RNN

Name	Nathan Varghese
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	Level	Completed
	Beginner	8
	Intermediate	2
	Advanced	0
	Expert	0

Goal	
5722	10

Total Completed
10

Recurrent Neural Network

CSCI 5722: Computer Vision

Fall 2024

Dr. Tom Yeh

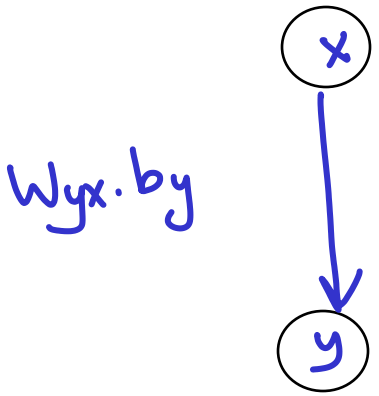
Evolution to RNNs

CSCI 5722 Computer Vision

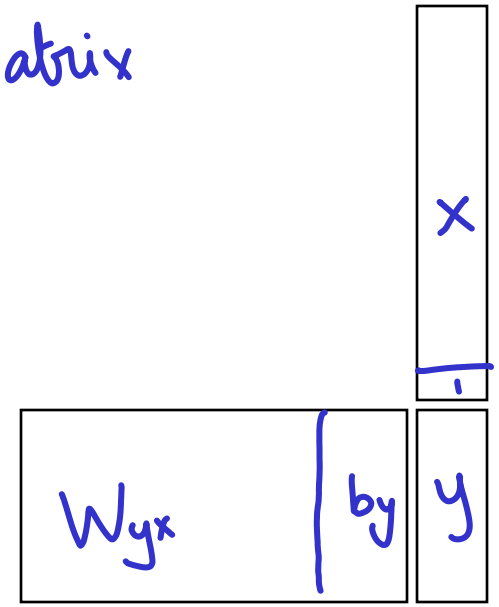
University of Colorado Boulder

Linear transformation

graphical



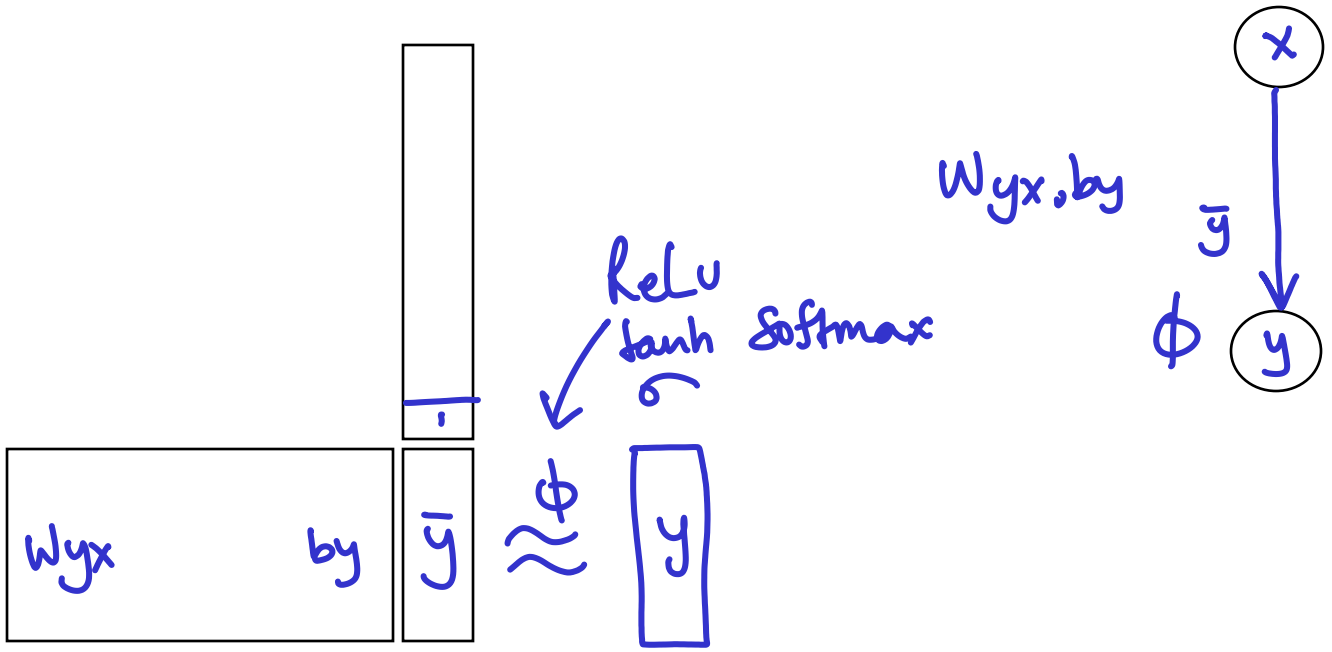
matrix



algebraic

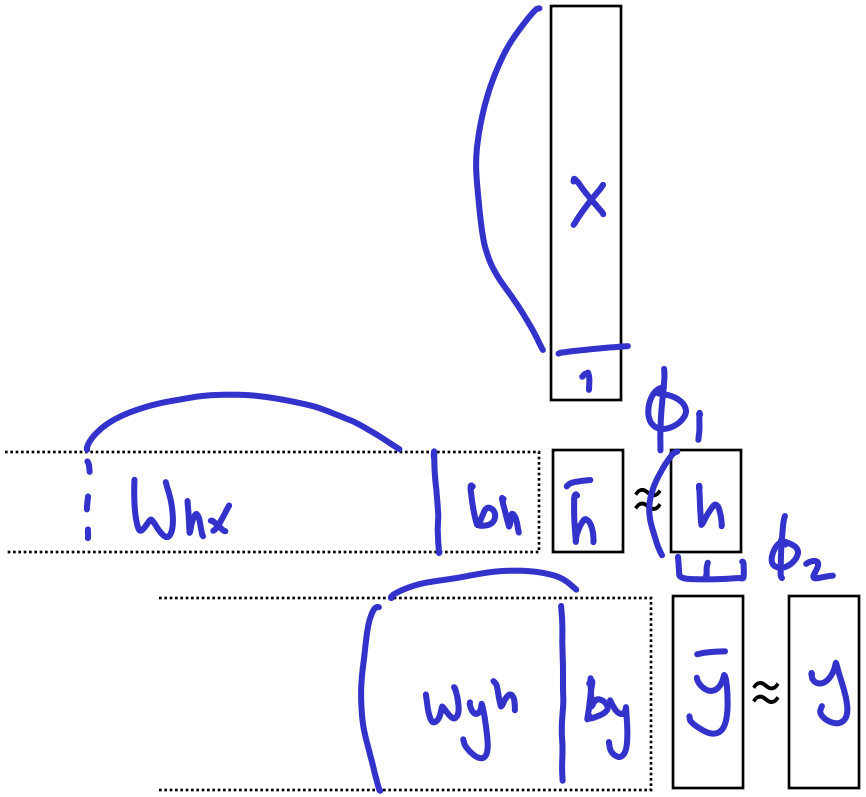
$$y = W_{yx} * X + b_y$$

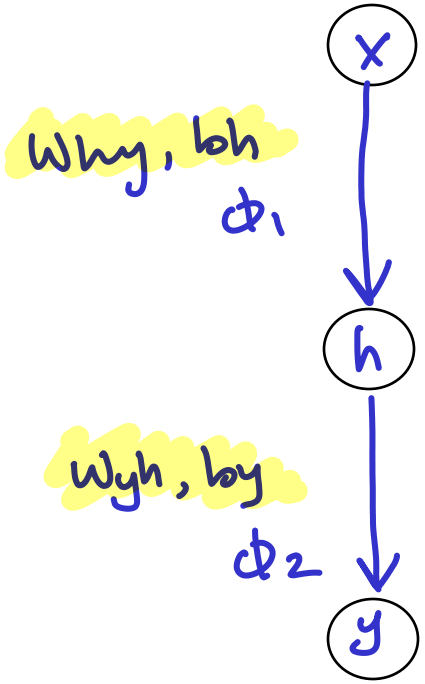
+ non-linearity



$$y = \phi(W_{yx} \cdot X + b_y)$$

+ hidden layer





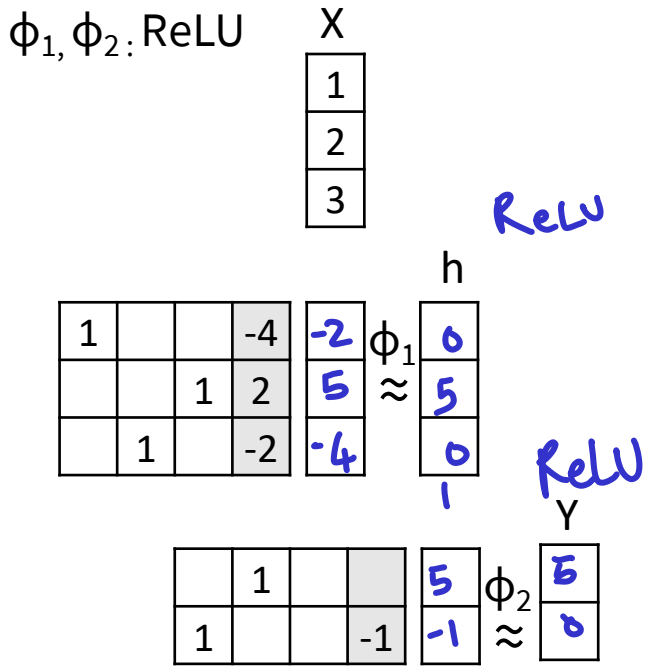
- trainable parameters

$h = \phi_1(W_{hx} \cdot X + b_h)$

$y = \phi_2(W_{yh} \cdot h + b_y)$

$= \phi_2(W_{yh} \cdot (\phi_1(W_{hx} \cdot X + b_h)) + b_y$

Calculate an NN with a hidden layer



$$Y = \phi_2(\mathrm{W}_2 \cdot \phi_1(W_1 \cdot X + b_1) + b_2)$$

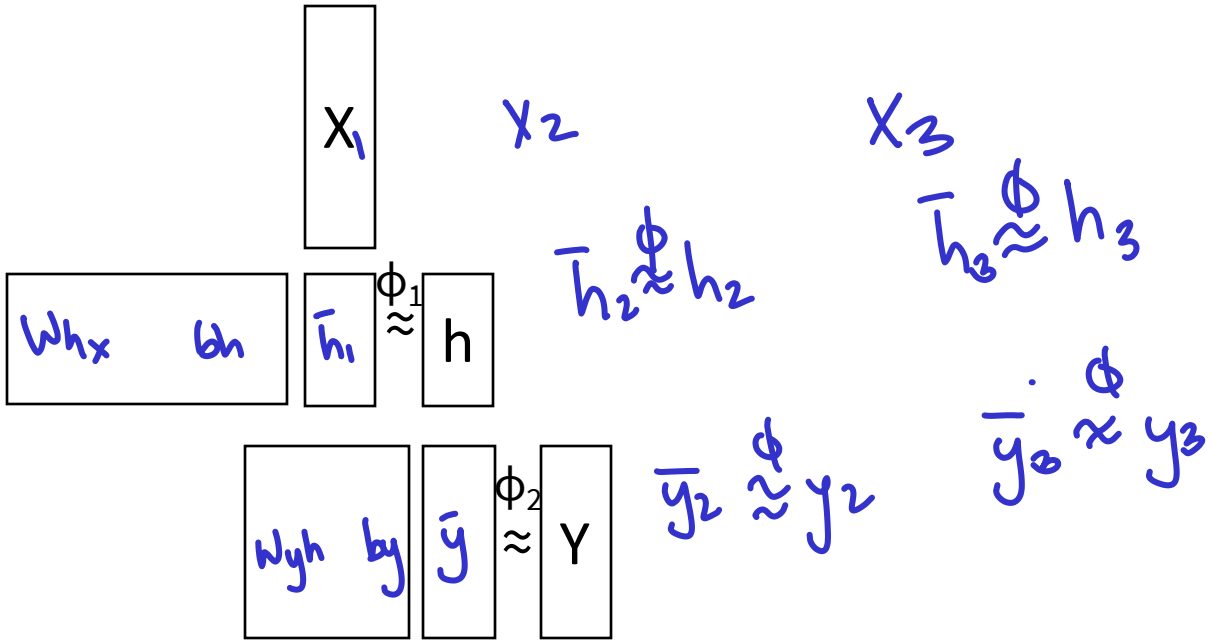
size(W_1) = 3 x 3

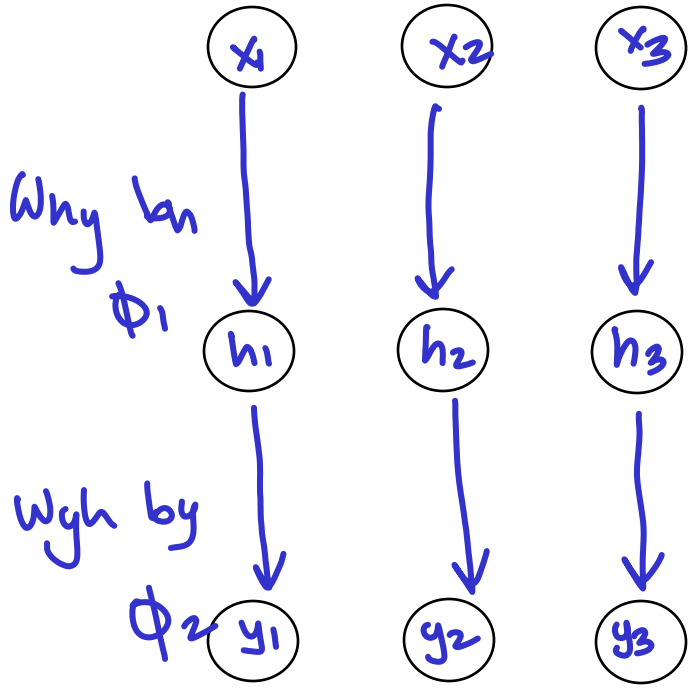
size(b_1) = 3 x 1

size(W_2) = 2 x 3

size(b_2) = 2 x 1

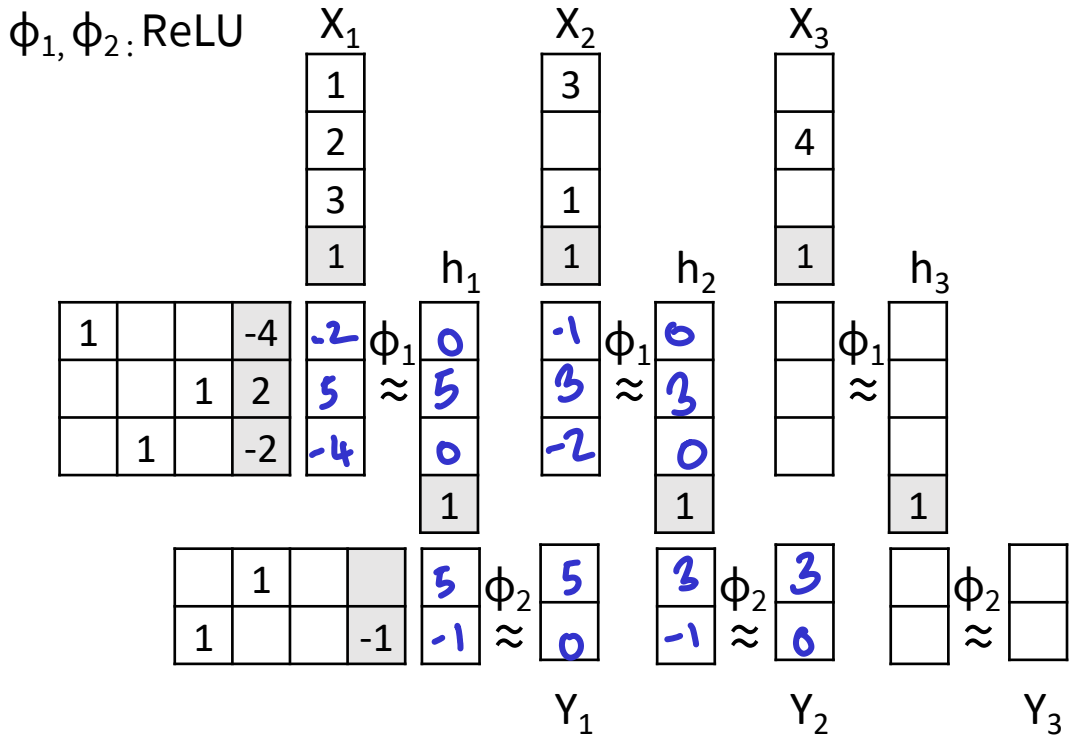
+ sequence





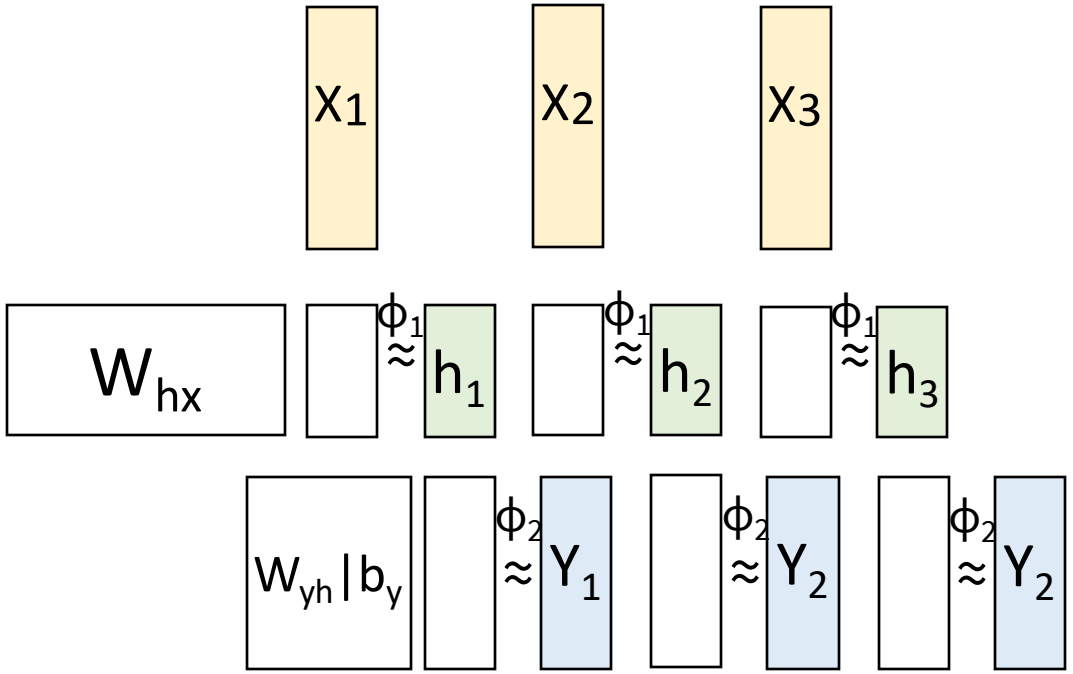
$$y_t = \Phi_2\left(W_{yh}\,\Phi_1(W_{hx}X_t + b_h) + b_y\right) \qquad t = 1,2,3\ldots$$

