Probability

Name	Nathan Varghese
Identity Key	nava3000

	Level	Completed
	Beginner	9
	Intermediate	5
	Advanced	0
	Expert	0

Goal	
5722	12

Total Completed
14

Probability

CSCI 5722: Computer Vision

Fall 2024

Dr. Tom Yeh

Probabilities

CSCI 5722 Computer Vision

University of Colorado Boulder

1: Correct, 0: Incorrect

A. How are you?	
B. How do you do?	
C. Howdy?	
D. How are you doing?	
E. How is you?	
F. How am you?	

3: very high, 2: high, 1: low, 0: wrong

A. How are you?	
B. How do you do?	
C. Howdy?	
D. How are you doing?	
E. How is you?	
F. How am you?	

Context: Texas +2

	A. How are you?	3
	B. How do you do?	2
	C. Howdy?	1
	D. How are you doing?	3
	A. How are you?	3
	B. How do you do?	2
	C. Howdy?	1
	D. How are you doing?	3

Context: Formal +1, Texas +2

Formal	Texas	A. How are you?	3
		B. How do you do?	2
		C. Howdy?	1
		D. How are you doing?	3
	!Texas	A. How are you?	3
		B. How do you do?	2
		C. Howdy?	1
		D. How are you doing?	3
!Formal	Texas	A. How are you?	3
		B. How do you do?	2
		C. Howdy?	1
		D. How are you doing?	3
	!Texas	A. How are you?	3
		B. How do you do?	2
		C. Howdy?	1
		D. How are you doing?	3

Language Model: Input and Output

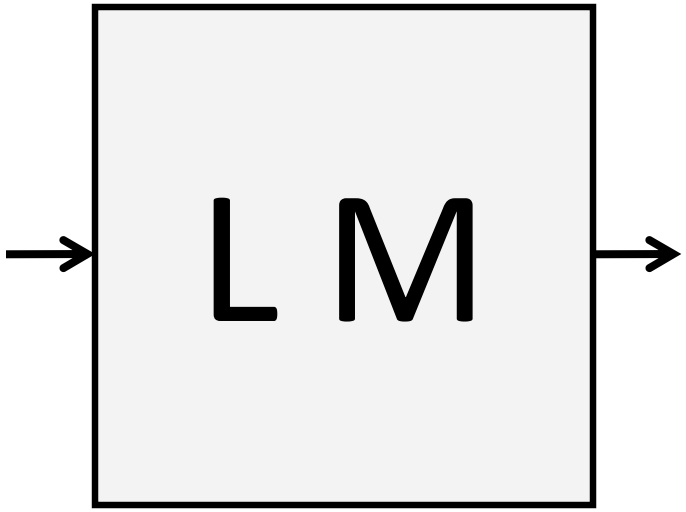
X_1: formal?

↓

X_2: Texas?

↓

X_3: sentence



Joint Probability

$$P(X_1, X_2, X_3)$$

$$= P(X_1)\, P(X_1 \mid X_2)\, P(X_3 \mid X_1, X_2)$$

a chain of conditional probability.

Aggregate

Formal	Texas	A. How are you?	3
		B. How do you do?	3
		C. Howdy?	3
		D. How are you doing?	3
	!Texas	A. How are you?	3
		B. How do you do?	3
		C. Howdy?	1
		D. How are you doing?	3
!Formal	Texas	A. How are you?	3
		B. How do you do?	2
		C. Howdy?	2
		D. How are you doing?	3
	!Texas	A. How are you?	3
		B. How do you do?	2
		C. Howdy?	1
		D. How are you doing?	3

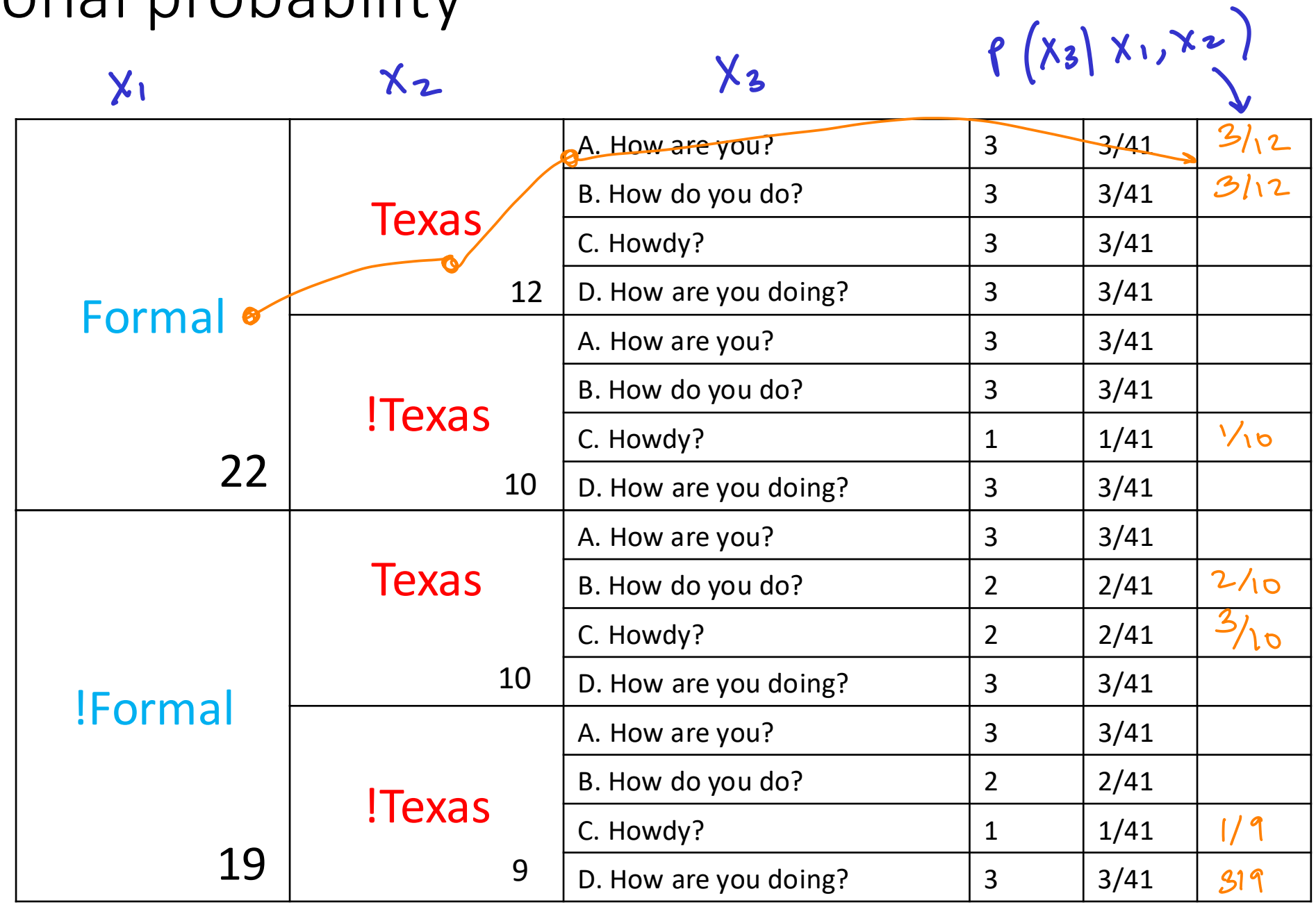
Joint probability

$P(X_1, X_2, X_3)$

X_1	X_2	X_3			
Formal 22	Texas 12	A. How are you?	3	3/41	
		B. How do you do?	3	3/41	
		C. Howdy?	3		
		D. How are you doing?	3		
	!Texas 10	A. How are you?	3		
		B. How do you do?	3		
		C. Howdy?	1	1/41	
		D. How are you doing?	3		
!Formal 19	Texas 10	A. How are you?	3		
		B. How do you do?	2	2/41	
		C. Howdy?	2	2/41	
		D. How are you doing?	3		
	!Texas 9	A. How are you?	3		
		B. How do you do?	2	2/41	
		C. Howdy?	1	1/41	
		D. How are you doing?	3	3/41	

41

Conditional probability



X_1	X_2	X_3			$P(X_3 \mid X_1, X_2)$
Formal 22	Texas 12	A. How are you?	3	3/41	3/12
		B. How do you do?	3	3/41	3/12
		C. Howdy?	3	3/41	
		D. How are you doing?	3	3/41	
	!Texas 10	A. How are you?	3	3/41	
		B. How do you do?	3	3/41	
		C. Howdy?	1	1/41	1/10
		D. How are you doing?	3	3/41	
!Formal 19	Texas 10	A. How are you?	3	3/41	
		B. How do you do?	2	2/41	2/10
		C. Howdy?	2	2/41	3/10
		D. How are you doing?	3	3/41	
	!Texas 9	A. How are you?	3	3/41	
		B. How do you do?	2	2/41	
		C. Howdy?	1	1/41	1/9
		D. How are you doing?	3	3/41	3/9

41

Joint probability

$p(x_3 | x_2, x_1)$

x_1	x_2	$p(x_1, x_2)$		x_3	$p(x_1, x_2, x_3)$		
Formal 22	Texas 12	12/41		A. How are you?	3	3/41	3/12
				B. How do you do?	3	3/41	3/12
				C. Howdy?	3	3/41	3/12
				D. How are you doing?	3	3/41	3/12
	!Texas 10	10/41		A. How are you?	3	3/41	3/10
				B. How do you do?	3	3/41	3/10
				C. Howdy?	1	1/41	1/10
				D. How are you doing?	3	3/41	3/10
!Formal 19	Texas 10	10/41		A. How are you?	3	3/41	3/10
				B. How do you do?	2	2/41	2/10
				C. Howdy?	2	2/41	2/10
				D. How are you doing?	3	3/41	3/10
	!Texas 9	9/41		A. How are you?	3	3/41	3/9
				B. How do you do?	2	2/41	2/9
				C. Howdy?	1	1/41	1/9
				D. How are you doing?	3	3/41	3/9

