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Fractal evaluations of fish school movements in simulations and real observations

Abstract Fish schools behave like a single organism, and this offers considerable survival advantages. In our simulations, a fish school is well organized, without a leader, and behaves like a single creature depending solely on the interactions among individuals. This kind of system can be said to be typical of "complex systems." In this article, it is shown that fractal evaluation is useful to understand the features of fish school movements. We make clear the validity of fractal analyses to quantify fish school movements through evaluations of simulated fish school movements and *sardine* movements. These fractal analyses show that we need two different fractal dimensions (D_1, D_2) to understand the features of fish school movements: D_1 corresponds to the *smaller coarsening levels*, and D_2 corresponds to the *large coarsening levels*. The linear analyses in log–log space give an excellent fit with both the simulated movements and the sardine school movements. In approaching complex systems or complex behaviors, fractal analyses have attracted wide attention in mathematics, physical sciences, and information science. The fractal evaluations here convince us that we are coming close to understanding the structure of complex movements of animals.

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1 Introduction

We find many types of "complex systems" in nature, all of which emerge spontaneously in systems of repeated individual interactions. The behaviors of a complex system, such as brain activities, self-similar structures in biology, and collective movements of creatures, are rich in diversity and contribute to adaptations to the environment. In studies about complex systems, and to understand the discipline of robustness and stability in creatures or nature, "fractal" is a significant concept, as well as "chaos" in the discipline of nonlinear dynamics. Fractal analyses have thrown new light on our understanding of complex systems in nature and in physiological structures.[1]

It is said that fish school movements offer considerable survival advantages such as reducing the risk of being eaten. A school of fish does not need a special individual as a leader since it can adapt to its surroundings. Fish school movements are based solely on the interactions among individuals. In other words, a fish school can be said to be an autonomous complex adaptive system. Behavior models of fish have been studied on the basis of biological facts, and these have shown that quite simple local rules can result in diverse behaviors.[2,3] In a simulation study with these behavior models, a fish school dominated by internal interactions only moved diversely like one big creature. This can be said to be a typical manifestation of complex systems found in nature.

Through our simulations, we can follow the minute changes in position and velocity which are almost impossible to measure under natural conditions. In our previous study, we successfully quantified the complexities of simulated fish school movements as fractal dimensions.[4] In this study, we applied fractal analyses not only to simulated movements, but also to video-recorded real fish (*sardine*) school movements. We examined the irregularity of each

movement by a time-resolution of the analyses, and obtained fractal dimensions reflecting the features of complicated fish school movements.

2 Models for simulations and real observations

2.1 Models for fish school simulations

The schooling of fish has been observed and investigated with various approaches in order to understand the mechanisms, why a school is formed and maintained. It has been considered that two basic behavior patterns contribute to schooling: one is a biosocial attraction, and the other is parallel orientation.[5] The Aoki model[2] is based on the biological concept that through repetitions of interactions with neighboring fish, there emerges an égalitarian leaderless society. In our simulation, we adopt the Aoki model in which a fish does not have any information about the school as a whole.

2.1.1 Behavior models of fish

Many researchers have tried to understand fish behavior, and it is clear that it is not just a simplified version of that seen in birds and mammals. One clear fact is that a fish perceives its surroundings with both eyes and with its lateral lines. It is known that the visual angle of fish eyes is often larger than 300°, and that their lateral lines detect water currents, vibrations, and pressure changes.[6] Therefore, to simulate a fish school, the local rules between neighboring fish should be set by considering the ranges perceived by the eyes and the lateral lines.

The fish behavior models studied by Aoki[2] are based on the real observation that a fish determines its next behavior only by knowing the distance to the perceived neighboring fish. A fish reacts differently (e.g., repulsion, parallel orientation, attraction, searching) according to the area in which the perceived neighbor fish is positioned. This model uses the basic assumptions that the speed and direction of individual movements are stochastic variables. An exact Γ distribution of fish velocity was observed in Aoki's experiment on real fish behavior.[7] The fish (i) takes a new direction α_i ($t + \Delta t$) from $\alpha_i(t)$ with turning angle $\beta_i(t)$ as follows:

$$\alpha_i(t + \Delta t) = \alpha_i(t) + \gamma_{ij}\beta_i(t) + \sqrt{2}\beta_0 \tag{1}$$

where the term $\sqrt{2}\beta_0$ is a fluctuation in determining a new direction; β_0 follows a gamma distribution $N(0,1)$. γ_{ij} is the constant which adjusts the degree of interaction.

