 when images 11 and 15 are the most dissimilar in this set. SE and functions using gray value correlation are less robust to scene translations. The Hausdorff-based measures are most sensitive to any noise, because of their extremal property. Less sensitive to this kind of image changes are functions based on accumulation of local distortions like AD, GD and SD (see Table 3).

Table 3
Comparison between human and automatic similarity evaluation

	AD, GD and SD error (%)	HG, CO and SE error (%)
Similarity	2	7
Dissimilarity	3	8

The new IDFs demonstrate better sensitivity in comparison of similar images and better separation between different image classes. In particular, SD and GD show a better similarity preservation of the same structural scene features when the scene has non-local changes presented in the compared images (images 02 and 03 in Fig. 2). Moreover, the SD measure is sensitive to the local geometric distribution of the pixel values, allowing to match point of interests in the images under comparisons.

Some other experiments with IDF of new types can be found in (Di Gesù and Valenti, 1996; Starovoitov and Marcelli, 1997; Zamperoni and Starovoitov, 1996). Additionally, we can report that the function AD is more robust to impulse noise, small image shifts (up to W pixels), small image rotations (up to 4°) and small intensity changes (up to W gray values). Locally, for neighboring images from one image sequence it satisfies the triangular inequality.

4. Conclusions

In the paper we present three new *image distance-based functions* for digital image comparison. We emphasize the digital nature of the images because by low-level methods we compare just digital images, but not scenes or objects presented in the images. Experimental results indicate better sensitivity of the functions by combining both global intensity and local structural features with respect to conventional intensity-based measures.

However, our study allows only to make a qualitative evaluation of their performance, because there are no formal criteria to compare various similarity measures and to define a universal (problem independent) function.

Moreover, the approach may depend on the nature of the treated problem. Depending on the data, even humans can find similarity with different meaning. For example, Daughman (1997) reported that in any real database of 100,000 face images, one would expect to find about 417 persons who have identical twins. In a database of 50,000,000 faces there should be nearly 250,000 persons who have identical twins! Hence it is impossible to find structural or other features for 100% correct recognition of the human photos from a large database. That gives a theoretical limit which equals approximately 99.5%. On the other hand problem oriented methods cannot be applied for general kind of images; while the distance-based approach is less problem dependent, less precise, but faster and more universal.

The work, presented here, is also related to preliminary experiments done by Starovoitov and Samal (1998) to extend a distance-based measure for color image comparison. The authors used the *averaged distance* (4) adapted to the RGB color space to compare color images considered as 2-d surfaces in 5-d spatial/color space.

Future work will address the problem of fully testing the usefulness of the presented functions to handle discrimination problems in large pictorial databases. Applications to low-level face analysis are also foreseen.

For further reading, see (Starovoitov, 1995; Stricker and Orengo, 1995).

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