Calculate an NN for a batch of inputs

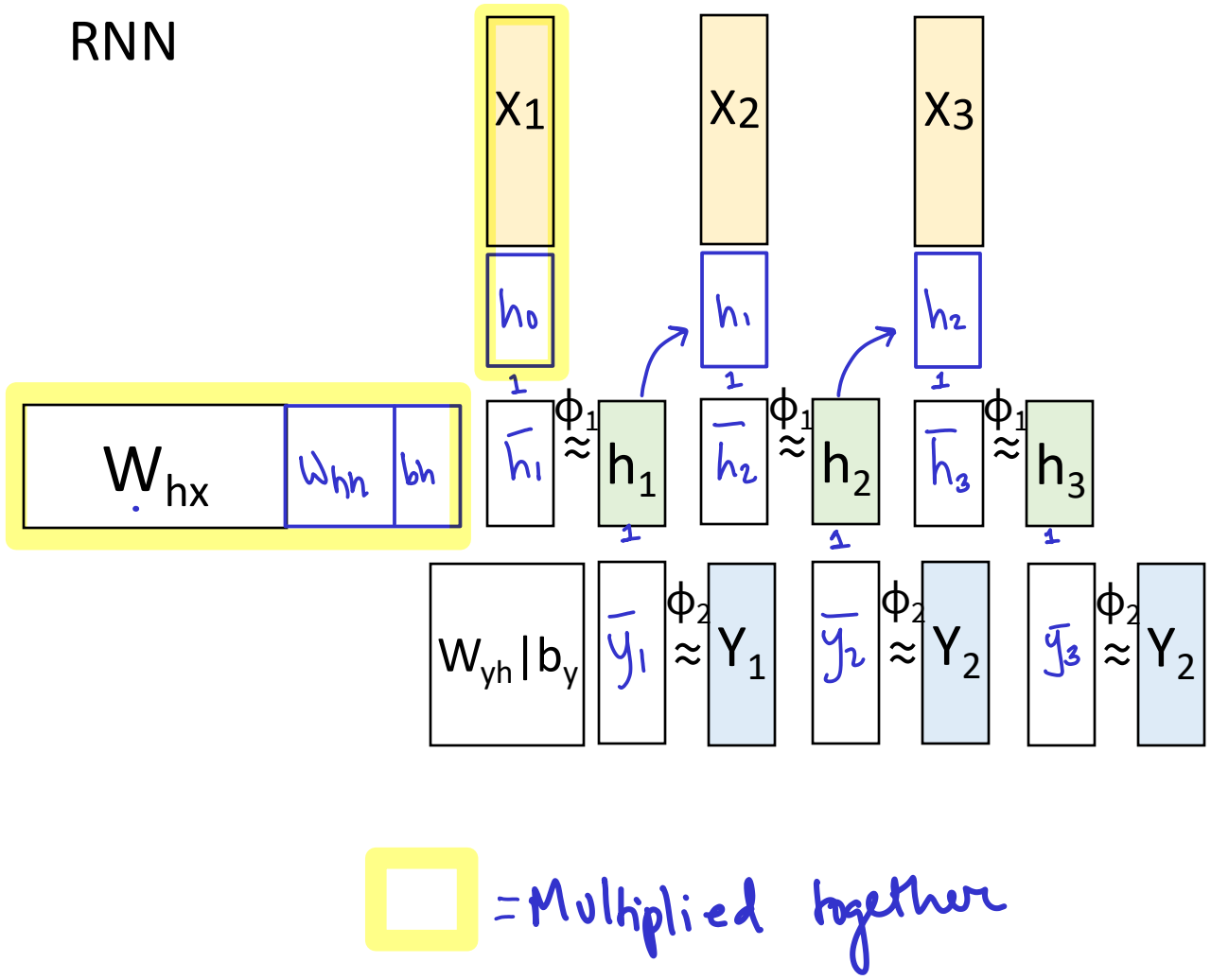


+ dependence on previous inputs

Independent NN



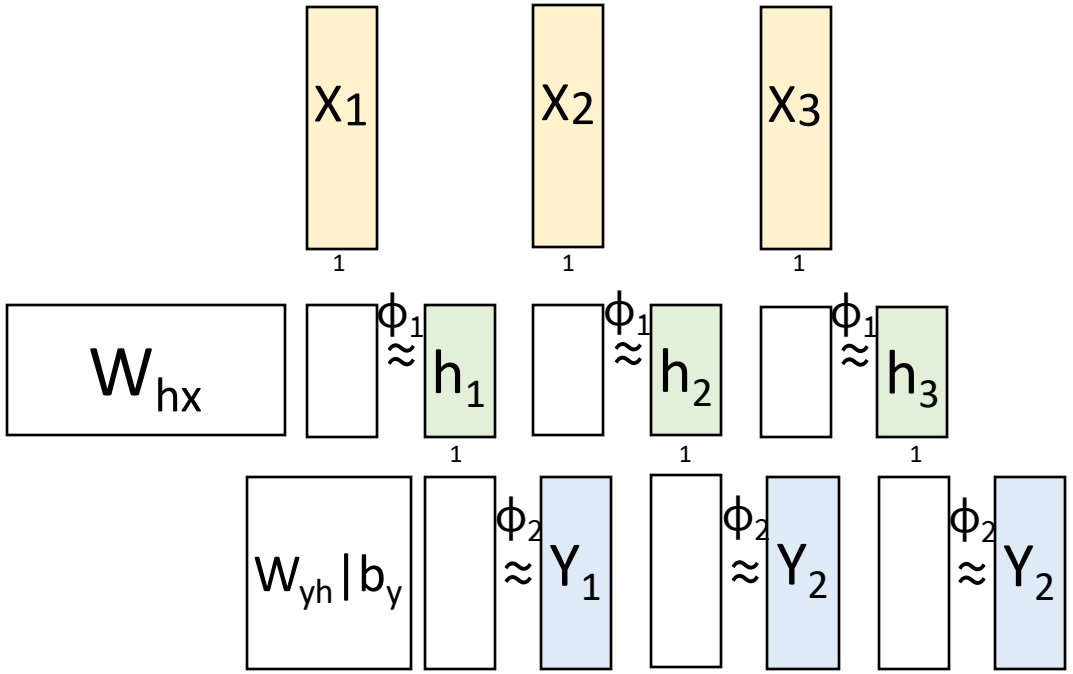
RNN



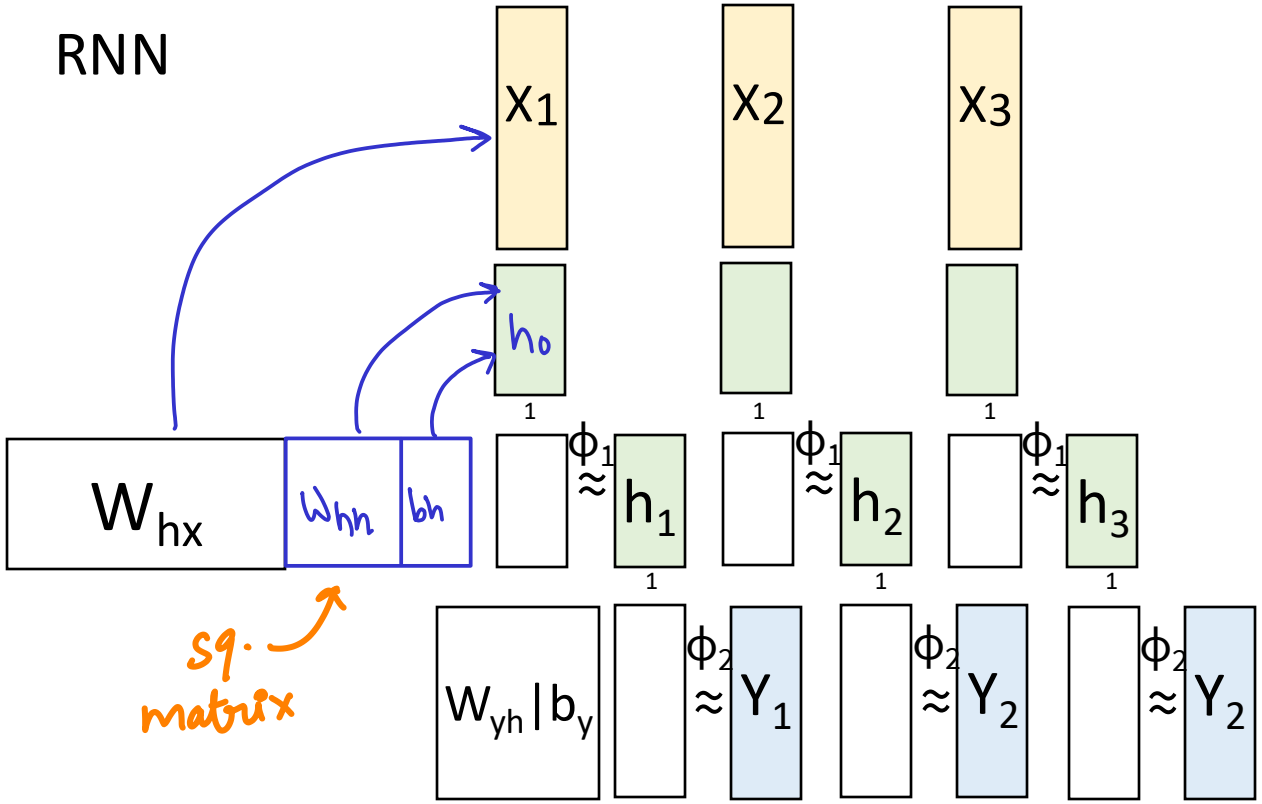
NN vs RNN: Math

- New term

Independent NN



RNN



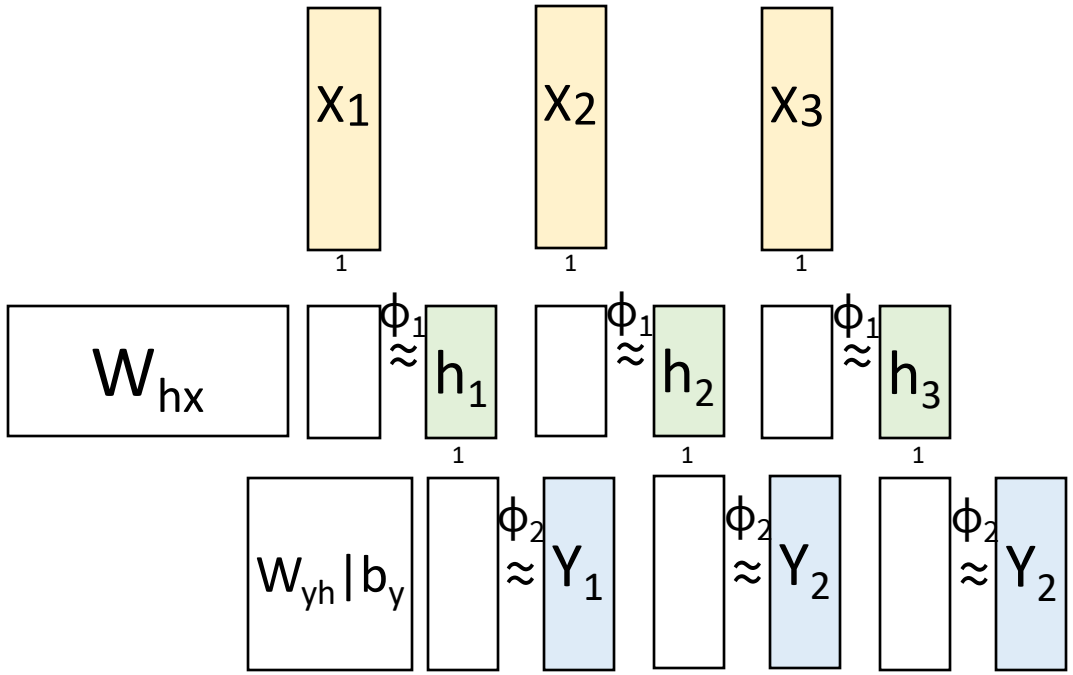
$$h_t = \phi_1(W_{hx} \cdot X_t + bh)$$

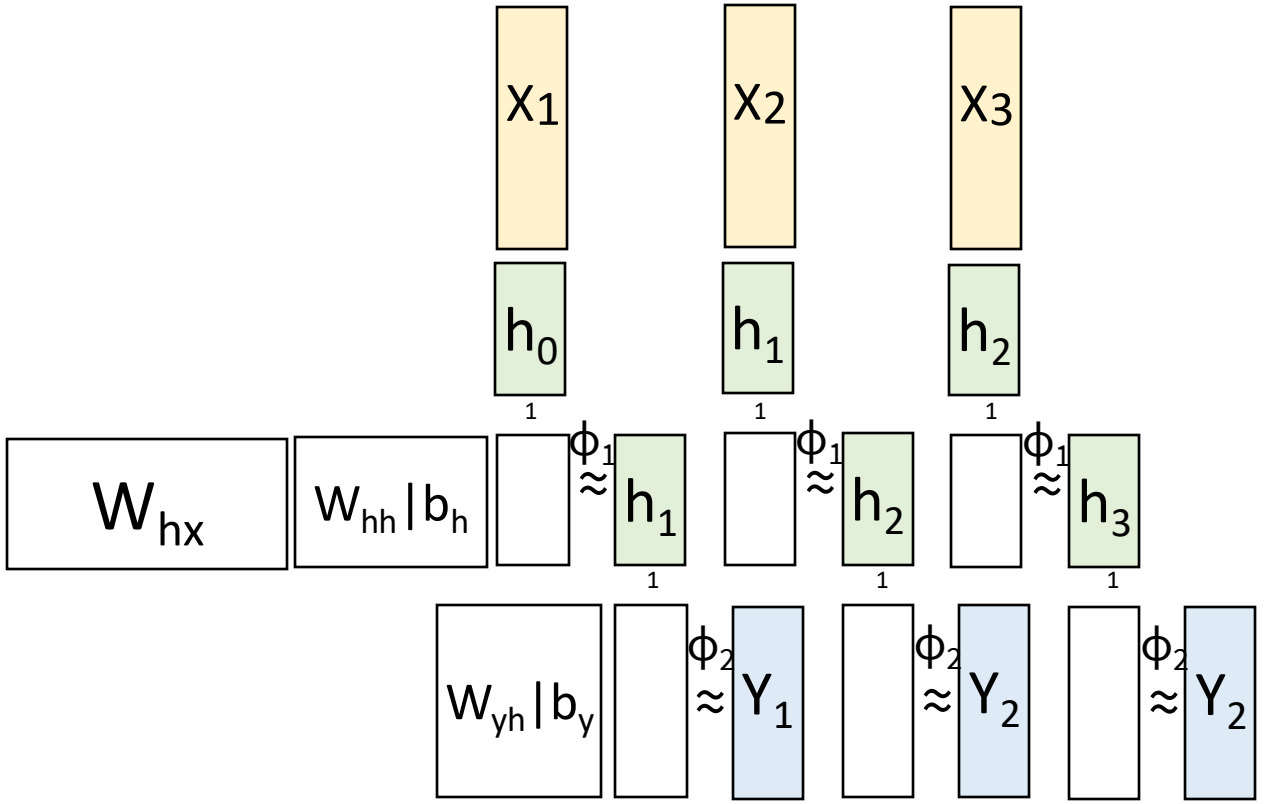
$$Y_t = \phi_2(W_{yh} \cdot h_t + by)$$

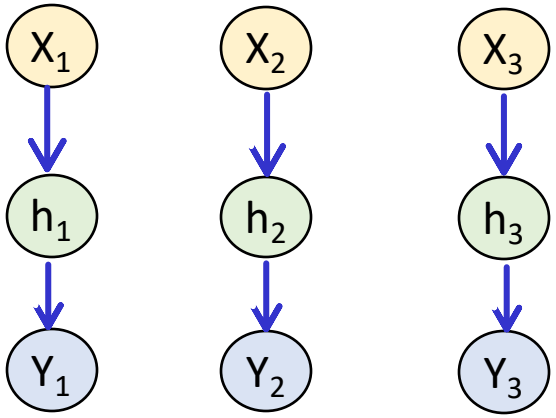
$h_t =$ $\phi_1 (W_{hx} \cdot X_t + W_{hh} \cdot h_{t-1} + bh)$