41

Conditional probability

$p(x_2|x_1)$

$p(x_3|x_2,x_1)$

		$p(x_1,x_2)$			$p(x_1,x_2,x_3)$	
Formal 22	Texas 12	$\frac{12}{41}$	$\frac{12}{22}$	A. How are you?	3	3/41	3/12
				B. How do you do?	3	3/41	3/12
				C. Howdy?	3	3/41	3/12
				D. How are you doing?	3	3/41	3/12
	!Texas 10	$\frac{10}{41}$	$\frac{10}{22}$	A. How are you?	3	3/41	3/10
				B. How do you do?	3	3/41	3/10
				C. Howdy?	1	1/41	1/10
				D. How are you doing?	3	3/41	3/10
!Formal 19	Texas 10	$\frac{10}{41}$	$\frac{10}{19}$	A. How are you?	3	3/41	3/10
				B. How do you do?	2	2/41	2/10
				C. Howdy?	2	2/41	2/10
				D. How are you doing?	3	3/41	3/10
	!Texas 9	$\frac{9}{41}$	$\frac{9}{19}$	A. How are you?	3	3/41	3/9
				B. How do you do?	2	2/41	2/9
				C. Howdy?	1	1/41	1/9
				D. How are you doing?	3	3/41	3/9

41

Probability

x_1	$p(x_1)$		$p(x_1, x_2)$	$p(x_2 \mid x_1)$		$p(x_3 \mid x_2, x_1)$	$p(x_1, x_2, x_3)$	
Formal 22	$\frac{22}{41}$	Texas 12	$\frac{12}{41}$	$\frac{12}{22}$	A. How are you?	3	3/41	3/12
					B. How do you do?	3	3/41	3/12
					C. Howdy?	3	3/41	3/12
					D. How are you doing?	3	3/41	3/12
		!Texas 10	$\frac{10}{41}$	$\frac{10}{22}$	A. How are you?	3	3/41	3/10
					B. How do you do?	3	3/41	3/10
					C. Howdy?	1	1/41	1/10
					D. How are you doing?	3	3/41	3/10
!Formal 19	$\frac{19}{41}$	Texas 10	$\frac{10}{41}$	$\frac{10}{19}$	A. How are you?	3	3/41	3/10
					B. How do you do?	2	2/41	2/10
					C. Howdy?	2	2/41	2/10
					D. How are you doing?	3	3/41	3/10
		!Texas 9	$\frac{9}{41}$	$\frac{9}{19}$	A. How are you?	3	3/41	3/9
					B. How do you do?	2	2/41	2/9
					C. Howdy?	1	1/41	1/9
					D. How are you doing?	3	3/41	3/9

41

Calculate Joint Probabilities $p(x_1, x_2)$

x_1	x_2	Freq	$p(x_1, x_2)$
!Angry 80	A. woof	40	40/100
	B. growl	20	20/100
	C. bark	20	20/100
Angry 20	A. woof	4	4/100
	B. growl	8	8/100
	C. bark	8	8/100

100

Calculate Conditional Probabilities $p(x_2|x_1)$

x_1	x_2	Freq	$p(x_2 \mid x_1)$
!Angry 80	A. woof	40	40/80
	B. growl	20	20/80
	C. bark	20	20/80
Angry 20	A. woof	4	4/20
	B. growl	8	8/20
	C. bark	8	8/20

Calculate p(home?)

(Simplify the fractions)

!Home 15	!Angry 10	A. woof	3
		B. growl	2
		C. bark	5
	Angry 5	A. woof	1
		B. growl	3
		C. bark	1

Home 30	!Angry 20	A. woof	12
		B. growl	4
		C. bark	4
	Angry 10	A. woof	2
		B. growl	5
		C. bark	3

45

$$p(\text{home}) = \frac{30}{45}$$

$$p(\text{!home}) = \frac{15}{45}$$

Calculate p(angry?, home?)

(Simplify the fractions)

!Home 15	!Angry 10	A. woof	3
		B. growl	2
		C. bark	5
	Angry 5	A. woof	1
		B. growl	3
		C. bark	1

Home 30	!Angry 20	A. woof	12
		B. growl	4
		C. bark	4
	Angry 10	A. woof	2
		B. growl	5
		C. bark	3

45

$$p(\text{angry} \mid \text{!home}) = \frac{5}{15}$$

$$p(\text{!angry} \mid \text{home}) = \frac{20}{30}$$

Calculate p(angry?, home?)

(Simplify the fractions)

!Home (15)	!Angry (10)	A. woof	3
		B. growl	2
		C. bark	5
	Angry (5)	A. woof	1
		B. growl	3
		C. bark	1

Home (30) (43)	!Angry (20)	A. woof	12
		B. growl	4
		C. bark	4
	Angry (10)	A. woof	2
		B. growl	5
		C. bark	3

p(angry, !home) = $\frac{5}{45}$

p(!angry, home) = $\frac{20}{45}$

Calculate p(sound?, angry?, home?)

(Simplify the fractions)

!Home (15)	!Angry (10)	A. woof	3
		B. growl	2
		C. bark	5
	Angry (5)	A. woof	1
		B. growl	3
		C. bark	1

Home (30)	!Angry (20)	A. woof	12
		B. growl	4
		C. bark	4
	Angry (10)	A. woof	2
		B. growl	5
		C. bark	3

(45)

p(growl, angry, !home) = $\frac{2}{45}$

p(woof, angry, !home) = $\frac{1}{45}$

p(woof, angry, home) = $\frac{2}{45}$

p(bark, !angry, home) = $\frac{4}{45}$

Calculate p(sound? | angry?, home?)

(Simplify the fractions)

!Home 15	!Angry 10	A. woof	3
		B. growl	2
		C. bark	5
	Angry 5	A. woof	1
		B. growl	3
		C. bark	1

Home 30	!Angry 20	A. woof	12
		B. growl	4
		C. bark	4
	Angry 10	A. woof	2
		B. growl	5
		C. bark	3

43

p(growl| angry, !home) = $\frac{3}{10}$

p(woof| angry, !home) = $\frac{1}{5}$

p(woof| angry, home) = $\frac{2}{10}$

p(bark| !angry, home) = $\frac{3}{20}$

The Chain Rule of Probabilities

CSCI 5722 Computer Vision

University of Colorado
Boulder

Conditional → Joint Probabilities

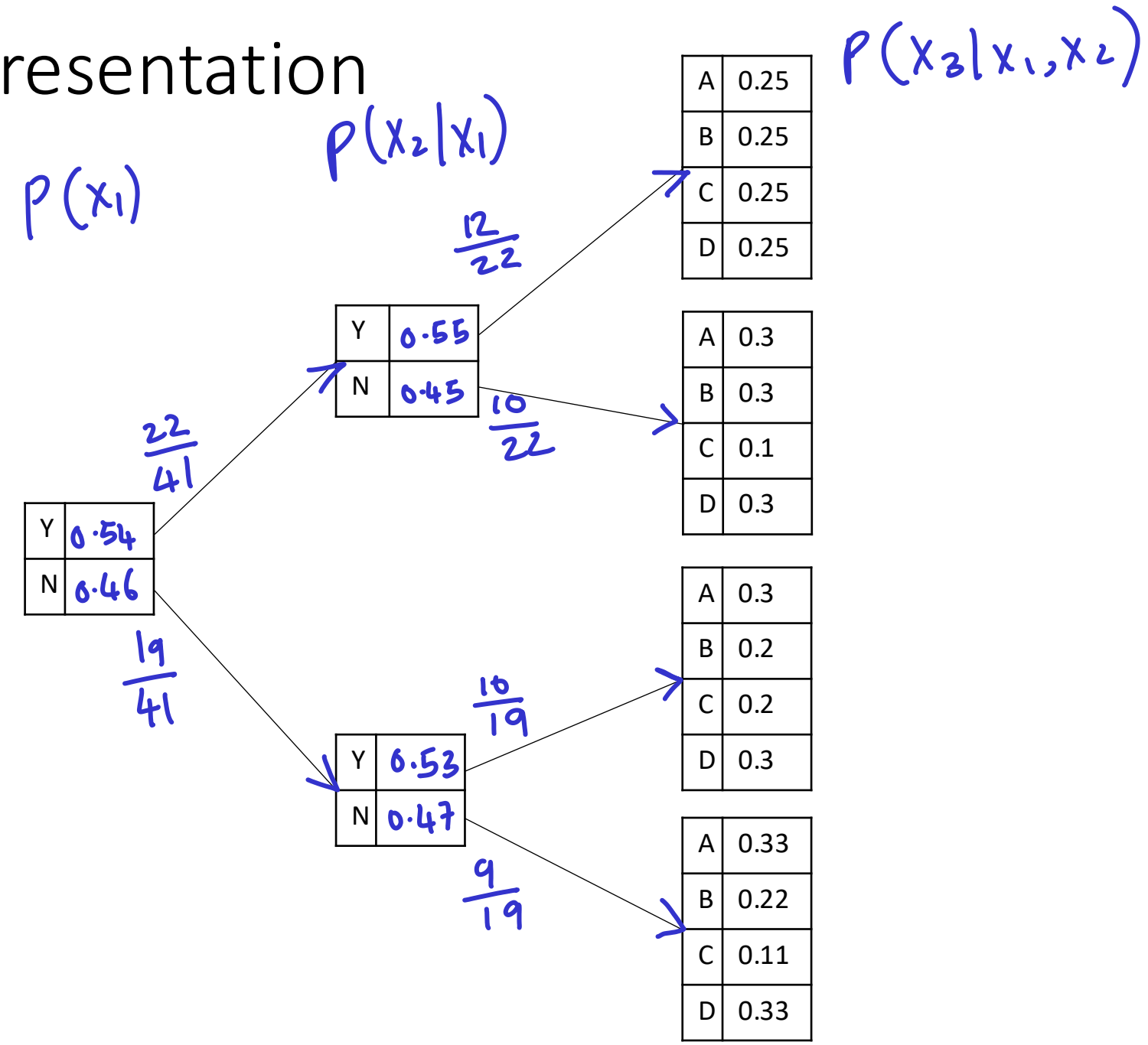
$p(x_3|x_2,x_1)$

	$p(x_1)$		$p(x_1,x_2)$	$p(x_2|x_1)$		$p(x_1,x_2,x_3)$	
Formal	$\frac{22}{41}$	Texas 12	$\frac{12}{41}$	$\frac{12}{22}$	A. How are you?	3	3/41	3/12
					B. How do you do?	3	3/41	3/12
					C. Howdy?	3	3/41	3/12
					D. How are you doing?	3	3/41	3/12
22		!Texas 10	$\frac{10}{41}$	$\frac{10}{22}$	A. How are you?	3	3/41	3/10
					B. How do you do?	3	3/41	3/10
					C. Howdy?	1	1/41	1/10
					D. How are you doing?	3	3/41	3/10

