CS-1390/PHY-1390-1: Introduction to Machine Learning
Assignment 0

Monsoon 2024

Questions

1. Two dice are tossed once. What is the probability of getting an even number on the first die or a total of 8?

2. Show that if two variables x and y are independent, then their covariance is zero.

3. Suppose that the two variables x and z are statistically independent. Show that:

- $E[x + z] = E[x] + E[z]$
- $\text{var}(x + z) = \text{var}(x) + \text{var}(z)$

4. You ask your neighbor to water a sickly plant while you are on vacation. Without water, it will die with probability 0.8; with water, it will die with probability 0.15. You are 90% certain that your neighbor will remember to water the plant.

1. What is the probability that the plant will be alive when you return?
2. If the plant is dead upon your return, what is the probability that your neighbor forgot to water it?

5. Urn A has 5 white and 7 black balls. Urn B has 3 white and 12 black balls. We flip a fair coin. If the outcome is heads, then a ball from urn A is selected, whereas if the outcome is tails, then a ball from urn B is selected. Suppose a white ball is selected. What is the probability that the coin landed tails?

6. Suppose A and B each randomly and independently choose 3 of 10 objects. Find the expected number of objects:

1. Chosen by both A and B;

2. Not chosen by either A or B;

3. Chosen by exactly one of A or B.

7. Find $P(1 \leq X \leq 4)$ if $X \sim N(2, 25)$.

8. Suppose X is normal with mean 6. If $P(X > 16) = 0.0228$, what is the standard deviation of X?

9. Suppose X is a normal random variable with mean 5. If $P(X > 0) = 0.8888$, approximately what is $\text{Var}(X)$?

10. Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal distribution with unknown mean μ and variance σ^2. Find maximum likelihood estimators of mean μ and variance σ^2.

11. Consider the points $(-1, 2), (0, 0), (1, 2)$ and $(2, 3)$. These almost follow a parabola. Suppose we want to find a function $f(x) = ax^2 + bx + c$ which does a good job of fitting these four points. Find a, b, and c.