CSE490R

Team Quiz

LAB2 P6 Non-linear Control

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1. Recall that $\dot{x} = v\cos(\theta), \dot{y} = v\sin(\theta), \dot{(\theta)} = \frac{V}{B}\tan(\delta)$. In our control law explained below, we have $\tan u = (-\frac{k_1 e_{ct} B}{\theta_e}\sin\theta_e - \frac{B}{V}k_2\theta_e)$. So, we readily arrive at

$\dot{x} = V\cos(\theta_e + \theta_{ref})$

$\dot{y} = V\sin(\theta_e + \theta_{ref})$

$\dot{\theta} = \frac{\dot{e_{ct}}}{B\sin(\theta_e)}(-\frac{k_1 e_{ct} B}{\theta_e}\sin\theta_e - \frac{B}{V}k_2\theta_e)$.

2. We use the Lyapunov value function $V(e_{ct}, \theta_e) = \frac{1}{2}k_1 e_{ct}^2 + \frac{1}{2}\theta_e^2$. and the control law is $u = \tan^{-1}(-\frac{k_1 e_{ct} B}{\theta_e}\sin\theta_e - \frac{B}{V}k_2\theta_e)$. We handle edge case when $\theta_e \to 0$ by setting $u = \tan^{-1}(-k_1 e_{ct} B)$. Observe that $V(e_{ct}, \theta_e) > 0$ and $\dot{V}(e_{ct}, \theta_e) = k_1 e_{ct} V\sin\theta_e + \theta_e \frac{V}{B}\tan u = -k_2\theta_e^2 < 0$ by our control law. Therefore, this is a valid Lyapunov method that provides stability almost everywhere. Now we show that it indeed is a saddle point $\dot{V} = 0$ and our control for $\theta_e \to 0$ would pull it out of the saddle point.

Observe that $\ddot{V} = k_1 \dot{e_{ct}} V\sin\theta_e + k_1\dot{\theta}_e e_{ct} V\cos\theta_e + \dot{\theta}_e \frac{V}{B}\tan u + \theta_e \frac{V}{B}(\tan u)'$ where $\tan u = -k_1 e_{ct} B$ and $\dot{\theta}_e = \frac{V}{B}\tan u = -k_1 V e_{ct}$. As $\theta_e \to 0$, $\cos\theta_e \to 1$ and $\sin\theta_e \to 0$, so it follows that $\ddot{V} = k_1 V e_{ct}\dot{\theta}_e + \frac{V}{B}\tan u\dot{\theta}_e = -\theta_e^2 + \theta_e^2 = 0$. Hence, it fails the second derivative test and thus only $\theta_e \to 0$ is not a local minimum. Observe that the control magnitude depends on the cross track error, which would change the heading and pull the value function out of the $\dot{V} = 0$ regime.

3. From above, our control law is $u = \tan^{-1}(-\frac{k_1 e_{ct} B}{\theta_e}\sin\theta_e - \frac{B}{V}k_2\theta_e)$ with an edge case when

$\theta_e \to 0, u = \tan^{-1}(-k_1 e_{ct} B)$. When heading error is 0 and cross track error is small, we are applying a control input depending on the cross track error to minimize the value function. However, when cross track error is large, theoretically we would apply a large torque, in practice the control would be clamped based on the limit. When heading error is $\pi/2$ and cross track error is small, our control is $\tan^{-1}(-\frac{B\pi}{2V}k_2)$ which is a constant. When heading error is $\pi/2$ and cross track error is large, we are again applying a large torque which would be clamped in real cases.