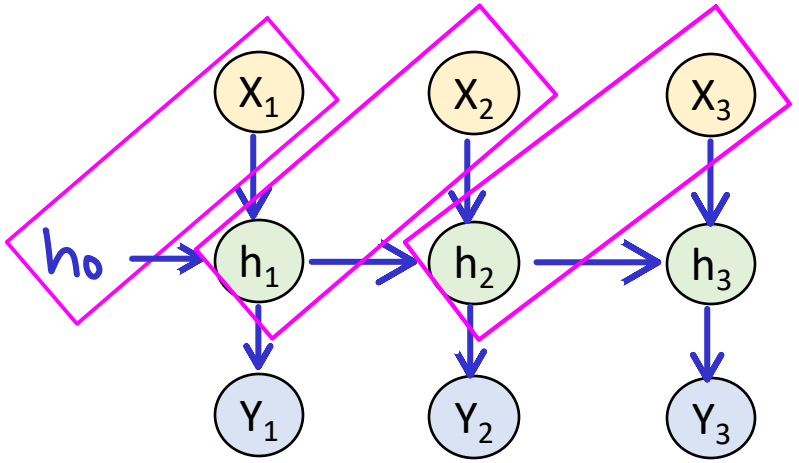
$Y_t =$ $\phi_2 (W_{yh} \cdot h_t + by)$

NN vs. RNN: Graph

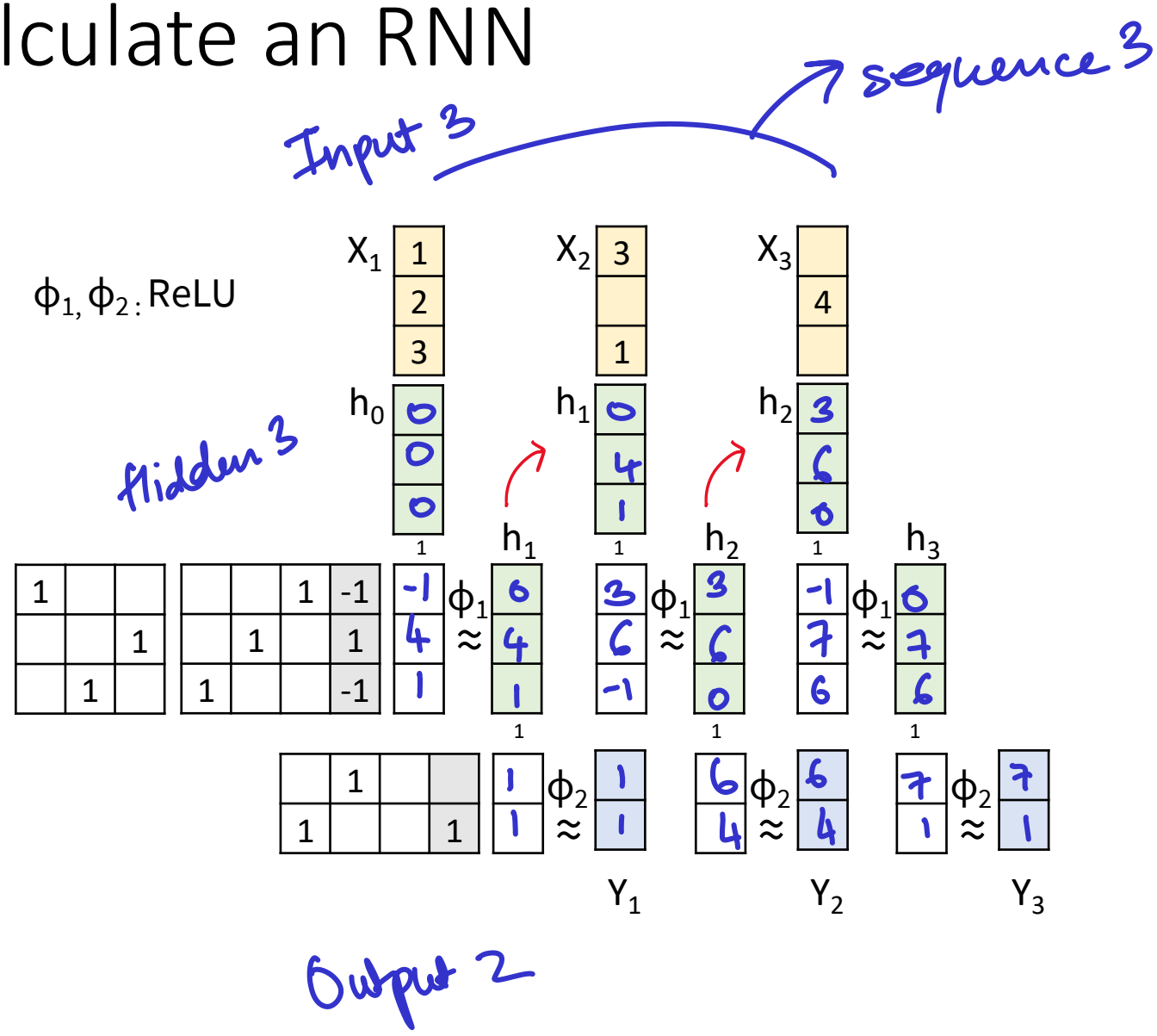




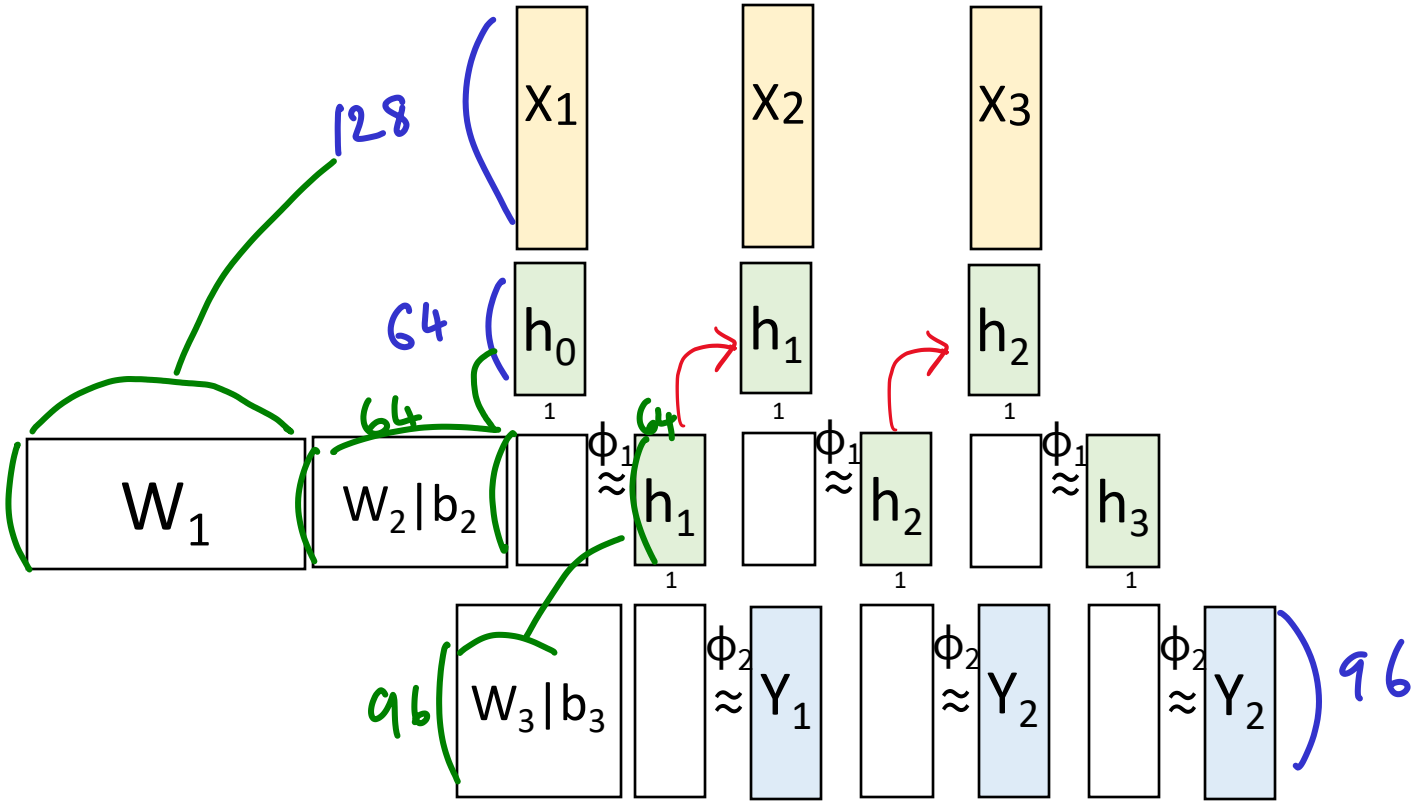




Calculate an RNN



Counting Parameters



Given :
size(X) = 128
size(h) = 64
size(Y) = 96

Determine:

size(W_1) = 64 x 128

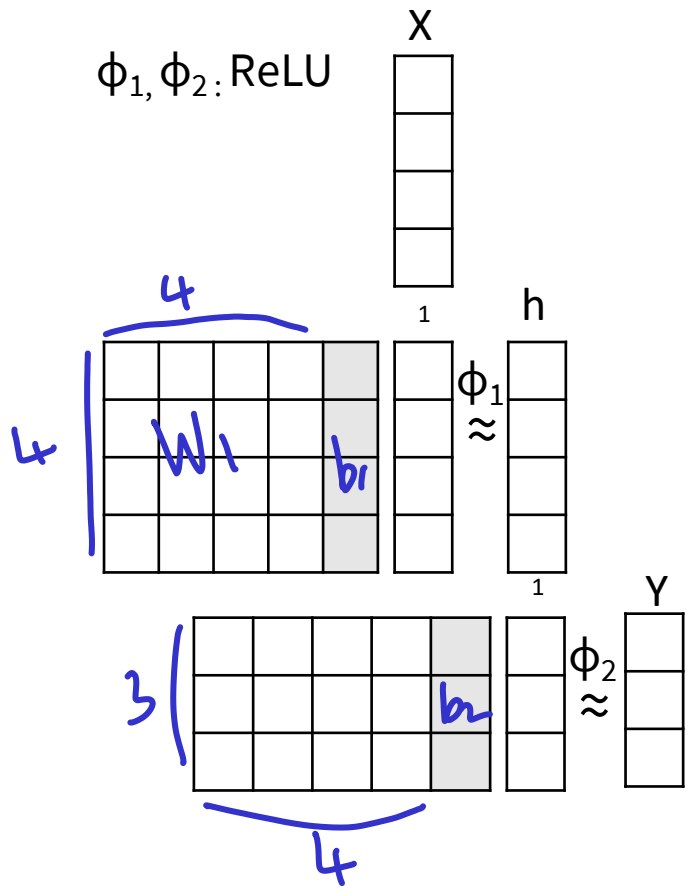
size(W_2) = 64 x 64

size(b_2) = 64 x 1

size(W_3) = 96 x 64

size(b_3) = 96 x 1

MLP Parameter Sizes



size(W_1) = 4 x 4

size(b_1) = 4 x 1

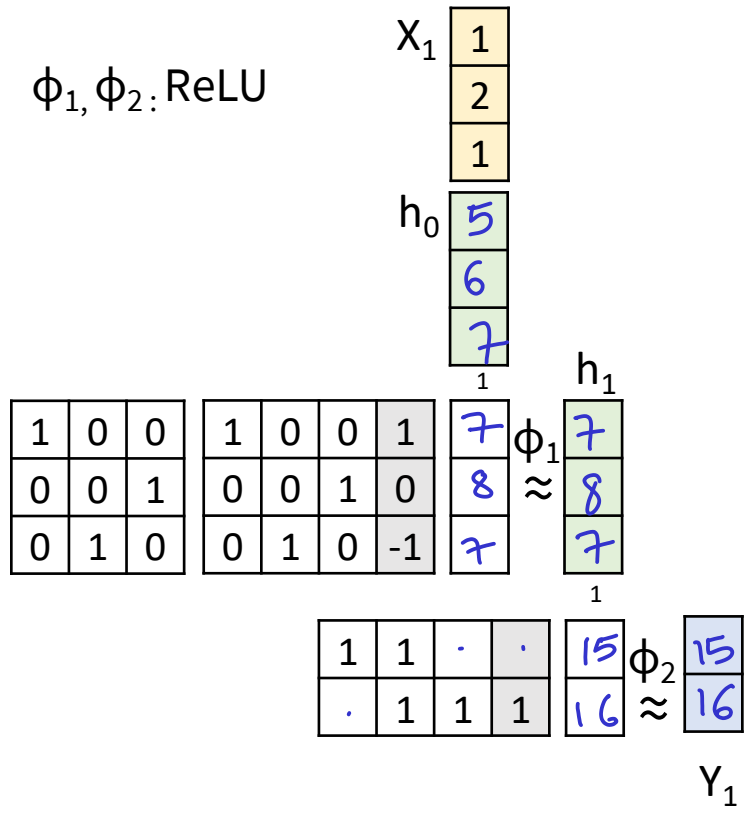
size(W_2) = 3 x 4

size(b_2) = 3 x 1

$$Y = \phi_2(\mathrm{W}_2 \cdot \phi_1(W_1 \cdot X + b_1) + b_2)$$

Calculate an RNN (t = 1)

This activity is standalone, not dependent on other activities.



$1+5+1=7$

$1+7=8$

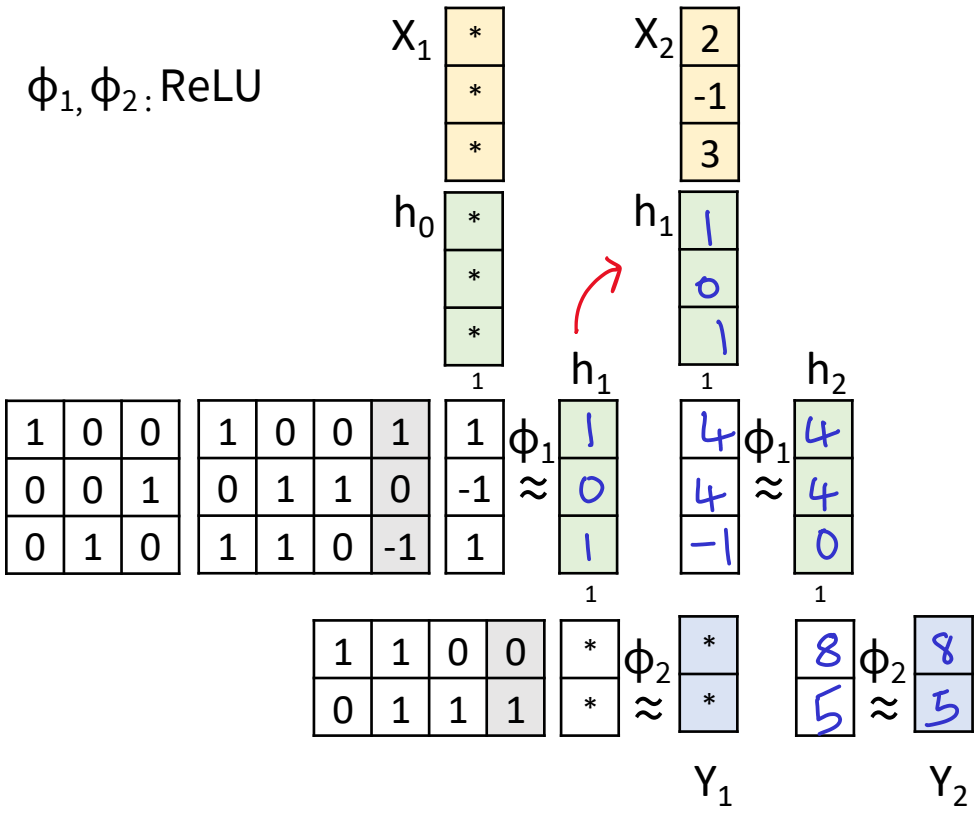
$2+6-1=7$

$7+8=15$

$8+7+1=16$

Calculate an RNN (t = 2)

This activity is standalone, not dependent on other activities.



