Math 441 (2020) – Homework 3 : 25 points

Due: Thursday, October 1, 2020, 11:59pm (HARD Deadline).

NAME: ____________________

1. [5pts] (Secant Method) The Secant method for finding roots of $f(x) = 0$ is defined by

$$x_{n+2} = x_{n+1} - \frac{f(x_{n+1})(x_{n+1} - x_n)}{f(x_{n+1}) - f(x_n)} \quad , \quad n = 1, 2, 3, \ldots$$

where x_1 and x_2 are specified initial guesses. Write matlab code to solve the following problem using Secant's method:

$$f(x) = \exp(x) - \ln(x+4) \qquad , \qquad x_1 = 1 \qquad x_2 = 0.5$$

Since the root is not known we can't compute the exact error. Instead we shall use the difference in successive x_n values as a measure of convergence.

$$E_n = |x_n - x_{n-1}|$$

A rough outline of the code is

```
x(1)=1;
x(2)=0.5;
n=2;
tolerance=1e-08;
E(n)=abs(x(n)-x(n-1));
R(1,:)=[n,x(n),E(n)];
while E(n)> tolerance
  x(n+1)=........
  E(n+1)=.......
  R(n,:)=.........
  n=n+1;
end;
R
```

Include the code and an output of R showing convergence. As before R has $n, x(n), E(n)$ as columns.

2. [5pts] (Sensitive systems) We illustrate by way of simple systems how solutions of $A\mathbf{x} = \mathbf{b}$ can change dramatically when we approximate $\mathbf{b}$. Below M is some very large number.

(a) Use Gauss Elimination to find the exact solution of $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} 2 & 2M \\ 1 & M+1 \end{bmatrix} \qquad \mathbf{b} = \begin{pmatrix} 2+6M \\ 4+3M \end{pmatrix}$$

(b) When M is large $\mathbf{b} \simeq (2,1)^T$. Re-solve the system with this new approximate $\mathbf{b}$. Are the solutions in (a) and (b) close??

Note: Simplify your answers. If done correctly the solutions $\mathbf{x}$ do not depend on M.

3. [10pts] (LU Factorizations) Consider the non-symmetric matrix:

$$A = \begin{bmatrix} 1 & 4 & 1 \\ 4 & 9 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

Throughout L, U, D are lower unit triangular, upper unit triangular and diagonal matrices, respectively. By convention an over $\tilde{}$ indicates the matrix need not have ones on the diagonal. Find the indicated factorizations and then answer c). DO NOT use Matlab or other software to find $L, U, \ldots$.

a) $A = L\tilde{U}$

b) $A = LDU$

c) Note $det(AB) = det(A)det(B)$ and the determinant of triangular matrices equals the product of its diagonal elements. Use these facts to compute $det(A) = det(L)det(\tilde{U})$.

4. [5pts] (Cholesky Factorizations) Below is a symmetric positive definite matrix:

$$A = A^T = \begin{bmatrix} 4 & -2 & 2 \\ -2 & 2 & 0 \\ 2 & 0 & 3 \end{bmatrix}$$

Find the Cholesky factorization $A = \tilde{L}^T\tilde{L}$.

DO NOT use Matlab or other software to find $L, U, \ldots$.