$p(x_1) \cdot p(x_2|x_1) \cdot p(x_3|x_2,x_1) = p(x_1,x_2,x_3)$

$\frac{22}{41} \cdot \frac{12}{22} \cdot \frac{3}{12} = \frac{3}{41}$

Tree Representation



Probability Distribution vs. Values

X_1

Y	0.54
N	0.46

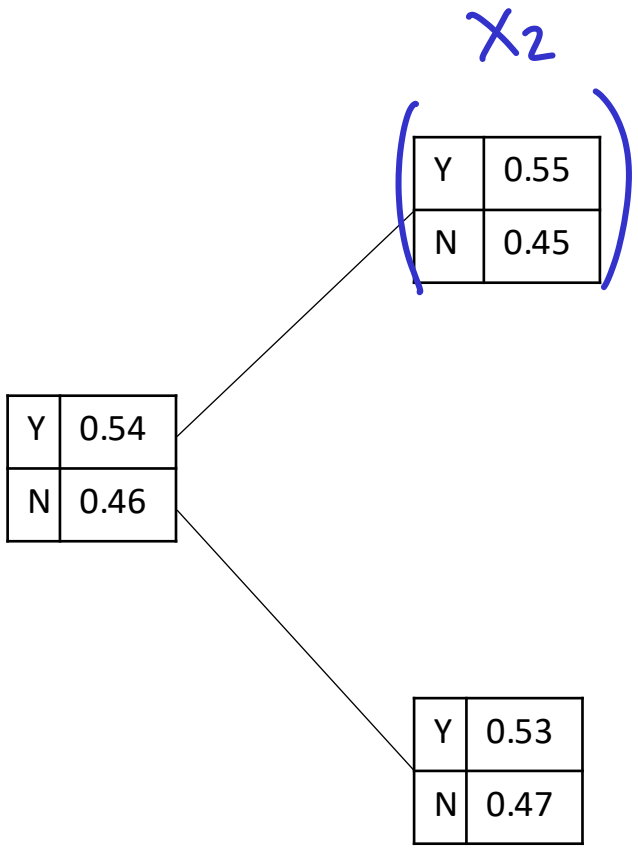
$P(x_1) =$ $\begin{pmatrix} 0.54 \\ 0.46 \end{pmatrix}$ distribution

$P(x_1 = Y) =$ 0.54

$P(x_1 = N) =$ 0.46

values

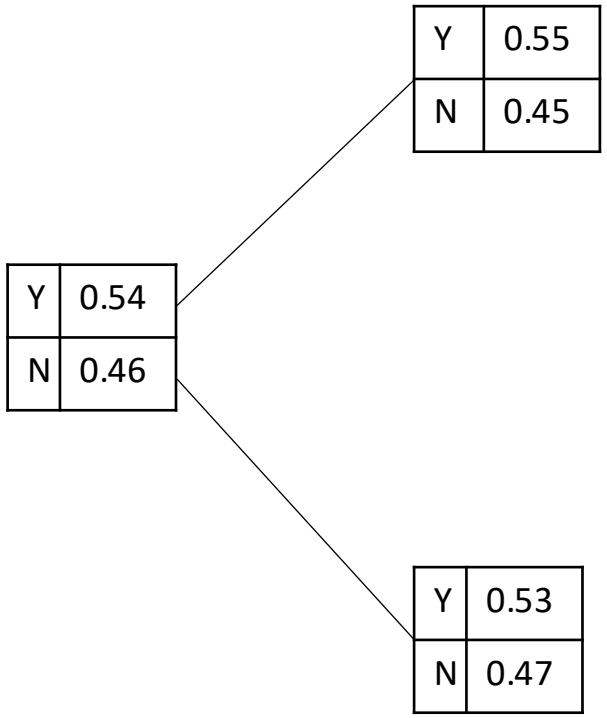
Conditional Probability Distributions



$$P(x_2|x_1 = Y) = \begin{pmatrix} 0.55 \\ 0.45 \end{pmatrix}$$

$$P(x_2|x_1 = N) = \begin{pmatrix} 0.53 \\ 0.47 \end{pmatrix}$$

Conditional Probability Values



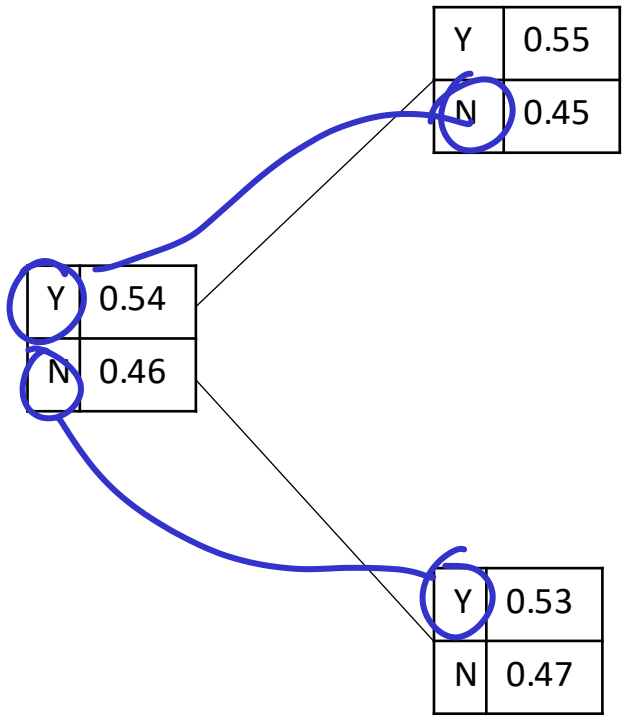
$$P(x_2 \mid x_1 = Y) = \begin{pmatrix} .55 \\ .45 \end{pmatrix}$$ dist.

$$P(x_2 = Y \mid x_1 = Y) = 0.55$$

$$P(x_2 \mid x_1 = N) = \begin{pmatrix} .53 \\ .47 \end{pmatrix}$$ dist

$$P(x_2 = N \mid x_1 = N) = 0.47$$

Joint Probability "Values" by the Chain Rule



Joint Probability of x1, x2

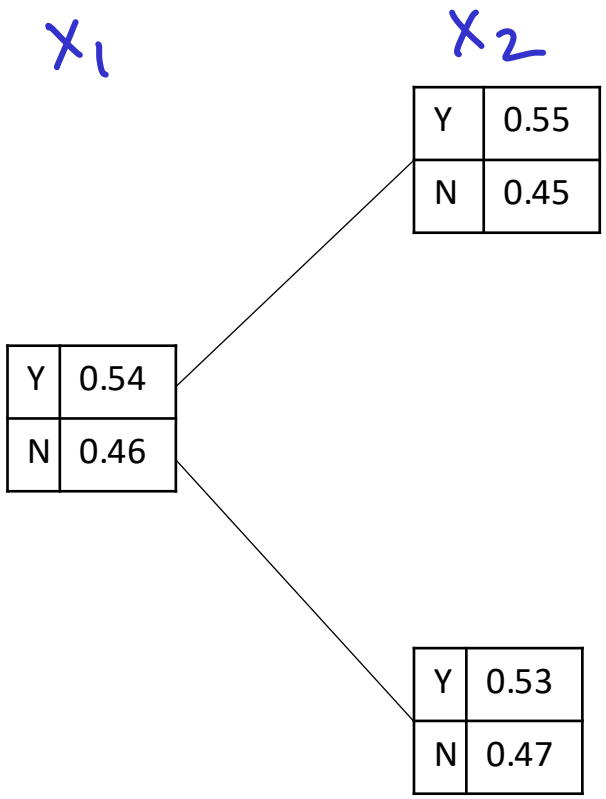
= p(X_1 , X_2)

= p(X_1) x p($X_2 \mid X_1$)

$P(x_1 = Y, x_2 = N) =$ 0.54×0.45

$P(x_1 = N, x_2 = Y) =$ 0.46×0.53

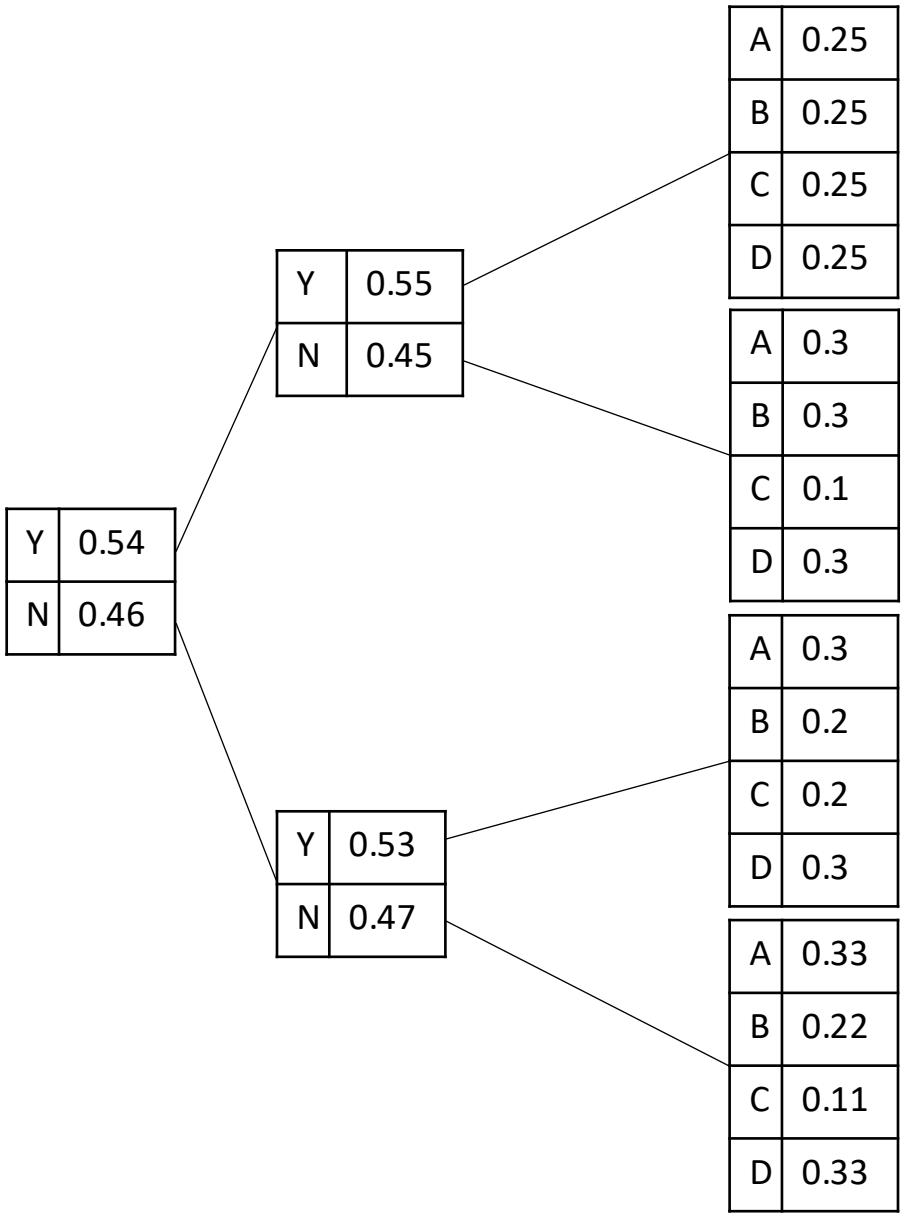
Joint Probability "Distributions" by the Chain Rule