Figure 1 shows a geometric drawing to illustrate the parameters specifying interactions. The turning angles $\beta_i(t)$ for actions (i)–(iv) are given below.

(i) Repulsion ($r < r_1$).
[If the perceived neighbor fish (j) is too close, the fish (i) tries to avoid a collision.]

$$\beta_i(t) = \min\left\{ \measuredangle\left(\overrightarrow{v_i(t)}, \overrightarrow{v_j(t)}\right) \pm 90° \right\}$$
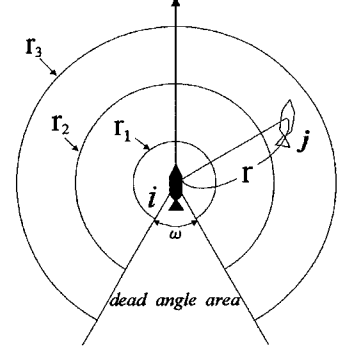

Fig. 1. Stages of the basic behavior patterns

(ii) Parallel orientation ($r_1 \leqq r < r_2$).
[If the perceived neighbor fish (j) is in a certain parallel orientation area, the fish (i) moves in the same direction as fish (j).]

$$\beta_i(t) = \measuredangle\left(\overrightarrow{v_i(t)}, \overrightarrow{v_j(t)}\right)$$

(iii) Attraction ($r_2 \leqq r < r_3$).
[If the perceived neighbor fish (j) is too far away, the fish (i) moves toward its neighbor.]

$$\beta_i(t) = \measuredangle\left(\overrightarrow{v_i(t)}, \overrightarrow{p_j(t) - p_i(t)}\right)$$

(iv) Searching ($r > r_3$, or dead angle area).
[If the fish (i) cannot perceive its neighbor, it begins to search for other fish by turning around by chance.]

$$\beta_i(t) = 0$$

where $\measuredangle(\vec{a}, \vec{b})$ denotes an angle between $\vec{a}$ and $\vec{b}$, $\min\{a \pm b\}$ denotes a smaller angle ($a + b$ or $a - b$), and p_j denotes a coordinate of fish (j).

The neighbor fish (j) is selected among N fish with a probability which is inversely proportional to the distance r. Then, the nearer fish is perceived as fish (j) with a higher probability. To simplify the models, the new velocity of the fish (v_i) is determined independently of other fish. The velocity v_i of every fish is computed by chance by a gamma distribution, which is based on observations of fish behaviors.

$$v_i = \text{chance}(p(v))$$

$$p(v) = \frac{A^K}{\Gamma(K)} \cdot \exp(-A_v) \cdot v^{K-1} \tag{2}$$

where v is velocity, K and A are constant parameters, $v \geq 0$, $K > 0$, $A > 0$, and $\Gamma(K)$ is a gamma function, and if K is an integer, $\Gamma(K) = (K - 1)!$.

2.1.2 Boundary conditions in a water tank

To compare the simulated fish school movements with real fish school movements in a water tank, actions near walls

[(v), (vi)] are added. A fish (i) will change its direction to avoid a collision with a wall. Then, the turning angle β_i for fish (i) is determined by the distance to a wall r_w.

(v) Avoid a collision with a wall ($r_w < r_0$).
[If a fish approaches very close to a wall, it moves along the wall to avoid a collision.]

$$\beta_i = \min\left\{\angle\left(\alpha_i(t),\theta_1\right),\angle\left(\alpha_i(t),\theta_2\right)\right\}$$

(vi) Avoid a collision with a corner ($r_{wa} < r_0$ and $r_{wb} < r_0$).
[If a fish approaches very close to a corner, it turns to move along one of the two walls.]

$$\beta_i = \min\left\{\angle\left(\alpha_i(t),\theta_a\right),\angle\left(\alpha_i(t),\theta_b\right)\right\}$$

where r_0 is the distance between the tank and an inner rectangle shown as the dotted line in Fig. 2, r_w is the distance between the fish (i) and a wall, r_{wa} and r_{wb} are the distances to walls a and b respectively, when fish (i) is in a corner, $\angle$ (α,θ) denotes the angle between α and θ, min{a, b} denotes the smallest angle a or angle b. In selecting one of the a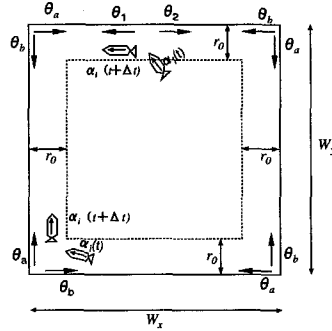ctions (i)–(vi), an action against another fish (i)–(iv) has priority over actions against a wall (v, vi), and action (vi) has priority over action (v).