$2+1+1=4$

$3+1=4$

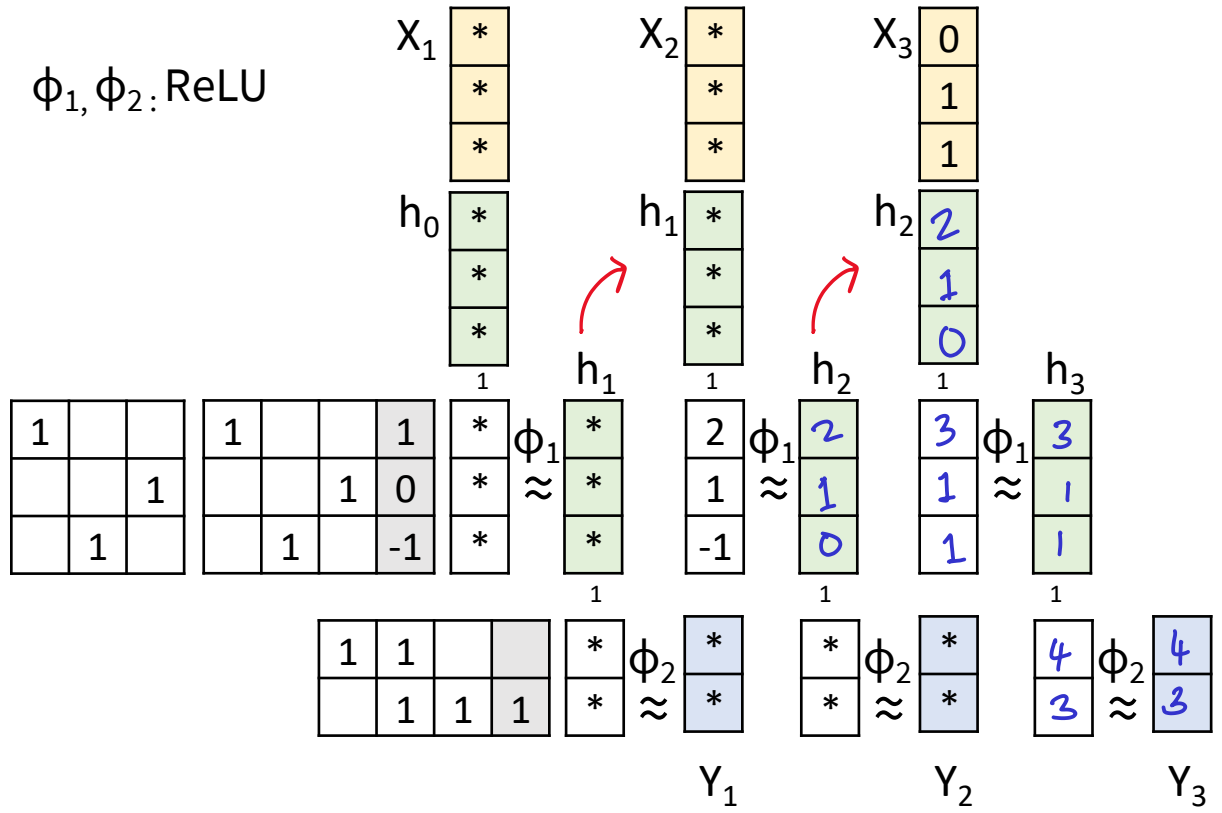
$-1+1-1=-1$

$4+4=8$

$4+1=5$

Calculate an RNN (t = 3)

This activity is standalone, not dependent on the previous one.



2 + 1 = 3

1 + 0 = 1

1 + 1 − 1 = 1

3 + 1 = 4

1 + 1 + 1 = 3

Variations of RNNs

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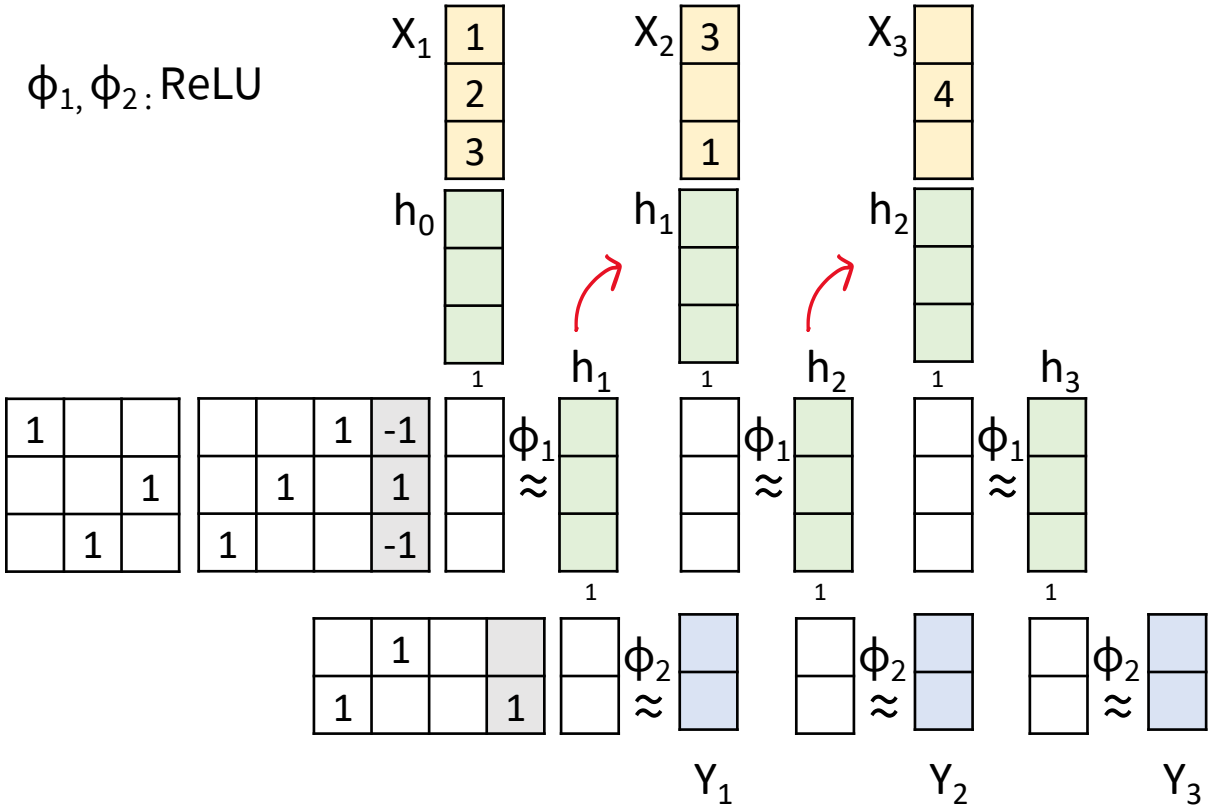
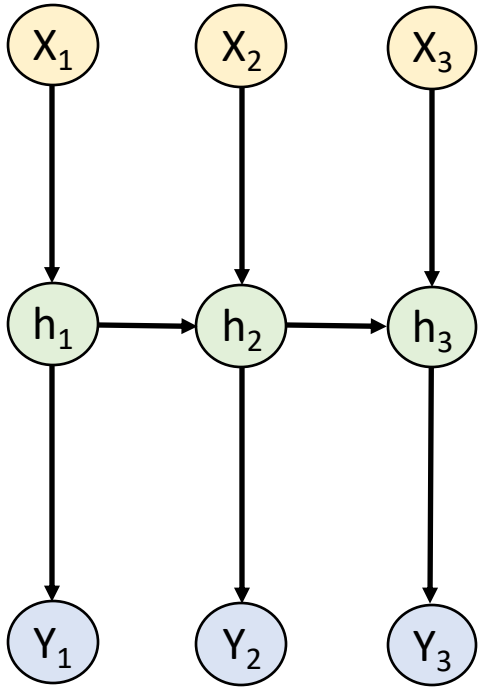
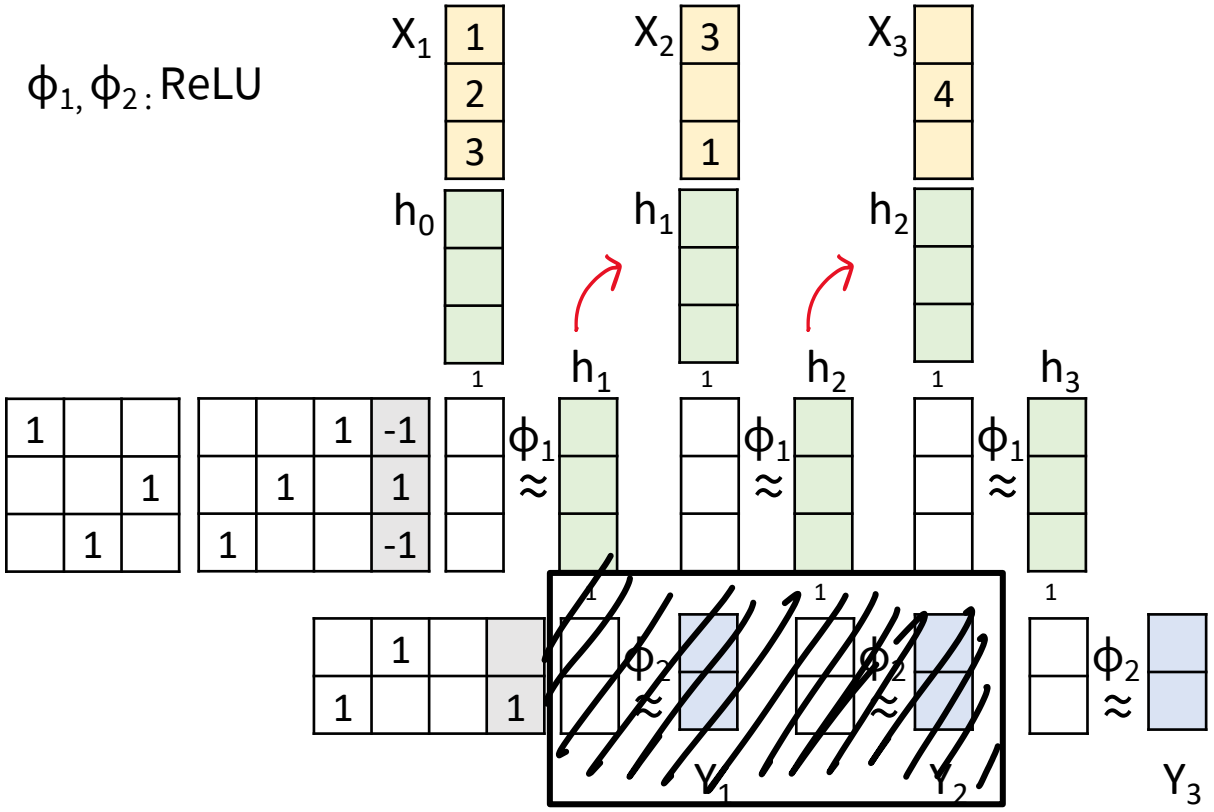
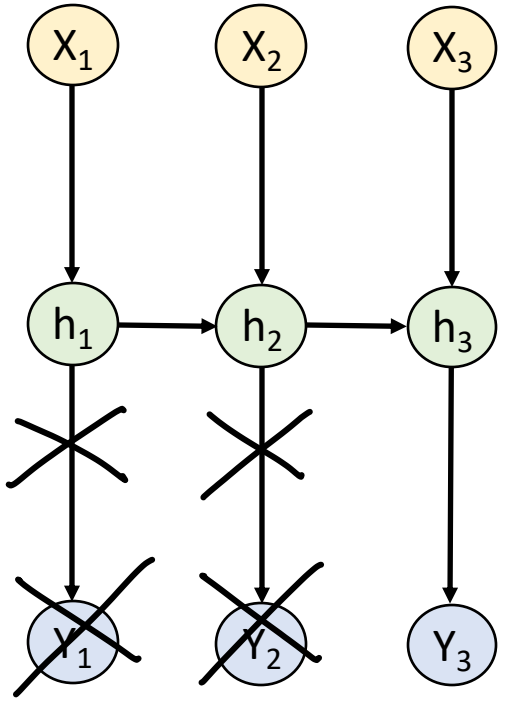
Many to Many