$$P(x_1 = Y, x_2) = 0.54 \times \begin{bmatrix} 0.55 \\ 0.45 \end{bmatrix}$$

$$P(x_1 = N, x_2) = 0.46 \times \begin{pmatrix} 0.55 \\ 0.45 \end{pmatrix}$$

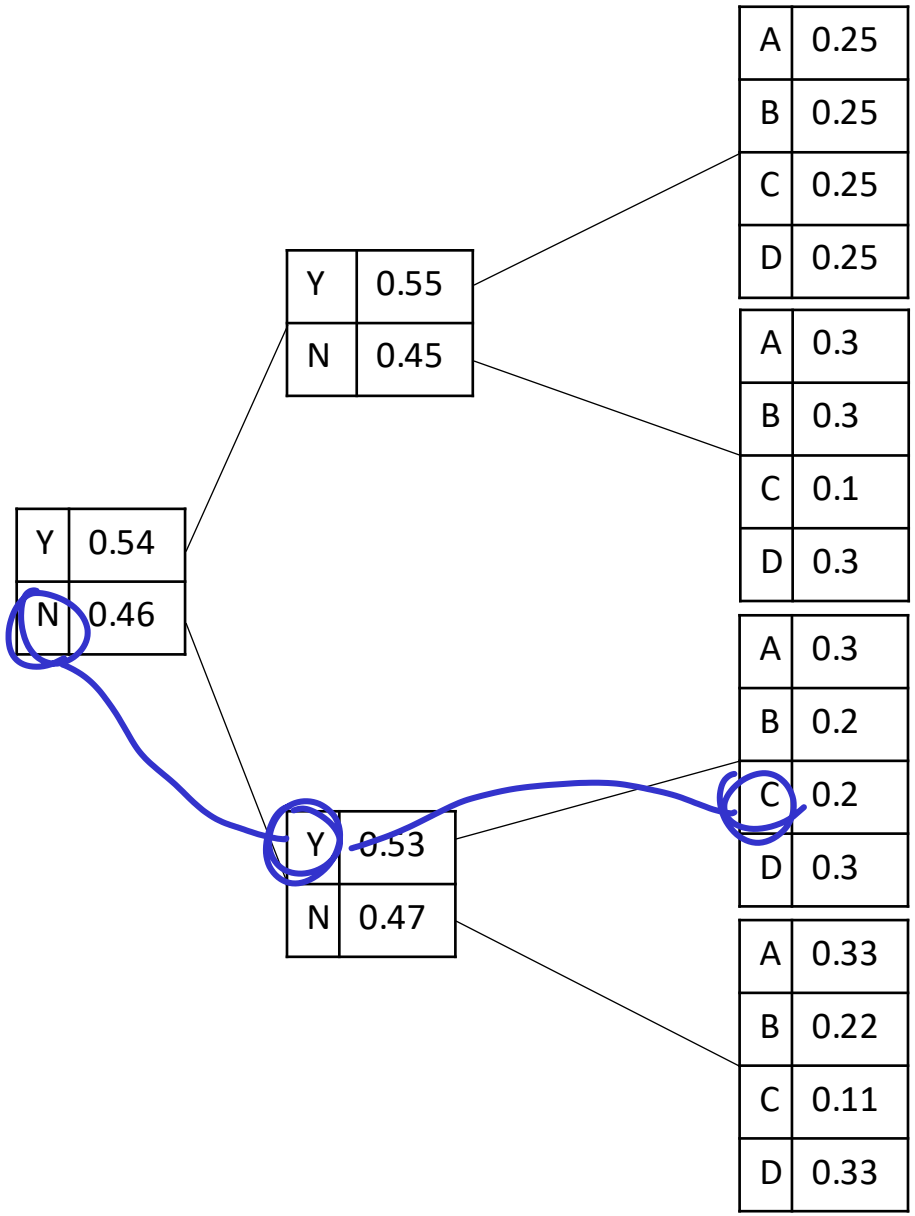
Conditional Probability Distributions



$$P(x_3 \mid x_1 = N, x_2 = Y) =$$

$$P(x_3 \mid x_1 = Y, x_2 = N) =$$

Conditional Probability Values



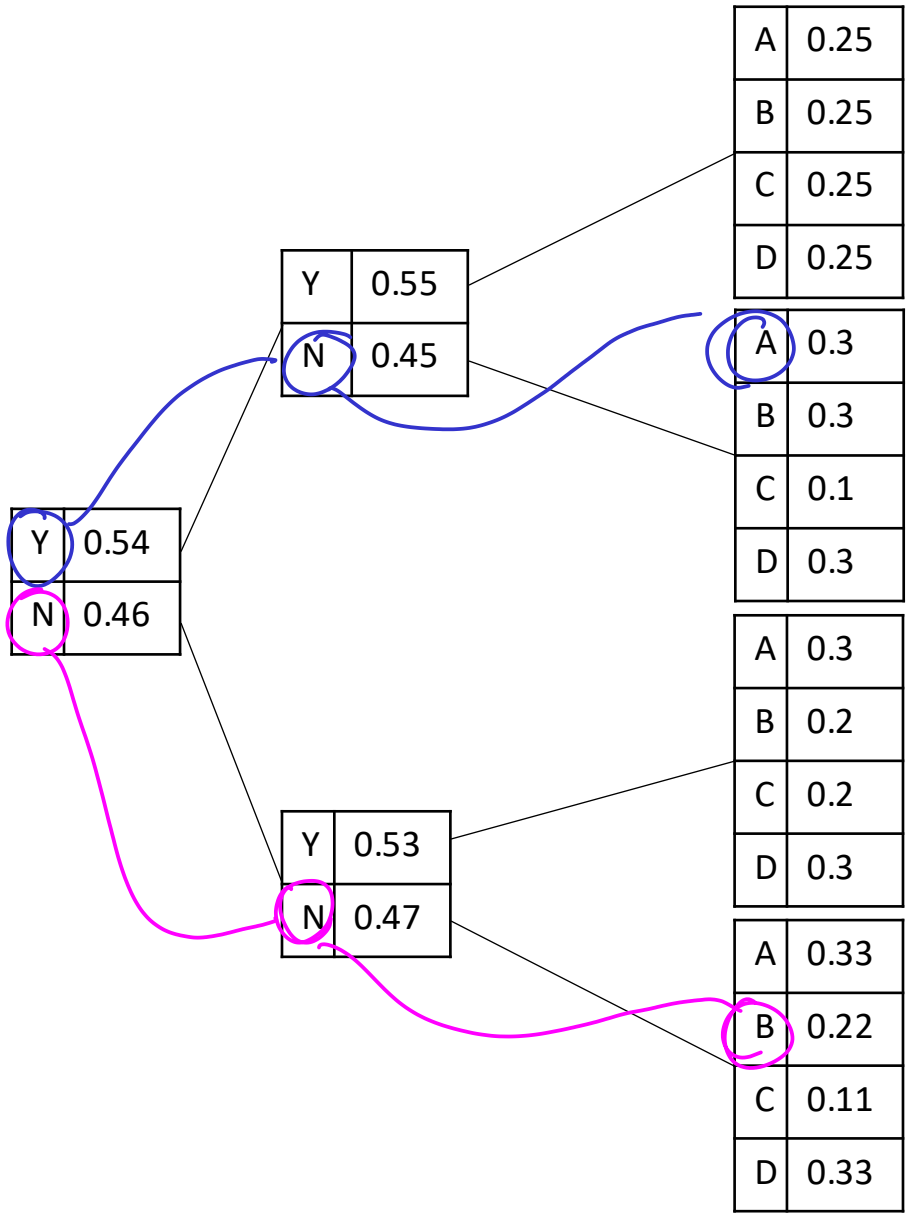
$$P(x_3 \mid x_1 = N, x_2 = Y) = \begin{pmatrix} .3 \\ .2 \\ .2 \\ .3 \end{pmatrix}$$

$$P(x_3 = C \mid x_1 = N, x_2 = Y) = 0.2$$

$$P(x_3 \mid x_1 = Y, x_2 = N) = \begin{pmatrix} .3 \\ .3 \\ .1 \\ .3 \end{pmatrix}$$

$$P(x_3 = D \mid x_1 = Y, x_2 = N) = 0.3$$

Joint Probability "Values" by the Chain Rule



Joint Probability of x1, x2, x3

= p(X1, X2, X3)

