ME457 - Assignment 2

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1 Conceptual Questions

1.1 Wind Triangle

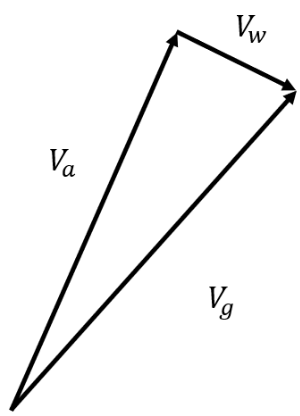

Figure 1: Wind Triangle

The wind triangle is an important visualization tool for understanding aircraft movement. The idea is that the velocity of the aircraft relative to the ground, V_g, is equal to the velocity of the aircraft relative to still air, V_a, plus the velocity of the wind relative to the ground V_w.

$$V_g = V_a + V_w$$

1.2 Flight Path & Angle of Attack

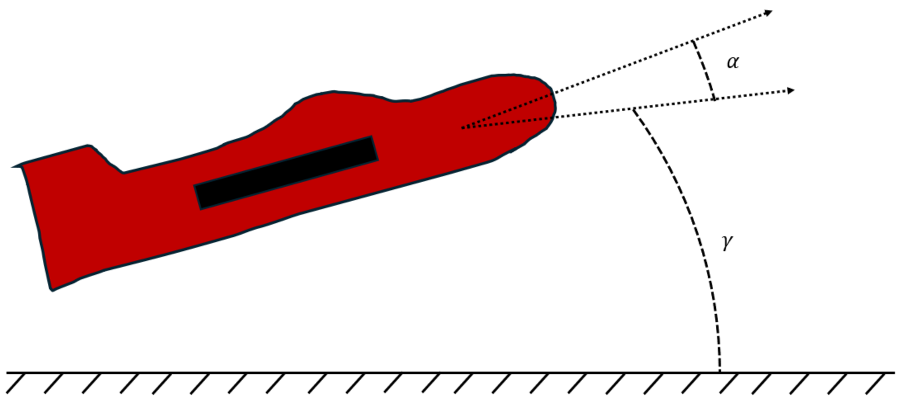

Figure 2: Flight Path Angle & Angle of Attack

1.3 Rotations & Moment of Inertia

The mass moment of inertia of a rigid body about the 2-axis of the body frame is as follows:

$$\int_V \rho(r)(x^2 + z^2)dV$$

The physical meaning of this principle is a body's resistance to rotational acceleration about the body's 2-axis.

1.4 Degrees of Freedom

A rigid body in three dimensions has a total of six degrees of freedom three for position and three for orientation. In order to predict the motion of a rigid body, we need to have knowledge of of the position and velocity of each of these parameters. Therefore, knowledge of twelve states are required to predict the motion of a rigid body. If we were interested in predicting the motion of the rigid body over a long stretch of time, we would require the differential equations which describe the change of each of these states over time.