2.2 Simulations of fish school movements

Here, we use the Aoki model with the following fixed parameters: $r_1 = 0.5\,BL$ (body length); $r_2 = 2.0\,BL$; $r_3 = 5.0\,BL$; $\omega = 30°$ (Fig. 1); $\gamma_{ij} = 0.3$; $r_0 = 1.0\,BL$; $K = 5.0$; $A = 1.0$. When action (v) or (vi) occurs, the new direction $\alpha(t + \Delta t)$ of fish (i) is determined with $\gamma_{ij} = 1.0$ and $\beta_0 = 0.0$. One simulation step corresponds to 0.5 s. The tank size and the velocity of the fish are set in accordance with real observations, which are described in Sect. 2.3. The tank size is set as 26 BL long (W_x) and 31 BL wide (W_y). The average velocities of ordinary movements without predators, and of movements after perceiving predators are set as 1.96 BL/s (0.98 BL/step) and 2.42 BL/s (1.21 BL/step), respectively.

In a tank, even if a few fish become separated from a school, or if a school is divided, they soon meet and are unified again. It is therefore assumed that individuals in a school draw almost the same locus as the school movement, except for the period immediately after a simulation starts. In this article, we analyze fish school movements by taking one fish at random as a sample in a school.

2.3 Real observation of fish schools

We evaluated real fish school movements in addition to the analyses of simulated fish school movements. To evaluate real fish school movements, we analyzed video-recorded pictures which were taken at the National Research Institute of Aquaculture, Mie Prefecture, Japan, in October 1990. The water tank was 5.0 m long and 6.0 m wide, and the school was composed of about 100 sardines (Fig. 3). In these
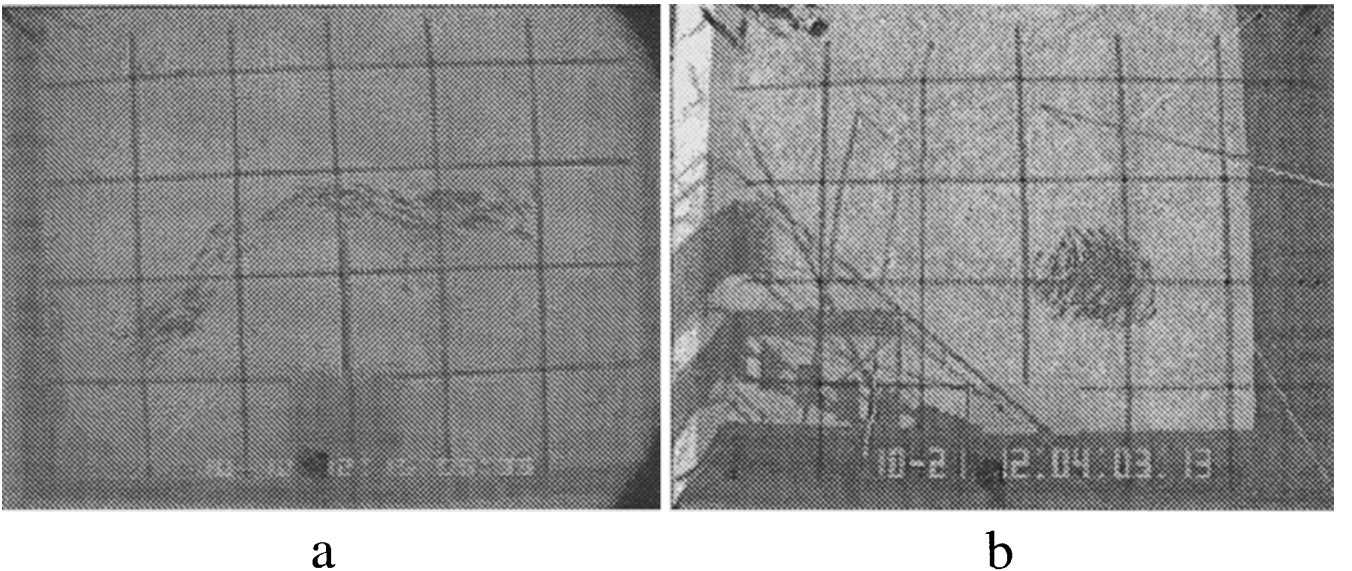

Fig. 2. Boundary conditions in a water tank

Fig. 3. Video pictures of fish school movements (N about 100). **a** The Normal fish school without predators. **b** The fish school after perceiving predators

