Assignment 3

CSC 520 Spring 2022 001

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1 Question 1

1.1 a.

1. $tea(BlackTea) \wedge (\forall x\ cheese(x) \Rightarrow \neg tea(x))$
2. $blend(BlackTea, GreenTea)$
3. $\forall x, y\ (\neg tea(x) \wedge tea(y)) \Rightarrow \neg blend(y, x)$
4. $\exists x\ tea(x) \wedge unoxidize(x) \wedge \neg wilted(x)$
5. $\exists x \forall y\ tea(x) \wedge ((tea(y) \wedge \neg unoxidize(y)) \Rightarrow blend(x, y))$

1.2 b.

1.2.1 1.

- $tea(BlackTea) \wedge (cheese(x) \Rightarrow \neg tea(x))$
- $tea(BlackTea) \wedge (\neg cheese(x) \vee \neg tea(x))$

1.2.2 2.

- $blend(BlackTea, GreenTea)$

1.2.3 3.

- $(\neg tea(x) \wedge tea(y)) \Rightarrow \neg blend(y, x)$
- $\neg(\neg tea(x) \wedge tea(y)) \vee \neg blend(y, x)$
- $(tea(x) \vee \neg tea(y)) \vee \neg blend(y, x)$
- $tea(x) \vee \neg tea(y) \vee \neg blend(y, x)$

1.2.4 4.

- $tea(T1) \wedge unoxidize(T1) \wedge \neg wilted(T1)$

1.2.5 5.

- $tea(T2) \wedge ((tea(y) \wedge \neg unoxidize(y)) \Rightarrow blend(T2, y))$
- $tea(T2) \wedge (\neg(tea(y) \wedge \neg unoxidize(y)) \vee blend(T2, y))$
- $tea(T2) \wedge ((\neg tea(y) \vee unoxidize(y)) \vee blend(T2, y))$
- $tea(T2) \wedge (\neg tea(y) \vee unoxidize(y) \vee blend(T2, y))$

1.3 c.

In order to prove by resolution, we introduce the following to our KB:

1. $blend(BlackTea, cheese(x))$

note that there is implied universal quantification

1.3.1 Also in our knowledge base

- 2. $tea(BlackTea)$
- 3. $\neg tea(cheese(x))$

1.3.2 Standardized for resolution

- 4. $tea(z) \vee \neg tea(y) \vee \neg blend(y, z)$

1.3.3 Resolution

- 3. $tea(z) \vee \neg tea(y) \vee \neg blend(y, z)$ resolves with 2. $tea(BlackTea)$
- this leaves 4. $tea(z) \vee \neg blend(BlackTea, z)$ after the substitution $y/BlackTea$
- 4. $tea(z) \vee \neg blend(BlackTea, z)$ resolves with 3. $\neg tea(cheese(x))$
- this leaves 5. $\neg blend(BlackTea, cheese(x))$ after the substitution $z/cheese(x)$
- 5. $\neg blend(BlackTea, cheese(x))$ resolves with 1. $blend(BlackTea, cheese(x))$
- this leaves the empty set
- this proves that cheese does not blend with BlackTea.

1. Note $cheese(x)$ is being used in place of any instatiation of x that would be true in our world. It is implied that we have at least one object C that maps to a cheese, so where I use $cheese(x), cheese(C)$ could be substituted. This could equivalently simply be C as the problem's use of "cheese" could simply be a constant. The problem's use of "cheese" is not clear in this respect. The resolution works conceptually either way. Similarly a Skolem function could be used, cheese(C(x)). If it is required that I pick one, simply "C" is the most clear because Sentence 1 in the question refers to cheese as a constant like BlackTea.

2 Question 2

- Note that I use PL-FOL to refer to propositionalization.
- Note that "mp" stands for Modus Ponens.

2.1 a.

