The following pages are proof that I did the derivation.

My handwriting is probably not readible though

Consider pairs of edges I, J
s.t. $I \cap J = \varnothing$

Sps $p = i_1$

$i = i_1 \quad j = j_1$:

Derivative of $\displaystyle \frac{1}{4} l_J \, l_I^{1-\alpha} \frac{\left| (\gamma_{i_2} - \gamma_{j_1}) \times \gamma_{i_1} - \gamma_{i_2} \times \gamma_{j_1} \right|^{\alpha}}{\left| \gamma_{i_1} - \gamma_{j_1} \right|^{\beta}}$

$$= \frac{1}{4} l_J \Bigg[(1-\alpha) l_I^{-\alpha} \frac{(\gamma_{i_1} - \gamma_{i_2})^T}{l_I} \frac{\left| (\gamma_{i_2} - \gamma_{j_1}) \times \gamma_{i_1} - \gamma_{i_2} \times \gamma_{j_1} \right|^{\alpha}}{\left| \gamma_{i_1} - \gamma_{j_1} \right|^{\beta}}$$

$$+ \frac{l_I^{1-\alpha}}{\left| \gamma_{i_1} - \gamma_{j_1} \right|^{2\beta}} \Bigg(\left| \gamma_{i_1} - \gamma_{j_1} \right|^{\beta} \alpha \left| (\gamma_{i_2} - \gamma_{j_1}) \times \gamma_{i_1} - \gamma_{i_2} \times \gamma_{j_1} \right|^{\alpha - 1} \frac{\left((\gamma_{i_2} - \gamma_{j_1}) \times \gamma_{i_1} - \gamma_{i_2} \times \gamma_{j_1} \right)^T}{\left| (\gamma_{i_2} - \gamma_{j_1}) \times \gamma_{i_1} - \gamma_{i_2} \times \gamma_{j_1} \right|} M_{\gamma_{i_2} - \gamma_{j_1}}$$

$$- \left| (\gamma_{i_2} - \gamma_{j_1}) \times \gamma_{i_1} - \gamma_{i_2} \times \gamma_{j_1} \right|^{\alpha} \beta \left| \gamma_{i_1} - \gamma_{j_1} \right|^{\beta - 1} \frac{(\gamma_{i_1} - \gamma_{j_1})^T}{\left| \gamma_{i_1} - \gamma_{j_1} \right|} \Bigg) \Bigg]$$

$i = i_2 \quad j = j_1$

Deriv of $\displaystyle \frac{1}{4} l_J \, l_I^{1-\alpha} \frac{\left| (\gamma_{i_2} - \gamma_{j_1}) \times \gamma_{i_1} - \gamma_{i_2} \times \gamma_{j_1} \right|^{\alpha}}{\left| \gamma_{i_2} - \gamma_{j_1} \right|^{\beta}}$

Only difference is that now denominator doesn't have i_1

$$= \frac{1}{4} l_J \Bigg[(1-\alpha) l_I^{-\alpha - 1} (\gamma_{i_1} - \gamma_{i_2})^T \frac{\left| (\gamma_{i_2} - \gamma_{j_1}) \times \gamma_{i_1} - \gamma_{i_2} \times \gamma_{j_1} \right|^{\alpha}}{\left| \gamma_{i_2} - \gamma_{j_1} \right|^{\beta}}$$

$$+ \frac{l_I^{1-\alpha}}{\left| \gamma_{i_2} - \gamma_{j_1} \right|^{\beta}} \alpha \left| (\gamma_{i_2} - \gamma_{j_1}) \times \gamma_{i_1} - \gamma_{i_2} \times \gamma_{j_1} \right|^{\alpha - 2} \left(\sim \right)^T M_{\gamma_{i_2} - \gamma_{j_1}} \Bigg]$$