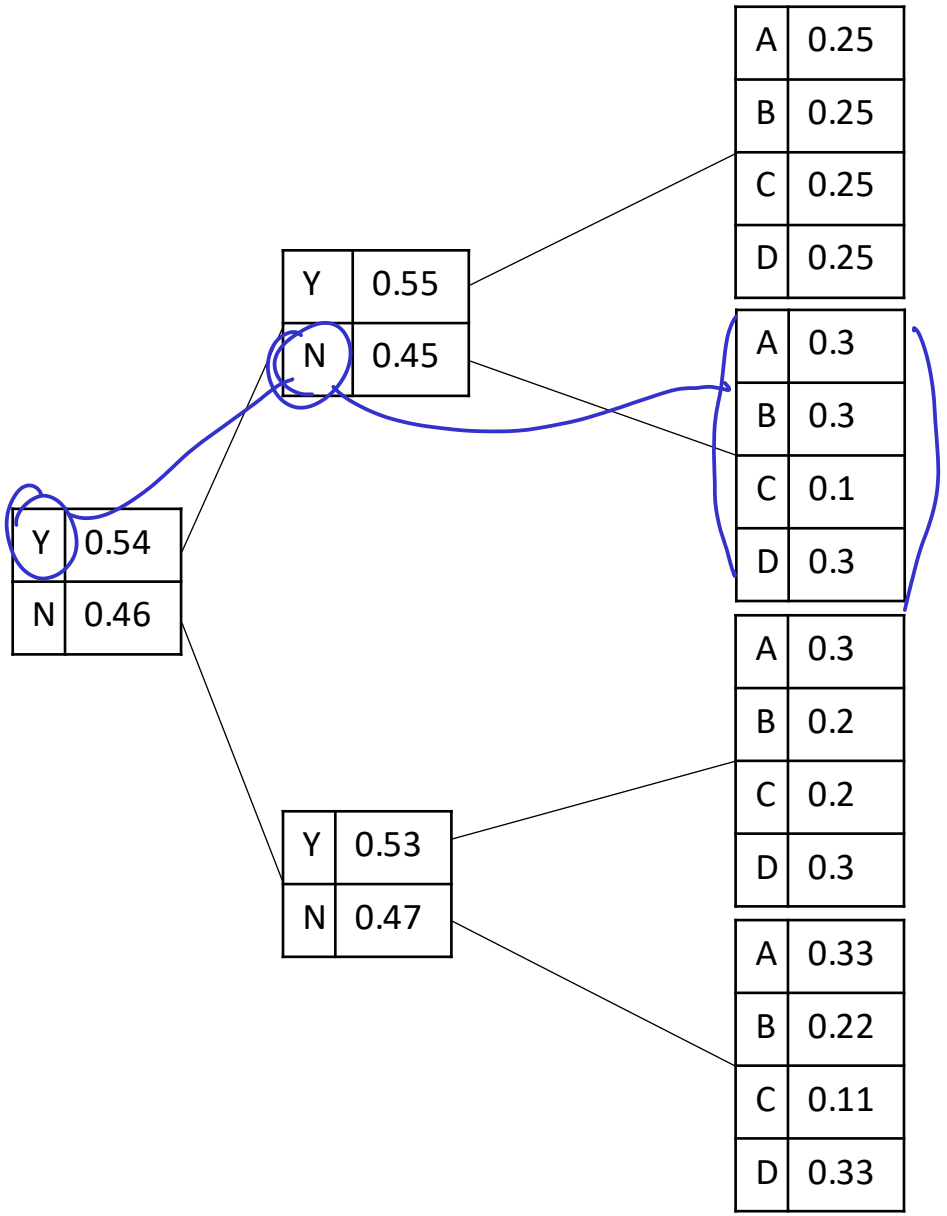
= p(X1) x p(X2|X1) x p(X3|X2 X1)

Joint P

$P(x_1 = Y, x_2 = N, x_3 = A)$ = 0.54 x 0.45 x 0.3

$P(x_1 = N, x_2 = N, x_3 = B)$ = 0.46 x 0.47 x 0.22

Joint Probability "Distributions" by the Chain Rule

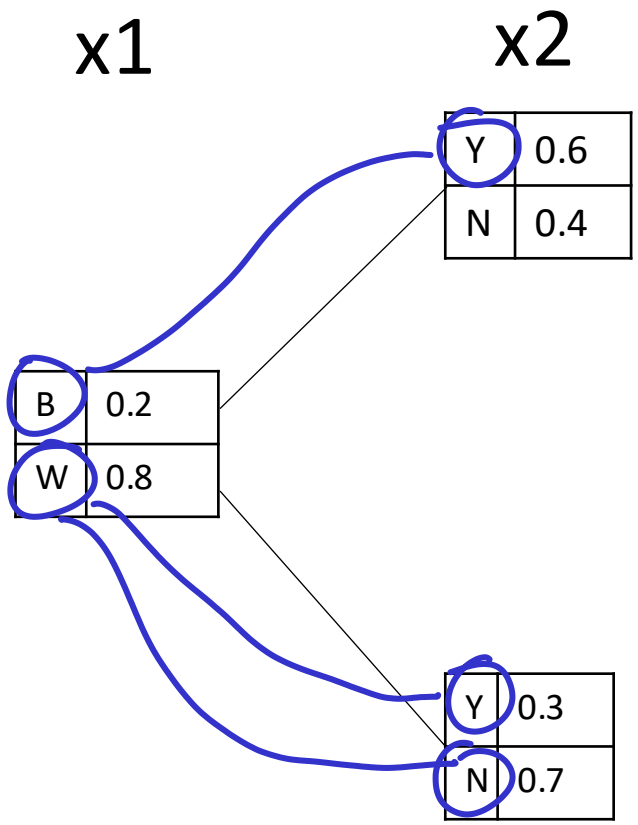


$$P(x_1 = Y, x_2 = N, x_3) = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix} \quad 0.54 \times 0.45 \times \begin{bmatrix} 0.3 \\ 0.3 \\ 0.1 \\ 0.3 \end{bmatrix}$$

$P(x_1 = Y, Y = 0 \times 0 \times [\ ])$

$P\ N\ Y = 0 \times 0 \times ( )$

Calculate Joint Probability

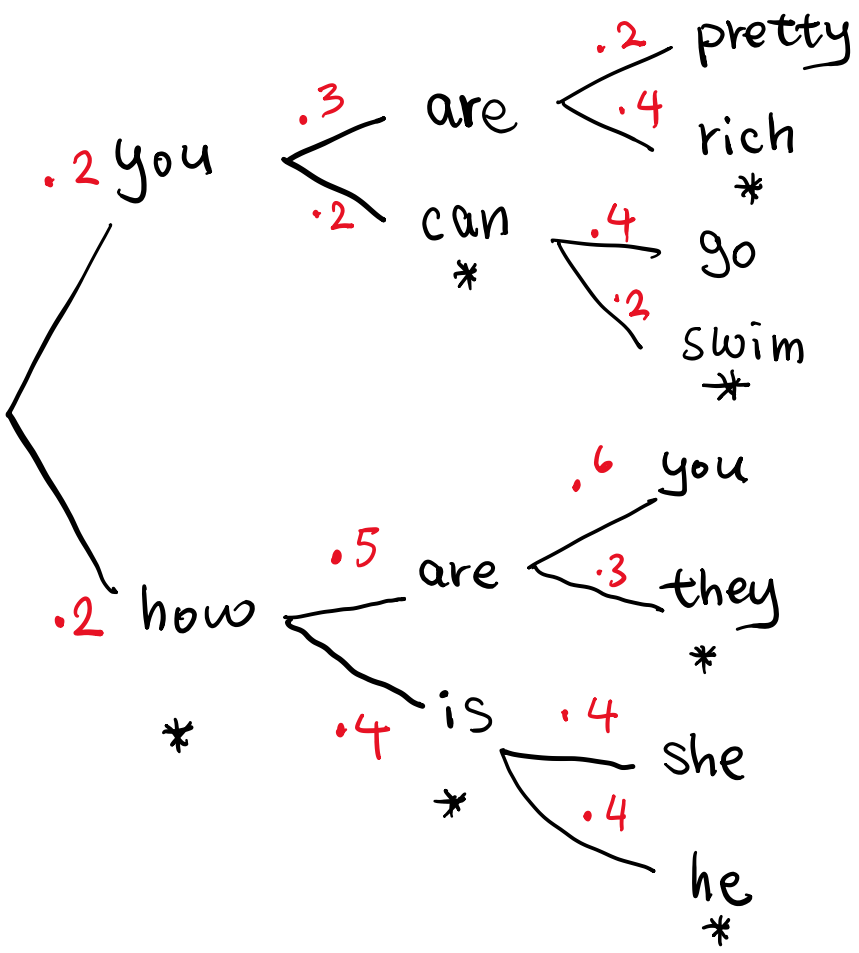


p(x1=B, x2=Y) = 0.2 x 0.6 = 0.12

p(x1=W, x2=N) = 0.8 x 0.7 = 0.56

p(x1=W, x2=Y) = 0.8 x 0.3 = 0.24

Calculate Conditional Probability



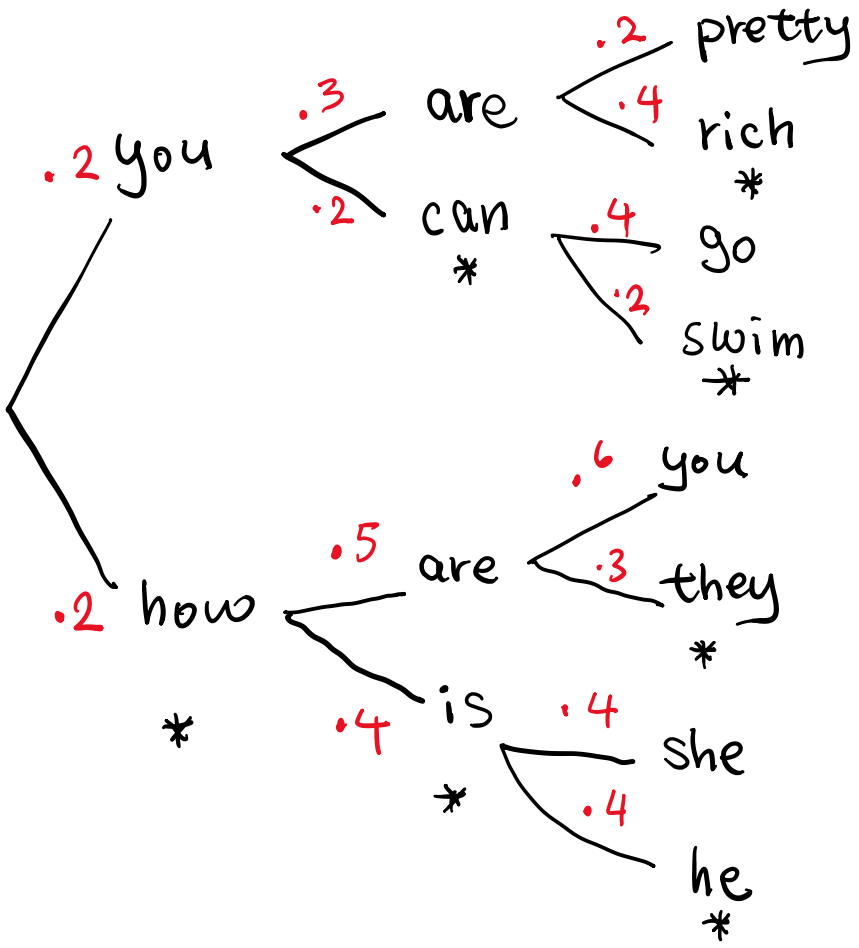
p("rich" | "you are") = 0.4

p("swim" | "you can") = 0.2

p("she" | "how is") = 0.4

p("he" | "how is") = 0.4

Calculate Joint Probability



p("you are rich") = 0.2 x 0.3 x 0.4 = 0.024

p("you can swim") = 0.2 x 0.2 x 0.2 = 0.008

p("how is she") = 0.2 x 0.4 x 0.4 = 0.032

p("how is he") = 0.032

Pixel-based Image Model

CSCI 5722 Computer Vision

University of Colorado Boulder

Color Image

C	C	B	A
B	B	B	A
B	C	B	A
B	B	B	B

Part Labels

1 = skin, 2 = hair

2	2	2	1
1	1	1	2
1	1	1	1
1	1	1	1

Generative Image Model (Joint Probability)

