M 441: Homework 3

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Problem 1

Problem. The Secant method for finding roots of $f(x) = 0$ is defined by

$$x_{n+2} = x_{n+1} - \frac{f(x_{n+1})f(x_{n+1} - x_n)}{f(x_{n+1} - f(x_n))}$$

where x_1 and x_2 are specified initial guesses. Write matlab code to solve the following problem using the Secant method:

$$f(x) = exp(x) - ln(x+4) \qquad\qquad x_1 = 1 \quad x_2 = 0.5$$

Since the root is not known, we can't compute the exact error. Instead, we shall use the difference in successive x_n values as a measure of convergence

$$E_n = |x_n - x_{n-1}|$$

Include the code and an output of R showing convergence. As before, R has $n, x(n), E(n)$ as the n^{th} row.

Answer.

Problem 2

Problem. We illustrate by way of simple systems how solutions of $A\mathbf{x} = \mathbf{b}$ can change dramatically when we approximate $\mathbf{b}$. Below M is some very large number.

(a) Use Gauss elimination to find the exact solution of $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} 2 & 2M \\ 1 & M+1 \end{bmatrix} \qquad \mathbf{b} = \begin{pmatrix} 2+6M \\ 4+3M \end{pmatrix}$$

Note: simplify your answers. If done correctly, the solutions $\mathbf{x}$ do not depend on M.

(b) When M is large $\mathbf{b} \approx \mathbf{b}_{new} = M \begin{pmatrix} 6 \\ 3 \end{pmatrix}$. Resolve the system with this new appproximate $\mathbf{b}_{new}$.
Are the solutions in (a) and (b) close?

Answer.

Problem 3

Problem. Consider the non-symmetric matrix

$$A = \begin{bmatrix} 1 & 4 & 1 \\ 4 & 9 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

L, U, D are lower unit triangular, upper unit triangular, and diagonal matrices respectively. By convention, a indicates that a matrix need not have ones on the diagonal.

(a) $A = L\tilde{U}$

(b) $A = LDU$ (c) Note that $det(AB) = det(A)det(B)$ and the determinant of triangular matrices equals the product of its diagonal elements. Use these face to compute $det(A) = det(L)det(\tilde{U})$.

Answer.

Problem 4

Problem. Below is a symmetric positive definite matrix:

$$A = A^T = \begin{bmatrix} 4 & -2 & 2 \\ -2 & 2 & 0 \\ 2 & 0 & 3 \end{bmatrix}$$

Find the Cholesky factorization $A = \tilde{L}^T \tilde{L}$

Answer.