



# Feature

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	Level	Completed	Goal
	Beginner	3	7
	Intermediate	4	
	Advanced	1	Total Completed
	Expert	0	8

# Feature

CSCI 5722: Computer Vision

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Dr. Tom Yeh

# Image Moments

CSCI 5722 Computer Vision



University of Colorado  
Boulder



# Large Vision Model's ability to compare image moments

Draw a binary image in the box. Take a screenshot. Use it in your prompt to get a large vision model to calculate image moment. Paste the result.

1	0	0	0	0	0	0
0	1	0	0	1	1	0
0	1	1	0	0	0	0
0	1	0	0	1	0	1
0	0	0	0	1	1	0
0	0	0	1	1	1	1
0	0	0	0	1	1	1

Calculate the image moment for the given image?

The image moments for the provided binary image are as follows:

**Spatial Moments:**

- $M_{00}$  (Area): 157,875
- $M_{10}$ : 39,124,310
- $M_{01}$ : 25,782,931
- $M_{20}$ : 12,954,565,612
- $M_{11}$ : 6,391,736,087
- $M_{02}$ : 5,627,029,523
- $M_{30}$ : 4,824,734,940,968
- $M_{21}$ : 2,116,581,245,683
- $M_{12}$ : 1,395,500,513,371
- $M_{03}$ : 1,381,647,053,725

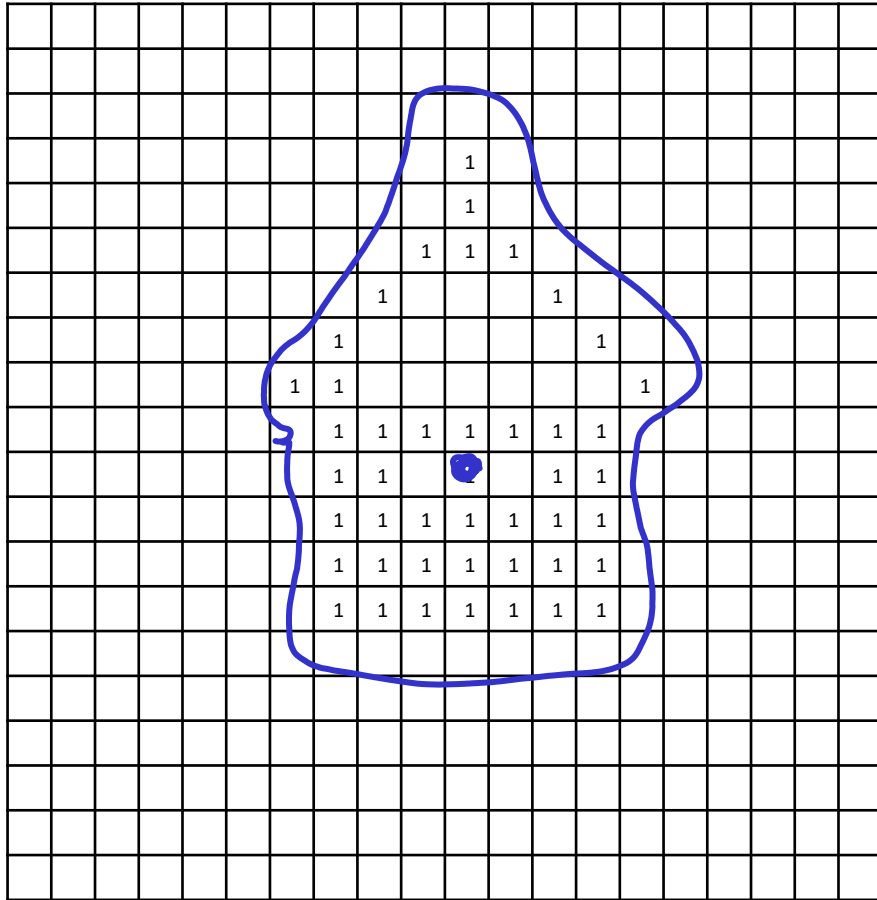
**Central Moments:**

- $\mu_{20}$ : 3,258,846,638.28
- $\mu_{11}$ : 2,254,629.18
- $\mu_{02}$ : 1,416,359,493.22
- $\mu_{30}$ : -846,646,720.04
- $\mu_{21}$ : -176,270,905.91
- $\mu_{12}$ : 283,352,107.99
- $\mu_{03}$ : 65,979,479.61

**Normalized Central Moments:**

- $\nu_{20}$ : 0.1307
- $\nu_{11}$ : 0.00009
- $\nu_{02}$ : 0.0568
- $\nu_{30}$ : -0.0000855
- $\nu_{21}$ : -0.0000178
- $\nu_{12}$ : 0.0000286
- $\nu_{03}$ : 0.00000666