ϕ_1, ϕ_2 : ReLU
X_1
X_2
X_3
h_0
h_1
h_2
h_3
ϕ_1
ϕ_2
Y_1
Y_2
Y_3

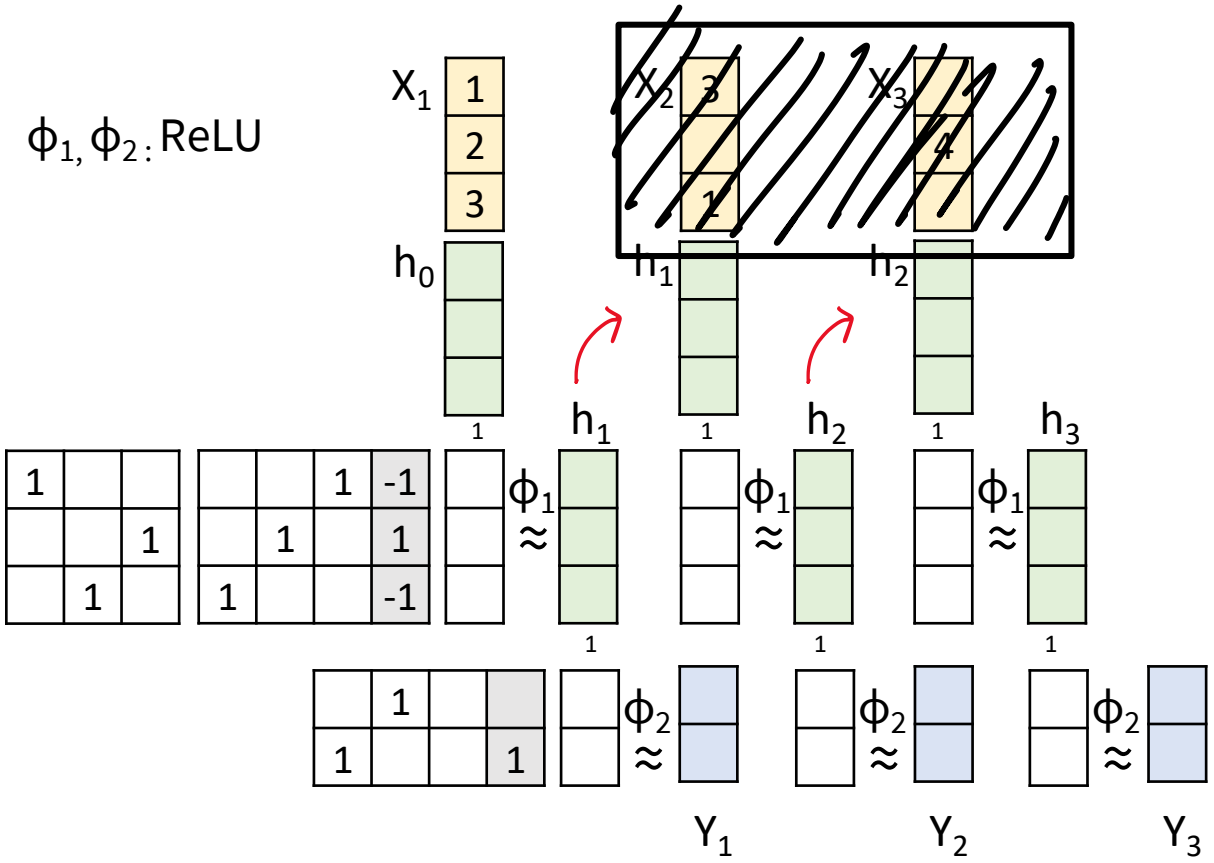

X_1
X_2
X_3
h_1
h_2
h_3
Y_1
Y_2
Y_3

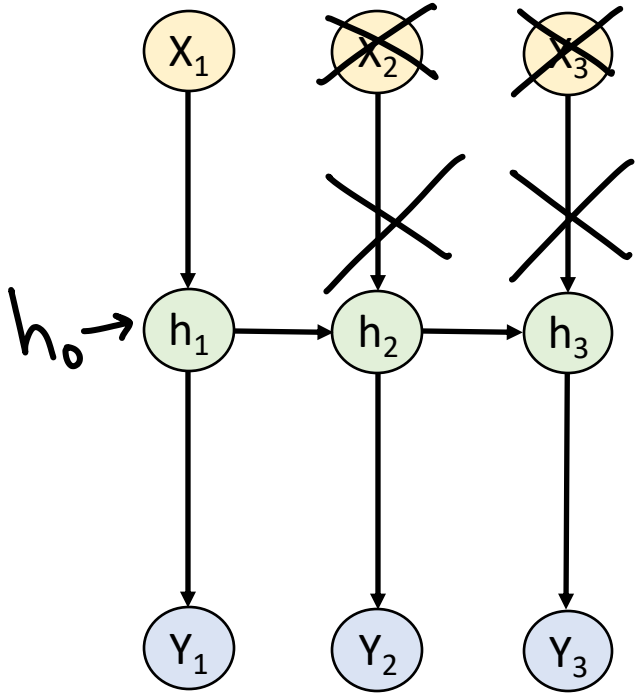
Many to One


ϕ_1, ϕ_2 : ReLU
X_1
X_2
X_3
h_0
h_1
h_2
h_3
ϕ_1
ϕ_2
Y_1
Y_2
Y_3

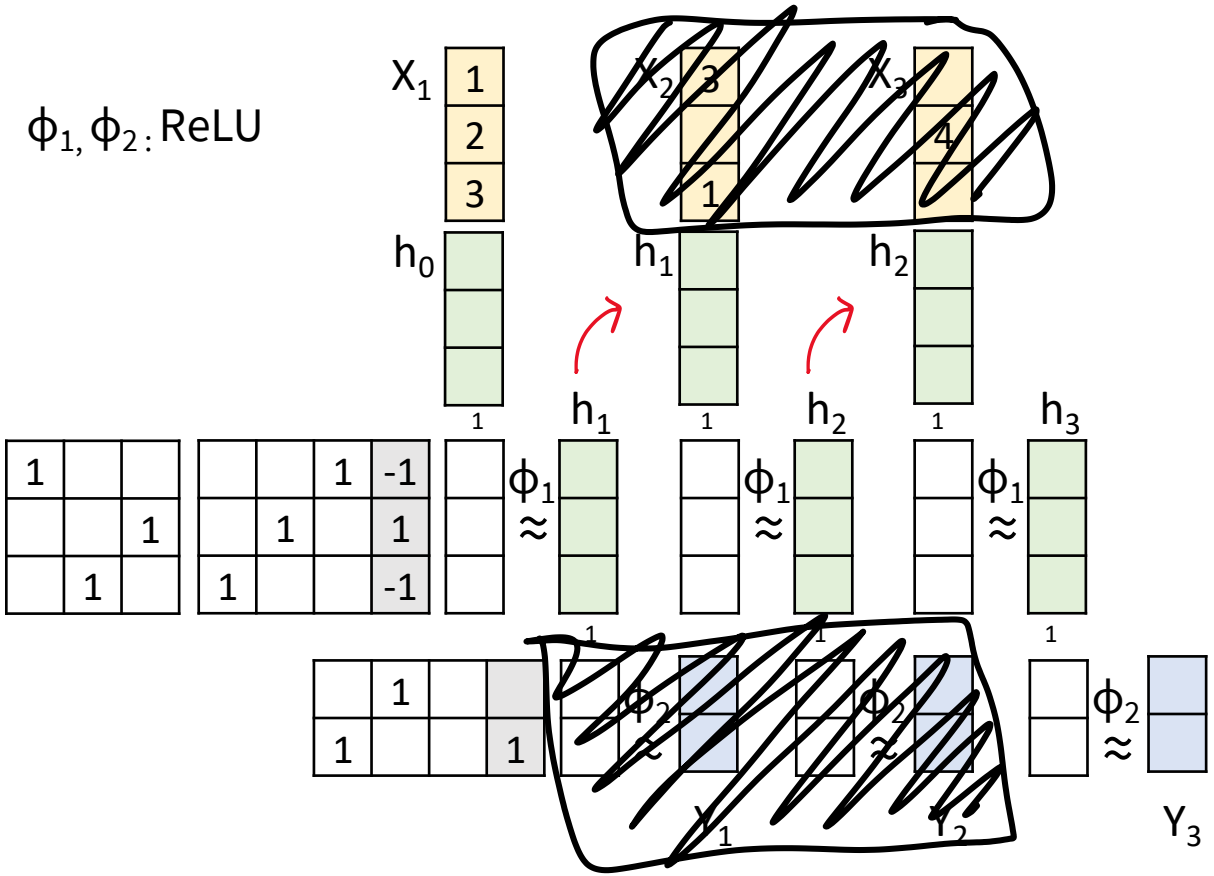

X_1
X_2
X_3
h_1
h_2
h_3
Y_1
Y_2
Y_3

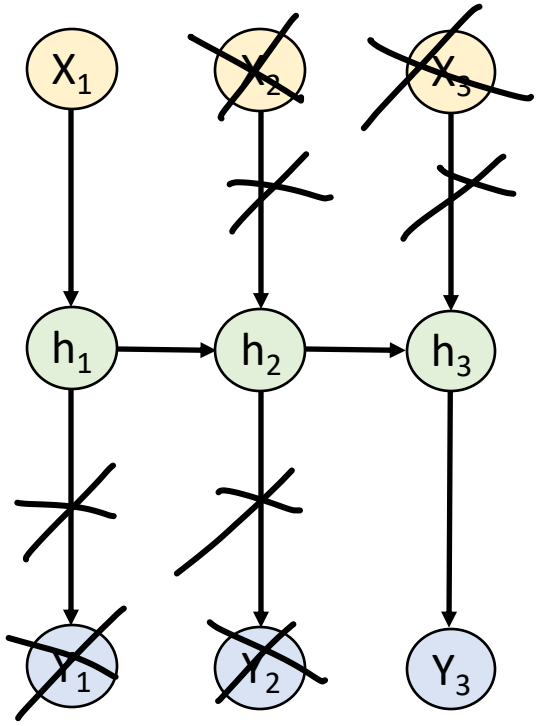
One to Many


ϕ_1, ϕ_2 : ReLU
X_1
X_2
X_3
h_0
h_1
h_2
h_3
ϕ_1
ϕ_2
Y_1
Y_2
Y_3


X_1
X_2
X_3
h_0
h_1
h_2
h_3
Y_1
Y_2
Y_3

One to One


ϕ_1, ϕ_2 : ReLU
X_1
X_2
X_3
h_0
h_1
h_2
h_3
ϕ_1
ϕ_2
Y_1
Y_2
Y_3


X_1
X_2
X_3
h_1
h_2
h_3
Y_1
Y_2
Y_3

Advanced RNN Architectures

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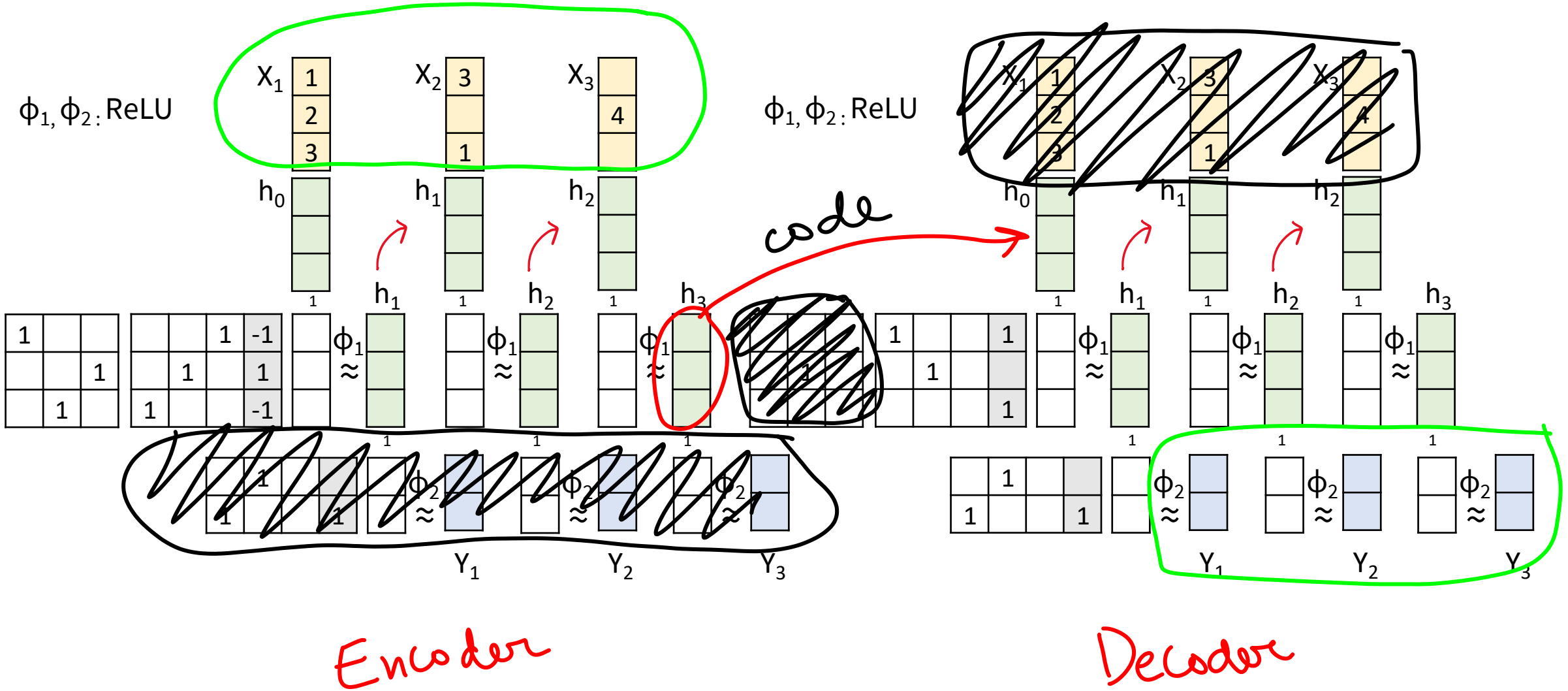
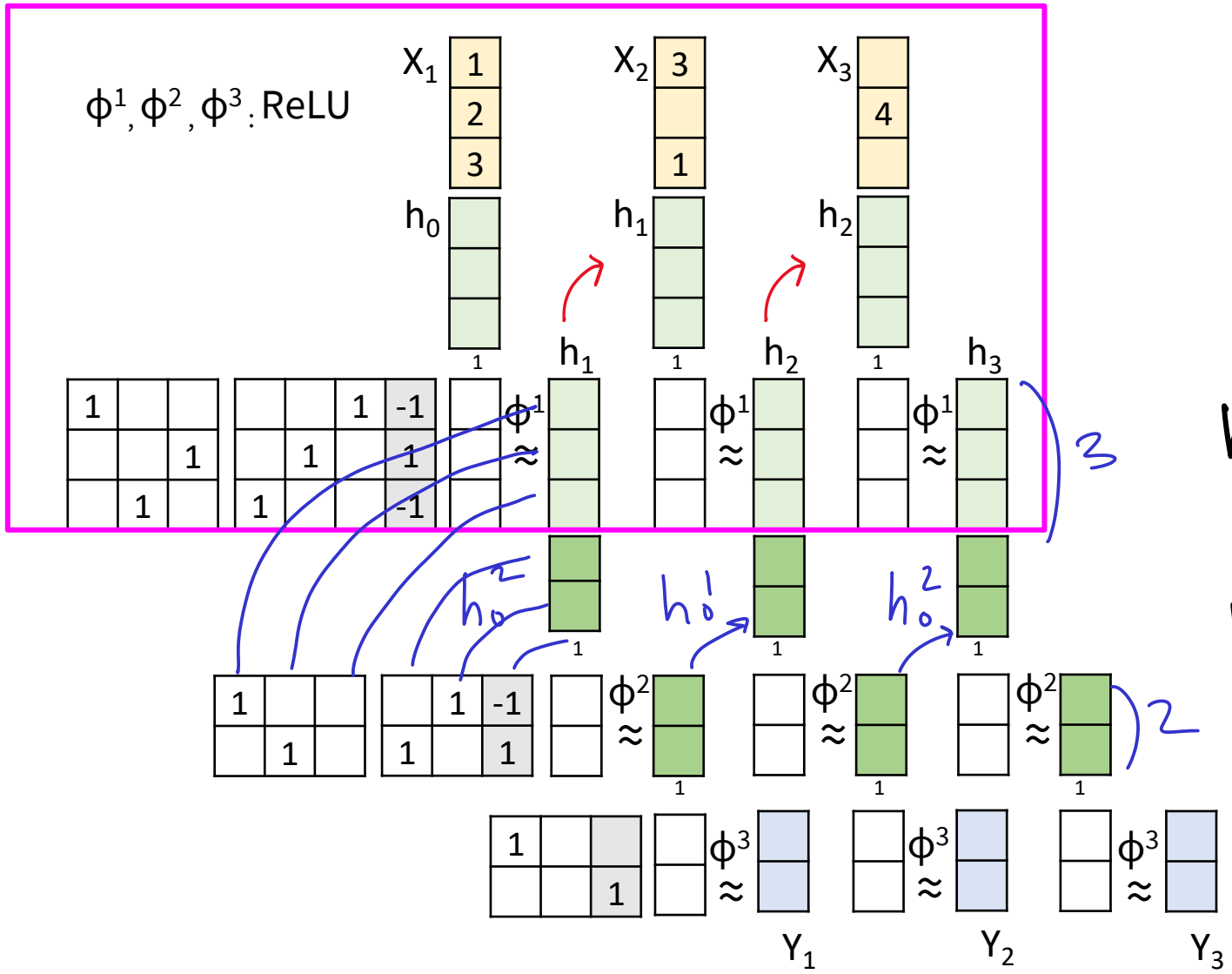
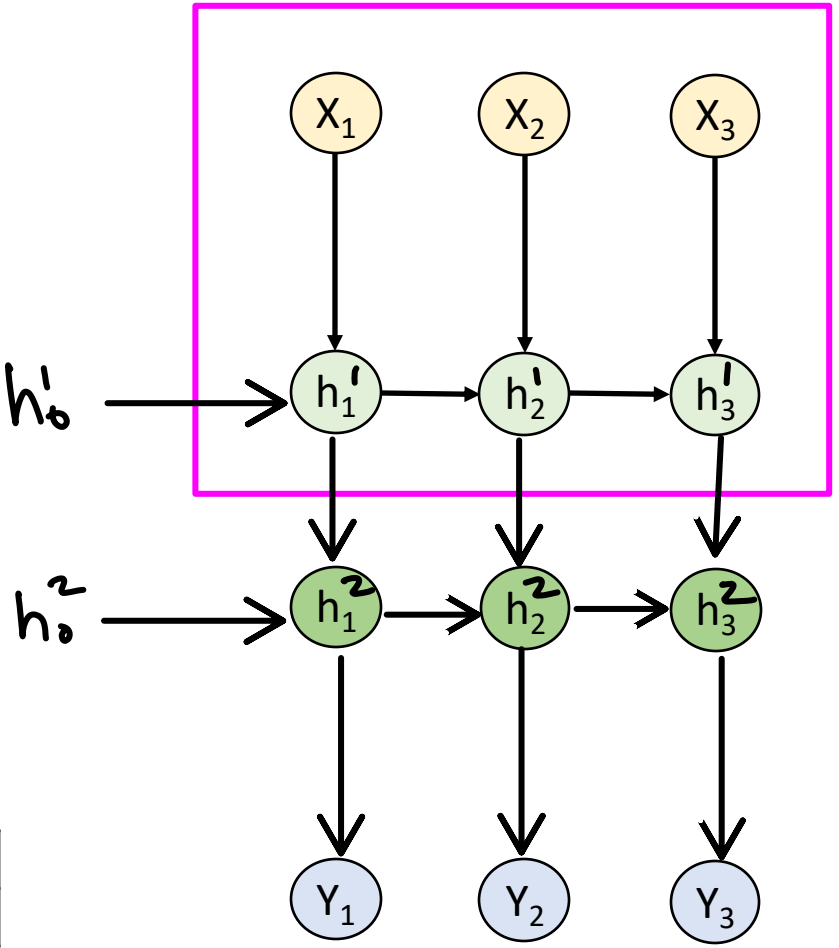
Sequence to Sequence

