

M 441: Homework 5

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Problem 1

Problem. Recall the Power Method algorithm is defined by:

$$\mathbf{y}_n = A\mathbf{x}_n$$

$$\lambda_n = \phi(\mathbf{y}_n)/\phi(\mathbf{x}_n)$$

$$\mathbf{x}_{n+1} = \mathbf{y}_n/\|\mathbf{y}_n\|_2$$

where $\phi(\mathbf{z}) = z_1 + z_2 + \cdots + z_n$ and $\mathbf{z} = (z_1, z_2, \dots, z_n)$. For most (but not all) initial guesses \mathbf{x}_0 , λ_n will approach the dominant eigenvalue λ of A with \mathbf{x}_n being an approximation of the associated (unit) eigenvector.

a) Let

$$A = \begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix} \quad \mathbf{x}_0 = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

Using hand calculations (no decimal approximations) only, compute in sequence $\mathbf{y}_0, \lambda_0, \mathbf{x}_1, \mathbf{y}_1, \lambda_1, \mathbf{x}_2$. You should find $\mathbf{x}_1 = -\mathbf{x}_2$. Is the method converging to the dominant eigenvalue? Give some reasonable explanation.

b) Let

$$A = \begin{bmatrix} 16 & 2 & 4 \\ 1 & 40 & -3 \\ 0 & 3 & 5 \end{bmatrix} \quad \mathbf{x}_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Use the posted *Power.m* and *phi.m* matlab scripts to approximate the dominant eigenvalue using $N = 15$ iterates. Include an extra column in the output matrix R as follows.

$$>> R = [x', \text{lambda}(k), \text{residual}(k), \text{abs}(r - \text{edom})]$$

where *edom* is the dominant eigenvalue. You may find this value using the matlab statement

$$>> \text{evals} = \text{eig}(A)$$

where *evals* is a vector of all the eigenvalues of A .

Answer.

Problem 2

Problem. Recall that the steepest descent algorithm for approximating the solution of $Ax = b$ (when $A = A^T$ is positive definite) is

$$\mathbf{r}_n = A\mathbf{x}_n - \mathbf{b}$$

$$\alpha_n = \frac{\mathbf{r}_n^T \mathbf{r}_n}{\mathbf{r}_n^T A \mathbf{r}_n}$$

$$\mathbf{x}_{n+1} = \mathbf{x}_n - \alpha \mathbf{r}_n$$

where \mathbf{r}_n is the residual vector and \mathbf{x}_n is the approximation of the solution $Ax = b$.

a) Let

$$A = \begin{bmatrix} 2 & 1/4 \\ 1/4 & 1/2 \end{bmatrix} \quad \mathbf{b} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \mathbf{x}_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

First prove A is positive definite and then use hand calculations (no decimal approximations) to compute in sequence $\mathbf{r}_0, \alpha_0, \mathbf{x}_1, \mathbf{r}_1, \alpha_1, \mathbf{x}_2$.

b) Use the code *steepest.m* to approximate the solution of $Ax = b$ where $A \in \mathbb{R}^{50 \times 50}$ is a tri-diagonal matrix. The iteration output in *steepest.m* is stored in the matrix R .

$$R(n, :) = [n, x_n', \text{norm}(en, 2)];$$

The last column of R is the 2-norm of the absolute error. Plot this absolute error versus iteration number for $N = 3000$ iterations. Use matlab to do this.

Answer.

a)