# Image and Image Moments (M)



	0	1	2
0	$M_{00}$	$M_{01}$	$M_{02}$
1	$M_{10}$	$M_{11}$	$M_{12}$
2	$M_{20}$	$M_{21}$	$M_{22}$

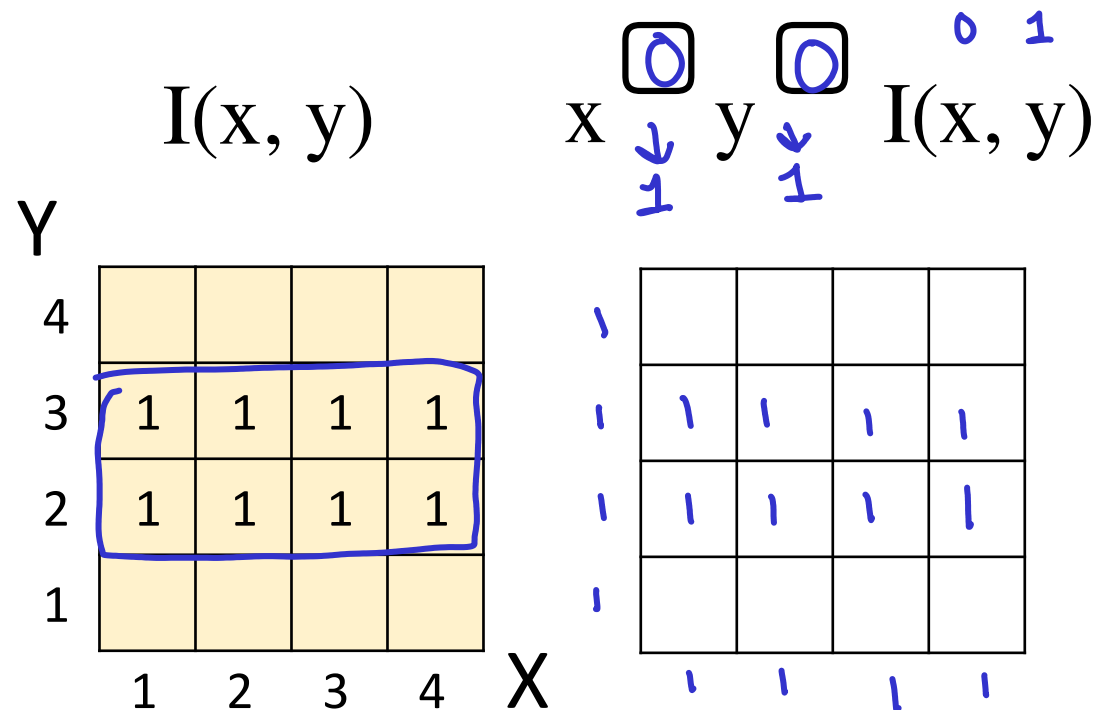
Math

Given:  $I(x, y)$

$$M_{ij} = \sum_x \sum_y x^i y^j I(x, y)$$

$M_{00}$

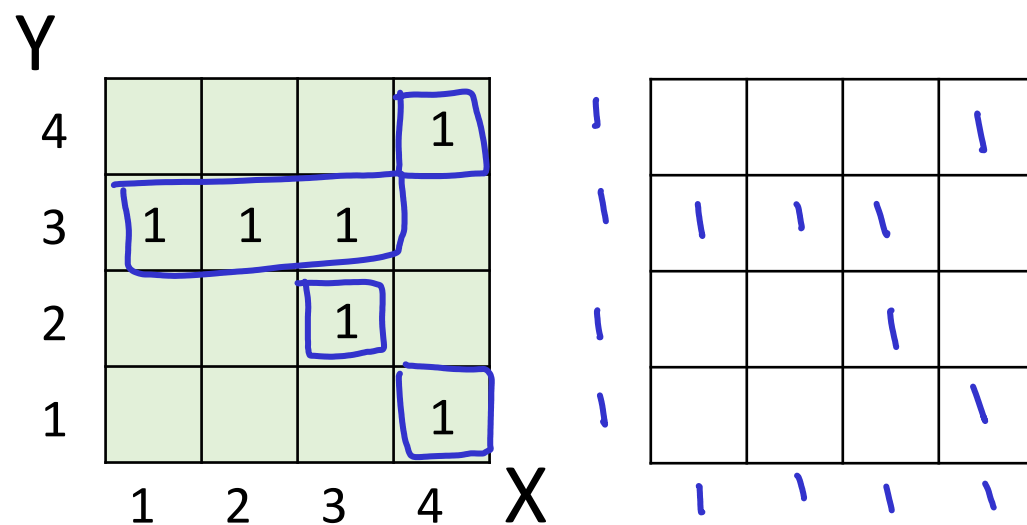
Ex. 1



$$\sum_x \sum_y$$

8

Ex. 2



6

$M_{10}$

$I(x, y)$

$x \begin{matrix} \boxed{1} \\ \end{matrix} y \begin{matrix} \boxed{0} \\ \end{matrix} I(x, y)$