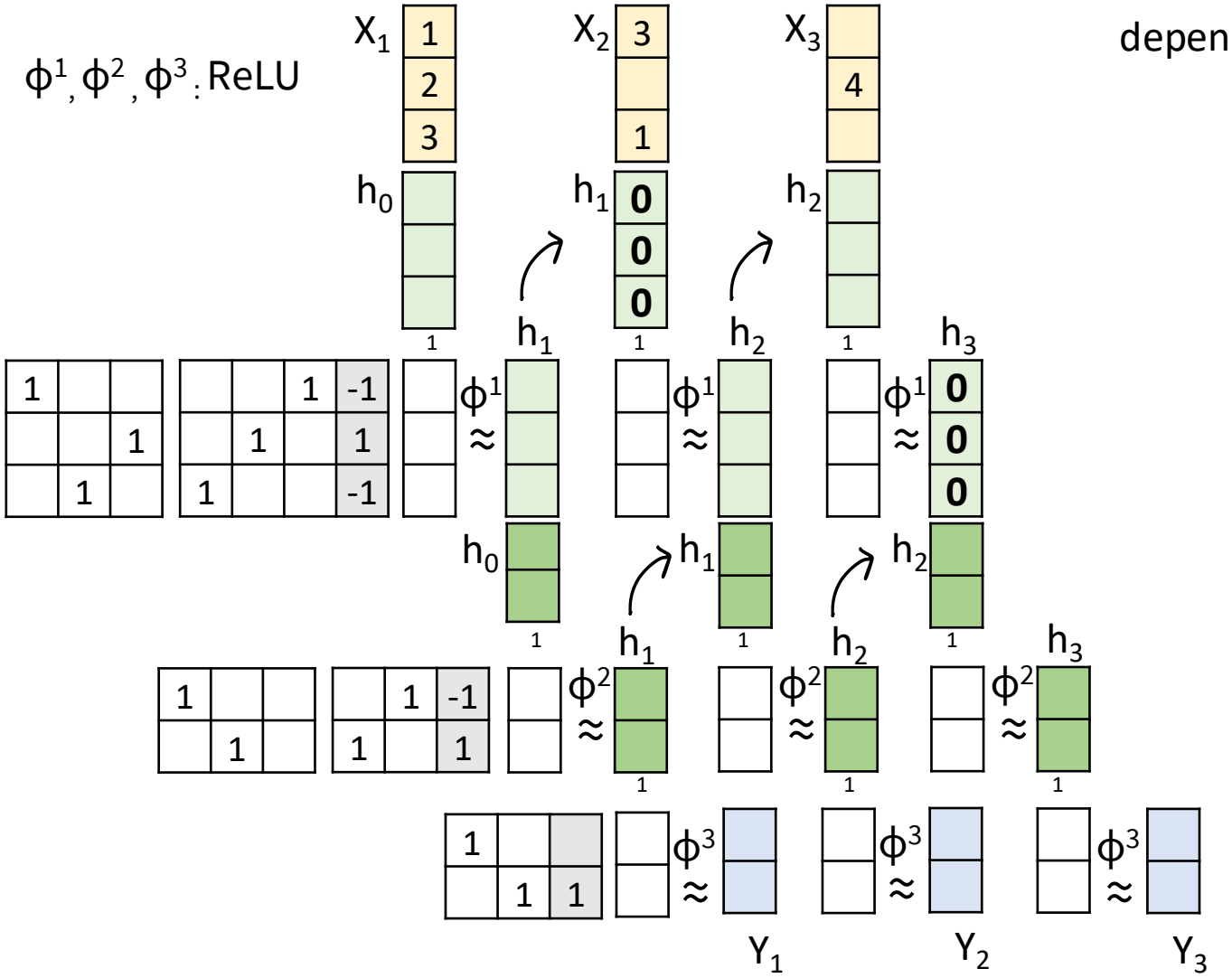


Multilayer RNN

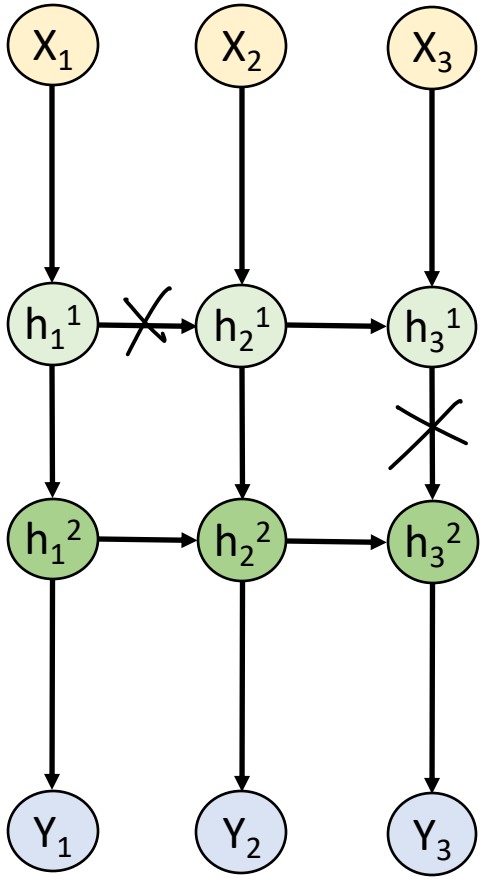




Identify links

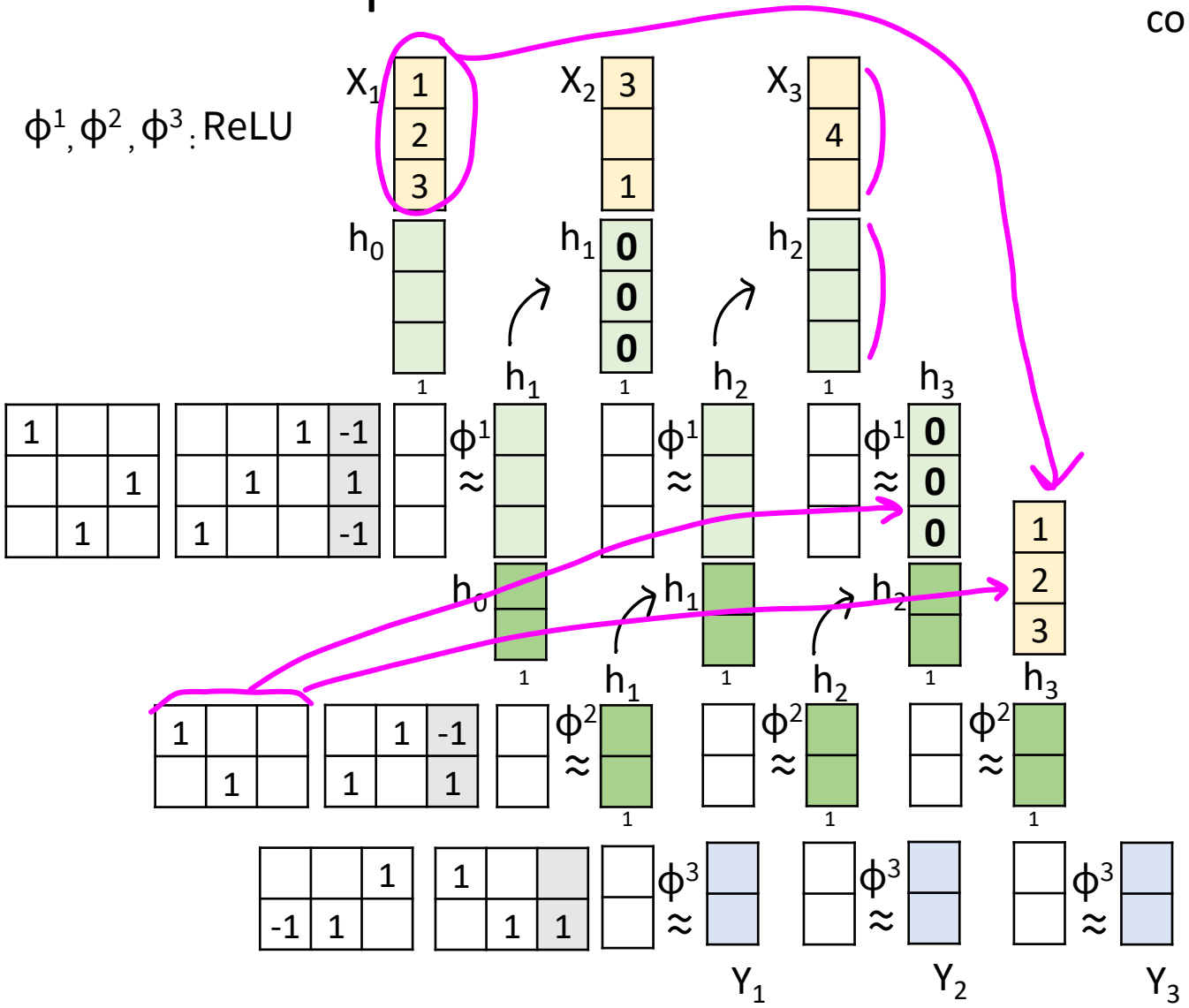


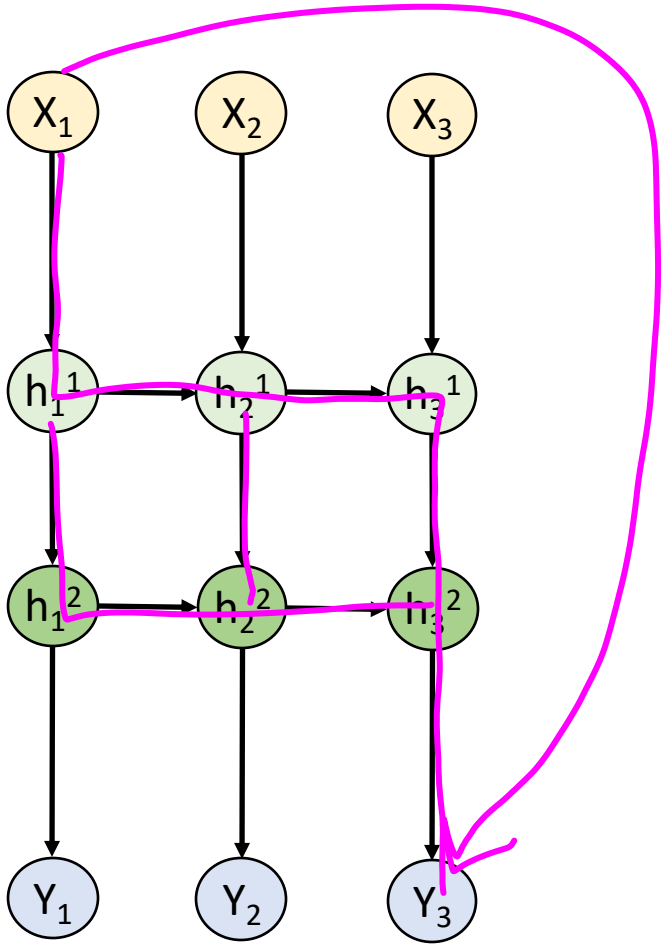
Suppose we set some hidden states to zeros on purpose. Cross out the affected dependency links.



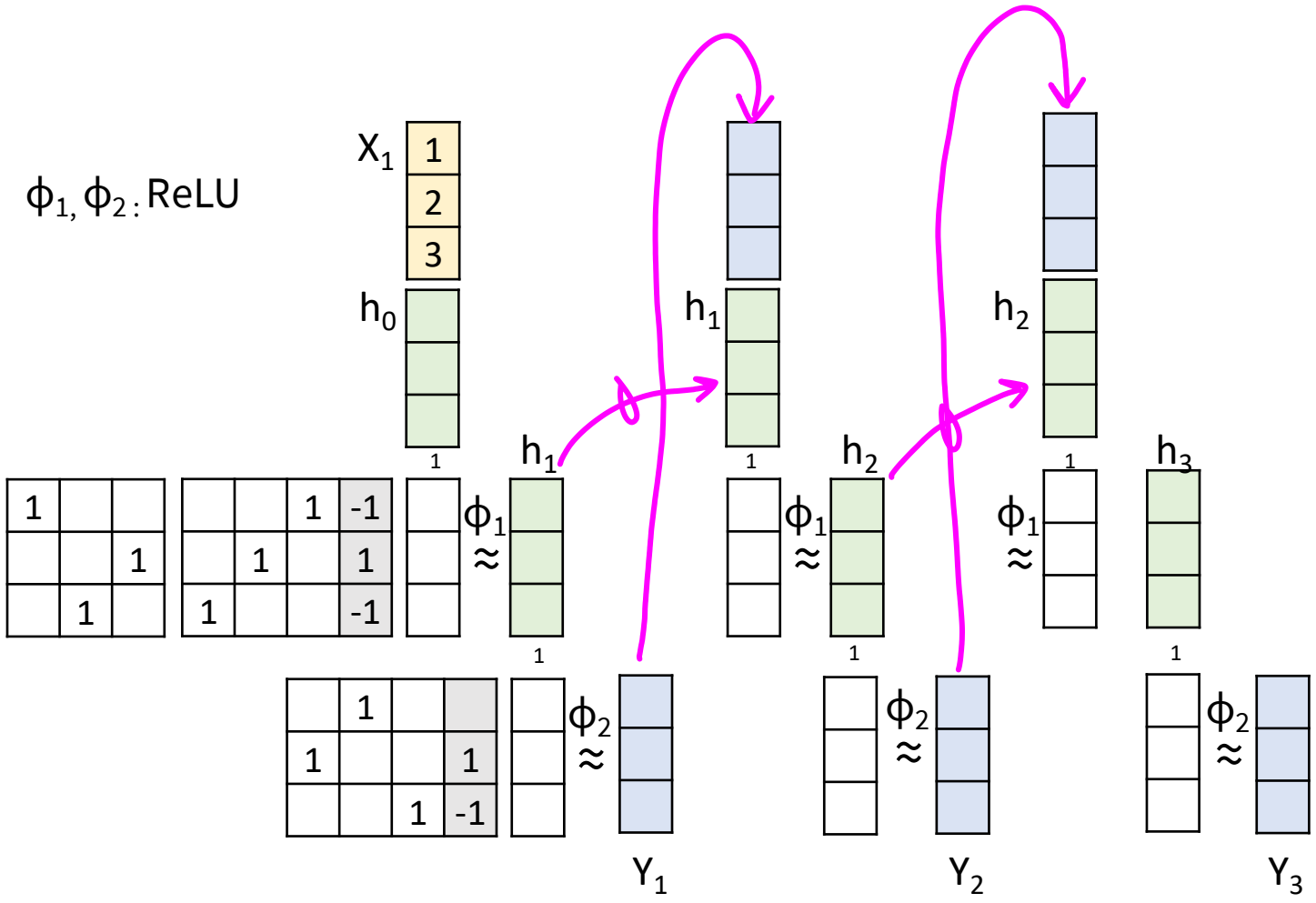
Add a skip connection

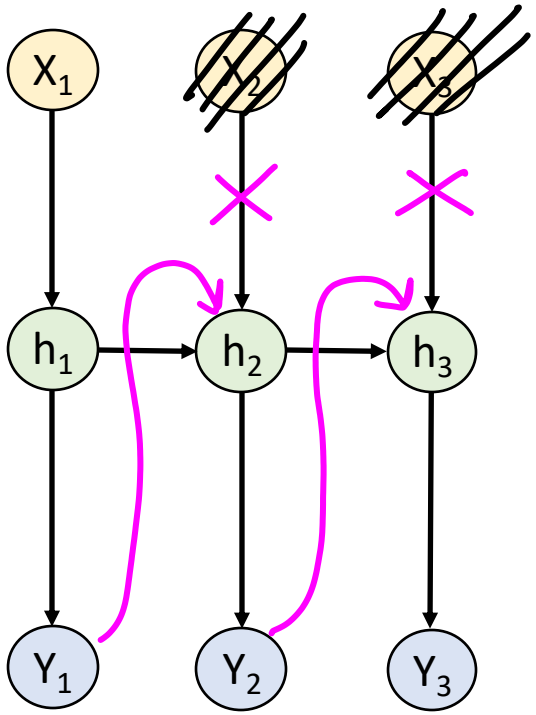
Draw a new arrow to illustrate the skip connection from X_1.