2 Runge Kutta Implementation

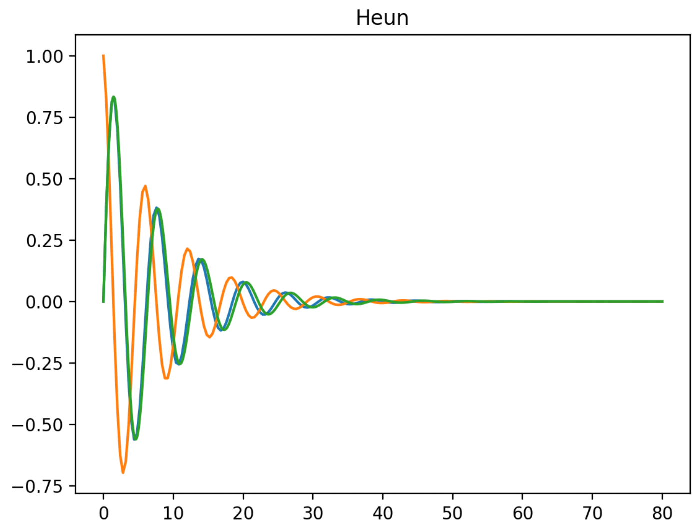

Figure 3: Heun Implementation

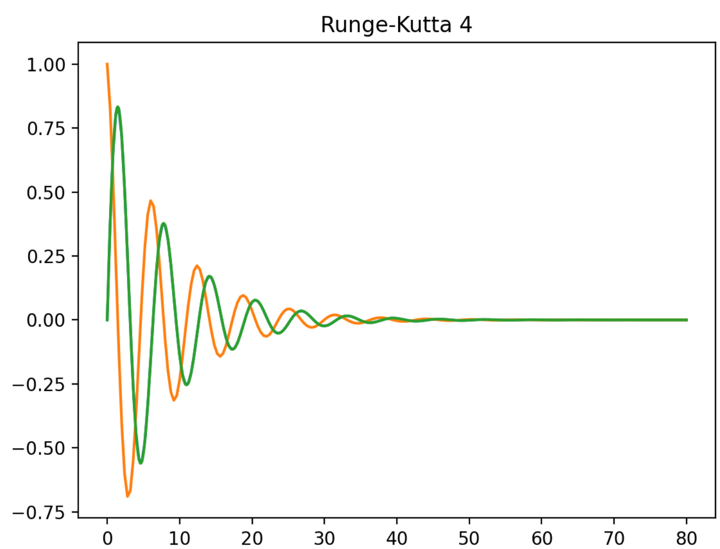

Figure 4: Runge Kutta Implementation

Both the Heun integrator and the Runge Kutta integrator successfully approximate the result of the analytical solution. With higher step sizes the performance falls off dramatically.

3 Inertial Coupling

3.1 Euler's Second Law

Euler's second law states the following about a rigid body

$$J_c^{(bb)} \dot{\omega}_{b/0}^{(b)} = M_c^{(b)} - \Omega_{b/0}^{(bb)} J_c^{(bb)} \omega_{b/0}^{(b)}$$

From this we can express the applied moments on the body about the center of mass as

$$M_c^{(b)} = J_c^{(bb)} \dot{\omega}_{b/0}^{(b)} + \Omega_{b/0}^{(bb)} J_c^{(bb)} \omega_{b/0}^{(b)}$$

This can be re-expressed as the following three equations:

$$M_1 = J_1 \dot{\omega}_1 + (J_3 - J_2)\omega_2\omega_3$$

$$M_2 = J_2 \dot{\omega}_2 + (J_1 - J_3)\omega_1\omega_3$$

$$M_3 = J_3 \dot{\omega}_3 + (J_2 - J_1)\omega_2\omega_1$$

3.2 Sketch

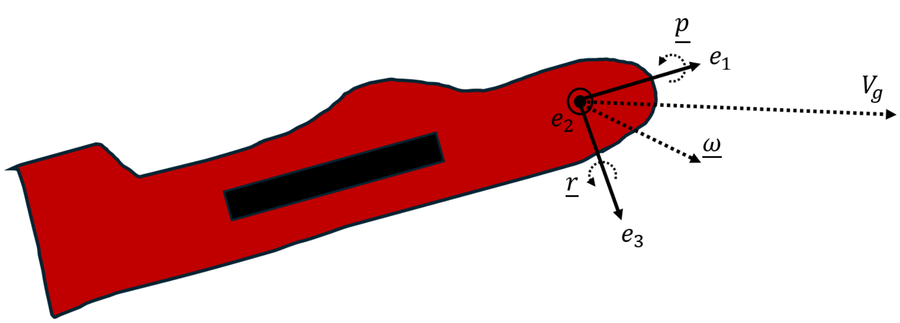

Figure 5: Diagram of the body frame with p, r, V_g

3.3 Stationary Vertical Spin

In order to achieve a stationary spin motion, a moment would need to be applied about the body's 3-axis. It is important to maintain a constant angular velocity about the 3-axis for a stationary spin. This means that the moment created by by the rudder must not continuously accelerate the angular velocity, but rather provide enough moment to cancel out other forces which would prevent the stationary vertical spin. This can be achieved by an input from the rudder.

3.4 Velocity Vector Roll

A velocity vector roll can be achieved by actuating the ailerons in opposite directions such that one creates lift and the other creates drag. This will cause a coupled moment to act on the aircraft and creating a velocity vector roll.

4 Rigid Body Class Implementation

See attached code for implementation