$$\sum_x \sum_y$$

Ex. 1

Y

4				
3	1	1	1	1
2	1	1	1	1
1				
	1	2	3	4

X

1	1	2	3	4
1	1	2	3	4
1				
1				
	1	2	3	4

20

Ex. 2

Y

4				1
3	1	1	1	
2			1	
1				1
	1	2	3	4

X

				4
1	1	2	3	
1			3	
1				4
1				
	1	2	3	4

17



$M_{01}$

$I(x, y)$

$x \begin{matrix} \boxed{0} \\ \boxed{1} \end{matrix} y \begin{matrix} \boxed{0} \\ \boxed{1} \end{matrix} I(x, y)$

$$\sum_x \sum_y$$

Ex. 1

Y

4				
3	1	1	1	1
2	1	1	1	1
1				
	1	2	3	4

X

Y

4				
3	2	2	2	2
2	1	1	1	1
1				
	1	1	1	1

24

Ex. 2

Y

4				1
3	1	1	1	
2			1	
1				1
	1	2	3	4

X

Y

4				4
3	3	3	3	
2			2	
1				1
	1	1	1	1

16

$M_{21}$ 
 $I(x, y)$ 
 $x^{\boxed{2}} y^{\boxed{1}} I(x, y)$ 

$$\sum_x \sum_y$$

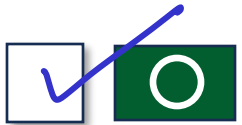
Y

4			1	
3	1			
2	1	1		
1				1
	1	2	3	4

X

4			36	
3	3			
2	2	8		
1				16
	1	4	9	16

55
----



$M_{20}$

$I(x, y)$

$x$   $\boxed{2}$   $y$   $\boxed{6}$   $I(x, y)$

$$\sum_x \sum_y$$

Y

4			1	
3	1			
2	1	1		
1				1
	1	2	3	4

X

1			$\boxed{b}$ 9	
1	1			
1	1	4		
1				$\boxed{c}$ 16
	1	4	9	16

$\boxed{a}$  31



$a + b + c = 56$



$M_{12}$

$I(x, y)$

$x^{\boxed{1}} y^{\boxed{2}} I(x, y)$

$$\sum_x \sum_y$$

Y

4			1	
3	1			
2	1	1		
1				1
	1	2	3	4

X

16			<b>28</b>	
9	9			
4	4	8		
1				<b>c 4</b>
	1	2	3	4

**a** **7 3** **a**



$a + b + c =$

125

$M_{2 \times 2}$

$x^i y^j I(x, y)$

$\sum_x \sum_y$

**X**

**I(x, y)**

**Y**

4	1			
3				
2		1		
1	1			1
	1	2	3	4

**X**

$j = 0$

$i = 0$

1			
	1		
1			1
	1	1	1

$j = 1$

4

3

2

1

4			
	2		
1			1
	1	1	1

$i = 1$

1			
	2		
1			4
	1	2	3

4

3

2

1

4			
	4		
1			4
	1	2	3

$j = 0$   $j = 1$

$i = 0$

$i = 1$

4	8
8	13

**M(X)**



$M_{2 \times 2}$

$$x^i y^j I(x, y)$$

**A**

$I(x, y)$

Y

4		1		
3	1			
2	1			
1		1		1
	1	2	3	4

**X**

$i = 0$

$j = 0$

		1		
1				
1				
		1		1

$j = 1$

4		4		
3	3			
2	2			
1		1		1

$i = 1$

		2		
1				
1				
		2		4

4		8		
3	3			
2	2			
1		2		4
	1	2	3	4

$$\sum_x \sum_y$$

	$j = 0$	$j = 1$		
$i = 0$	a	5	11	b
$i = 1$	c	10	19	d
	M(A)			



$a + b + c + d =$



$M_{2 \times 2}$

$$x^i y^j I(x, y)$$

B

$I(x, y)$

Y

4		1		
3				
2	1			1
1		1	1	
	1	2	3	4

X

$i = 0$

$j = 0$

	1		
1			1
	1	1	

$j = 1$

4		4		
3				
2	2			2
1		1	1	

$i = 1$

	2		
1			4
	2	3	
1	2	3	4

4		8		
3				
2	2			8
1		2	3	
	1	2	3	4

$$\sum_x \sum_y$$

$j = 0 \quad j = 1$

	a		b
$i = 0$	5	10	
$i = 1$	12	23	
	c		d

$M(B)$



$a + b + c + d =$



Compare X to A and B by L1 of their 2x2 Moments

M(X)

4	8
8	13

M(A)

5	11
10	19

$|M(X) - M(A)|$

1	3
2	6

$\Sigma$

12
----

a

argmin



You must solve the previous two activities to calculate M(A) and M(B) before you can solve this.