Fig. 4. The coarsened trail pattern of simulated fish school movements ($N = 100$, $v_{avg} = 1.96\,BL/s$, 50 lines). **a** $k = 5$ ($t = 500$–750); **b** $k = 10$ ($t = 500$–1000); **c** $k = 50$ ($t = 500$–3000); **d** $k = 100$ ($t = 500$–5500)

3 Coarsened trail patterns
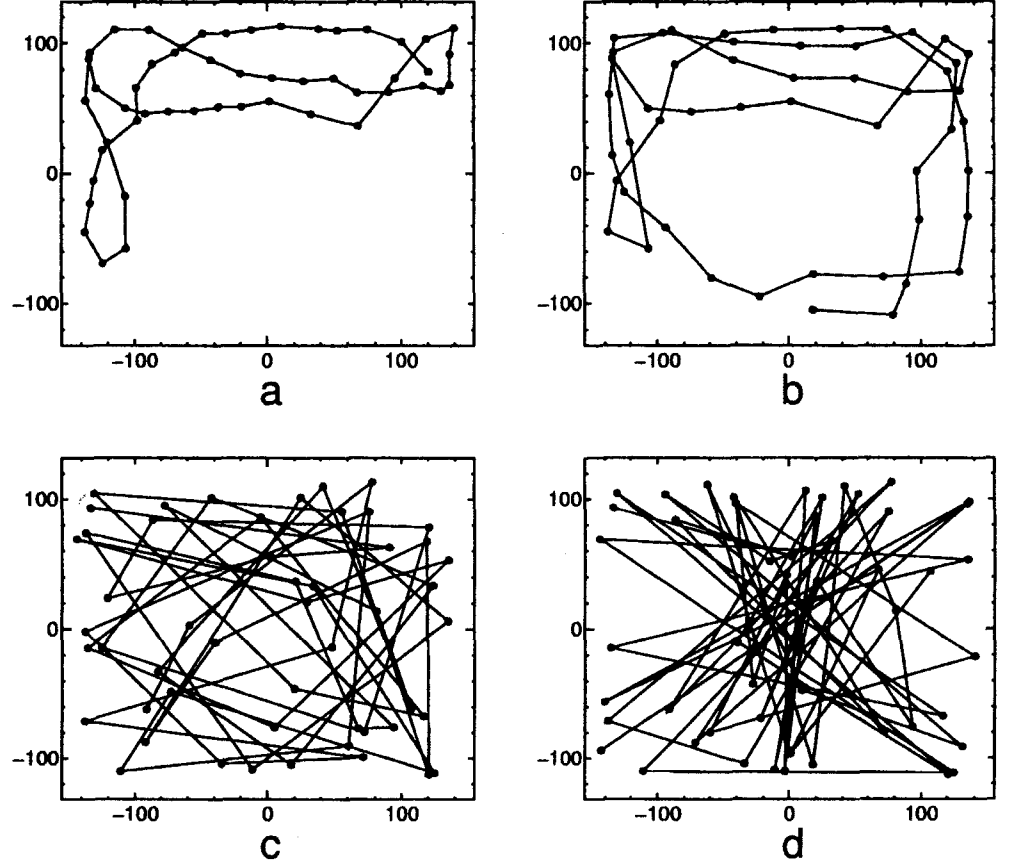
observations, the water tank was only 0.75 m deep so that we could examine the fish behavior in a two-dimensional space. In this shallow water tank, fish seldom move vertically except for the moment they are attacked by predators. To record their movements, straight lines are marked every meter in both directions on the floor of the tank. Fish movements in the tank were recorded with a CCD camera which was set 6 m above the center of the tank. Sardines that had been raised for about one year in a 25-m^3 outdoor tank were moved into the experimental tank a few days before the observations. The mean body length of the sardines was 10.4 cm. Figure 3a shows the normal sardine school movements when no predator is present. Figure 3b shows the highly polarized movements after predators have been perceived. The predators were juvenile yellowtails, which had been kept in the neighboring tank and separated by a net.

To obtain the time-series position coordinates of fish movements, we displayed the pictures on a computer screen, and traced the movements of one fish with a hand-operated mouse. The coordinates of the mouse are saved on the computer every 0.5 s. We found that the average velocities of the school as shown in Fig. 3a, b were 1.96 BL/s and 2.42 BL/s, respectively.

The behavior of a fish school is often influenced by the size of the tank, because the fish frequently change direction to avoid collisions with the walls. Therefore, the trail pattern of a fish school in a tank is much more complicated than it would be in a free space. To understand the complexities of fish school movements, we began by analyzing fish school movements with several different time-resolutions. We let k be the coarsening level which determines the time-resolution. We evaluated the coarsened trail pattern in terms of the coarsening level k, which consists of lines linking position coordinates every k simulation steps. Figures 4–6 show the coarsened trail patterns, which are composed of 50 straight lines.