We can easily replace j_1 w/ j_2 in these to get no difference

Now, say $p = j_1$

$i = i_1 \quad j = j_1$

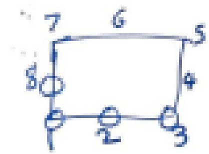
~~Deriv of~~ $\frac{1}{4} l_J l_I \frac{|T_I \times (\gamma_{i_1} - \gamma_{j_1})|^\alpha}{|\gamma_{i_1} - \gamma_{j_1}|^\beta} = \frac{1}{4} l_I l_J \frac{|T_I \times (\gamma_{j_1} - \gamma_{i_1})|^\alpha}{|\gamma_{j_1} - \gamma_{i_1}|^\beta}$

$$\text{Deriv} = \frac{1}{4} l_I \Bigg[\frac{(\gamma_{j_1} - \gamma_{j_2})^T}{l_J} \frac{|T_I \times (\gamma_{j_1} - \gamma_{i_1})|^\alpha}{|\gamma_{j_1} - \gamma_{i_1}|^\beta} + \frac{l_J}{|\gamma_{j_1} - \gamma_{i_1}|^{2\beta}} \Big(|\gamma_{j_1} - \gamma_{i_1}|^\beta \alpha |T_I \times (\gamma_{j_1} - \gamma_{i_1})|^{\alpha-2} (T_I \times (\gamma_{j_1} - \gamma_{i_1}))^T M_{T_I}$$
$$- |T_I \times (\gamma_{j_1} - \gamma_{i_1})|^\alpha \beta |\gamma_{j_1} - \gamma_{i_1}|^{\beta-2} (\gamma_{j_1} - \gamma_{i_1})^T \Big) \Bigg]$$

$i = i_1 \quad j = j_2$

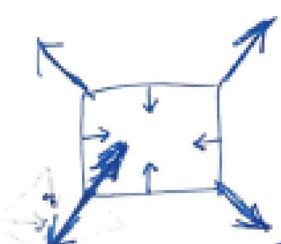
$$\frac{1}{4} l_I l_J \frac{|T_I \times (\gamma_{j_2} - \gamma_{i_1})|^\alpha}{|\gamma_{j_2} - \gamma_{i_1}|^\beta}$$

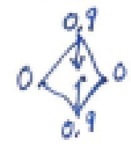
$$\text{Deriv} = \frac{1}{4} l_I \frac{|T_I \times (\gamma_{j_2} - \gamma_{i_1})|^\alpha}{|\gamma_{j_2} - \gamma_{i_1}|^\beta} \left(\frac{(\gamma_{j_1} - \gamma_{j_2})^T}{l_J} \right)$$



1st 3
& last

Exchanging i_1 w/ i_2 makes no difference





Case 1 $$\frac{1}{4} l_J \Bigg[\frac{(\gamma_{i_1} - \gamma_{i_2})^T}{l_I} \frac{|T_J \times (\gamma_{i_1} - \gamma_{j_1})|^\alpha}{|\gamma_{i_1} - \gamma_{j_1}|^\beta}$$
$$+ \alpha l_I |\gamma_{i_1} - \gamma_{j_1}|^{-\beta} |T_J \times (\gamma_{i_1} - \gamma_{j_1})|^{\alpha-2} (T_J \times (\gamma_{i_1} - \gamma_{j_1}))^T M_{T_J}$$
$$- \beta l_I |\gamma_{i_1} - \gamma_{j_1}|^{-\beta-2} |T_J \times (\gamma_{i_1} - \gamma_{j_1})|^\alpha (\gamma_{i_1} - \gamma_{j_1})^T \Bigg]$$

switch I & J

but use these new formulas...

$$\frac{\frac{1}{4} l_J}{l_I} \frac{|T_J \times (\gamma_{i_2} - \gamma_{j_1})|^\alpha}{|\gamma_{i_2} - \gamma_{j_1}|^\beta} (\gamma_{i_1} - \gamma_{i_2})^T$$

0.848 081