2.1.1 Provided Sentences in FOL

- Sentence 1: $\forall x\ o(x) \Rightarrow l(x)$
- Sentence 2: $\forall x\ o(x) \Rightarrow d(x)$
- Sentence 3: $\forall x\ (o(x) \land i(x)) \Rightarrow s(x)$
- Sentence 3 simplified: $\forall x\ i(x) \Rightarrow s(x)$
- The remaining sentences will take this simplified approach. It is a safe assumption that our model does not have any assignments that would complicate these simplifications.
- From Sentence 5: $\forall x\ o(x) \Rightarrow (g(x) \lor si(x))$
- From Sentence 8: $\forall x\ (g(x) \land i(x)) \Rightarrow pre(x)$
- From Sentence 8: $\forall x\ si(x) \Rightarrow \neg pre(x)$

2.1.2 Provided sentences that can be interpreted in terms of PL-FOL

- Sentence 4: $\neg s(C)$
- Sentence 6: $o(C)$

- Sentence 7: $g(C) \land \neg i(C) \land \neg s(C) \land \neg pre(C)$

2.1.3 FOL Sentences instantiated into PL (Carol is the only customer, so she is the only instatiation that matters)

- Sentence 1 PL-FOL: $o(C) \Rightarrow l(C)$
- Sentence 2 PL-FOL: $o(C) \Rightarrow d(C)$
- Sentence 3 PL-FOL: $i(C) \Rightarrow s(C)$
- Sentence 5 PL-FOL: $o(C) \Rightarrow (g(C) \lor si(C))$
- From Sentence 8 PL-FOL: $(g(C) \land i(C)) \Rightarrow pre(C)$
- From Sentence 8 PL-FOL: $si(C) \Rightarrow \neg pre(C)$

2.1.4 Example: Carol can collect loyalty points (Sentence 1. combined with Sentence 6.)

- Sentence 1. is a FOL statement: $\forall x\, o(x) \Rightarrow l(x)$
- Sentence 6. can be interpreted as a PL statement: $o(C)$ (This is true in our truth table for the consistent model.)
- Since Carol is the only object that's a customer in our model, the only instantiation of Sentence 1. that adds value is the following: $o(C) \Rightarrow l(C)$
- Thus, in our model's truth table, $l(C)$ is true.
- This is an example of how we will use the provided sentences to prove that a our model is consistent.
- Note that this example is not critical because there are no other statements that involve $l(C)$ in PL-FOL

2.1.5 Truth table to show a consistent model.

No statement contradicts another statement. S0 referes to "Statement 0" not "Sentence 0"

Ref	PL-FOL	T/F
S0	$\neg s(C)$	T
S1	$o(C)$	T
S2	$g(C) \wedge \neg i(C) \wedge \neg s(C) \wedge \neg pre(C)$	T
S3	$o(C) \Rightarrow l(C)$	T
S1 mp S3	$l(C)$	T
S4	$o(C) \Rightarrow d(C)$	T
S1 mp S4	$d(C)$	T
S5	$i(C) \Rightarrow s(C)$	T (S2)
S6	$o(C) \Rightarrow (g(C) \vee si(C))$	T (S2)
S7	$(g(C) \wedge i(C)) \Rightarrow pre(C)$	T (S2)
S8	$si(C) \Rightarrow \neg pre(C)$	T (S2)

1. Note As FOL statements have either true/false values, they could have been used in place of their propositionalized counterparts. I have universally instantiated with C because the language makes more sense and it doesn't change the values for their corresponding T/F.

2.2 b.

Assuming SO has been replaced with $s(C)$, we can prove that there is a contradiction with the following resolution: Consider S0* to be $s(C)$

2.2.1 Resolution

- With decomposition we can move from S2 to $\neg s(C)$
- Consider S2* to be $\neg s(C)$
- S2* resolves with S0* to the empty set, thus we have a contradiction and our model is not consistent

3 Question 3

Execution instructions and solutions locations explained:

./README.pdf

4 VCL

Execution was tested on the Linux Lab machine.