CSE490R

Team Quiz

LAB2 EX1

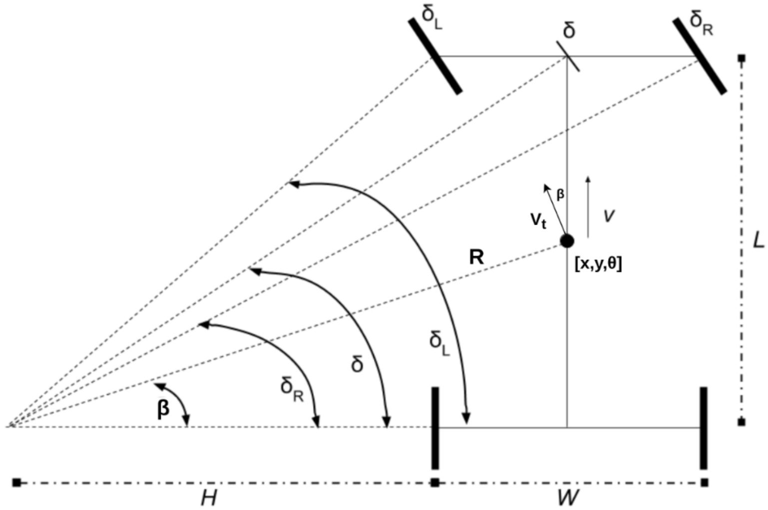

Figure 1: Visual representation of the kinematic car model

Figure 1: Car Kinematics Model

Car model for center of mass: From Fig 1, we have that $\dot{x} = v\cos(\theta), \dot{y} = v\sin(\theta), \dot{\theta} = w$ assuming a rigid body of the car. Observe that we can write $\omega = \frac{v}{R} = \frac{v}{\sqrt{(\frac{L}{\tan(\delta)})^2 + (\frac{L}{2})^2}} = \frac{v}{L\sqrt{\cot^2(\delta)+\frac{1}{4}}}$. So, $\dot{\theta} = w = \frac{v}{L\sqrt{\cot^2(\delta)+\frac{1}{4}}}$.

Change in θ: $\theta_{t+1} - \theta_t = \Delta\theta = \int_{\theta_t}^{\theta_{t+1}} d\theta = \int_t^{t+1} \frac{v}{L\sqrt{\cot^2(\delta)+\frac{1}{4}}} dt = \frac{v}{L\sqrt{\cot^2(\delta)+\frac{1}{4}}} \Delta t$.

Change in x: $x_{t+1} - x_t = \int_{x_t}^{x_{t+1}} dx = \int_t^{t+\Delta t} v\cos(\theta) dt = \int_t^{t+\Delta_t} v\cos(\theta) L\frac{1}{v}\sqrt{\cot^2(\delta)+\frac{1}{4}} d\theta = L\sqrt{\cot^2(\delta)+\frac{1}{4}} \int_{\theta_t}^{\theta_{t+1}} \cos(\theta) d\theta = L\sqrt{\cot^2(\delta)+\frac{1}{4}}(\sin(\theta_{t+1}) - \sin(\theta_t))$.

Change in y: $y_{t+1} - y_t = \int_{y_t}^{y_{t+1}} dy = \int_t^{t+\Delta t} v\sin(\theta) dt = \int_t^{t+\Delta_t} v\sin(\theta) L\frac{1}{v}\sqrt{\cot^2(\delta)+\frac{1}{4}} d\theta = L\sqrt{\cot^2(\delta)+\frac{1}{4}} \int_{\theta_t}^{\theta_{t+1}} \sin(\theta) d\theta = L\sqrt{\cot^2(\delta)+\frac{1}{4}}(-\cos(\theta_{t+1}) + \cos(\theta_t))$.

Putting all of them together, we have:

$\dot{x} = v\cos(\theta)$

$\dot{y} = v\sin(\theta)$

$\dot{\theta} = w = \frac{v}{L\sqrt{\cot^2(\delta)+\frac{1}{4}}}$

$x_{t+1} = x_t + L\sqrt{\cot^2(\delta)+\frac{1}{4}}(\sin(\theta_{t+1}) - \sin(\theta_t))$

$y_{t+1} = y_t + L\sqrt{\cot^2(\delta)+\frac{1}{4}}(-\cos(\theta_{t+1}) + \cos(\theta_t))$

$\theta_{t+1} = \theta_t + \frac{v}{L\sqrt{\cot^2(\delta)+\frac{1}{4}}}\Delta t$

Car model for front axle: From Fig 1, we have that $\dot{x} = v\cos(\theta), \dot{y} = v\sin(\theta), \dot{\theta} = w$ assuming a rigid body of the car. Observe that we can write $\omega = \frac{v}{R} = \frac{v}{L/\sin(\delta)} = \frac{v\sin(\delta)}{L}$. So, $\dot{\theta} = w = \frac{v\sin(\delta)}{L}$.

Change in θ: $\theta_{t+1} - \theta_t = \Delta\theta = \int_{\theta_t}^{\theta_{t+1}} d\theta = \int_t^{t+1} \frac{v\sin(\delta)}{L} dt = \frac{v\sin(\delta)}{L}\Delta t$.

Change in x: $x_{t+1} - x_t = \int_{x_t}^{x_{t+1}} dx = \int_t^{t+\Delta t} v\cos(\theta) dt = \int_t^{t+\Delta_t} v\cos(\theta)\frac{L}{v\sin(\delta)} d\theta = \frac{L}{\sin(\delta)}\int_{\theta_t}^{\theta_{t+1}} \cos(\theta) d\theta = \frac{L}{\sin(\delta)}(\sin(\theta_{t+1}) - \sin(\theta_t))$.

Change in y: $y_{t+1} - y_t = \int_{y_t}^{y_{t+1}} dy = \int_t^{t+\Delta t} v\sin(\theta) dt = \int_t^{t+\Delta_t} v\sin(\theta)\frac{L}{v\sin(\delta)} d\theta = \frac{L}{\sin(\delta)}\int_{\theta_t}^{\theta_{t+1}} \sin(\theta) d\theta = \frac{L}{v\sin(\delta)}(-\cos(\theta_{t+1}) + \cos(\theta_t))$.

Putting all of them together, we have:

$\dot{x} = v\cos(\theta)$

$\dot{y} = v\sin(\theta)$

$\dot{\theta} = w = \frac{v\sin(\delta)}{L}$

$x_{t+1} = x_t + \frac{L}{v\sin(\delta)}(\sin(\theta_{t+1}) - \sin(\theta_t))$

$y_{t+1} = y_t + \frac{L}{v\sin(\delta)}(-\cos(\theta_{t+1}) + \cos(\theta_t))$

$$\theta_{t+1} = \theta_t + \frac{v \sin(\delta)}{L} \Delta t$$