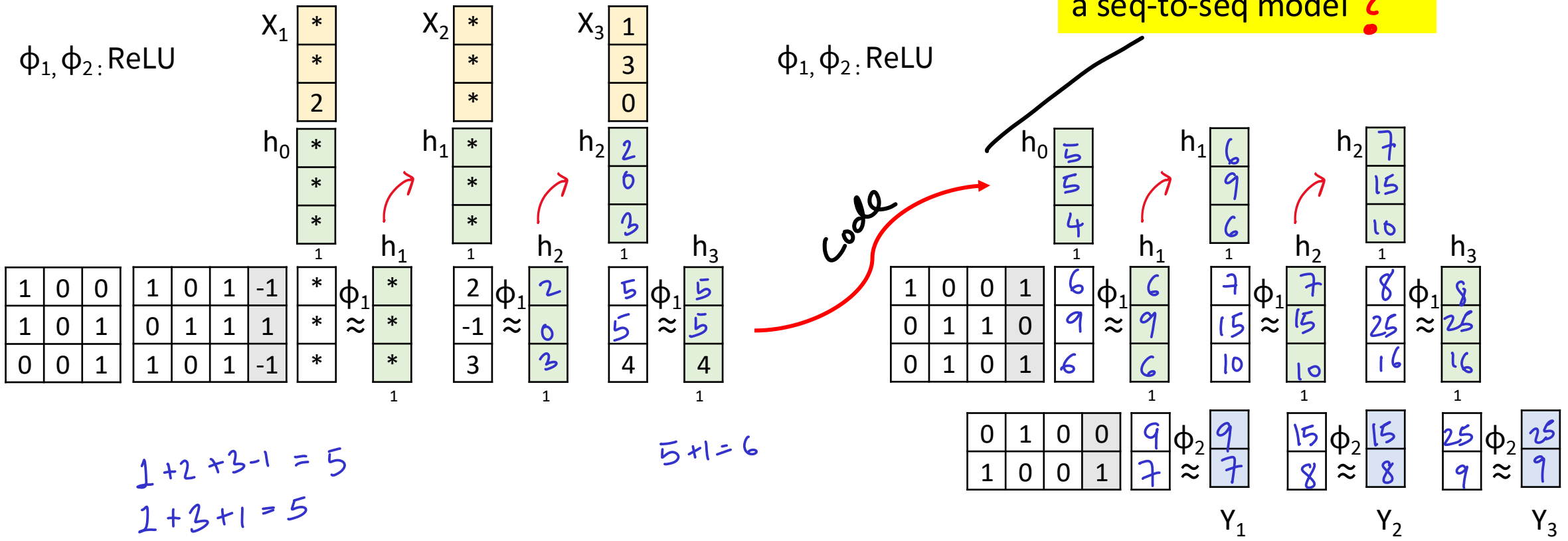


Auto-Regressive RNN

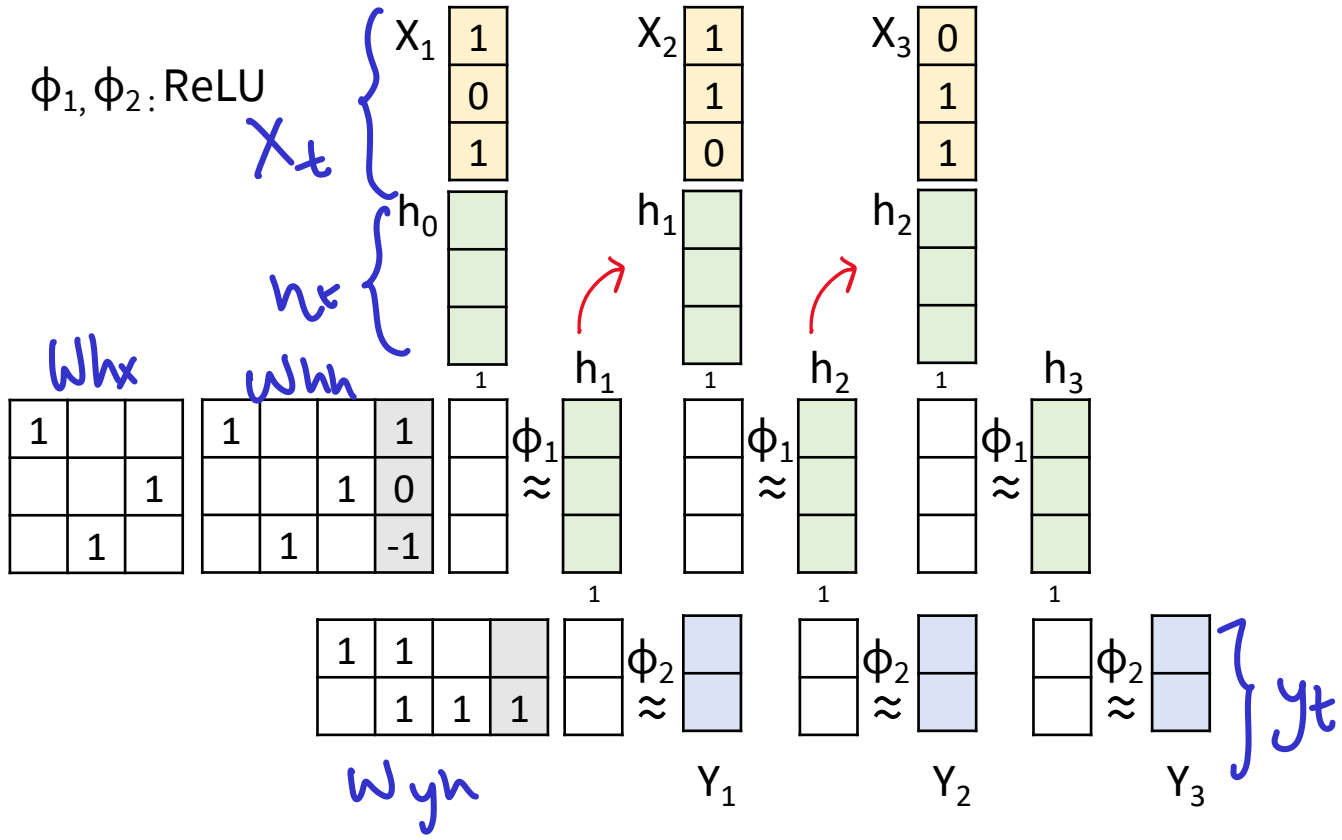




Sequence to Sequence



Counting Parameters (small)



size(X_t) = 3x1

size(h_t) = 3x1

size(y_t) = 2x1

size(W_{hx}) = 3x3

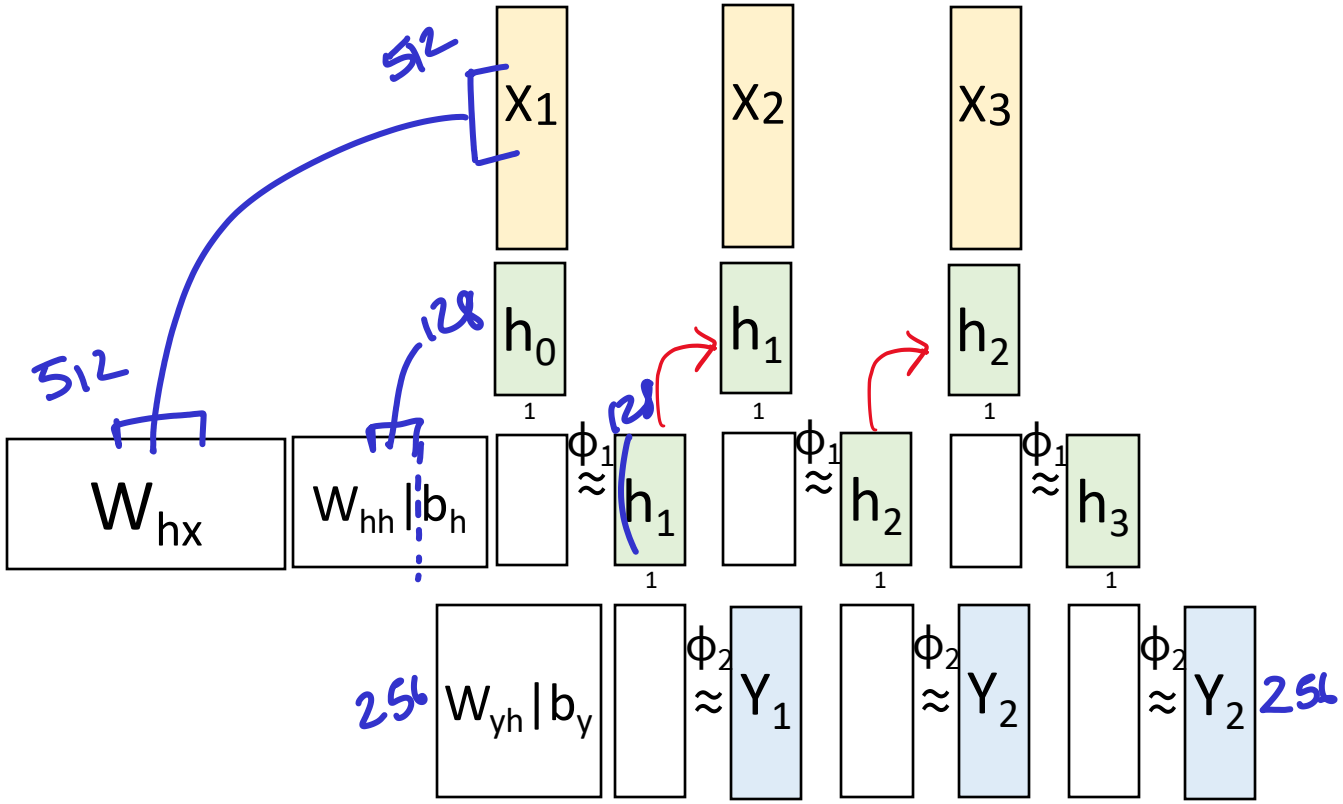
size(b_h) = 3x1

size(W_{hh}) = 3x3

size(W_{yh}) = 2x3

size(b_y) = 2x1

Counting Parameters (large)



size(X_t) = 512 x 1

size(h_t) = 128 x 1

size(Y_t) = 256 x 1

size(W_{hx}) = 512 x 128

size(b_h) = 128 x 1

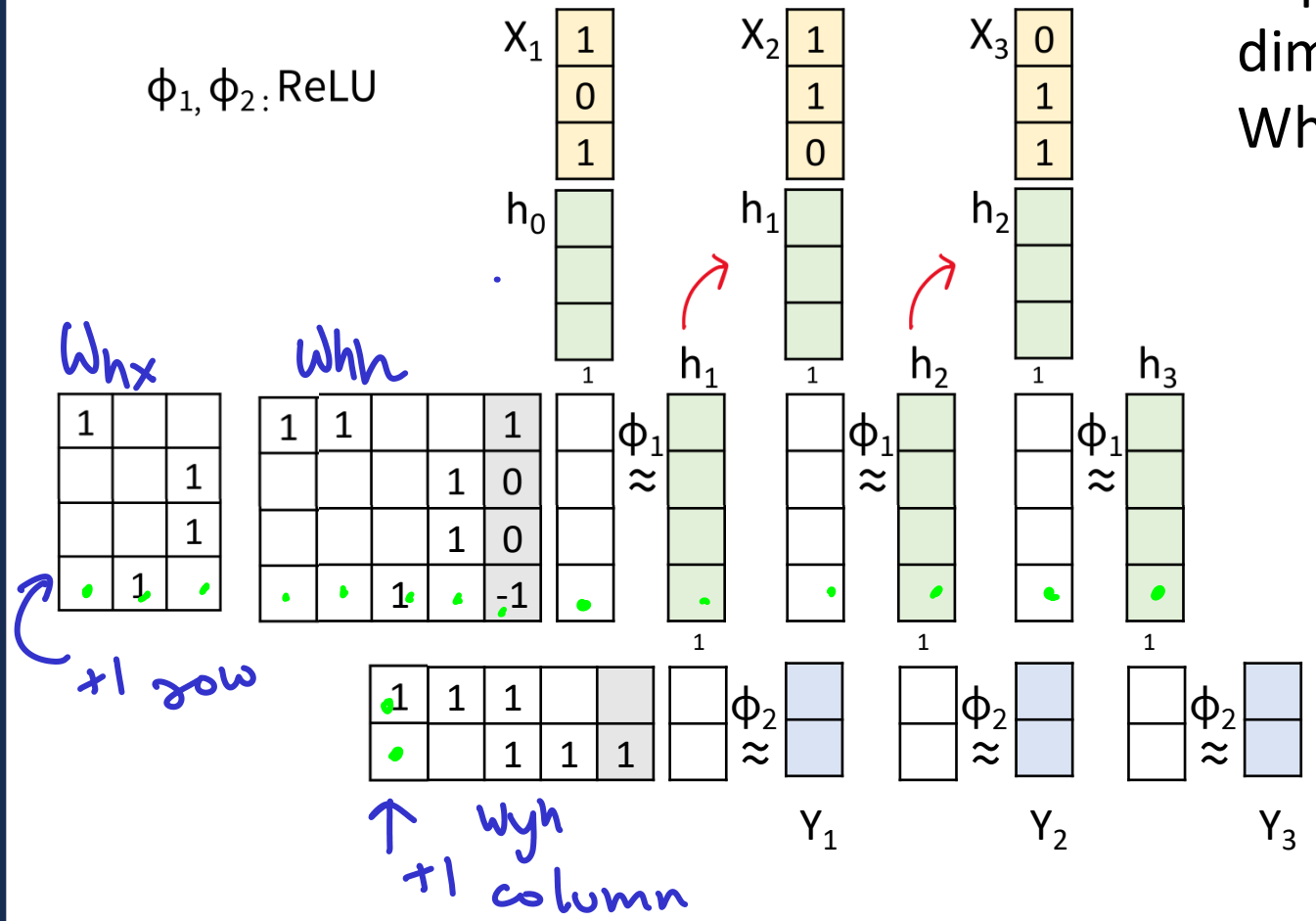
size(W_{hh}) = 128 x 128

size(W_{yh}) = 256 x 128

size(b_y) = 256 x 1

Adding Parameters

I copy pasted a screenshot



Suppose we increase the hidden state's dimension by 1.
What would be the new parameter sizes?

size(W_{hx}) = 4x3

size(b_h) = 4x1

size(W_{hh}) = 4x4

size(W_{yh}) = 2x4

size(b_y) = 2x1

# of New Parameters = 16

(Hint: You can try to draw the extra cells as visual aid)

Modeling Probabilities

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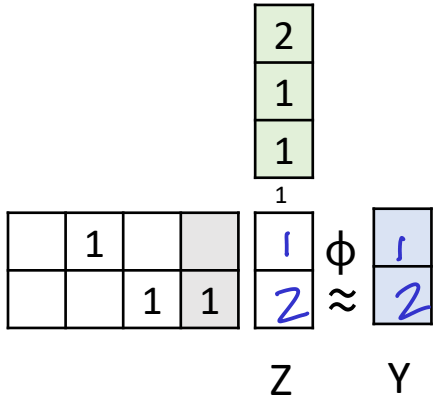
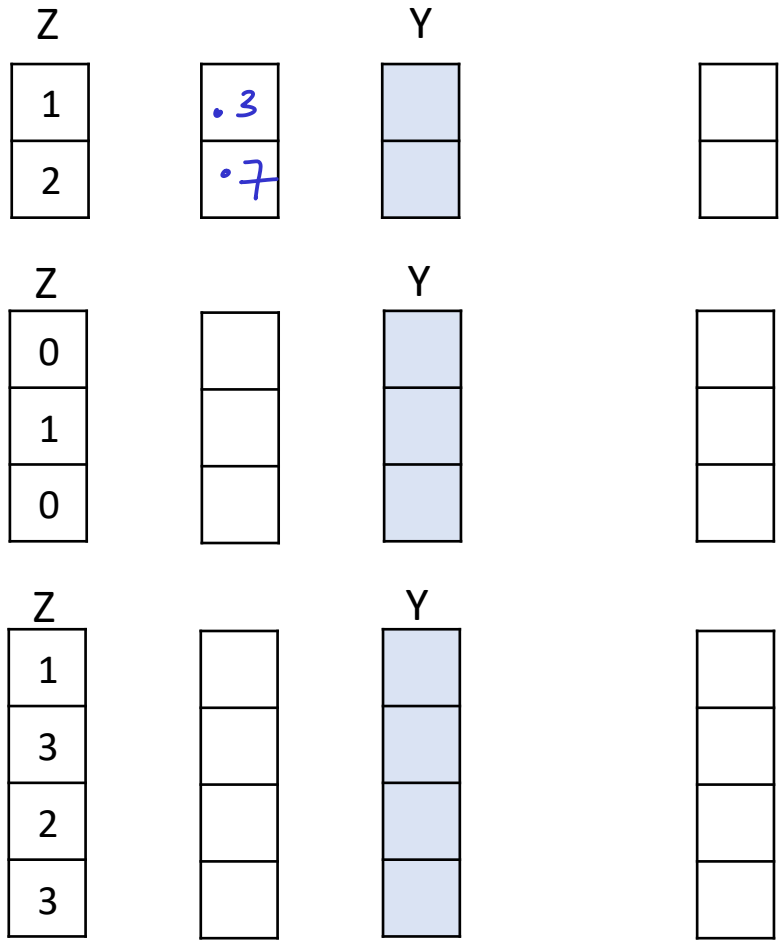
Output Values → Probability Distribution

ϕ = ReLU



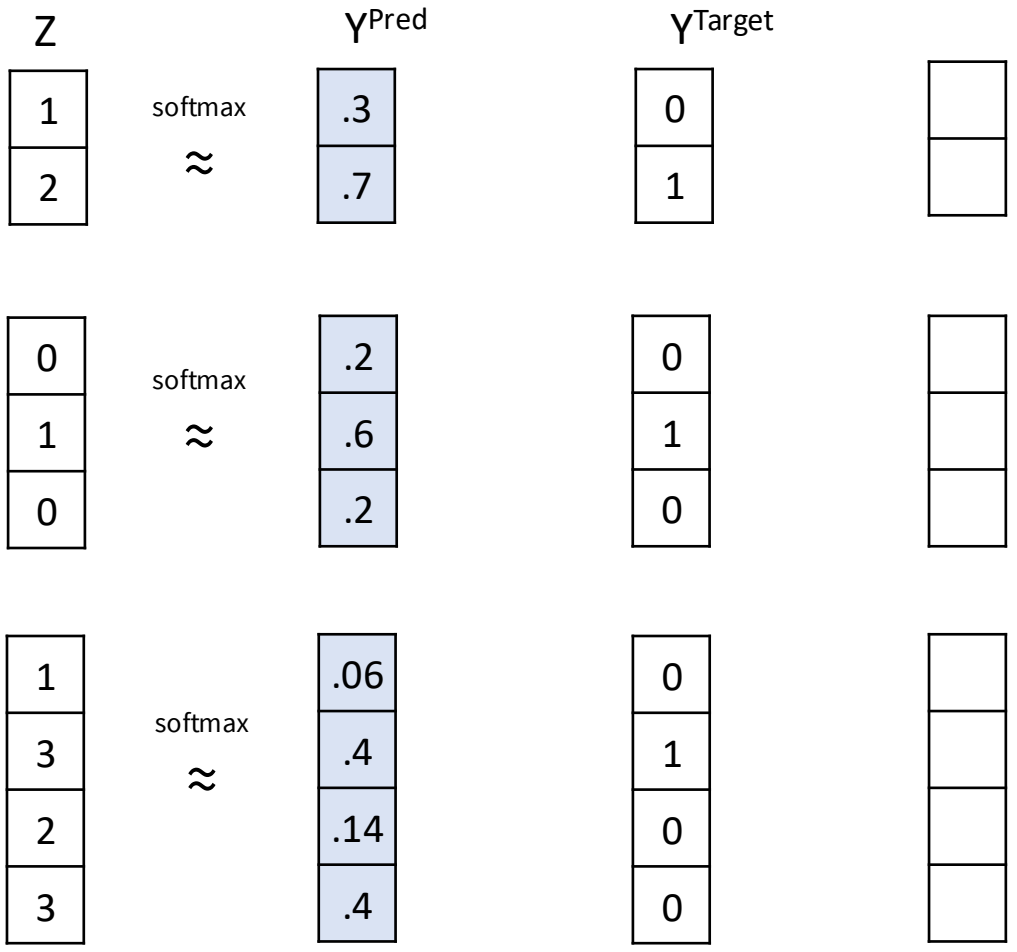
$Y = \phi(Z) =$



x	e^x	round
0	1	1
1	2.71828	3
2	7.38906	7
3	20.08554	20
4	54.58715	55
5	148.41316	148