3.1 Coarsened trail patterns of simulated movements

Figure 4 shows the coarsened trail patterns of simulated movements with an average speed of 1.96 BL/s. The coarsening levels in Fig. 4a,b,c, and d are $k = 5$, 10, 50, and 100, respectively. The higher the coarsening level, the rougher the time-resolution. In patterns with low coarsening levels ($k = 5$, 10) (Fig. 4a,b), the fish school moves smoothly along the walls of the tank. On the other hand, in patterns with high coarsening levels ($k = 50$, 100) (Fig. 4c,d), we find much more complicated trace patterns. For the pattern

Fig. 5. The coarsened trail patterns of normal sardine school movements with no predators, as shown in Fig. 3a (N about 100; v_{avg} = 1.96 BL/s, 50 lines). **a** k = 5 (t = 0–250); **b** k = 10 (t = 0–500); **c** k = 50 (t = 0–2500); **d** k = 100 (t = 0–5000)
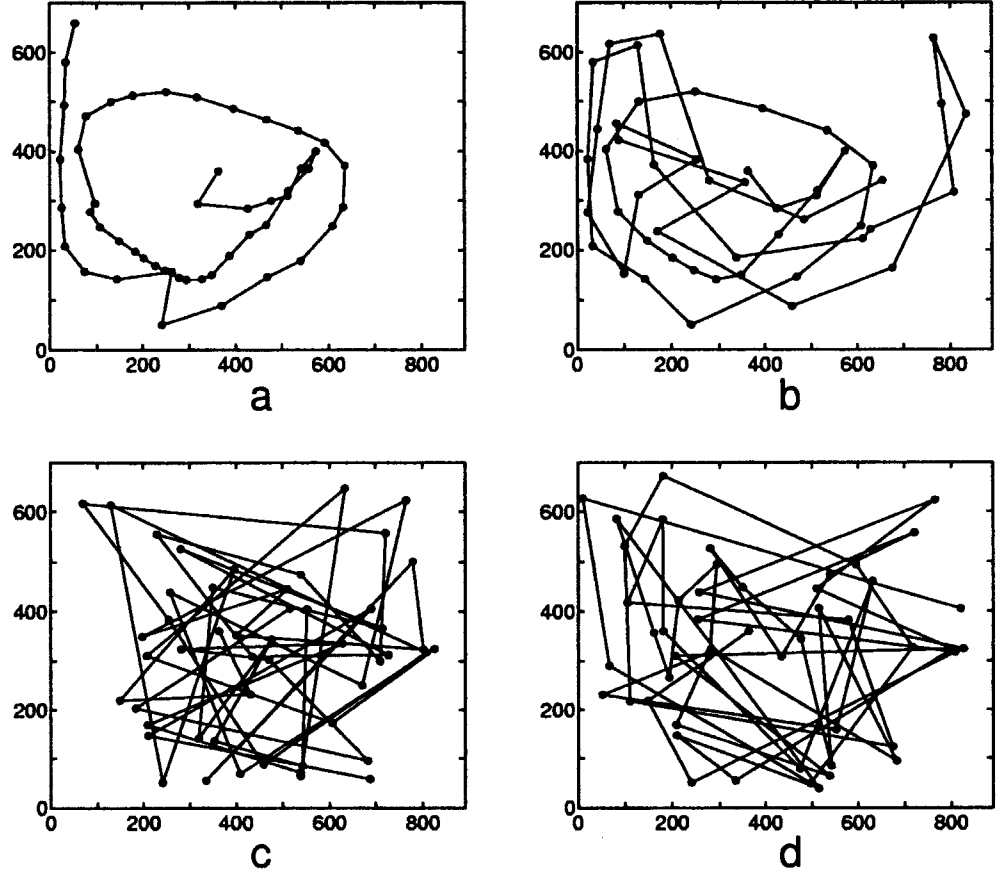
shown in Fig. 4c, the length of almost every line is nearly 49 BL, which is longer than the side length of the tank. Because of direction changes near walls, it is supposed that the correlation between two linked positions in Fig. 4c is lost. Consequently, the coarsened trail turns at an acute angle. Figure 4c,d are much influenced by the walls of the tank, but Fig. 4a,b are not.