4. The test graphs from ipython book are attached.

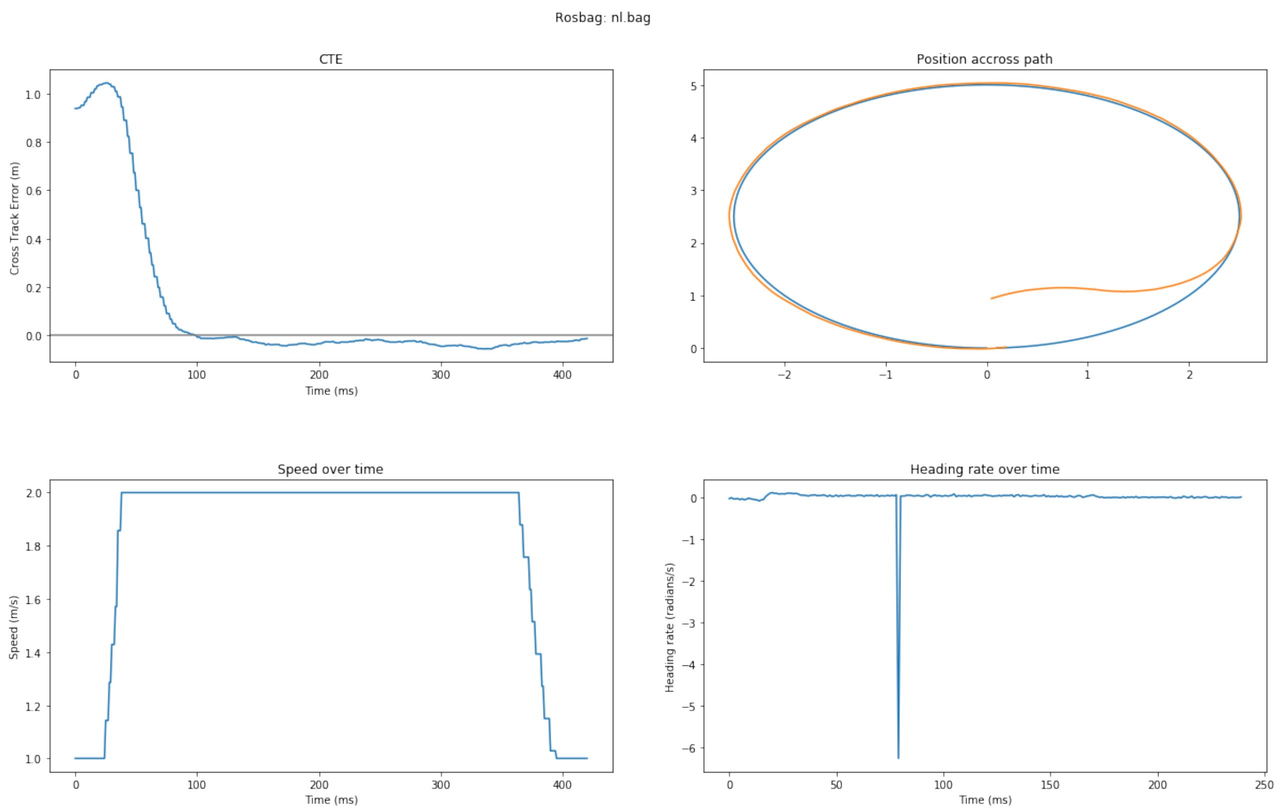

Figure 1: Lyapunov Plot for Circle

5. The controller is provably always stable in theory. However, in real world, some assump-

tions exist such as the limit for the steering angle that we can give for the car, around 0.4 in this racecar for instance, which can break the proof. Assuming the reference position is the closet way point. Because our test waypoints are discrete, we end up not being able to fail the controller. However, given a continuous way point in a circle, to get it to diverge, we initialize the pose to have large cross track error $e_{ct} \to \infty, \theta = \pi$, and $u = \tan^{-1}(-e_{ct}\frac{k_1 B \sin\theta_e}{\theta_e} - \frac{B}{V}k_2\theta_e)$. In this way, the reference point is always the intersection between the car and the origin. So, the reference point's heading is the tangent vector of the reference point, which can be always π heading error with our car. However, in practice, it still ends up converging.