colour

C	C	B	A
B	B	B	A
B	C	B	A
B	B	B	B

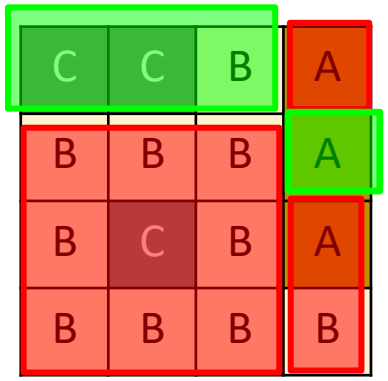
part

1 = skin, 2 = hair

2	2	2	1
1	1	1	2
1	1	1	1
1	1	1	1

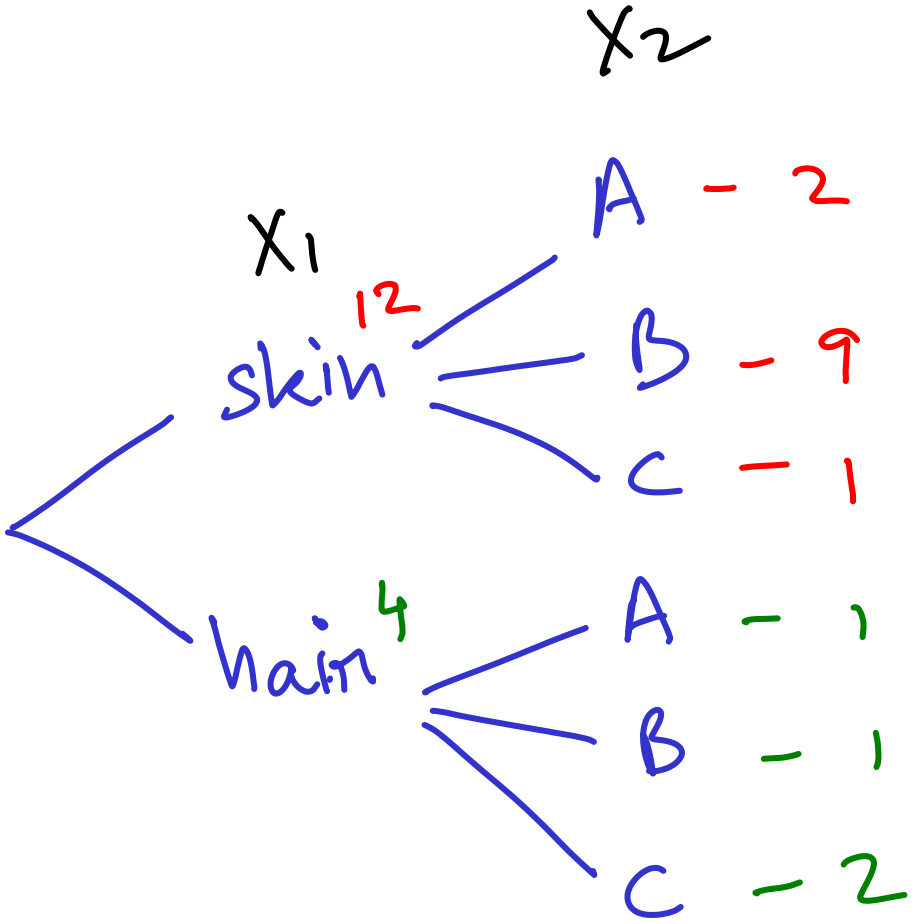
$$P(\text{Colour} = \{A, B, C\}, \text{part} = \{\text{skin}, \text{hair}\})$$

Tree Representation

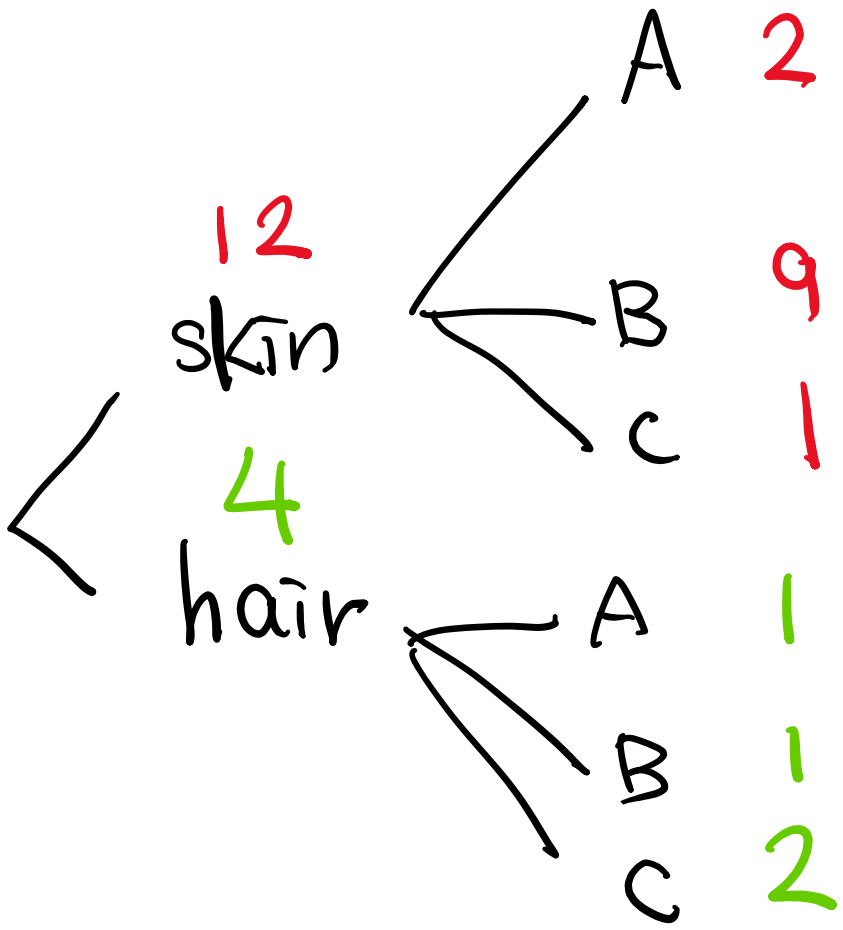


1 = skin, 2 = hair

2	2	2	1
1	1	1	2
1	1	1	1
1	1	1	1



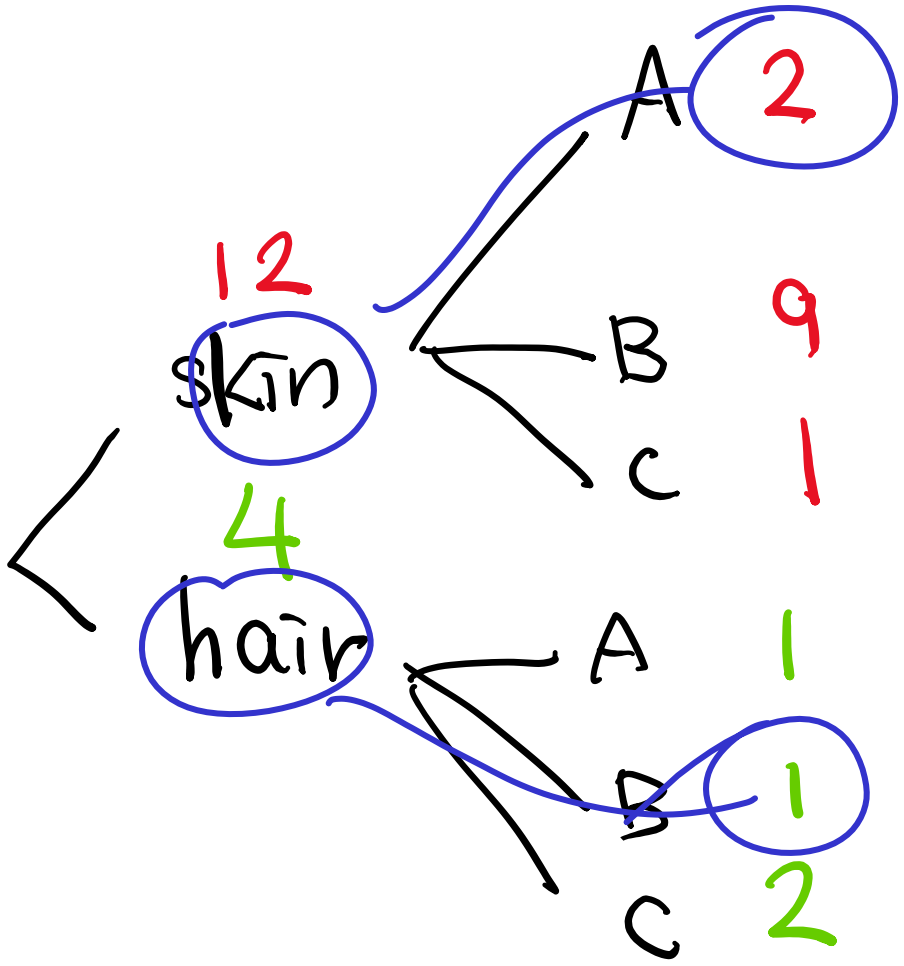
Conditional Probability Distribution



$$p(\text{color} \mid \text{part} = \text{skin}) = \frac{1}{12}\begin{pmatrix} 2 \\ 9 \\ 1 \end{pmatrix}$$

$$p(\text{color} \mid \text{part} = \text{hair}) = \frac{1}{4}\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