3.2 Coarsened trail patterns of observed movements

Figures 5 and 6 show the coarsened trail patterns of a normal fish school without predators, as shown in Fig. 3a, and a tense fish school after perceiving predators, as shown in Fig. 3b. For normal fish movements with coarsening levels k = 5 and k = 10 (Fig. 5a,b), the school moves smoothly along the walls. This is similar to the results of coarsening shown in Fig. 4a,b. The coarsened trail patterns with coarsening levels k = 50 and k = 100 show a highly complicated structure, which is also very similar to the results shown in Fig. 4c,d. By comparing these coarsened patterns, it can be seen that real school movements without predators are well reproduced by computer simulations.

On the other hand, tense school movements after perceiving predators shows a different distribution in its behavior. After perceiving predators, the fish school does not move over the whole area of the tank, but keeps to a favorite area or route, shown in Fig. 6. Therefore, even in patterns with coarsening levels k = 5 and k = 10, a tense school

changes direction frequently compared with the patterns shown in Fig. 5a,b.
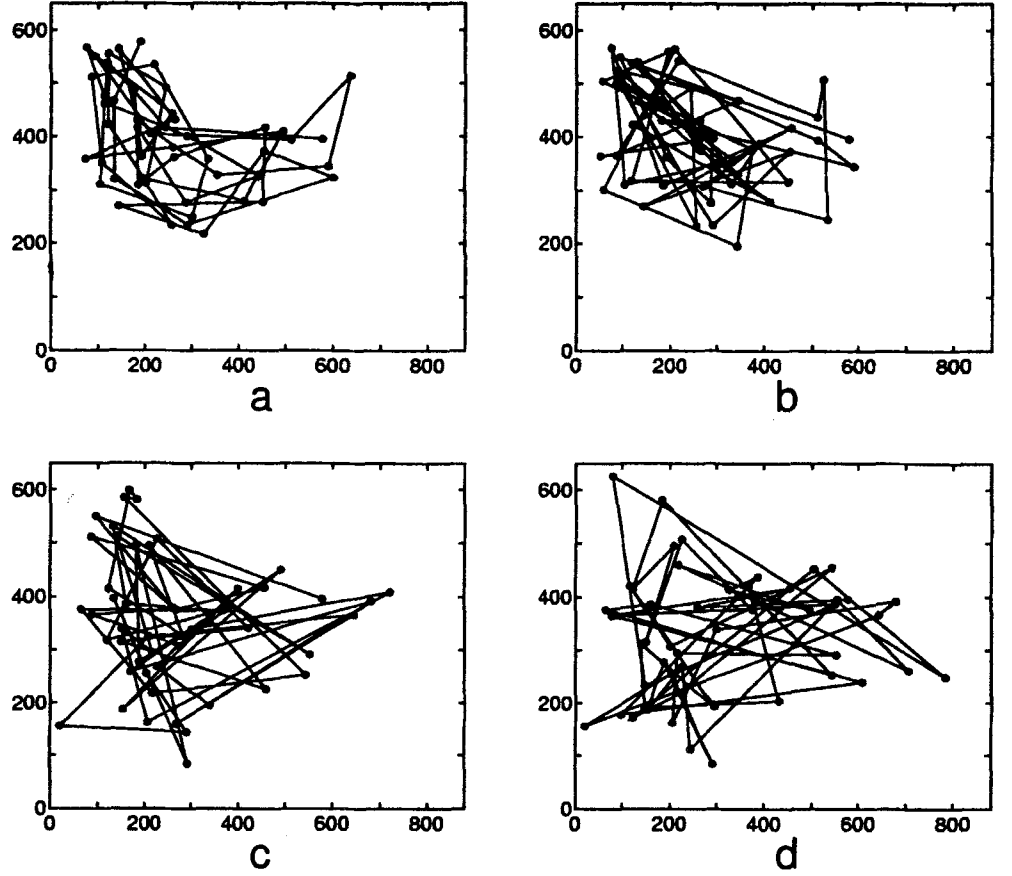
4 Fractal analyses

4.1 Fractal dimension

Here, we quantitatively evaluate the coarsened patterns shown in Figs. 4–6. In general, when we examine features of complex structures or behaviors, the resolution in the analyses is the significant factor. If even finer features are revealed when the object is magnified, it can be said that the object has the property of a fractal. In our analyses, the length of the trail pattern of a fish school is measured at different resolutions, which correspond to the coarsening level of k. In other words, the fish school movement is evaluated at each time-interval k used for the coarsening. If the measured length $< L(k) >$ is related to the coarsening level of k, as

$$< L(k) > \propto k^{-D} \qquad (3)$$

the system is said to have fractal dimension D. Generally, an assigned fractal dimension is a non-integer, and shows how much complexity is being repeated at each scale (k). Thus, an object evaluated with a higher fractal dimension is more

Fig. 6. The coarsened trail pattern of sardine school movements after perceiving predators, as shown in Fig. 3b (N about 100, v_{avg} = 2.42 BL/s, 50 lines). Δt corresponds to the time interval 0.5 s. **a** k = 5 (t = 0–250); **b** k = 10 (t = 0–500); **c** k = 50 (t = 0–2500); **d** k = 100 (t = 0–5000)

complicated. Fish school movements in a two-dimensional area gives D in the range 1.0–2.0.

