Math 441 (2020)
Homework 2 - max 30
Due: Thur. September 17, 2020

. NAME: ______________________________

1. [5pts] (Big O) Use Theorem 2.4 of the notes to prove

$$h - \sin h = O(h^3) \qquad as\ h \to 0$$

Note: multiple applications of L'Hopital's rule are required.

2. [5pts] (Second Derivative Approximation) Using Taylor series of $f(x+h)$ and $f(x-h)$ show

$$f(x+h) - 2f(x) + f(x-h) = f''(x)h^2 + O(h^4) \quad .$$

Then, solve for $f''(x)$ above to get an approximation for $f''(x)$ stating the order of the truncation error. You may use Example 2.6 of the notes as guide where (for example)

$$f(x+h) = f(x) + f'(x)h + \frac{1}{2!}f''(x)h^2 + \frac{1}{3!}f^{(3)}(x)h^3 + O(h^4)$$

3. [5pts] (Bisection Method) Let $f(x) = (x-1)(x-3)(x-4)$. What root of $f(x)$ does the Bisection Method converge to on the interval $[a,b] = [0,5]$. Sketch $f(x)$ and label (on the x axis) the first two midpoint values x_1, x_2. Example 4.2 of the notes is a guide.

4. [10pts] (Newton's Method) For the following three problems use modified versions of the posted *Newton.m*, *f.m* and *df.m* Matlab files to find and approximation of a root of $f(x)$ using the indicated starting guess x_1. For each case print your output in three columns:

$$n \qquad x_n \qquad E_n \qquad , \qquad (1 \le n \le 10).$$

where E_n is the (exact) absolute error (you'll to find the exact value of the root). Use "format long" in Matlab. Lastly, state if the convergence rate is linear, superlinear or quadratic.

a) Newton's Method, $x_1 = 4$, $f(x) = (x-1)^2 - 2$

b) Newtons Method $x_1 = 4$, $f(x) = (x-1)^2$

c) Accelerated Newton's Method $x_1 = 4$, $f(x) = (x-1)^2$

The last method is $\lambda = 2$ on page 40 (Nonsimple roots) of the class notes.

5. [5pts] Steffensen method for solving $f(x) = 0$ is defined by:

$$x_{n+1} = x_n - \frac{f(x_n)}{g(x_n)} = x_n - F(x_n)$$

where

$$g(x) = \frac{f\left(x + f(x)\right) - f(x)}{f(x)}$$

For simple roots $\bar{x}$ where $f(\bar{x}) \neq 0$, one can show the method has quadratic convergence.

Write a Matlab file Steffensen.m who's output is

$$n \qquad x_n \qquad E_n$$

for $1 \leq n \leq 10$ for the case $f(x) = x^2 - 4$ and $x_0 = 1$. Include your code Steffensen.m and output.