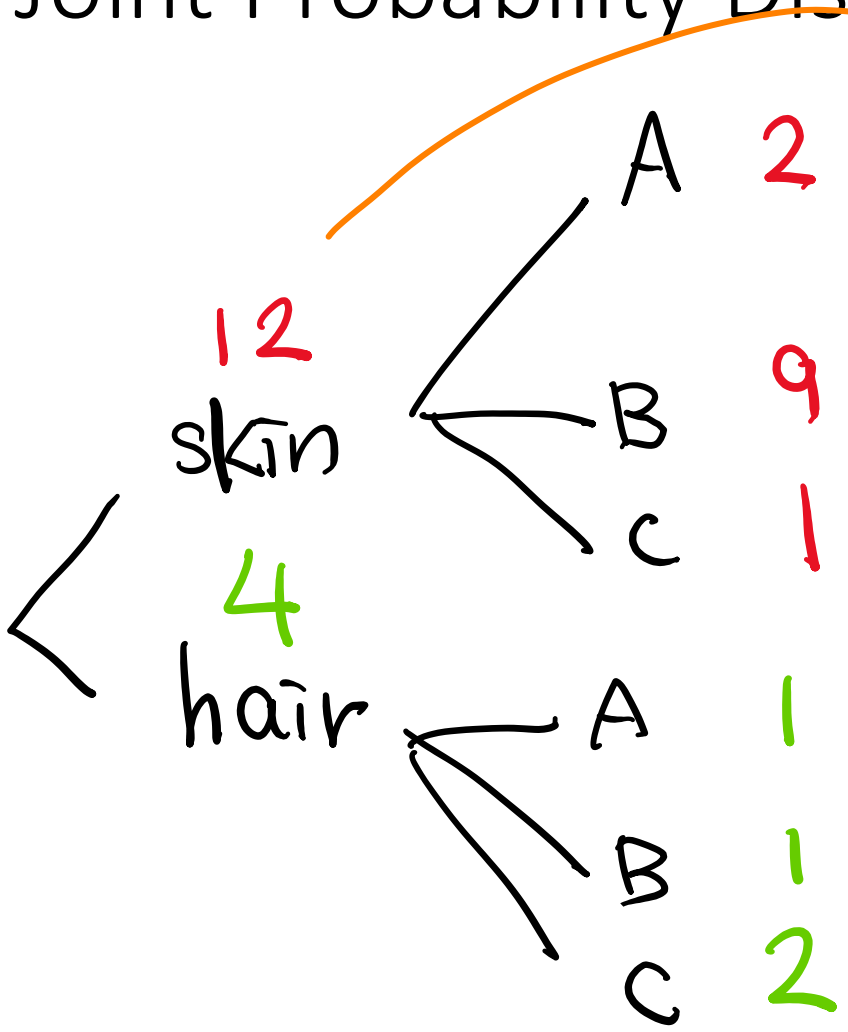
Conditional Probability Values



p(color = A | part = skin) = $\frac{2}{12}$

p(color = B | part = hair) = $\frac{1}{4}$

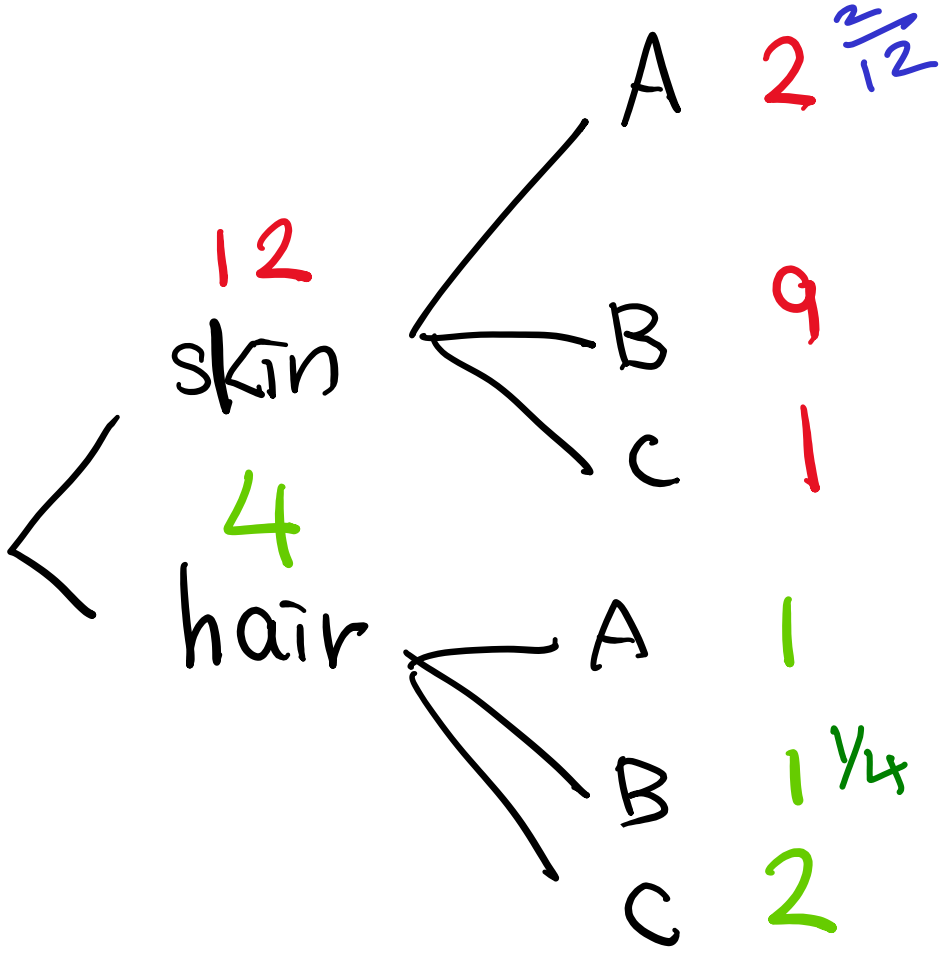
Joint Probability Distribution



p(color, part = skin) = $\frac{12}{16} \times \begin{pmatrix} 2 \\ 9 \\ 1 \end{pmatrix} \frac{1}{12}$

p(color, part = hair) = $\frac{4}{16} \times \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \frac{1}{4}$

Joint Probability Values



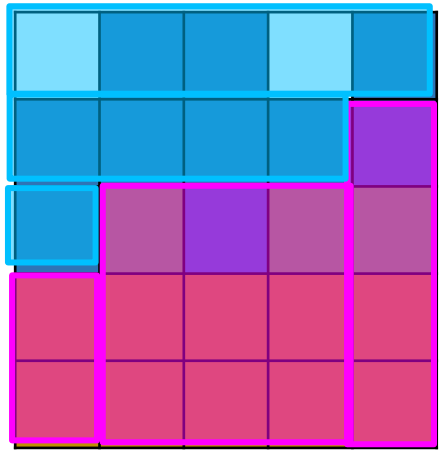
p(color = A, part = skin) = $\frac{12}{16} \times \frac{2}{12}$

p(color = B, part = hair) = $\frac{4}{16} \times \frac{1}{4}$

Fill the aggregation tree

1 = sky, 2 = land

1	1	1	1	1
1	1	1	1	2
1	2	2	2	2
2	2	2	2	2
2	2	2	2	2



sky	10
land	15

white	2
blue	8
green	0
brown	0

white	X 0
blue	X 2
green	3
brown	10

Conditional Probability Values

white	2
blue	8
green	0
brown	0

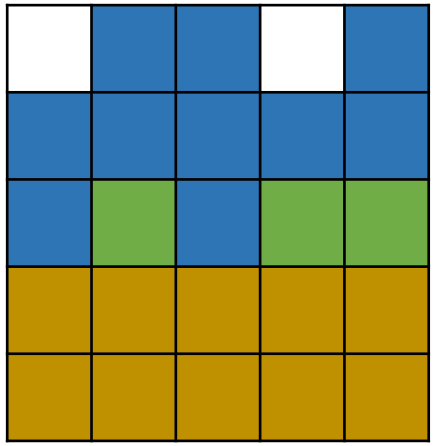
total=25

sky	10
land	15

white	~~X~~ 0
blue	~~10~~ 2
green	3
brown	10

1 = sky, 2 = land

1	1	1	1	1
1	1	1	1	2
1	2	2	2	2
2	2	2	2	2
2	2	2	2	2



p(color = blue | part = sky) = 8/10

p(color = blue | part = land) = 2/15

p(color = green | part = sky) = 0/10 = 0

p(color = green | part = land) = 3/15

Joint Probability Values

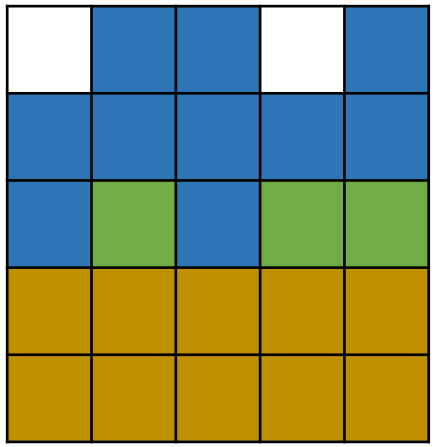
white	2
blue	8
green	0
brown	0

sky	10
land	15

white	~~X~~ 0
blue	~~10~~ 2
green	3
brown	10

1 = sky, 2 = land

1	1	1	1	1
1	1	1	1	2
1	2	2	2	2
2	2	2	2	2
2	2	2	2	2



p(color = blue, part = sky) = $\frac{8}{10} \times \frac{10}{25} = \frac{8}{25}$

p(color = blue, part = land) = $\frac{2}{15} \times \frac{15}{25} = \frac{2}{25}$

p(color = green, part = sky) = $\frac{0}{25} = 0$

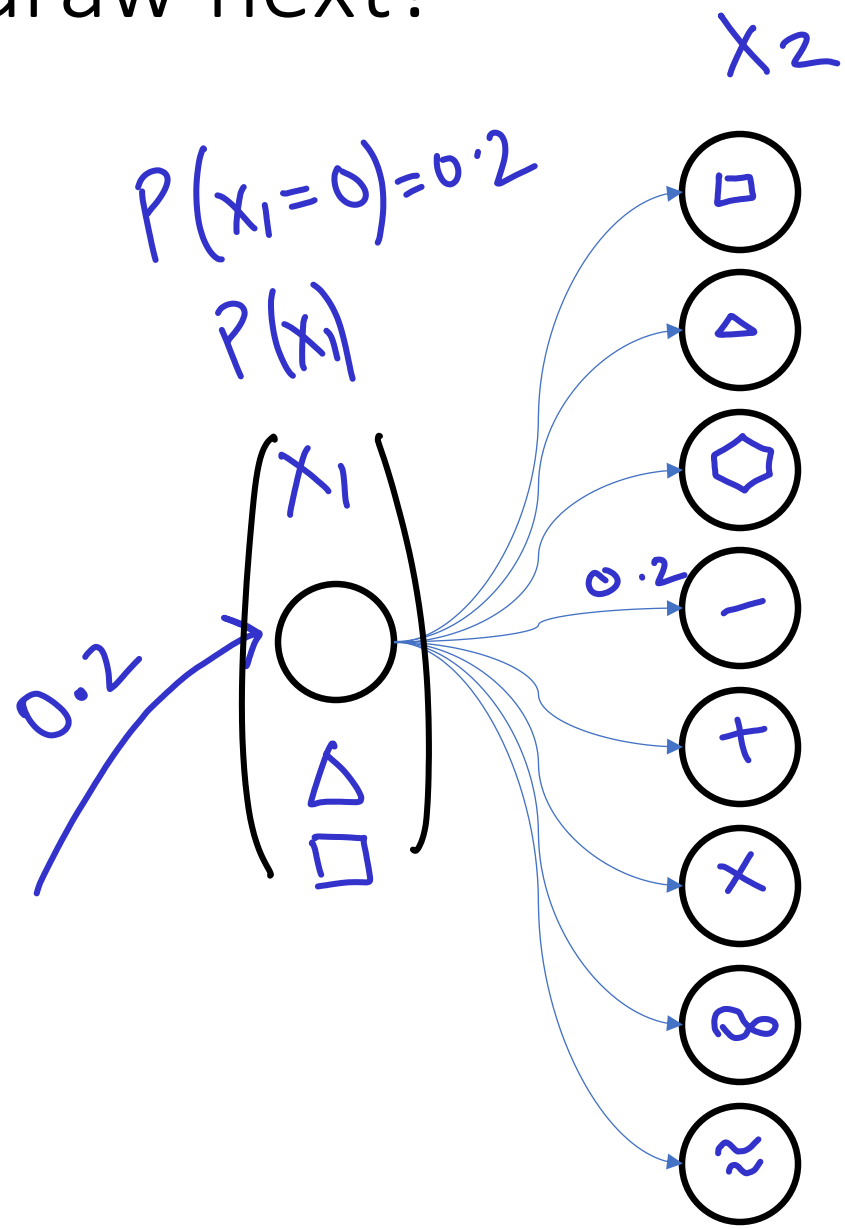
p(color = green, part = land) = $\frac{3}{25}$