In general fractal analyses, fractal dimension D is determined from the relation between the number of circles needed to cover the object and the radius r of the circles

$$\left(D = \lim_{r \to \infty} \frac{\log N(r)}{\log(1/r)} \right).$$ In our fractal analyses with time-series data, we use the time-interval k instead of radius r.

The coarsened length $< L\ (k) >$ at the coarsening level of k is defined as

$$<L(k)> = \frac{1}{k}\left[\sum_{t=1}^{T-k} \sqrt{(x_{t+k} - x_t)^2 + (y_{t+k} - y_t)^2} \cdot \frac{T}{(T-k)k} \right]$$

$$(4)$$

where x_t and y_t are the coordinates of the position $P(t)$ at t. The whole is multiplied by $1/k$ because $< L(k) >$ is found from k sets of time-series, each with a different starting point. T is the total time of a fish school simulation, and the term $T/((T - k)k)$ normalizes the differences in the number of subsets.[8,9]

4.2 Fractal analyses of simulated movements

We obtain the coarsened length $< L(k) >$ from the time-series position coordinates which gave the coarsened
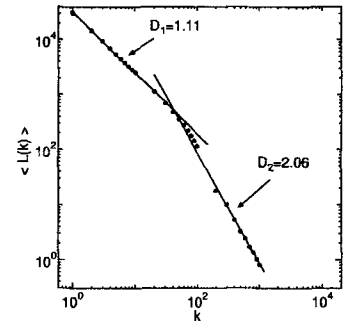

Fig. 7. Fractal analyses for simulated fish school movements (N = 100)

patterns in Fig. 4. Figure 7 shows double logarithmic points of the coarsened length $< L(k) >$ as a function of the coarsening level k. Generally, as the coarsening level k increases, the measured coarsened length $< L(k) >$ becomes short. However, it is clear from Fig. 7 that the rate of decrease of $< L(k) >$ is not constant for all k. It has a sudden change at around k = 40. Therefore, to satisfy Eq. 3, two straight lines need to be fitted to all points. One line (1 $\leq k \leq$ 40) has a gradual inclination, and the other line (40 $\leq k \leq$ 1000) has a steep inclination. Each straight line is fitted to the group of points by the least-squares method. The absolute value of the slope represents the fractal dimension D. Then, D_1 = 1.11 and D_2 = 2.06 (in Fig. 7) represent fractal dimensions of simulated fish school movements.

Fig. 8. Fractal analyses for real fish (sardine) school movements (N about 100). The average fractal dimension is shown in parentheses. These were obtained from five sets of results (Table 1). **a** Normal fish school with no predators (Fig. 3a, Fig. 5). **b** Fish school after perceiving predators (Fig. 3b, Fig. 6)
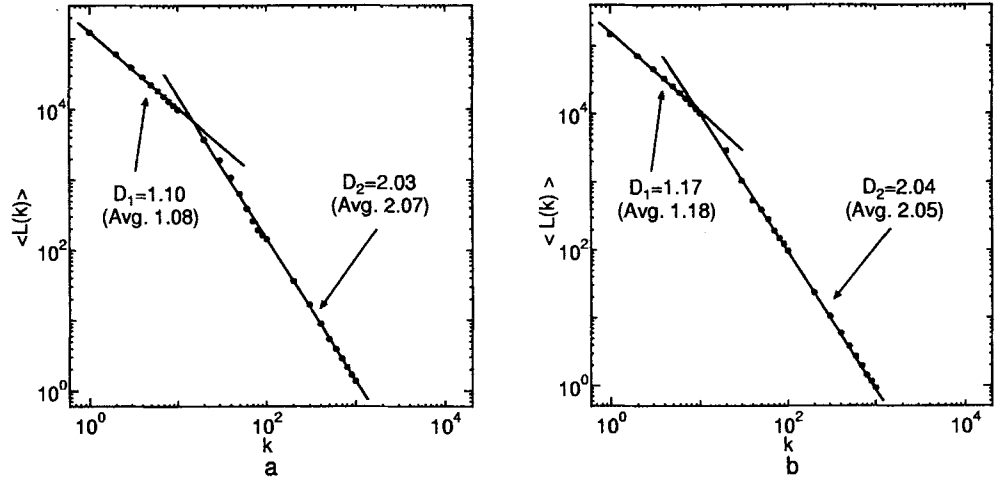
Fractal dimensions D_1 and D_2 are quantified indices which indicate the characteristics of fish school movements. D_1, with a low value (1.11), corresponds to the low level of complexity shown in Fig. 4a,b, and D_2, with a high value (2.06), corresponds to the high level of complexity shown in Fig. 4c,d. Since points on a straight line indicate the same rank of complexity, it can be concluded that the fish school has two types of fractal structure in the complexities.

