

ME - 457 - Assignment 1

Nathan Wolf-Sonkin

January 2025

1 Controlled Flight

Unmanned aircrafts are usually steered around using a combination of three different types of control surfaces as shown in figure 1. The roll, pitch, and yaw of the aircraft are controlled by the ailerons, elevator, and rudder respectively.

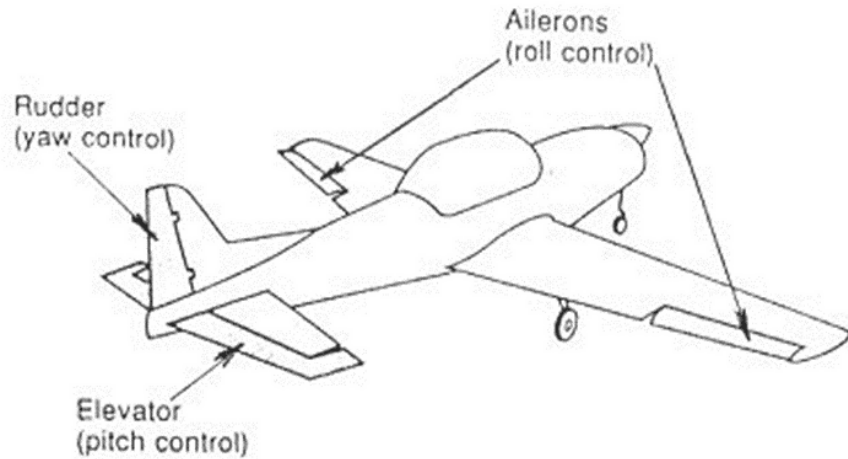


Figure 1: Control surfaces on a small aircraft

A coordinated turn is a maneuver in which the aircraft's ailerons and rudder are used together to maintain balanced flight during a turn, preventing adverse yaw and ensuring that the aircraft's nose points along the flight path. This is performed by actuating the ailerons to achieve a roll while simultaneously utilizing the rudder to counteract any undesirable yaw movement.

2 System Architecture

Below is a block diagram of the system architecture of an unmanned aerial vehicle. This type of diagram is useful as an overview of what the required

inputs and outputs to a system will be. This is a powerful tool for planning the setup of a large scale project like a simulation. By knowing the required inputs and outputs of any given system, designers are able to make sections of the simulation more modular and compact. This increases the scalability and maintainability of the project.

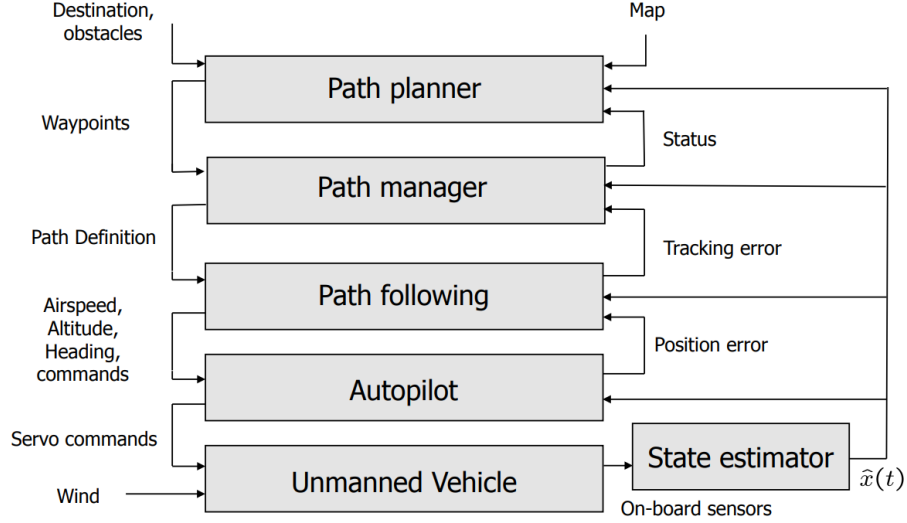


Figure 2: System architecture of an unmanned aerial vehicle

3 Simulation Value

Simulations offer both distinct advantages and limitations. On the positive side, they enable reduced costs, quicker design iterations, and the ability to conduct non-destructive, risk-free testing. However, a key drawback is that simulations rely on models, which are inherently an approximation of reality. No model can fully capture every nuance of a system or object. The best a simulation can do is focus on the most influential factors that affect the relevant outputs for the designer. In the context of a single system, one engineer might prioritize inertial properties, while another might focus on thermal characteristics. It is the role of the simulation designer to identify which factors are most critical to the system and the specific outcomes they are seeking to measure.