Autoregressive Image Model

CSCI 5722 Computer Vision

University of Colorado
Boulder

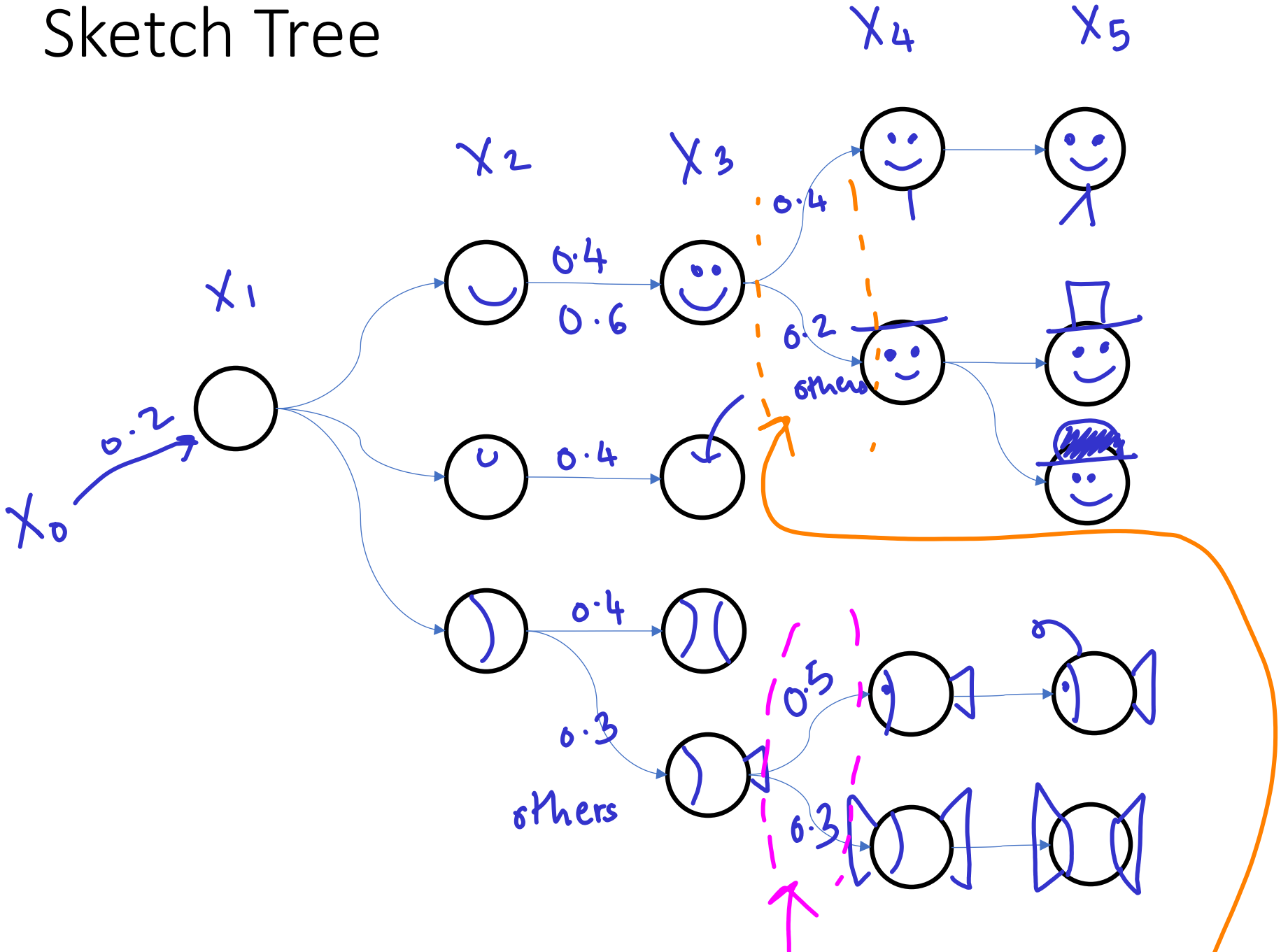
What to draw next?



$P(X_2|X_1=0)$

$P(X_2=-1|X_1=0)=0.2$

Sketch Tree

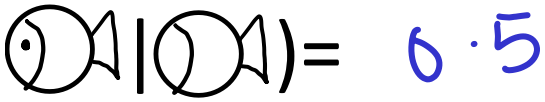


Conditional Probability

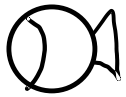
$= 0.2$

$p(x_4 \mid$  $) =$

$$\begin{pmatrix} 0.4 \\ 0.2 \\ \vdots \\ \vdots \\ \vdots \end{pmatrix}$$

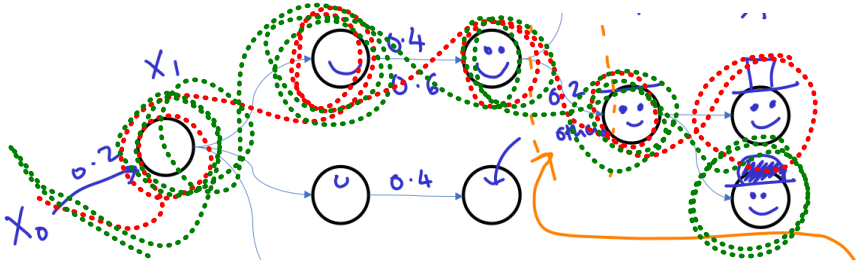


$= 0.5$

$p(x_4 \mid$  $) =$

$$\begin{pmatrix} 0.5 \\ 0.3 \\ \vdots \\ \vdots \\ \vdots \end{pmatrix}$$

Joint Probability



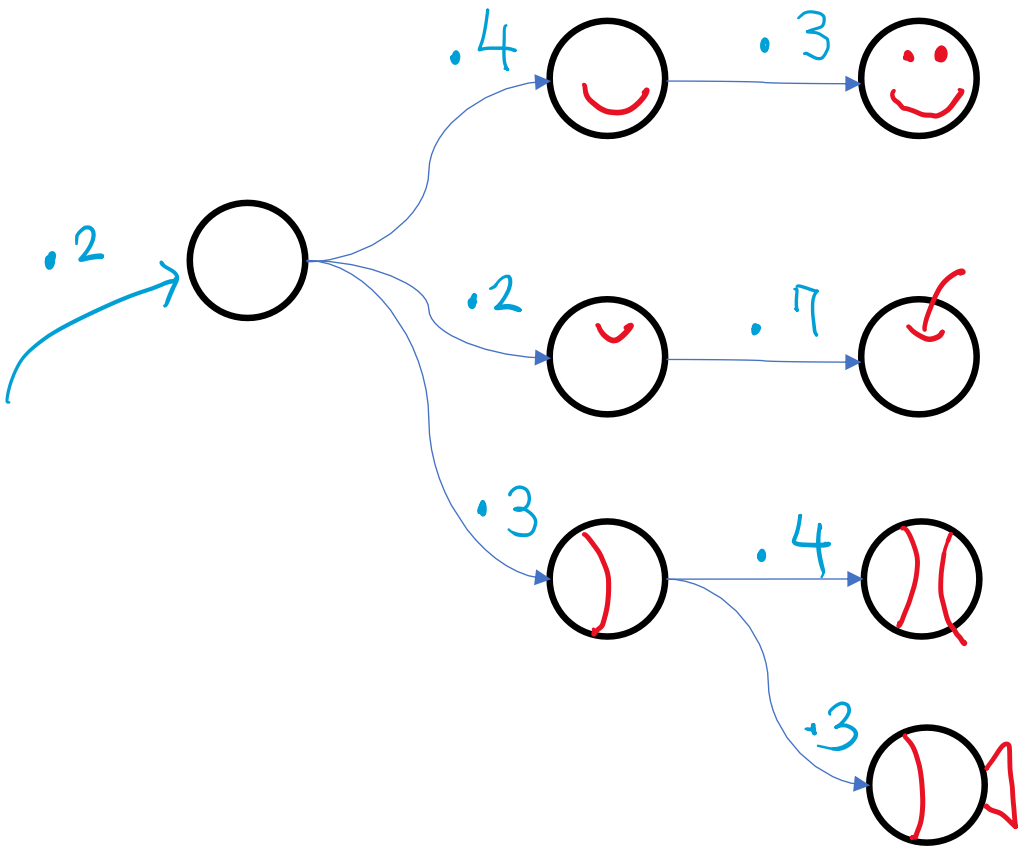
p( )= 0.2 x 0.5 x 0.4 x 0.2 x 0.3

p( )=

p( )= 0.2 x 0.1 x 0.3 x 0.5 x 0.4

p( )= 0.2 x 0.2 x

Joint Probability



p( ) = 0.3 × 0.4 × 0.2 = 0.024

p( ) = 0.7 × 0.2 × 0.2 = 0.028

p( ) = 0.4 × 0.3 × 0.2 = 0.024

p( ) = 0.3 × 0.3 × 0.2 = 0.018