4.3 Fractal analyses of observed movements

The fractal analyses for a simulated fish school were applied to two types of real fish school movements: one is a sardine school without predators (Fig. 8a), and the other is a sardine school after perceiving predatory juvenile yellowtails (Fig. 8b). In Fig. 8, we found two linear structure as for the simulated movements, and we also found that the declination rates D_1 and D_2 fitted the points well.

To get reliable results from the analyses, we took the time-series position coordinates five times from the same video-recorded picture. In each observation, we traced a fish for about 30 min. The five observations were performed to reduce the error in data traced with a mouse, which are influenced by the choice of the target fish. Table 1 shows the five sets of results for both the sardine movements without predators and the sardine movements after perceiving predators. These results indicate the reliability of this procedure.

Comparing Fig. 8a with Fig. 8b, we find a difference in D_1 (1.10 in Fig. 8a; 1.17 in Fig. 8b), while D_2 has similar values (2.03 in Fig. 8a; 2.04 in Fig. 8b). D_2 approximately 2.0 in both Fig. 8a and Fig. 8b may explain this complicated behavior as being the result of repeated direction changes near walls (Figs. 5c,d, 6c,d). These complexities in D_2 were also found in our analyses of simulated movements (see Fig. 7).

D_1 in the fractal analyses of normal sardine school movements (Fig. 8a) also indicates similar complexities. On the other hand, D_1 of movements after perceiving juvenile yellowtails (predators) (see Fig. 8b) indicates a highly complicated feature. Since, the sardine school is on its guard

Table 1. Fractal dimensions from analyses of position data obtained from the same 30-min video-tape

No.	Without predator		With predator	
	D_1	D_2	D_1	D_2
1	1.086	2.069	1.186	2.047
2	1.084	2.074	1.175	2.049
3	1.079	2.088	1.182	2.050
4	1.077	2.094	1.178	2.045
5	1.065	2.034	1.162	2.045
Avg.	1.078	2.072	1.177	2.047
SD	0.007	0.021	0.008	0.002

SD, standard deviation

against attacks by juvenile yellowtails, and frequently changes direction completely, the movements have a large fractal dimension ($D_1 = 1.17$) compared with $D_1 = 1.10$ of the normal fish school.

5 Summary

In our precedence study of fractal analyses for simulated fish school movements, we have found that two straight lines are needed to fit the coarsening length $< L(k) >$ against the coarsening level k. Here, it has been shown that real fish movements also need two straight lines for fitting. The fractal dimensions D_1 and D_2 fit both the simulated movements and the real movements very well. Normal sardine school movements give a fractal dimension which is similar to that of simulated movement. In other words, fractal analyses are an effective way to evaluate the irregularities not only of simulated movements, but also of real movements. Although understanding the behavior of many animals has been a difficult task, we have shown that fractal analyses and computer simulations are an effective way to evaluate their complex nature quantitatively. These quantitative evaluations have also allowed us to examine the validity of the simulations. The fact that the fractal dimensions

of the simulated movements coincide with those of movements without having perceived predators shows the efficacy of our simulation model to describe complex fish school movements. Since "fractal" is such a significant concept in understanding stability and robustness in nature, fractal analyses of complex behavior are expected to clarify the irregularities and complexities of adaptive functions. The fractal dimensions obtained from tense movements reflect the fast, complicated defence measures taken against predators.

While the fractal analyses gave a good representation of normal fish school movements, there is a small discrepancy between simulated movements and real movements in cases where fish have perceived predators. It is assumed that the velocity of the fish influences the complexities of the movements. Therefore, the rules of a behavior model for determining the velocity need to be improved in order to adapt to diverse conditions. Here, the velocity was determined stochastically based on Γ distributions. To reproduce more realistic movements, it is expected that the velocity of a fish must be determined by considering interactions with neighbors or with external conditions.

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