M(B)

5	10
12	23

$|M(X) - M(B)|$

1	2
4	10

17
----

b



$a + b = 27$



# Image Gradients

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# Draw Image Gradients

8	2		
4	2		
4	2	2	2
4	4	4	8


# Calculate Image Gradient in X-direction

$y \uparrow$ 

0	0	6	0
2	4	0	2
0	0	4	0
0	0	2	0

 $\rightarrow x$

$I(x, y)$

$I(x+1, y)$ 

0	6	0	0
4	0	2	0
0	4	0	0
0	2	0	0

$I(x-1, y)$ 

0	0	0	6
0	2	4	0
0	0	0	4
6	0	0	2

$\Delta x = 2$

$\frac{\partial I}{\partial x}$

$=$ 

0	6	0	-6
4	-2	-2	0
0	4	0	-4
0	2	0	-2

$\frac{1}{2}$ 

0	3	0	0
2	-1	-1	0
0	2	0	-2
0	1	0	-1



# Calculate Image Gradient in Y-direction

I

0	0	6	0
2	4	0	2
0	0	4	0
0	0	2	0

I(x,y+1)

0	0	0	0
0	0	6	0
2	4	0	2
0	0	4	0

I(x,y-1)

2	4	0	2
0	0	4	0
0	0	2	0
0	0	0	0

I(x,y+1) - I(x,y-1)

-2	-4	0	-2
0	0	2	0
2	4	-2	2
0	0	4	0

$\frac{\partial I}{\partial y}$

-1	-2	0	-1
0	0	1	0
1	2	-1	1
0	0	2	0

$$\frac{\partial I}{\partial y} = \frac{I(x, y + 1) - I(x, y - 1)}{2}$$

# Calculate Image Gradient Directions

$$\nabla I \left( \frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right)$$

Tuple

7			0
	-2	3	
		4	
1			

0			-5
	-2	3	
		1	
0			

$$\Theta = \tan^{-1} \left( \frac{\partial I / \partial y}{\partial I / \partial x} \right)$$

$\uparrow \pi/2$			$\tan^{-1}(0)?$
	$\swarrow \pi + \pi/4$	$\xrightarrow{0}$	
		$\tan^{-1}(4)?$	
$\xrightarrow{0}$			

# Calculate Image Gradient Magnitude

$$\text{Magnitude} = \sqrt{\left(\frac{\partial I}{\partial x}\right)^2 + \left(\frac{\partial I}{\partial y}\right)^2}$$

$$\frac{\partial I}{\partial x}$$

<sup>49</sup> 7			<sup>0</sup> 0
	<sup>4</sup> -2	<sup>9</sup> 3	
		<sup>16</sup> 4	
1			

$$\frac{\partial I}{\partial y}$$

<sup>0</sup> 0			<sup>25</sup> -5
	<sup>4</sup> -2	<sup>9</sup> 3	
		<sup>1</sup> 1	
0			

<sup>49</sup> $\sqrt{49}$			<sup>25</sup> $\sqrt{25}$
	$\sqrt{4}$	$\sqrt{9}$	
		$\sqrt{16}$	
1			

# Calculate Image Gradient in X-direction

0	0	6	0
2	4	0	2
0	0	4	0
0	0	2	0





# Calculate Gradient using a Filter (X direction)

I

0	0	6	0
2	4	0	2
0	0	4	0
0	0	2	0

$I(x+1,y)$

		2	

$I(x-1,y)$

		4	

$I(x+1,y) - I(x-1,y)$

		-2	

$\frac{\partial I}{\partial x}$

		-1	

filter

dot product

4	0	2
.	.	.
$\frac{1}{2}$	0	$-\frac{1}{2}$
=	=	=
-2	0	1

$\Sigma = -1$





# Calculate Gradient using a Filter (Y direction)

I

0	0	6	0
2	4	0	2
0	0	4	0
0	0	2	0

$I(x, y+1)$

		6	
		0	

$I(x, y-1)$

		4	
		2	

$I(x, y+1) - I(x, y-1)$

		2	
		-2	

$\frac{\partial I}{\partial y}$

		1	
		-1	

Answer:

0	6
4	0
2	4

$$\begin{aligned} &\cdot -\frac{1}{2} = \begin{bmatrix} 0 & 3 \\ 0 & 0 \\ -1 & -2 \end{bmatrix} \\ &\cdot 0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \\ &\cdot \frac{1}{2} = \begin{bmatrix} 0 & 3 \\ 0 & 0 \\ -1 & -2 \end{bmatrix} \end{aligned}$$

What is the gradient filter for the Y-direction? Show your calculations using at least two image locations.