4 Framing and Rotations

The rotation from the inertial frame (using North-East-Down convention) to the aerodynamic frame is generated using a yaw pitch roll Euler sequence. This sequence is defined as follows:

$$R^{(01)} = R_3(\psi)R_2(\theta)R_1(\phi) = \begin{bmatrix} C_\psi & -S_\psi & 0 \\ S_\psi & C_\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_\psi & 0 & S_\psi \\ 0 & 1 & 0 \\ -S_\psi & 0 & C_\psi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\psi & -S_\psi \\ 0 & S_\psi & C_\psi \end{bmatrix}$$

4.1 Battery Position

The position of the battery in the inertial frame can be defined as

$$P_{bat} = P_{com}^{(0)} + P_{com_bat}^{(1)} = \begin{bmatrix} 0 \\ 0 \\ -10 \end{bmatrix} + R^{(01)} \begin{bmatrix} 0.2 \\ 0 \\ 0 \end{bmatrix}$$

where P_{com} is the position of the center of mass of the plane and P_{com_bat} is the vector pointing from the center of mass of the plane towards the battery. The position of the battery in the inertial frame can then be expressed as follows:

$$P_{bat}^{(0)} = P_{com}^{(0)} + R^{(01)} P_{com_bat}^{(1)}$$

The resulting positional vector is as follow:

$$\begin{bmatrix} 0.06983539 \\ -0.15259312 \\ -9.89119578 \end{bmatrix}$$

4.2 Velocity in the Inertial Frame

The velocity of the plane in the body frame is as follows:

$$v^{(1)} = \begin{bmatrix} 15 \\ 1 \\ .5 \end{bmatrix}$$

Rotating this into the inertial frame results in

$$v^{(0)} = R^{(01)} v^{(1)} = \begin{bmatrix} 6.18821856 \\ -11.42898431 \\ 7.57193956 \end{bmatrix}$$

4.3 Flight Path Angle

The flight path angle is defined as the angle between the direction that the aircraft is moving and the horizon. This can be calculated as follows:

$$\gamma = \arctan\left(\frac{v^{(0)}[3]}{v^{(0)}[1]}\right) = 50.74235727$$

4.4 Angle of Attack

The angle of attack is the angle between the chord line of the air foil and the direction of oncoming air. Since the wind is still, there is no angle of attack.

4.5 Heading Angle / Course Angle

Heading angle represents the direction that the nose of the aircraft is facing in reference to true north. Course angle represents the direction that the aircraft is moving in reference to true north.

5 Simulation

Different numerical integrators will yield different results under the same conditions. This is simply because a numerical integrator is taking an approximation of what the system would look like using some finite resolution time step. Naturally each integrator attempts to yield a result as close to the analytical truth as possible. Differences between the results of these integrators can be seen in the following plots

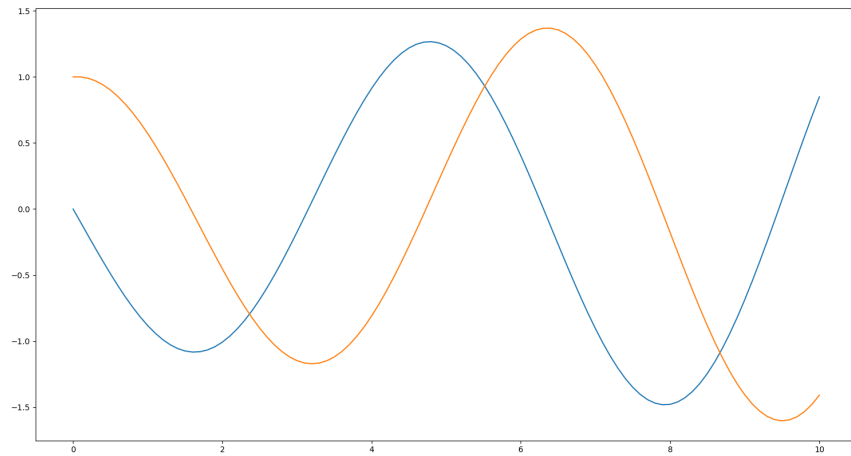


Figure 3: Result of the simulation using the Euler method

Following the orange line, it can be seen that the response is slightly, albeit noticeably more damped when the system is integrated using the Euler method.