Gradient of Softmax + CE Loss

$$\frac{\partial L}{\partial Z} = \_\_\_\_\_$$

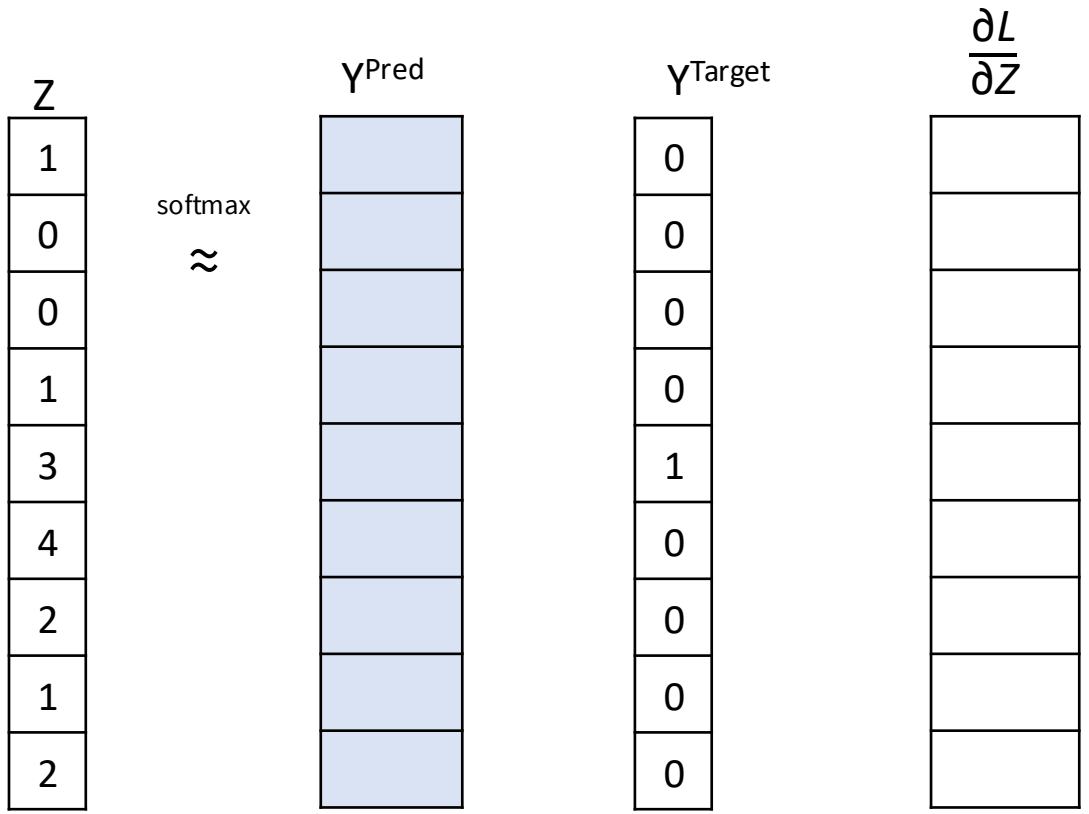


$$Softmax = S(y_i) = \frac{e^{y_i}}{\sum e^{y_j}}$$

$$CE\ loss = \frac{-1}{N} \sum y_i \log(\hat{y}_i)$$

Calculate the Gradient of Softmax + CE Loss

x	e^x	round
0	1	1
1	2.71828	3
2	7.38906	7
3	20.08554	20
4	54.58715	55
5	148.41316	148



Calculate the Gradient of Softmax + CE Loss

$$\text{CE loss} = -\frac{1}{N}\sum y_i \log(\hat{y}_i)$$

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0	1	1
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2	7.38906	7
3	20.08554	20
4	54.58715	55
5	148.41316	148

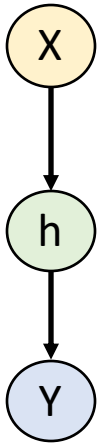
Z	softmax ≈	Y^{Pred}	Y^{Target}	$\frac{\partial L}{\partial Z}$
0		0.01	0	0.01
1		0.03	0	0.03
4		0.55	0	-0.55
1		0.03	0	0.03
0		0.01	1	0.01
3		0.20	0	0.20
2		0.07	0	0.07
2		0.07	0	0.07
1		0.01	0	0.02

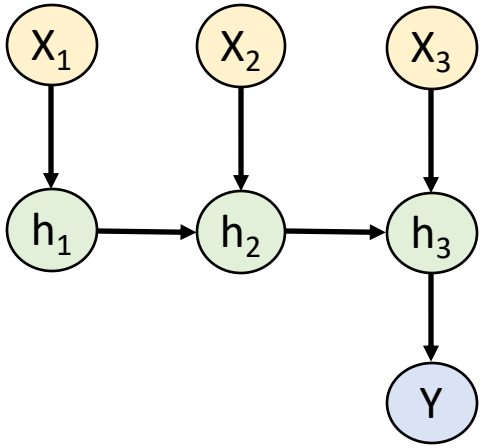
$2e^0 + 3e^1 + 2e^2 + 1e^3 + 1e^4$

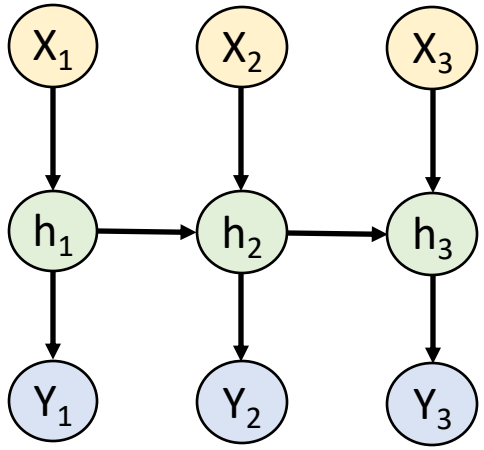
$= 2(1) + 3(3) + 2(7) + 20 + 55$

$= 100$

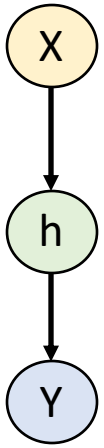
Model → Function

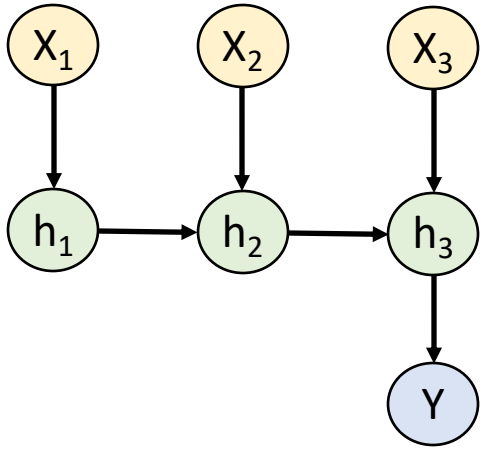


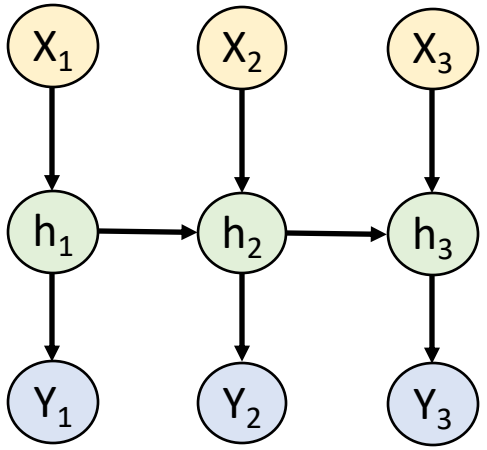




Function → Conditional Probability Function







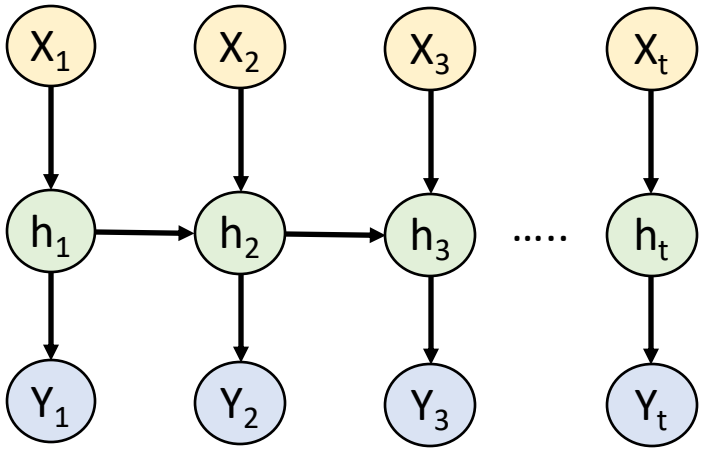
$Y = f(X)$

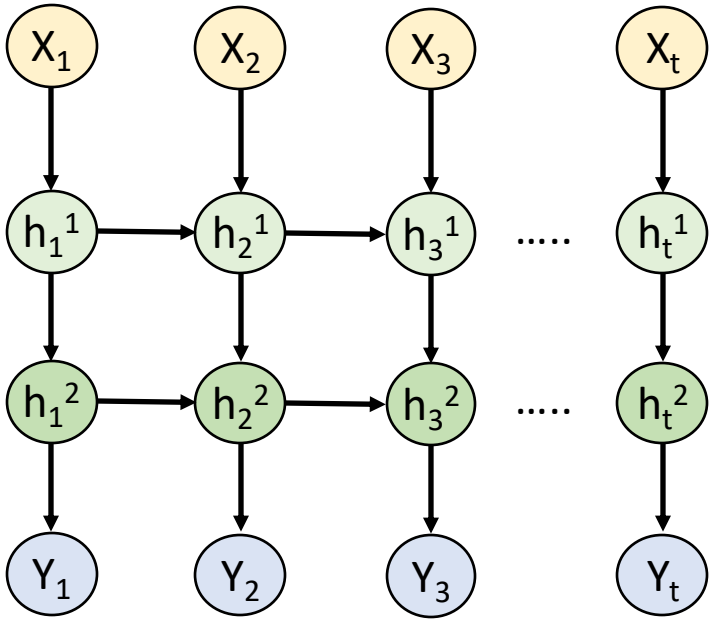
$Y = f(X_1, X_2, X_3)$

$Y = f(Y_t \mid X_t, X_{t-1}, \ldots X_1)$

Model → Conditional Probability Distributions

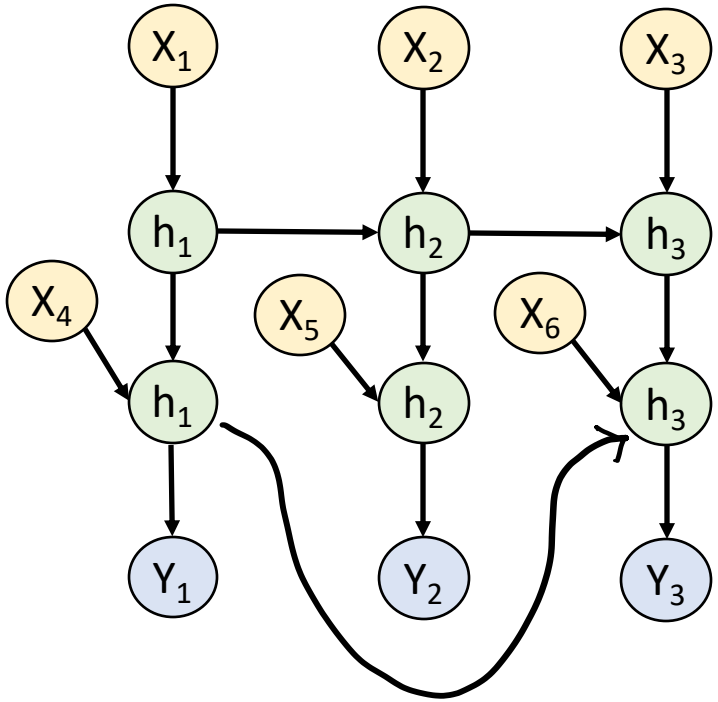
What conditional probability distributions could these two models estimate?





$P(Y_t \mid \quad)$ $P(Y_t \mid \quad)$

Model → Conditional Probability Distributions



What conditional probability distributions could this model learn to estimate?

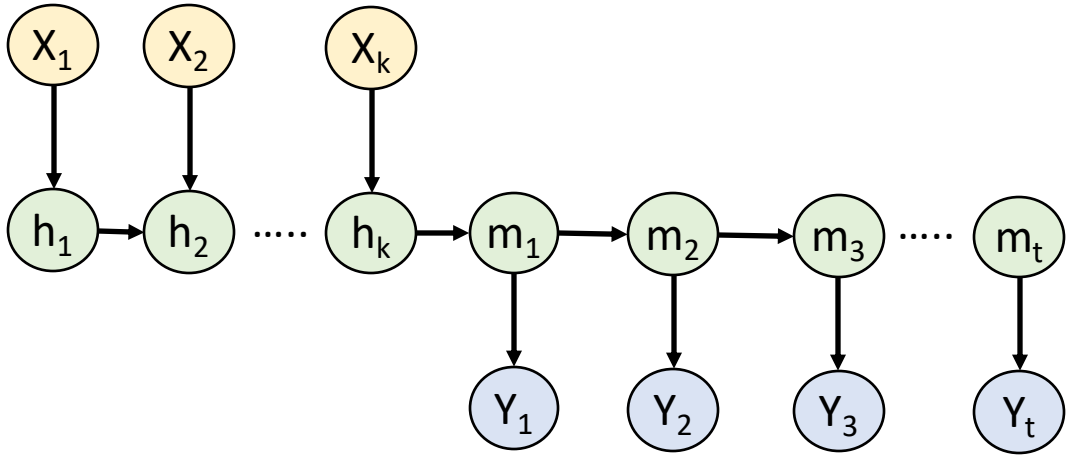
$P(Y_1 \mid \quad)$

$P(Y_2 \mid \quad)$

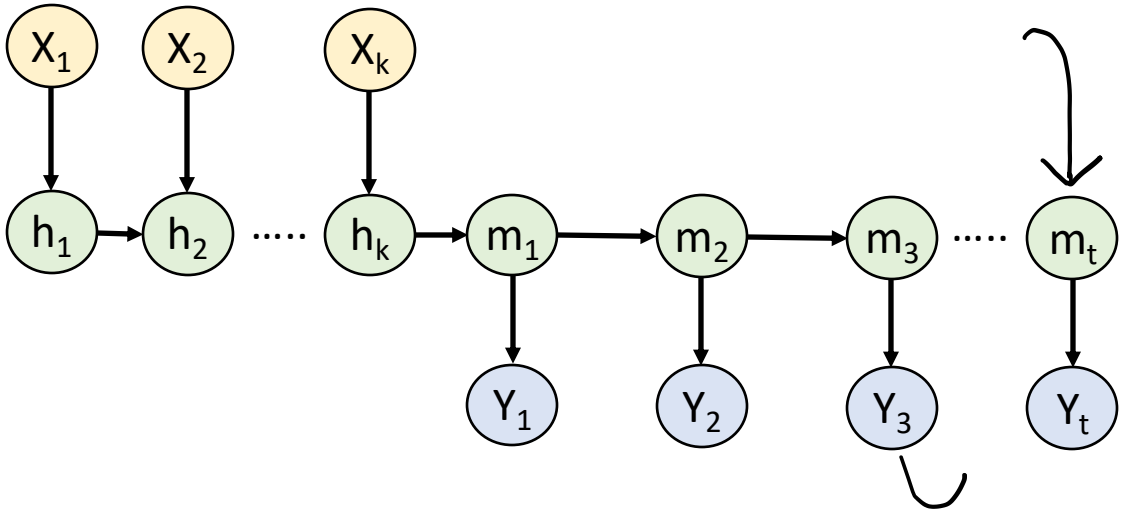
$P(Y_3 \mid \quad)$

Model → Conditional Probability Distributions

What conditional probability distributions could these two seq-to-seq models estimate?



$P(Y_t \mid \quad)$



$P(Y_t \mid \quad)$

Identify "No Dependency" Links

Cross out the dependency links to match the matrix form.