Next, a simple mass spring damper system was integrated to generate the response below

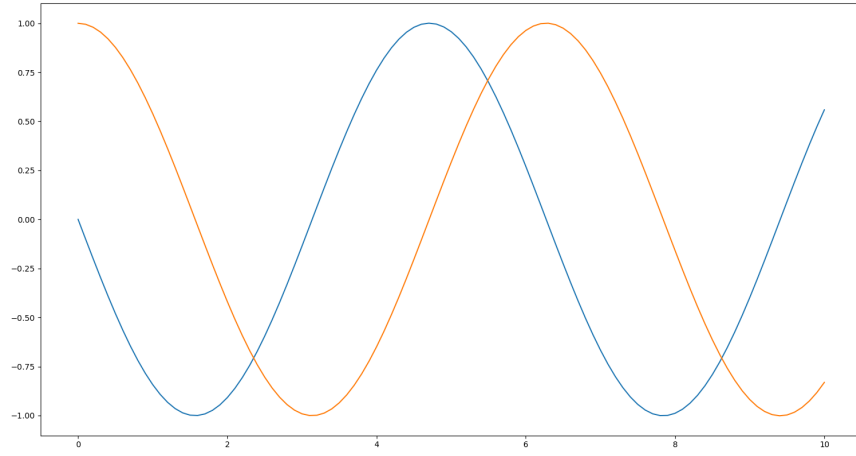


Figure 4: Result of the simulation using the Heun method

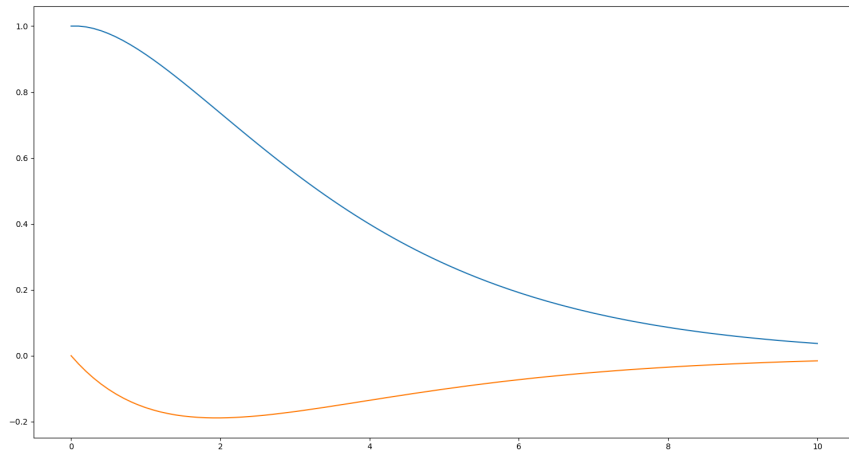


Figure 5: Results of a mass spring damper simulation using the Euler method

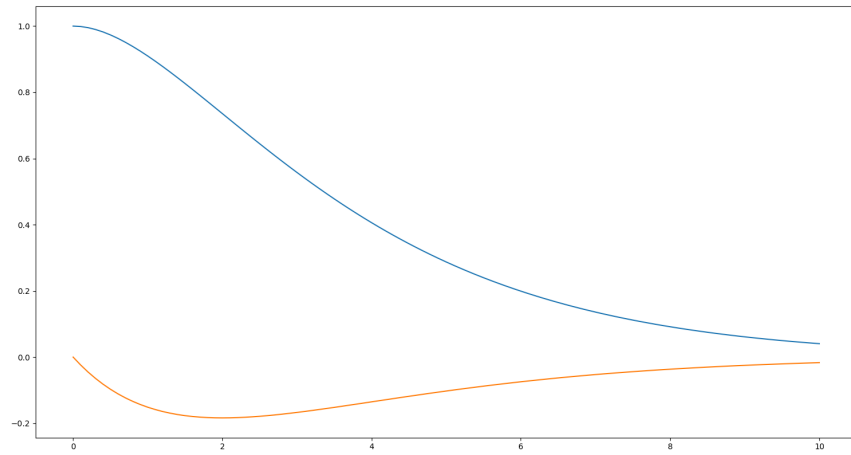


Figure 6: Results of a mass spring damper simulation using the Heun method